1 Q2. Inventory Pooling

Primitives
D = demands
Q = quantity ordered
P = price
h = inventory costs = c-s
b = backholding costs = p-c

First we will show that $Q_p^* = \sqrt{n}Q^* + \mu(n-\sqrt{n})$

$$\begin{aligned} P(\sum_{L=1}^{n} D_i \leqslant Q_p^*) =& \frac{b}{b+R} \\ &= P(\sqrt{n}D_i + \mu(n-\sqrt{n}) \leqslant Q_p^*) = \frac{b}{b+R} \\ &= P(D_i \leqslant \frac{1}{\sqrt{n}}(Q_p^* - \mu(n-\sqrt{n}))) = \frac{b}{b+h} \end{aligned}$$

which implies that this is equal to Q^*. Thus, we can use the hint to find the following:

$$\begin{aligned} G(Q) &= CQ - PD + h(Q-D)^+ + b(D-Q)^+ \\ G(Q) &= CQ - PE[D] + hE[Q-D]^+ bE[D-Q]^+ \\ G'(Q^*) &= C + hP(D \leqslant Q^*) - b(1 - P[D \leqslant Q^*]) = 0 \\ P(D \leqslant Q^*) &= \frac{-c+b}{h+b} \end{aligned}$$

By using the hint, we can prove the desired result:

$$\begin{aligned} nG(Q^*) &= n[CQ^* - PE[D] + hE[(Q^*-D)^+] + bE[(D-Q^*)^+]] \\ G(Q_p^*) &= [CQ_p^* - PE[\sum_{i=1}^{n} D_i] + hE[(Q_p^* - \sum_{i=1}^{n} D_i)^+] + bE[(\sum_{i=1}^{n} D_i - Q_p^*)^+]] \end{aligned}$$

Since $\sum_{i=1}^{n} D_i = \sqrt{n}D_i + \mu(n-\sqrt{n})$ and $Q_P^* = \sqrt{n}Q^* + \mu(n-\sqrt{n})$, we can show that:

$$\begin{aligned} G(Q_P^*) =& [C\sqrt{n}Q^* + \mu(n-\sqrt{n}) - PE[\sqrt{n}D_i + \mu(n-\sqrt{n})^+] \\ &+ hE[(\sqrt{n}Q^* + \mu(n-\sqrt{n}) - \sqrt{n}D_i - \mu(n-\sqrt{n}))^+] \\ &+ bE[\sqrt{n}D_i + \mu(n-\sqrt{n}) - \sqrt{n}Q^* + \mu(n-\sqrt{n})^+]] \end{aligned}$$

$$\begin{aligned} G(Q_P^*) =& [C\sqrt{n}Q^* + \mu(n-\sqrt{n}) - PE[\sqrt{n}D_i + \mu(n-\sqrt{n})^+] \\ &+ hE[(\sqrt{n}Q^* - \sqrt{n}D_i] \\ &+ bE[\sqrt{n}D_i - \sqrt{n}Q^*]] \end{aligned}$$

Problem 4 (An Investment Problem)