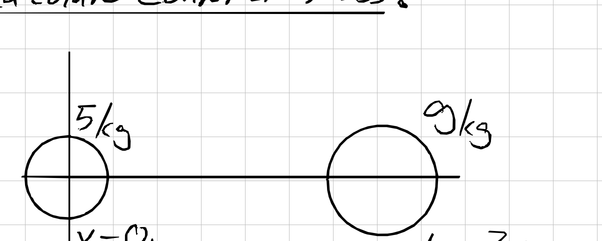
Concepts & Formulae

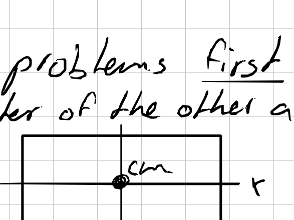
Calculate Center of Mass:



Formula to determine center of mass:

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \quad \text{or} \quad = \frac{md}{M} \text{ (conceptual)}$$

For xy problems first calculate one axis (e.g. x), and then the center of the other axis.



Torque and Moment

Moment ≙ the turning effect of a force about a pivot point / axis (Nm)

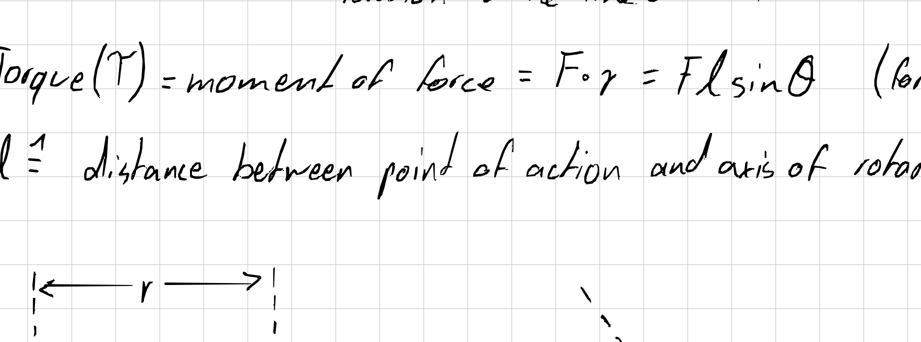
Torque ≙ the moment of force (Nm)

difference: Moment includes bending forces and even the moment of inertia, while torque refers to rotation (forces) only

moment arm (r) ≙ perpendicular distance from the axis of rotation to the line of action

Torque (τ) = moment of force = $F \cdot r = F l \sin\theta$ (for $\sin 90° = 1$)

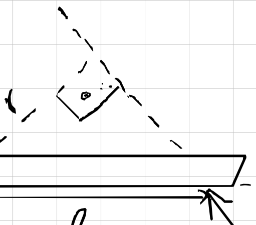
l ≙ distance between point of action and axis of rotation



Basically:

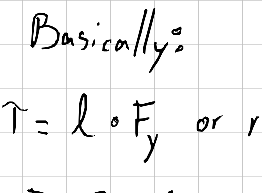
$\tau = l \cdot F_y$ or $r \cdot F$

$F_y = F \sin\theta$

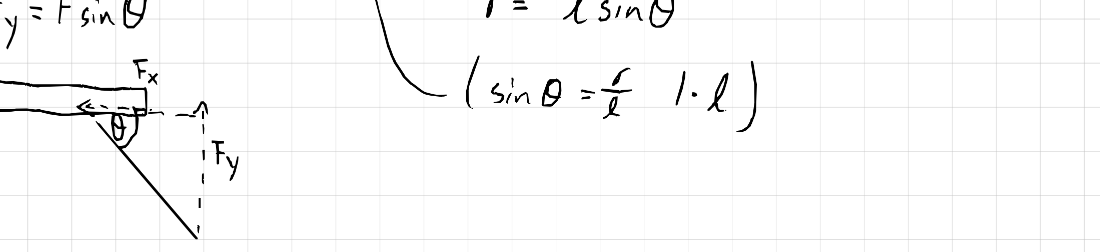


$r = l \sin\theta$

$(\sin\theta = \frac{r}{l} \quad \cdot l)$

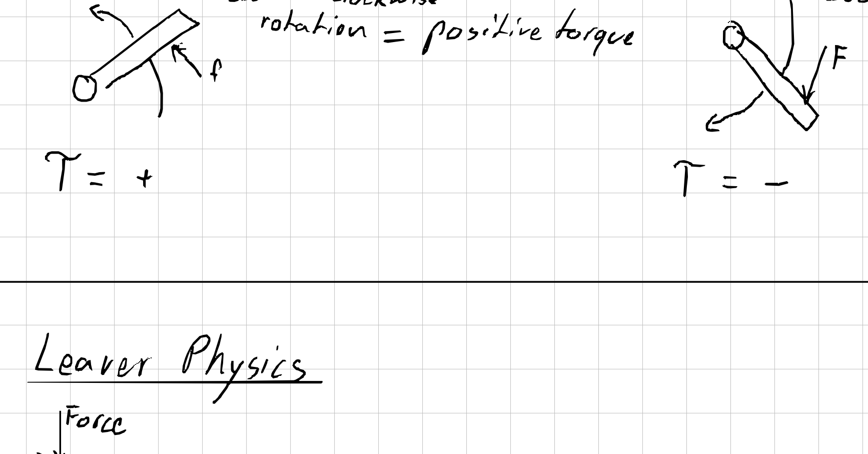


Sign of Torque:



$\tau = +$ $\tau = -$

Leaver Physics



if $\sum \tau = 0$: $\tau_1 = -\tau_2$

The net torque will be the same everywhere in the system

Mechanical Advantage = $\frac{F_{out}}{F_{in}}$ ≙ the ratio between the output and the input force

IMA ≙ $\frac{l_{in}}{l_{out}}$ ≙ the Ideal Mechanical Advantage describes a zero-loss scenario

Equation of the Best Fit Line:

(not in book)

$$a = \frac{n\sum(xy) - \sum(x)\sum(y)}{n\sum(x^2) - (\sum(x))^2} \qquad b = \bar{y} - a\bar{x}$$

$$y = ax + b \qquad b = \frac{\sum(y)}{n} - a \cdot \frac{\sum(x)}{n}$$

Gravitational Potential Energy

$\Delta PE_{grav} = mg\Delta h$

ΔPE_{grav} ≙ gravitational potential energy in J (Joules)
m ≙ mass
g ≙ Gravitational field strength ($N\,kg^{-1}$)

! Actually (most) gravitational fields aren't constant, though usually differ insignificantly over smaller distances, depending on the planets / objects mass of course.

Kinetic Energy

$E_k = \frac{1}{2}mv^2$

E_k ≙ Kinetic Energy (J)
m ≙ mass (kg)
v = velocity (ms^{-1})

Kinetic energy grows at the square of speed!

Mechanical Energy

An object's mechanical energy is the sum of the object's kinetic and potential energy.

Kinetic Energy
- the energy of motion
- includes:
  - Rotational Kinetic Energy
  - Translational Kinetic Energy

Potential Energy
- the energy of position
- includes:
  - Gravitational Potential Energy
  - Elastic Potential Energy
  - Electrical Potential Energy

Kinetic and Gravitational Energy Conversion / Conservation

- Energy always stays conserved:

$\frac{1}{2}mv^2 = mg\Delta h$

→ as Galileo Galilei discovered, mass is insignificant to the speed of a falling object, leaving the equation:

$v = \sqrt{2gh}$

Work (J) or Nm

Work describes an amount of energy transferred, and therefore the change of energy of an object. For mechanical energy work can be defined as force applied over a distance:

$\Delta W = F\Delta s$ If F acts at an angle to Δs:

W ≙ work (J; Nm) $\Delta W = F\Delta s \cos\theta$
F ≙ force ($kg\,m\,s^{-2}$)
s ≙ distance and direction travelled (taking the line of the force) (m)
θ ≙ angle between the direction of movement and direction of force

!! work is a scalar, the force causing the work has to be ...

Work is only done if there is displacement and the force and direction of movement are not perpendicular.

Power (W) Watts or $\frac{J}{s}$

Power describes the rate at which work is done, therefore the rate of energy transfer. Rate = 'how fast'

$$P = \frac{\Delta E}{t} = \frac{\Delta W}{t} = \underbrace{\frac{F\Delta s}{t}}_{\text{for mechanical energy}} = Fv \quad \text{as } \frac{\Delta s}{\Delta t} = v_{elocity}$$

$P(t) = W'(t)$

Efficiency (ratio)

Describes the ratio of useful energy output to the energy input. This ratio also works for useful power output over power input as units cancel:

$$\text{efficiency} = \frac{\text{useful energy output (J)}}{\text{total energy input (J)}}$$

$$\text{efficiency} = \frac{\text{useful power output } (\frac{J}{s}; W)}{\text{total energy input } (\frac{J}{s}; W)}$$

this value is often given as percent

Work - Energy Theorem

The net work applied at an object allways equals the change of kinetic energy of that object.
If the net work is zero both components of kinetic energy stay conserved by themselves. It will therefore always require work to convert between translational and rotational energy.

$W_{net} = \Delta E_k$ or $Fs\cos\theta = \frac{1}{2}mv^2$ (if rotational energy is 0)

if $W_{net} = 0$: no change in E_k and its components
if $W_{net} > 0$: object is accelerating
if $W_{net} < 0$: object is decelerating

This principle is practical if you have to find the net work but don't have a force

Momentum

Describes the quantity of mass in motion.
(It is even more fundamental than mass itself.)

$\vec{p} = m \cdot \vec{v}$

p = momentum ($kg\,m\,s^{-1}$; for impulse: Nm)
m = mass
v = velocity

$F = \frac{\Delta p}{\Delta t} = \frac{\Delta(m \cdot v)}{\Delta t}$ so $F(t) = p'(t)$

Deriving momentum:

$F = m \cdot a$ $a = \frac{\Delta v}{\Delta t}$ (derivative of v)

$F = m \cdot \frac{\Delta v}{\Delta t}$

requires constant mass $F = \frac{\Delta(m \cdot v)}{\Delta t}$

$F = \frac{\Delta p}{\Delta t}$

Varying mass & momentum

A force applied on a body is directly proportional to the rate of change of the momentum of that object.
While $F = ma$ requires a constant mass, $F = \frac{\Delta p}{\Delta t}$ allows for both a varying mass and velocity.

Conservation of momentum

The total momentum of a system is constant (given that the system is closed). Therefore, momentum is always conserved

Types of momentum

- Linear momentum : $\vec{p} = m\vec{v}$
  - tendency of objects to continue moving in a line
- Angular momentum : $\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{p})$
  - tendency of objects to keep spinning

Linear and angular momentum stay conserved independently

Impulse

Impulse describes the accumulation of force applied to an object over time. As a force applied over time causes a change in momentum, the impulse is also equal to the objects change in momentum.

$J = F\Delta t$ $J = \int F(t)\,dt$

J ≙ Impulse (Nm; technically also $kg\,m\,s^{-1}$)
F ≙ Force (N)
Δt ≙ change in time (s)

Impulse-Momentum Theorem

... Derive this formula:

$J = \Delta p$

or: $F\Delta t = m\Delta v$

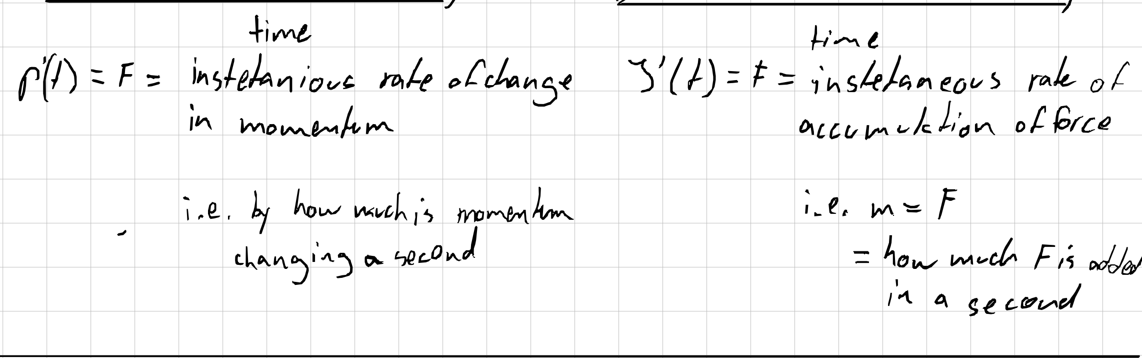
or $F\Delta t = \Delta(mv)$ really.

$F = m \cdot a$
$F = m \cdot \frac{\Delta v}{\Delta t}$ as $a = \frac{\Delta v}{\Delta t}$ $v_f = v_i + at$, $a = \frac{v_f - v_i}{t}$
$\underbrace{\Delta t F}_{\Downarrow\, J} = \underbrace{m\Delta v}_{\Downarrow\, \Delta p}$

could be $\Delta(m \cdot v)$, $\Delta m\,v$, $m\Delta v$ really no difference (maybe look that up, lol)

Impulse in Relation to Momentum

$\Delta p = J$ Impulse essentially describes the accumulation of force over a given time which only accumulates to a portion of the object's entire moment.



$p'(t) = F$ = instantanious rate of change in momentum

i.e. by how much is momentum changing a second

$J'(t) = F$ = instantaneous rate of accumulation of force

i.e. $m = F$
= how much F is added in a second

Conservative Forces

=> Forces for which the work done is independent of the path taken when it moves an object

Conservative Forces:
- Weight
- Electromagnetic Force
- Spring Force ...

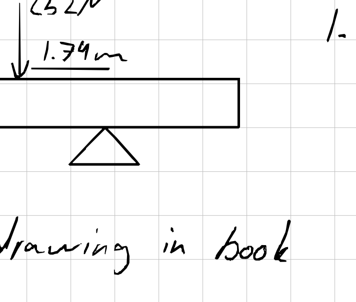
Non-Conservative Forces*
- Friction
- Driving Force
...

* non-conservative forces result in the dissapation of mechanical energy into other forms, e.g. thermal

1.5 problems

p. 19

1.



$1.74 \times 252 = 438.48 \approx 438\ Nm$

2. drawing in book

$l_1 mg = l_2 F$

$2.75 \cdot 46 \times 10 = l_2 \cdot 824$

$1265 = 824 l_2$

$l_2 \approx 1.5m$

3. Due to friction with the fingers it comes to a halt.

4. $r_1F_1 = r_2F_2 + r_3F_3$ Dinosour: $r = 0.2\ m$, $F = 5.8\ N$ Banana: $r = ?$, $F \doteq 1.4$ Beam: $r = 0.3\ m$, $F = 1.3N$

$0.2 \times 5.8 = 0.3 \times 1.3 + 1.4 r_3$

$r_3 = \frac{0.2 \times 5.8 - 0.3 \times 1.3}{1.4} = 0.55m$

1.6 Problems

1. a) The book will sit stationary because the table is acting with an opposing force to the weight of the book

b) This would result in a negative normal force, meaning that the book would lift off the table. (→ acceleration)

c) Because the weight of the book and the partner force of you lifting the book would act on my hand.

2. a) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0.4 - 0.2}{0.2 - 0.1} = \frac{.2}{.1} = 2$

$F = ma$

why should I do this?

$0.5 = m \cdot 1$

$m = 0.5kg$

b)

$a = \frac{F}{m}$ explains the '2'

so $4a = \frac{2F}{m}$

↳ therefore the gradient

$f(x): m = \frac{2F}{4a}$

$\text{gradient} = f'(x) = \frac{2}{4} = \frac{1}{2} kg$ Understood.

3. a) $F = ma$ $12kg = m$ $785N = F$

$785N = 12kg\ a$

$a = \frac{785N}{12kg} \approx 65\ ms^{-2}$

b) $F = 22.2N$ $m = 3.1kg$

$22.2 = 3.1a$

$a = \frac{22.2N}{3.1\frac{m}{s^2}} \approx 7.2ms^{-2}$

c) $F = ma$ $a = \frac{F}{m}$

$a = \frac{19.6N}{2.0kg} = 9.8\ ms^{-2}$

d) $|210N - 287N| = F$ $m = 0.430\ kg$

$F = 77N$

$a = \frac{77N}{0.403kg} = 179\ ms^{-2}$

1.7. (P. 25)

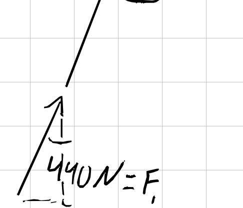
5. $V_i = 41.6\ m/s$ $t = 0.6s$ $\Delta s = 3m$

$\Delta x = v_i t + \frac{1}{2} a t^2$

$\bar{a} = \frac{\Delta x - V_i t}{\frac{1}{2} t^2} = \frac{3m - 41.6\ m/s \cdot 0.6s}{0.5 \cdot 0.6s^2} = -122\ m/s^2$ (down)

1.8 problems p. 27

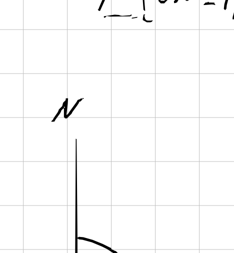
1. b)



$V_x = 10\cos(45°) = 7.1\ m/s$

$V_y = 10\sin(45°) \approx 7.1\ m/s$

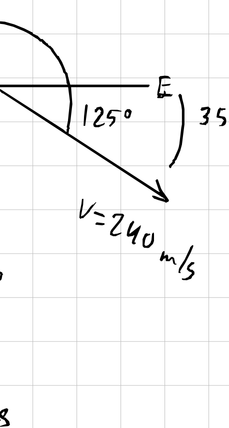
2.



$v_x = 16\cos(35°) = 13.1\ m/s$

$v_y = 16\sin(35°) = 9.2\ m/s$

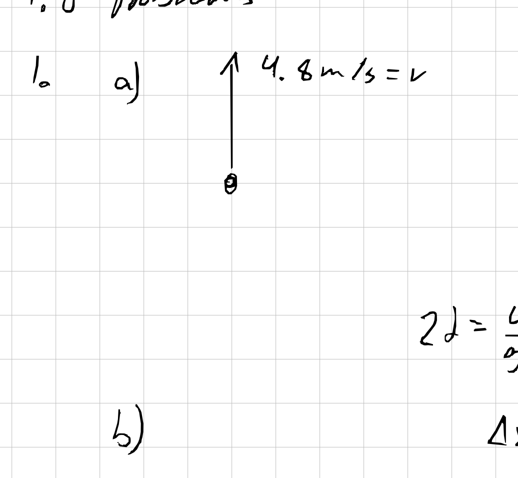
3.



$F_x = 440 \sin\alpha = 440 \sin 28° = 206.6N$

$F_y = 440 \cos\alpha = 440 \cos 28° = 388.5N$

4.



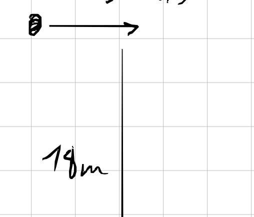
$125° - 90° = 35°$

$V_S = 240 \sin 35° = 137.7\ m/s$

$V_E = 240 \cos 35° = 196.6\ m/s$

1.8 Problems

1. a)



$V_f = V_i + at$

$0 = 4.8 - 9.81t$

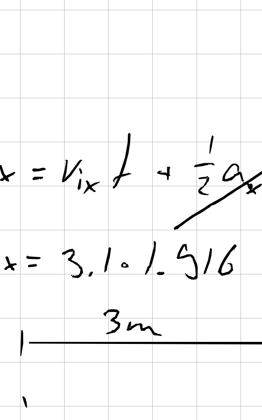
$4.8 = 9.81t$

$2t = \frac{4.8}{9.81} \cdot 2 = 0.45 \times 2 = 0.97s$

b) $\Delta x = (v_i - v_f)\frac{t}{2}$

$= \frac{4.8}{2} \cdot 0.489 = 1.17m$

2. a)



$V_{yi} = 0$

$\Delta y = -18m$

$t = ?$

$a = -9.81m/s^2$

$\Delta y = \cancel{V_i t} + \frac{1}{2} a_y t^2$

$\frac{\Delta y}{\frac{1}{2}a} = t^2$

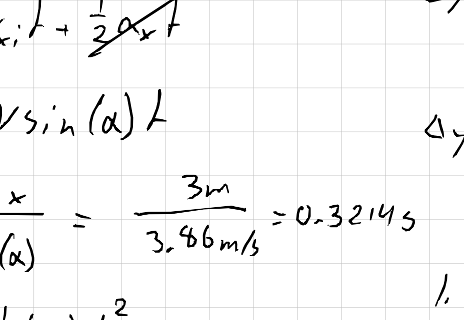
$t = \pm\sqrt{\frac{2\Delta y}{a}}$

$t_1 = 1.92$ $\cancel{t_2 = -1.92}$

b) $\Delta x = V_{ix} t + \cancel{\frac{1}{2} a_x t^2}$

$\Delta x = 3.1 \cdot 1.916 = 5.94m$

3.



a) $V_{yi} = v\cos(\alpha)$

$V_{yf} = 0$

$V_{yf}^2 = V_{yi}^2 + 2a\Delta y$

$\frac{V_{yf}^2 - V_{iy}^2}{2a} = \Delta y$

$\Delta y = \frac{V_{yf}^2 - (v\cos(\alpha))^2}{2a}$

$\Delta y = \frac{0 - (4.596)^2}{-2 \times 9.81} = 1.077m$

$1.08 > 0.9$ so yes, it flies high enough.

$\Delta y = 0.9m$

b) $V_x = v\sin(\alpha)$

$\Delta x = 3m$

$a_x = 0$

$\Delta x = V_{xi} t + \cancel{\frac{1}{2} a_x t^2}$

$\Delta x = V\sin(\alpha) t$

$t = \frac{\Delta x}{v\sin(\alpha)} = \frac{3m}{3.86m/s} = 0.3214s$

$\Delta y = V_{yi}t + \frac{1}{2}(-g)t^2$

$\Delta y = (v\cos(\alpha))t + \frac{1}{2}(-g)t^2$

$\Delta y = \frac{1}{2}(-g)t^2 + v\cos\alpha t$

$\Delta y = t^2 + \frac{2v\cos\alpha}{-g} t$

$\Delta y = 0.3214^2 + \frac{2 \times 6 \times \cos(40°)}{-9.81} \times 0.3214 = -0.1979 \approx -0.198m$

So now, the ball would be to low to make it into the basket

P. 31

~~1. a)~~

2. a) $F = ma$ $F = 9300N$ $m = 12kg$

$a = \frac{F}{m} = \frac{9300N}{12kg} = 775\ m/s^2$

b) $a = 775\ m/s^2$ $\Delta t = 0.05s$ $v_i \doteq 0$

$V_f = V_i + at$

$V_f = 0 + 775m/s^2 \cdot 0.05 = 38.75m/s$

c) $v = 38.75\ m/s$ at 45° to the horizontal

$V_x = V_y = V \cdot \sin(\alpha) = 38.75 \cdot \sin(45°) = 27.4\ m/s$

d) $\Delta x = 150m$ $\Delta t = ?$

$\Delta x = V_i t + \cancel{\frac{1}{2}at^2}$

$t = \frac{\Delta x}{V_i} = \frac{150m}{27.4m/s} = 5.474s$

$V_f = \frac{F}{m} \cdot t_1$

$V_{x,y} = \frac{F}{m} t_1 \sin\alpha$

$t_2 = \frac{\Delta x}{\frac{F}{m} t_1 \sin\alpha}$

$t_3 = \frac{\frac{F}{m} t_1 \sin\alpha}{\frac{\Delta x}{\frac{F}{m} t_1 \sin\alpha}}$

$t_3 = \frac{(\frac{F}{m} t_1 \sin\alpha)^2}{\Delta x}$

$t_4 = \frac{(\frac{F}{m} t_1 \sin\alpha)^2 - (\frac{F}{m} t_1 \sin\alpha)}{}$

e) $V_f = 0$ $V_i = 27.4\ m/s$

$V_f = V_i + at$

$0 = 27.4 - 9.81t$

$\frac{27.4}{9.81} = t = 2.793s$

f) $5.474 - 2.793 = 2.681s$

g) $\Delta x = \cancel{V_i t} + \frac{1}{2}at^2$

$\Delta x = -\frac{1}{2}(9.81)\ 2.681^2 = -35.256m$

h) $\Delta y = v_i t + \frac{1}{2}at^2$

$\Delta y = 27.4 \times 2.793 + \frac{1}{2}(-9.81)\ 2.793^2 = 38.27m$

$38.27m - 35.256 = 3.0m$

3. The Canon has a height of zero and there is no air resistance

4. $\Delta x = 150m$ $F = 9300N$ $\alpha = 45°$ $m = 10kg, 14kg$

$F = m \cdot a$

$a = \frac{F}{m}$

$V_f = V_i + at$ $v_i = 0$

$V_f = \frac{F}{m} \cdot t_1$ $t_1 = 0.05s$

$V_f = V_{i2}$

find time in x

$V_{xi}, V_{yi} = V_i \sin(45°) = \frac{F}{m} t_1 \sin(45)$

$\Delta x = V_{xi} t_2 + \cancel{\frac{1}{2} a t_2^2}$

$\Delta y =$

$\Delta x = \frac{F}{m} t_1 \sin(45)\ t_2 + \cancel{\frac{1}{2} a t_2^2}$

$\frac{\Delta x}{t}$

$t_2 = \frac{\Delta x}{\frac{F}{m} t_1 \sin(45°)}$

$\Delta y = V_{iy} t_2 + \frac{1}{2}(-g) t_2^2$

$\Delta y = \cancel{\frac{F}{m} t_1 \sin(45°)} \cdot \frac{\Delta x}{\cancel{\frac{F}{m} t_1 \sin(45°)}} + \frac{1}{2}(-9.81)\left(\frac{\frac{F}{m} t_1 \sin(45°)}{}\right)^2$

$\Delta y = 150m + \frac{1}{2}(-9.81)\left(\frac{150m}{\frac{9300N}{m} \cdot 0.05s \cdot \sin(45°)}\right)^2$

$m = 10kg$: $\Delta y =$

2.1 Problems

[illegible]

Weight $= m \cdot g = 0.1 \cdot 10 = 1N$

Work $= F \cdot \Delta s = m \cdot g \cdot 1m = 1Nm = 1J$

Power $= \frac{\Delta W}{t} = \frac{1J}{1s} = 1W$

1. a) $p = 240\ kgms^{-1}$

   b) $p = 588\ kgms^{-1}$

   c) $0.000001 \times 0.05 = 5 \times 10^{-8}\ kgms^{-1}$

2. $60km/h : 3.6 \approx 17ms^{-1}$

   $m \approx 300kg$

   $p = 300 \cdot 17 = 5100\ kgms^{-1}$

3. $F = m \cdot a$

   The acceleration can be smaller, as there is a larger distance over which the object is (de)accelerated. This in return means the force will also be smaller.

4. $F = \frac{\Delta p}{\Delta t}$ 1s for Δt and $\Delta p = m \cdot v \approx 0.5kg \cdot 3m/s$

   $F = \frac{0.5kg \cdot 3ms^{-1}}{1s} \approx 1.5N$

ISAAC

u U M m velocity $= v$ $M+m$

$mu - UM = v(M+m)$

$E_{ki} = \frac{1}{2}mu^2 + \frac{1}{2}MU^2$

$E_{kf} = \frac{1}{2}(M+m)v^2$

$mu - UM = v(M+m)$

$v = \frac{mu - MU}{m+M}$

$E_{kf} = \frac{1}{2}(M+m)\left(\frac{mu-MU}{m+M}\right)^2$

$= \frac{1}{2}\frac{(mu-MU)^2}{m+M}$

$E_d = E_{kf} - E_{ki}$

$E_d = \frac{(mu-MU)^2}{2(m+M)} - \frac{1}{2}(mu^2 + MU^2)$

$E_d = \frac{m^2u^2 - 2muMU + M^2U^2}{2m+2M} - \frac{mu^2 + MU^2}{2} \cdot \frac{m+M}{m+M}$

$E_d = \quad - \quad \frac{m^2u^2 + mMU^2 + Mmu^2 + M^2U^2}{2m+2M}$

$E_d = \frac{\cancel{m^2u^2} - 2muMU + \cancel{M^2U^2} - \cancel{m^2u^2} - mMU^2 - Mmu^2 - \cancel{M^2U^2}}{2m+2M}$

$E_d = \frac{-2muMU - mMU^2 - mMu^2}{2m+2M}$

$E_d = \frac{\cancel{mM}(-2 - U^2 - u^2)}{\cancel{mM}\left(\frac{2}{M} + \frac{2}{m}\right)}$ $\qquad \frac{2m}{Mm} + \frac{2M}{Mm}$

$E_d = (-2 - U^2 - u^2)$ $\qquad \frac{2(m+M)}{Mm}$

Checkpoint

1. $(\cancel{50kg} + 1kg) \cdot 0 + 4.1ms^{-1} \times 3.4kg = (50 + 1 + 3.4)v$

   $13.94\ kgms^{-1} = 54.4kg \cdot v$

   $v = 0.148\ ms^{-1}$

2. a) $0 \cdot (60 + 40 + 5) = (60+40)v + 5 \cdot 4$

   $0 = 100v + 20$

   $v = -\frac{20}{100} = -0.2ms^{-1}$

   b) $F = \frac{\Delta p}{\Delta t} = \frac{5 \cdot 4}{0.2} = 100N$

3. By pushing back on the boat with his feet to accelerate foreward the boy exerts a force on the boat. As the third law states every force between two objects has a reaction force, therefore the boat also exerts a force on the boy, just in the opposite direction. As a result both objects drift appart.

   → discurse to momentum and how it is directly derived from newton's third law of motion

4. a)

   200kg $v = 3ms^{-1}$ 100kg $v = 12ms^{-1}$ 80° Find v_{x_f}

   $v_x = 12\cos 80° \approx 2.084ms^{-1}$

   $100kg \cdot 2.084 + 200kg \cdot 3 = 300kg \cdot v$

   $v_{xf} = 2.69ms^{-1}$

   $v_y = 12\sin 80° \approx 11.82$

   $\frac{11.82 \cdot 100 + 0}{300} = \cancel{300} \cdot v_{yf}$

   $v_{yf} = 3.94ms^{-1}$

   $v_f = \sqrt{3.94^2 + 2.69^2} \approx 4.77ms^{-1}$ $\qquad 4.77 \cdot 300kg = 1430kgms^{-1}$

   $\theta = \tan^{-1}\left(\frac{3.94}{2.69}\right) \approx 55.7$

   $1200kgms^{-1}$ 80° 100kg 200kg $\rightarrow 600kgms^{-1}$

   1200 1430 56° 600

   $\Delta x = v_i t + \cancel{\frac{1}{2}at^2}$

   $t = \frac{\Delta x}{v_i}$

   $t_y = \frac{16}{3.94} \approx 4.06s$ $\qquad$ $4.06s > 37.17s$, there is plenty of time

   $t_x = \frac{100}{2.69} \approx 37.17s$

P. 55

1. [illegible]

2. $70.25kmh^{-1} : 3.6 \approx 19.51ms^{-1}$

   $19.51ms^{-1} \times 0.16kg \approx 3.12\ kgms^{-1}$

   $106kmh^{-1} : 3.6 \approx 29.44ms^{-1}$ $\qquad$ range: $3.12kgms^{-1}$ to $4.71kgms^{-1}$

   $29.44ms^{-1} \times 0.16kg \approx 4.71kgms^{-1}$

3. a) This would only be the case if no other force was acting on the ball between shot and impact. Also the duration of the cannon aplying force and duration of the impact have to be of equal time. Due $p = m \cdot v$ which can be derived from Newton's second law of motion the momentum gained through the cannon is the same required to bring the ball to rest again. In both cases F is the slope of Δp in respect to time, therefore both times equal.

   b) As the Δp increases due to the higher velocity the force applied increases too due to $F = \frac{\Delta p}{\Delta t}$, given that Δt changes insignificantly. The different slopes can be explained by the different deflection rates of the material. They are both differently proportional to the Force applied on the material

4. (The collision is perfectly inelastic, meaning that the ball looses all its kinetic energy $\approx$ remember Δt I guess) relevant?

   when taking the player as the refference frame.
   The combined momentum of the ball and keeper is countered by the muscle force of the keeper pushing his entire body with the ball forward. The momentum is canceled

   momentum of ball + ~~momentum of player~~

   $=$ (mass of player + mass of ball + mass of earth) v

   where v is approaching 0 due to the enorme mass

   The player passes on the momentum to the earth.

