Explanations

Solving Functions of the form ...

1. $\sqrt{ax+b} = \sqrt{dx^2+ex+f}$
   $\Rightarrow ax+b = dx^2+ex+f$
   $\Rightarrow 0 = dx^2 + (e-a)x + (f-b)$
   $\Rightarrow$ ...
2. $\sqrt{ax^2+bx+c}$
   $\Rightarrow ax^2+bx+c = 0$
3. $\sqrt{ax^2+bx+c}\sqrt{dx^2+ex+f}$
   $\Rightarrow ax^2+bx+c = 0$
   $\Rightarrow dx^2+ex+f = 0$
   $\Rightarrow$ Apply solutions from solving ax^2+bx+c and dx^2+ex+f and see if the solutions $\Rightarrow$ ...
   $\Rightarrow$ ...
   $\Rightarrow$ (x^2+x-2)

Intermediate Quadratic Example

$$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Intermediate number of rational solutions

$0 < b^2-4ac \Rightarrow$ 2 distinct solutions
$0 = b^2-4ac \Rightarrow$ 1 repeat solution
$0 > b^2-4ac \Rightarrow$ no real solutions

Domain and Range definition

$f(x) = \sqrt{x+2}$ $\quad f: \{x \in \mathbb{R} \mid x \geq 2\} \to \mathbb{R}$ — output of the function

Domain: $\{x \in \mathbb{R} \mid x \geq 2\}$
Range: $\{y \in \mathbb{R} \mid y \geq 0\}$

Function Transformations

$y = f(x+c) + d$

$(x, y) \to (x-c, y+d)$

Vertical transformations

$y = f(x) + c$ translates $\begin{pmatrix} 0 \\ c \end{pmatrix}$

$y = -f(x)$ reflection in the x-axis

$y = f(-x)$ reflection in the y-axis

$y = -f(-x)$ reflection in both axes

Horizontal transformations

$y = f(x-c)$ translates $\begin{pmatrix} c \\ 0 \end{pmatrix}$

$y = f(-x)$ reflection in the y-axis

$y = -f(-x)$ horizontal stretch factor $\frac{1}{c}$

Vertex Addition

$f(x) = \frac{1}{x}$, $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$f(x+3, y+2)$

$(1, 0)$

$\Rightarrow \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

Lines & Harmonic

Exponential:
- Linearly than linear
- Exponential than linear
- Power and linear

Log:
- Linear than log
- Power than log

$\frac{d}{dx} = \frac{dy}{dx} \cdot \frac{dz}{dy}$

Reflection in the Length

$\sqrt{x^2} = |x|$ reflect
$\sqrt{x^2} = x$ no reflect

$\frac{\sqrt{x^2+1}}{x} = \frac{x}{x}$ $\Rightarrow$ $\frac{x}{x} = 1$

$\Rightarrow$ $\frac{x}{x}$ is only defined on $\mathbb{R}$

$\frac{\sqrt{x^2+1}}{x} = \frac{x}{x}$ $\Rightarrow$ $\frac{x}{x} = 1$

The length $= 1$ $\Rightarrow$ length of the number.

The length $\frac{\sqrt{x^2}}{x} = 0 \Rightarrow x = 0$ is a degree

The same definition $\Rightarrow$ $\frac{x}{x}$ $\Rightarrow$ $\frac{x}{x}$ $\Rightarrow$ $\frac{x}{x}$ $\Rightarrow$ $\frac{x}{x}$ $\Rightarrow$ $\frac{x}{x}$ $\Rightarrow$ $\frac{x}{x}$

$\sqrt{x^2} = |x|$ reflect

$\sqrt{x^2} = x$ reflect

Cosine Rule

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$

Sine Rule

Slope Regions

$f(-3) = 2$ $\quad$ $f(-3) = 2$ $\quad$ $f(-3) = 2$

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Function (Domain)

There are three types of functions, which are based on the input of the function. Linear, Quadratic, Cubic. Linear functions are the simplest, and are of the form $ax + b$. Quadratic functions are of the form $ax^2 + bx + c$. Cubic functions are of the form $ax^3 + bx^2 + cx + d$.

Cubic Function (Domain)

Cubic functions are the most complicated of the three functions. Their domain is all real numbers, but they can be restricted because values. For example, the function $f(x) = \sqrt{x}$ is only defined for values $x \geq 0$.

Quadratic Function

Quadratic functions, which function in slightly less complicated than cubic functions. Quadratic functions are of the form $ax^2 + bx + c$.

Linear Function

The coefficient of function is a predetermined constant, different for each function. Linear functions are of the form $ax + b$. Linear functions are the most simple of the three functions, and are of the form $ax + b$.

Angle of Function

$f(x) = \frac{1}{x}$ $\quad$ $f(x) = \frac{1}{x}$ $\quad$ $f(x) = \frac{1}{x}$

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Asymptote Function

$f(x) = \frac{1}{x}$ $\quad$ $f(x) = \frac{1}{x}$ $\quad$ $f(x) = \frac{1}{x}$

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$f(x) = \frac{1}{x}$ $\quad$ $f(x) = \frac{1}{x}$ $\quad$ $f(x) = \frac{1}{x}$

Inverted Asymptote

$f(x) = \frac{1}{x}$ $\quad$ $f(x) = \frac{1}{x}$ $\quad$ $f(x) = \frac{1}{x}$

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Inverted Asymptote Function

$f(x) = \frac{1}{x}$ $\quad$ $f(x) = \frac{1}{x}$ $\quad$ $f(x) = \frac{1}{x}$

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Inverted Domain

$f(x) = \frac{1}{x}$ $\quad$ $f(x) = \frac{1}{x}$ $\quad$ $f(x) = \frac{1}{x}$

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Parts of a Circle

Inverted

6.1 Velocity and Acceleration

some note: 1. Check the accelerations system by continuous

2. Consider the boundary line moving

3. Use $\frac{dv}{dt}$ to big !!

Kinematic equations or SUVAT-Equations:

1) displacement s, Δx

2) initial velocity u

3) velocity at time t: v

4) acceleration a

5) time t

$v_f = v_i + at$ $\qquad v_f^2 = v_i^2 + 2a\Delta x$

$\Delta x = \frac{1}{2}(v_i + v_f)t$ $\qquad \Delta x = v_i t + \frac{1}{2}at^2$

End of flight Review

F = ma = 3.5

To $v_i = 80\,m/s$ $\quad \Delta x = ?$ $\quad$ $t_f = 10\,s$ $\quad$ $v_f = 0\,m/s$ $\quad$ $a = ?$

$a = -2.0\,m/s^2$ $\quad$ $t_f = 0$ $\qquad$ $a = 2.0\,m/s^2$ for 3s, then 0

$\Delta x = \frac{1}{2}(v_i + v_f)t$ $\qquad v_f = v_i + at$

$\Delta x = \frac{1}{2}(80 + 0)$ $\qquad \Delta x = \frac{1}{2}(v_i + v_f)t$

Could be $v_i = v_f = 0$

$\Delta x = 40 \cdot (-0.5)$

$\Delta x = 20\,m$

$\Delta x = \frac{1}{2}(v_i + v_f)t$

$\Delta x = \frac{1}{2}(\frac{1}{2}(v_i + v_f)t + \frac{1}{2}(v_i + v_f)t)$

$\Delta x = \frac{1}{2}(\frac{1}{2}(0 + 3) + 3) = 3m$

$\Delta x = \frac{1}{2}(3 + 3) = 3m$

$2s = 3 = 3m$

$\Delta x = 3\,m$

Ex $v_i = 40\,m/s$ $\quad$ $t = 4\,s$

$v_f = 0\,m/s$

$a = \frac{v_f - v_i}{t}$

$a = \frac{0 - 40}{4}$

$a = -10\,m/s^2$

a) $\Delta x = ?$

$\Delta x = v_i t + \frac{1}{2}at^2$

$v_f = v_i + at$

$0 = 40 + (-10) \cdot t$

$t = 4$

$\Delta x = 40 \cdot 4 + \frac{1}{2} \cdot (-10) \cdot 4^2$

$= 160 + (-20 \cdot 16)$

$= 160 - 320$

$= -160\,m$

b) $v_f = ?$ $\quad$ $t = 2\,s$

$v_f = v_i + at$

$v_f = 40 + (-10) \cdot 2$

$v_f = 20\,m/s$

Ex $v_i = 0\,m/s$ $\quad$ $\Delta x = 20\,m$ $\quad$ $t = 2\,s$ $\quad$ $a = ?$

$\Delta x = \frac{1}{2}(v_i + v_f)t$

$20 = \frac{1}{2}(0 + v_f) \cdot 2$

$20 = 0 + v_f$

$v_f = 20\,m/s$

$v_f = v_i + at$

$20 = 0 + a \cdot 2$

$a = 10\,m/s^2$

$\Delta x = v_i t + \frac{1}{2}at^2$

$\Delta x = 0 \cdot 2 + \frac{1}{2} \cdot 10 \cdot 2^2$

$\Delta x = 20\,m$

Ex $v_i = 0\,m/s$ $\quad$ $\Delta x = 20\,m$ $\quad$ $t = 2\,s$ $\quad$ $a = ?$

$\Delta x = \frac{1}{2}(v_i + v_f)t$

$20 = \frac{1}{2}(0 + v_f) \cdot 2$

$20 = 0 + v_f$

$v_f = 20\,m/s$

$v_f = v_i + at$

$20 = 0 + a \cdot 2$

$a = 10\,m/s^2$

$\Delta x = v_i t + \frac{1}{2}at^2$

$\Delta x = 0 \cdot 2 + \frac{1}{2} \cdot 10 \cdot 2^2$

$\Delta x = 20\,m$

$\frac{\Delta x}{\Delta t} = \frac{\Delta x}{\Delta t}$ $\qquad \frac{\Delta x}{\Delta t} = \frac{\Delta x}{\Delta t}$

$\frac{20 - 0}{2 - 0} = \frac{20 - 0}{2 - 0}$

$10 = 10$

$\frac{\Delta x}{\Delta t} = \frac{\Delta x}{\Delta t}$

$\frac{20 - 0}{2 - 0} = \frac{20 - 0}{2 - 0}$

$10 = 10$

4. a) $v_i = 0\,m/s$ $\quad$ $v_f = ?$

$v_f^2 = v_i^2 + 2a\Delta x$

$v_f^2 = 0^2 + 2 \cdot 10 \cdot 100$

$v_f^2 = 2000$

$v_f = \sqrt{2000} = \sqrt{(10 \cdot 200)} = 10 \cdot \sqrt{20} = 44.7\,m/s$

5. $v_i = 0\,m/s$

$a = 10\,m/s^2$

$\Delta x = 100\,m$

$v_f = ?$

$v_f^2 = v_i^2 + 2a\Delta x$

$v_f^2 = 0^2 + 2 \cdot 10 \cdot 100$

$v_f^2 = 2000$

$v_f = \sqrt{2000} = \sqrt{(10 \cdot 200)} = 10 \cdot \sqrt{20} = 44.7\,m/s$

6. $\Delta x = \frac{v_f + v_i}{2} \cdot t$

$\Delta x = \frac{0 + 40}{2} \cdot 4$

$\Delta x = 20 \cdot 4$

$\Delta x = 80\,m$

$v_f = v_i + at$

$0 = 40 + a \cdot 4$

$a = -10\,m/s^2$

$\Delta x = v_i t + \frac{1}{2}at^2$

$\Delta x = 40 \cdot 4 + \frac{1}{2} \cdot (-10) \cdot 4^2$

$\Delta x = 160 - 80$

$\Delta x = 80\,m$

7. $v_i = 0\,m/s$ $\quad$ $\Delta x = 40\,m$

$v_f = ?$

$v_f^2 = v_i^2 + 2a\Delta x$

$v_f^2 = 0^2 + 2 \cdot 10 \cdot 40$

$v_f^2 = 800$

$v_f = \sqrt{800}$

$v_f = 28.3\,m/s$

8. $\Delta x = \frac{v_f + v_i}{2} \cdot t$

$\Delta x = \frac{0 + 40}{2} \cdot 4$

$\Delta x = 20 \cdot 4$

$\Delta x = 80\,m$

$v_f = v_i + at$

$0 = 40 + a \cdot 4$

$a = -10\,m/s^2$

$\Delta x = v_i t + \frac{1}{2}at^2$

$\Delta x = 40 \cdot 4 + \frac{1}{2} \cdot (-10) \cdot 4^2$

$\Delta x = 160 - 80$

$\Delta x = 80\,m$

9. $\Delta x = \frac{v_f + v_i}{2} \cdot t$

$\Delta x = \frac{0 + 40}{2} \cdot 4$

$\Delta x = 20 \cdot 4$

$\Delta x = 80\,m$

$v_f = v_i + at$

$0 = 40 + a \cdot 4$

$a = -10\,m/s^2$

$\Delta x = v_i t + \frac{1}{2}at^2$

$\Delta x = 40 \cdot 4 + \frac{1}{2} \cdot (-10) \cdot 4^2$

$\Delta x = 160 - 80$

$\Delta x = 80\,m$

10. $\Delta x = \frac{v_f + v_i}{2} \cdot t$

$\Delta x = \frac{0 + 40}{2} \cdot 4$

$\Delta x = 20 \cdot 4$

$\Delta x = 80\,m$

$v_f = v_i + at$

$0 = 40 + a \cdot 4$

$a = -10\,m/s^2$

$\Delta x = v_i t + \frac{1}{2}at^2$

$\Delta x = 40 \cdot 4 + \frac{1}{2} \cdot (-10) \cdot 4^2$

$\Delta x = 160 - 80$

$\Delta x = 80\,m$

11. $\Delta x = \frac{v_f + v_i}{2} \cdot t$

$\Delta x = \frac{0 + 40}{2} \cdot 4$

$\Delta x = 20 \cdot 4$

$\Delta x = 80\,m$

$v_f = v_i + at$

$0 = 40 + a \cdot 4$

$a = -10\,m/s^2$

$\Delta x = v_i t + \frac{1}{2}at^2$

$\Delta x = 40 \cdot 4 + \frac{1}{2} \cdot (-10) \cdot 4^2$

$\Delta x = 160 - 80$

$\Delta x = 80\,m$

12. $\Delta x = \frac{v_f + v_i}{2} \cdot t$

$\Delta x = \frac{0 + 40}{2} \cdot 4$

$\Delta x = 20 \cdot 4$

$\Delta x = 80\,m$

$v_f = v_i + at$

$0 = 40 + a \cdot 4$

$a = -10\,m/s^2$

$\Delta x = v_i t + \frac{1}{2}at^2$

$\Delta x = 40 \cdot 4 + \frac{1}{2} \cdot (-10) \cdot 4^2$

$\Delta x = 160 - 80$

$\Delta x = 80\,m$

13. $\Delta x = \frac{v_f + v_i}{2} \cdot t$

$\Delta x = \frac{0 + 40}{2} \cdot 4$

$\Delta x = 20 \cdot 4$

$\Delta x = 80\,m$

$v_f = v_i + at$

$0 = 40 + a \cdot 4$

$a = -10\,m/s^2$

$\Delta x = v_i t + \frac{1}{2}at^2$

$\Delta x = 40 \cdot 4 + \frac{1}{2} \cdot (-10) \cdot 4^2$

$\Delta x = 160 - 80$

$\Delta x = 80\,m$

14. $v = \frac{\Delta x}{\Delta t}$

$v = \frac{100\,m}{10\,s}$

$v = 10\,m/s$

15. $v = \frac{\Delta x}{\Delta t}$

$v = \frac{100\,m}{10\,s}$

$v = 10\,m/s$

16. $v = \frac{\Delta x}{\Delta t}$

$v = \frac{100\,m}{10\,s}$

$v = 10\,m/s$

Circle and Intersects

Formula:

$x^2+y^2=r^2$ e.g. for unit circle: $x^2+y^2=1$

or $(x-h)^2+(y-k)^2=r^2$

$h \hat{=}$ horizontal shift
$k \hat{=}$ vertical shift
$r \hat{=}$ radius

Completing the square to find r and coordinates of center:

$x^2+y^2-6x+8y=11$ 1. group x an y terms

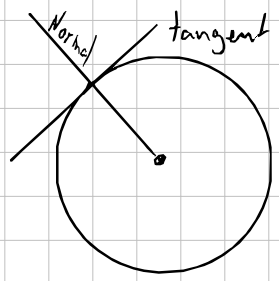
$x^2-6x+y^2+8y=11$ 2. Complete the square

$(x^2-6x+3^2)-3^2+(y^2+8y+4^2)-4^2=11$

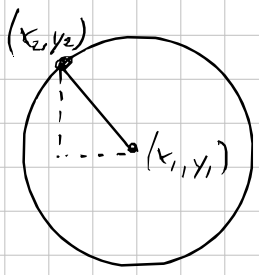
$(x-3)^2+(y+4)^2-3^2-4^2=11$ $11+16+9=36$

