

1 Q2. Inventory Pooling

Primitives

D = demands

Q = quantity ordered

P = price

h = inventory costs = c-s

b = backholding costs = p-c

First we will show that $Q_p^* = \sqrt{n}Q^* + \mu(n - \sqrt{n})$

$$\begin{aligned} P\left(\sum_{i=1}^n D_i \leq Q_p^*\right) &= \frac{b}{b+R} \\ &= P(\sqrt{n}D_i + \mu(n - \sqrt{n}) \leq Q_p^*) = \frac{b}{b+R} \\ &= P(D_i \leq \frac{1}{\sqrt{n}}(Q_p^* - \mu(n - \sqrt{n}))) = \frac{b}{b+h} \end{aligned}$$

which implies that this is equal to Q^* . Thus, we can use the hint to find the following:

$$\begin{aligned} G(Q) &= CQ - PD + h(Q - D)^+ + b(D - Q)^+ \\ G(Q) &= CQ - PE[D] + hE[(Q - D)^+] + bE[(D - Q)^+] \\ G'(Q^*) &= C + hP(D \leq Q^*) - b(1 - P[D \leq Q^*]) = 0 \\ P(D \leq Q^*) &= \frac{-c + b}{h + b} \end{aligned}$$

Next, we will apply the hint to prove the desired result:

$$\begin{aligned} nG(Q^*) &= n[CQ^* - PE[D] + hE[(Q^* - D)^+] + bE[(D - Q^*)^+]] \\ G(Q_p^*) &= [CQ_p^* - PE[\sum_{i=1}^n D_i] + hE[(Q_p^* - \sum_{i=1}^n D_i)^+] + bE[(\sum_{i=1}^n D_i - Q_p^*)^+]] \end{aligned}$$

Since $\sum_{i=1}^n D_i = \sqrt{n}D_i + \mu(n - \sqrt{n})$ and $Q_p^* = \sqrt{n}Q^* + \mu(n - \sqrt{n})$, we can show that:

$$\begin{aligned} G(Q_p^*) &= [C\sqrt{n}Q^* + \mu(n - \sqrt{n}) - PE[\sqrt{n}D_i + \mu(n - \sqrt{n})^+] \\ &\quad + hE[(\sqrt{n}Q^* + \mu(n - \sqrt{n}) - \sqrt{n}D_i - \mu(n - \sqrt{n}))^+] \\ &\quad + bE[\sqrt{n}D_i + \mu(n - \sqrt{n}) - \sqrt{n}Q^* + \mu(n - \sqrt{n})^+]] \\ G(Q_p^*) &= [C\sqrt{n}Q^* + \mu(n - \sqrt{n}) - PE[\sqrt{n}D_i + \mu(n - \sqrt{n})^+] \\ &\quad + hE[(\sqrt{n}Q^* - \sqrt{n}D_i) \\ &\quad + bE[\sqrt{n}D_i - \sqrt{n}Q^*]] \end{aligned}$$

Problem 4 (An Investment Problem)