Exam practice

1. C ✓
2. B ✓
3. A ✓
4. $\frac{404}{10} = 8.04kg$ so B ✓
5. A ✓
6. By ejecting slime the backteria have to apply a force on the slime to accelerate it. As stated by Newton's third Law of motion forces between two objects always have a reaction force, meaning that the slime also exerts an equal force on the backeria. This in return also moves the backeria. Also could add $F = \frac{\Delta p}{\Delta t}$ meaning that the force causes momentum.
7. a) The student can check if the max velocity of the other ball is the same as the initial ball, due to $p = mv$ ✓ Pref =0
   (Also, to simplify things he could see if the height is the same for both balls. This would mean that energy was conserved)

   b) 1. nearly all kinetic energy is transfered between the two outer balls and momentarilly converted to gravitational potential energy.
   2. Some of the kinetic energy is lost to the middle balls due to imperfect elastic collisions.
   3. All kinetic energy has transformed into other types of energy, eg. sound or heat. ✓

8. a) When releasing the weight a pulling force is applied on the right of the trolly. Due to $F = m \cdot a$ there has to be an acceleration, therefore the velocity must be increasing. Because $p = m\Delta v$ the trolly now has a momentum. ✓

   b) $F = m \cdot a$ $\qquad$ $F = m \cdot a$ $\qquad$ $2.89 = mg$

   $2.85 = 0.35 \cdot a$ $\qquad$ $a = \frac{F}{m} = \frac{mg}{m_1 + m_2}$ $\qquad$ $\frac{2.89}{9.81} \approx 0.295kg$

   $a = 8.14ms^{-2}$ ✗ $\qquad = \frac{2.85}{0.35 + 0.295} \approx 4.48ms^{-2}$

   $a =$ rate of change of velocity

   c) $11.1ms^{-1} \times 0.35kg \approx 3.89\ kgms^{-1}$ ✓

   d) Both linear and rotational momentum are conserved individually, ie. the net momentum of a system stays the same as long as there are no external forces acting upon the system. ✓ Stays the same before and after the collision

   e) This must be a perfectly elastic collision where both trollys have exactly the same mass. The moving trolly transfers all of its momentum to the stationairy trolly. if $p = 0$ then $p = mv$ must be zero but only v can change so the velocity has to be zero. Therefore the trolly stops. The once stationairy trolly moves now at the same speed as the other one initially ✓

   0 f) This is a perfectly inelastic collision ✓ ~~where the moving trolly looses all of its kinetic energy~~, taking the other trolly as the frame of refference. Explain that trollies stick together and how their momentum is conserved.

   0 g) The two trollies could be placed together with a force that forces both trollies appart, e.g. two magnets.
   add technical details for measurements, eg. lightgates (Also a collision could be simulated backwards, i.e. moving both trollies at such speeds that they come to a rest when they collide.) * only works for perfectly inelastic collisions

9. Each force between two objects has a reaction force. Newton's third law of motion (✓)
   Putting momentum into terms of Force:

   $F = \frac{\Delta p}{\Delta t}$ $\qquad$ any force applied over some time ✓ results in some momentum, or at least its change. This in return means that ther is a reaction momentum to any reaction force that exists over time. equal change in (✓)

   $F\Delta t = \Delta p$ ✓ $\qquad$ Any collision results in an reaction momentum but might be more or less noticable due to the other object's mass. Though it always prevails. + The total change in momentum is zero

10. a) Between 0 and 4 ms the momentum stayed at a constant 0.8 $kgms^{-1}$. Then between 4ms and 8ms it decreased steadilly uniformly by $-0.5kgms^{-1}$ to $-1.2\ kgms^{-1}$.

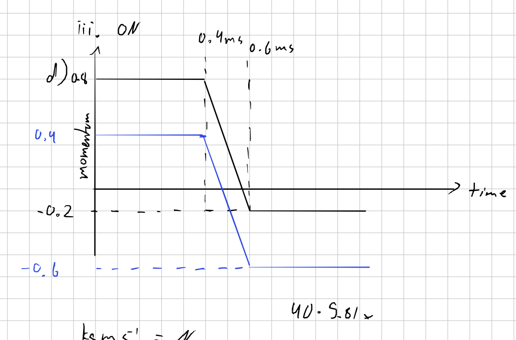
    b) Due to Newton's third law to every force between two objects there exists an reaction force, which in this case causes the ball to accelerate in the opposite direction. This means that the ball reaches approximately the same speed into the opposite direction, and because $p = m \cdot v$ there exists an equal opposite momentum. The player must have been moving towards the ball, else the magnitude of the ball's momentum wouldn't end up larger. ✓

    c) i. 0N ✓

    ii. $\frac{-1.2 - 0.8}{0.008 - 0.004} = 500N$ ✗

    iii. 0N

    d)



$kgms^{-1} = N$ $\qquad$ $40 \cdot 9.81 \cdot$

$0.5 \cdot kg \cdot \left(\frac{m}{s}\right)^2$

$\Delta x = \cancel{v_i t} + \frac{1}{2}at^2$ $\qquad$ kgm^2s^{-2}

$v_f^2 = \cancel{v_i^2} + 2\Delta x a$

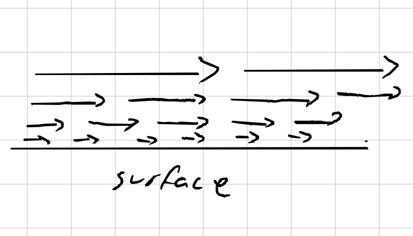
$v_f = \sqrt{2 \cdot 9 \cdot 9.81}$

3.1 Fluids, Density & Upthrust

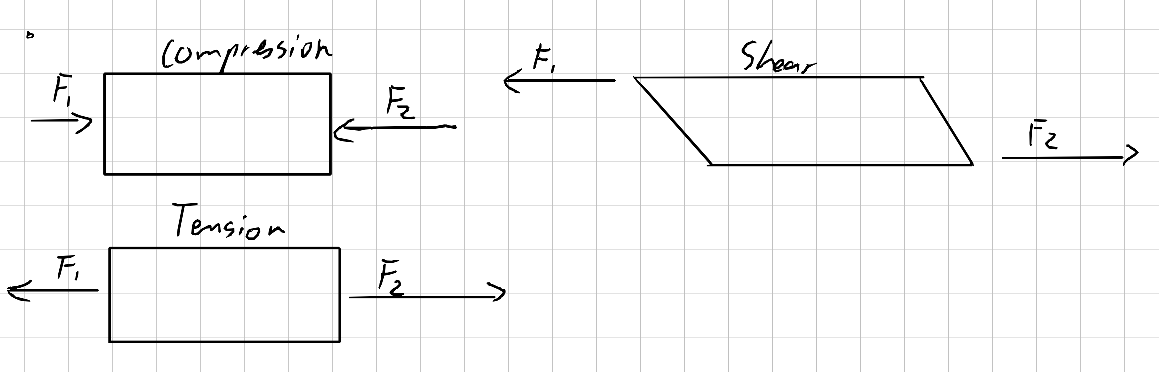
Fluids

- A fluid is a substance that deforms continuously when forces are applied to it, i.e. when it's subject to shearing forces.
- when a fluid is at rest then there are no shearing forces acting on it.
- Fluids can be gas, liquid or granular solid substances (e.g. sand)

Shear(ing) Forces



- Shear forces emerge when two forces act in opposite directions but on different planes.
- Shear forces occur parallel to surface and inbetween layers of fluids that move at different speeds.
- usually causes deformation



Density

$$\rho = \frac{m}{V}$$

ρ ≙ density ($kg\,m^{-3}$)
m ≙ mass (kg)
V ≙ volume (m^3)

Basically:
- How close are the particles packed?
- How heavy are the particles?

Practical Conversion:

$1\,g\,cm^{-3} = 1000\,kg\,m^{-3}$

Upthrust

- The upwards force an object experiences when submerged in water.
- Defined by Archimedes' Principle:

Upthrust = weight of water displaced

$$W = V \cdot \rho \cdot g$$

where $V\rho = m$

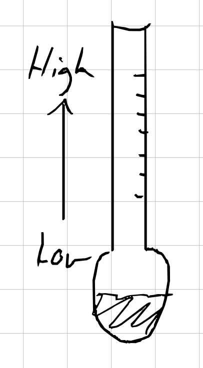
V = volume (m^3)
ρ = density of fluid ($kg\,m^{-3}$)
g = gravitational constant
W = weight of liquid displaced (N)

Floating

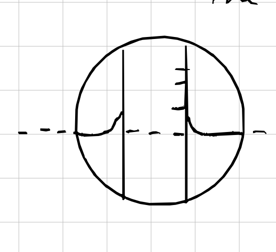
- An object floats when it has displaced its own weight in water.

Hydrometer

- measures the density of a liquid



- a weighted glass tube with readings at the side, indicating the weight of the liquid
- when it sinks deeper it has to displace more liquid, meaning that the liquid is less dense
- it floats when the water displaced matches the weight of the tube



! The test tube in which the hydrometer is submerged has to be wide enough so that surface tension to the walls doesn't influence the measurement.

Sphere Volume & Area

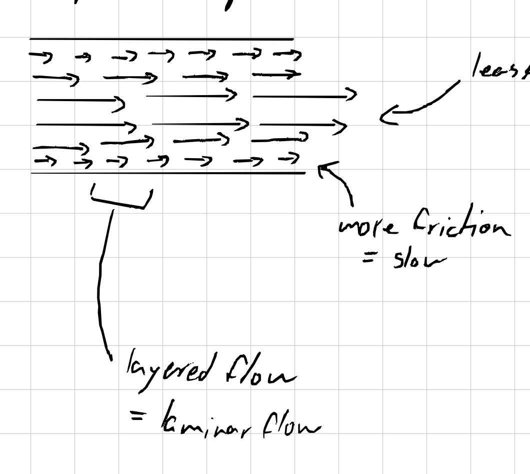
r ≙ radius, l ≙ length

$V = \frac{4}{3}\pi r^3$ $A = 4\pi r^2$

Laminar Flow & Turbulent Flow

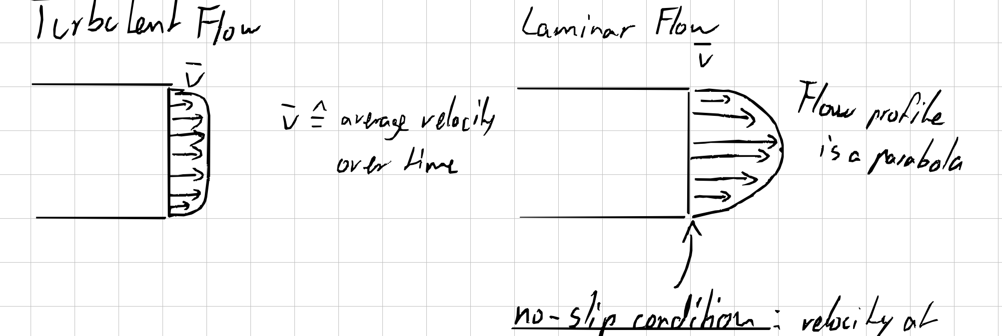
Laminar/Streamline Flow:
- occurs at lower speeds
- changes to turbulent flow when reaching a certain speed
  - ↳ this value is determined by the properties of the fluid and the shape of the surroundings
- 'laminar' means layered, laminae = layers

Pipe example:



- layers of laminar flow are called streamlines or laminae
  - ↳ velocity is constant for each layer over time

Turbulent Flow:
- velocity continuously changes, it's chaotic
- it consists of swirling eddies
  - ↳ eddies are rotating patches of fluid



Newtonian fluids:
- can be described with Newton's formulas for fluid dynamics

Viscosity

- Viscosity causes friction within a fluid
- Coefficient of viscosity: η
- Rate of flow (pipe) is inversely proportional to the fluid's viscosity
- increased temperature
  - decreases viscosity for liquids
  - increases viscosity for gasses

Reynold's Number

partially required

calculate if a fluid is turbulent or laminar

$$Re = \frac{\rho u L}{\mu}$$

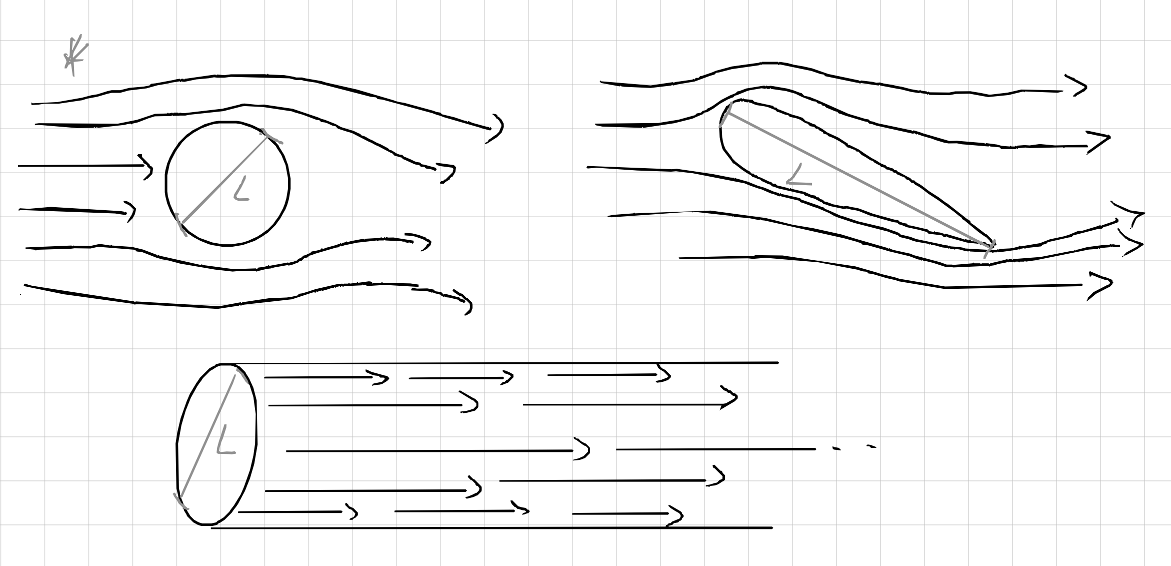
Re ≙ Reynold's Number (none)
ρ ≙ fluid density ($kg\,m^{-3}$)
u ≙ velocity (ms^{-1})
L ≙ characteristic length (m)
μ ≙ dynamic viscosity ($Pa\,s$)

simplified:

$$Re = \frac{uL}{\nu}$$

u ≙ velocity
* L ≙ characteristic length
ν ≙ kinematic viscosity where $\nu = \frac{\mu}{\rho}$
↳ $m^2 s^{-1}$

*



Theory: $Re = \frac{\rho u L}{\mu} = \frac{\text{inertial forces}}{\text{viscous forces}}$

inertial forces ↳ causes fluid to move
viscous forces ↳ causes friction due to viscosity

- Depending on which forces dominate, the flow will be turbulent, transitional, or laminar

$$\underbrace{\text{laminar flow}}\;\underbrace{\text{transitional flow}}\;\underbrace{\text{turbulent flow}}$$

Reynold's Number Range

*$Re = 2000$ *$Re = 4000$

* for flow through a pipe

Calculate Preassure Drop in Pipe

not required

Darcy-Weisbach Equation:

$$\Delta p = f\frac{\rho u_{avg}^2 L}{2D}$$

Δp ≙ change in preassure (Pa)
f ≙ darcy friction factor (none)
L ≙ pipe segment length (m)
ρ ≙ fluid density ($kg\,m^{-3}$)
v_{avg} ≙ mean fluid velocity
D ≙ pipe diameter

calculate 'f' for laminar flow:

$$f = \frac{64}{Re}$$

calculate 'f' for turbulent flow:

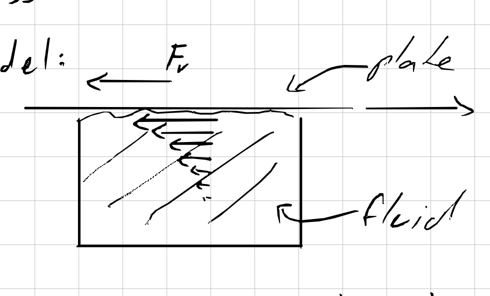
don't even try (use Colebrook equation)

Viscous Force (viscous drag)

partially required

calculate the force experienced by an object dragged over a fluid

model:



$$F_v = \eta A \frac{v}{d}$$

F_v ≙ Viscous Force (N)
η ≙ coefficient of viscosity ($Pa\,s$)
A ≙ area of contact
v ≙ velocity/speed (ms^{-1})
d ≙ depth of the fluid

* η sometimes measured in 'Poise'
$1\,Pa\,s = 10$ Poise

Volume Flowing through Pipe

not required

- how much volume passes a crossection of a pipe over time

Poiseuille's Law:

$$\frac{V}{t} = \frac{\Delta p \pi R^4}{8\eta L}$$

V ≙ volume (m^3)
t ≙ time (s)
Δp ≙ difference in preassure (Pa)
R ≙ diameter of tube (m)
η ≙ coefficient of viscosity (none)
L ≙ total tube length (m)

laminar flow only!

Exercises

1. $\rho = \frac{m}{V} = \frac{3.38}{0.23 \times 0.1 \times 0.07} \approx 2099 \ kgm^{-3}$

2. $0.2m = r$

$\rho = \frac{m}{V}$ $\quad V = \frac{4}{3}\pi \cdot 0.2^3 \approx 0.335$

$\rho = \frac{0.006}{0.335} = 0.179 \ kgm^{-3}$

3. $W = V\rho g$

$1500\,000 \times 9.81 = 10 \cdot 60 \cdot h \cdot 1000 \cdot 9.81$
$14715\,000 = 5886000\,h$
$h = 2.5m$

1. $\rho = \frac{m}{V} = \frac{1.265}{0.1^3} = 1265 \ kgm^{-3}$

2. $V = \frac{4}{3}\pi \cdot 0.11^3 \approx 0.00558 \ m^3$

$\rho = \frac{m}{V} = \frac{7.26}{0.00558} \approx 1302 \ kgm^{-3}$

$V = \frac{4}{3}\pi \cdot 11^3 \approx 5575 \ cm^3$

$\rho = \frac{m}{V} = \frac{7260}{5575} \approx 1302 \ gcm^{-3}$

3. $\rho = \frac{m}{V}$

$m = \rho \cdot V = 1.2 \cdot 4.8^2 \approx 30.7 kg$

4. $d = 4.27cm$ $\quad r = 2.135$

a) $W = V\rho g = \frac{4}{3}\pi \cdot 0.002135^3 \cdot 1000 \cdot 9.81$

$W = 0.0004 N$ up

b) $-0.045 \cdot 9.81 + 0.0004 = -0.441 N$ down

c) due to $F = ma$ and the resultant force being negative it can be said that the ball will accelerate downwards.

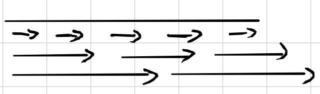
5. $0.18 kg \cdot 9.81 - 2.25 \times 10^{-5} \times 800 \cdot 9.81 = 1.589 \ N$ Down

6. Slightly above $1000 \ kg \ m^{-3}$

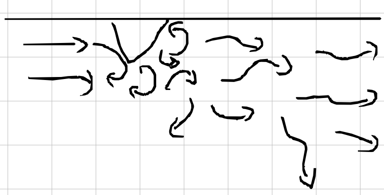
$V \cdot \rho = m$
$V = \frac{m}{\rho} = \frac{0.18}{8000} = 2.25 \times 10^{-5}$

p. 64

1. cars, trains, planes

2. 

laminar flow: Streamlines flowing orderly past eachother, with constant velocity



turbulent flow: There are no layers and the velocity changes continuously. There are swirling eddies all over the place.

3. (This is because the streamlines themselves are bent over the car. Within those streamlines the flow is at a constant velocity), not changing direction, at least relative to the streamlines.
(Fluid is constant only velocity for every point individually)

4. The left picture shows a flat surface as in laminar flow, while the second image shows a turbulent surface as in turbulent flow. → moving fast past obstacle

moving slow

3.4 Terminal Velocity

Terminal Velocity

When the weight of an object falling through a fluid cancels with the drag and upthrust, it falls with terminal velocity. ! V_{term} only describes vertical motion!

Condition:

$W = F_{upthrust} + F_{drag}$ or $F_{net} = 0$

Stoke's Law

$F_D = 6\pi r \eta v$

$r \hat{=}$ radius
$\eta \hat{=}$ coefficient of viscosity
$v \hat{=}$ velocity

calculates drag force on a sphere at low speeds

$F_{drag} \propto r$
and $F_{drag} \propto v$

Calculate terminal velocity

W = upthrust + drag
mg = weight of displaced fluid + Stoke's Force

$m_{sphere} \cdot g = \frac{4}{3}\pi r^3 \rho V_{term}$

⋮

$V_{term} = \frac{2r^2 g(\rho_{sphere} - \rho_{fluid})}{9\eta}$

$V_{term} \hat{=}$ terminal velocity
$V_{term} \propto r^2$

- Predictions of any kind are though for turbulent flow, which is usually caused by
  - fast moving objects
  - irregular shapes
  - large objects
- Though larger objects generally fall faster ↳ (if they're massive)

Check understanding:

$F_{net} = 0$
$F_G = F_B + F_D$

$F_B = V \cdot \rho \cdot g$ $F_B = \frac{4}{3}\pi r^3 \cdot \rho \cdot g$
$V = \frac{4}{3}\pi r^3$
$F_D = 6\pi r \eta v$
$\rho = \frac{m}{V} \Rightarrow m = \rho \cdot V$
$m = \rho \frac{4}{3}\pi r^3$

$mg = \frac{4}{3}\pi r^3 \rho g + 6\pi r \eta v$

$\rho_s \frac{4}{3}\pi r^3 g = \frac{4}{3}\pi r^3 \rho_f g + 6\pi r \eta v$

$\rho_s \frac{4}{3} r^{\cancel{3}2} g - \frac{4}{3} r^{\cancel{3}2} \rho_f g = 6\eta v$

$\frac{4}{3} r^2 g(\rho_s - \rho_f) = 6\eta v$

$\frac{2}{9} r^2 g(\rho_s - \rho_f)\frac{1}{\eta} = v$

$v = \frac{2r^2 g(\rho_s - \rho_f)}{9\eta}$

Exercise

1. $F_D = ?$ $r = 0.001 m$ $v = 0.001 ms^{-1}$ $\eta = 100\ Pa \cdot s$

$F_D = 6\pi r \eta v$

$F_D = 6\pi \cdot 0.001 \cdot 100 \cdot 0.001 \approx 1.88 \times 10^{-3} N$ ✓

2. The cat is shaped irregularly, while also falling very fast, so the flow will be turbulent. Turbulent flow is chaotic, therefore hard to predict. (Stoke's Law doesn't apply) ✓

3. a) $r = 2m$ $V_{term} = ?$ $\eta = 1.7 \times 10^{-5}\ Pa \cdot s$ $\rho_s = 7850\ kg\,m^{-3}$ $\rho_f = 1.2\ kg\,m^{-3}$

$v = \frac{2 \cdot r^2 g(\rho_s - \rho_f)}{9\eta} = \frac{2 \cdot 2^2 \cdot 10(7850 - 1.2)}{9 \cdot 1.7 \times 10^{-5}}$

$v = 4.1 \times 10^9\ ms^{-1}$ (✓) use 9.81 for g

The ball just traveled dimensions xD

b) $r = 2m$ $\eta = 10^{-3}\ Pa \cdot s$ $\rho_s = 7850$ $\rho_f = 1000$

$v = \frac{2 \cdot 2^2 \cdot 10(7850 - 1000)}{9 \cdot 10^{-3}} \approx 6.1 \times 10^7\ ms^{-1}$
(must be) (✓) use 9.81 for g

c) Flow is laminar, which it isn't.
also to fast speeds and to large objects (✓)

4. ~ 45 ms^{-1} (✓) explain... even if not asked for it lol

5. a) i as temperature increases, density decreases ✓

ii same ✓

b) $F_B = V\rho \cdot g$

where a decrease in 'ρ' due to temperature would result in a smaller force. Note that V is increasing though. ✓

c) It becomes less viscous (roughly halves) ✓ V_{term} is inversely proportional

d) ~~It's linear, as V_{term} is proportional to r^2~~

$V_{term} = \frac{2gr^2(\rho_s - \rho_f)}{9\eta}$ the decrease in viscosity overweighs the decrease in density

e) It's viscosity is way to low for accurate measurements. ✓

Exam Practice

1. B ✓

2. $r = 0.025 m$ $\rho = 2900 kg\,m^{-3}$

$\rho = \frac{m}{V} \Rightarrow m = \rho \cdot V$

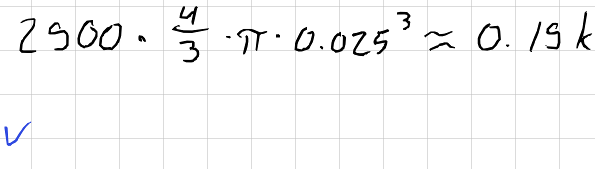
$m = 2900 \cdot \frac{4}{3} \cdot \pi \cdot 0.025^3 \approx 0.19 kg$ so C ✓

3. C ✓

4. B ✓

5. $2.88 N - 0.58 \approx 2.25 N$ so C ✓

6. a) i



✓

ii) Laminar flow describes flow that occurs in layers (laminae) that move at different speeds. ✓ At any point in the fluid the velocity stays constant.
Turbulent flow describes flow where the velocity at any given point changes continuously, ✓ it's chaotic ✓ and hard to predict. Contains eddies (extra)
layers mix/cross (extra)

b) i with lower viscosity it should flow faster. (✓) = higher velocity

ii Viscosity of liquids decreases and viscosity of gasses increases with temperature. I expect the overall viscosity to drop, as the blood mainly consists of water. Therefore velocity should increase. ✓

7. The oil would spread out more as an increase in temperature causes a lower viscosity of the oil which in return results in lower surface tension. ✓ (higher flow rate)

8. a) i Laminar flow is shown by the continuous streamlines ✓ infront and on the sides of the ball. All points within those streamlines have a constant velocity. ✓
Turbulent flow is chaotic and consists of many spinning eddies. This is shown by the randomly spinning arrows behind the ball.

ii At the top air is being dragged towards the opposing laminar flow, causing a constant turbulence in that area. ✓

b) If air is being dragged upwards, that means the ball must be dragged downwards. This can be explained by Newton's third law of motion: every action force has a reaction force. ✓

c) i $\Delta x = vt$
$2.7 = 31t$
$t = \frac{27}{310}$
$\Delta y = \cancel{v_1 t}^{0} + \frac{1}{2}at^2 = \frac{1}{2} \cdot (-10) \cdot \left(\frac{27}{310}\right)^2$
$\Delta y \approx -0.0379 m \approx -0.04 m$ ✓

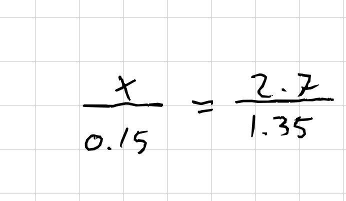
ii

$\frac{x}{0.15} = \frac{2.7}{1.35}$

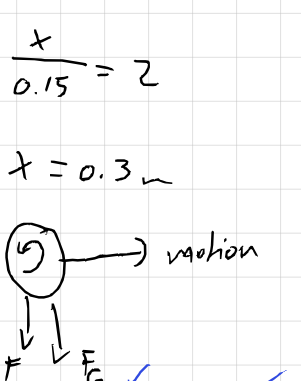
$\frac{x}{0.15} = 2$

$x = 0.3 m$

The ball with no spin would have to be hit from a height of around 30cm. If the ball spins (backwards) it experiences an additional force and would come down faster. The initial height can be less.



9. a) i



come up with smaller

ii That's because the forces acting on it reach an equilibrium. Drag increases with speed until Upthrust + Drag equal the weight of the object.

iii $F_g = F_{Buoyant} + F_{Drag}$

b) i $F_B = \rho \cdot V \cdot g$ It's not explore

$F_B = 1000 \cdot \frac{4}{3}\pi (1.6 \times 10^{-3})^3 \cdot 10 \approx 1.7 \times 10^{-4} \approx 2 \times 10^{-4}$

$V_{term} = \frac{2r^2 g(\rho_s - \rho_f)}{9 \cdot \eta} = \frac{2 \cdot (1.6 \times 10^{-3})^2 \cdot 10(2.7 \times 10^3 - 10^3)}{9 \cdot 1.2 \times 10^{-3}}$

$V_{term} \approx 8.06\ ms^{-1}$

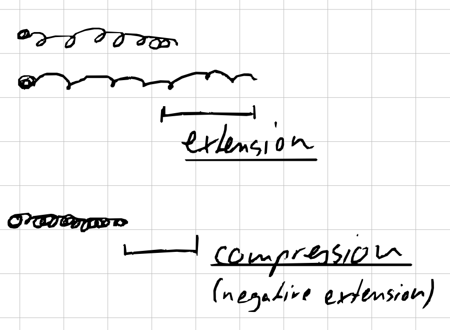
c) The terminal velocity is proportional to the square of the radius, meaning that the larger particles sink faster.

10. When the stone is dropped it accelerates until the drag has increased to a point where drag + Buoyant force cancel with the weight, so that $F_{net} = 0$.
When the temperature increases the drag decreases, meaning the equilibrium will be reached at higher speeds in summer.

Higher temperature → lower viscosity in liquid → less friction → less drag for same speed

3.5 Hooke's Law

Terms of deformation:



When Hooke's Law applies:

$\Delta x \propto F_s$

until limit of proportionality is passed.

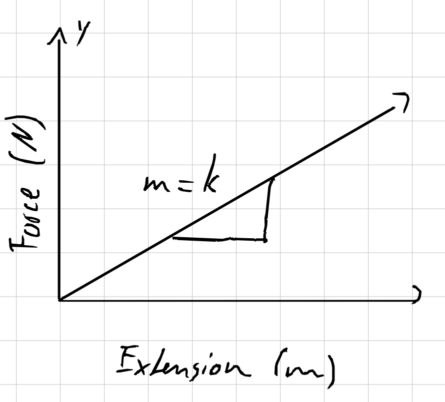
If the elastic limit is passed, the spring won't fully return.

! they're not the same

Hooke's Law

$\Delta F_s = k \Delta x$

$F_s \hat{=}$ Spring Force / Force applied (N)
$k \hat{=}$ spring constant (Nm^{-1})
$\Delta x \hat{=}$ extension (m)



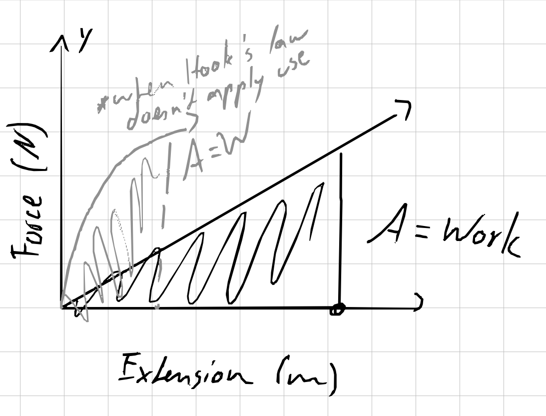
i.e $\frac{\Delta F}{\Delta x} = k$

Elastic Strain Energy

i.e. elastic potential energy

$(\Delta PE_{el} = \frac{1}{2} F_s \Delta x)$

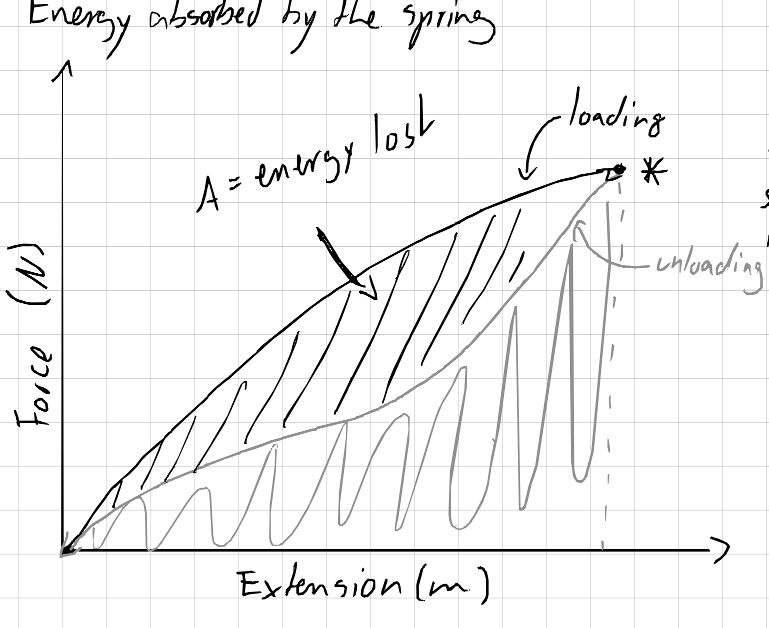
$\Delta PE_{el} = \frac{1}{2} k \Delta x^2$



Hysteresis

ganize

Energy absorbed by the spring



Explanation:

The force increases differently when unloading, as some enegy gets transfered into e.g. heat. Both starting and finishing point can be the same
↳ no permanent deformation

* They converge here because unloading started here

Worked examples:

1. $k = 50 Nm^{-1}$ $\quad l_i = 0.03m$
$l = ?$ when $m = 0.2kg$

$F = k\Delta x$
$mg = k\Delta x$
$\Delta x = \frac{mg}{k} = \frac{0.2 \times 9.81}{50} = 0.039\,m$

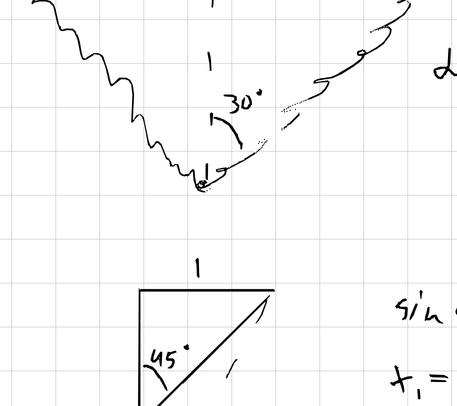
$l = 0.03 + 0.039 = 0.069\,m$

2. $6 \times 0.025 = 0.075\,J$

$PE_{el} = \frac{1}{2} F \Delta x$

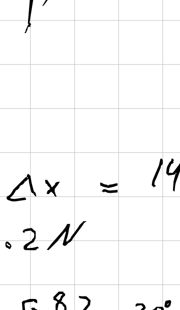
3. $A = \frac{0.04 \times 4}{2} + 0.08 \times 4 + \frac{2 \times 0.04}{2} = 0.44\,J$

Sping Triangle



$k = 14\,Nm^{-1}$

$\alpha_i = 45°$



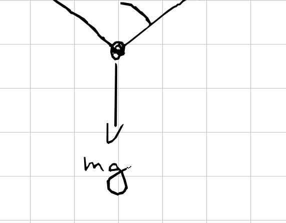
$\sin 45° = \frac{1}{x}$

$x_1 = \frac{1}{\sin 45°} = 1.414$

$x_2 = \frac{1}{\sin 30°} = 2$

$2 - 1.414 \approx 0.586\,m$

$F = k\Delta x = 14 \cdot 0.586$
$F = 8.2\,N$



$F_{net} = 0$

$\cos 30° \cdot 8.2 \cdot 2 = 14.2\,N$

$mg = 14.2\,N$

$m = \frac{14.2}{9.81} = 1.45\,kg$

3.6 Stress, Strain, Young Modulus

Stress

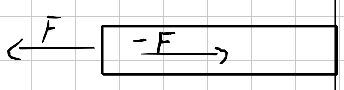
- Tensile Stress: force due to tension / extension
- Compressive Stress: force due to pressure / compression

Stress is calculated as the force acting within an object over its perpendicular cross-section.

$$\sigma = \frac{F}{A}$$

$\sigma \hat{=}$ stress (Pa)
$F \hat{=}$ force acting within the object * (N)
$A \hat{=}$ cross-sectional area of object (m^2)

*



The force acting inside the object is the reaction force to the one acting outside. At all points within the object it's equal.