$(x-3)^2+(y+4)^2=36$

Find normal and tangent to a square



$m_t \times m_n = -1$

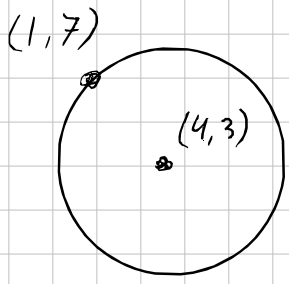


Step #1:

find the gradient of the normal

! you require the coordinates of the center and the point on the circle

$m_n = \frac{y_2-y_1}{x_2-x_1}$



Step #2:

$m_n \times m_t = -1$

so $m_t = -\frac{1}{m_n}$

$m_n = \frac{7-3}{1-4} = -\frac{4}{3}$

$m_t = -\frac{1}{-\frac{4}{3}} = \frac{3}{4}$

Step #3:

m_n for normal
m_t for tangent

Plug (x_2,y_2) and m_n or m_t into $y=mx+b$ and solve for b

$y=mx+b$ $(1,7)$

$7=\frac{3}{4}(1)+b$ $7-\frac{4}{3}=\frac{17}{3}$

$\frac{25}{3}=b$ so $y=\frac{3}{4}x+\frac{17}{3}$

now for the normal:

$(1,7)$ $m=-\frac{4}{3}$ $y=mx+b$

$7=-\frac{4}{3}(1)+b$ $1+\frac{4}{3}$

$\frac{25}{3}=b$

$y=-\frac{4}{3}x+\frac{25}{3}$

Some calculations for 2.1

End of chapter review p. 67 - 69

1. $g(x) = 3(3x-1) = 5(3x-1) - (3x-1)^2$

$= 15x - 5 - (9x^2 - 6x + 1)$

$= 15x - 5 - 9x^2 + 6x - 1 = 21x - 9x^2 - 6$

$= -(9x^2 - 21x) - 6$

$= -9(x^2 - \frac{7}{3}x) - 6$

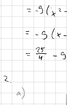
$= -9\left(x^2 - \frac{7}{3}x + \left(\frac{7}{6}\right)^2\right) - 6 + \left(\frac{7}{6}\right)^2 9$

$= -9\left(x - \frac{7}{6}\right)^2 + \frac{25}{4}$

$= \frac{25}{4} - 9\left(x - \frac{7}{6}\right)^2$

2. $y = -f\left(\frac{1}{2}x\right)$

a)



b) $y = f(x) \to y = f(\frac{1}{2}x)$

$-1x \times 2$

1. Reflection over y-axis

2. shift left by 3

3. $y = x^2 + 6x + 8$

a) $y = (x^2 + 6x + 3^2) + 8 - 3$ (translation by $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$)

$y = (x+3)^2 - 1$ $V(-3,-1)$, concave up

$y = ax^2 + bx + c$: $y = 0^2 + 6 \cdot 0 + 8 = 8$

x-int: $x^2 + 6x + 8 = 0$ Alt M3

$(x+2)(x+4) = 0$ 4 2

$x_1 = -2$ $x_2 = -4$

b) $+\begin{pmatrix} -3 \\ 0 \end{pmatrix}$, $3f(x)$

$y = (x+3)^2 - 1$

$y = (x+1)^2 - 1$

$y = 3(x+1)^2 - 3$

$y = 3(x^2 + 2x + 1) - 3$

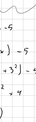
$y = 3x^2 + 6x$

$y = 3x^2 + 6x$ ✓

4. $f(x) = x^{3/2}$, $x \geq 0$

a) $f^{-1}(x) = \frac{1}{x^2 - 2}$, $\{-2 \leq x \leq 4, x \neq \pm\sqrt{2}, x \in \mathbb{R}\}$

b) Hmm, let's turn those calculators on



Fine

5. i $-x^2 + 6x - 5$

$= -(x^2 - 6x) - 5$

$= -(x^2 - 6x + 3^2) - 5 + 3^2$

$= -1(x-3)^2 + 4$

ii $V(3,4)$

iii $-1(x-3)^2 + 4$, $x \geq 5$ $\quad -x^2 + 6x - 5 = 0$

$\frac{1}{-(x-3)^2 - 4}$, $x \neq 1, x \neq 5$ $\quad x^2 - 6x + 5 = 0$

$x_{1,2} = 3 \pm \sqrt{3^2 - 5}$

Domain: $\{x \neq -5, x \neq 1, x \in \mathbb{R}\}$

$x_{1,2} = 3 \pm 2$

6. $x^2 - 4x + k$, $x \geq p$ $\quad x_1 = 1 \quad x_2 = 5$

i $(x+a)^2 + b + k$

$(x^2 - 4x) + k$

$(x^2 - 4x + 2^2) + k - 2^2$

$(x-2)^2 - 4 + k$

ii Range: $\{-4 + k \leq x, x \in \mathbb{R}\}$

iii $V(2, k-4)$

7. i $3x - 2$ max between -1 and 1

$3(1) - 2 = 1$ min between -1 and 1

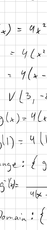
$3(-1) - 2 = -5$

$\frac{4}{5-x}$ max / min between 1 and 4

$\frac{4}{5-1} = 1 \quad \frac{4}{5-4} = 4$

global max = 4, global min = -5

Range: $\{-5 \leq x \leq 4, x \in \mathbb{R}\}$



iii $g(x) = \begin{cases} \frac{1}{3x-2} & \text{for } -1 \leq x \leq 1 \\ \frac{5-x}{4} & \text{for } 1 \leq x \leq 4 \end{cases}$

8. i $f(x) = 4x^2 - 24x + 11$, $x \in \mathbb{R}$

$= 4(x^2 - 6x + 3^2) + 11 - 4 \cdot 3^2$

$= 4(x-3)^2 - 25$

$V(3, -25)$

ii $g(x) = 4(x-3)^2 - 25$, $x \leq 1$

$g(1) = 4(1-3)^2 - 25 = 16 - 25 = -9$

Range: $\{g(x) \geq -9, g(x) \in \mathbb{R}\}$

iii $g^{-1}(x) = \frac{1}{4(x-3)^2 - 25}$

Domain: $\{x \geq -9, x \in \mathbb{R}\}$

9. i $2x^2 - 12x + 13$

$= 2(x^2 - 6x)$

$= 2(x^2 - 6x + 3^2) + 13 - 2 \cdot 3^2$ $\quad 13 - 18 = -5$

$= 2(x-3)^2 - 5$

ii $f(x) = 2(x-3)^2 - 5$, $x \geq k$

$x = 2x^2 - 12x + 13$

$0 = 2x^2 - 13x + 13$

$0 = x^2 - \frac{13}{2}x + \frac{13}{2}$

$x_{1,2} = \frac{13}{4} \pm \sqrt{\left(\frac{13}{4}\right)^2 - \frac{13}{2}}$

$x_1 \approx 3.52 \quad x_2 \approx 4.48$

so $k = -4.48$

iii $2(x-3)^2 - 5$, $x \geq 7$

$V(3, -5)$

x^2

V Domain

$2(7-3)^2 - 5 = 32 - 5 = 27$

Range: $\{f(x) \geq 27, f(x) \in \mathbb{R}\}$

iv. $f^{-1}(x) = \frac{1}{2(x-3)^2 - 5}$, $x \geq 7$, ~~$x \geq 1.42$~~, ~~$x \leq 4.48$~~ below 7

$0 = 2x^2 - 12x + 13$

$0 = x^2 - 6x + \frac{13}{2}$

$x_{1,2} = 3 \pm \sqrt{3^2 - \frac{13}{2}}$

$x_1 \approx 1.42 \quad x_2 \approx 4.58$

10. i $x^2 - 2x - 15$

$(x^2 - 2x + 1^2) - 15 - 1$

$(x-1)^2 - 16$

ii Range: $c \leq f(x) \leq d$

$V(1, -16)$ so $c \geq -16$

iii $f(x) = (x-1)^2 - 16$

Range: $0 \leq f(x) \leq 65$

Domain: $p \leq x \leq q$, find p & q

$9 = (x-1)^2 - 16$

$0 = x^2 - 2x + 1 - 25$

$0 = x^2 - 2x - 24$

End of Chapter review 2

Ms. Ng's suggested homework examples of mistakes

1. $f(x) = 3x^2$

$x \geq 0$

$y^2(x) = \sqrt{y} \quad y(x) = \pm\sqrt{x}$ $\quad 5(3x-1) - (3x-1)^2$

$y = \sqrt{x}$ $\quad 5(3x-1)$

$y = 3x^2 - 6x + 3$

$= (3x-1)(5-3x+1)$

$= 3x^2 - 6x + 3$

$= 3(x^2 - 2x + 1)$

$= 3(x-1)^2$

2. a)

$y = f^{-1}(x)$

shaded in $-\frac{1}{2} \leq x$

$f(x)$ $\quad$ $f(x) = x$ meet

b) $y = f(x)$ & $y = f(2-x)$ $\quad -(x-2)$

$f(x) = x$ $\quad f(-x)$

2: shift right by 2

Reflection by y

$f(2-x)$

$f(x)$ & $f(2-x)$ meet

$x = 1$

$x = 3$

$f(x) = f(2-x)$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$x = 2$

$f(x) = f(2-x)$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$x = 2$

3. $f(x) = x^2 - 6x + 8$

$= (x-3)^2 - 9 + 8$

$= (x-3)^2 - 1$

$= (x-4)(x-2)$

$x = 3 \pm \sqrt{1}$

$x = 3 \pm 1$

$x = 4, 2$

Axis of $f^{-1}(x)$ is $x = 3$

$f(3) = -1$ $\quad (1)$

Range: $\{f(x) \in \mathbb{R}, f(x) \in \mathbb{R}\}$

4. $f(x) = x^2 - 6x + 8$, $x \geq 3$ $\quad$ Range: $\{y \in \mathbb{R}, y \geq 0\}$

$= (x-3)^2 - 1$

$x - 3 = y$

$(x-3)^2 = y + 1$

$x = \sqrt{y+1} + 3$

$f^{-1}(x) = \sqrt{x+1} + 3$

Range: $\{f(x) \in \mathbb{R}, f(x) \in \mathbb{R}\}$

5. $f(x) = \frac{1}{x-2}$

$y = \frac{1}{x-2}$

$x = \frac{1}{y} + 2$

$y = \frac{1}{x-2} + 2$

$f^{-1}(x) = \frac{1}{x-2} + 2$

6. $f(x) = 4x^2 - 20x + 11$

$= 4(x^2 - 5x) + 11$

$= 4(x^2 - 5x + \frac{25}{4}) - 4 \times \frac{25}{4} + 11$ $\quad 11 - 25 = -14$

$f(x) = 4(x - \frac{5}{2})^2 - 14$

$(x - \frac{5}{2})^2 = \frac{y+14}{4}$

$x - \frac{5}{2} = \sqrt{\frac{y+14}{4}}$

$x = \frac{5}{2} \pm \sqrt{\frac{y+14}{4}}$

$f^{-1}(x) = \frac{5}{2} \pm \sqrt{\frac{x+14}{4}}$

7. a) $f(x) = 4x^2 - 20x + 11$

$= 4(x^2 - 5x) + 11$

$= 4(x^2 - 5x + \frac{25}{4}) - 4 \times \frac{25}{4} + 11$

$f(x) = 4(x - \frac{5}{2})^2 - 14$

$(x - \frac{5}{2})^2 = \frac{y+14}{4}$

$x - \frac{5}{2} = \pm\sqrt{\frac{y+14}{4}}$

$x = \frac{5}{2} \pm \sqrt{\frac{y+14}{4}}$

$f^{-1}(x) = \frac{5}{2} \pm \sqrt{\frac{x+14}{4}}$

Range: $\{y \geq -14, y \in \mathbb{R}\}$

b) $f(x) = \frac{1}{x-2}$

$y = \frac{1}{x-2}$

$x = \frac{1}{y} + 2$

$y = \frac{1}{x-2} + 2$

$f^{-1}(x) = \frac{1}{x-2} + 2$

Range: $\{y \in \mathbb{R}, y \in \mathbb{R}\}$

8. $f(x) = \frac{x+1}{x-2}$

$y = \frac{x+1}{x-2}$

$xy - 2y = x + 1$

$xy - x = 2y + 1$

$x(y-1) = 2y + 1$

$x = \frac{2y+1}{y-1}$

$y = \frac{2x+1}{x-1}$

$f^{-1}(x) = \frac{2x+1}{x-1}$

9. $f(x) = \frac{x+1}{x-2}$

$x \neq 2$

$x \neq 1$

$f(x) = \frac{x+1}{x-2}$ $\quad$ completely regular not inverse

Range: $\{y \in \mathbb{R}, y \in \mathbb{R}\}$ $\quad$ $y \neq 1$ for proper function

$f(x) =$

$2x + 1 = 0$

$x = -\frac{1}{2}$

$x - 2 = 0$

$x = 2$

$y = \frac{2x+1}{x-2} = 1$

$2x + 1 = x - 2$

$x = -3$

$f^{-1}(x)$

$x = \sqrt{x+1}$

$x^2 = x + 1$

$y = \sqrt{x+1}$

10. $f(x) = \sqrt{x^2 - 6x + 8}$

$= \sqrt{(x-3)^2 - 1}$

$x - 3 = y$

$(x-3)^2 = y^2 + 1$

$x = \sqrt{y^2 + 1} + 3$

$f^{-1}(x) = \sqrt{x^2 + 1} + 3$

Range: $\{f(x) \in \mathbb{R}, f(x) \in \mathbb{R}\}$

11. $f(x) = \sqrt{x^2 - 6x + 8}$, $x \geq 3$ $\quad$ Range: $\{y \in \mathbb{R}, y \geq 0\}$

$= \sqrt{(x-3)^2 - 1}$

$x - 3 = y$

$(x-3)^2 = y^2 + 1$

$x = \sqrt{y^2 + 1} + 3$

$f^{-1}(x) = \sqrt{x^2 + 1} + 3$

Range: $\{f(x) \in \mathbb{R}, f(x) \in \mathbb{R}\}$

12. Always $(-1, 0)$ & $(1, 0)$

$y = \frac{x+1}{x-1}$

$x = \frac{y+1}{y-1}$

$xy - x = y + 1$

$xy - y = x + 1$

$y(x-1) = x + 1$

$y = \frac{x+1}{x-1}$

$f^{-1}(x) = \frac{x+1}{x-1}$

13. $y = \frac{x+1}{x-2}$ $\quad$ Range: $\{y \in \mathbb{R}, y \in \mathbb{R}\}$

$x = \frac{y+1}{y-2}$

$y = \frac{2x+1}{x-1}$

$f^{-1}(x) = \frac{2x+1}{x-1}$

14. $y = \frac{x+1}{x-2}$ $\quad$ Range: $\{y \in \mathbb{R}, y \in \mathbb{R}\}$

