数字图像与视频处理 王桥

复习提要 Version3

本学期课程主要围绕一些图像处理的原理、Python/C++试验为主进行. 所讲内容并不多.考试侧重基础知识、基础算法与基础程序设计.

细胞功能(补充)

视杆细胞	视锥细胞
亮度(黑白)	色彩(颜色)

图像的一般性知识、JPEG/PNG/GIF格式的差异、数据压缩的性能、使用的场景

JPEG	GIF	PNG
有损压缩	无损压缩	无损压缩

以下全部百度,仅供参考

- GIF全称Graphic Interchange Format ,图像交换格式,最多可使用256种颜色,适合导航条,按钮,图标等具有统一色调的图像,可以制作动态图像.当网速慢的时候可以将图像从模糊到清晰慢慢显示.
- JPEG全称Joint Photographic Experts Group ,图像压缩模式,可以包含数百万种颜色,文件大小和加载时间很大,不支持透明图和动态图,但能够保留全真的色调版格式,如果图像需要全彩模式才能表现效果,最好使用JPEG.
- PNG格式 全称Portable Network Graphics 提供了将图像文件以最小的方式压缩又不造成图像失真的技术.支持48位的色彩,更快地交错显示,跨平台的图像亮度控制,更多层的透明度设置.

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1. Gif格式特点

- 透明性: Gif是一种布尔透明类型,既它可以是全透明,也可以是全不透明,但是它并没有半透明(alpha透明).
- 动画: Gif这种格式支持动画.
- 无损耗性: Gif是一种无损耗的图像格式,这也意味着你可以对gif图片做任何操作也不会使 得图像质量产生损耗.
- 水平扫描: Gif是使用了一种叫作LZW的算法进行压缩的,当压缩gif的过程中,像素是由上到下水平压缩的,这也意味着同等条件下,横向的gif图片比竖向的gif图片更加小.例如500-10的图片比10-500的图片更加小
- 间隔渐进显示: Gif支持可选择性的间隔渐进显示

由以上特点看出只有256种颜色的gif图片不适合照片,但它适合对颜色要求不高的图形(比如说图标,图表等),它并不是最优的选择,我们会在后面中看到png是最优的选择.

2. Jpeg格式特点

- 透明性: 它并不支持透明.

- 动画: 它也不支持动画.
- 损耗性: 除了一些比如说旋转(仅仅是90、180、270度旋转),裁切,从标准类型到先进类型,编辑图片的原数据之外,所有其它操作对jpeg图像的处理都会使得它的质量损失.所以我们在编辑过程一般用png作为过渡格式.
- 隔行渐进显示: 它支持隔行渐进显示(但是ie浏览器并不支持这个属性,但是ie会在整个图像信息完全到达的时候显示).

由上可以看出Jpeg是最适web上面的摄影图片和数字照相机中.

3. Png格式特点

- 类型: Png这种图片格式包括了许多子类,但是在实践中大致可以分为256色的png和全色的png,你完成可以用256色的png代替gif,用全色的png代替jpeg
- 透明性: Png是完全支持alpha透明的(透明,半透明,不透明),尽管有两个怪异的现象在ie6(下面详细讨论)
- 动画: 它不支持动画
- 无损耗性: png是一种无损耗的图像格式,这也意味着你可以对png图片做任何操作也不会使 得图像质量产生损耗.这也使得png可以作为jpeg编辑的过渡格式
- 水平扫描: 像GIF一样,png也是水平扫描的,这样意味着水平重复颜色比垂直重复颜色的图片更小
- 间隔渐进显示: 它支持间隔渐进显示,但是会造成图片大小变得更大

4. 其它图片格式与PNG比较

众所周知GIF适合图形,JPEG适合照片,PNG系列两种都适合.

- 相比GIF
  - PNG 8除了不支持动画外,PNG8有GIF所有的特点,但是比GIF更加具有优势的是它支持alpha透明和更优的压缩.所以,大多数情况下,你都应该用PNG8不是GIF(除了非常小的图片GIF会有更好的压缩外).
- 相比JPEG
  - JPEG比全色PNG具有更加好的压缩,因此也使得JPEG适合照片,但是编辑JPEG过程中容易造成质量的损失,所以全色PNG适合作为编辑JPEG的过渡格式.