Strain

- Tensile strain: deformation caused by stretching
- Compressive strain: deformation caused by compression lol

strain describes the ratio of extension to original length:

$$\varepsilon = \frac{\Delta l}{l_i}$$

$\varepsilon \hat{=}$ strain (none)
$\Delta l ; \Delta x \hat{=}$ extension (m)
$l_i ; x_i \hat{=}$ initial length (m)

Strain is often represented as a persentage, e.g.

$\varepsilon = -0.25$ => The objet has a compresive strain of 25%.

Young Modulus

The Young modulus describes the stiffness constant of a material. Its value is independent of an object's shape or size, as it generalizes those atributes.

$$E = \frac{\sigma}{\varepsilon} = \frac{F l_i}{A \Delta l}$$

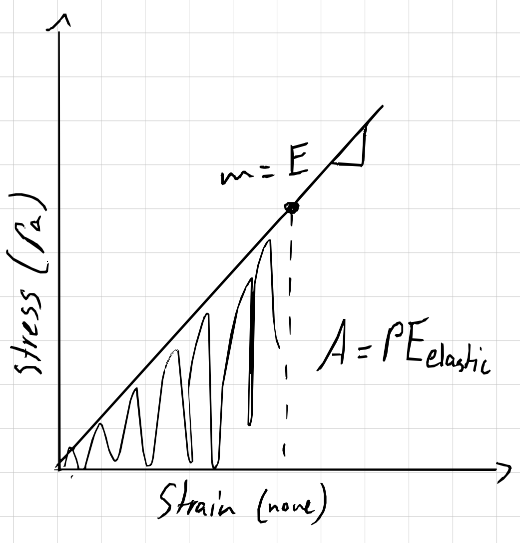
$E \hat{=}$ Young Modulus (Pa)
$\sigma \hat{=}$ Stress (Pa)
$\varepsilon \hat{=}$ strain (none)

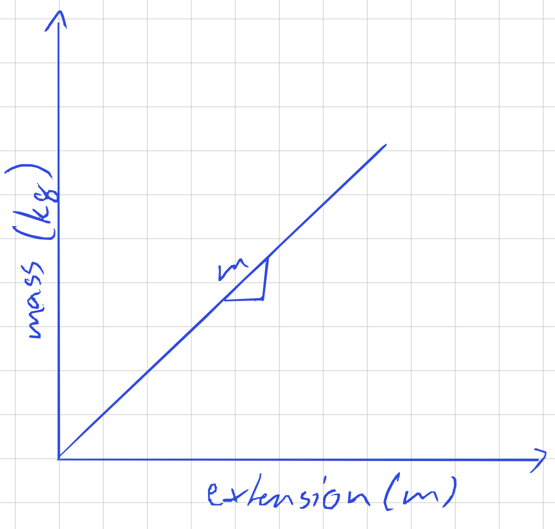
! elastic deformation only

Difference of k and E:
- k relates to the stiffness of a particular object
- E describes the stiffness of a material, regardless to its shape

The Young modulus (E) actually measures the work done along the object, over the potential energy gained per volume extended

$$E = \frac{\sigma}{\varepsilon} = \frac{F l_i}{A \Delta l} = \frac{F \cdot d}{V} = \frac{W}{V}$$





$$E = \frac{F l_i}{A \Delta l} = \frac{m}{\Delta l} \cdot \frac{g l_i}{A}$$

$$E = \text{gradient} \cdot \frac{g l_i}{A}$$

Exercises

1. $r = 0.3\,m \qquad F = 2500\,N$ compression

$$\sigma = \frac{F}{A} = \frac{2500}{\pi 0.3^2} \approx 8840\,Pa$$ compressive stress

2. $l_i = 1.76\,m \qquad l_f = 1.8\,m \qquad \varepsilon = ?$

$$\varepsilon = \frac{\Delta l}{l_i} = \frac{1.8 - 1.76}{1.76} \approx 0.0227 = 2.27\%$$ tensile strain

3. $r = 0.00011\,m \qquad F = 100\,N \qquad \varepsilon = 0.0227$

$$A = \pi r^2 = \pi \cdot 0.00011^2 \approx 3.80 \times 10^{-8}$$

$$\sigma = \frac{F}{A} = \frac{100}{3.8 \times 10^{-8}} = 2.631 \times 10^9$$

$$E = \frac{\sigma}{\varepsilon} = \frac{2.631 \times 10^9}{0.0227} = 1.16 \times 10^{11}\,P$$

$l_i = 0.05\,m \qquad A = 2.5 \times 10^5\,m^2 \qquad \Delta l = 2 \times 10^{-6}\,m \qquad F = 8829\,N$

$$E = \frac{F l_i}{A \Delta l} = \frac{8829 \times 0.05}{2.5 \times 10^{-5} \times 2 \times 10^{-6}}$$

$A = 1.5 \times 10^{-6}\,m \qquad \Delta l = 0.005\,m$

$\sigma = 70 \times 10^6\,Pa \qquad E = 130\,Pa$

$$E = \frac{F l_i}{A \Delta l} = \frac{\sigma}{\varepsilon}$$

$$130 \times 10^9 = \frac{70 \times 10^6}{\varepsilon}$$

$$\varepsilon = 5.38 \times 10^{-4}$$

$$\varepsilon = \frac{\Delta l}{l_i} = \frac{0.005}{l_i}$$

$$5.38 \times 10^{-4} = \frac{0.005}{l_i}$$

$$l_i = 9.28\,m$$

1. $\varepsilon = \frac{\Delta x}{x} = \frac{104 - 97}{\cancel{104}\ 97} = \cancel{0.067}\ 0.072 \approx \cancel{6.7\%}$

2. $r = \frac{0.2}{1000}\,m \qquad F = 9.81\,N \qquad \sigma = ?$

$\sigma = \frac{F}{A} \qquad \text{Area} = \pi r^2 = \pi \cdot \left(\frac{0.2}{1000}\right)^2 = 1.26 \times 10^{-7}$

$\sigma = \frac{9.81}{1.26 \times 10^{-7}} \approx 7.81 \times 10^7\,Pa$ ✓

3. $E = \frac{F l_i}{A \Delta l} = \frac{0.6 \cdot \frac{12}{100}}{3.14 \times 10^{-8} \cdot \frac{1.8}{1000}}$

$\text{Area} = \pi r^2 = \pi\left(\frac{0.1 \cdot \frac{1}{2}}{1000}\right)^2 = \cancel{3.14 \times 10^{-8}}$

$E = \cancel{1.27 \times 10^{10}}\,Pa \quad 5.09 \times 10^9\,Pa$

4. a) $\sigma = \frac{F}{A}$ $\qquad \frac{7000}{4} = 1750\,kg$

$\sigma = \frac{17168}{0.0491} \approx 3.50 \times 10^5\,Pa$ $\qquad 1750 \times 9.81 = 17168$

$A = \pi\left(\frac{25}{200}\right)^2 = 0.0491$

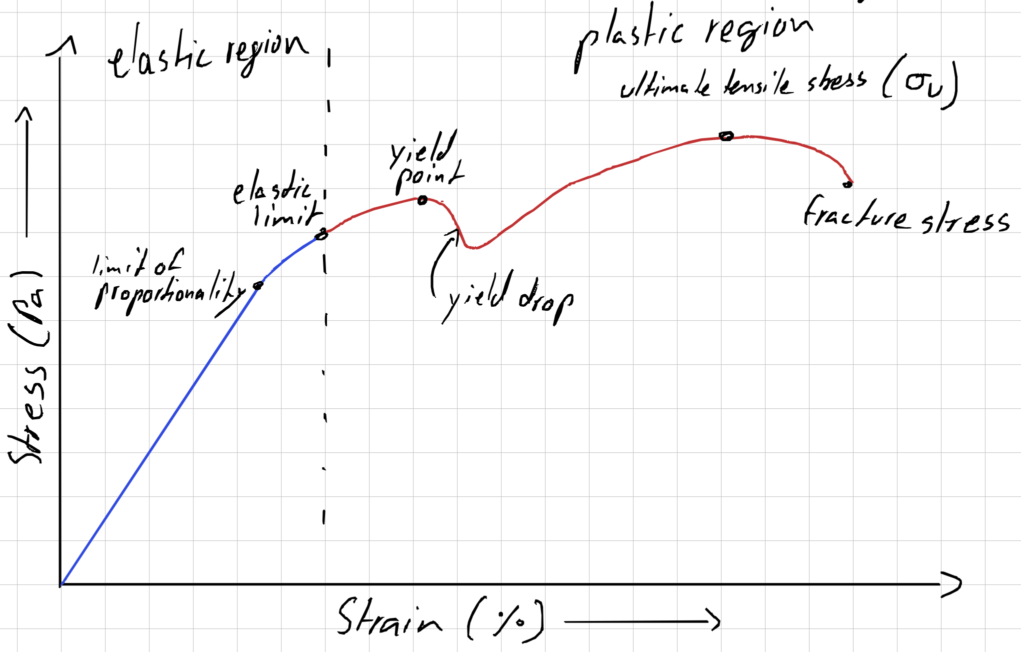
b) $E = \frac{\sigma}{\varepsilon}$

$$19 \times 10^9 = \frac{3.5 \times 10^5 \cdot 2}{\varepsilon}$$

$$\varepsilon = 3.68 \times 10^{-5}$$

$\varepsilon = \frac{\Delta l}{l_i} \qquad 3.68 \times 10^{-5} = \frac{\Delta l}{0.95} = 3.5 \times 10^{-5}\,m$ ✓

3.7 Stress - Strain Graphs



<u>Elastic region:</u> object undergoes elastic deformation and returns to initial shape

<u>Plastic region:</u> object experiences plastic deformation and won't return to its initial shape
↳ after elastic limit was reached

<u>limit of proportionality:</u> marks the end of where $\sigma \propto \varepsilon$, usually shortly before elastic limit..

<u>Elastic limit:</u> Point after which object experiences plastic deformation

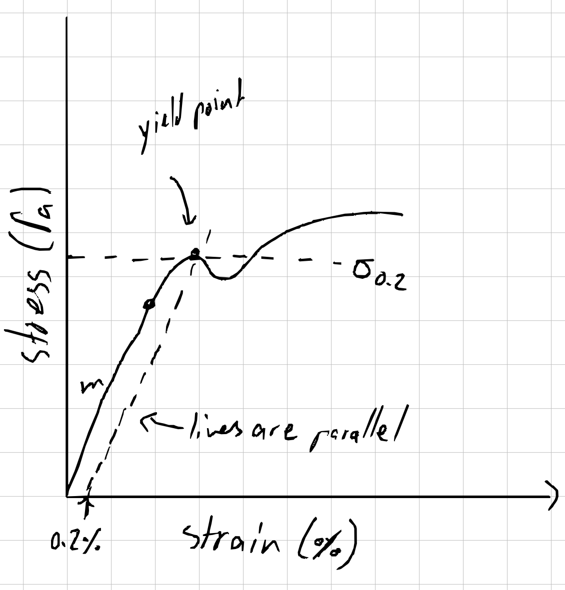
<u>Yield point:</u> Point before molecules in the objet rapidly rearrange to reduce internal stress → therefore the drop

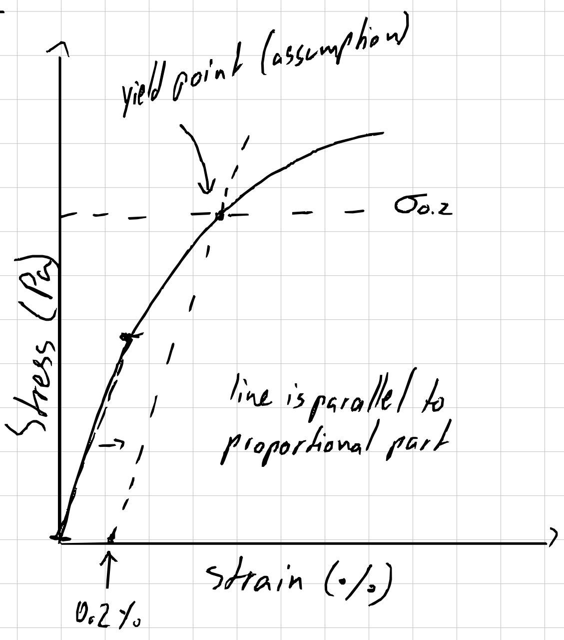
<u>Ultimate Tensile Stress (σ_U):</u> The ultimate tensile stress an object can withstand

<u>Fracture stress:</u> The value of stress where an object will break

! Often the limit of proportionality, elastic limit, and yield point are <u>the same</u>

0.2% Offset Rule





- The .0% offset rule is used to estimate a material's yield point. Some industries might use more strict estimations, e.g. using 0.1%.
- The value is denoted as '$\sigma_{0.2}$'

<u>Practice</u>

1. ~470 MPa

2. The wire might extend or retract due to a change in temperature, but when taking the difference to the control wire this effect is canceled. All extension due to heat will also occur in the control wire, therefore temperature change doesn't influence the difference.

1. D ✓ $1 Pa = 1 Nm^{-2}$

2. $F = k \Delta x$

$k = \frac{F}{\Delta x} = \frac{2}{0.8}$ ~~A~~ $k = \frac{F}{\Delta x} = \frac{0.2 \times 9.81}{0.008} \approx 250 Nm^{-1}$

C

3. A

4. ~~5.~~ D ✓

5. $\Delta x = \frac{F}{k}$ $\Delta E = kx^2$ C ✓

6. a) I would use a long piece of wire made out of the metal, which should be as pure as possible.

I would measure:
- initial length
- mean of the diameter from three positions
- The weight of my hook for the weights

✓ The setup would be as follows:
- Clamp down the wire at the short side of the table
- Run the wire over the table and over a pully
- put a paper mark on the wire and allign a fixed ruler with it
- Place the hook onto the wire

✓ The experiment is executed as follows
- ensure the flag alligns with zero on the ruler
- add a 100g mass and note down the weigh (mass × 9.81)
- measure the total displacement of the flag and note it down

Repeat

Use the data:
- Plot the mass on thy y-axis and the strain on the x-axis
- Determine the slope before the limit of proportionality
- calculate the Young Modulus as follows:

use venir scale and use a micrometer to determine the diameter

$$E = \frac{\sigma}{\varepsilon} = \frac{F l_i}{A \Delta l} = \frac{m}{\Delta l} \cdot \frac{g l_i}{A}$$

$$E (Pa) = \text{gradient} \cdot \frac{g l_i}{A}$$ ✓

where $A = \pi \left(\frac{D}{2}\right)^2$

b) I would put a drop-box below the weights ✓

(+ wear glasses for eye protection)

c) I would repeat the experiment twice, as inaccuracies will become less significant ✓

7. a) the spring obeys hook's law, as the length of the spring, or more specifically its change in length ✓ is proportional (in a linear relation) to the force applied.

↳ k, the gradient stays constant

b) $k = \frac{\Delta F}{\Delta x} = \frac{1.6}{0.1} = 16 Nm^{-1} \approx 20 Nm^{-1}$

c) i $0.41 - 0.09 = 0.32 m$

$\Delta x = 0.32 m$ $k = 20 Nm^{-1}$ $F = ?$

$F = k \Delta x$

$F = 20 \cdot 0.32 = 6.4 N$ they want you to use $16 Nm^{-1}$

$PE_{el} = \frac{1}{2} F \cdot \Delta x$

(✓)

ii $PE_{el} = \frac{1}{2} \cdot 6.4 \cdot 0.32 \approx 1N$

d) As the spring is compressed even more, it stores even more potential elastic energy. When the cann is opened, all that elastic energy is converted into ✓ kinetic energy, therefore the speed will be slightly higher.

8. a) $F_T = F_D$ ✓ $F_T \mathrel{\hat{=}}$ Thrust

$W = F_U$ ✓ $F_{Dx} \mathrel{\hat{=}}$ drag viscous drag

$W \mathrel{\hat{=}}$ Weight

$F_U \mathrel{\hat{=}}$ Upthrust

b) $W = F_U$

$W = V \cdot \rho \cdot g$

$W = 7100 \cdot 1030 \cdot 9.81 \approx 7.2 \times 10^7 \approx 7 \times 10^7 N$ ✓

c) i It means that the submarine slightly decreases in size as it gets compressed by the deeper water.

ii The weight should also be decreased, as ✓

$W = V \rho g$ has to stay true. → Pump out water

iii A much smaller Young Modulus means a lot less stiffness. High stiffness is required to hold up against the preassure from outside. → else yield point might be exceeded.

9. a)

$\Delta F = k \Delta x$

$k = \frac{\Delta F}{\Delta x}$ $k = \frac{7.9}{0.008} = 987.5 Nm^{-1} \approx 1000 Nm^{-1}$ ✓

b) $\Delta x = 0.063 - 0.016 = 0.047 m$ $k = 1000 Nm^{-1}$

$EP_{el} = \frac{1}{2} k x^2$

$EP_{el} = \frac{1}{2} \cdot 1000 \cdot 0.047^2 \approx 1.1 J \approx 1 J$

c) i $EP_{el} = KE$ $KE = \frac{1}{2} m v^2$

$1 = \frac{1}{2} \cdot 0.0094 \cdot v^2$

$1 = 0.0047 v^2$ forgot vertical movement

$v^2 = 212.77$

$v = 14.6 ms^{-1} \approx 15 ms^{-1}$

ii There is no air resistance.
The spring doesn't 'produce' thermal energy ✓

d) i $\Delta x = v \cdot t$

$3 = 15 \cdot t$

$t = 0.2 s$

$V_{fy} = \cancel{V_{iy}}^{0} + at$

$V_{fy} = 9.81 \cdot 0.2 = 1.962 ms^{-1}$

$c = \sqrt{a^2 + b^2} = \sqrt{15^2 + 1.962^2} \approx 15.1 ms^{-1}$ forgot angle

ii $\Delta y = \cancel{ut}^{0} + \frac{1}{2} a t^2$

$\Delta y = \frac{1}{2} \cdot 9.81 \cdot 0.2^2 \approx 0.196 m$

$0.196 + 0.03 = 0.226 m$

$0.196 - 0.03 = 0.166 m$

$0.226 m > 0.2 m > 0.166 m$ ✓

e) A solid disk would fly at a slower speed with the same kinetic energy provided due to $EK = \frac{1}{2} m v^2$. Therefore the lighter disk flies faster. ✓

4.1 Physics Math Skills

Significant Figures

1. Leading zeros NEVER count
2. Trailing zeros only count if there is a decimal point somewhere in the number
3. All numbers that arent leading or trailing zeros ALWAYS count

e.g.

0032 : 2sf	140.0 : 4sf
320 : 2sf	012030. : 5sf
320. : 3sf	010.01 : 4sf

4.2 Plan & Conduct Experiments

1. Formute an Experimental Question:
   - What question will this experiment answer?
   - Be clear and specific
   - You should probably state dependent and independent variables

2. Define Experimental Variables
   - be clear about the type of variable you are using
   - include <u>all</u> variables

   Independent Variables:
   - the variables I determine, e.g. the load on a spring
   - those variables are varied in steps to uncover their influence on other variables

   Dependent Variables:
   - the variables I measure
   - dependent variables depend on the values of the independent variables

   Control Variables:
   - the variables which I keep constant
   - such variables include any factors that might interfer or add unwanted complexity to our experiment, e.g. gravity
   - This may include variables such as temperature or air moisture which I should record

3. Create a Data Table (can be done after step 4 also)
   - for each measurement and value calculated a column
   - for each variation in an independent variable a row
   - Labels should include: name, symbol used and units

Example:

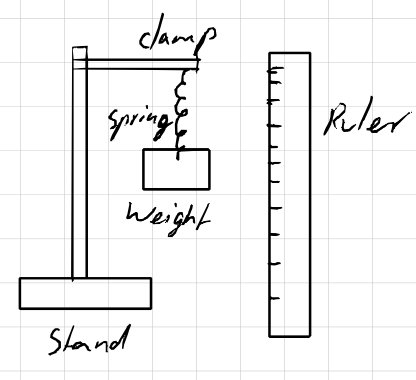
Weight force F (N)	Length l (mm)	Extension x (cm)
0.00	50	0
0.98		
2.13		
2.98		

! always also measure the independent variable

The weights might not be perfect (1N)

4. Planning and Recording Aperatus Setup
   - Create a simple labled sketch
   - Use the correct terminology for describing the setup

Example:



5. Safety Hazard Assessment
   - Following objects and conditions are considered hazardeous:
     - Voltages > 12V
     - Large electric currents
     - Temperatures > 40°C
     - brittle/sharp materials
     - Flammable and reactive substances
     - Toxic substances
     - High preassure or large amounts of energy
   - Create a table as follows to determine mitigations

Hazard	Mitigation
- Heavy mass falling on people	- Place the apperatus on a stable workbench, clamp it to the bench
- flying fragments (e.g. when spring breaks)	- Wear safety glasses

6. Conducting the Experiment
   - follow a constant procedure to keep results constant
     ↳ both for setup and changind the independent variable
   - Regularly measure control variables ! e.g. temperature
   - Create multiple data tables for each run (if you do multiple)
   - also note faulty/unexpected measurements
     ↳ explain what happened in the foot notes
     ↳ don't introduce a bias

7. Identify Soures of Error
   - Determine what might have caused errors
   - How can thoes errors be reduced?

8. Analyze and Interpret the Data
   - Draw graphs and find correlations
   - Answer your research/experimental question
   - differentiate between causation and correlation

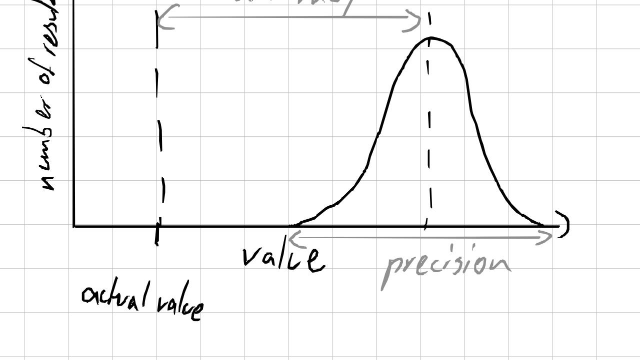
Note: Calculations
   - Note down formulae that you may need
   - Rearrange the formulae if needed

Scientific Method Breakdown

1. Make observation
2. Create a model or hypothesis to predict future observations
3. Test if the model holds true
4. Observe the result
5. Revise the model or hypothesis
6. Repeat if needed

A scientific theory:
- makes predictions
- can be tested
- can be falsified

Accuracy and Precision



Accuracy: How close are my results to the correct value?

Precision: How often will I get the same result when measuring? what is the spread (random error)?

4.3 Uncertainty & Errors

Errors in Experimentation

- Error = difference between recorded value and the actually underlying value
- uncertainty = the range of possible errors around a measured value (e.g. ±5)
- Human Error = when the human makes a mistake

Random and Systematic Error

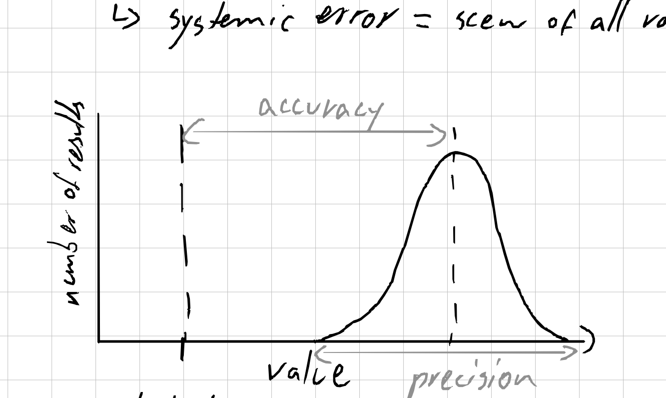
Random Errors
- show no pattern
  ↳ values scattered randomly around true value
- Random errors cancel (are reduced) when repeating measurements
  ↳ drawing a best-fit line works too
- For A-levels repeat an experiment 3-5 times
- Can be reduced by using instruments with better precision and higher resolution

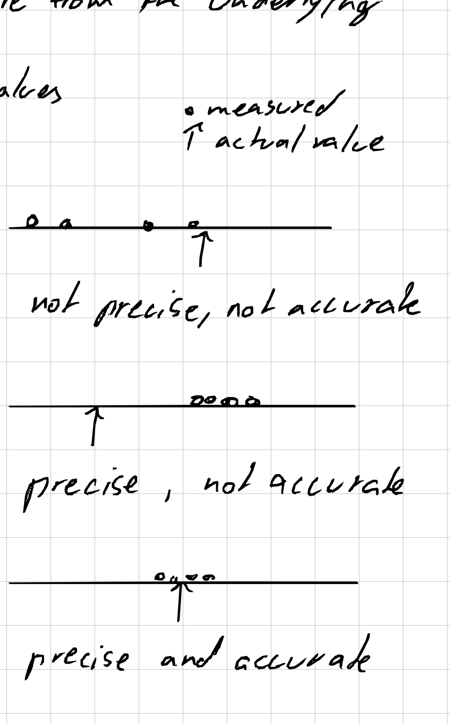
Systematic Error
- values are consistently skewed
- no pattern can be recognized
  ↳ all measurements will be too high or too low
- Offset shows up as "zero error"
  ↳ values are all off by a fixed value
  ↳ where the value should be zero it's not
  Fix: subtract/add the offset to each value
- Values can also be off by a fraction (e.g. × 1.06%)
  ↳ just multiply each value by a correcting factor
  ↳ ! This is only possible if the instrument can be tested against actually known values

Precision & Accuracy

Precision:
- for one measurement: how many significant digits recorded
  ↳ resolution = smallest difference in the scale
- for multiple measurements: how scattered measurements are
  ↳ how large is the random error

Accuracy:
- how far the (mean) measurements are from the underlying (true) value
  ↳ systemic error = skew of all values





Timing Errors

Human reaction time:
- Measurement delays are inconsistent
  ↳ start delay will be different from stop delay

Fix: Record multiple cycles (e.g. pendulum) and devide time by cycle

Automatic Systems:
- delay is small
- start-stop delay is more consistent

Parallax Errors

- caused by a change in observing angle
  ↳ when an object moves e.g. when on a spring

Fix: e.g. ruler closer to the object

Meniscus Errors

- caused by surface tension pulling of liquid on walls
  ↳ creates a meniscus on all sides

Fix: measure slightly below the meniscus

A-Level Instruments & Resolutions

Instrument	Resolution (accuracy)
Metre rule	1mm
Standard Masses	1g
Ammeters & Voltmeters	Significant figures displayed
Thermometers	1°C
Micrometer	0.01mm
Vernir callipers	0.1mm
Measuring Cylinders and beakers	smallest measuring devision (depends)

Expressing Uncertainties

For test results: Always indicate the uncertainty of results

Rule: Uncertainty $= \pm \frac{\text{least significant figure}}{2}$

Example 340mm : 2 SF
↑
decimal place so least significant figure = 10mm
↳ 10 : 2 = 5mm
↳ 340mm ± 5mm

Rule #2: • Never quote a number to more significant figures then known

Calculate with Uncertainties

Multiplication and Division:
- combined uncertainty = sum of percentage uncertainties
- The final uncertainty is given with the same number of significant figures as the initial uncertainty with least s.f.

Example: 3.75 ± 0.005 • 5.6 ± 0.05

percentage uncertainties: $\frac{0.005}{3.75} \approx 0.13 \approx 0.1\%$ (1sf)

$\frac{0.05}{5.6} \approx 0.89\% \approx 0.9\%$ (1sf)

combined uncertainty $= 0.1\% + 0.9\% = 1.0\% \approx 1\%$ (1sf)

because initial uncertainties had 1sf

Addition & Subtraction:

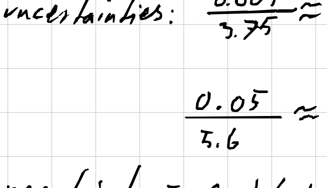
Rule: Take the worst uncertainty

Example: $1m \pm 0.03m + 5.4m \pm 0.5m = 6m \pm 0.5m$

Estimated Figure

- You are allowed to estimate one magnitude of 10 lower than the resolution of an instrument

Example:



$l \approx 3.6cm$ where .6 is the estimated figure

- Estimated figures do count as significant figures

Systematic Uncertainty for Instruments

- Usually instruments have a systemic error of 1× their smallest unit of measurement for their entire length

Example: Meter Stick with milimeter marks:
systemic error = ± 1mm for 1m measurements

! This is added to the random uncertainty

Calculate Error for Repeated Measurements

Rule: Error $= \frac{\text{max\_measured} - \text{min\_measured}}{2}$

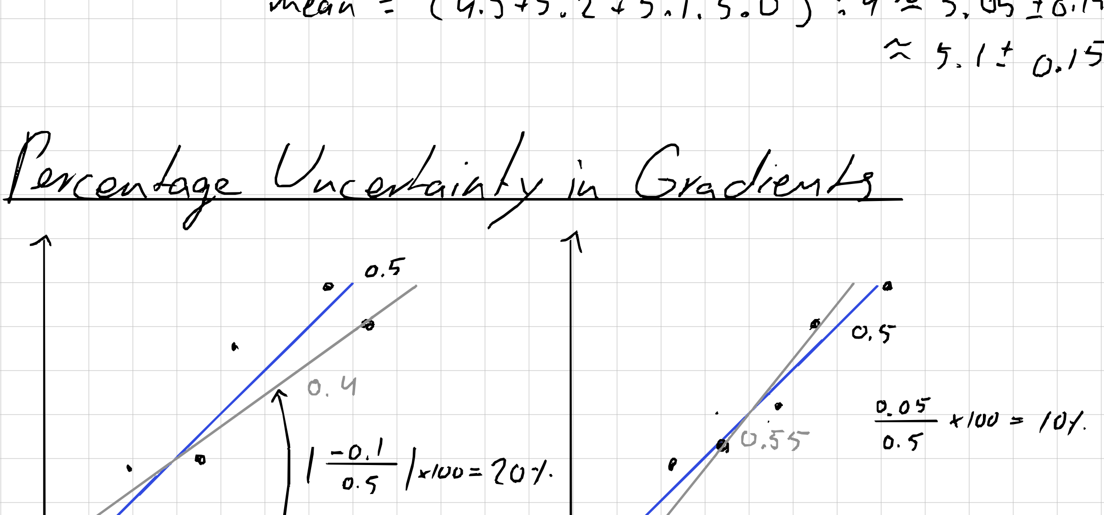
Example: Measurements:

4.9, 5.2, 5.1, 5.0

error $= \frac{5.2 - 4.9}{2} = 0.15$

mean $= (4.9 + 5.2 + 5.1 + 5.0) : 4 \approx 5.05 \pm 0.15$
$\approx 5.1 \pm 0.15$

Percentage Uncertainty in Gradients



% uncertainty in gradient $= \left|\frac{m_{best} - m_{worst}}{m_{best}}\right| \times 100$

Human vs. Experimental Error

Human error: • errors in measurements due to human mistakes
↳ e.g. reading a value wrong
↳ using the wrong unit

Experimental error: • either systematic or random error
↳ caused by measurement instruments and setup of the apparatus

Repeatable vs. Reliable

Repeatable: • An experiment is repeatable if it gives similar values when repeated under the same conditions
↳ related to random error

Reliable: • An experiment is reliable when it consistently produces accurate results (values that are close to the true value) and has low scatter
↳ related to systemic error and random error.