$x = \frac{y+1}{y-2}$

$y = \frac{2x+1}{x-1}$

$f^{-1}(x) = \frac{2x+1}{x-1}$

15. a) $y = \frac{x+1}{x-2}$

$x = \frac{y+1}{y-2}$

$y = \frac{2x+1}{x-1}$

$f^{-1}(x) = \frac{2x+1}{x-1}$

b) $f(x) = \frac{x+1}{x-2}$

$f(-1) = \frac{0}{-3} = 0$

$f(0) = \frac{1}{-2} = -\frac{1}{2}$

$f(1) = \frac{2}{-1} = -2$

$f(2) = \frac{3}{0}$ $\quad$ undefined

$f(3) = \frac{4}{1} = 4$

$f(4) = \frac{5}{2} = 2.5$

$f(5) = \frac{6}{3} = 2$

$f(6) = \frac{7}{4} = 1.75$

$f(7) = \frac{8}{5} = 1.6$

$f(8) = \frac{9}{6} = 1.5$

$f(9) = \frac{10}{7} = 1.4$

$f(10) = \frac{11}{8} = 1.375$

Range: $\{y \in \mathbb{R}, y \in \mathbb{R}\}$

16. $y = \frac{x+1}{x-2}$ $\quad x \neq 2, x \neq 1$ $\quad f(x) \in \mathbb{R}$

Range: $\{y \in \mathbb{R}, y \in \mathbb{R}\}$

$x = \frac{y+1}{y-2}$

$y = \frac{2x+1}{x-1}$

$f^{-1}(x) = \frac{2x+1}{x-1}$

17. a) $f(x) = \frac{x+1}{x-2}$

$f(-1) = \frac{0}{-3} = 0$

$f(0) = \frac{1}{-2} = -\frac{1}{2}$

$f(1) = \frac{2}{-1} = -2$

$f(2) = \frac{3}{0}$ undefined

$f(3) = \frac{4}{1} = 4$

$f(4) = \frac{5}{2} = 2.5$

$f(5) = \frac{6}{3} = 2$

$f(6) = \frac{7}{4} = 1.75$

$f(7) = \frac{8}{5} = 1.6$

$f(8) = \frac{9}{6} = 1.5$

$f(9) = \frac{10}{7} = 1.4$

$f(10) = \frac{11}{8} = 1.375$

Range: $\{y \in \mathbb{R}, y \in \mathbb{R}\}$

b) $y = \frac{x+1}{x-2}$ $\quad x \neq 2, x \neq 1$ $\quad f(x) \in \mathbb{R}$

$x = \frac{y+1}{y-2}$

$y = \frac{2x+1}{x-1}$

$f^{-1}(x) = \frac{2x+1}{x-1}$

18. a) $f(x) = \frac{x+1}{x-2}$

$f(-1) = \frac{0}{-3} = 0$

$f(0) = \frac{1}{-2} = -\frac{1}{2}$

$f(1) = \frac{2}{-1} = -2$

$f(2) = \frac{3}{0}$ undefined

$f(3) = \frac{4}{1} = 4$

$f(4) = \frac{5}{2} = 2.5$

$f(5) = \frac{6}{3} = 2$

$f(6) = \frac{7}{4} = 1.75$

$f(7) = \frac{8}{5} = 1.6$

$f(8) = \frac{9}{6} = 1.5$

$f(9) = \frac{10}{7} = 1.4$

$f(10) = \frac{11}{8} = 1.375$

Range: $\{y \in \mathbb{R}, y \in \mathbb{R}\}$

b) $y = \frac{x+1}{x-2}$ $\quad x \neq 2, x \neq 1$ $\quad f(x) \in \mathbb{R}$

$x = \frac{y+1}{y-2}$

$y = \frac{2x+1}{x-1}$

$f^{-1}(x) = \frac{2x+1}{x-1}$

19. a) $y = \frac{x+1}{x-2}$

$x = \frac{y+1}{y-2}$

$y = \frac{2x+1}{x-1}$

$f^{-1}(x) = \frac{2x+1}{x-1}$

b) $f(x) = \frac{x+1}{x-2}$

$f(-1) = \frac{0}{-3} = 0$

$f(0) = \frac{1}{-2} = -\frac{1}{2}$

$f(1) = \frac{2}{-1} = -2$

$f(2) = \frac{3}{0}$ undefined

$f(3) = \frac{4}{1} = 4$

$f(4) = \frac{5}{2} = 2.5$

$f(5) = \frac{6}{3} = 2$

$f(6) = \frac{7}{4} = 1.75$

$f(7) = \frac{8}{5} = 1.6$

$f(8) = \frac{9}{6} = 1.5$

$f(9) = \frac{10}{7} = 1.4$

$f(10) = \frac{11}{8} = 1.375$

Range: $\{y \in \mathbb{R}, y \in \mathbb{R}\}$

20. a) $y = \frac{x+1}{x-2}$ $\quad x \neq 2, x \neq 1$ $\quad f(x) \in \mathbb{R}$

$x = \frac{y+1}{y-2}$

$y = \frac{2x+1}{x-1}$

$f^{-1}(x) = \frac{2x+1}{x-1}$

b) $y = 2(x-1)^2 + 2$

$x = 2(y-1)^2 + 2$

$x - 2 = 2(y-1)^2$

$\frac{x-2}{2} = (y-1)^2$

$\sqrt{\frac{x-2}{2}} = y - 1$

$y = \sqrt{\frac{x-2}{2}} + 1$

$f^{-1}(x) = \sqrt{\frac{x-2}{2}} + 1$

21. a) $y = 2(x-1)^2 + 2$

$x = 2(y-1)^2 + 2$

$x - 2 = 2(y-1)^2$

$\frac{x-2}{2} = (y-1)^2$

$\sqrt{\frac{x-2}{2}} = y - 1$

$y = \sqrt{\frac{x-2}{2}} + 1$

$f^{-1}(x) = \sqrt{\frac{x-2}{2}} + 1$

b) $f(x) = 2(x-1)^2 + 2$

$f(0) = 2(-1)^2 + 2 = 4$

$f(1) = 2(0)^2 + 2 = 2$

$f(2) = 2(1)^2 + 2 = 4$

$f(3) = 2(2)^2 + 2 = 10$

$f(4) = 2(3)^2 + 2 = 20$

$f(5) = 2(4)^2 + 2 = 34$

$f(6) = 2(5)^2 + 2 = 52$

$f(7) = 2(6)^2 + 2 = 74$

$f(8) = 2(7)^2 + 2 = 100$

$f(9) = 2(8)^2 + 2 = 130$

$f(10) = 2(9)^2 + 2 = 164$

Range: $\{y \geq 2, y \in \mathbb{R}\}$

22. a) $y = 2(x-1)^2 + 2$

$x = 2(y-1)^2 + 2$

$x - 2 = 2(y-1)^2$

$\frac{x-2}{2} = (y-1)^2$

$\sqrt{\frac{x-2}{2}} = y - 1$

$y = \sqrt{\frac{x-2}{2}} + 1$

$f^{-1}(x) = \sqrt{\frac{x-2}{2}} + 1$

b) $f(x) = 2(x-1)^2 + 2$

$f(0) = 2(-1)^2 + 2 = 4$

$f(1) = 2(0)^2 + 2 = 2$

$f(2) = 2(1)^2 + 2 = 4$

$f(3) = 2(2)^2 + 2 = 10$

$f(4) = 2(3)^2 + 2 = 20$

$f(5) = 2(4)^2 + 2 = 34$

$f(6) = 2(5)^2 + 2 = 52$

$f(7) = 2(6)^2 + 2 = 74$

$f(8) = 2(7)^2 + 2 = 100$

$f(9) = 2(8)^2 + 2 = 130$

$f(10) = 2(9)^2 + 2 = 164$

Range: $\{y \geq 2, y \in \mathbb{R}\}$

23. a) $y = 2(x-1)^2 + 2$

$x = 2(y-1)^2 + 2$

$x - 2 = 2(y-1)^2$

$\frac{x-2}{2} = (y-1)^2$

$\sqrt{\frac{x-2}{2}} = y - 1$

$y = \sqrt{\frac{x-2}{2}} + 1$

$f^{-1}(x) = \sqrt{\frac{x-2}{2}} + 1$

b) $f(x) = 2(x-1)^2 + 2$

$f(0) = 2(-1)^2 + 2 = 4$

$f(1) = 2(0)^2 + 2 = 2$

$f(2) = 2(1)^2 + 2 = 4$

$f(3) = 2(2)^2 + 2 = 10$

$f(4) = 2(3)^2 + 2 = 20$

$f(5) = 2(4)^2 + 2 = 34$

$f(6) = 2(5)^2 + 2 = 52$

$f(7) = 2(6)^2 + 2 = 74$

$f(8) = 2(7)^2 + 2 = 100$

$f(9) = 2(8)^2 + 2 = 130$

$f(10) = 2(9)^2 + 2 = 164$

Range: $\{y \geq 2, y \in \mathbb{R}\}$

24. $f(x) = 3x^2$

$g(x) = x^2$

$x \geq 0$

$f(x) = f(x^2) + 2(x^2 - 1)^2$

$= 3x^2 + 2x^4$

$= 2x^4 + 3x^2$

$g(f(x)) = g(2x^4) + 2$

$= 2x^8 + 4x^4 + 2$

$= 2x^8 + 4x^4 + 2$

$f^{-1}(x) = \sqrt{x}$

$g^{-1}(x) = \sqrt{x}$

$g^{-1}(f(x)) = \sqrt{2x^4 + 2}$

$= 2(2x^4 - 1)^2 + 2$

$= 2(2x^4 - 1)(2x^4 - 1) + 2$

$= 2(4x^8 - 4x^4 + 1) + 2$

$= 8x^8 - 8x^4 + 2$

$2x^8 + 4x^4 + 2 = 0$

$2x^8 - 2x^4 = 0$

$2x^4(x^4 - 1) = 0$

$2x^4 = 0$

$x^4 = 0$

$x = 0$

$x^4 - 1 = 0$

$x^4 = 1$

$x = \pm\sqrt[4]{1}$ $\quad \sqrt[4]{1} = \frac{1}{2}$

$x = \frac{1}{2}$

$x = -\frac{1}{2}$

$f(x) = \frac{1}{2}, -\frac{1}{2}$

25. a) $f(x) = \frac{3x^2}{2}$

$y = \frac{3x^2}{2}$

$x = \sqrt{\frac{2y}{3}}$

$y = \sqrt{\frac{2x}{3}}$

$f^{-1}(x) = \sqrt{\frac{2x}{3}}$

b) $f(x) = \frac{3x^2}{2}$

$f(0) = \frac{3 \times 0^2}{2} = 0$

$f(1) = \frac{3 \times 1^2}{2} = \frac{3}{2}$

$f(2) = \frac{3 \times 2^2}{2} = 6$

$f(3) = \frac{3 \times 3^2}{2} = \frac{27}{2}$

$f(4) = \frac{3 \times 4^2}{2} = 24$

$f(5) = \frac{3 \times 5^2}{2} = \frac{75}{2}$

$f(6) = \frac{3 \times 6^2}{2} = 54$

$f(7) = \frac{3 \times 7^2}{2} = \frac{147}{2}$

$f(8) = \frac{3 \times 8^2}{2} = 96$

$f(9) = \frac{3 \times 9^2}{2} = \frac{243}{2}$

$f(10) = \frac{3 \times 10^2}{2} = 150$

Range: $\{y \geq 0, y \in \mathbb{R}\}$

26. $f(x) = 3x^2$

$g(x) = x^2$

$x \geq 0$

$f(x) = f(x^2) + 2(x^2 - 1)^2$

$= 3x^2 + 2x^4$

$= 2x^4 + 3x^2$

$g(f(x)) = g(2x^4) + 2$

$= 2x^8 + 4x^4 + 2$

$= 2x^8 + 4x^4 + 2$

$f^{-1}(x) = \sqrt{x}$

$g^{-1}(x) = \sqrt{x}$

$g^{-1}(f(x)) = \sqrt{2x^4 + 2}$

$= 2(2x^4 - 1)^2 + 2$

$= 2(2x^4 - 1)(2x^4 - 1) + 2$

$= 2(4x^8 - 4x^4 + 1) + 2$

$= 8x^8 - 8x^4 + 2$

$2x^8 + 4x^4 + 2 = 0$

$2x^8 - 2x^4 = 0$

$2x^4(x^4 - 1) = 0$

$2x^4 = 0$

$x^4 = 0$

$x = 0$

$x^4 - 1 = 0$

$x^4 = 1$

$x = \pm\sqrt[4]{1}$ $\quad \sqrt[4]{1} = \frac{1}{2}$

$x = \frac{1}{2}$

$x = -\frac{1}{2}$

$f(x) = \frac{1}{2}, -\frac{1}{2}$

27. a) $f(x) = \frac{3x^2}{2}$

$y = \frac{3x^2}{2}$

$x = \sqrt{\frac{2y}{3}}$

$y = \sqrt{\frac{2x}{3}}$

$f^{-1}(x) = \sqrt{\frac{2x}{3}}$

b) $f(x) = \frac{3x^2}{2}$

$f(0) = \frac{3 \times 0^2}{2} = 0$

$f(1) = \frac{3 \times 1^2}{2} = \frac{3}{2}$

$f(2) = \frac{3 \times 2^2}{2} = 6$

$f(3) = \frac{3 \times 3^2}{2} = \frac{27}{2}$

$f(4) = \frac{3 \times 4^2}{2} = 24$

$f(5) = \frac{3 \times 5^2}{2} = \frac{75}{2}$

$f(6) = \frac{3 \times 6^2}{2} = 54$

$f(7) = \frac{3 \times 7^2}{2} = \frac{147}{2}$

$f(8) = \frac{3 \times 8^2}{2} = 96$

$f(9) = \frac{3 \times 9^2}{2} = \frac{243}{2}$

$f(10) = \frac{3 \times 10^2}{2} = 150$

Range: $\{y \geq 0, y \in \mathbb{R}\}$

28. a) $y = 2(x-1)^2 + 2$

$x = 2(y-1)^2 + 2$

$x - 2 = 2(y-1)^2$

$\frac{x-2}{2} = (y-1)^2$

$\sqrt{\frac{x-2}{2}} = y - 1$

$y = \sqrt{\frac{x-2}{2}} + 1$

$f^{-1}(x) = \sqrt{\frac{x-2}{2}} + 1$

b) $f(x) = 2(x-1)^2 + 2$

$f(0) = 2(-1)^2 + 2 = 4$

$f(1) = 2(0)^2 + 2 = 2$

$f(2) = 2(1)^2 + 2 = 4$

$f(3) = 2(2)^2 + 2 = 10$

$f(4) = 2(3)^2 + 2 = 20$

$f(5) = 2(4)^2 + 2 = 34$

$f(6) = 2(5)^2 + 2 = 52$

$f(7) = 2(6)^2 + 2 = 74$

$f(8) = 2(7)^2 + 2 = 100$

$f(9) = 2(8)^2 + 2 = 130$

$f(10) = 2(9)^2 + 2 = 164$

Range: $\{y \geq 2, y \in \mathbb{R}\}$

29. a) $y = 2(x-1)^2 + 2$

$x = 2(y-1)^2 + 2$

$x - 2 = 2(y-1)^2$

$\frac{x-2}{2} = (y-1)^2$

$\sqrt{\frac{x-2}{2}} = y - 1$

$y = \sqrt{\frac{x-2}{2}} + 1$

$f^{-1}(x) = \sqrt{\frac{x-2}{2}} + 1$

b) $f(x) = 2(x-1)^2 + 2$

$f(0) = 2(-1)^2 + 2 = 4$

$f(1) = 2(0)^2 + 2 = 2$

$f(2) = 2(1)^2 + 2 = 4$

$f(3) = 2(2)^2 + 2 = 10$

$f(4) = 2(3)^2 + 2 = 20$

$f(5) = 2(4)^2 + 2 = 34$

$f(6) = 2(5)^2 + 2 = 52$

$f(7) = 2(6)^2 + 2 = 74$

$f(8) = 2(7)^2 + 2 = 100$

$f(9) = 2(8)^2 + 2 = 130$

$f(10) = 2(9)^2 + 2 = 164$

Range: $\{y \geq 2, y \in \mathbb{R}\}$

30. $f(x) = \frac{3x^2}{2}$, $x \geq 0$ $\quad$ Range: $\{y \geq 0, y \in \mathbb{R}\}$

a) $y = \frac{3x^2}{2}$

$x = \sqrt{\frac{2y}{3}}$

$y = \sqrt{\frac{2x}{3}}$

$f^{-1}(x) = \sqrt{\frac{2x}{3}}$

b) $f(x) = \frac{3x^2}{2}$

$f(0) = \frac{3 \times 0^2}{2} = 0$

$f(1) = \frac{3 \times 1^2}{2} = \frac{3}{2}$

$f(2) = \frac{3 \times 2^2}{2} = 6$

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$f(9) = \frac{3 \times 9^2}{2} = \frac{243}{2}$

$f(10) = \frac{3 \times 10^2}{2} = 150$

Range: $\{y \geq 0, y \in \mathbb{R}\}$

31. a) $y = 2(x-1)^2 + 2$

$x = 2(y-1)^2 + 2$

$x - 2 = 2(y-1)^2$

$\frac{x-2}{2} = (y-1)^2$

$\sqrt{\frac{x-2}{2}} = y - 1$

$y = \sqrt{\frac{x-2}{2}} + 1$

$f^{-1}(x) = \sqrt{\frac{x-2}{2}} + 1$

b) $f(x) = 2(x-1)^2 + 2$

$f(0) = 2(-1)^2 + 2 = 4$

$f(1) = 2(0)^2 + 2 = 2$

$f(2) = 2(1)^2 + 2 = 4$

$f(3) = 2(2)^2 + 2 = 10$

$f(4) = 2(3)^2 + 2 = 20$

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Range: $\{y \geq 2, y \in \mathbb{R}\}$

32. $f(x) = 3x^2$

$g(x) = x^2$

$x \geq 0$

$f(x) = f(x^2) + 2(x^2 - 1)^2$

$= 3x^2 + 2x^4$

$= 2x^4 + 3x^2$

$g(f(x)) = g(2x^4) + 2$

$= 2x^8 + 4x^4 + 2$

$= 2x^8 + 4x^4 + 2$

$f^{-1}(x) = \sqrt{x}$

$g^{-1}(x) = \sqrt{x}$

$g^{-1}(f(x)) = \sqrt{2x^4 + 2}$

$= 2(2x^4 - 1)^2 + 2$

$= 2(2x^4 - 1)(2x^4 - 1) + 2$

$= 2(4x^8 - 4x^4 + 1) + 2$

$= 8x^8 - 8x^4 + 2$

$2x^8 + 4x^4 + 2 = 0$

$2x^8 - 2x^4 = 0$

$2x^4(x^4 - 1) = 0$

$2x^4 = 0$

$x^4 = 0$

$x = 0$

$x^4 - 1 = 0$

$x^4 = 1$

$x = \pm\sqrt[4]{1}$ $\quad \sqrt[4]{1} = \frac{1}{2}$

$x = \frac{1}{2}$

$x = -\frac{1}{2}$

$f(x) = \frac{1}{2}, -\frac{1}{2}$

33. a) $f(x) = \frac{3x^2}{2}$

$y = \frac{3x^2}{2}$

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Range: $\{y \geq 0, y \in \mathbb{R}\}$