图像格式的基本常识、像素级检索的编程基础(矩阵)

RGB:(row,col,3) Gray:(row,col)

彩图灰度图转化: $Gray = R \times 0.299 + G \times 0.587 + B \times 0.114$

```
def rgb2gray(rgb):
    return np.dot(rgb[...,:3], [0.299, 0.587, 0.114])
```

像素级操作的时候,索引[y,x] ([行值,列值])

图像质量评价(补充)

精确的原始图像$f(x,y)$,从它的相关数据D(D可以是f的模糊版本),得到估计图像$\tilde{f}(m,n)$

均方误差

$$\mathrm{MSE} = \frac{1}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N}|\tilde{f}(m,n) - f(m,n)|^2$$

峰值信噪比

$$\mathrm{PSNR}=10\lg\frac{255^2}{\mathrm{MSE}}\quad(\mathrm{dB})$$

图像边缘检测的算法基础、一阶微分算子型与二阶微分算子型边缘检测算法的优劣对比、数据驱动的边缘检测算法,对应的程序设计

一阶算子(差分值较大的区域为边缘)

对$|R_x f|+|R_y f|$取阈值判决,人工调阈值参数

- Roberts 交叉梯度算子

$$\begin{bmatrix}1&0\\0&-1\end{bmatrix}\quad\begin{bmatrix}0&-1\\1&0\end{bmatrix}$$

- Prewitt 算子

$$\begin{bmatrix}-1&-1&-1\\0&0&0\\1&1&1\end{bmatrix}\quad\begin{bmatrix}-1&0&1\\-1&0&1\\-1&0&1\end{bmatrix}$$

- Sobel算子

$$\begin{bmatrix}-1&-2&-1\\0&0&0\\1&2&1\end{bmatrix}\quad\begin{bmatrix}-1&0&1\\-2&0&2\\-1&0&1\end{bmatrix}$$

```
grad_x = cv2.filter2D(gray,  -1 , ker_x )
grad_y = cv2.filter2D(gray,  -1 , ker_y )
grad = abs(grad_x)  + abs(grad_y)
#  调阈值
grad = (grad > 100) * 255
plt.imshow(grad,cmap='gray')
```

二阶算子(二阶差分的跨零点为边缘)

原则上不需要取阈值,跨零点即边界. 但是我自己编程用二阶差分较大的地方(二阶差分极大极小)近似代替跨零点,所以代码里面也有阈值需要调整.

- Laplace算子Δ $\Delta f=\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}$

$$\frac{\partial^2 f}{\partial x^2}\to f(i+1,j)-2f(i,j)+f(i-1,j)$$
$$\frac{\partial^2 f}{\partial y^2}\to f(i,j+1)-2f(i,j)+f(i,j-1)$$

所以得到第一个模板,第二、三为拓展模板

$$\begin{bmatrix}0&1&0\\1&-4&1\\0&1&0\end{bmatrix}\quad\begin{bmatrix}1&1&1\\1&-8&1\\1&1&1\end{bmatrix}\quad\begin{bmatrix}1&4&1\\4&-20&4\\1&4&1\end{bmatrix}$$

- LOG算子(带噪声的图像边沿检测)

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 1 & 2 & -16 & 2 & 1 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$LOG(f)(x,y) = \Delta(G_\sigma * f) = (\Delta G_\sigma) * f(\text{微分、卷积算子次序可换})$$

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2} exp\left[-\frac{x^2+y^2}{2\sigma^2}\right]$$

$$\Delta G_\sigma = \frac{x^2+y^2-2\sigma^2}{2\pi\sigma^6} exp\left[-\frac{x^2+y^2}{2\sigma^2}\right]$$