Correction on Uncertainty

Assume all measurements have an uncertainty equal to the smallest SF of the instrument

e.g. ruler: 10.3 ± 0.1cm ← 1mm

Double Readings

=> When reading from an instrument twice, the uncertainty will be twice the smallest unit of the instrument

E.g. when taking two readings from the ruler, uncertainty = ± 2mm

Formatting Uncertainties

value ± uncertainty unit
3.5 ± 0.2 cm

Addition & Subtraction

=> Add the absolute uncertainties

$8.5 \pm 0.2cm - 5.1 \pm 0.1cm = 3.4 \pm 0.3cm$

Multiplication and Division

=> Add percentage uncertainties

$100 \pm 1cm \times 5 \pm 0.5cm$

$= 100cm\ (1\%) \times 5cm\ (10\%) = 500\ (15\%)$

Exponents

=> Multiply percentage uncertainty by exponent

$\pi(3 \pm 0.1cm)^2 = \pi(3cm\ (3\%))^2 = \pi\ 3cm\ (6\%)$

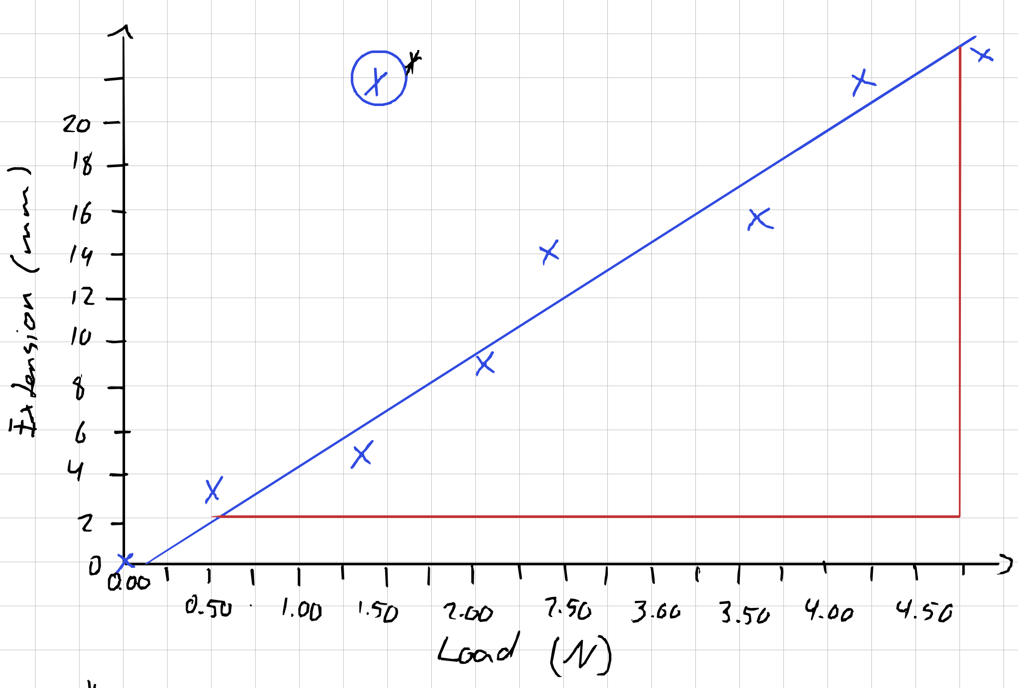
4.4 Plotting Data

How to Plot Graphs

1. determine the range of variables displayed and the grid's resolution
2. Put independent variable on the x and dependent variable on the y
   ↳ Exceptions:
   - Hooke's Law $x: \Delta l$ y: Force
   - Ohm's Law x: Voltage y: current
3. The graph should cover at least half the given space on the paper
4. Chose grid steps that are either in units of 1, 2, 5 or 10^n
5. Label the x and y axis with a name and unit
6. Plot points precisely using 'x' or '+'
7. Plot anomalous points!
   ↳ circle them and add a note, stating: 'this point is anomalous because...'
   ↳ Doesn't have to be included in best line of fit or calculations
8. Draw a line connecting the dots, exceptions:
   - line of best fit instead if relation should be linear ↗ ! doesn't have to pass origin
   - connect with curves if dots approximate a curve
9. Add a graph title: $\boxed{\text{y-variable vs x-variable}}$
10. You're allowed to interpolate.
    ↳ Don't use the line of best fit far from it's range of data
11. Always show the gradient triangle (when calculating gradients)
    ↳ triangle should be larger than 8cm

Example

Extension vs Load
for a plastic spring



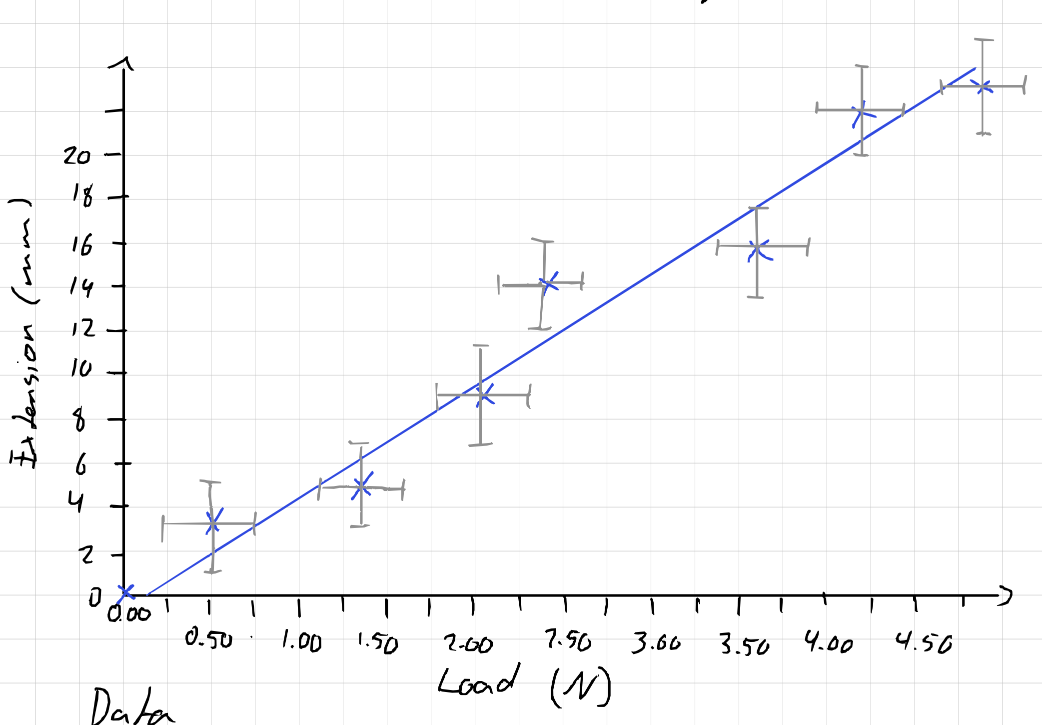
* anomalous measurement, researcher bumped into the weight

Error Bars

⇒ used to indicate a possible error for data points

Example

Extension vs Load
for a plastic spring



Data

Extension (mm)	Load (N)
0 ± 2mm	0 ± 0.25N
⋮	⋮

Line of best fit should pass each data point inside the area:

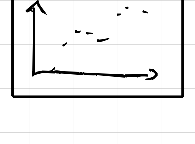
Line of Best Fit

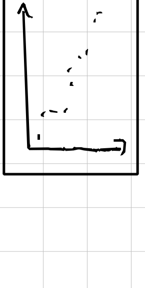
- line that best approximates a linear relationship
- doesn't have to pass origin
  ↳ should instead best match data
- somewhat equal amounts of points on both sides
- draw a confident line in one go

Checklists for Experimentation

After executing experiment: record...
- all steps for the experiment execution
- define and categorize dependent, independent, and control variables
- major dificulties
- Ideas for improvement

Plotting Graphs:
- does the plotted data take up more than half of the sheet?
- does the plot have a title?
- are both axies labled with
  - variable name
  - unit





- are my points either 'x' or '+'?
- are my data points accurate?
- are all lines straight and confident?
- Is my gradient triangle existent and larger then 8cm?
  ↳ did I show gradient calculations?

No:

#1 : 1.85 mm ✓

#2 : 4.40mm ✓

#3 : 6.65 mm ✓

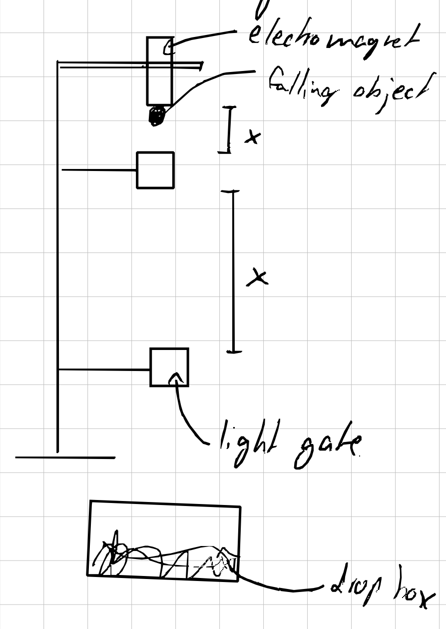
#4 : 8.05 mm ✓

#5 : 0.20 mm ✓

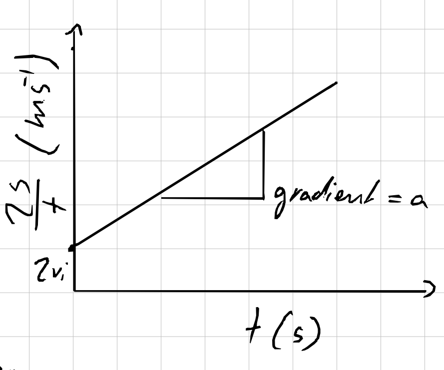
4.6 Practical Notes

Note:
- techniques used
- which technique will produce the most reliable result
- potentials for error and uncertainty

Determine 'g'



$$s = v_i t + \frac{1}{2}at^2$$
$$2s = 2v_i t + at^2$$
$$\frac{2s}{t} = 2v_i + at$$
$$y = c + mt$$

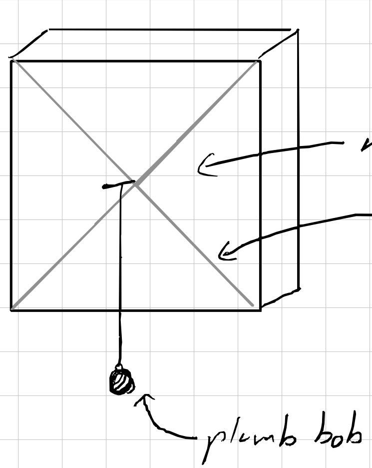


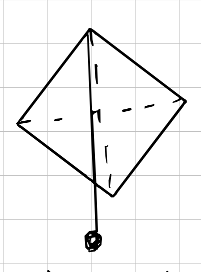
Reason for error:
- wobbeling
- air resistance
- light gates not perfectly vertical

Safety improvements:
- use a cushioned drop box to catch the ball
- clamp down the aparatus

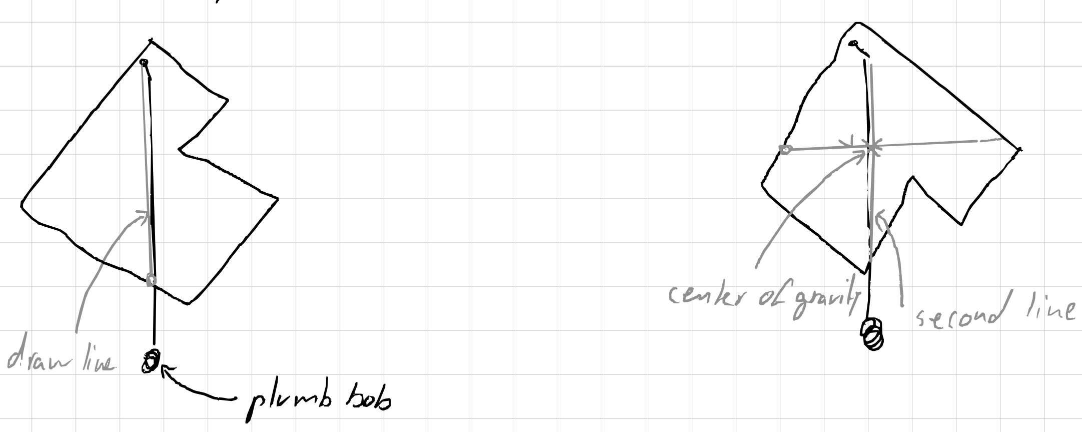
Center of Mass

A block of uniform density and shaped like a perfect rectangle with the same thickness throughout has its center of mass at its geometric center.





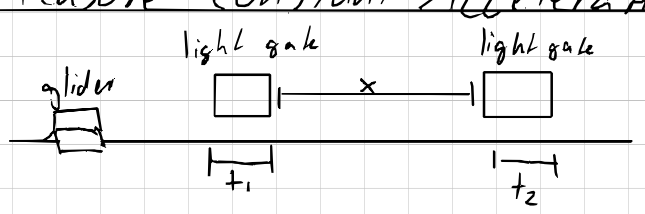
when the string passes the line then the block tips



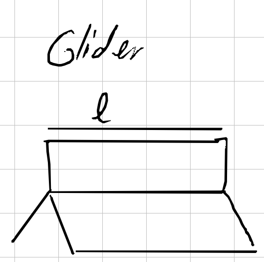
Room for Error:
- friction between plumb bob and block
- no perfect constant thickness
- displacement of material by nail

Safety precactions:
- drop box below the table
- protective glasses when hammering nail

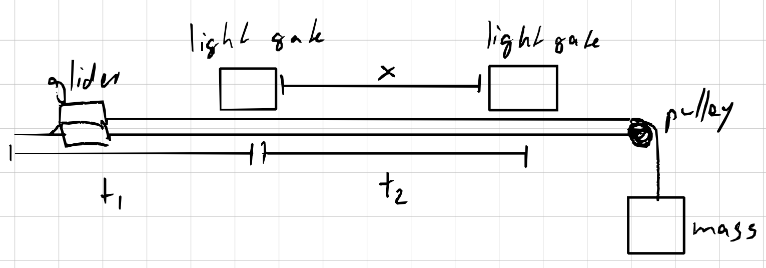
Measure Constant Acceleration



Glider



$t_1 = t_2$ (adjust x)



$t_1 > t_2$

Here light gates measure how long the beam didn't return.

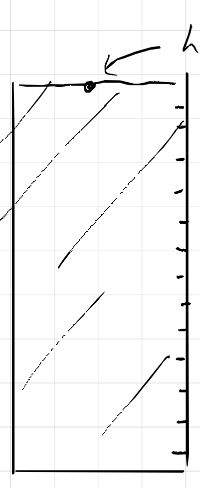
$$v_i = \frac{l}{t_1} \qquad v_f = \frac{l}{t_2}$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$a = \frac{v_f^2 - v_i^2}{2x} \approx g$$

Viscosity of Liquid

use terminal velocity

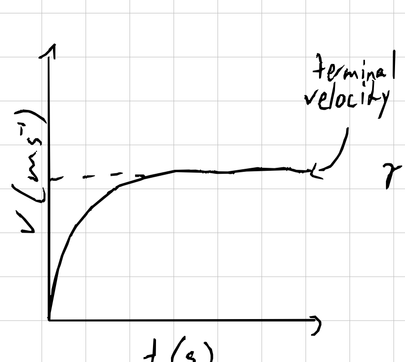


measure in regular time intervals

$$\eta = \frac{mg - \frac{4}{3}\pi r^3 g\rho}{6\pi r v}$$

$$\rho = \frac{\text{liquid mass}}{\text{liquid volume}}$$

measure but subtract beaker weight from mass

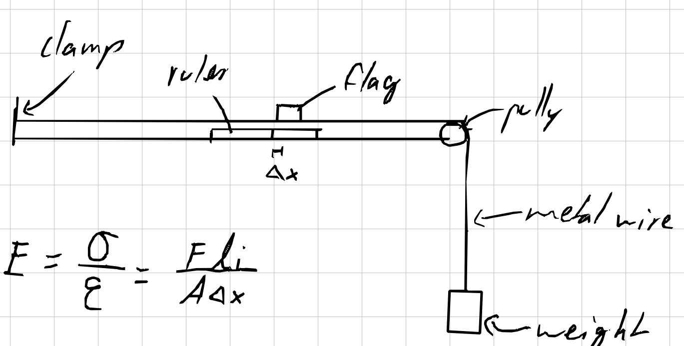


r = radius, measure 3x

calculate v for each step with

$$v = \frac{\Delta s}{\Delta t}$$

Young's Modulus



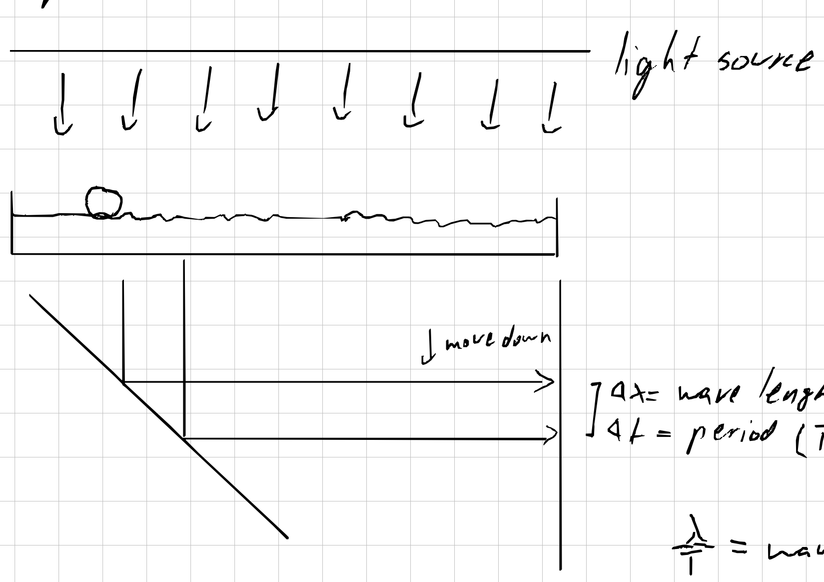
$$E = \frac{\sigma}{\varepsilon} = \frac{F l_i}{A\Delta x}$$

measure diameter 3x

$$A = \pi\left(\frac{d}{2}\right)^2$$

measure total length of wire (l_i)

Properties of Waves (Ripple Tank)



Δx = wave length (λ)
Δt = period (T)

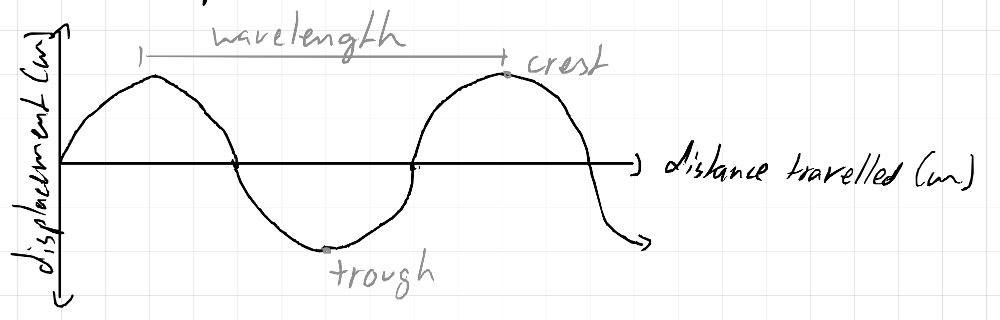
$$\frac{\lambda}{T} = \text{wave speed}$$

5.1 Wave Basics

Waves transfer energy over oscilation
mechanical waves: require a medium
electromagnetic waves: don't require a medium
↳ slow down in a medium

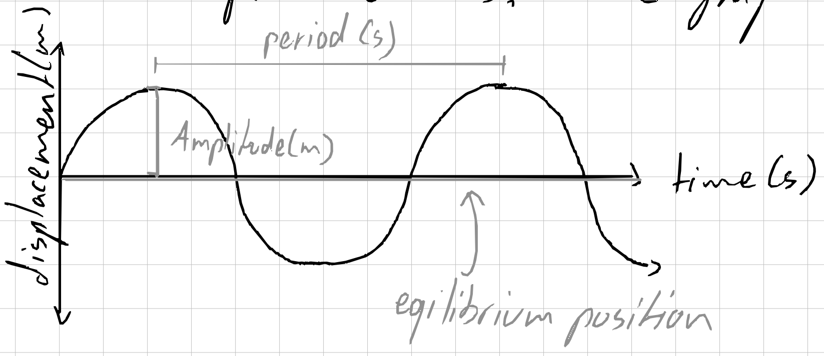
Showing physical representation:

displacement vs. distance travelled



- describes the displacement from the equilibrium position at any point of the wave
- is an accurate representation of the pysical wave
- doesn't conclude the speed of the wave

displacement vs. time graph



Find Wave Speed

$v = f\lambda$ 	v = speed
f = frequency = $\frac{1}{T}$
λ = wavelength
T = period

or $v = st$

Phase

phase describes where a point is relative to a complete cycle.

- 2π is a complete cycle
- Phase is an offset so there is no absolute starting point
- Measured as an angle radiants (sometimes degrees)

Transverse waves

- displacement is perpendicular to direction of movement
  ↳ e.g. light, water waves

Longitudinal Waves

- Displacement is parallel to direction of movement
  ↳ e.g. sound

---

1. 0.2m, 80m, 5.5m

2. $v = st$

$s = \frac{v}{t} = \frac{330}{3.75} = 88m$

3. $v = f\lambda$ 	$v = st$

$st = f\lambda$

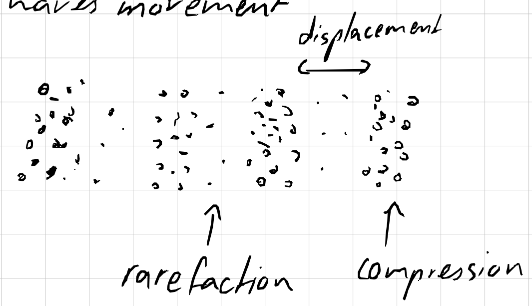
distance × times = cycles per time × distance per cycle

$st = c \cdot t \cancel{\cdot s \cdot c}$

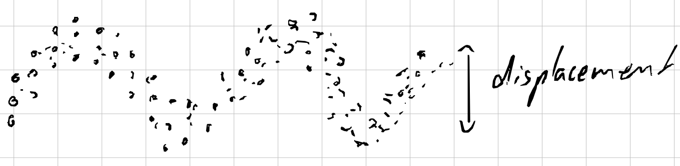
$st = \cancel{c(st)}$

5.2 Types of Waves

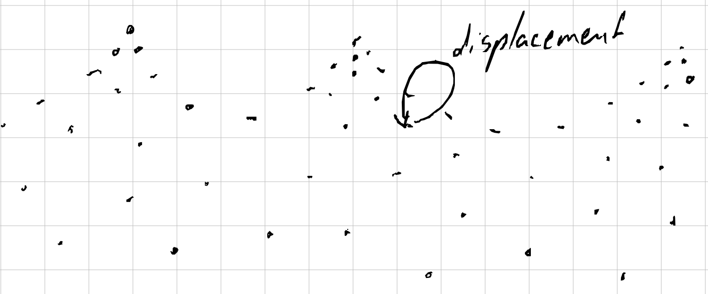
Longitudinal Waves
- Displacement is parallel to wave's movement



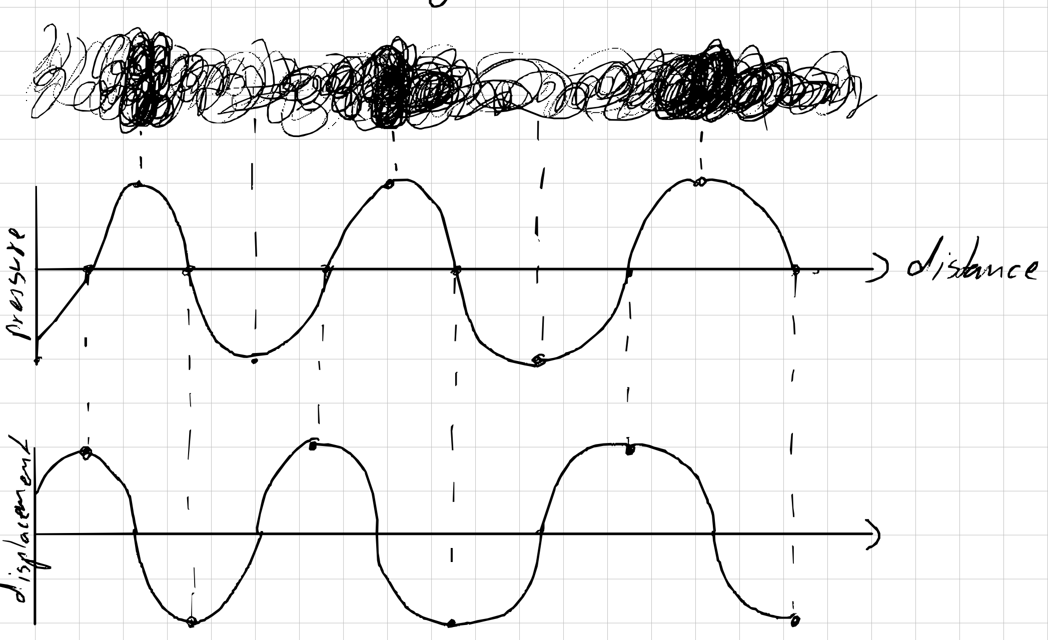
Transverse Waves
- Displacement is perpendicular to wave's movement.



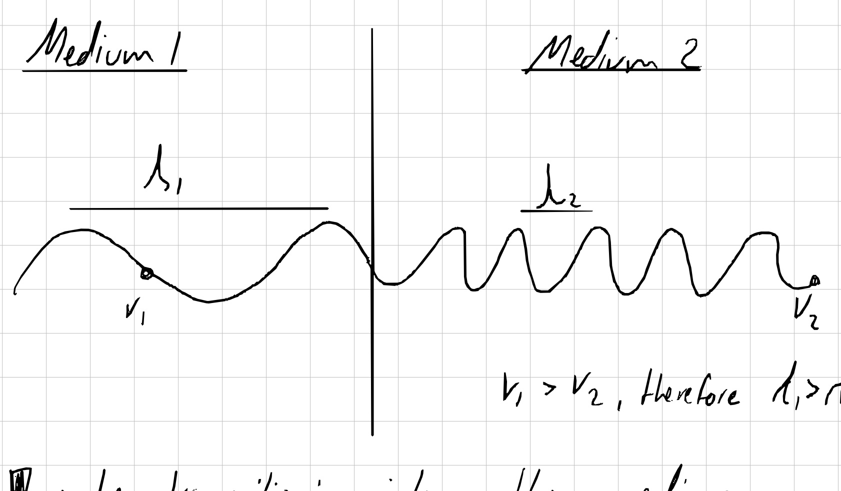
Surface Waves
- Involve a combination of transverse and longitudinal waves
- particles spiral
  ↳ e.g. water (surface) waves



Graphing Longitudinal Waves



Transitioning between Media



$v_1 > v_2$, therefore $\lambda_1 > \lambda_2$, but $f_1 = f_2$

- when transitioning into another medium,
- the wave speed and wavelength change, the frequency stays the same!

---

1. $A = 0.5\,cm$ $\lambda = 4\,cm$

2. a) Because it causes displacement in the medium that is perpendicular to direction of the wave's movement

   b) Seismic waves are longitudinal waves, meaning that the displacement occurs in the direction of motion.

3. The displacement is higher, causing more movement inside our ear. Higher displacement means larger differences in preassure, as shown in fig. 6.

Exam Practice

1. B ✓

2. $v = f\lambda$

$f = \frac{v}{\lambda} = \frac{2.8}{4.4} \approx 0.64$

D ✓

3. A ✓

4. $1.500 \cdot 0.87 \cdot \frac{1}{2} = 652.5$ A ✓

5. a) Some energy got transformed into e.g. heat ✓

b) $50\mu s \cdot 5900 \cdot \frac{1}{2}$

$50 \times 10^{-6} \cdot 5900 \cdot \frac{1}{2} = 0.1475m \approx 0.15m$ ✓

6. a) Sound waves travel through the air as longitudinal ✓ pressure waves. Atoms are bumping into eachother and forming areas of compression and rarefaction. ✓

b) The frequency refers to completed wave cycles per second. ✓

c) $v = f\lambda = 2 \cdot 1500 = 3.0 \times 10^3 ms^{-1}$ ✓

d) The primary wave moves faster than the more powerful secondary wave, which probably caused the wave. Also waves travel faster through solids, so the wave might have been noticable through the ground. ✓

7. $343 ms^{-1}$ $f = 55.8$ MH use speed of light $(3 \times 10^8$

$\lambda = \frac{v}{f} = \frac{\cancel{343}}{55800 \times 10^3} = \cancel{3.148 \times 10^{-3}}$

8. a) i. Frequencies are higher than what the human ear can detect. ✓

ii. $s = vt = 1500 \cdot 1.6 \times 10^{-4} = 0.24m$ $\frac{0.24}{2} = 0.12$

iii Because one measures only the start of a pulse, sending more would waste energy and wouldn't allow for a second measurement. ✓ Pulse must return before next is sent

b) i. They can cause cancer by altering one's DNA ✓

ii. X-rays are:
- an electromagnetic wave ✓
- significantly faster (speed of light in medium ✓)

Ultrasound is:
- a mechanical wave ✓
- rather slow (speed of sound in medium)

9. a) The reflected waves will have flipped amplitudes that are also a bit smaller, as some energy was dissipated into the plate ✓

b) i. ~~The phases of both waves must be perfectly overlapping at the microphone~~ There is only one microphone

ii. Put the plate directly to the microphone and slowly put it back until both waves perfectly overlap. The distance between the plate and the microphone is ~~wavelength · 1.5~~ add second mic lol

iii Previous question to vague, I now understand...

Remove the metal plate to prevent any reflections, ✓ add a second microphone and connect it to one input on the oscilloscope. ✓

Put both microphones together directly ✓ and move one back slowly until both waves overlap perfectly again. ✓

Distance between microphones $= \lambda = (1m)$ example

Speed of sound $= \cancel{343 ms^{-1}}$

$f = \frac{v}{\lambda} = \frac{343}{1m} = 343 Hz$

λ = measured value f = known value

$v = f \cdot \lambda = \ldots$

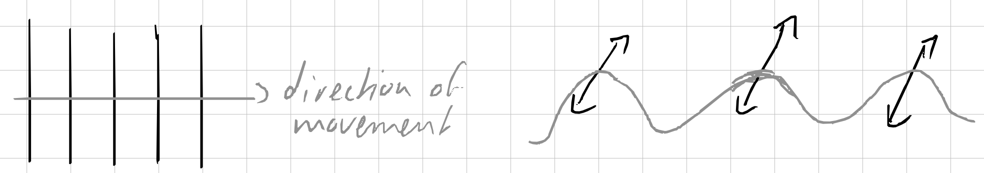
5.3 Wave Phase & Superposition

Wavefronts

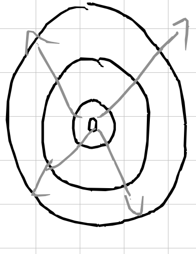
=> a way of visualizing wave patterns

A line is drawn through all points that currently have a chosen phase (not amplitude!)

Phase drawing:



or

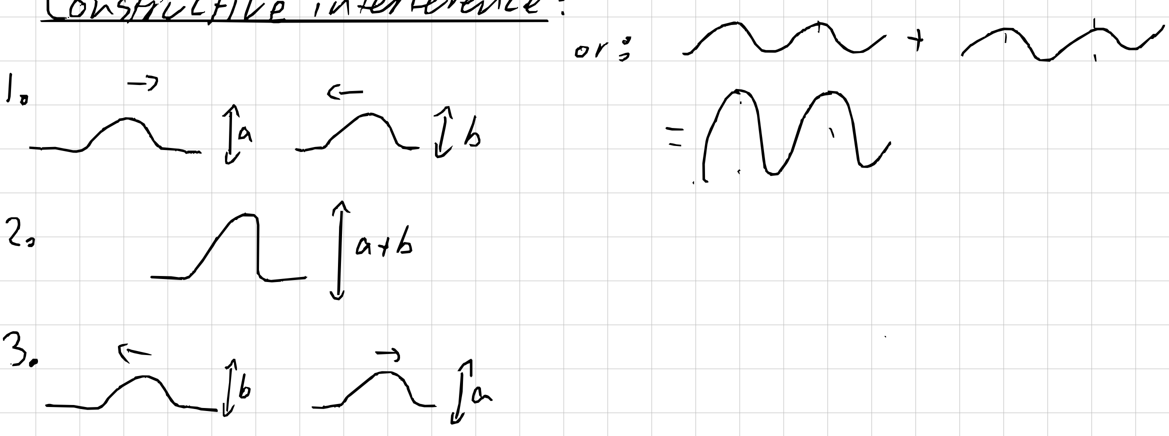


Wave Superposition

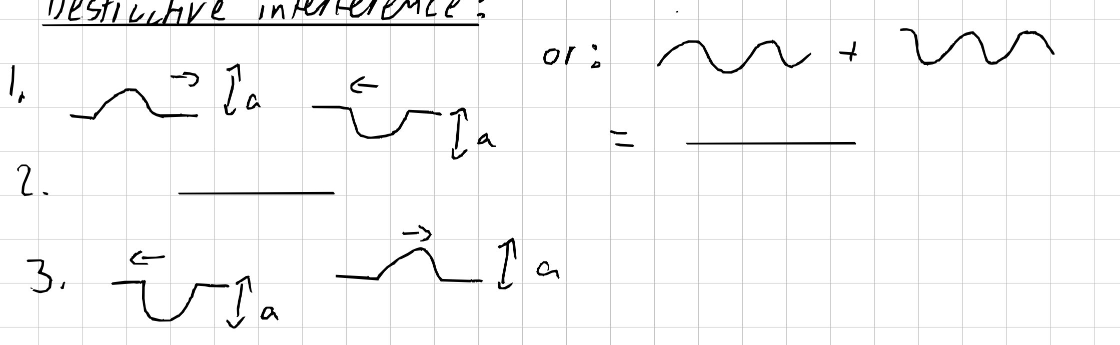
=> The resulting wave pattern when multiple waves collide
  → The sum of all wave's displacement for every point on the pattern

Interference

Constructive interference:



Destructive interference:



Note: There is partial and complete destructive/constructive interference

---

Questions

1. a ray only gives the direction in which the wave travels while a wavefront shows all points that currently have the same phase. → It shows the wave pattern. ✓ both are always at right angles

2. π or $\pi + 2\pi n$ ✓

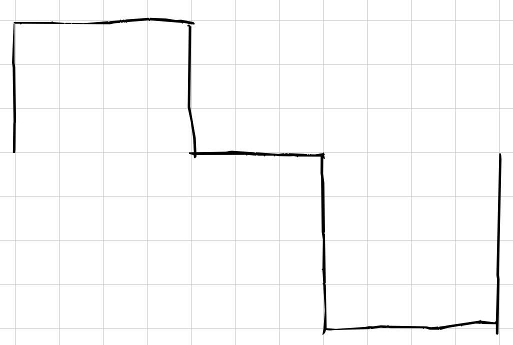
3. a) 0 or 2π ✓

   b) π ✓

   c) $\pi + \pi(2) \cdot 2 = 5\pi$ ✓

4.

