Stochastic Models and Optimization: Problem Set 4

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Q1

Q2

Q3

Asset selling w/offer estimation

Primitives

- $w_0, w_1, \ldots w_{n-1}$ of iid offers with unknown distribution

- an underlying distribution (i.e. the hidden state): F_1 or F_2, where $F_1(w) = \mathbb{P}(W = w | F_1$ is the true distribution), thus $x_k = x^1$ if true distribution is F_1 and x^2 if the true distribution is F_2

- constraints (if seller sells or not): $\left\{ \begin{array}{l} u^1, u^2 \;\; \text{if} \;\; x_k \neq \mathrm{T} \\ 0, \text{otherwise} \end{array} \right\}$

- rewards: $g_n() = \left\{ \begin{array}{l} x_n, \text{if} \;\; x_n \neq \mathrm{T} \\ 0, \text{otherwise} \end{array} \right\}$

  $g_k(x_k, u_k, w_k) = \left\{ \begin{array}{l} (1+r)^{n-k} x_k, \text{if} \;\; x_k \neq \mathrm{T} \text{ and if } u_k = u^1 \\ 0, \text{otherwise} \end{array} \right\}$

- P = prior belief that F_1 is true

- $P_{k-1} = \frac{\mathbb{P}\{x_1 = x_1\} \cap \{w_1 = w_1\}}{\mathbb{P}(w_1 = w_1)} = \frac{q_k F_1(w_k)}{q_k F_1(w_k) + (1-q) F_2(w_k)}$

Now, we can apply the DP algorithm to find an optimal asset selling policy

$J_{n-1}(P_{n-1}) = \left\{ \begin{array}{l} (P_{n-1}\mathbb{E}_{F_1}[w_{n-1}] + (1 - P_{n-1})\mathbb{E}_{F_2}[w_{n-1}])(1+r)^{n-k} \\ 0, \text{otherwise} \end{array} \right\}$ if $x_{n-1} \neq T$

$J_k(x_k) = \left\{ \begin{array}{l} \max(P_k\mathbb{E}_{F_1}[w_k] + (1 - P_k)\mathbb{E}_{F_2}[w_k])(1+r)^k, \mathbb{E}[J_{k+1}(w_k)] \\ 0, \text{otherwise} \end{array} \right\}$ if $x_k \neq \mathrm{T}$

Thus, the threshold for selling an asset will be: $P_k\mathbb{E}_{F_1}(w_k) + (1 - P_k)\mathbb{E}_{F_2}(w_k) \geqslant \frac{\mathbb{E}[J_{k+1}(w_k)]}{(1+r)^{n-k}}$

And the optimal asset selling policy: $\mu^*(x_k) = \left\{ \begin{array}{l} u^*, \frac{\mathbb{E}[J_{k+1}(w_k)]}{(1+r)^{n-k}} \\ u^2, \text{otherwise} \end{array} \right\}$

1 Q4

This problem is basically the same as the inventory management considering the demand as a random variable following an unknown distribution. It is a case with imperfect state information, in which the distribution of demand will be either F_1 or F_2. The probability that the demand follows F_1 is updated at each period k after observing the realization of the demand. That will effect the way the expectation of the demand is computed.

Primitives:
x_k: items in the inventory at period k.
u_k: quantity ordered at period k.
w_k: demand during period k. w_k are iid with probability distribution either F_1 or F_2.
q_k: probability that w_k follows distribution F_1.
$q_0 = q$: a priori probability that demand follows the distribution F_1.

Dynamics:
$x_{k+1} = x_k + u_k - w_k$
$q_{k+1} = \frac{q_k f_1(w_k)}{q_k f_1(w_k) + (1-q_k) f_2(w_k)}$, where $f_i(w)$ is the pdf of the distribution F_i.

Cost:
$g_N(x_N) = 0$.
$g_k(x_k, u_k, w_k) = cu_k + h \max\{0, w_k - x_k - u_k\} + p \max\{0, x_k + u_k - w_k\}$, where c, h, p are positive and $p > c$.

DP algorithm:

$J_N(x_N) = 0$
$J_k(x_k) = \min_{u_k \geqslant 0} \mathbb{E} \, []$