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$x = 2(y-1)^2 + 2$

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$\frac{x-2}{2} = (y-1)^2$

$\sqrt{\frac{x-2}{2}} = y - 1$

$y = \sqrt{\frac{x-2}{2}} + 1$

$f^{-1}(x) = \sqrt{\frac{x-2}{2}} + 1$

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Range: $\{y \geq 2, y \in \mathbb{R}\}$

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Range: $\{y \geq 2, y \in \mathbb{R}\}$

40. $f(x) = \frac{3x^2}{2}$, $x \geq 0$ $\quad$ Range: $\{y \geq 0, y \in \mathbb{R}\}$

a) $y = \frac{3x^2}{2}$

$x = \sqrt{\frac{2y}{3}}$

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$f(10) = \frac{3 \times 10^2}{2} = 150$

Range: $\{y \geq 0, y \in \mathbb{R}\}$

1) $\frac{x}{x-1}$

$\frac{x}{x-1} \cdot \frac{(x-1)}{(x-1)} = \frac{x}{x-1}$

$\frac{x}{x-1} = \frac{x}{x-1}$

$\frac{x}{x-1} = \frac{x}{x-1}$

$\frac{x}{x-1} = \frac{x}{x-1}$

Corrections:

3. $A(a,3)$ $B(4,b)$

$\overline{AB} = 4\sqrt{5}$ $m_{AB} = -\frac{1}{2}$

$(4\sqrt{5})^2 = (a-4)^2 + (3-b)$

$-\frac{1}{2} = \frac{b-3}{4-a}$

$-\frac{1}{2} = \frac{b-3}{4-a}$

$-\frac{1}{2}(4-a) + 3 = b$

$\frac{1}{2}a - 2 + 3 = b$

$\frac{1}{2}a + 1 = b$

$-\frac{1}{2}(4-a)$

$-2 + \frac{a}{2} = b-3$

$a - 4 = 2b - 6$

$a = 2b - 2$

$(4\sqrt{5})^2 = (a-4)^2 + \left(3 - \left(\frac{1}{2}a + 1\right)\right)^2$

$= 16 \cdot 5 = a^2 - 8a + 16 + \left(2 - \frac{1}{2}a\right)^2$

$= 80 = a^2 - 8a + 16 + 4 - 2a + \frac{1}{4}a^2$

$0 = \frac{5}{4}a^2 - 10a - 60$

$0 = 5a^2 - 4a - 240$

$(4\sqrt{5})^2 = (2b-2-4)^2 + (3-b)^2$

$80 = (2b-6)^2 + 9 - 6b + b^2$

$80 = 4b^2 - 24b + 36 + 9 - 6b + b^2$

$80 = 5b^2 - 30b + 45$

$0 = 5b^2 - 30b - 35$

$0 = b^2 - 6b - 7$

$M -7$ $A -6$

-7 1

$0 = (b-7)(b+1)$

$b_1 = 7$ $b_2 = -1$

$\frac{a}{2} + 1 = b$

$\frac{a}{2} + 1 = 7$ $\frac{a}{2} + 1 = -1$

$\frac{a}{2} = 6$ $\frac{a}{2} = -2$

$a_1 = 3$ $a_2 = -4$

$a = 3$ when $b = 7$

$a = -4$ when $b = -1$

Class Test Review

1) $y = \frac{x}{x-2}$

$y = \frac{x}{x-2}$

$y(x-2) = x$

$yx - 2y = x$

$yx - x = 2y$

$x(y-1) = 2y$

$x = \frac{2y}{y-1}$ $y \neq 1$

$f^{-1}(x) = \frac{2x}{x-1}$

2) $f(x) = \frac{x}{x-2}$

$f(x)$

3) $f(x) = \frac{x}{x-2}$

$f(x)$

4) $f(x) = \frac{x}{x-2}$

$f(x) = \frac{2}{x-2} + 1$

$y = \frac{2}{x-2} + 1$

$y - 1 = \frac{2}{x-2}$

$(y-1)(x-2) = 2$

$yx - 2y - x + 2 = 2$

$yx - x = 2y$

$x(y-1) = 2y$

$x = \frac{2y}{y-1}$ $y \neq 1$

$f^{-1}(x) = \frac{2x}{x-1}$

5) $f(x) = \frac{x}{x-2}$

$f(x) = \frac{2}{x-2} + 1$

$f(x) = \frac{2}{x-2} + 1$

$f(x) = \frac{2}{x-2} + 1$

$f(x) = \frac{2}{x-2} + 1$

6) Indicate the domain and range of $f(x) = \frac{x}{x-2}$

$D: \{x \in \mathbb{R} \mid x \neq 2\}$

$R: \{y \in \mathbb{R} \mid y \neq 1\}$

7) Indicate the domain and range of $f(x) = \frac{x}{x-2}$

$D: \{x \in \mathbb{R} \mid x \neq 2\}$

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$D: \{x \in \mathbb{R} \mid x \neq 2\}$

$R: \{y \in \mathbb{R} \mid y \neq 1\}$

11) Indicate the domain and range of $f(x) = \frac{x}{x-2}$

$D: \{x \in \mathbb{R} \mid x \neq 2\}$

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25) Indicate the domain and range of $f(x) = \frac{x}{x-2}$

$D: \{x \in \mathbb{R} \mid x \neq 2\}$

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$R: \{y \in \mathbb{R} \mid y \neq 1\}$

43) Indicate the domain and range of $f(x) = \frac{x}{x-2}$

$D: \{x \in \mathbb{R} \mid x \neq 2\}$

$R: \{y \in \mathbb{R} \mid y \neq 1\}$

44) Indicate the domain and range of $f(x) = \frac{x}{x-2}$

$D: \{x \in \mathbb{R} \mid x \neq 2\}$

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45) Indicate the domain and range of $f(x) = \frac{x}{x-2}$

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$R: \{y \in \mathbb{R} \mid y \neq 1\}$

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18. a) $f(x) = 2x+1$

$g(x) = 8 - ax - bx^2$

$fg(x) = 17 - 24x - 4x^2$

$f(8 - ax - bx^2)$

$= 2(8 - ax - bx^2) + 1$

$= 16 - 2ax - 2bx^2 + 1$

$\Rightarrow 17 - 2ax - 2bx^2 = 17 - 24x - 4x^2$

$24 = 2a \qquad 4 = 2b$

$a = 12 \qquad b = 2$

b) $g(x) = 8 - 12x - 2x^2$

$= -2x - 12x + 8$

$= -2(x + 6x + 3^2) + 8 + 2 \cdot 3^2$

$= -2(x+3)^2 + 26$

$V(-3, 26)$

$k \geq -3$

c) $8 - 12x - 2x^2 = y$

$8 - 12y - 2y^2 = x$

$-2y^2 - 12y + 8 - x = 0$

$y = \dfrac{12 + \sqrt{12^2 - 4 \cdot (-2)(8-x)}}{2 \cdot (-2)}$ neglecting negative root

$y = \dfrac{12 + \sqrt{12^2 + 64 - 8x}}{-4}$

$y = \dfrac{12 + \sqrt{208 - 8x}}{-4} \rightarrow \sqrt{16(13 - \frac{1}{2}x)}$

$y = \dfrac{12 + 4\sqrt{13 - \frac{1}{2}x}}{-4}$

$y = -3 - \sqrt{13 - \frac{1}{2}x}$

19. $C(8, 3) \qquad P(13, 5)$

a) $(13-8)^2 + (5-3)^2 = r^2 = 29$

$(x-8)^2 + (y-3)^2 = 29$

$m = \dfrac{5-3}{13-8} = \dfrac{2}{5}$

$m_t = -\dfrac{5}{2}$

$y - 5 = -\frac{5}{2}(x - 13)$

$y = -\frac{5}{2}x + 32.5 + 5$

$y = -\frac{5}{2}x + 37.5$

$2y + 5x = 75$

20.

7.1 Forces in Two Dimensions

1. $F_x = 8\cos(60°) = 4.0\ N$
$F_y = 8\sin(60°) = 6.9\ N$

2. $F_x = 15\cos 45°$ N/J
$F_y = 15\sin 45°$ N/J

3. $F_x = 5\cos 30°$ N/J
$F_y = 5\sin 30°$ N/J

4. $F_x = 10\cos 30°$ N/J
$F_y = 10\sin 30°$ N/J

5. $F_x = 15\cos 30°$ N/J
$F_y = 15\sin 30°$ N/J

6. $F_x = 10\cos(-30°)$ N/J
$F_y = 10\sin(-30°)$ N/J

7. $F_x = 5\cos 30°$ N/J
$F_y = 5\sin 30°$ N/J

8. $F_x = 5\cos 30°$ N/J
$F_y = 5\sin 30°$ N/J

9. $F_x = 5\cos 30°$ N/J
$F_y = 5\sin 30°$ N/J

10. $F_x = 5\cos 30°$ N/J
$F_y = 5\sin 30°$ N/J

11. $F_x = 5\cos 30°$ N/J
$F_y = 5\sin 30°$ N/J

12. $F_x = 5\cos 30°$ N/J
$F_y = 5\sin 30°$ N/J

13. $F_x = 5\cos 30°$ N/J
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14. $F_x = 5\cos 30°$ N/J
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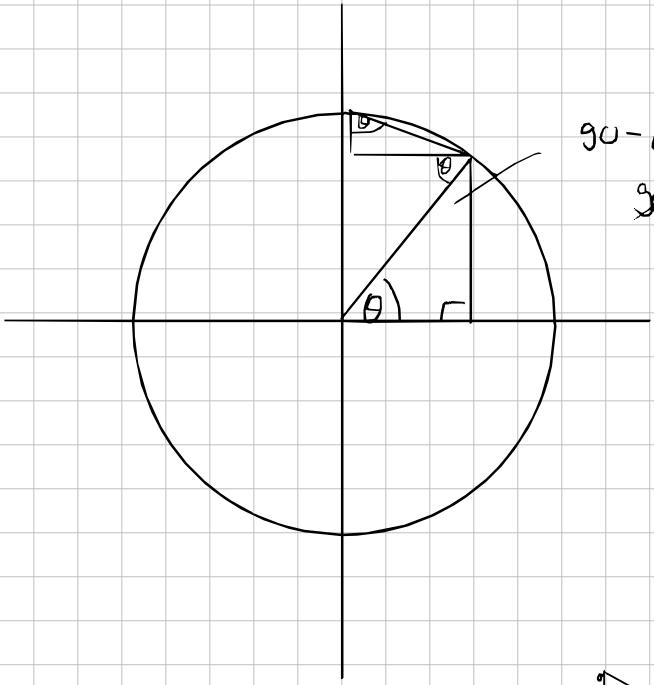


$90 - \theta$

$\cancel{90} - (\cancel{90} - \theta) = \theta$

$17^2 = 170 + 85 + 34$

$= 204 + 85 = 289$

3A

10. $\sin\theta \cdot F = \sin\rho \cdot G$

$\frac{3}{5} \cdot F = \frac{15}{17} \cdot G$

$\cos\theta \cdot F = \cos\rho \cdot G$

$\frac{4}{5} \cdot F = \frac{8}{17} \cdot G$

$F = \frac{8}{17} \cdot \frac{5}{4} \cdot G$

$F = \frac{40}{68} G = \frac{10}{17} G$

$\frac{3}{5}\left(\frac{10}{17}\right) G = \frac{15}{17} G$

$\cos^2\theta + \left(\frac{3}{5}\right)^2 = 1$

$\cos\theta = \sqrt{1 - \frac{9}{25}}$

$\cos\theta = \sqrt{\frac{16}{25}} = \frac{4}{5}$

$\cos^2\rho + \left(\frac{15}{17}\right)^2 = 1$

$\cos\rho = \sqrt{1 - \left(\frac{15}{17}\right)^2}$

$\cos\rho = \sqrt{\frac{64}{289}}$

$\cos\rho = \frac{8}{17}$

$\frac{225}{289}$

$289 - 225 = 064$

End of Chapter review 7.2 Friction

1. $\mu = 0.45$, $m = 75\,kg$

$F_f = F_p$

$\mu mg = 0.45 \cdot 75 \cdot 9.8$

$F_f = 331.2\,N$

2. $F_{net} = 0$, $\mu = 0.3$

$F_{gx} = F_p + F_f$

$mg\sin\theta = F_p + \mu mg\cos\theta$

$m \cdot 9.8 \cdot \sin 25° = 15 \cdot 10 \cos 25° + 0.3 \cdot 15 \cdot 10 \cos 25° \cdot 0.3$

$g = 132.7\,N$

$F_f = F_p + F_{gx}$

$F_f = \mu mg\cos\theta + mg\sin\theta = 15 \cdot 10 \cos 25° \cdot 0.3 + 15 \cdot 10 \sin 25°$

$F_f = 132.7\,N$

$132.8\,N \approx F_f < 164.7\,N$

3. $\mu = 0.04$

$a = 0.04 \cdot 9.8 = 0.392\,m/s^2$

find v_f $\quad F_{net} = F_f$

$a = \frac{F}{m} = \frac{-F_f}{m} = -0.4\,m/s^2$

$v_f^2 = v_i^2 + 2ad$

$v_f = \sqrt{3^2 - 2 \cdot 0.4 \cdot 55} = 2.19\,m/s$

4. $\mu = 0.09$

find a

$F_p = mg\cos 15° \mu = 5 \cdot 10 \cos 15° \cdot 0.09$

$F_p = 4.34\,N$

$a = \frac{F}{m} = \frac{4.34}{5} = 0.87\,m/s^2$

5. $\mu = 0.03$

find v_f ... find a

$F_p = mg\cos 5° \mu = 5000 \cdot 10 \cos 5° \cdot 0.03 = 3.49\,kN$

$3.49 - mg\sin 5° = 3.49 - 5000 \cdot 10 \sin 5° = 3.13\,kN$

$a = \frac{F}{m} = \frac{3.13}{5000} = 0.626\,m/s^2$

$v_f^2 = v_i^2 + 2ad$

$v_f = \sqrt{0 + 2 \cdot 0.626 \cdot 10} = 2.44\,m/s$

6. $\mu = 0.3$

$F_p = mg\cos 5° \mu = 5000 \cdot 10 \cos 5° \cdot 0.3 = 3.49\,kN$

$3.49 - mg\sin 5° = 3.49 - 5000 \cdot 10 \sin 5° = 3.13\,kN$

$a = \frac{F}{m} = \frac{3.13}{5000} = 0.626\,m/s^2$

$v_f^2 = v_i^2 + 2ad$

$v_f = \sqrt{0 + 2 \cdot 0.626 \cdot 10} = 2.44\,m/s$

7. $F_{net} = 0$, $\mu = 0.3$

$R = 60\,N - F_p$

$F_f = 0.3 \cdot 60 \cdot \cos 25° = 15.4\,N$

$\mu = \tan^{-1} 0.3 = 16.7°$

8. $F_{net} = 0$, $\mu = 0.45$

a) $F_p = mg\cos 15° \mu = 1.5 \cdot 10 \cos 15° \cdot 0.45 = 5.82\,N$

$F_f = mg\sin 15° = 1.5 \cdot 10 \sin 15° = 3.93\,N$

$F_f > F_p$, hold mountain bicycle

b) $F_{net} = F_{gx} - F_p = mg\sin\theta - mg\cos\theta\,\mu$

$= 1.5 \cdot 10 (\sin 15° - \cos 15° \cdot 0.45) = 0.55\,N$

9.

a: $\frac{F}{2m} = \frac{0.025}{0.5 \cdot 9.8} = 0.51\,m/s^2$

$\mu = \frac{a}{g} = \frac{0.51}{9.8} = 0.05$

10.