1s



✓

2s

✓

3s

✓

5.4 Stationary Waves

=> also known as standing wave

Requirements:
- two waves traveling in opposite direction
- same speed
- same frequency

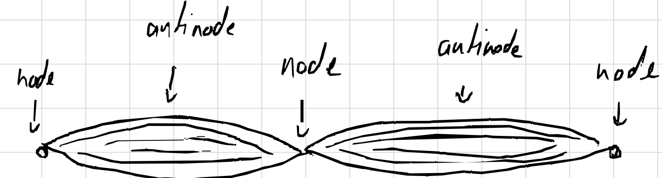
=> then they're coherent

note: actually the phase between both waves does change, but if one imagines one wave flipped, then the phase shift is constant

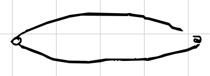
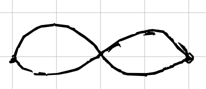
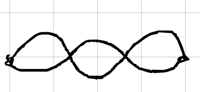
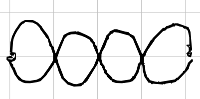
Energy
- Standing waves don't transmit energy
- Progressive waves do transmit energy
  ↳ waves that 'move' in a direction

Nodes
=> Points on the wave that always perfectly cancel
↳ amplitude is always zero

Antinodes
=> Points (lines) that experience the most variation in amplitude



Modes
=> shape of the standing wave (number of nodes)

shape	mode	wavelength (λ)
	fundamental / 1st harmonic	$= 2L$
	1st overtone / 2nd harmonic	$= L$
	2nd overtone / 3rd harmonic	$= \frac{2L}{3}$
	3rd overtone / 4th harmonic	$= \frac{L}{2}$

$\lambda = \frac{2L}{n}$

$n \hat{=}$ # harmonic

String Wave Speed

$$\boxed{v = \sqrt{\frac{T}{\mu}}}$$

$v \hat{=}$ travel speed of wave (ms^{-1})
$T \hat{=}$ tension in string (N)
$\mu \hat{=}$ mass of string per meter (kgm^{-1})

find frequency for standing wave:

$$\boxed{f = \frac{1}{\lambda} \cdot \sqrt{\frac{T}{\mu}}}$$ where $\lambda = 2L, L, \frac{2}{3}L, \ldots$

Note: fundamental frequency $\hat{=}$ $\boxed{f_0}$

---

1. $75cm = l$ 2nd harmonic

$v = 420ms^{-1}$ $\lambda = 1 \times l = 0.75m$ ✓

$v = \lambda \cdot f$
$420 = 0.75 \cdot f$
$f = 560Hz$ ✓

2. a) $T = 50N$ $\mu = 4.8 \times 10^{-4} kgm^{-1}$

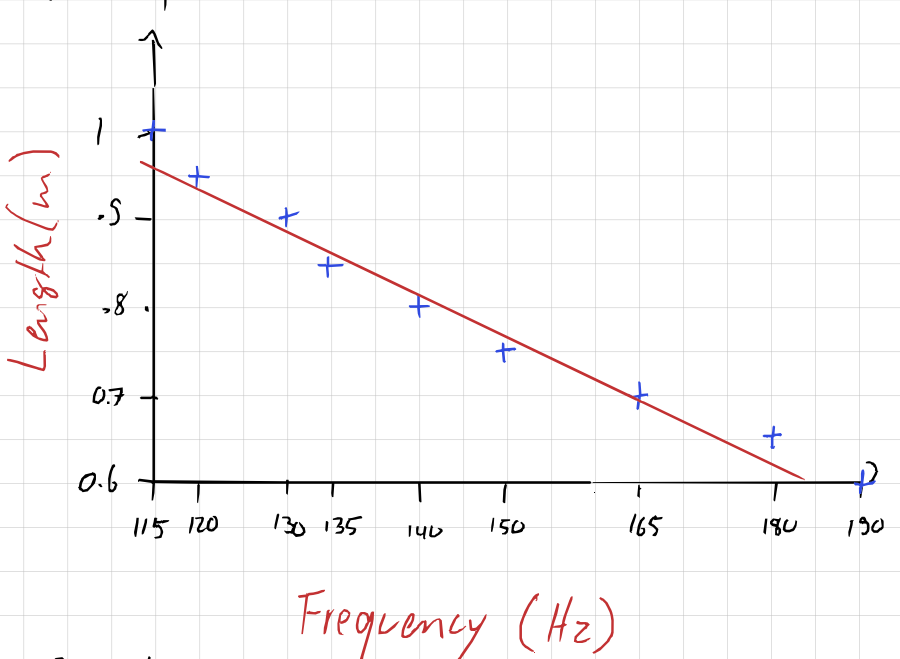
$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{50}{4.8 \times 10^{-4}}} = 400ms^{-1}$ ✓

b) $l = 0.74m$

$f = \frac{v}{\lambda}$

$f_0 = \frac{1}{2 \cdot 0.74} \cdot 433 = 293 \approx 300Hz$ ✓

3.



f_0 $(0.7, 165)$ closest point to the line
$l = 0.7m$ $f = 165Hz$

$f_0 = \frac{1}{2 \cdot l} \sqrt{\frac{T}{\mu}}$

$2 f_0 l = \sqrt{\frac{T}{\mu}}$

$(2 f_0 l)^2 = \frac{T}{\mu}$

$\mu = \frac{T}{4f^2 l^2} = \frac{23.5}{4 \cdot 165^2 \cdot 0.7^2} = 4.4 \times 10^{-4} kgm^{-1}$ ✓

4.

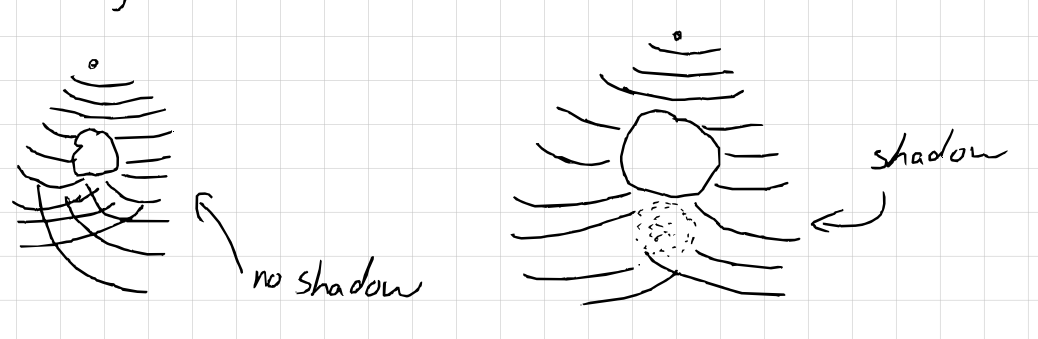
$\mu = \frac{50}{4 \cdot 3000^2 \cdot 2^2} = 3.47^{-5} kgm^{-1}$

5.5 Diffraction

=> Spreading of wave energy through a gap or around an obstacle

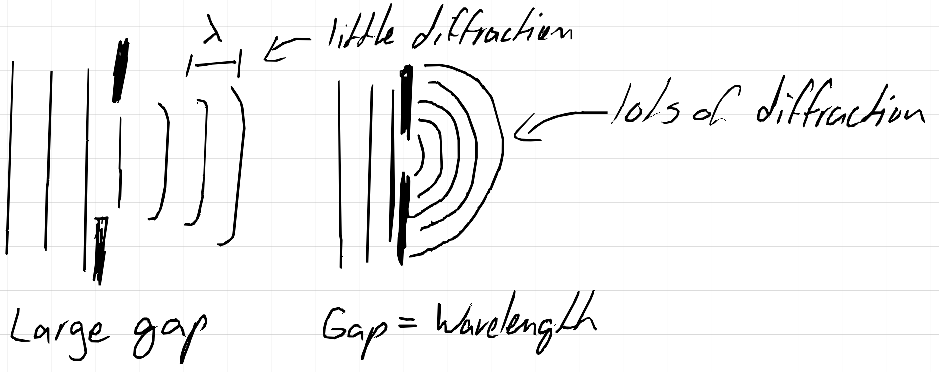
Diffraction Shadows

- depending on the difference of the wavelength and obstacle size the wave might, or might not fully surround the object



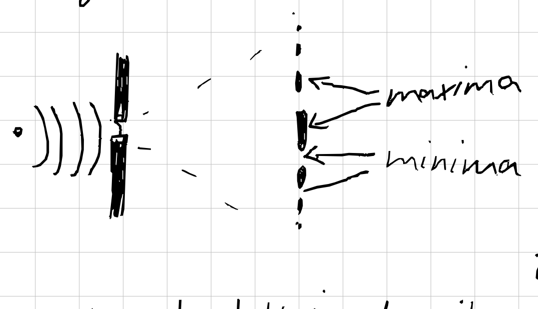
Diffraction Intensity

- Diffraction happens most when the gap size equals the wavelength

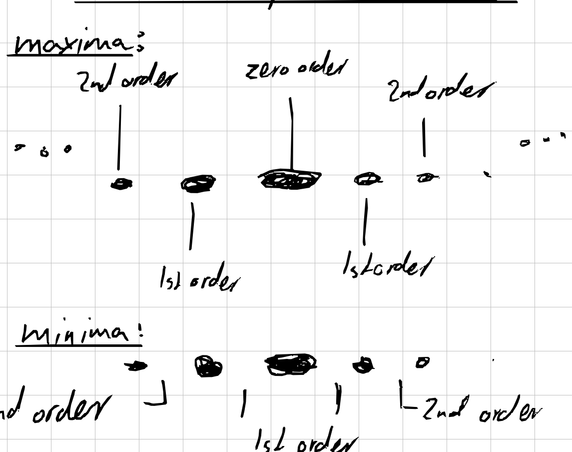


Interference Pattern

Single - Slit Interference



Interference pattern orders

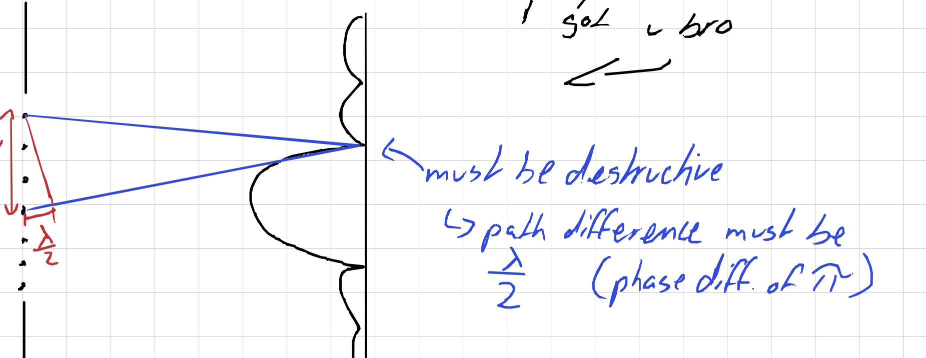


Monochromatic light is used, as it contains only one frequency (color) of electromagnetic waves

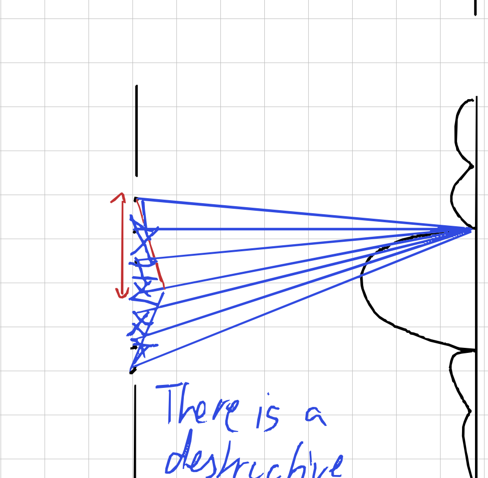
- Huygen's Priciple

=> Each wave front consists of an infinite amount of point wave sources

- usually those sources interfere and add up back to the original wave
- when removing the outer sources nothing will cancel the inner sources
  ↳ the wave diffracts, spreads out

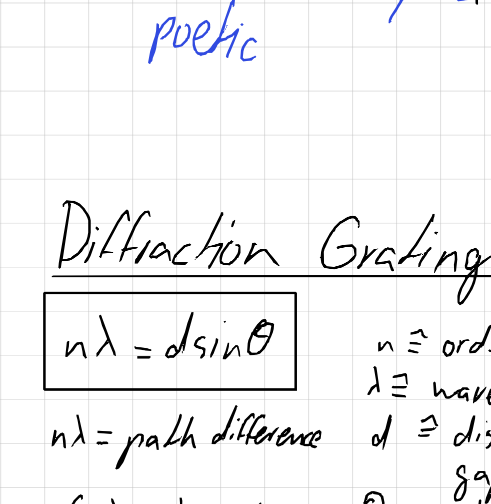


* this only shows two points but actually there are infinite... got u bro



must be destructive
↳ path difference must be $\frac{\lambda}{2}$ (phase diff. of π)

Ooh, but why do all points interfere destructively at the minima?



There is a destructive partner for everyone
poetic

calculate maxima positions

two slits: $d\sin\theta = n\lambda$
grating: $d\sin\theta = n\lambda$

single slit, minima only: $d\sin\theta = m\lambda$

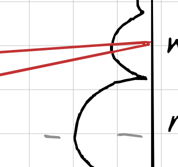
Diffraction Grating Formula

$\boxed{n\lambda = d\sin\theta}$

$n\lambda$ = path difference

$n \mathrel{\hat{=}}$ order of maxima
$\lambda \mathrel{\hat{=}}$ wavelength
$d \mathrel{\hat{=}}$ distance between gaps
θ: angle between initial direction and maxima

Grating:

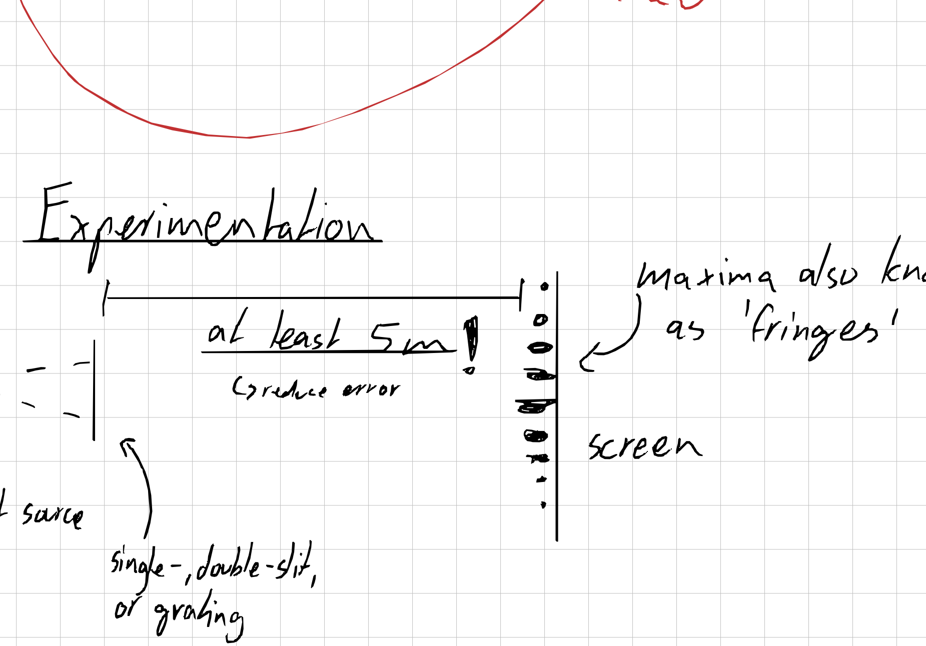


sheet with many gaps

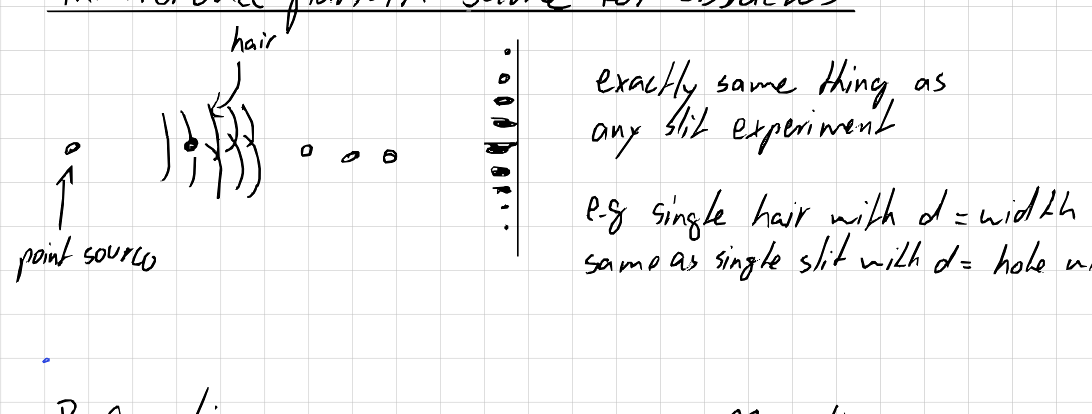
if $n\lambda = 1, 2,$ etc
↳ maximum (constructive)

if $n\lambda = 0.5, 1.5,$ etc
↳ minimum (destructive)

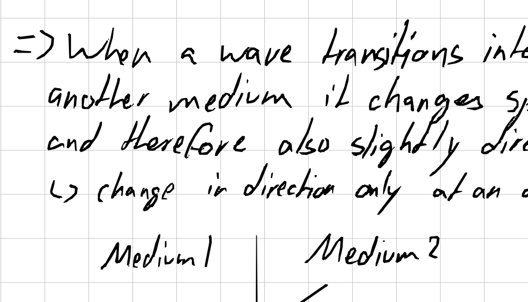
θ visualization



Experimentation



Interference pattern same for obstacles

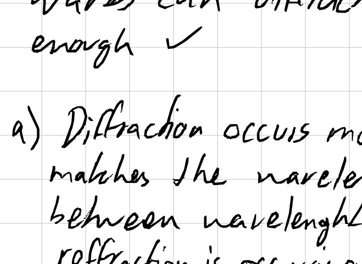


exactly same thing as any slit experiment

e.g single hair with d = width same as single slit with d = hole width

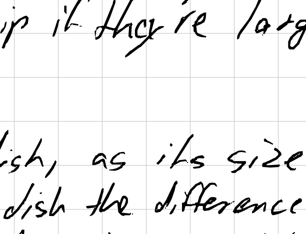
Refraction

=> When a wave transitions into another medium it changes speed and therefore also slightly direction
↳ change in direction only at an angle



Diffraction

=> When a wave encounters a corner or squeezes through a gap it spreads out.



Questions

1. Waves can diffract around the ship if they're large enough ✓

2. a) Diffraction occurs more for the first dish, as its size matches the wavelength. For the second dish the difference between wavelenght and dish width must be large, as little reffraction is occuring ✓
   b) 1. might be useful for broadcasting and 2. for satilite communicatio ✓ explaination...

3. 26.6° 30.1° 36.0° $\quad \frac{1}{1000 \times 10^3} = 10^{-6}\,m = d$ ✓

$\lambda n = d\sin\theta \qquad n = 1$

$\lambda = \frac{d\sin\theta}{n} = 10^{-6}\sin\theta$

26.6: $\lambda = 10^{-6}\sin 26.6 = 4.48\times10^{-7}\,m$ ✓

30.1: $\lambda = 10^{-6}\sin 30.1 = 5.02\times10^{-7}\,m$ ✓

36.0: $\lambda = 10^{-6}\sin 36.0 = 5.88\times10^{-7}\,m$ ✓

4. $\lambda n = d\sin\theta$

$n = \frac{d\sin\theta}{\lambda} = \frac{10^{-6}\sin(50°)}{\lambda}$

$\lambda = 4.48\times10^{-7}$ $\quad n = 2.23$ so $n = 2$ ✓
$\lambda = 5.02\times10^{-7}$ $\quad n = 1.55$ so $n = 1$ ✓ ~~let's go~~
$\lambda = 5.88\times10^{-7}$ $\quad n = 1.701$ so $n = 1$ ✓

X-Ray

$\lambda = 1.51\times10^{-10} \qquad n = 2$ at $0.0867\,m$

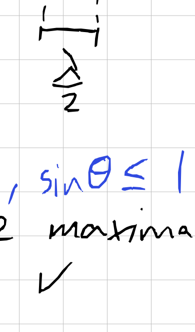
$\theta = \tan^{-1}\left(\frac{0.0867}{0.5}\right) \approx 9.84°$

$n\lambda = d\sin\theta$

$d = \frac{n\lambda}{\sin\theta} = \frac{2 \cdot 1.51\times10^{-10}}{\sin(9.84)} = 1.77\times10^{-5}\,m$ ✓

1. Coherent waves have a consistent phase relationship with each other. This requires them to have the same frequency, wavelength and travel at the same speed! ✓

2. a) 0 rad. ✓
   b) path difference $= \frac{\lambda}{2}$ ✓
   phase difference $= \pi$ rad. ✓
   c) path difference $= \lambda$ ✓
   phase difference $= 2\pi$ rad. ✓
   d) The pattern ~~would~~ might show more maxima, be fainter but larger. → stretched out ✓
   → other factors, $\sin\theta \leq 1$



Exam - Style

1. $0.74 \times 2 = \lambda$

   $\lambda = 1.48\,m$ C ✓

2. B ✓

3. B ✓

4. C ✓

5. D ✓

6. When light hits the oil, part of it reflects and another part defracts due to the change of medium. The refracted light then reflects off the water's surface, defracts again and re-emerges at the air but at a different position. This means that there are many individual light sources due to the splitting that interfere with eachother and cause the pattern. ✓

   ↳ want direct reference to path difference; missing: light is coherent

7. a) $500 \cdot 1000 = 500\,000$ per meter

   $\frac{1}{500\,000} = 2 \times 10^{-6}\,m$ ✓

   b) $n = 1 \quad \theta = 15°$

   $\lambda n = d \sin\theta$

   $\lambda = \frac{d\sin\theta}{n} = \frac{2 \times 10^{-6} \sin(15°)}{1} = 5.18 \times 10^{-7}\,m$ ✓

   c) $\lambda = 450\,nm = 450 \times 10^{-9} \quad \theta = 90$

   $\lambda n = d \sin\theta$

   $n \approx \frac{d\sin\theta}{\lambda} = \frac{2 \times 10^{-6} \sin(90°)}{450 \times 10^{-9}} \approx 4.4 \approx 4$ ✓

8. a) $v = \sqrt{\frac{T}{\mu}}$ $\quad F = m \cdot a$

   $N = kg \cdot ms^{-2}$

   $ms^{-1} = \sqrt{\frac{N}{kgm^{-1}}} = \sqrt{\frac{kg\,ms^{-2}}{kg\,m^{-1}}}$ ✓

   $ms^{-1} = \sqrt{\frac{m^2}{s^2}} = \frac{m}{s} = ms^{-1}$

   b) i. The wave sent by the vibration generator is reflected off the pully and interferes with itself. ✓ As both waves are coherent, meaning their phase difference has a consistent relationship, a standing wave is produced. → want reference to nodes and antinodes

   ii. $\lambda = 2m \cdot 2 = 4m$ ✓ (1st harmonic)

   iii. $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{150}{0.005}} = 170\,ms^{-1}$ ✓

   iv. As the frequency is increased, every time it hits a multiple of the 1st harmonic frequency, a standing wave is formed. Each new standing wave gains an additional node and antinode. At some point the string will snap ✓ some more info, e.g. lower amplitude → more nodes

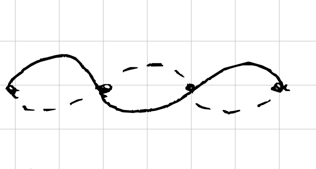
9. a) Diffraction describes the process of wave energy spreading ✓ around corners or through slits. ✓

   b) 2 Superpositions are the patterns that result from multiple waves (wave sources) colliding. ✓

   A superposition is the ~~sum~~ of all (amplitudes) displacements at a given point, for all points ✓

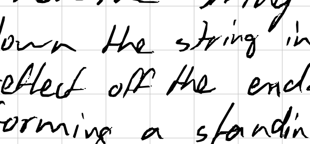
   c) When the waves incide on the rocks, they diffract, i.e. spread around the rocks. As each wave re-emerges on the other side of the obstacle, it acts as a new wave source that can interfere with the other waves. ✓ This interference creates a superposition - a wave pattern. Within this pattern there are areas where the waves interact constructively ✓ and destructively ✓. At the edges of the pattern this causes the waves to have different forces ✓ when acting on the sand. Over time the shape of the interference pattern becomes visible on the sand. ✓

10. a)



   ✓

   b)



   c) When the string is released, two waves travel down the string in opposite directions. Both waves reflect off the ends and interfere with each other, forming a standing wave, if the frequency is just right.

   R they're coherent and therefore produce a standing wave

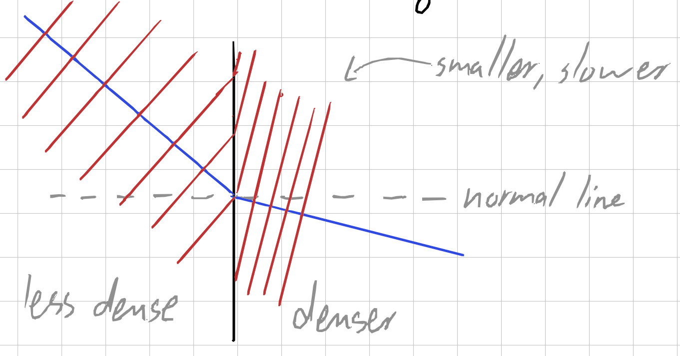
   the

5.7 Refraction

=> The process of a wave changing direction when entering another medium

Explaination

- a wave entering a different medium changes speed
- due to $v = \lambda f$ the wavelength changes
- when traveling at an angle, parts of the wave change speed early
  ↳ the direction changes



Refractive Index

=> describes the ratio of light speed inside a vacuum to the speed inside the medium

- denoted 'n':

$$n = \frac{c}{v}$$

$n \hat{=}$ refractive index of medium
$c \hat{=}$ speed of light (vacuum)
$v \hat{=}$ speed of light (medium)

$n_{Air} \approx 1.000$

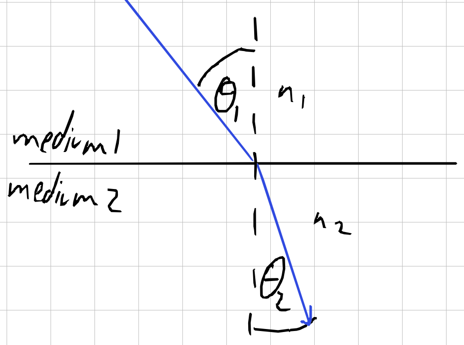
Density correlation

- higher density usually means higher refraction index
  ↳ the relationship depends on more factors

Angle Prediction (Snell's Law)

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$n_{1,2} \hat{=}$ refractive index of medium$_{1,2}$
$\theta_{1,2} \hat{=}$ angle between ray$_{1,2}$ and normal



note: θ_1 will always be larger in the less-dense medium

θ_1: incident angle
θ_2: angle of refraction

Dispersion

formulae for medium 1 = air (1.00)	
$\sin\theta_1 = \frac{c \sin\theta_2}{f\lambda}$	$\sin\theta_2 = \frac{f\lambda \sin\theta_1}{c}$

=> A phenomena where different wave frequencies refract at slightly different angles.

How Dispersion Works

- different frequencies actually have slight variations in their refraction index for a medium
- due to $n = \frac{c}{v}$ we know light at different frequencies will move at different speeds
  ↳ refract differently
- smaller initial wavelength → higher refraction index
  ↳ larger angle of refraction outside medium
  ↳ smaller angle of refraction inside medium

Derive Dispersion Formulae

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad n = \frac{c}{v} = \frac{c}{f\lambda}$$

$$\frac{c}{f\lambda_1} \sin\theta_1 = \frac{c}{f\lambda_2} \sin\theta_2 \quad \Big| \cdot \frac{c}{f}$$

$$\frac{\sin\theta_1}{\lambda_1} = \frac{\sin\theta_2}{\lambda_2}$$

---

1. a) dec. b) dec. c) stays the same
2. Refraction (change in medium for light waves)
3. $n_1 = 1.00$ $\quad n_2 = 1.50$
   $\theta_1 = 25°$ $\quad \theta_2 = ?$

$$\cancel{n_1} \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_2 = \frac{\sin\theta_1}{n_2}$$

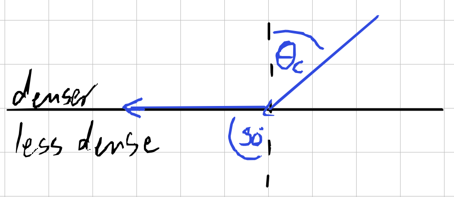
$$\theta_2 = \sin^{-1}\left(\frac{\sin\theta_1}{n_2}\right) = \sin^{-1}\left(\frac{\sin 25°}{1.5}\right) = 16.4°$$

5.8 Total Internal Reflection

=> Short 'TIR', occurs when a wave tries to move from a dense medium to a less dense medium past a specific angle of incident

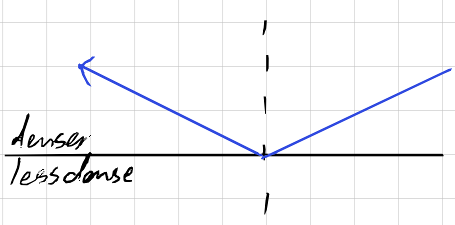
Critical Angle

- At the critical angle the angle of refraction is perpendicular to the interface
  ↳ 'Interface' -> border between two media



$\theta_c \hat{=}$ critical angle

- Past the critical angle total internal refraction occurs:



Calculating the Critical Angle

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ (Snell's Law)
$\theta_2 = 90°$, $\sin 90° = 1$

$\boxed{n_1 \sin\theta_1 = n_2}$

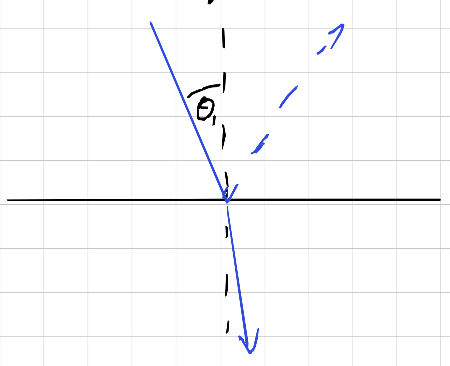
$n_1 \hat{=}$ refraction index (denser)
$n_2 \hat{=}$ refraction index (less dense)
$\theta_1 \hat{=}$ incident angle (critical angle)

$\boxed{\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$

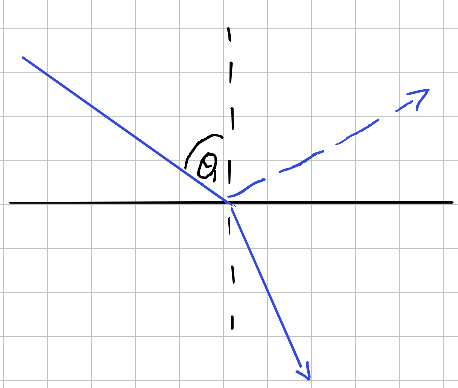
Partial Internal Reflection

=> Always occurs when the angle of incidence is smaller or equal to the critical angle

steep incident angle = less reflection



larger incident angle = more reflection



Questions

1. Critical angle describes the incident angle at which the angle of refraction is $90°$, therefore parallel to the interface. Past this angle total internal reflection occurs.

2. The angle of incident must surpass the critical angle.

3. $n_{water} = 1.33$ $n_{air} = 1.00$

$n_1 \sin\theta_c = n_2$

$n_{water} \sin\theta_c = n_{air}$

$\sin\theta_c = \frac{n_{air}}{n_{water}}$

$\theta_c = \sin^{-1}\left(\frac{n_{air}}{n_{water}}\right) = \sin^{-1}\left(\frac{1}{1.33}\right) = 48.8°$

4. Due to the higher density of water, the angle of refraction in the less dense air will be greater than the incident angle. The fish gets to see a larger range.

