Stochastic Models and Optimization: Problem Set 4

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March 16, 2017

Q1

Q2

Q3

Asset selling w/offer estimation

Primitives

- $w_0, w_1, ...w_{n-1}$ of iid offers with unknown distribution
- an underlying distribution (i.e. the hidden state): F_1 or F_2, where $F_1(w) = \mathbb{P}(W = w|F_1$ is the true distribution), thus $x_k = x^1$ if true distribution is F_1 and x^2 if the true distribution is F_2
- constraints (if seller sells or not): $\left\{ \begin{array}{l} u^1, u^2 \text{ if } x_k \neq \text{T} \\ 0, \text{otherwise} \end{array} \right\}$
- rewards: $g_n() = \left\{ \begin{array}{l} x_n, \text{ if } x_n \neq \text{T} \\ 0, \text{otherwise} \end{array} \right\}$
  $g_k(x_k, u_k, w_k) = \left\{ \begin{array}{l} (1+r)^{n-k} x_k, \text{ if } x_k \neq \text{T and if } u_k = u^1 \\ 0, \text{otherwise} \end{array} \right\}$
- P = prior belief that F_1 is true
- $P_{k-1} = \frac{\mathbb{P}\{x_1 = x_1\} \cap \{w_1 = w_1\}}{\mathbb{P}(w_1 = w_1)} = \frac{q_k F_1(w_k)}{q_k F_1(w_k) + (1-q) F_2(w_k)}$

Now, we can apply the DP algorithm to find an optimal asset selling policy

$$J_{n-1}(P_{n-1}) = \left\{ \begin{array}{l} (P_{n-1}\mathbb{E}_{F_1}[w_{n-1}] + (1 - P_{n-1})\mathbb{E}_{F_2}[w_{n-1}])(1+r)^{n-k} \\ 0, \text{otherwise} \end{array} \right\} \text{if } x_{n-1} \neq T$$

$$J_k(x_k) = \left\{ \begin{array}{l} \max(P_k\mathbb{E}_{F_1}[w_k] + (1 - P_k)\mathbb{E}_{F_2}[w_k])(1+r)^k, \mathbb{E}[J_{k+1}(w_k)] \\ 0, \text{otherwise} \end{array} \right\} \text{if } x_k \neq \text{T}$$

Thus, the threshold for selling an asset will be: $P_k\mathbb{E}_{F_1}(w_k) + (1 - P_k)\mathbb{E}_{F_2}(w_k) \geqslant \frac{\mathbb{E}[J_{k+1}(w_k)]}{(1+r)^{n-k}}$

And the optimal asset selling policy: $\mu^*(x_k) = \left\{ \begin{array}{l} u^*, \frac{\mathbb{E}[J_{k+1}(w_k)]}{(1+r)^{n-k}} \\ u^2, \text{otherwise} \end{array} \right\}$