$F_{net} = 0$

$F_p + 5 \cdot 9.8 \sin 30° = 5 \cdot 9.8 \cos 30° \mu$

$F_p = 24.5\,N$

$5 \cdot 9.8 \cos 30° \mu = 50.3\,N$

$\mu = 0.6$

11.

a) $F_p = 0.4 \cdot 10 \cos 15° \cdot 0.2 = 3.83\,N$ down

$a = \frac{F}{m} = \frac{5.24 + 0.4 \cdot 10 \sin 15°}{6} = 1.3\,m/s^2$ down

$v_f^2 = v_i^2 + 2ad$

$0 = \frac{v_i^2}{2a} - \frac{1.3}{2} \cdot \frac{v_i^2}{0.4 \cdot 9.8} = 1.33 - 1.33 \times 1.33 = 1.33$

b) $a = \frac{F}{m} = \frac{5.24 - 0.4 \cdot 10 \sin 15° - 0.4 \cdot 10 \cos 15° \cdot 0.2}{6}$

$= 0.49\,m/s^2$

$v_f = \sqrt{2 \cdot 0.49 \cdot 1.33} = 1.14\,m/s$

c) $a = \frac{F}{m} = \frac{5.24 - 0.4 \cdot 10 \sin 15° - 0.4 \cdot 10 \cos 15° \cdot 0.2}{6}$

$= 2.32\,m/s^2$

$v_f^2 = v_i^2 + 2ad$

$v_f = \sqrt{2 \cdot 2.32 \cdot 1.33} = 2.47\,m/s$

12. a) $\mu = 0.05$

b) find a:

$a = \frac{F}{m}$

$F_{gx} = mg\sin 15° = 5 \cdot 9.8 \sin 15° = 20.3\,N$

$F = F_{gx} - (mg\sin 15° - 5 \cdot 9.8 \cos 15°)\,\mu = (5 \cdot 9.8 \cos 15° - 20.3 \cdot 0.05)\,N$

$F_f = 5.35\,N$

$a = \frac{20.3 - 0.05 \cdot 5.35}{5} = 3.92\,m/s^2$

$v_f^2 = v_i^2 + 2ad$

$v_f = \sqrt{0 + 2 \cdot 3.92 \cdot 5} = 3.96\,m/s$

c) $v_f = 0 + 0.05 \cdot 5 = 0.25\,m/s$

$10 \cdot 9.8 = 98\,m/s^2 > 0.25$

d) $v_f = 0 + 0.05 \cdot 5 = 0.25\,m/s$

$v_f = 3.96\,m/s$

13. a) $\mu = 0.05$

$a = \frac{F}{m}$

$F_{gx} = mg\sin 15° = 5 \cdot 9.8 \sin 15° = 20.3\,N$

$F = F_{gx} - (mg\sin 15° - 5 \cdot 9.8 \cos 15°)\,\mu = (5 \cdot 9.8 \cos 15° - 20.3 \cdot 0.05)\,N$

$F_f = 5.35\,N$

$a = \frac{20.3 - 0.05 \cdot 5.35}{5} = 3.92\,m/s^2$

$v_f^2 = v_i^2 + 2ad$

$v_f = \sqrt{0 + 2 \cdot 3.92 \cdot 5} = 3.96\,m/s$

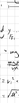
b) $v_f = 0 + 0.05 \cdot 5 = 0.25\,m/s$

$10 \cdot 9.8 = 98\,m/s^2 > 0.25$

c) $v_f = 0 + 0.05 \cdot 5 = 0.25\,m/s$

$v_f = 3.96\,m/s$

14.



$F_{net} = 0$, $\mu = 0.3$

$a = \frac{F_p + F_{gx}}{m} = \frac{\mu mg\cos\theta + mg\sin\theta}{m} = g(\mu\cos\theta + \sin\theta)$

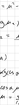
$= 9.8 \cdot (0.3 \cos 30° + \sin 30°) = 7.7$

find v_f ... find F_{net}

$v_f^2 = v_i^2 + 2ad$

$v_f = \sqrt{0 + 2 \cdot 7.7 \cdot 2} = 3.9\,m/s$

15.



$F_{net} = 0$, $\mu = 0.2$

$a = \frac{v_f^2 - v_i^2}{2d} = \frac{0 - 4 \cdot 4}{2 \cdot 2.5} = 3.44\,m/s^2$

16.

$F_{net} = 0$, $\mu = 0.2$

$a = \frac{v_f^2 - v_i^2}{2d} = \frac{0 - 4 \cdot 4}{2 \cdot 2.5} = 3.44\,m/s^2$

17.

$F_{net} = 0$, $\mu = 0.32$

a) $F_{gx} = F_f = F_p$

$F_f = F_p$

$F_f = 0.32 \cdot 10 \cos 5° \cdot 5 = 1.6\,N$

$F_p = 1.6\,N$

b) $F_{gx} = F_f + F_p$

$F_f = F_{gx} - F_p$

$F_f = 5 \cdot 9.8 \sin 5° - 1.6 = 0.17\,N$

$0.17 < F_f < 1.6$ $\quad$ 4.49 N $\quad$ 1.2

$A = \frac{1}{2} \cdot \frac{1}{2}$

$\cos\theta = \frac{x - x_0}{d}$ $\quad$ $\cos\theta = \frac{y - y_0}{d}$

$\cos\theta = \frac{x - 0}{d}$ $\quad$ $\cos\theta = \frac{y - 0}{d}$

$\cos\frac{\pi}{3} = \frac{x - 0}{d}$ $\quad$ $\cos\frac{\pi}{3} = \frac{y - 0}{d}$

1. a

11.39

$r = 5 \sin 60°$

$\cos\theta = \frac{x - x_0}{d}$

$\cos\theta = \frac{(-2) - 2}{5} = -0.8$

Radians and Trig

1. a) $\frac{20^\circ \pi}{180} = \frac{1}{9}\pi$ b) $\frac{40^\circ \pi}{180} = \frac{2}{9}\pi$

c) $\frac{25^\circ \pi}{180^\circ} = \frac{5}{36}$ d) $\frac{50^\circ \pi}{180^\circ} = \frac{5}{18}\pi$

e) $\frac{5^\circ \pi}{180^\circ} = \frac{1}{36}\pi$ f) $\frac{150^\circ \pi}{180^\circ} = \frac{5}{6}\pi$

l) $\frac{540\pi}{180} = \frac{27}{9}\pi = \frac{3}{2}\pi$ m) $\frac{2^\circ \pi}{180^\circ} = \frac{1}{90}\pi$

n) $\frac{35^\circ \pi}{180^\circ} = \frac{7}{36}\pi$ o) $\frac{600\pi}{180^\circ} = \frac{20}{3}\pi = \frac{10}{3}\pi$

2. radians to degrees:

$\theta = \phi \cdot \frac{180^\circ}{\pi}$

a) $\frac{\pi^\circ}{2} \cdot \frac{180^\circ}{\pi} = 90^\circ$ b) $\frac{\pi}{3} \cdot \frac{180}{\pi} = 60^\circ$

c) $\frac{\pi}{4} \cdot \frac{180^\circ}{\pi} = 30^\circ$ d) $\frac{\pi}{12} \cdot \frac{180}{\pi} = \frac{30^\circ}{2} = 15^\circ$

l) $\frac{5}{4}\pi \cdot \frac{180}{\pi} = \frac{4 \cdot 180}{4} = 45^\circ$ m) $\frac{5\pi}{4} \cdot \frac{180}{\pi} = 225^\circ$

n) $\frac{7\pi}{3} \cdot \frac{180}{\pi} = 420^\circ$ o) $\frac{5\pi}{6} \cdot \frac{180}{\pi} = 202.5^\circ$

7. $\tan(1) = \frac{x}{5}$

$5\tan(1) = x$

$x = 7.77\,cm$

Worked Example 4.4.

a) $arc = 0.9 \cdot 8 = 7.2\,cm$

b) $\pi - 0.9 - 0.9 = 1.34$

$\frac{8}{\sin(0.9)} = \frac{x}{\sin(1.34)}$

$x = \frac{8 \sin 1.34}{\sin 0.9} = 9.95\,cm$

W

42) $9.95 - 8 = 1.95\,cm$

$\frac{9.95 \cdot 8 \sin(0.9)}{2} \cdot \frac{1}{2} = 33.55\,cm^2$

4B

6. a) $\phi = \tan^{-1}\left(\frac{8}{6}\right) = 0.927$

b) $\sqrt{8^2+6^2} = 10$

$10 - 6 = 4\,cm$

c) $arc = 0.927 \cdot 6 = 5.56\,cm$

$5.56 + 4 + 4 = 13.56\,cm$

7. a) $2 \cdot 7 = 14\,cm$

b) $\pi = 2\phi + 2$

$\phi = \frac{\pi - 2}{2} = 0.57$ $\frac{x}{\sin(2)} = \frac{7}{\sin(0.57)}$

c) $11.78 + 14 = 25.78\,cm$ $x = \frac{7 \sin 2}{\sin 0.57} \approx 11.78\,cm$

9. $4 \cdot 10 + \pi 10 = 2 \cdot 10 + 10\phi + (2 \cdot 10 + 10\phi)$

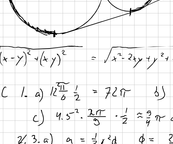
$40 + \pi 10 = 20 + 10\phi + 40 + 20\phi$

$40 + \pi 10 = 60 + 30\phi$

$\frac{10\pi - 20}{30} = 30\phi$

$\frac{\pi - 2}{3} = \phi$

10.



$\sqrt{(x-y)^2 + (x+y)^2} = \sqrt{x^2 - 2xy + y^2 + x^2 + y^2}$

4C 1. a) $12\frac{\pi}{6} \cdot \frac{1}{2} = 2\pi$ b) $10^2 \frac{2\pi}{5} \cdot \frac{1}{2} = 20\pi$

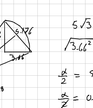
c) $4.5^2 \cdot \frac{3\pi}{5} \cdot \frac{1}{2} \approx \frac{9}{4}\pi$ d) $5^2 \cdot \frac{4\pi}{3} \cdot \frac{1}{2} = 5\pi$

3. a) $a_1 = \frac{1}{2} r^2 \phi$ $\phi = \frac{2 \cdot 9}{4} = \frac{9}{2}$

$\phi = \frac{2a}{r^2}$

b) $\phi = \frac{2 \cdot 27}{6} = 9$

9.



$10^2 = 8^2 + 6^2$

$100 = 100$ (right triangle) ✓ $\sin^{-1}\left(\frac{6}{10}\right) \approx 0.965$

$\sin^{-1}\left(\frac{8}{10}\right) \approx 0.707$

$A_{triangle} - A_{sec1} - A_{sec2} - A_{sec3} = A_{inside}$

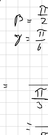
$\frac{8 \cdot 6}{2} - \frac{1}{2}(6^2 \cdot 0.965 + 8^2 \cdot \frac{\pi}{4} + 4^2 \cdot 0.707) = \underline{4.36}$

12. $x^2 + y^2 = 10^2$ $(5, y)$

$5^2 + y^2 = 10^2$

$y = \sqrt{10^2 - 5^2} = 5\sqrt{3}$ neglect negative root

$(5, 5\sqrt{3})$ and $(5\sqrt{3}, 5)$



$5\sqrt{3} - 5 \approx 3.66$

$\sqrt{3.66^2 + 3.66^2} = 5.176$

$\frac{x}{2} = \sin^{-1}\left(\frac{5.176}{2 \cdot 10}\right)$

$\frac{x}{2} = 0.5294$

$\frac{A}{4} = \frac{1}{2} 10^2 \cdot 0.5294 - \frac{1}{2}\left(\frac{3.66 \cdot 2.588}{x}\right)$

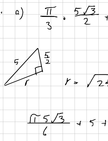
$\frac{A}{4} = 1.2 \cdot 9 = 9.8$

$c = \sqrt{100 - 2.588^2} = 9.66$

$4 \cdot 9 + 5.176^2 = \underline{31.59\,cm^2}$ ✓

nice

13.



$\alpha = \frac{2\pi}{3} \cdot 2 = \frac{\pi}{3}$

$\cos(\alpha) = \frac{r}{x}$

$x = \frac{r}{\cos\left(\frac{\pi}{3}\right)} = \frac{r}{\frac{1}{2}} = 2r$

b)

$2r + r = 3r$

$\beta = \frac{\pi}{2} - \frac{\pi}{3} = \frac{3\pi}{2 \cdot 3} - \frac{2\pi}{3 \cdot 2} = \frac{\pi}{6}$

$\gamma = \frac{\pi}{6} \cdot 2 = \frac{\pi}{3}$

$\frac{2}{3} = \frac{\frac{\pi r^2}{3} - \frac{1}{2} \cdot (3r)^2}{\frac{\pi}{3} \cdot \frac{1}{2} \cdot (3r)^2}$

$= \frac{\frac{\pi}{3} - 9}{\frac{\pi}{6} \cdot 9}$

$= \frac{\frac{\pi}{3} - 9}{\frac{9\pi}{6}} = \frac{\pi}{3} - \frac{6}{3\pi}$

$= \frac{6\pi - 18}{3\pi} = \frac{2}{3}$

$\frac{2}{3} = \frac{2}{3}$

Exam style:

4. a) $\frac{\pi}{3} + \frac{5\sqrt{3}}{2} + 5 + 2\left(\frac{5 - 5\sqrt{3}}{2}\right)$

$r = \sqrt{25 - \frac{25}{4}} = \sqrt{\frac{25}{4}} = \frac{5\sqrt{3}}{2}$

$\frac{\pi 5\sqrt{3}}{6} + 5 + 10 - 5\sqrt{3}$

$\frac{\pi 5\sqrt{3}}{6} + 15 - 5\sqrt{3}$

$\frac{\pi 5\sqrt{3}}{6} + 15 - 5\sqrt{3}$ $\sqrt{5^2 - 2.5^2} =$

b)

$\frac{5\sqrt{3}}{2} \cdot \frac{5}{2} = 2f$

$\frac{25\sqrt{3}}{4} - \frac{25\pi}{6}$ $\pi \cdot \frac{1}{2} \cdot 5^2$

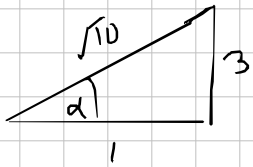
$\frac{150\sqrt{3} - 100\pi}{24}$

$\frac{75\sqrt{3} - 50\pi}{12}$

End of Chapter Review 5

1. $a = 1$ $b = 2$ (stretch by factor of 2)
(shift up by 1)

2. $\sin^{-1}(x-1) = \tan^{-1}(3)$



$c = \sqrt{1^2 + 3^2} = \sqrt{10}$

$x - 1 = \frac{3}{\sqrt{10}}$

$x = \frac{3}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} + 1$

$x = \frac{3\sqrt{10}}{10} + 1$

3 $\sin\theta =$

1.

9.1 Forces & Moment of Systems

Force
Moment
Force
Force
Force

8.2 Calculus in Kinematics

let's go!! ... etc. distance, velocity, acceleration. Look, you are seeing it's not a constant!!

acceleration = $\frac{\text{velocity}}{\text{time}} = \frac{dv}{dt}$

velocity = $\frac{\text{displacement}}{\text{time}} = \frac{ds}{dt}$ — derivative where all quantities 0

$\frac{dv}{dt} = v'(t)$, also, acceleration velocity, $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

Worked example:

1. $s(t) = t^3 - 18t$ $\quad \frac{ds}{dt}$

$v(t) = 3t^2 - 18$

2. $s(t) = 2t^3 - 10t^2$

$v(t) = 6t^2 - 20t$

find speed $t = 2$

$v(2) = v'(2)$

$v(2) = 6(2)^2 - 20t$

$v(2) = 6(4) - 20(2)$

$v(2) = 24 - 40 = -16\ m/s$

speed, 16 m/s

3. $s = 2t^3 - 12t^2 + 18t$

$v(t) = 6t^2 - 24t + 18$

$0 = 6t^2 - 24t + 18$

$0 = 6t^2 - 4t + 3$ $\quad$ A $\to$ AB 3

$0 = (t-3)(t-1)$ $\quad$ -3 -1

$t_1 = 3 \quad t_2 = 1$

$s(3) = 2 \cdot 3^3 - 12 \cdot 3^2 + 18 \cdot 3 = 0$ $\quad s(4) = 2 \cdot 4^3 - 12 \cdot 4^2 + 18 \cdot 4$

$s(1) = 2 \cdot 1^3 - 12 \cdot 1^2 + 18 \cdot 1 = 8$ $\quad = 40$

$s(0) = 0 - 0 + 0 = 0$

0 $\to$ 8

8 $\to$ 40 $\quad$ 8 + 8 + 40 = 56 m

4. $s(t) = 20t - 4$ find v

$v(t) = s'(t)$

$v(t) = 20$

$t = 20 - 5$

4) a) $s_0 = 6t^2 + 20t$

$v(t) = 12t + 20$

$v(0) = 20$

The acceleration due to the earth is assumed to be a constant 10 m/s² down.

There are no resistance instructions so based for the way up and down.