证明:

$$\begin{aligned} G_\sigma(x,y) &= \frac{1}{2\pi\sigma^2} exp\left[-\frac{x^2+y^2}{2\sigma^2}\right] \\ \frac{\partial G_\sigma}{\partial x} &= \frac{1}{2\pi\sigma^2} exp\left[-\frac{x^2+y^2}{2\sigma^2}\right]\left(-\frac{x}{\sigma^2}\right) \\ \frac{\partial^2 G_\sigma}{\partial x^2} &= \frac{1}{2\pi\sigma^2} exp\left[-\frac{x^2+y^2}{2\sigma^2}\right]\left(\frac{x^2}{\sigma^4}-\frac{1}{\sigma^2}\right) \\ \frac{\partial^2 G_\sigma}{\partial y^2} &= \frac{1}{2\pi\sigma^2} exp\left[-\frac{x^2+y^2}{2\sigma^2}\right]\left(\frac{y^2}{\sigma^4}-\frac{1}{\sigma^2}\right) \\ \Delta G_\sigma &= \frac{1}{2\pi\sigma^2} exp\left[-\frac{x^2+y^2}{2\sigma^2}\right]\left(\frac{x^2+y^2}{\sigma^4}-\frac{2}{\sigma^2}\right) \\ &= \frac{x^2+y^2-2\sigma^2}{2\pi\sigma^6} exp\left[-\frac{x^2+y^2}{2\sigma^2}\right] \end{aligned}$$

注意,$\Delta G_\sigma(x,y)$ 是一个小波,即:$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\Delta G_\sigma(x,y)dxdy = 0$

证明:

$$\begin{aligned} & \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\Delta G_\sigma(x,y)dxdy \\ = & \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{x^2+y^2-2\sigma^2}{2\pi\sigma^6} exp\left[-\frac{x^2+y^2}{2\sigma^2}\right]dxdy \\ = & \int_0^{2\pi}\int_0^{+\infty}\frac{r^2-2\sigma^2}{2\pi\sigma^6} exp\left[-\frac{r^2}{2\sigma^2}\right]rdrd\theta \\ = & \frac{1}{2\pi\sigma^6}\int_0^{2\pi}d\theta\int_0^{+\infty}\left(r^3-2\sigma^2 r\right)exp\left[-\frac{r^2}{2\sigma^2}\right]dr \\ = & \frac{1}{\sigma^6}\int_0^{+\infty}\left(r^3-2\sigma^2 r\right)exp\left[-\frac{r^2}{2\sigma^2}\right]dr \\ = & \frac{-\sigma^2}{\sigma^6}\left[(r^2-2\sigma^2)exp\left[-\frac{r^2}{2\sigma^2}\right]\Bigg|_0^{+\infty} - \int_0^{+\infty} exp\left[-\frac{r^2}{2\sigma^2}\right]dr^2\right] \\ = & \frac{-1}{\sigma^4}\left[2\sigma^2+2\sigma^2 exp\left[-\frac{r^2}{2\sigma^2}\right]\Bigg|_0^{+\infty}\right] \\ = & \frac{-1}{\sigma^4}\left[2\sigma^2-2\sigma^2\right] = 0 \end{aligned}$$

```
laplace = np.array([[1, 1, 1],
                    [1,-8, 1],
                    [1, 1, 1]])
grad = cv2.filter2D(gray,  -1 , laplace )
# 取二阶导数较大的点替代二阶导数接近 0 的位置
grad = (np.abs(grad) > 90) * 255
plt.imshow(grad,cmap='gray')
```

数据驱动算法

- Canny

与二阶差分方法的区别(这里 =0代表跨零点)

Canny	二阶算子Laplace	二阶算子LOG
$\frac{\partial^2}{\partial n^2}(G_\sigma * f) = 0$	$\Delta f = \frac{\partial^2}{\partial n^2} f + \frac{\partial^2}{\partial s^2} f = 0$	$\Delta(G_\sigma * f) = \frac{\partial^2}{\partial n^2}(G_\sigma * f) + \frac{\partial^2}{\partial s^2}(G_\sigma * f) = 0$

