

Stochastic Models and Optimization: Problem Set 4

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Q1

Q2

Q3

Asset selling w/offer estimation

Primitives

- w_0, w_1, \dots, w_{n-1} of iid offers with unknown distribution
- an underlying distribution (i.e. the hidden state): F_1 or F_2 , where $F_1(w) = \mathbb{P}(W = w|F_1)$ is the true distribution), thus $x_k = x^1$ if true distribution is F_1 and x^2 if the true distribution is F_2
- constraints (if seller sells or not): $\begin{cases} u^1, u^2 & \text{if } x_k \neq T \\ 0, & \text{otherwise} \end{cases}$
- rewards: $g_n() = \begin{cases} x_n, & \text{if } x_n \neq T \\ 0, & \text{otherwise} \end{cases}$
- $g_k(x_k, u_k, w_k) = \begin{cases} (1+r)^{n-k}x_k, & \text{if } x_k \neq T \text{ and if } u_k = u^1 \\ 0, & \text{otherwise} \end{cases}$
- P = prior belief that F_1 is true
- $P_{k-1} = \frac{\mathbb{P}\{x_1=x_1\} \cap \{w_1=w_1\}}{\mathbb{P}(w_1=w_1)} = \frac{q_k F_1(w_k)}{q_k F_1(w_k) + (1-q) F_2(w_k)}$

Now, we can apply the DP algorithm to find an optimal asset selling policy

$$J_{n-1}(P_{n-1}) = \begin{cases} (P_{n-1} \mathbb{E}_{F_1}[w_{n-1}] + (1 - P_{n-1}) \mathbb{E}_{F_2}[w_{n-1}]) (1+r)^{n-k} \\ 0, & \text{otherwise} \end{cases} \text{ if } x_{n-1} \neq T$$

$$J_k(x_k) = \begin{cases} \max(P_k \mathbb{E}_{F_1}[w_k] + (1 - P_k) \mathbb{E}_{F_2}[w_k]) (1+r)^k, \mathbb{E}[J_{k+1}(w_k)] \\ 0, & \text{otherwise} \end{cases} \text{ if } x_k \neq T$$

Thus, the threshold for selling an asset will be: $P_k \mathbb{E}_{F_1}(w_k) + (1 - P_k) \mathbb{E}_{F_2}(w_k) \geq \frac{\mathbb{E}[J_{k+1}(w_k)]}{(1+r)^{n-k}}$

And the optimal asset selling policy: $\mu^*(x_k) = \begin{cases} u^*, & \frac{\mathbb{E}[J_{k+1}(w_k)]}{(1+r)^{n-k}} \\ u^2, & \text{otherwise} \end{cases}$

Q4

Q5