The ball finally stops partially straight up

b) $v(t) = 12t + 20$

$v(0) = 12 \cdot 0 + 20 = 20\ m/s$

$v(2) = 12 \cdot 2 + 20 = 44\ m/s$

5. $s(t) = 4t + t^2$

a) $s(18) = 4 \cdot 18 + 18^2 = 324\ m$

b) $v(t) = 4 + 2t$

$v(18) = 4 + 2 \cdot 18 = 36\ m/s$

c) $s_0 = 1.2t^2$ $\quad 3.20 = 1.2t^2$

$v(t) = 2.4t$ $\quad \frac{3.20}{1.2} = t^2$

$t = 1.633$

$v_1(1.633) = 2.4 \cdot 1.633 = 3.9\ m/s$

d) $s(18) = 1.2 \cdot 18^2 = 389.2\ m$

$324 - 389.2 = 12.8\ m$

10. $s(t) = At^2 + Bt$

$s(4) = A \cdot 4^2 + B \cdot 4$

$s(4) = 2A + 4B$

$s(7) = 5A^2$

$s(7) = 49 + 7B = 30$

$30 = 2A + 4B$

$40 = A + 2B$

because the displacement of the pendulum has to be continuous, that I need to of the pendulum have to have the same result same they intersection.

b) $s_1(t) = 5t^2$ $\quad s_2(t) = At^2 + Bt$

$v_1(t) = 10t$ $\quad v_2(t) = \frac{1}{2}At^{-\frac{1}{2}} + B$

$v_1(4) = 40 = x$ $\quad v_2(4) = \frac{1}{2}A \cdot 4^{-\frac{1}{2}} + B$

$40 = x = 0.25A + B$

c) $s = At^{\frac{1}{2}} + Bt$ $\quad s = 0.25t + 30$

$At^{\frac{1}{2}} = 0.25t + 0.25t + 30$

$v = \frac{1}{2}At^{-\frac{1}{2}} + B$ $\quad v = C$

$v = \frac{1}{2}At^{-\frac{1}{2}} + B$

$= 0.1A + B$

$0.1A + B = C$

$5 \cdot A + 25B = 25(0.1A + B) + 30$

$5 \cdot A + 25B = 2.5A + 25B + 30$

$2.5A = 2.5A + 30$

$2.5A = 30$

$A = 12$

$A + 2B = 40$

$12 + 2B = 40$

$2B = 28$

$B = 14$

$0.25t + 14 = 0.1t + 30$

$0.25t + 14 = 0.1t + 30$

$t = 40 - 14$

$t = 25$ ✓

Find my baby!

11. $s(t) = 0.6t^3 - 7.4t^2 - 3.6t^2 + 2.4t$

$v(t) = -2.4t^2 - 7.2t^2 - 7.2t + 2.4$

a) $v(0) = 0 + 0 - 0 - 2.4 = -2.4\ m/s$ Heading negative

b) $0 = -2.4t^2 - 7.2t^2 - 7.2t + 2.4$

$t = 1$ $\quad$ types only one solution

$s(1) =$

6.6

$s_1 = \frac{1}{3}t^3 - 6t^2 + 16t$

$v(t) = \frac{1}{3}t^2 - 12t + 16$

$v(t) = t^2 - 12t + 16$

$0 = t^2 - 12t + 16$

$x(t^2 - 12t + 6^2) = 16 - 6^2$

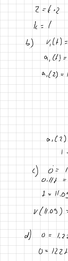
$x_{1,2} = \frac{12}{2} \pm \sqrt{\left(\frac{12}{2}\right)^2 - 16}$

$= (t - 6)^2 = 21$

$x_1 = 1.92 \quad > 10.58$

$x(0, -21)$

$v(10) = 10^2 - 12 \cdot 10 + 16 = -4$



$V(6, -21)$

6.8

11. $s(t) = t^2 - 2t^3 + 75t$

$v(t) = 2t - 6t^2 + 75$

$= -6(t^2 - \frac{1}{3}t + (\frac{1}{6})^2) + 75 - (-6 \cdot (\frac{1}{6})^2)$

$= -6(t - \frac{1}{6}) + 75.1$

a) after 1/6 hours (after 10 minutes)

b) max vel = 75.1 km/h

c) $3(\frac{1}{6}) = (\frac{1}{6})^2 - 2(\frac{1}{6})^3 + 75(\frac{1}{6}) = 12.52$ km

3.a) $s = 4t^3 - 0.05t^4$

$v(t) = 12t^2 - 0.05t^3$

$= -0.05(t^4 - \frac{12}{0.05}t) + (\frac{12}{2})^2 - (-0.05)$

$= -0.05(t - \frac{12}{2})^2 + 43.2$

$v_{max} = 21.33\ m/s$

a) $\frac{12}{0.05} = 5.3$

$t = 5.3$

$9.75 - 0.05 \cdot 10^3 = -12$ $\quad 12t^2 - 0.05t^3 = 0$

$12t^2 - 0.05t^3 = 0$

$t(12 - 0.05t) = 0$

$t_1 = 0 \quad 12 - 0.05t = 0$

$\frac{12}{0.05} = t$

$t = 240$

$t_2 = 240$

$t_1 = 0$

So 0, 5.3 and 5.3?

$10.67 - 5.3 = 5.37$

13. a)

$2 = 1.2 - 0.0005(t-2)^2$

$2 = 1 - 2$

$k = 1$

b) $v_1(t) = 1$ $\quad v_2(t) = 1.2 - 0.005(t-2)^2$

$a_1(t) = 1$ $\quad = 1 - 0.005(t^2 - 4t + 4)$

$a_2(t) = 1$ $\quad = 1 - 0.5t^2 + 0.01t - 0.02$

$= 1.2 - 0.005t^2 - 0.02$

$a_1(t) = 1.2 - 0.1t$

$a_2(t) = 1.2 - 0.01t - 1$

$a_1(t) = a_2(t)$

$1 = 1$

c) $0 = 1.2 - 0.1t$

$0.1t = 1.2$

$t = 10.5$

$v(10.5) = 11.05 - 0.005(10.5 - 2)^2 \approx 6.5\ m/s$

d) $0 = 1.2 - 0.005t^2 - 0.02$

$0 = 1.2 - 0.5t^2 - 0.2$

$t_{1,2} = \frac{-1.2 \pm \sqrt{(1.2)^2 - 4(-0.5)(-0.2)}}{2 \cdot (-0.2)}$

$t_1 = 5.95$ $\quad t_2 = -5.5$

5.95

e) $v(t) = 1.2 - 0.005t^2 - 0.02$

$s(t) = 0.6t^2 - \frac{0.01}{3}t^3 - 0.02t$

$s(5.5) = 0.6(5.5)^2 - \frac{1.1}{300}(5.5)^3 - 0.02(5.5)$

$s(5.5) = 19.19$

6C 10 12 min ≈ 10 min

Read 6.4 6 min ≈ 5 min

Worked example 6.11 5 min ≈ 4 min

6.12 10 min ≈ 12 min

6D 1 4 min ✓

2 4 min ✓

5 10 min ✓

7 10 min Fail → look if know a derivative rule

12 15 min

Chapter Review

8. 7 min

10. 12 min

11, 19 min

End of Chapter review 4.1 (6)

1. find coefficient of x^2 in $\left(2x+\frac{3}{x^2}\right)^5$

$0 + (-10)$
$1 + (-8)$
$2 + (-6)$
$3 + (-4)$
$r=4 \quad 4 + (-2) = 2$

$$\binom{5}{4}(2x)^4 \cdot \left(\frac{3}{x^2}\right)^1 = 5 \cdot 2^4 x^4 \cdot 3 \cdot x^{-2} = 240x^2$$

$a = 240$

2.

a^6	0	
a^5	x	
a^4	x^2	$\leftarrow 3$

$$\binom{6}{2} \cdot a^4 \cdot (2x)^2 = 60a^4x^2$$
$$= 60a^4$$

$$\binom{6}{1} \cdot a^5 \cdot 2x = 12a^5x$$
$$= 12a^5$$

$12a^5 = 60a^{\cancel{4}}$
$12a^5 = 60$
$a = 5$

3. $\binom{6}{2} \cdot 5^4 x^2 + \binom{6}{1} \cdot 5^5 x \cdot \left(-\frac{x}{a}\right)$

$= 15 \cdot 5^4 x^2 - 6 \cdot 5^5 x^2 \frac{1}{a}$

$= \frac{-9375x^2}{a}$

$0 = \frac{-9375}{a}$

$a = -9375$ w/f

9.1 Momentum

$$\vec{p} = m \cdot \vec{v}$$

$p \hat{=}$ momentum ($kg\,ms^{-1}$)
$m \hat{=}$ mass
$v \hat{=}$ velocity

Momentum is described as the impetus possessed by a moving object.

<u>Conservation of Momentum</u>

$$\underbrace{m_1 v_1 + m_2 v_2}_{\text{before collision}} = \underbrace{m_1 v_1' + m_2 v_2'}_{\text{after collision}}$$

Momentum stays conserved, even if mechanical energy does not.

(only when no external forces act)
always conserved in impacts

<u>Perfectly Elastic Collision:</u> Both momentum and kinetic energy stay conserved.
↳ objects split up

<u>Inelastic Collision:</u> Part of the kinetic energy is lost (transformed)
↳ momentum still stays the same

<u>Perfectly Inelastic Collision:</u> Objects stick together and continue as one object.

Practice

$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$

$4\cdot 3 + 0\cdot 6 = 0.4 + 6\cdot v$

$12 = 6v$

$v = 2\,ms^{-1}$

$3\cdot 4 - 2\cdot 4 = (3+2)v$

$4 = 5v$

$v = \frac{4}{5}\,ms^{-1}$

3. find a and v_f

$m_1 = 40\,000 + 50\,000 \cdot 2 = 140\,000\,kg$

$m_2 = 50\,000 \cdot 2 = 100\,000\,kg$

$(140\,000 + 100\,000)\cdot 5 = 140\,000\cdot .7 + 100\,000\,v$

$1\,100\,000 = 98\,000 + 100\,000$

$1\,200\,000 = 1\,00\,000$

$v = 2.2\,ms^{-1}$

$v_f^2 = v_i^2 + 2a\Delta x$

$0 = 2.2^2 + 2a\cdot 100$

$-4.84 = 200a$

$a = -0.0242\,ms^{-2}$

$v_f = v_i + at$

$0 = 2.2 + (-0.0242)t$

$2.2 = 0.0242t$

$t = 90.91\,s$

4.

$4\cdot 3 + 0 = -v + 4v$

$12 = 3v$

$v = 4\,ms^{-1}$

5.

$2\cdot 3 - 1\cdot 5 = 2\cdot(-v) + 5(v)$

$1 = 3v$

$v = \frac{1}{3}$

$v_f = \frac{1}{3}\,ms^{-1}$

6.

a) $m_1v + 2\cdot 0.2 =$ total momentum

total momentum $= 5m\cdot 0.4$

b)

$5m + 0.4 = (m+0.2)\,3$

$5m + 0.4 = 3m + 0.6$

$2m = 0.2$

$m = 0.1\,kg$

7.

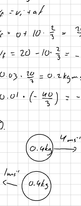
$3\cdot 4 - 2\cdot 4 = 3\cdot 0.4 + 2\cdot v$

$4 = 1.2 + 2v$

$2.8 = 2v$

$v = 1.4\,ms^{-1}$

8. a)



$\Delta x = \frac{1}{2}v_i^0 t + \frac{1}{2}at^2 = v_i t + \frac{1}{2}at^2$

$30 = \frac{1}{2}(20)t^2 + 20t + \frac{1}{2}(-10)t^2$

$30 = 20t$

$t = \frac{3}{2}\,s$

$v_f = v_i + at$

$v_f = 0 + 10\cdot\frac{3}{2} = \frac{20}{3}\,ms^{-1}$ down

$v_f = 20 - 10\cdot\frac{3}{2} = -\frac{60-20}{3} = \frac{40}{3}\,ms^{-1}$ up

$0.03\cdot\frac{20}{3} = 0.2\,kgms^{-1}$

$0.01\cdot(-\frac{40}{3}) = -\frac{2}{15}\,kgms^{-1}$

9.

$0.4\cdot 4 + 0 = 0.4(-1) + mv$

$1.6 = mv - 0.4$

$2 = mv$

$mv + 0.25\cdot 0.8 = mv + 0.25\cdot 1$

$2 + 0.2 = mv + 0.25$

$m\in 1.75$

$2 = mv$ $\quad m\in = 1.75$

$v > 0.8$ $\quad v < 1$

$2 = m\cdot 0.8$ $\quad m\cdot 1 = 1.75$

$m < 2.5$ $\quad m > 1.75$

$v > -1$

$m(-1) = 1.75$

$m > -1.75$

$1.75 < m < 2.5$

10.

a) $v_f^2 = v_i^2 + 2a\Delta x$

$v_f = \sqrt{2\cdot 10\cdot 1.8} = 6$

$v' = 6\cdot 0.9 = 5.4$

$0.6\cdot 6 - 0.6\cdot 5.4 = 0.36\,kgms^{-1}$

b) $v_f^2 = v_i^2 + 2a\Delta x$

$\Delta x = \frac{-v_i^2}{2a}$

after 1st bounce:

$\frac{-(5.4)^2}{2\cdot(-10)} = 1.46\,m$

bounce #2: $v = 5.4\cdot 0.9 = 4.86$

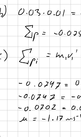
$\frac{-(4.86)^2}{2\cdot(-10)} = 1.18$

bounce #3: $v = 4.86\cdot 0.9 = 4.37$

$\frac{-(4.37)^2}{2\cdot(-10)} = 0.957\,m$

c) the ball always loses exactly 10% of its speed. the ball is dropped on earth, and has a radius of 0.

11.



$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$

$0.03\cdot 0.5 + 0 = 0.03\cdot 0.01 + 0.06\mu$

$0.0153 = 0.0003 + 0.06\mu$

$0.06\mu = 0.015$

$\mu = \frac{15}{4}$

when holding the wall, it absorbs 1.5s initial momentum with 0.5s

make the sign clear

a) $\frac{1}{4}\cdot 0.06\cdot(-0.5) = -0.025\,kgms^{-1} = p_f$

b) $0.03\cdot 0.01 = 0.0003\,kgms^{-1} = p_i$

$\sum p = -0.025 - 0.0003 = -0.0247\,kgms^{-1}$, so negative

c) $\sum p = m_1v_1' + m_2v_2'$

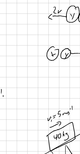
$-0.0247 = 0.03\cdot(-0.15) + 0.06\mu$

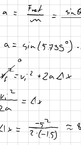
$-0.0247 = -0.0045 + 0.06\mu$

$-0.0202 = 0.06\mu$

$\mu = -1.17\,ms^{-1}$ ↓

12.





a) find a

b) Δx

c) ΔPE_g

$\theta = \sin^{-1}(10) =$

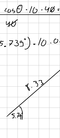
$a = \frac{F_{net}}{m} = \frac{\sin\theta\cdot 10\cdot 40 + \cos\theta\cdot 10\cdot 40\cdot 0.05}{40}$

$a = \sin(5.735°)\cdot 10 + \cos(5.735°)\cdot 10\cdot 0.05 = 1.50\,ms^{-2}$

$v_f^2 = v_i^2 + 2a\Delta x$

$\frac{v_i^2}{2a} = \Delta x$

$\Delta x = \frac{-5^2}{2\cdot(-1.5)} \approx 8.33\,m$



$\sin(5.735)\cdot 8.33 = 0.833\,m$

$PE_g = 0.833\cdot 40\cdot 10 = 333\,J$

$KE = \frac{1}{2}mv^2$

$= \frac{1}{2}\cdot 40\cdot 5^2 \approx 500\,J$

$500\,J - 333\,J = 167\,J$

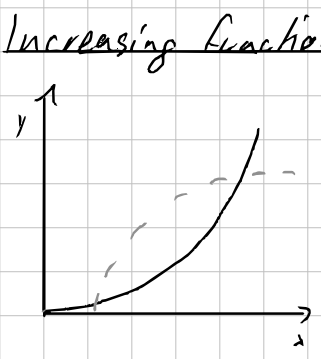
5.1 Derivatives

Finding a function that describes the slope of another function (its instantaneous rate of change) at any given point.