蕴含一个旋转不变性(各向同性)
当退化为离散形式时,该算子最多只能保证45度旋转的无关性

(**Def 旋转不变性**): Laplace算子是各向同性的,即假定(x', y')是(x, y)的旋转,则:

$$\Delta = \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

证明:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$x = x' \cos\theta - y' \sin\theta$$
$$y = x' \sin\theta + y' \cos\theta$$

$$\begin{aligned}
\nabla^2 f &= \frac{\partial^2 f}{\partial x'^2} + \frac{\partial^2 f}{\partial y'^2} \\
&= \frac{\partial}{\partial x'}\left(\frac{\partial f}{\partial x'}\right) + \frac{\partial}{\partial y'}\left(\frac{\partial f}{\partial x'}\right) \\
&= \frac{\partial}{\partial x'}\left(\frac{\partial f}{\partial x}\frac{\partial x}{\partial x'} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial x'}\right) + \frac{\partial}{\partial y'}\left(\frac{\partial f}{\partial x}\frac{\partial x}{\partial y'} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial y'}\right) \\
&= \frac{\partial}{\partial x'}\left(\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta\right) + \frac{\partial}{\partial y'}\left(-\sin\theta\frac{\partial f}{\partial x} + \cos\frac{\partial f}{\partial y}\right) \\
&= \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta\right)\frac{\partial x}{\partial x'} + \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta\right)\frac{\partial y}{\partial x'} \\
&\quad + \frac{\partial}{\partial x}\left(-\sin\theta\frac{\partial f}{\partial x} + \cos\frac{\partial f}{\partial y}\right)\frac{\partial x}{\partial y'} + \frac{\partial}{\partial y}\left(-\sin\theta\frac{\partial f}{\partial x} + \cos\frac{\partial f}{\partial y}\right)\frac{\partial y}{\partial y'} \\
&= \frac{\partial}{\partial x}(\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta)\cos\theta + \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta\right)\sin\theta \\
&\quad + \frac{\partial}{\partial x}\left(-\sin\theta\frac{\partial f}{\partial x} + \cos\frac{\partial f}{\partial y}\right)(-\sin\theta) + \frac{\partial}{\partial y}\left(-\sin\theta\frac{\partial f}{\partial x} + \cos\theta\frac{\partial f}{\partial y}\right)\cos\theta \\
&= \frac{\partial}{\partial x}\frac{\partial f}{\partial x} + \frac{\partial}{\partial y}\frac{\partial f}{\partial y} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \nabla^2 f
\end{aligned}$$

工程上,对于图像, ***n的方向只取4个:*** $0, \pi/4, \pi/2, 3\pi/4$

$\frac{\partial^2}{\partial n^2}(G_\sigma * f)=0 \Leftrightarrow \frac{\partial}{\partial n}(G_\sigma * f)_{\text{取极值}} \Rightarrow \left|\frac{\partial}{\partial n}(G_\sigma * f)\right|_{\text{取极大值}}$