$v = (gd)^{\frac{1}{2}}$

$n_1 \sin\theta_c = n_2$

$\sin\theta_c = \frac{n_2}{n_1} = \frac{\frac{c}{v_2}}{\frac{c}{v_1}} = \frac{\cancel{c}}{v_2} \cdot \frac{v_1}{\cancel{c}} = \frac{v_1}{v_2}$

$v_1 = (gd_1)^{\frac{1}{2}}$

$v_2 = (gd_2)^{\frac{1}{2}}$

$\sin\theta_c = \left(\frac{\cancel{g}d_1}{\cancel{g}d_2}\right)^{\frac{1}{2}}$

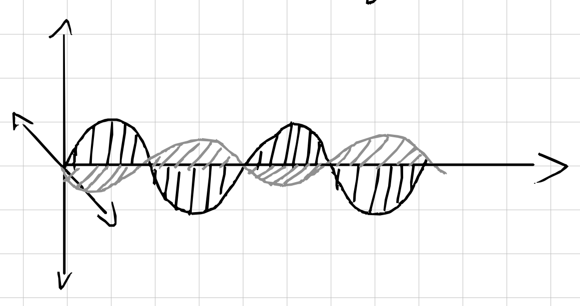
$\theta_c = \sin^{-1}\left(\left(\frac{d_1}{d_2}\right)^{\frac{1}{2}}\right)$

5.9 Polarization

=> The orientation of the plane along which transverse waves oscilate

Light Composition

- any electromagnetic waves oscilate in two directions perpendicular to each other:
  - vertical: electric field
  - horizontal: magnetic field



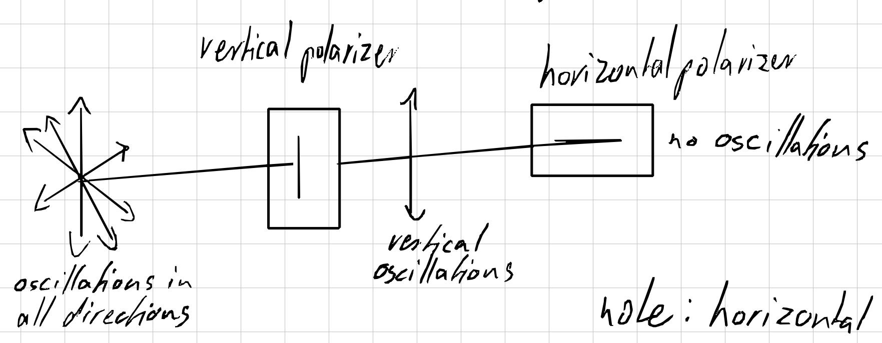
Plane Polarization

- a wave is said to be '<u>plane polarized</u>' when its transverse displacement happens along one plane
- natural light consists of many differently oriented waves
  ↳ each wave individually is plane polarized, but together they're not
- <u>only transverse waves can be polarized</u>

Polarizing Filters

=> allow only one plane polarization to pass and absorb or reflect the rest

- Types of filters:
  - sheet with slit for string polarization
  - 'polaroid sheet' for electromagnetic waves

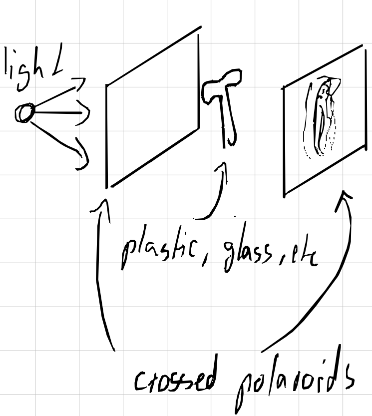


note: horizontal polarizing filter
↳ allows horizontally polarized light to pass

Crossed Polaroids

=> Two polaroids that polarize light both vertically and horizontally, therefore blocking all light

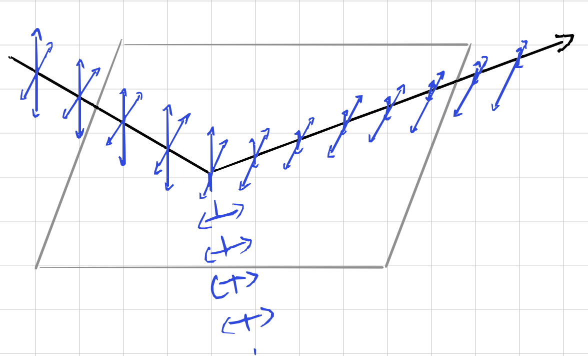
Stress Analyzis with Crossed Polaroids:



- all light that doesn't get twisted by the transparent object gets blocked
- some light gets twisted by the object and travells slower through areas of high stress
- light traveling at different speeds carses an interference pattern related to the stress
- also different wavelenghts are twisted differently by the object

Polarization By Reflection and Refraction

=> Vertically polarized light has a higher tendency to refract into the surface
↳ Reflections tend to have horizontal plane polarization



- the degree of polarization depends on the angle of incidence
  ↳ steeper angle => less reflection

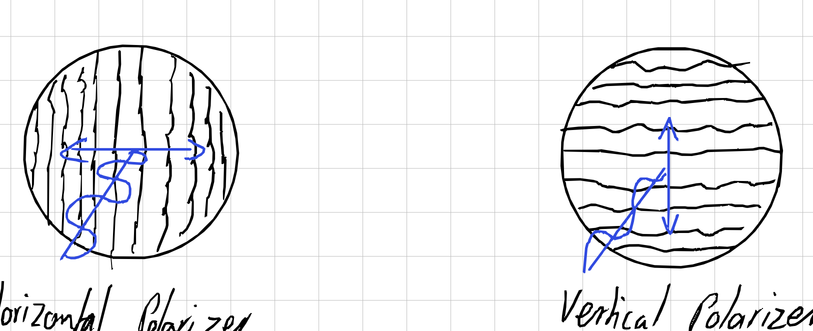
Polarization by Sugar Solutions

=> Sugar solutions are able to rotate the angle of the plane of polarization

- Factors that determine the emerging color:
  - sugar to water ratio -> how much twisted
  - distance for light in sugar solution -> more distance = more twisted
- crossed polaroids are required to see the effects

Note: These notes only consider plane polarization, other types include:
- circular polarization -> rotation direction

Relation between Long-Chain Molecules and Polarization



Horizontal Polarizer   Vertical Polarizer

Radio Transmitter Polarization

- radio transmitters are often used for practical exams
- almost always are plane polarized
- this includes e.g wifi, communication, radio

Questions

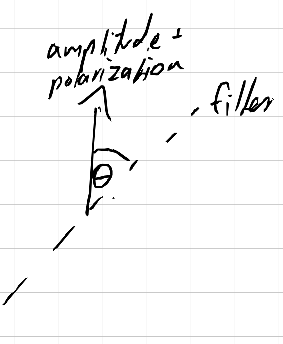
1. Only transverse waves can be polarized, sound consists of longitudinal waves ✓

2. It means that the waves doesn't oscillate in a specific plane -> beam of randomly rotated electromagnetic waves

3. To block the strong reflections that are contained in the horizontal component of all reflected light. Surfaces tend to horizontally polarize light ✓

4. One can look at stress within values where no pressure sensor could reach in. Engineers can tell where they need reenforcement ✓

a_i a_f



3C Exam Practice

1. D

2. $n_1 \sin\theta = n_2$
   $1.48 \sin\theta = 1$
   $\theta = \sin^{-1}\left(\frac{1}{1.48}\right) \approx 42.5°$ C

3. C

4. B

5. b) i $n = \frac{c}{v} = \frac{3.00 \times 10^8}{1.96 \times 10^8} = 1.53$

   ii $\theta = \sin^{-1}\left(\frac{1}{1.53}\right) \approx 40.8°$

6. a) A beam of unpolarized light consists of many individual photons that are ~~momentarily~~ plane polarized in many random directions.

   b) Reflections are strongly horizontally plane polarized, and this 'glare' can be gotten rid of by vertically plane polarizing the light.

   c) They're longitudinal waves which can't be polarized!

7. a) $\sin\theta_1 = n_2 \sin\theta_2$
   $\frac{\sin\theta_1}{\sin\theta_2} = n_2$

   $n_2 = \frac{\sin 48°}{\sin 30°} \approx 1.49 \approx 1.5$

   b) i The student has reached the critical angle at which total internal reflection occurs. This confirms that the refraction index of the glass must be larger then that of air.

   ii $n_2 \sin\theta_c = n_1$
   $1.49 \sin\theta_c = 1$
   $\theta_c = \sin^{-1}\left(\frac{1}{1.49}\right) \approx 42.2° \approx 42°$ haha

8. a) $n_1 = \frac{3.00 \times 10^8}{1.96 \times 10^8} \approx 1.53$

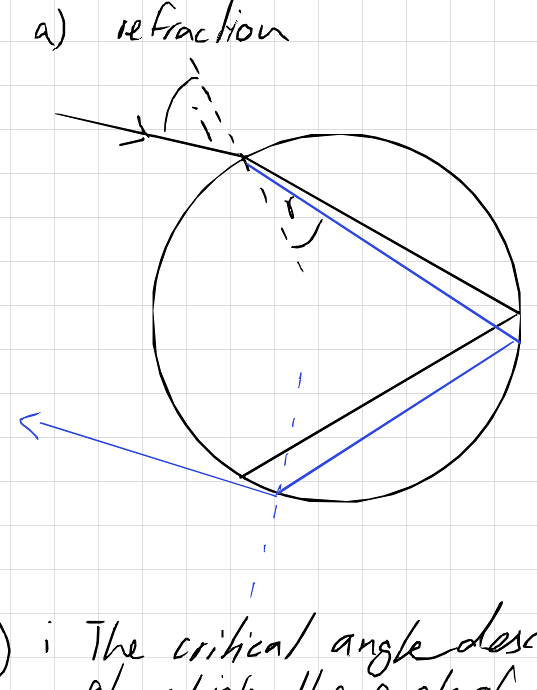
   $n_2 = \frac{3.00 \times 10^8}{2.03 \times 10}\approx 1.48$

   $\theta_c = \sin^{-1}\left(\frac{1.48}{1.53}\right) \approx 75.3°$

   b) It is going to be reflected at 80° to the normal, as it's past the critical angle.

   c) Glare increases with a larger angle of incidence. Due to the light source being at the top, the angle of incidence will increase (relative to the normal) the lower you get. ?

9. a) refraction



   c) i The critical angle describes the angle of incidence at which the angle of refraction is 90° to the normal, just before total internal reflection occurs.

   ii $n_1 \sin\theta_c = n_2$

   $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1}{1.3}\right) \approx 50.°$

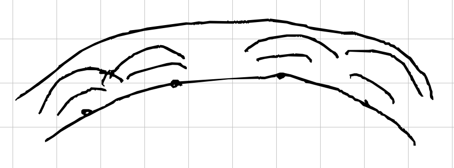
   d) $v = f\lambda$
   $\lambda = \frac{v}{f} = \frac{2.2 \times 10^8}{4.2 \times 10^{14}} = 5.2 \times 10^{-7}$ m

10. Infrared experiences total internal reflection inside the optical fibre, under the condition that it entered rather parallel to the cable and the fibre doesn't have any steep turns. This is because optic fibre has a refraction index larger than air or cladding which is usually used in optic fibre cables. Therefore a critical angle exists, past which total internal reflection occurs.
    Applications are e.g. super-high-bandwidth data transmission cables, as many different frequencies can be used simultaneously to send signals.

6.1 Wave-Particle Duality

Huygens' Principle

=> infinite points on a wavefront are treated as wave sources that form the next wavefront



- points constructively and destructively interfere to form another wave front
- This model explains refraction and diffraction
  ↳ e.g. no sources at the edges to cancel spreading waves = diffraction

isn't exclusive to e.m. radiation

Wave-Particle Duality

=> Electromagnetic radiation (photons) can act as waves and as particles under certain circumstances

Quantisation:

=> Energy can only exist in certain increments in our universe, the concept of such steps is called quantization
- particle-like behaviour of electromagnetic waves can be explained by quantisation:
  -> in place of the wave there are moving energy packets (photons) with energy proportional to that of the wave

Calculate Photon Energy:

Photons have no mass or charge

$E = hf$

$E \hat{=}$ energy of photon (J)
$h \hat{=}$ planck's constant (Js) $\approx 6.63 \times 10^{-34}$ J·s
$f \hat{=}$ frequency of electromagnetic wave (Hz)

Planck's constant: smallest possible increment for energy in the universe

Evidence that electromagnetic radiation is...

a wave:
- interference
- diffraction
- polarization
  ↳ only waves can be polarized

a particle:
- photoelectric effect

-> particles cannot superpose to a constant pattern

Electron Wave-Particle-Duality

=> If electrons move at very high speeds they act as waves
- at high speeds electrons interfere and diffract
  ↳ they can even form an interference pattern

Evidence that electrons are ...

waves:
- diffraction and interference at high speeds

particles:
- when ionizing atoms (changing its charge) the charge changes in steps
  ↳ must be quantisation which hints to packets of energy
  = particles

Questions

134.

1. a) Interference pattern caused by a double slit when unobserved ✓

   b) Very fast travelling electrons passing through a double slit will also produce an interference pattern when not observing the slits ✓

2. $v = f \cdot \lambda$

$f = \frac{v}{\lambda} = \frac{3 \times 10^8}{2 \times 10^{-7}} = 1.5 \times 10^{15}$ Hz

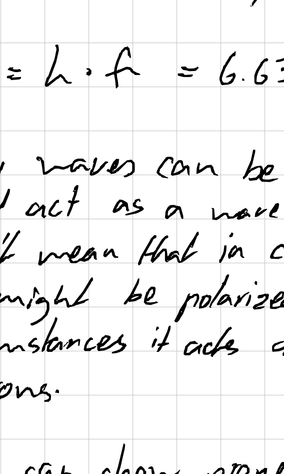
(✓)

$E = h \cdot f = 6.63 \times 10^{-34} \cdot 1.5 \times 10^{15} \approx 9.95 \times 10^{-19}$ J

3. Only waves can be polarized, meaning that electrons would act as a wave under those circumstances. But this doesn't mean that it cannot also act as particles. Light e.g. might be polarized sometimes, but under certain circumstances it acts as particles, the same counts for electrons.

4. Light can show properties of a wave and of a particle, depending on the circumstances. It would be wrong to say that it's exclusively one of both. ✓

5.



(at all times)

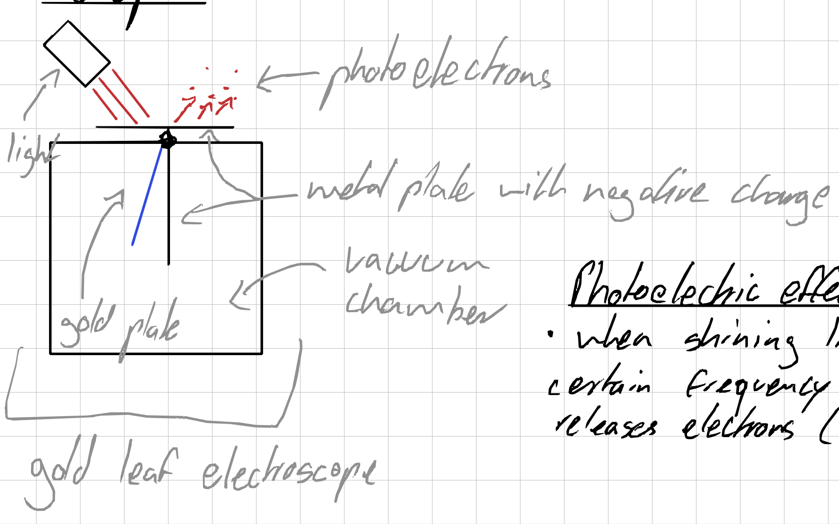
very detailed, less sources and only wavefronts next to and after the obstacle would be enough

no waves to negatively interfere with at the edges -> diffraction occurs

6.2 Photoelectric Effect

=> Showed how under certain circumstances light will exclusively act as particles

Setup:



Photoelectric effect:
- when shining light above a certain frequency onto a metal, it releases electrons (called 'photoelectrons')

How it works:
- light travels as photons, their energy is proportional to their frequency
- a photon collides with the electron
  - the photon's energy is greater or equal to the work function (= energy required to free the electron)
  - -> photon is 'used up' and the electron is released as a photoelectron

Creation of Photoelectrons

- photoelectrons = electrons released due to interaction with a photon
- work function = minimum energy required to free an electron from the surface of a material (not the atom!)
  ↳ ϕ
- threshold frequency = frequency at which one photon carries energy equal to the work function

Atomic Interaction

- photon collides with electron
  - if photon energy >= work function -> electron probably released
  - if photon energy < work function -> photon continues it's journey

Model expectations

Light as Wave
- any frequency will remove the metal's charge with enough time and intensity
  ↳ wrong
- more intense (brighter) light will provide the released electrons with more energy so they will be faster
  ↳ wrong

Light as Particle
- only certain frequencies provide photons with sufficient energy to create a photoelectron on a collision
  ↳ correct
- more photons will cause more photoelectrons to be emitted due to more collisions
  ↳ correct

Photoelectric Effect Equation

=> Determine the energy and speed of photoelectrons

$$\frac{1}{2}mv_{max}^2 = hf - \phi$$

final energy = photon energy − energy for freeing the electron

- m ≙ mass of electron (kg)
- v_{max} ≙ maximum possible speed of electron (m/s)
- h ≙ Planck's Constant (Js)
- f ≙ frequency of el.mag. wave (Hz)
- ϕ ≙ work function (J)

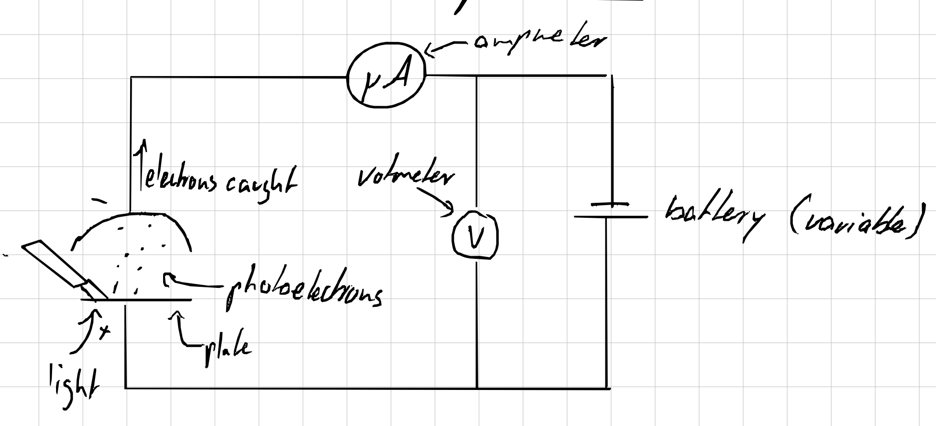
Stopping Voltage

=> The voltage of the electric field required to prevent electrons from freeing while under a certain frequency of electromagnetic radiation

$$\frac{1}{2}mv_{max}^2 = e \times V_s$$

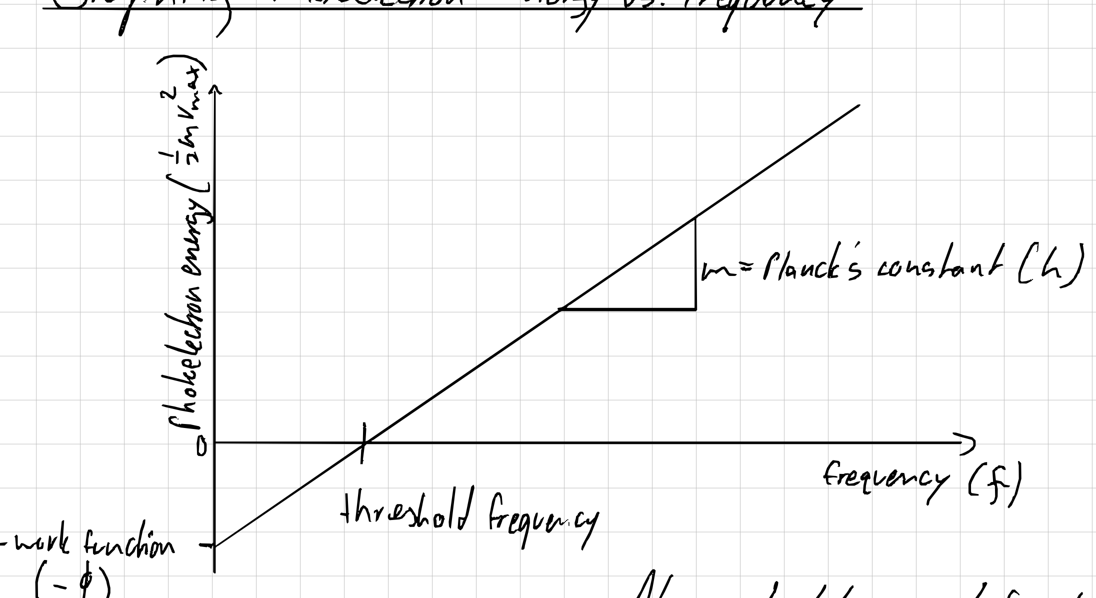
- e ≙ single electron charge (C)
- V_s ≙ stopping voltage (V)

Photoelectric Cell Experiment



- increase the voltage of the battery until no electrons are able to detatch from the plate
  ↳ Ampmeter shows zero
  ↳ voltmeter shows 'stopping voltage'

Graphing Photoelectron Energy vs. Frequency



$$\underset{y}{\frac{1}{2}mv_{max}^2} = \underset{m \cdot x}{hf} - \underset{+b}{\phi}$$

Also calculate work function with x-intercept:

$$\frac{1}{2}mv_{max}^2 = hf - \phi$$

$$0 = hf - \phi$$

$$\phi = hf$$

Questions

1. $\frac{3.68 \times 10^{-15}}{1.602 \times 10^{-15}} \simeq 2.30\ eV$

2. a) red light has a too long frequency, so the photons won't have enough energy to free the electrons. Photon's energy is proportional to the frequency so under the threshold frequency the energy of the photon won't exceed the working function.

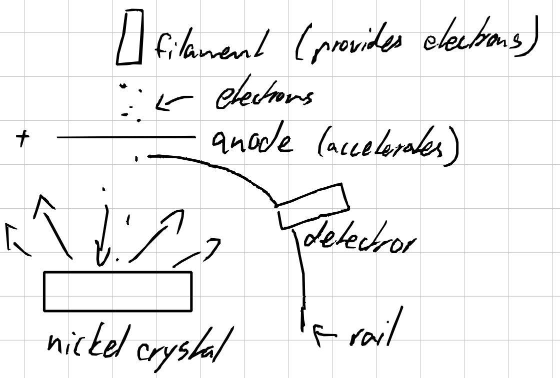
6.3 Electron diffraction & interference

=> All particles, especially small ones like electrons show wave-particle duality under certain circumstances

- electrons shot through a double slit one-by-one will still create an interference pattern

Davisson-Germer Diffraction Experiment

=> When shooting electrons at a nickel crystal they reflect off in a diffraction pattern



- the detector is moved around the crystal to detect changes in reflected intensity
- a diffraction pattern is measured

$\lambda = \frac{h}{p}$

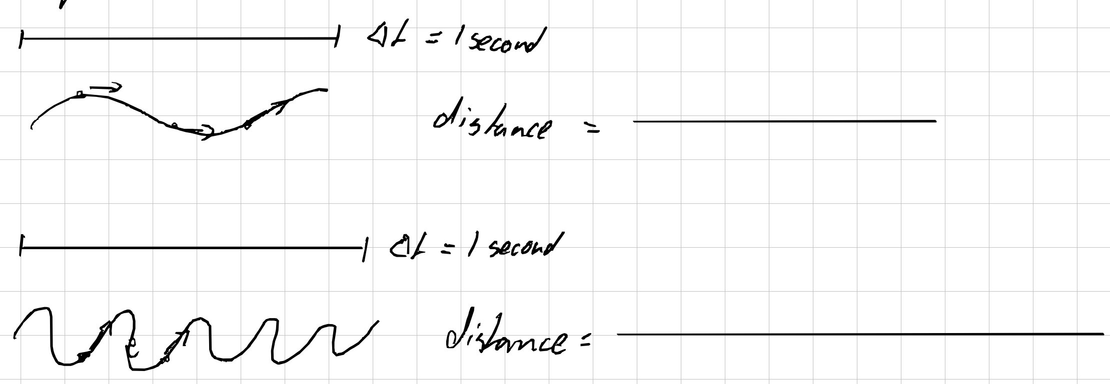
De Broglie Equation

=> the wavelength of an electron wave is inversilly proportional to the electrons momentum
↳ therefore inversilly proportional to the speed (except close to speed of light.)

$\lambda = \frac{h}{p}$

$\lambda \hat{=}$ wavelength (m)
$h \hat{=}$ Planck's constant (J.s)
$p \hat{=}$ momentum ($kgms^{-1}$)

Explaination:



↳ more distance traveled in the same time, smaller wavelength has particles that travel faster

Electron Microscope

=> The faster electrons are accelarated, the smaller their wavelength

=> The wave length has to be approximately equal or smaller then an object in order to be reflected

---

Questions:

1. When electrons are shot through a double slit one-by-one, they will still produce a diffraction pattern ✓

2. Because the wavelength has to be roughly equal or smaller to the object in order to be reflected off of it. By accelerating the electron more it will have a smaller wavelength. ✓

3. $E_{ele} = qV$ (energy = charge x potential difference)

$\frac{1}{2}mv^2 = E_{kin}$

$qV = \frac{1}{2}mv^2$

$1.602 \times 10^{-15} \cdot 400 = \frac{1}{2} \cdot 9.11 \times 10^{-31} v^2$
$3.52 \times 10^{11} = v^2$

$v = 5.93 \times 10^{5} ms^{-1}$

$\lambda = \frac{h}{p} = \frac{1.62 \times 10^{-35}}{9.11 \times 10^{-31} \times 5.93 \times 10^{5}} = 3.00 \times 10^{-11} m$

4. 3kg $20ms^{-1}$

$\lambda = \frac{1.62 \times 10^{-35}}{20 \cdot 3} \approx 2.7 \times 10^{-37} m$ ✓

Test

$E_{el} = Vq$ $E_{kin} = \frac{1}{2}mv^2$

$Vq = \frac{1}{2}mv^2$

$m2Vq = v^2$

$v = \sqrt{2Vqm}$

$\lambda = \frac{h}{m \cdot \sqrt{2Vqm}} =$

$\frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \cdot \sqrt{2 \cdot 800 \cdot 1.602 \times 10^{-19} \times 9.11 \times 10^{-31}}} = 4.76 \times 10^{13} m$

$\frac{2Vq}{m} = v^2$

$v = \sqrt{\frac{2Vq}{m}}$ $\lambda = \frac{h}{m\sqrt{\frac{2Vq}{m}}}$

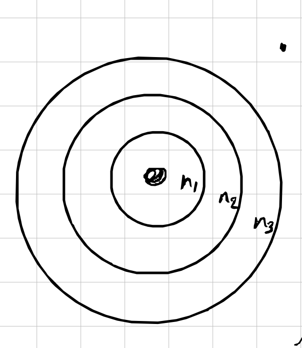
$4.34 \times 10^{-11} m \simeq 0.0434 nm$ ✓

Isac Test

$E = hf = 6.626 \times 10^{-34} \cdot 6 \times 10^{14} = 5.7 \times 10^{-21} J$

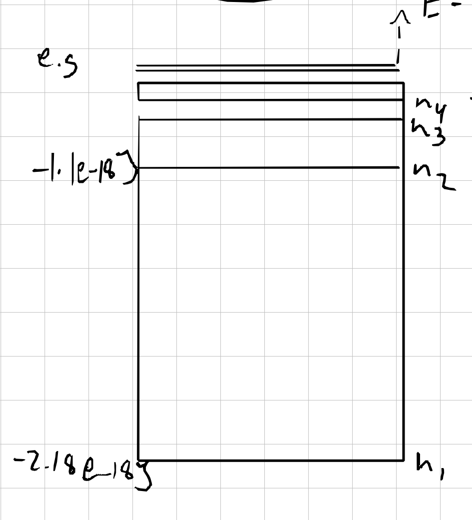
6.4 Atomic Electron Energies

Electron Energy Levels



- n_1 = ground state, the electron has least possible energy
- n_n = energy levels of the entire atom! (not electron orbits)

$E = 0$ when $n = \infty$



$E \to 0$

- energy approaches zero as the energy level becomes infinite
- Providing an electron with more energy will make its energy less negative
- different energy levels have large ranges of possible energy values
  ↳ ranges decrease as atoms become more free (e.g. gas)

Excitation

=> The electron is provided with exactly the right amount of energy to jump to the next energy level
↳ for isolated atoms with seperate electrons too fast photons just pass by

- A photon with an energy different to what would be required for a clean jump doesn't interact with the electron
- The photon will completely disappear and give all its energy to the electron, if it carries the exactly right amount
- collision with a particle (not photon) can also result in excitation
  ↳ e.g. when a high voltage is applied.

De-Excitation

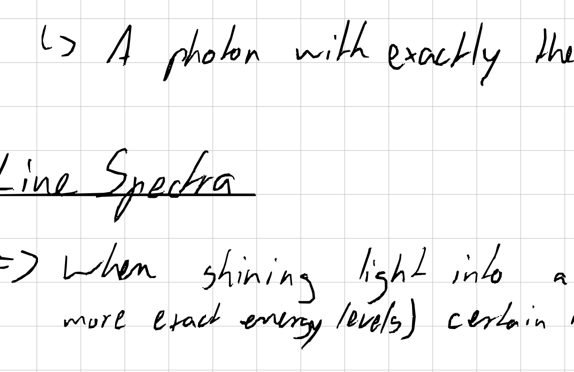
=> After a random amount of time the electron will drop down to some intermediate energy level or directly to the ground state

↳ A photon with exactly the freed energy is released

Line Spectra

=> When shining light into a substance (e.g. gas due to more exact energy levels) certain frequencies will be absorbed

=> When exciting atoms with a large potential difference many atoms will de-excite and release very specific wavelengths
↳ fast electrons slam into neutral electrons
↳ a line spectra can be created with a prism



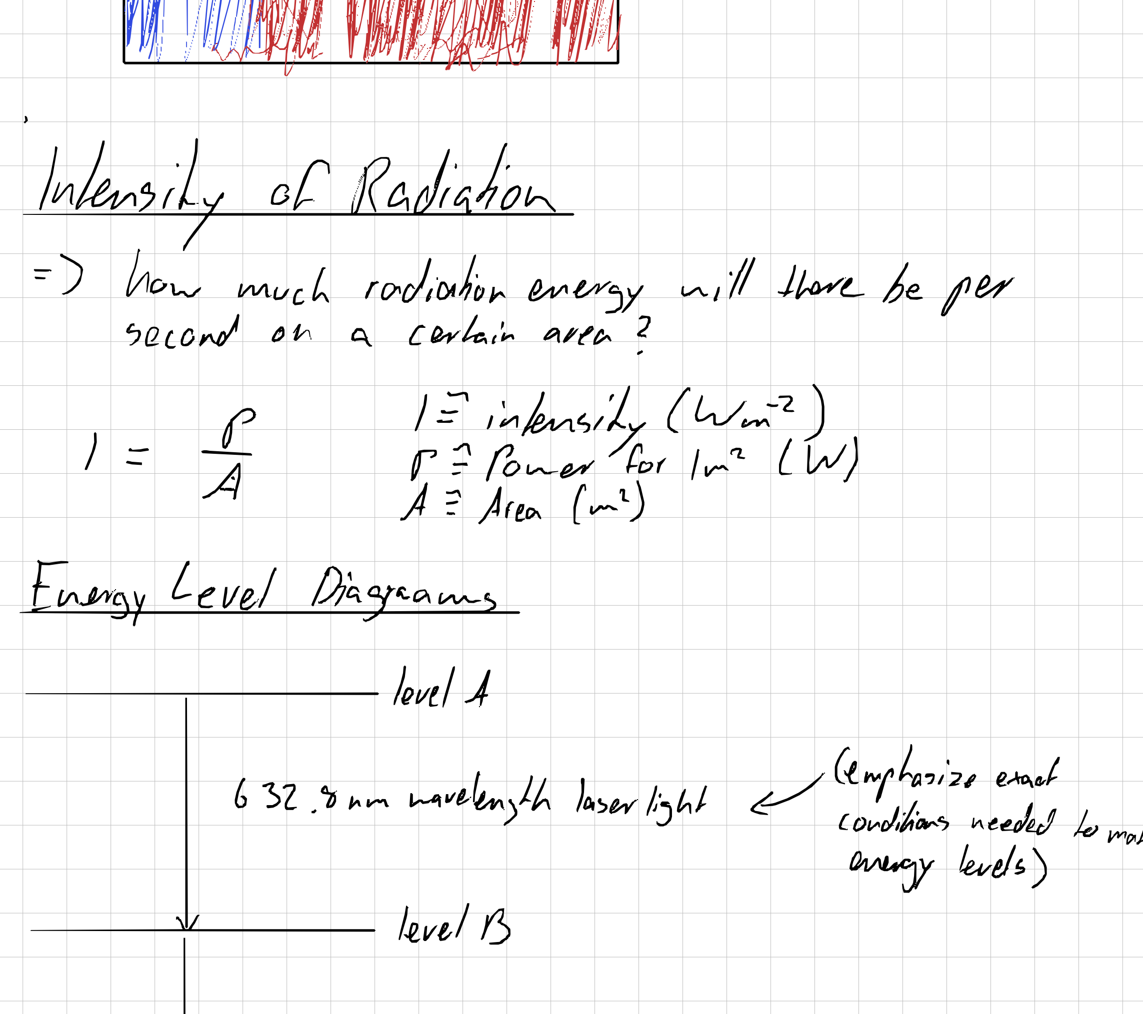
Intensity of Radiation

=> how much radiation energy will there be per second on a certain area?

$$I = \frac{P}{A}$$

$I \equiv$ intensity (Wm^{-2})
$P \equiv$ Power for $1m^2$ (W)
$A \equiv$ Area (m^2)

Energy Level Diagrams



Ionisation

=> When the electron leaves the atom ($n=\infty$), the atom will be ionised. The energy level for the electron is zero

=> the minimum energy required to move an electron from the atom's ground state outside of the atom is called <u>ionisation energy</u>
↳ the ionization energy is like the work function, but for an isolated atom, instead of a continuous metal surface

---

1. $2.18E-18 - 5.45E-19 = 1.635E-18$

   $E = hf \quad f = \frac{v}{\lambda}$

   $E = h\frac{v}{\lambda}$

   $\lambda = \frac{hv}{E} = \frac{6.63E-34 \cdot 3E8}{1.635E-18} = 1.22 \times 10^{-7} m$

2. Because $1.43E-18J$ is smaller than the required $1.635E-18J$

3. $E = hf \quad f = \frac{v}{\lambda}$

   $E = h\frac{v}{\lambda} = \frac{6.63 \times 10^{-34} \cdot 3 \times 10^{8}}{632.8 \times 10^{-9}} \div (1.6 \times 10^{-19})$

   $\approx 2.0 eV$

4. When colliding with other particles, electrons can be excited too. One could apply a current to the gas. (send)

5. Mercury has many different energy levels, and when atoms change between them many different colors arrise.

6. It must be a number extreamly large

Test

1. $E = hf \quad f = \frac{v}{\lambda}$

   $E = \frac{hv}{\lambda} = \frac{6.63 \times 10^{-34} \cdot 3 \times 10^{8}}{550 \times 10^{-9}} = 3.6 \times 10^{-19}$

   C

2. $2.18 \times 10^{-18} - 1.36 \times 10^{-19} = 20.44 \times 10^{-19}$

   $\lambda = \frac{hv}{E} = \frac{6.63 \times 10^{-34} \cdot 3 \times 10^{8}}{20.44 \times 10^{-19}} \approx 9$

   $f = \frac{E}{h} = \frac{20.44 \times 10^{-19}}{6.63 \times 10^{-34}} \approx 3.08 \times 10^{15}$

   C

3. D

4. D   1. Green 2. Orange 3. Red

5. no blue
   ~~1. Blue 2. Green 3. Red~~

6. That frequency has to exceed a certain level to produce photoelectrons tells us that light has to travel as many individual packages of energy. The energy of the single colliding particle has to be sufficient. Increasing the amount would just result in more photoelectrons, as more collisions occur. (more detail)

7. a) The energy of the photons isn't sufficient to free any electrons

   b) $m = h \qquad 0 = 10^{15} \cdot 6.63 \times 10^{-34} + b$

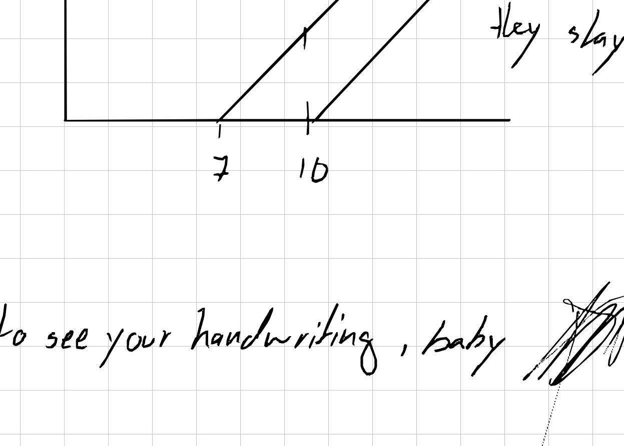
   $0 =$

   $b = -6.63 \times 10^{-19}$

   $\phi = 6.63 \times 10^{-19} J$ (✓) convert to eV

   c) Plancks constant, the minimum possible change in energy (in the universe)

   d)



I want to see your handwriting, baby 桐

徐栢桐 phillipp

phillipp philipp

philipp

I like philipp.