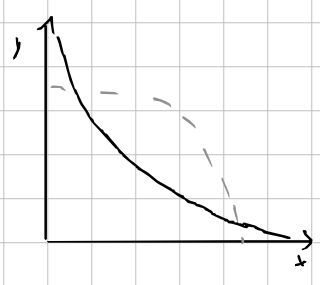
A function is:

- Decreasing if the derivative is always negative
- Increasing if the derivative is always positive

Increasing function:

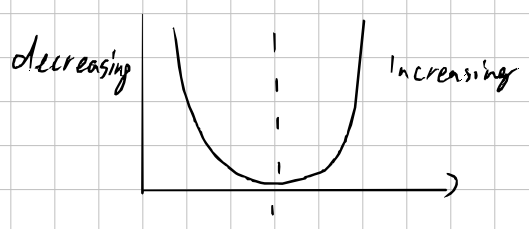


Decreasing function:



--- just a variation

Functions can also increase/decrease over sections:



Derivative Notation

- $y = x^2$ becomes $\frac{dy}{dx} = 2x$
- $f(x) = x^2$ becomes $f'(x) = 2x$
- x^2 becomes $\frac{d}{dx}(x^2) = x^2$

note: $f'(x)$ or $\frac{dy}{dx}$ is sometimes called the gradient function

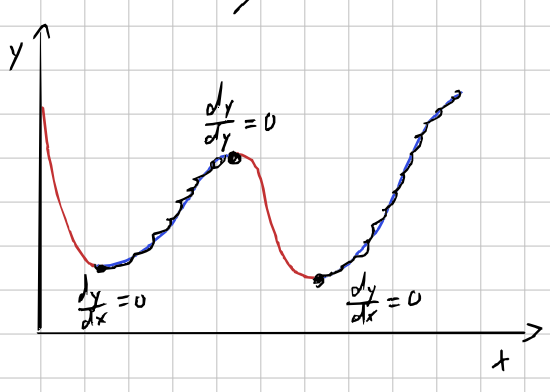
Second Derivative Notation

First derivative: $\frac{dx}{dy}(\ldots)$

Second derivative: $\frac{d}{dx}\left(\frac{dx}{dy}(\ldots)\right) \Rightarrow \frac{d^2y}{dx^2}(\ldots)$

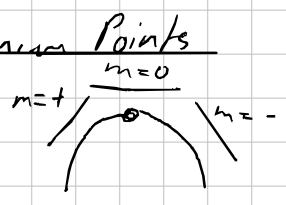
so $\boxed{\frac{d^2y}{dx^2}}$ or $f''(x)$

Stationary Points



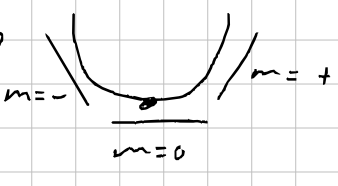
Maximum Points

$\frac{dy}{dx} = 0$



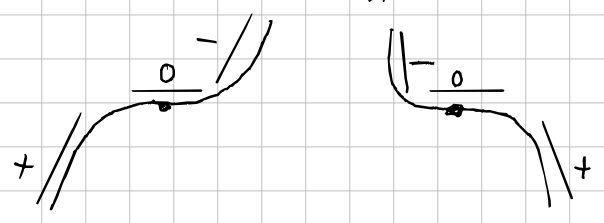
Minimum Points

$\frac{dy}{dx} = 0$



Points of Inflection

$\frac{dy}{dx} = 0$



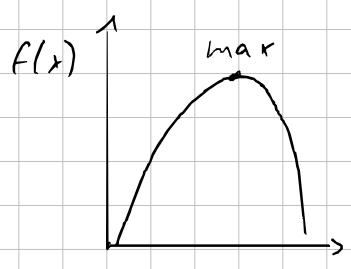
First Derivative Test

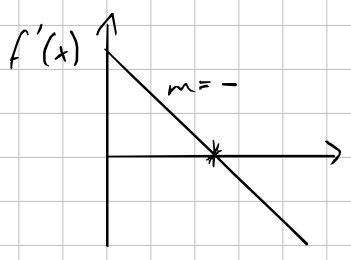
1. Find the derivative (gradient function)
2. Solve for $f'(x) = 0$
3. For each found value of x plug in a number slightly larger and smaller:

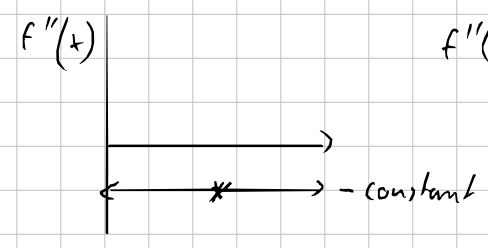
   e.g if $f'(3) = 0$ then plug in $f'(3.1) = \ldots$ and $f'(2.9) = \ldots$
4. Compare found pattern (e.g. +, 0, −) with known stationary point behaviour

Second Derivative Test

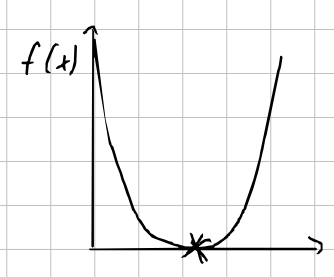
Local Maximum

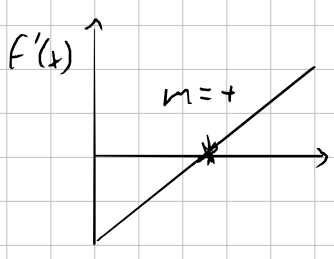


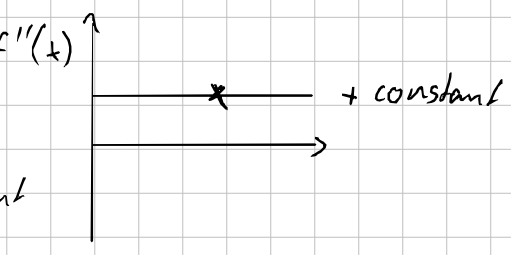




Local Minimum







Local Maximum: $f''(x) < 0$ as +, 0, −

Local Minimum: $f''(x) > 0$ as −, 0, +

if $f''(x) = 0$, use first derivative test!

Simplified Chain Rule

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$\frac{dy}{dt}$ ≙ ratio of the change in y over change in t

$\frac{dy}{dx}$ ≙ ratio of change in y over change in x

$\frac{dx}{dt}$ ≙ ratio of change in x over change in t

where the function has three variables:

eg $y = x^2$ $\quad x = 3t + 1$

$\frac{dy}{dx} = 2x$

$\frac{dx}{dt} = 3$

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = 2x \cdot 3 = 6x$

$\frac{dy}{dt} = 6x$ | substitute $3t+1$ for x

$\frac{dy}{dt} = 6(3t+1) = 18t + 6$

Inverse for Derivatives

$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$$

meaning that $\frac{dy}{dx}$ is the inverse of $\frac{dx}{dy}$

$2x^2 + 4x - 2 = 0$

$7x^2 - 4x - 51 = 0$

$x_{1,2} = \frac{4 \pm \sqrt{4^2 - 4 \cdot 7 \cdot (-51)}}{2 \cdot 7}$

$x_1 = -\frac{12}{7} \qquad x_2 = 3$ ✓

10. $\binom{5}{4} \cdot (2x)^4 =$

$\binom{5}{1} \cdot (2x)^4 =$

$\binom{5}{2} \cdot (2x)^3 = 32x^5 + 16x^4 + 8x^3 \quad 1:30/2:00$

Fuck, according

10. i $\binom{5}{0} \cdot 1 \cdot 1 = 1$

$\binom{5}{1} \cdot 1 \cdot (2x) = 10x$

$\binom{5}{2} \cdot 1 \cdot (2x)^2 = \underset{\underset{\binom{5}{3}}{\downarrow}}{\underset{10}{\cancel{20x}}}$ $\quad 1:14 / 2:00$

$40x^2$

ii $\binom{5}{5} \cdot 1 \cdot (-x)^5 = -x^5$

$\binom{5}{4} \cdot 3 \cdot (-x)^4 = 3x^4 \cdot 5 = 15x^4$

$\binom{5}{3} \cdot 3^2 \cdot (-x)^3 = 10 \cdot 9 \cdot (-x^3) = -90x^3$

$1:40 : 2:00$

$\frac{n}{2}\left[2 \cdot 1.75 + (n-1) \cdot (-0.25)\right] = -n$

$\frac{n}{2}\left[3.5 - 0.25n + 0.75\right] = \frac{-n2}{4}$

$3.75 - 0.25n = -2$

$5.75 = 0.25n$

$n = 23$ ✓ $\qquad 4:30 \ / \ 4:00$

Arithmetic sequences

$u_n = u_1 + (n-1)d$ $\qquad \cancel{S_n = \frac{n}{2}[u_1 + (n-1)d]}$

$S_n = \frac{n}{2}\left[2u_1 + (n-1)d\right]$

$S_n = \frac{n}{2}(u_1 + u_n)$

Geometric

$u_n = u_1 \cdot r^{n-1}$ $\qquad S_n = \frac{u_1(1-r^n)}{1-r}$

infinite sum

$S_n = \frac{u_1}{1-r}$

$S_n = \frac{u_1}{1-r}$ for converging

for diverging, if $r > 1$ then $+\infty$

if $r < -1$ then $-\infty$

again sum for infinite converging is

$u_1 = \frac{u_1}{1-r}$

sum for u_n is

$S_n = \frac{u_1(r^n - 1)}{r-1} = \frac{u_1(1-r^n)}{1-r}$

Arithmetic Series

$u_n = u_1 + (n-1)d$

$S_n = \frac{n}{2}\left[2 \cdot u_1 + (n-1)d\right]$

$S_n = \frac{n}{2}\left[u_1 + u_n\right]$

back to geometric lol :)

$u_n = u_1 \cdot r^{n-1}$

$S_n = \frac{u_1(1-r^n)}{1-r} \qquad S_\infty = \frac{u_1}{1-r}$ for converging

for diverging:

if $r > 1$ then $S_n = +\infty$

if $r < -1$ then $S_n = -\infty$

And now all rules for differentiation (know!)

power rule: $x^n \to n x^{n-1}$

add-subtract rule or whatever: $f(x) \pm g(x) \to f'(x) \pm g'(x)$

product rule: $f(x)g(x) \to g(x)f'(x) + g'(x)f(x)$

division rule: $\frac{f(x)}{g(x)} \to \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

chain rule: $f(g(x)) \to f'(g(x)) \cdot g'(x)$

$d = u_n - u_{n-1}$

$r = \frac{u_n}{u_{n-1}}$

12. $r = \left(\frac{-1458}{432}\right)^{\frac{1}{2-5}} = -\frac{3}{3}$ ✓

$u_n = u_1 \cdot r^{n-1}$

$432 = u_1 \cdot (-\frac{3}{3})^{5-1}$

$\cancel{\frac{432}{(\frac{3}{3})}} \quad u_1 = \frac{432}{(\frac{3}{3})^4} = 2187$ ✓

$u_1 = -32805$

converging

$S_\infty = \frac{u_1}{1-r} = \frac{2187}{1-(-\frac{3}{3})} = 1312.2$ ✓

$r = \left(\frac{u_n}{u_m}\right)^{\frac{1}{n-m}}$

19. i $d = 1000 \quad n = 40$

$1000 \cdot 0.05 = 50$

$2000 \cdot 0.05 = 0.5$

$r = 0.5$

$S_n = \frac{1000(1-0.5^{40})}{1-0.5} = 2000\$$

ii 1000

$1000 \cdot 1.1 = 1100$

$1100 \cdot 1.1 = 1210$

$1000 \cdot 0.05 = 50$

$1100 \cdot 0.05 = 55$

$1210 \cdot 0.05 = 60.5$

$\frac{60.5}{55} = 1.1 \qquad \frac{55}{50} = 1.1$

$u_1 = 50 \quad r = 1.1 \quad n = 40$

$S_n = \frac{u_1(1-r^n)}{1-r} = \frac{50(1-1.1^{40})}{1-1.1} \approx 22130\$$

15. a) $1, \cos^2 x \to a - b\sin^2 x$

$S_n = \frac{n}{2}\left[2 \cdot u_1 + (n-1)d\right] = \frac{10}{2}\left[2 \cdot 1 + (10-1)(\cos^2 x - 1)\right]$

$= 5\left[2 \cdot 1 + (10\cos^2 x - 10 - \cos^2 x + 1)\right]$

$= 10 + 5(9\cos^2 x - 9)$

$= 10 + 45\cos^2 x - 45$

$= 10 + \cancel{45}(1 - \sin^2 x) - 46$

$= 10 + \cancel{45 - 45}\sin^2 x$

$S_n = 10 - 45\sin^2 x$

$a = 10 \quad b = 45$

b) i $1, \frac{1}{3}\ln^2\theta$ geometric

$u_n = u_1 \cdot r^{n-1}$

$\frac{1}{3}\ln^2\theta = 1 \cdot r^1$

$\frac{1}{3}\ln^2\theta = r$

$-1 < r < 1$ but $\ln^2 \geq 0 \quad 0 \leq r < 1$

$1 < \frac{1}{3}\ln^2\theta$

$3 < \ln^2\theta$

$\sqrt{3} < \ln\theta$

$\ln^{-1}(\sqrt{3}) = \underline{60^\circ}$

$0 \leq r < 1$

quadrantable but only?

End of Chapter 8

1. $\frac{dV}{dt} = 40$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$

$40 = 4\pi r^2 \cdot \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{10}{\pi r^2} = \frac{10}{\pi \cdot .15^2} = 0.0141$ cm a second ✓

Integration Rules Rev.

Indefinite Integrals: - Have infinite solutions
- Written: $\int (\ )\,dx$ *without top or bottom numbers

$$\frac{dy}{dx} = x^n \qquad y = \frac{1}{n+1}x^{n+1} + C$$

$$\int k f(x)\,dx = k\int f(x)\,dx$$

$$\int \left(f(x) \pm g(x)\right)dx = \int f(x)\,dx \pm \int g(x)\,dx$$

$$\int (ax+b)^n\,dx = \int \frac{1}{a(n+1)}(ax+b)^{n+1} + C$$

Definite Integrals:

Integrals that have a specified range

The notation is:

$$\int_a^b f(x)\,dx$$

Rules specifically for indefinite Integrals:

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

Evaluating Integrals:

$$\int_a^b f(x)\,dx = \Big[F(x)\Big]_a^b = F(b) - F(a)$$

Area between two curves:

$$A = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx \qquad \text{or}$$

$$A = \int_a^b \left[f(x) - g(x)\right]dx$$

Improper Integrals

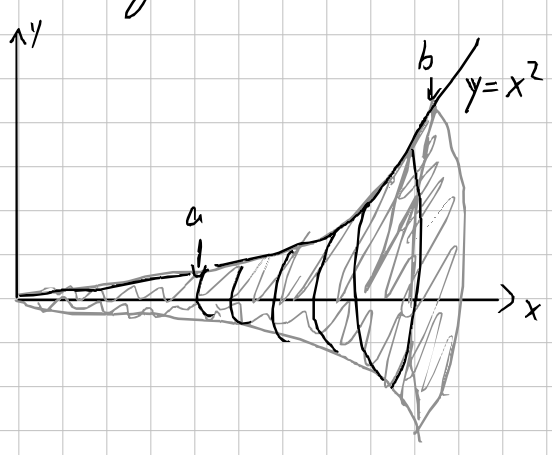
- Integrals with one or two limits being infinite
- Integrals that approach infinity (in any way really)

Solving Integrals with infinite limits:
- 1. find the Integral
- 2. evaluate the limit of the integrated function

$$\int_a^\infty f(x)\,dx = \Big[F(x)\Big]_a^\infty$$

$$= \lim_{x\to\infty} F(x) - \Big[F(x)\Big]_0^a$$

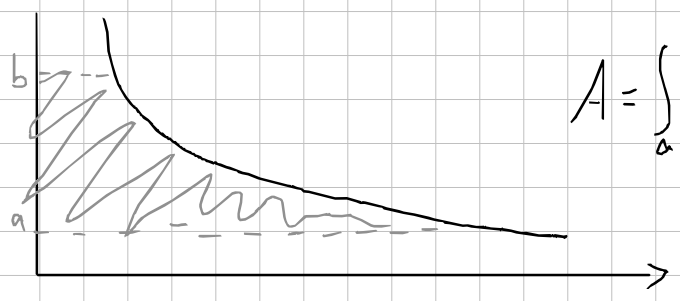
Finding Volumes of Rotated Areas:



$$V = \int_a^b \pi y^2\,dx$$

Integrals on the Y-Axis

if $f(x) \geq 0$ (for every point between a and b)



$$A = \int_a^b f(y)\,dy$$

$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b| + C$$

$$\int \frac{1}{x}\,dx = \ln x + C$$

$$\int \frac{f(x)'}{f(x)} = \ln f(x) + C$$

$$\int e^x\,dx = e^x + C$$

$$\int e^{(ax+b)}\,dx = \frac{1}{a}e^{(ax+b)}$$ only for lineal functions

$$\int a^{(bx+d)}\,dx = \frac{a^{(bx+d)}}{b \cdot \ln a}$$

Practice :

1. $\int (12x^3 + 10x)\,dx + c = 3x^4 + 5x^2 + c$

$f(-1) = 1$

$3(-1)^4 + 5(-1)^2 + c = 1$

$3 + 5 + c = 1$

amoni

$8 + c = 1$

$c = 7$

2. $\int \left(5x - \frac{2}{x}\right)^2 dx + c$

$\int fg(x)\,dx + c$

$\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$

2. $\int \left(5x - \frac{2}{x}\right)^2 dx + c$

$1\left(5x - \frac{2}{x}\right)^2 \cdot 1$

$\left[\frac{1}{5(2+1)}\left(5x - \frac{2}{x}\right)^{(}\right.$

$f'(x) = x^2 \quad g'(x) = 1$

$f(x) = \frac{1}{3}x^3 + c \quad g(x) = x$

$f(g(x)) = \frac{1}{3}x^3 + c$

wrong :)