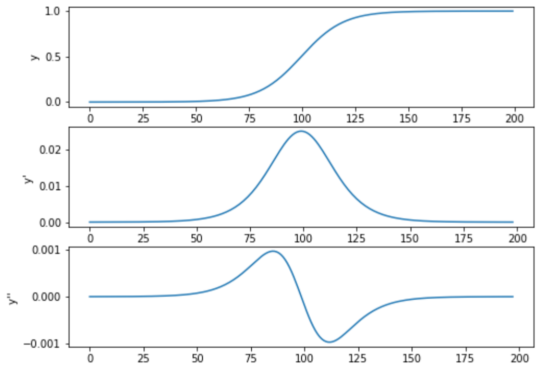
编程就先按梯度最大选出法向n，再给$\left|\frac{\partial^2}{\partial n^2}(G_\sigma * f)\right|$设阈值求边缘.

```
# 自编函数版
def my_canny(src):
    # 高斯低通滤波器
    gass = cv2.GaussianBlur(src, (3,3), 0)
    # canny 核 四个角度 一阶
    ker = np.zeros((3,3,4))
    ker[...,0] = np.array([[0,0,0],
                          [0,-1,1],
                          [0,0,0]])
    ker[...,1]  = np.array([[0,0,1],
                           [0,-1,0],
                           [0,0,0]])
    ker[...,2]  = np.array([[0,1,0],
                           [0,-1,0],
                           [0,0,0]])
    ker[...,3]  = np.array([[1,0,0],
                           [0,-1,0],
                           [0,0,0]])
    # 核 四个角度 二阶
    ker2 = np.zeros((3,3,4))
    ker2[...,0] = np.array([[0, 0,0],
                           [1,-2,1],
                           [0, 0,0]])
    ker2[...,1]  = np.array([[0, 0,1],
                            [0,-2,0],
                            [1, 0,0]])
    ker2[...,2]  = np.array([[0, 1,0],
                            [0,-2,0],
                            [0, 1,0]])
    ker2[...,3]  = np.array([[1, 0,0],
                            [0,-2,0],
                            [0, 0,1]])

    # 四个角度的差分
    d = np.zeros(src.shape +(4,))
    dd = np.zeros(src.shape +(4,))
    for i in range(4):
        d[...,i] = cv2.filter2D(src, -1 , ker[...,i])
        dd[...,i] = cv2.filter2D(src, -1 , ker2[...,i])

    n_f = np.ones_like(src)
    for row in range(src.shape[0]):
        for col in range(src.shape[1]):
            # 法向
            n = np.where( np.abs(d[row,col,:]) ==\
                         np.max(np.abs(d[row,col,:]) ))[0][0]

            n_f[row,col] = dd[row,col,n]
    return n_f

canny_out = my_canny(gray)
# 取二阶导数较大的点替代二阶导数接近 0 的位置
canny_out = (canny_out > 25) * 255
```

```
plt.imshow(canny_out,cmap='gray')
```

图像增强的Laplace算子方法,程序设计基础

- 一维边缘增强 $res = y' - \lambda y''$



- 二维边缘增强 $u_t(x,y) = u(x,y) - \lambda\Delta u(x,y)$

  Δ正中间值(这里是-4或-8)的符号与修正的正负号对应,符号反的话就是反向模糊

$$\Delta = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{或} \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

```
Laplace = np.array([[1, 1, 1],
                    [1,-8, 1],
                    [1, 1, 1]])
deta_f = cv2.filter2D(f,-1,Laplace)
labd = .5
re_build = f - (labd * deta_f).astype(int)
plt.imshow(re_build,cmap='gray')
```

Gabor图像复原第一算法、第二算法的原理与实现(图像各向异性增强)

Gabor适用的场景: 含噪声的图像增强

- Laplace增强(法向、切向都增强,用于不含噪的图像增强)

$$f_t(x,y) = f(x,y) - t(\frac{\partial^2 f}{\partial n^2} + \frac{\partial^2 f}{\partial s^2})$$

- 第一算法(法向增强)

$$f_t(x,y) = f(x,y) - t\frac{\partial^2 f}{\partial n^2}$$

编程取 $\left|\frac{\partial}{\partial n} f\right|$最大值的方向为法向$n$,切向$s$为$n + \pi/2$.

```
def gabor1(src,t):
    # 核 四个角度 一阶
    ker = np.zeros((3,3,4))
```

```
    ker[...,0] = np.array([[0,0,0],
                           [0,-1,1],
                           [0,0,0]])
    ker[...,1]  = np.array([[0,0,1],
                            [0,-1,0],
                            [0,0,0]])
    ker[...,2]  = np.array([[0,1,0],
                            [0,-1,0],
                            [0,0,0]])
    ker[...,3]  = np.array([[1,0,0],
                            [0,-1,0],
                            [0,0,0]])
    # 核 四个角度 二阶
    ker2 = np.zeros((3,3,4))
    ker2[...,0] = np.array([[0, 0,0],
                            [1,-2,1],
                            [0, 0,0]])
    ker2[...,1]  = np.array([[0, 0,1],
                             [0,-2,0],
                             [1, 0,0]])
    ker2[...,2]  = np.array([[0, 1,0],
                             [0,-2,0],
                             [0, 1,0]])
    ker2[...,3]  = np.array([[1, 0,0],
                             [0,-2,0],
                             [0, 0,1]])

    # 四个角度的差分
    d = np.zeros(src.shape +(4,))
    dd = np.zeros(src.shape +(4,))
    for i in range(4):
        d[...,i] = cv2.filter2D(src, -1 , ker[...,i])
        dd[...,i] = cv2.filter2D(src, -1 , ker2[...,i])

    n_f = np.ones_like(src)
    for row in range(src.shape[0]):
        for col in range(src.shape[1]):
            # 法向
            n = np.where( np.abs(d[row,col,:]) ==\
                         np.max(np.abs(d[row,col,:]) ))[0][0]

            n_f[row,col] = dd[row,col,n]
    return src - t * n_f

gabor = gabor1(gray,0.5)
```