Philipp like Xu Baitong.

very much ♡

徐栢桐

徐栢桐

8. a) $v = f\lambda$

   i. $f = \frac{v}{\lambda} = \frac{3 \times 10^{8}}{6.56 \times 10^{-7}} = 4.57 \times 10^{14} Hz$

   ii. $\frac{3.03 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.89 eV \qquad 3.4eV - 1.5eV = 1.9 eV$

   $1.89 \approx 1.9$ so from <u>n3 to n2</u>

   b) $E = h \cdot f \quad f = \frac{v}{\lambda}$

   $E = h \cdot \frac{v}{\lambda} = \frac{6.63 \times 10^{-34} \cdot 3 \times 10^{8}}{656 \times 10^{-9}} = 3.03 \times 10^{-19}$

   ↳ same as energy required for a drop from n_3 to n_2

   calculation not expected

   The hydrogen absorbes this frequency. It's precisely the energy needed to drop from n_3 to n_2.

9. a) Ultraviolet light is a very short wave that provides each photon with sufficient energy to free an electron from an atom in its ground state. This means that the photon must have roughly equal ore more energy than the zink's work function
   ↳ Charge is reduced and the leave falls (✓)

   b) Visible light has longer wavelengths that carry less energy. The energy each photon carries doesn't exceed the Zink's work function

   c) $E = hf \qquad v = \lambda f$

   $f = \frac{v}{\lambda}$

   $E = \frac{hv}{\lambda} = \frac{6.63 \times 10^{-34} \cdot 3 \times 10^{8}}{2 \times 10^{-7}} = 9.945 \times 10^{-19} J$

   $(9.945 - 6.88) \times 10^{-19}$

   $3.065 \times 10^{-19} J$

   $E = \frac{1}{2}mv^2$

   $v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \cdot 3.065 \times 10^{-19}}{9.11 \times 10^{-31}}} = 8.20 \times 10^{5} ms^{-1}$

   let's goo

   d) The speed will stay the same, as the wavelength doesn't change.

150 nm

$$\frac{1}{2}mv^2 = hf - \phi$$

$$\frac{1}{2}mv^2 = h\frac{v}{\lambda} - \phi$$

$$\frac{1}{2}mv^2 - h\frac{v}{\lambda} =$$

$$eV = fh$$

$$\frac{eV}{h} = f$$

$$eV = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2eV}{m}}$$

$$\lambda = \frac{v}{f}$$

7.1 Electric Current

Electric Charge

- measured in coulombs (C)
- one electron charge equals:
  $e = -1.602 \times 10^{-19}$ C
- The total charge always stays conserved

Electromagnetic Force

- one of the four fundamental forces in the universe
- Opposite charges repell, same charges attract

Electric Current

=> The rate of movement of electric charge
- Occurs when a charge experiences an electric force
  ↳ the charge will accelerate, creating a current (if it's a conduction electron)
- Conduction electrons: electrons in a solid that are able to move
- The unit for current is 'A' for ampere, or $\frac{\text{coulombs}}{\text{second}} = 1$

Conventional Current vs Flow of Electrons

- Conventional current assumes the moving charges are positive,
  ↳ Particles move from positive to negative
- Actual electrons have a negative charge
  ↳ They move from negative to positive

Amount of Charge moving through a Component over Time:

$\Delta Q = I \Delta t$

ΔQ ≙ amount of charge (C)
I ≙ current (A) - ampere
Δt ≙ change in time (s)

Positive Charge Carriers

- If ions are suspended e.g. in a liquid, it's possible that positively charged particles act as charge carriers
  ↳ This is also a respected circuit
- Charge on a proton = $-e$ $(1.602 \times 10^{-19}$ C$)$

Questions

1. a) $I = \frac{C}{s} = \frac{12.5 \times 10^{18} \times 1.602 \times 10^{-19}}{3.2}$

   $I = 0.626$ A ✓

   b) $Q = I \cdot \Delta t = 0.95 \cdot 8 = 7.6$ C ✓

   c) $\Delta t = \frac{Q}{I} = \frac{1.602 \times 10^{-19} \times 10^{11}}{0.68} = 2.36 \times 10^{-8}$ s ✓

2. a) $\Delta t = \frac{Q}{I} = \frac{20 \times 10^{-9}}{0.12 \times 10^{-6}} = 0.167$ s ✓

   b) $\frac{20 \times 10^{-9}}{1.602 \times 10^{-19}} = 1.25 \times 10^{11}$ electrons ✓

3. $0.01 \times 2 \times 6.02 \times 10^{23} = 1.2 \times 10^{22}$ e

   $I = \frac{\Delta Q}{\Delta t} = \frac{1.2 \times 10^{22} \times 1.602 \times 10^{-19}}{23 \times 60} = 1.398$ A ≈ 1.40 A ✓

7.2 Electrical Energy Transfer

Voltage

=> Also known as "Potential Difference", describes the energy transfered per unit charge

$V = \frac{E}{Q}$ V ≙ voltage (V)
E ≙ energy transfered (J)
Q ≙ Coulombs (C)

- Voltage is used for a theoretical transfer of energy, for actual energy transfered e.g. in circuits, potential difference is used.

Potential Difference

- Voltage accross electrical components that actually do work is refered to as "potential difference", or "pd"

$V = \frac{W}{Q}$ V ≙ Voltage (V)
W ≙ Work (J)
Q ≙ Coulombs

basically the same as for volts

Electromotive Force

=> (emf) is the energy supplied per unit charge

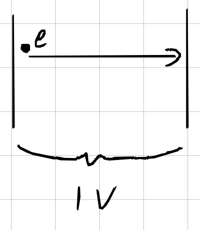
=> not a force! Describes a supply voltage, e.g. of a battery that provides a circuit with energy

↳ often refered to as "emf"

$\varepsilon = \frac{E}{Q}$ ε ≙ emf (V)
E ≙ energy provided (J)
Q ≙ charge passing (C)

Electron Volt

=> Describes a tiny amount of energy equal to the energy a single electron gains when being accelerated through a potential difference of 1 volt



$V = \frac{E}{Q}$ $E = QV = 1.602 \times 10^{-19} \times 1$ J
$E = 1.602 \times 10^{-19}$ J

$1 eV = 1.602 \times 10^{-19}$ J

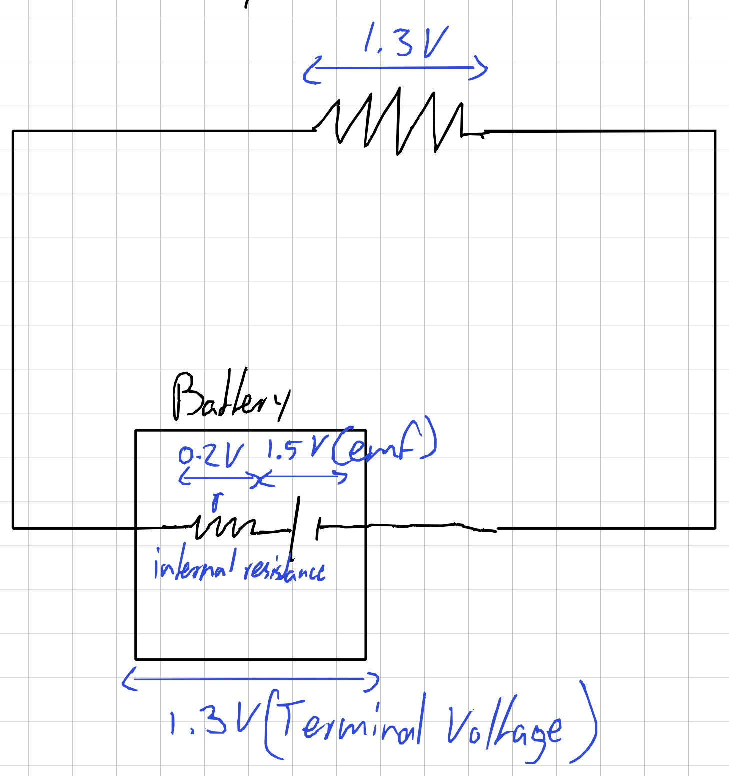
Potential Difference vs Electromotive Force

Pd voltage: for energy taken out of the system

Emf voltage: for energy that is put into the system

Terminal Voltage

=> Batteries usually have some internal resistance



Voltmeters & Ammeters

- Voltmeters have infinite resistance
- Ammeters have negligible resistance

(The rest you know)

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1. Potential difference describes the drop of voltage over components in the circuit, while the emf refers to the increase in voltage over e.g. a battery.

2. a) $\varepsilon = \frac{E}{Q} = \frac{120}{76} = 1.6 V$

b) $\Delta Q = I \cdot \Delta t = 2.4 \cdot 5.4 = 12.96 C$

$V = \frac{E}{Q} = \frac{120}{12.96} \approx 9.3 V$

3. a) $\frac{9.6 \times 10^{-19}}{1.602 \times 10^{-19}} \approx 6.0 eV$

b) $4.8 \times 10^{6} \cdot 1.602 \times 10^{-19} = 7.7 \times 10^{-13}$ J

4. Usually there is only one type of charge carrier

The skiers m

7.3 Current - Voltage Relationship

emf - current relationship

- the emf pushes electrons through the electric force
- a higher emf voltage results in a stronger electric force
  ↳ electrons move faster
  ↳ the current is higher

$I \propto V$ (usually)

Ohm's Law

$R = \frac{V}{I}$

$R \mathrel{\hat=}$ resistance (Ω)
$V \mathrel{\hat=}$ voltage (V)
$I \mathrel{\hat=}$ current (A)

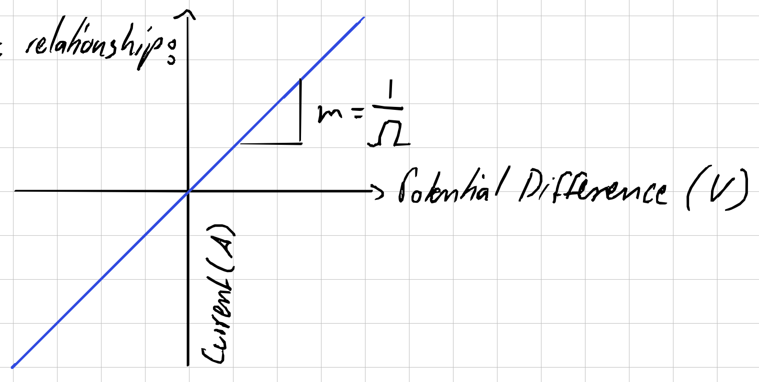
Ohmic Conductor: • conductors for which Ohm's law applies:
↳ the current is proportional to the voltage across the conductor
↳ provided the temperature stays constant

Temperature and Resistance

→ Increased temperature reduces the conductivity
↳ resistance increases

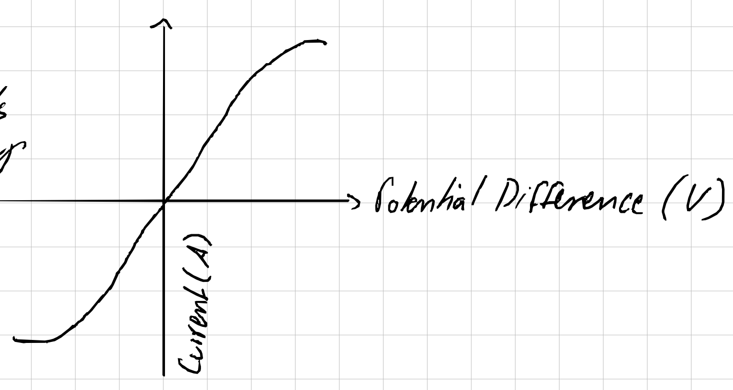
Graph Relationship - Ohm's Law

standard ohmic relationship:



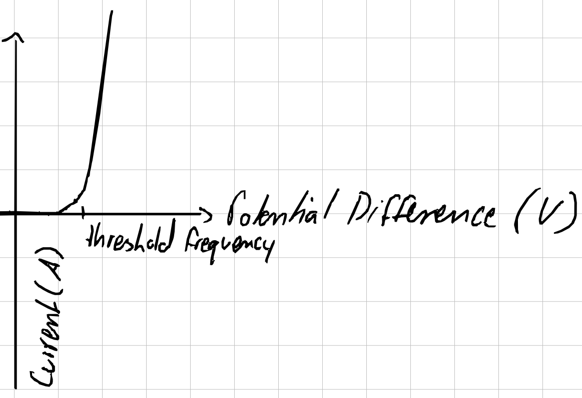
Filament Bulb:

- non-ohmic
- at high currents the bulb heats up and resistance increases
  ↳ less current



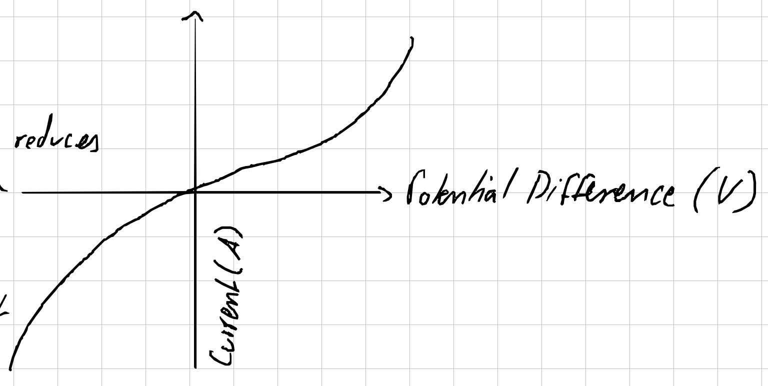
Diode (e.g. LED):

- A diode only allows current to pass in one direction
- A threshold voltage has to be passed first
  ↳ then there is little resistance



Thermistor:

- Aka thermal resistor reduces its resistance at high temperatures
- many semiconductors become less resistive at higher temperatures



Questions

1. $V = I \cdot R$ The voltage across two points is proportional to the current flowing between them.

2. $V = IR$
   $R = \frac{V}{I} = \frac{6}{0.05} = 120\,\Omega$

3. The ohm's law only applies for the ohmic conductor, the non-ohmic conductor will show behaviour where $V \propto I$ isn't the case.

4. Because the slope would equal to the resistance, when pd is on the y.

5. Component a has higher resistance as $m = \frac{1}{\Omega}$, meaning a less steep slope indicates higher resistance

7.4 Resistivity

=> A fundamental property to each material, describing its tendency to resist the flow of current

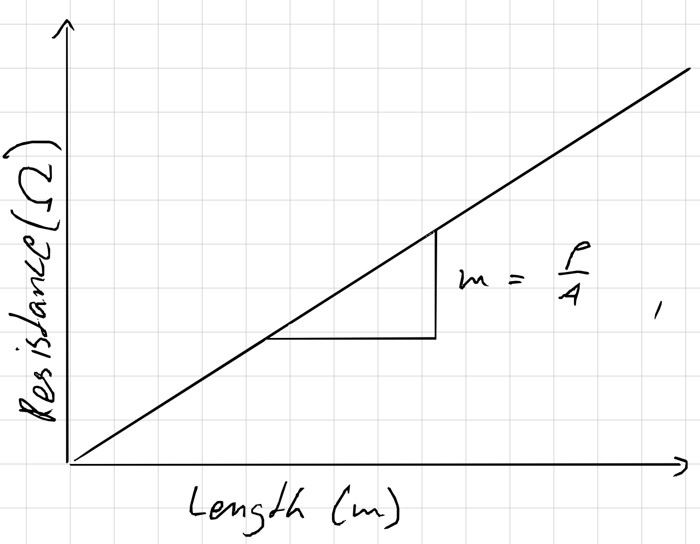
- denoted by symbol ρ
- unit: Ωm
- ρ describes the resistance current experiences which when flowing through $1 m^3$ of the material

$$R = \frac{\rho l}{A}$$

$R \hat{=}$ Resistance (Ω)
$\rho \hat{=}$ resistivity of material (Ωm)
$A \hat{=}$ area of crossection (m^2)
$l \hat{=}$ length of conductor (m)

- Resistivity is highly dependent on a material's density and temperature

Graphing Resistivity



$$R = \frac{\rho}{A} l$$
$$y = m x \; \cancel{+b}^{0}$$

$m = \frac{\rho}{A}$, $\rho = m \cdot A$

Questions

1. $l = 0.8$ $d = 0.00084 m$

$$A = \frac{\pi (0.00084)^2}{4} \approx 5.54 \times 10^{-7}$$

$$R = \frac{\rho l}{A} = \frac{4.9 \times 10^{-7} \cdot 0.8}{5.54 \times 10^{-7}} = 0.7 \Omega$$

2. - Measure the thickness of the wire multiple times
   + at different angles
   - Use the gradiant of the line of best fit for calculation to compensate for systemic errors
   - use a low voltage and low current so that that only little heat is produced and resistivity doesn't varry too much.
   + use a long wire to reduce percentage error

3. $\rho_{copper} = 1.7 \times 10^{-8} \Omega m$

$$\frac{1}{1.7 \times 10^{-8}} = 5.9 \times 10^7 Sm^{-1}$$ -> very conductive

$$\rho_{polyethene} = 2 \times 10^{11}$$

$$\frac{1}{2 \times 10^{11}} = 5 \times 10^{-12} Sm^{-1}$$ -> not conductive at all

4. a) $d = 0.02 m$ $A = 0.0005 \times 0.003 \times 2 = 3 \times 10^{-6} m^2$

$$l = \frac{\pi d}{2} = \frac{\pi \cdot 0.02}{2} = 0.314$$

$$R = \frac{\rho l}{A} \qquad \rho = \frac{RA}{l}$$

$V = IR$

$$R = \frac{V}{I} = \frac{4.2 \times 10^{-3}}{18} \approx 2.33 \times 10^{-4} \Omega$$

$$\rho = \frac{2.33 \times 10^{-4} \cdot 3 \times 10^{-6}}{0.0314} = 2.23 \times 10^{-8} \Omega m$$

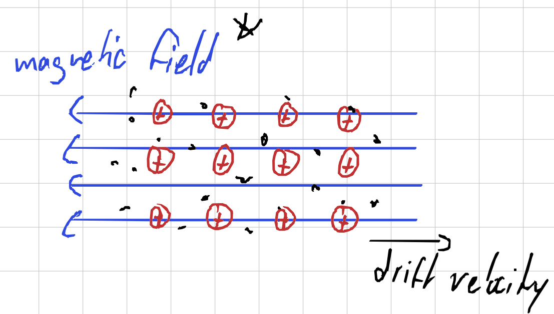
b) (gold is a better conductor than the copper inside the cables of a common voltmeter - this will influence the result ?)

High current might result in a changing temperature and therefore changing resistivity of the material

7.5 Conduction & Resistance

Conduction

- conductors have free electrons in the outer layer of their atoms
  - ↳ electrons not required for bonding can move freely through the material
- electrons move thousands of kilometers a second in random directions at room temperature → random thermal motion
- An electric field will create a tendency for the electrons to move to one direction
  - ↳ this average velocity is called drift velocity
  - ↳ the drift velocity usually is only milimiters per second

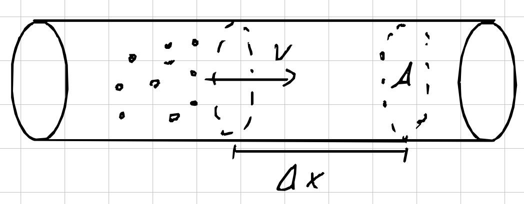


* magnetic field lines show the direction in which positive particles would mov

Transport Equation

$I = nAve$

$I \hat{=}$ current (A)
$n \hat{=}$ number of electrons per m^3 of material
aka "number density"
$A \hat{=}$ crossectional area of material (m^2)
$v \hat{=}$ drift velocity (ms^{-1})
$e \hat{=}$ electron charge $(1.602 \times 10^{-19} C)$



volume = Area × Δx
volume = Area × v × Δt
= AvΔt

$$I = \frac{Q}{\Delta t} = \frac{n \times \text{volume} \cdot e}{\Delta t} = \frac{n \times Av\cancel{\Delta t} \times e}{\cancel{\Delta t}}$$

Resistance - Atom Respective

- electrons experience high random motion, colliding with many atoms
  - ↳ this is resistance: creating a general drift of electrons through the lattice is difficult
- at high currents electrons collide more with atoms → heat is produced
  - ↳ the heat then causes greater resistance

Resistivity - Atom Perspective

- for most materials resistivity increases at higher temperatures due to incresed collisions
- semiconductors have lower resistivity at higher temperatures
  - ↳ electrons require energy to move from the atom's valence band to the conduction band
  - ↳ more heat means more free electrons per m^3

$\hat{I} = \hat{n} Ave$

Questions

1. The local average velocity of electrons moving through the conductor.

2. $I = nAve$

$$v = \frac{I}{nAe} = \frac{2.2}{18.2 \times 10^{28} \times \frac{(0.0009)^2 \pi}{4} \cdot 1.602 \times 10^{-19}}$$

$V = 6.0 \times 10^{-4}\ ms^{-1}$

3. There are more atoms that the electrons can collide with.

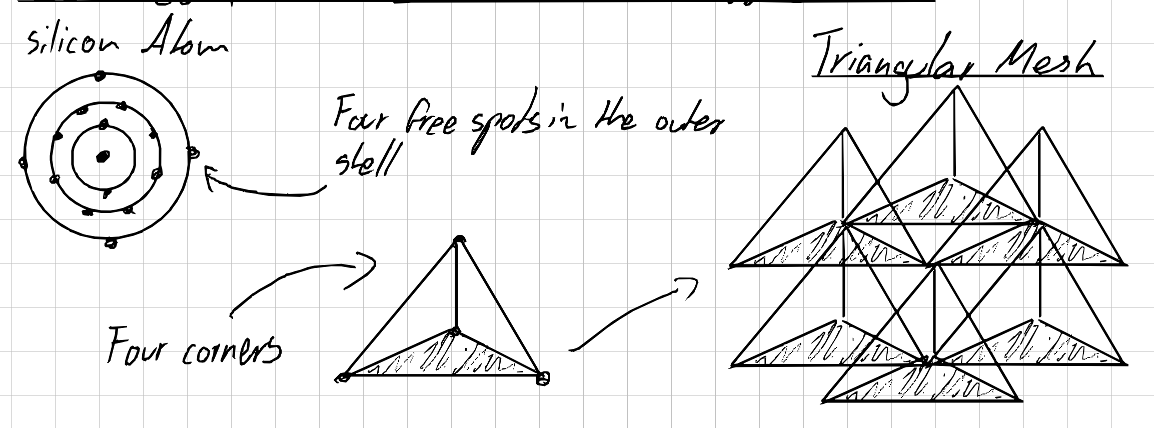
~~4.~~ 5.

$$V \cdot n = \frac{(0.02)^2 \pi}{4} \cdot 0.002 \times 8.42 \times 10^{28}$$

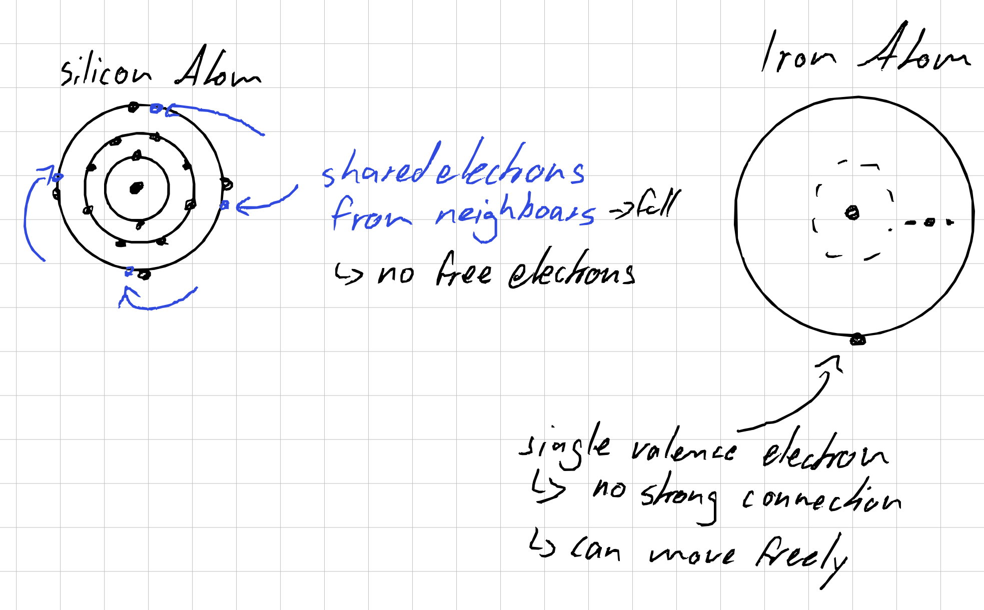
number of electrons ≈ 5.29×10^2

7.6 Semiconductors

Semiconductors - Atomic Structure



Conductivity of silicone



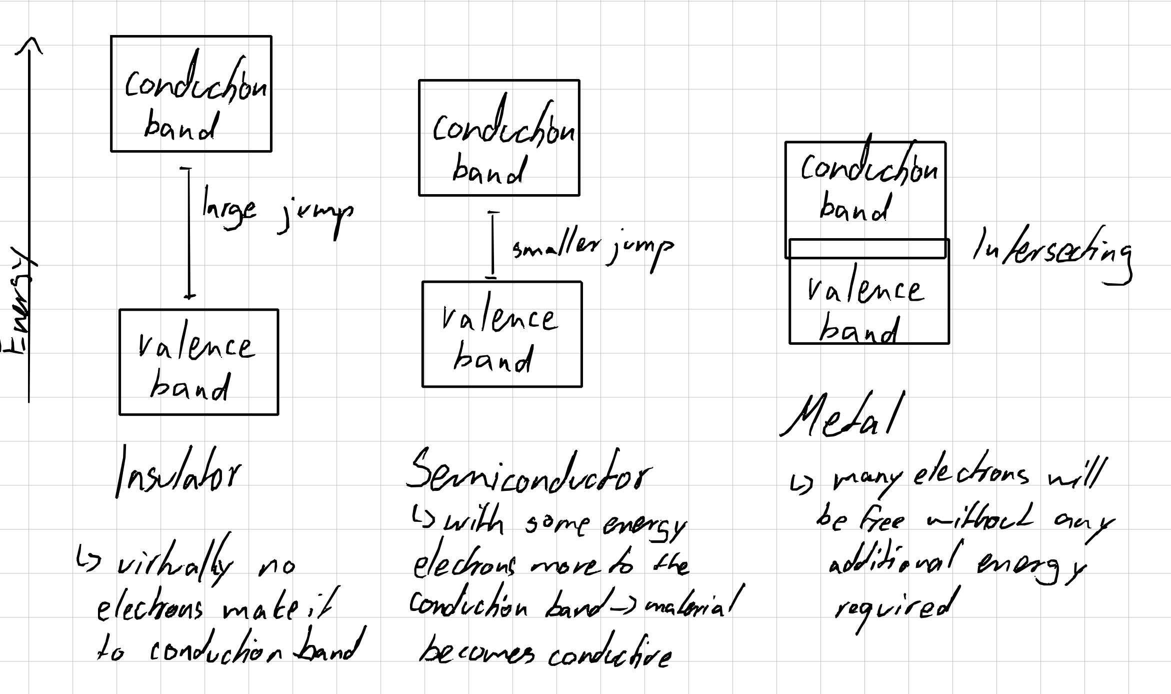
Energy Bands

- Energy levels of single atoms can supperpose with the ones from atoms in close proximity

↳ This resuts of bands that contain many similar but specific energy levels

↳ Those bands exist throughout the entire conductor

- The two outer bands are:
  - valence band: electrons still stuck to their atoms
  - conduction band: electrons can move freely through the lattice



Semiconductor and Heat

- high temperature causes:
  - increase in free electrons
  - increase in resistance due to atom vibration

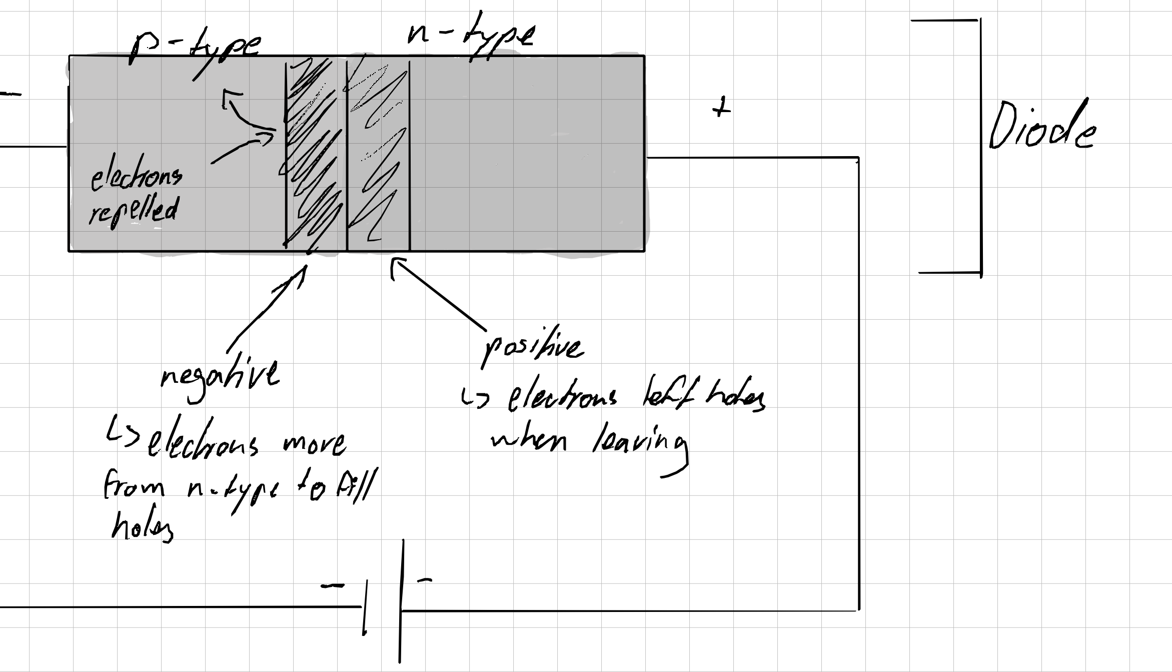
↳ increase in free electrons significantly overpowers
↳ resistivity overall drops