13. $y = 2 - \frac{18}{2x+3}$

i $\quad f(x) = -18 \qquad g(x) = 2x + 3$

$f'(x) = 0 \qquad g'(x) = 2$

$$\frac{dy}{dx} = \frac{g(x) \cdot f'(x) - f(x)g'(x)}{g(x)^2}$$

$$\frac{dy}{dx} = \frac{(2x+3)\,0 - (-18) \cdot 2}{(2x+3)^2} = 36(2x+3)^{-2}$$

$0 = 2 - \frac{18}{2x+3}$

$2 = \frac{18}{2x+3}$

$2(2x+3) = 18$

$4x + 6 = 36$

$$y = x^n \rightarrow y = \frac{1}{n+1} x^{(n+1)} + C$$

$$y = \int k f(x)\,dx = k \int f(x)\,dx$$

$$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

$$(ax+b)^n \rightarrow \frac{1}{a(n+1)} (ax+b)^{(n+1)} + C$$

! only works for linear expressions

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$V = \int \pi y^2\,dx$$

Lab Exp 8

```
clear;
x=[0:0.1:2*pi];
y=sin(x);
plot(x,y)
xlabel('x')
ylabel('y')
title('sin(x)')
grid on
x=[0:0.1:2*pi];
y=cos(x);
plot(x,y)
xlabel('x')
ylabel('y')
title('cos(x)')
grid on
x=[0:0.1:2*pi];
y=tan(x);
plot(x,y)
xlabel('x')
ylabel('y')
title('tan(x)')
grid on
x=[0:0.1:2*pi];
y=cot(x);
plot(x,y)
xlabel('x')
ylabel('y')
title('cot(x)')
grid on
x=[0:0.1:2*pi];
y=sec(x);
plot(x,y)
xlabel('x')
ylabel('y')
title('sec(x)')
grid on
x=[0:0.1:2*pi];
y=csc(x);
plot(x,y)
xlabel('x')
ylabel('y')
title('csc(x)')
grid on
```

End of Chapter 9 (Mechanics)

1. $F = (F_x, F_y)$

$F_x = m \cdot a = 1750 \cdot 0.25 = 437.5$

$F_y = m \cdot g = 1750 \cdot 9.81 = 17167.5$

$(437.5, 17167.5) = (F_x, F_y) = F$

$1000 \cdot 9.5 \cdot 9.81 = 93195$ N

$11 F_x = 93195$

$F_x = 93.195$

$F = (93.195, 17167.5) = 17.5500 \cdot 10^3$

2. a) $F = 1 \cdot 10 \cdot 10 = 10$ N

$\frac{F}{2} = 10 \cdot 10 \quad \text{Non-T} = ma$

$\frac{F}{2} = 10 \cdot 10 \quad 10 = \frac{F}{2} \cdot a$

$\frac{F}{2} = 10 \cdot 10$

$\frac{F}{2} = 10 \cdot 10$

$F(\frac{1}{2} + \frac{1}{2}) = 20$

$F = \frac{20}{1/2 + \frac{1}{2}}$

$F = \frac{20}{\frac{1}{2} + \frac{1}{2}}$

$F = \frac{20}{1} = 20 = \frac{20 \cdot 2}{2} = \frac{40}{2} = \frac{80}{4}$

$F = \frac{20}{1} = 20 = \frac{20 \cdot 2}{2} = \frac{40}{2} = \frac{80}{4}$

$\text{Wed} = \Delta KE$

$\text{Wed} F_d = \left(\frac{\text{Mass}}{2}\right) \cdot v^2$

$\text{Wed} F_d = \frac{1}{2} \cdot m \cdot (v_f^2 - v_i^2) = 2 \cdot (0.5 \cdot 3)$

$\left(\frac{\text{Mass}}{2}\right) \cdot (0.5 \cdot 3 \cdot 3)$

$\frac{\text{Mass}}{2} = 0.5 \cdot 3 \cdot 3$

$\frac{\text{Mass}}{2} = 0.5 \cdot 3 \cdot 3$

$\frac{\text{Mass}}{2} = 4.5$

$\frac{\text{Mass}}{2} = 4.5 \cdot 2$

$\frac{\text{Mass}}{2} = 9$

3. $\text{Mass} = 4$

$\text{Wed} = \Delta KE$

$F_d = \text{Mass} \cdot a \cdot d = 1500 \cdot 10 \cdot 1500$

$\text{Wed} = m \cdot (1000 - 350 - 1500) \cdot 1500$

$F_{max}^2 = m \cdot \text{Wed}$

$\sqrt{1500 \cdot 1 \cdot 1500}$

$2500^2 = 1 \cdot 1500$

$v = \sqrt{2 \cdot 1 \cdot 1500}$

$v = \sqrt{3000}$

$v = 54.77$

4.

a) $\frac{1}{2} \cdot \frac{F_{max} - \text{Wed} - \text{Wed}}{\text{Mass}} \cdot t^2$

$1.2 = \frac{F_{max} - 350}{1500} \cdot t^2$

$1800 = F_{max} - 350$

$F_{max} = 1500$

$\text{Wed} = 1500 \cdot 1 \cdot 1500$

$F_{max} = 2150$

$50 \cdot 000 = 2150 \cdot v$

$v = \frac{50 \cdot 000}{2150}$

$v = 23.25 \text{ m s}^{-1}$

5. The speed would increase because the friction decreases and because it is given by $\frac{F}{m}$

6. $F = F_x$ in general, but in this case, the force, the speed would increase, because friction is less than the pull force. Because with each second, a little additional is applied.

7.

When $F_{max} = 1500$ N

$a = 1.2 \text{ m s}^{-2}$

For the speed

$F = F_x$

$F = \frac{F_{max} - \text{Wed} - \text{Wed}}{\text{Mass}}$

$F = \frac{F_{max} - 350 - 350}{1500}$

$F = \frac{F_{max} - 700}{1500}$

$F = 1.2 \cdot 1500 = 1800$

$F = 1800 + 700 = 2500$

$F = 2500$

$50 \cdot 000 = 2500 \cdot v$

$v = \frac{50 \cdot 000}{2500}$

$v = 20 \text{ m s}^{-1}$

8. $\text{Wed} = \Delta KE$

$\text{Wed} F_d = \Delta KE$

$\text{Wed} F_d = \frac{1}{2} \cdot m \cdot (v_f^2 - v_i^2)$

$\text{Wed} F_d = \frac{1}{2} \cdot 1 \cdot (0 - 3.5^2)$

$\text{Wed} F_d = \frac{1}{2} \cdot 1 \cdot (-3.5^2)$

$\text{Wed} F_d = -\frac{1}{2} \cdot 3.5^2$

$\text{Wed} F_d = -6.125$

$F_d = \frac{-6.125}{1}$

$F_d = -6.125$

9. $F_{max} = 1500$

$F_{max} = 1500 \cdot 9.81$

$F_{max} = 14715$

$\text{Wed} = 1500 \cdot 1 \cdot 1500$

$\text{Wed} = 2250$

$\text{Wed} = 14715 - 2250$

$\text{Wed} = 12465$

$12465 = 1500 \cdot v$

$v = \frac{12465}{1500}$

$v = 8.31 \text{ m s}^{-1}$

10. $F_{max} = 1500$

$F = F_x$

$F = \frac{F_{max} - \text{Wed} - \text{Wed}}{\text{Mass}}$

$F = \frac{F_{max} - 350 - 350}{1500}$

$F = \frac{F_{max} - 700}{1500}$

$F = 1.2 \cdot 1500 = 1800$

$F = 1800 + 700 = 2500$

$F = 2500$

$50 \cdot 000 = 2500 \cdot v$

$v = \frac{50 \cdot 000}{2500}$

$v = 20 \text{ m s}^{-1}$

11. a) $\text{Wed} = \Delta KE$

$\text{Wed} F_d = \Delta KE$

$\text{Wed} F_d = \frac{1}{2} \cdot m \cdot (v_f^2 - v_i^2)$

$\text{Wed} F_d = \frac{1}{2} \cdot 1 \cdot (0 - 3.5^2)$

$\text{Wed} F_d = \frac{1}{2} \cdot 1 \cdot (-3.5^2)$

$\text{Wed} F_d = -\frac{1}{2} \cdot 3.5^2$

$\text{Wed} F_d = -6.125$

$F_d = \frac{-6.125}{1}$

$F_d = -6.125$

b) $F_{max} = 1500$

$F_{max} = 1500 \cdot 9.81$

$F_{max} = 14715$

$\text{Wed} = 1500 \cdot 1 \cdot 1500$

$\text{Wed} = 2250$

$\text{Wed} = 14715 - 2250$

$\text{Wed} = 12465$

$12465 = 1500 \cdot v$

$v = \frac{12465}{1500}$

$v = 8.31 \text{ m s}^{-1}$

12.

$\text{Wed} = \Delta KE$

$\text{Wed} F_d = \Delta KE$

$\text{Wed} F_d = \frac{1}{2} \cdot m \cdot (v_f^2 - v_i^2)$

$\text{Wed} F_d = \frac{1}{2} \cdot 1 \cdot (0 - 3.5^2)$

$\text{Wed} F_d = \frac{1}{2} \cdot 1 \cdot (-3.5^2)$

$\text{Wed} F_d = -\frac{1}{2} \cdot 3.5^2$

$\text{Wed} F_d = -6.125$

$F_d = \frac{-6.125}{1}$

$F_d = -6.125$

$F_{max} = 1500$

$F_{max} = 1500 \cdot 9.81$

$F_{max} = 14715$

$\text{Wed} = 1500 \cdot 1 \cdot 1500$

$\text{Wed} = 2250$

$\text{Wed} = 14715 - 2250$

$\text{Wed} = 12465$

$12465 = 1500 \cdot v$

$v = \frac{12465}{1500}$

$v = 8.31 \text{ m s}^{-1}$

13. $F = \frac{F_{max} - \text{Wed} - \text{Wed}}{\text{Mass}}$

$F = \frac{F_{max} - 350 - 350}{1500}$

$F = \frac{F_{max} - 700}{1500}$

$F = 1.2 \cdot 1500 = 1800$

$F = 1800 + 700 = 2500$

$F = 2500$

$50 \cdot 000 = 2500 \cdot v$

$v = \frac{50 \cdot 000}{2500}$

$v = 20 \text{ m s}^{-1}$

14. $F = F_x$

$F = \frac{F_{max} - \text{Wed} - \text{Wed}}{\text{Mass}}$

$F = \frac{F_{max} - 350 - 350}{1500}$

$F = \frac{F_{max} - 700}{1500}$

$F = 1.2 \cdot 1500 = 1800$

$F = 1800 + 700 = 2500$

$F = 2500$

$50 \cdot 000 = 2500 \cdot v$

$v = \frac{50 \cdot 000}{2500}$

$v = 20 \text{ m s}^{-1}$

15. a) $\text{Wed} = \Delta KE$

$\text{Wed} F_d = \Delta KE$

$\text{Wed} F_d = \frac{1}{2} \cdot m \cdot (v_f^2 - v_i^2)$

$\text{Wed} F_d = \frac{1}{2} \cdot 1 \cdot (0 - 3.5^2)$

$\text{Wed} F_d = \frac{1}{2} \cdot 1 \cdot (-3.5^2)$

$\text{Wed} F_d = -\frac{1}{2} \cdot 3.5^2$

$\text{Wed} F_d = -6.125$

$F_d = \frac{-6.125}{1}$

$F_d = -6.125$

b) $F_{max} = 1500$

$F_{max} = 1500 \cdot 9.81$

$F_{max} = 14715$

$\text{Wed} = 1500 \cdot 1 \cdot 1500$

$\text{Wed} = 2250$

$\text{Wed} = 14715 - 2250$

$\text{Wed} = 12465$

$12465 = 1500 \cdot v$

$v = \frac{12465}{1500}$

$v = 8.31 \text{ m s}^{-1}$

16. $F = F_x$

$F = \frac{F_{max} - \text{Wed} - \text{Wed}}{\text{Mass}}$

$F = \frac{F_{max} - 350 - 350}{1500}$

$F = \frac{F_{max} - 700}{1500}$

$F = 1.2 \cdot 1500 = 1800$

$F = 1800 + 700 = 2500$

$F = 2500$

$50 \cdot 000 = 2500 \cdot v$

$v = \frac{50 \cdot 000}{2500}$

$v = 20 \text{ m s}^{-1}$

17. a) $\text{Wed} = \Delta KE$

$\text{Wed} F_d = \Delta KE$

$\text{Wed} F_d = \frac{1}{2} \cdot m \cdot (v_f^2 - v_i^2)$

$\text{Wed} F_d = \frac{1}{2} \cdot 1 \cdot (0 - 3.5^2)$

$\text{Wed} F_d = \frac{1}{2} \cdot 1 \cdot (-3.5^2)$

$\text{Wed} F_d = -\frac{1}{2} \cdot 3.5^2$

$\text{Wed} F_d = -6.125$

$F_d = \frac{-6.125}{1}$

$F_d = -6.125$

b) $F_{max} = 1500$

$F_{max} = 1500 \cdot 9.81$

$F_{max} = 14715$

$\text{Wed} = 1500 \cdot 1 \cdot 1500$

$\text{Wed} = 2250$

$\text{Wed} = 14715 - 2250$

$\text{Wed} = 12465$

$12465 = 1500 \cdot v$

$v = \frac{12465}{1500}$

$v = 8.31 \text{ m s}^{-1}$

18. $F = F_x$

$F = \frac{F_{max} - \text{Wed} - \text{Wed}}{\text{Mass}}$

$F = \frac{F_{max} - 350 - 350}{1500}$

$F = \frac{F_{max} - 700}{1500}$

$F = 1.2 \cdot 1500 = 1800$

$F = 1800 + 700 = 2500$

$F = 2500$

$50 \cdot 000 = 2500 \cdot v$

$v = \frac{50 \cdot 000}{2500}$

$v = 20 \text{ m s}^{-1}$

19. a) $\text{Wed} = \Delta KE$

$\text{Wed} F_d = \Delta KE$

$\text{Wed} F_d = \frac{1}{2} \cdot m \cdot (v_f^2 - v_i^2)$

$\text{Wed} F_d = \frac{1}{2} \cdot 1 \cdot (0 - 3.5^2)$

$\text{Wed} F_d = \frac{1}{2} \cdot 1 \cdot (-3.5^2)$

$\text{Wed} F_d = -\frac{1}{2} \cdot 3.5^2$

$\text{Wed} F_d = -6.125$

$F_d = \frac{-6.125}{1}$

$F_d = -6.125$

b) $F_{max} = 1500$

$F_{max} = 1500 \cdot 9.81$

$F_{max} = 14715$

$\text{Wed} = 1500 \cdot 1 \cdot 1500$

$\text{Wed} = 2250$

$\text{Wed} = 14715 - 2250$

$\text{Wed} = 12465$

$12465 = 1500 \cdot v$

$v = \frac{12465}{1500}$

$v = 8.31 \text{ m s}^{-1}$

20. $F = F_x$

$F = \frac{F_{max} - \text{Wed} - \text{Wed}}{\text{Mass}}$

$F = \frac{F_{max} - 350 - 350}{1500}$

$F = \frac{F_{max} - 700}{1500}$

$F = 1.2 \cdot 1500 = 1800$

$F = 1800 + 700 = 2500$

$F = 2500$

$50 \cdot 000 = 2500 \cdot v$

$v = \frac{50 \cdot 000}{2500}$

$v = 20 \text{ m s}^{-1}$

21. $F = F_x$

$F = \frac{F_{max} - \text{Wed} - \text{Wed}}{\text{Mass}}$

$F = \frac{F_{max} - 350 - 350}{1500}$

$F = \frac{F_{max} - 700}{1500}$

$F = 1.2 \cdot 1500 = 1800$

$F = 1800 + 700 = 2500$

$F = 2500$

$50 \cdot 000 = 2500 \cdot v$

$v = \frac{50 \cdot 000}{2500}$

$v = 20 \text{ m s}^{-1}$

22. $F = F_x$

$F = \frac{F_{max} - \text{Wed} - \text{Wed}}{\text{Mass}}$

$F = \frac{F_{max} - 350 - 350}{1500}$

$F = \frac{F_{max} - 700}{1500}$

$F = 1.2 \cdot 1500 = 1800$

$F = 1800 + 700 = 2500$

$F = 2500$

$50 \cdot 000 = 2500 \cdot v$

$v = \frac{50 \cdot 000}{2500}$

$v = 20 \text{ m s}^{-1}$

23. $F = F_x$

$F = \frac{F_{max} - \text{Wed} - \text{Wed}}{\text{Mass}}$

$F = \frac{F_{max} - 350 - 350}{1500}$

$F = \frac{F_{max} - 700}{1500}$

$F = 1.2 \cdot 1500 = 1800$

$F = 1800 + 700 = 2500$

$F = 2500$

$50 \cdot 000 = 2500 \cdot v$

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27. $F = F_x$

$F = \frac{F_{max} - \text{Wed} - \text{Wed}}{\text{Mass}}$

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28. $F = F_x$

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$F = \frac{F_{max} - 700}{1500}$

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29. $F = F_x$

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