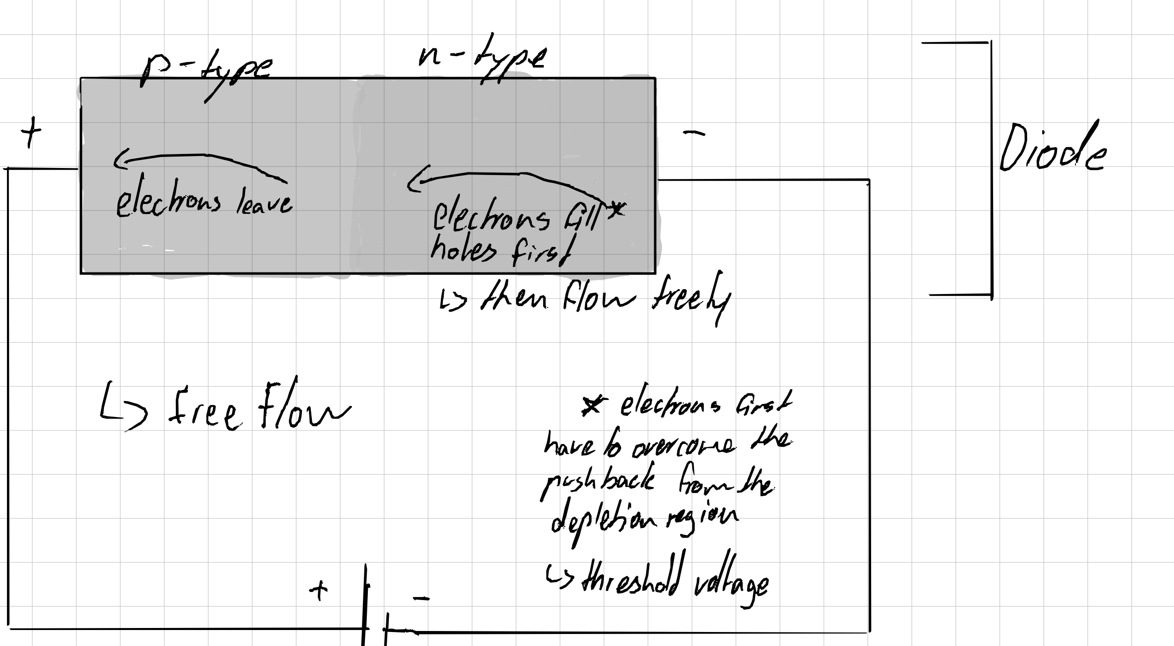
Electron Holes as Positive Charges

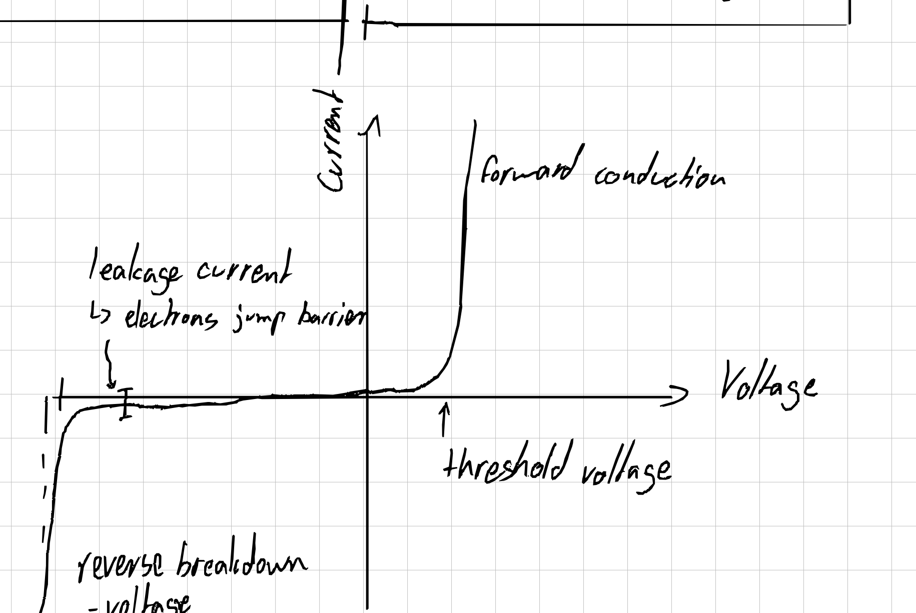
- When electrons jump between atoms they leave free spots
  ↳ "holes" are the absence of electrons
- Those holes move from pos. to neg.
  ↳ holes act like positive charges

Semiconductor Diode

- a diode consists of two semiconductor components;
  - n-type semiconductor:
    - neutral charge
    - more electrons than needed for bonding
  - p-type semiconductor:
    - neutral charge
    - missing electrons to complete lattice bonding







Variable Resistors

⇒ When energy is added to the semiconductor, the resistance decreases

Light Dependent Resistor (LIDRS)

- light excites electrons from valence band to conduction band

Thermistor

- with a rise in temperature more electrons are freed

↳ This is called "negative temperature coefficient thermistor"

↳ as temperature goes up, property goes down

Insulators

⇒ The energy gap between the valence-and conduction band are so large, virtually no electrons are free

↳ usually the required energy to move enough electrons so the material become conductive would melt it

Superconductivity

⇒ When cooling a conductor below its critical temperature, the electrons pair up (to cooper pairs - not needed)

↳ some weird quantum shit happens where electrons experience no resistance

---

1. The temperature increase provides enough energy to excite the electrons in the valence band. More electrons move to the conduction band as the temperature increases. The semiconductor becomes a better conductor.

2. This means that when lead is cooled to 7.2K or lower, current will flow through it without experiencing resistance

3. Atoms could be embeded into the lattice structure which have more electrons in their outer shell then are needed to bond with its neighbours. This will result in free electrons that can travel through the material freely.

   + The energy band gap for the impurities will have to be low enough so that the electrons are on the conduction band at room temperature

$I = nAve$ $\qquad R = \frac{\rho l}{A}$

$v = \frac{1}{nAe}$ $\qquad A = \frac{\rho l}{R}$

$P = IV$ $\qquad A = \frac{\pi d^2}{4}$

$I = \frac{P}{V}$ $\qquad 4A = \pi d^2$

$I = nAve$ $\qquad \sqrt{\frac{4A}{\pi}}$

$\rho = \frac{RA}{L}$

4A Exam Practice

1. $V = \frac{E}{Q} = \frac{J}{C}$ C x

2. $\frac{1.42}{1.602\times10^{-19}} \cdot 60^2 = 3.19\times10^{22}$ D ✓

3. C ✓

4. $R = \frac{L\rho}{A} = \frac{2L\rho}{2A}$ B ✓

5. A potential difference describes a drop in voltage across a component which makes use of electric energy. Electromotive force describes the voltage across a component which adds energy to the system. ✓ ✓

6. $I = \frac{Q}{\Delta t} = \frac{2.6\times10^{26} \times e}{15} = 2.78\times10^{6}\,A$

7. a) $I = nAve$

$v = \frac{I}{nAe} = \frac{8\times10^{-3}}{6\times10^{28} \times 3\times10^{-6} \times e} = 2.77\times10^{-7}\,ms^{-1}$

$\approx 3\times10^{-7}\,ms^{-1}$ ✓

b) $I = nAve$

$v = \frac{I}{nAe}$

Gold has much more free electrons, meaning for a current of 8mA many free electrons shift a tiny bit.
Silicon has few free electrons, therefore electrons have to move much faster to achieve the same current.

for silicon n is smaller, so v is larger ✓

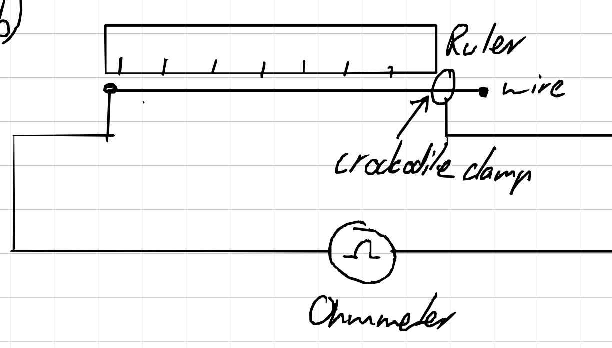
c) The resistance decreases, as more electrons move into the conduction band ✓

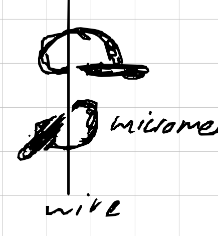
8. a) Resistivity is a fundamental property of each material, describing the resistance that a current of 1A experiences when traveling through a meter cubed of the material.
Resistance describes how much the flow of current is opposed. and varies with shape ✓

b) $R = \frac{\rho l}{A} = \frac{1.7\times10^{-8} \times 0.5}{1\times10^{-6}}$ $R = \frac{\rho l}{A}$ $8.5\times10^{-3}\,\Omega$

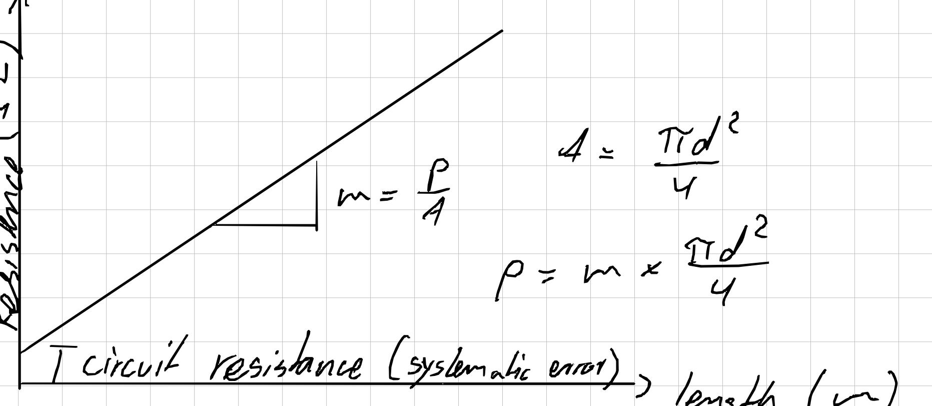
9. a) The resistivity is a fundamental property to the material, while resistance varies with shape. ✓

b)





I would measure the resistance in Ohm with an Ohmmeter.
I would measure the wire's diameter in multiple places.



$A = \frac{\pi d^2}{4}$

$\rho = m \times \frac{\pi d^2}{4}$

$R = \frac{\rho l}{A}$

$R = \frac{\rho}{A} \times l$ + resistance of rest circuit

$y = m \quad x \quad + b$

10. a) $V = I \cdot R$ X Current between two points is proportional to the voltage across both points

c) The filament gets very hot so the resistance drastically increases, as the atoms move arround alot more and more collisions occur with the electrons ✓

11. a) Diode ✓

b) virtually infinite ✓

c) $I = 0.42\,A$

$R = \frac{V}{I} = \frac{0.7}{0.42} = 1.67\,\Omega$ ✓

d) It can be used to restrict the flow of current in one direction and is often used to protect intrecate circuits ✓

12. a) $R = \frac{V}{I}$ Connect both ends to a small power supply which provides e.g. 0.1V of potential difference.

Include an ammeter and maybe another resistor for security, whose resistance can be subtracted later.

Measure the current and the terminal voltage. ✓

$R = \frac{V}{I}$ - security resistor

b) $R = \frac{\rho l}{2A}$ It would roughly half ✓

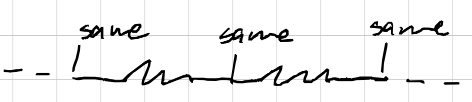
7.7 Series & Parallel Circuits

Conservation of Current

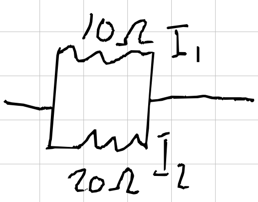
=> Throughout the entire circuit current stays conserved

Current

Series:



Parallel:



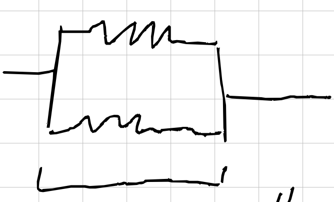
$I_1 = 2 \times I_2$

=> The current flowing through a resistor in a parallel circuit is inversly proportional to the resistance

Voltage

Series:

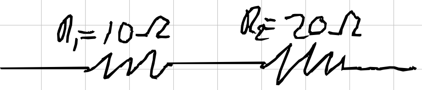
Parallel:



5V allways the same for both

Resistance

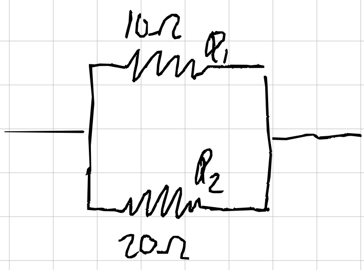
Series:



$R_{total} = R_1 + R_2$

$= 10 + 20 = 30Ω$

Parallel:



$$R_{total} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$= \frac{1}{\frac{1}{10} + \frac{1}{20}} \approx 6.67Ω$$

---

1. The charge stays the same at every point in a series circuit

2. $1.5 + 4.5 + 3 + 3 = 6 + 6 = 12V$ ✓

3. $V = IR$

$I = \frac{V}{R} = \frac{6}{24} = \frac{1}{4}A$ ✓

4. $V = IR = 0.12 \cdot 100 = 12V$

$0.24 \cdot 50 = 12V$ ✓

5. $R = 1000\left(100 + \frac{1}{\frac{1}{100} + \frac{1}{100}}\right) = 150kΩ$

$V = IR$

$I = \frac{V}{R} = \frac{230}{150000} = 1.5 \times 10^{-3} A$ ✓

6. a) $R_{total} = 8Ω$ $I = \frac{240}{8}$ $30A$ ✓

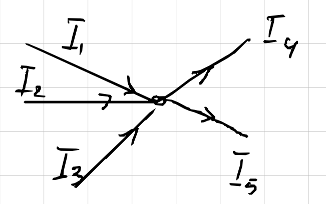
b) $I = \frac{V}{R} = \frac{240}{48} = 5A$ ✓

7.8 Electric Circuit laws

Kirchhoffs Current Law

=> The current entering a junction must equal the current leaving the junction

Or: The sum of current at a junction is zero



$$\sum I = 0$$

$$I_1 + I_2 + I_3 = I_4 + I_5$$

↳ This applies to any point in the circuit actually
↳ charge entering = charge leaving

Kirchhoff's Votage Law

- The sum of the emfs in a closed loop is equal to the sum of the potential differences in that closed loop

$$\sum \varepsilon = \sum V$$

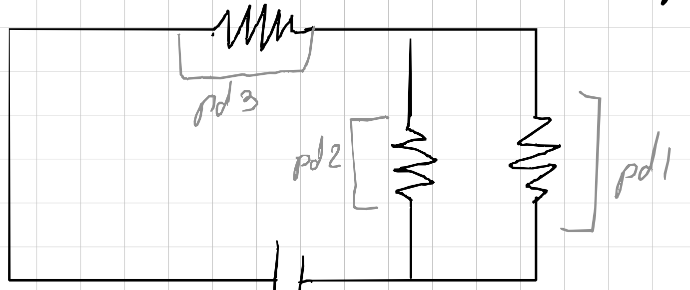
ε = emf (V)
V = potential difference (V)

$$\sum \varepsilon = \sum (IR)$$

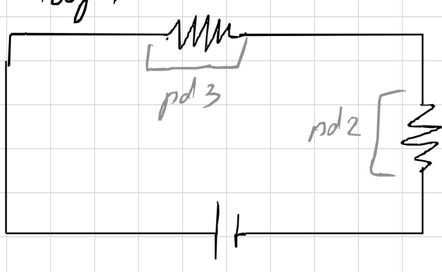
as $V = IR$

$$\sum \Delta V = 0$$

The net work done by the electric field on the charges must add up to zero in a closed loop.

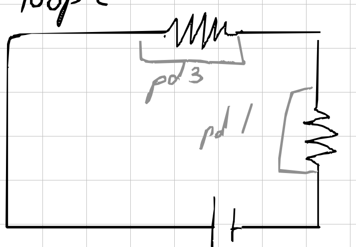


loop 1



emf = pd 3 + pd 2

loop 2



emf = pd 3 + pd 1

---

1. $V = IR$
   $I = \frac{V}{R} = \frac{4.5}{5} = 0.9\,A$ ✓

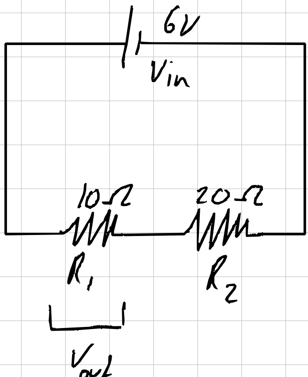
2. $1.35 - 0.45 - 0.9 = 0$ ✓

3. Net work done by electric field must be zero for a closed loop.
   $4.5 - 5 \cdot 0.9 = 0$ ✓
   $\sum \Delta V = 0$

4. $\varepsilon_{net} = V_{net} = IR = 0.38 \cdot (10 + 20) = 11.4\,V$ ✓

7.9 Potential Dividers

Potential Divider Circuit:



- used to split the voltage across two resistors
  - ↳ creates a desired potential difference over a component

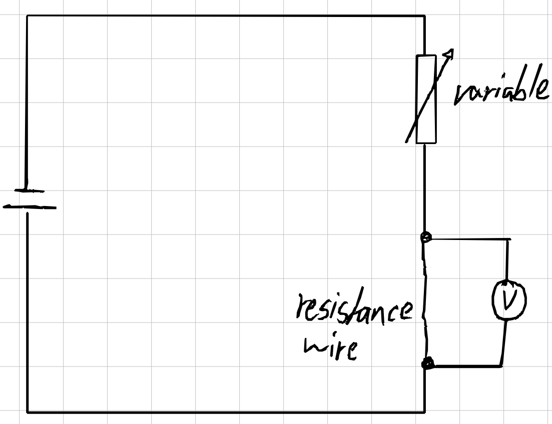
Formula:

$$V_{out} = V_{in} \frac{R_1}{R_1 + R_2}$$

↳ calculate voltage over desired component

$$V_{out} = I \cdot R_1 = \frac{V_{emf}}{R_{net}} \cdot R_1 = V_{emf} \frac{R_1}{R_1 + R_2}$$

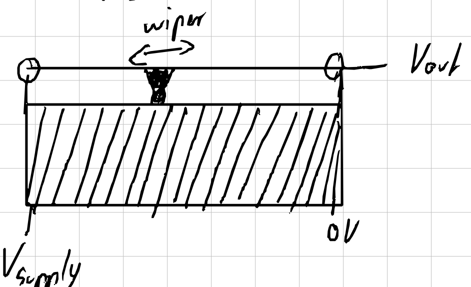
Example of Usefull Voltage Devider

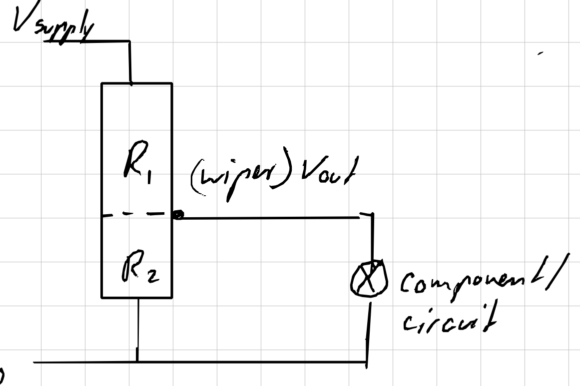


- resistor can be adjusted so that a specific voltage is reached across the resistance wire

* can e.g. be a thermistor (or any sensor)

Potentiometer

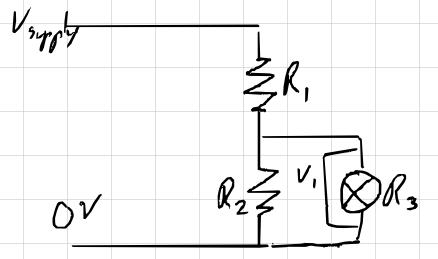




- note that the component and R_2 are in parallel
  - ↳ V_{out} will be different when there is a load applied

I personally think potentiometers aren't very practical as complete voltage dividers, as the second resistor in parallel adds unecessary complexity

but wait...



The resistance of a potentiometer is usually much less than the load:

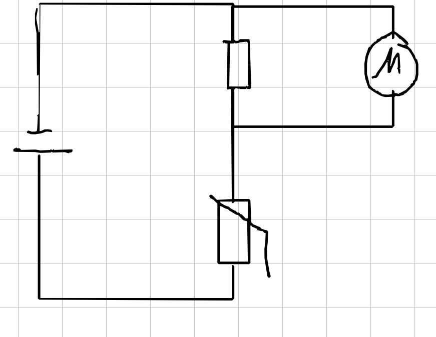
$$R_1, R_2 \ll R_3$$

e.g. R_2 and R_3 → $\dfrac{1}{\frac{1}{1\Omega} + \frac{1}{1000\Omega}}$

$$R_{2\,parallel\,3} = 0.999\,\Omega \approx 1\,\Omega$$

Questions

1. $V_2 = V_{emf} \dfrac{R_2}{R_1 + R_2} = 8 \dfrac{135}{125 + 135} = 4.15\,V$ ✓
2. $V_{out} = V_{in} \dfrac{R_1}{R_1 + R_2} = 15 \dfrac{30}{40} \approx 11.25\,V$ ✓
3.



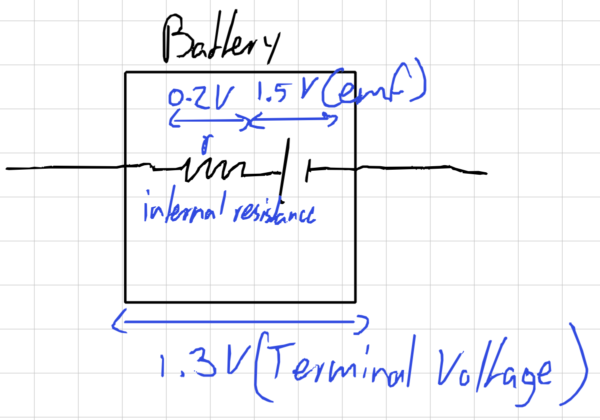
4. We have one potentiometer acting as a variable resistor and a lamp which acts as the second resistor.
   By changing the resistance of the potentiometer we can controll the voltage over the lamp ✓

7.10 EMF & Internal Resistance

Terminal Voltage

$V_{terminal} = \varepsilon - Ir$

$V_{terminal} \hat{=}$ terminal voltage (V)
$\varepsilon \hat{=}$ emf (electromotive force) (V)
$I \hat{=}$ current
$r \hat{=}$ internal resistance (Ω)

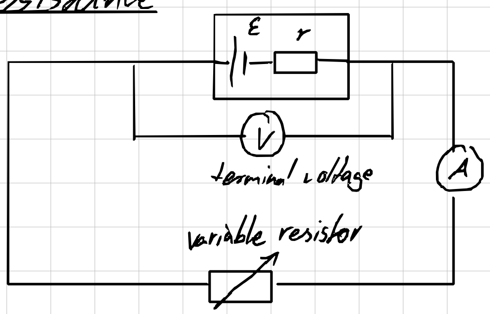


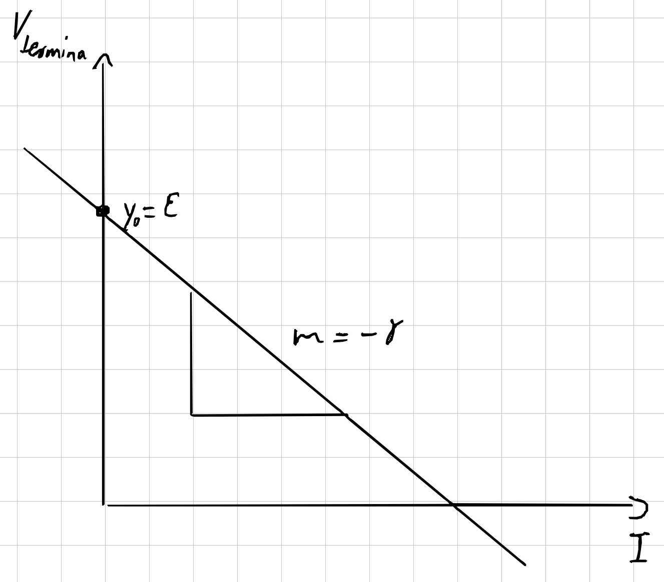
Investigating Internal Resistance

$V_{terminal} = \varepsilon - Ir$

$V_{terminal} = -r \cdot I + \varepsilon$

$y = m\,x + b$





ElectroMotive Force - Definition

emf $\hat{=}$ The work done per unit charge

as $\varepsilon = \dfrac{\text{work done by non-electrostatic forces}}{\text{charge moved}} = \dfrac{\text{Work on charges}}{Q}$

$\varepsilon = \dfrac{W}{Q}$

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1. $V_{ter.} = \varepsilon - Ir$

$V_{ter} - \varepsilon = -Ir$

$r = \dfrac{\varepsilon - V_{ter}}{I} = \dfrac{6 - 5.8}{0.4} = 0.5\,\Omega$ ✓

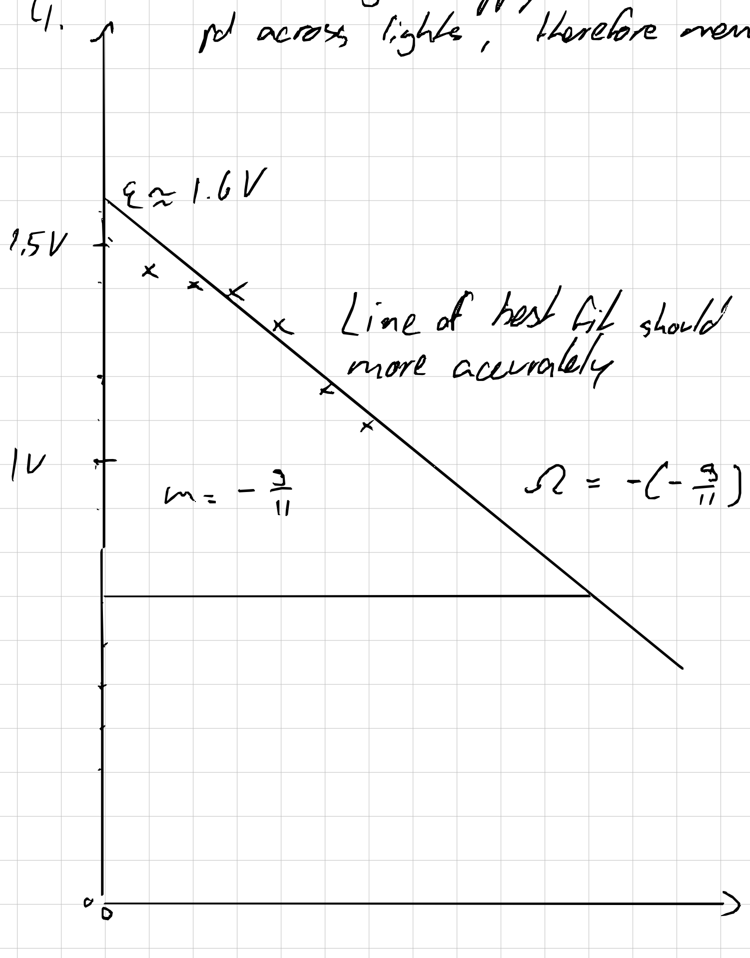
2. $I = \dfrac{V}{R} = \dfrac{1.3}{5} = 0.26\,A$

$r = \dfrac{\varepsilon - V_{terminal}}{I} = \dfrac{1.5 - 1.3}{0.26}\ 0.77\,\Omega$ ✓

3. a) At high currents the internal resistance becomes very noticable. due to $V = IR$ when the current is very high the voltage drop over the internal resistance will be large.
→ Pd across motor maybe be low

b) They must be in ~~paralel~~ with the starter motor as the motor draws a lot of current. Therefore the current passing through the lights will momentarily decrease.
→ Same might apply for series circuit with lower pd across lights, therefore mention power

4.



7.11 Power in Electric Circuits

Electrical Work

=> The energy transfered when an electric force moves a charge through a potential difference

$W = V \times Q$

$\boxed{W = VIt}$

W = work (J)
V = potential difference (V)
Q = charge (C)
I = current (A)
t = time (s)

Electrical Power

=> The rate at which electrical work is done

$\boxed{P = VI}$

P = power (W)
V = potential difference (V)
I = current (A)

$P = \frac{W}{t} = \frac{VI\cancel{t}}{\cancel{t}} = VI$

$\boxed{P = I^2 R}$

R = resistance (Ω) $P = VI = (IR)I = I^2R$

$\boxed{P = \frac{V^2}{R}}$

$P = VI = V\left(\frac{V}{R}\right) = \frac{V^2}{R}$

Efficiency

=> Ratio of the useful energy/power output to the total energy output

units cancel anyway

$$\text{efficiency} = \frac{\text{useful power output}}{\text{total power output}} = \frac{\text{useful energy output}}{\text{total energy output}}$$

=> efficiency describes how good a machine is at using energy usefully

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Questions

1. $W = VIT = 1.03 \times 1.8 \cdot 11 \approx 20.4$ J ✓

2. $P = IV = 1.8 \times 1.03 \approx 1.85$ W ✓

3. a) $P = I^2R$
$= 1.76^2 \cdot 470$
$= 1456$ W ✓

b) $P = \frac{V^2}{R}$
$= \frac{9^2}{470} = 0.172$ W ✓

4. a) $\text{efficiency} = \frac{W_{useful}}{W_{total}} = \frac{95}{100} = 0.95$ ✓

b) This is because if energy is supplied continuously the ratio of energy spent usefully in relation to the total energy stays the same.

Exam Practice

1. B ✓
2. A ✓
3. $V_{ter} = \mathcal{E} - Ir$

   $r = \frac{\mathcal{E} - V_{ter}}{I} = \frac{6 - 5.8}{0.44} = 0.45\,\Omega$ B ✓
4. C ✓
5. A ✓ 6a) ✓

6. ~~a)~~ b) Bulb A will be brighter, as it has less resistance so more current flows through it. Both have the same pd across them, so current difference will be the only significant factor + $P = \frac{V^2}{R}$

7. The voltage across both lamps is the same. Because B has a lower resistence, more current will flow through B. Due to $P = IV$ we know that when current increases the power will be larger for the component, so B will be brighter.

8. a) $V_{out} = V_{in}\frac{R_1}{R_1 + R_2} = 9\frac{40}{40+80} = 3V$ ✓ (3.0V)

   b) As the thermistor is in parallel with the 40 Ω resistor, an increase in resistance through the thermistor means the overall resistance of the two resistors increases. ✓ The total resistance in the circuit increases, less current flows at the same voltage ✓

9. a) $P = \frac{V^2}{R}$

   $R = \frac{V^2}{P} = \frac{220^2}{1000} = 48.4\,\Omega$ ✓

   b) $1000 \times 60 \cdot 3 = 180\,000\,J$ ✓

   c) i. $P = \frac{110^2}{48.4} = 250\,W$

   $P \cdot t = W$

   $t = \frac{W}{P} = \frac{180\,000}{250} = 720\,s$ or 12 min ✓

   ii. Probably A fuse inside it would burn. Or the heater might burn if there is no fuse, as the power output would drastically increase due to $P = \frac{V^2}{R}$ ✓

10. a) $P = IV$

    $I = \frac{P}{V} = \frac{4.8}{230} = 0.021\,A$ ✓

    b) i $V \cdot A = W$

    $\frac{E}{Q} \cdot \frac{Q}{t} = W$

    $\frac{E}{t} = W$

    $W = W$ ✓ → use units next time

    ii $P_{out} = IV = 5 \cdot \cancel{0.5}\ 0.1 = \cancel{2.5W}\ 0.5W$

    efficiency $= \frac{\cancel{2.5}\ 0.5}{4.8} = \cancel{0.52}$ 0.1042 or ~~52%~~ 10.42% ✗

    iii a lot of energy will be transfered into heat ✓

11. a) i) $2 \cdot 0.2 + 3.6 = 4\,\Omega$ ✓

    ii. $I = \frac{V}{R} = \frac{3}{4} = 0.75\,A$ ✓

    iii. $P = I^2R = 0.75^2 \cdot 3.6 \approx 2.0\,W$ ✓

    b) As the internal resistance increases, a larger share of the power will be lost within the cell and the radiator will be but so lightly less hot. This is due to a decreased terminal voltage, and the equation $P = \frac{V^2}{R}$ stating that with less of a potential difference the power decreases too.

12. Hoffmann's Current Law states that the sum of all current entering a junction must equal the sum of currents leaving the junction. No current appears or disappeares out of nowhere, current, and therefore charge, stay conserved. ✓

    Hoffman's Voltage Law states that the sum of all changes in voltage around a closed loop in a circuit must equal zero. The emf voltages perfectly cancel with all pd's, as energy within the system has to stay conserved, ✓ and energy added must equal energy removed from the system. ✓