- 第二算法(法向增强[-],切向平滑[+])

$$f_t(x,y) = f(x,y) - t(\frac{\partial^2 f}{\partial n^2} - \frac{1}{3}\frac{\partial^2 f}{\partial s^2}) = f(x,y) - t\frac{\partial^2 f}{\partial n^2} + \frac{t}{3}\frac{\partial^2 f}{\partial s^2}$$

```
def gabor2(src,t):
    # 核 四个角度 一阶
    ker = np.zeros((3,3,4))
    ker[...,0] = np.array([[0,0,0],
                           [0,-1,1],
                           [0,0,0]])
```

```
    ker[...,1]  = np.array([[0,0,1],
                            [0,-1,0],
                            [0,0,0]])
    ker[...,2]  = np.array([[0,1,0],
                            [0,-1,0],
                            [0,0,0]])
    ker[...,3]  = np.array([[1,0,0],
                            [0,-1,0],
                            [0,0,0]])
    # 核 四个角度 二阶
    ker2 = np.zeros((3,3,4))
    ker2[...,0] = np.array([[0, 0,0],
                            [1,-2,1],
                            [0, 0,0]])
    ker2[...,1]  = np.array([[0, 0,1],
                             [0,-2,0],
                             [1, 0,0]])
    ker2[...,2]  = np.array([[0, 1,0],
                             [0,-2,0],
                             [0, 1,0]])
    ker2[...,3]  = np.array([[1, 0,0],
                             [0,-2,0],
                             [0, 0,1]])

    # 四个角度的差分
    d = np.zeros(src.shape +(4,))
    dd = np.zeros(src.shape +(4,))
    for i in range(4):
        d[...,i] = cv2.filter2D(src, -1 , ker[...,i])
        dd[...,i] = cv2.filter2D(src, -1 , ker2[...,i])

    n_f = np.ones_like(src)
    s_f = np.ones_like(src)
    for row in range(src.shape[0]):
        for col in range(src.shape[1]):
            # 法向
            n = np.where( np.abs(d[row,col,:]) ==\
                             np.max(np.abs(d[row,col,:]) ))[0][0]
            # 切向
            s = (n+2)%4
            n_f[row,col] = dd[row,col,n]
            s_f[row,col] = dd[row,col,s]
    return src - t * (n_f - s_f /3 )

gabor = gabor2(gray,0.5)
```

Malik-Perona算法的原理与实现

PM针对图像的边缘部分和非边缘部分进行不同性质的处理 图像边缘内部区域,c远离0,图像可以平滑;边缘附近,c接近0,图像不再平滑

$$\text{径向函数}: g(s) = exp\left[-(\frac{s}{K})^2\right] \text{或} \, g(s) = \frac{1}{1+(\frac{s}{K})^{1+\alpha}}, \alpha > 0 (g:\text{单调下降})$$

$$\text{扩散函数}: c(x,y,t) = g(|\bigtriangledown u|)$$

$$u(x,y,t) = u(x,y,0) + t\frac{\partial u}{\partial t}(\text{泰勒展开})$$

$$\frac{\partial u}{\partial t} = \Delta u(\text{热传导方程})$$

$$\frac{\partial u}{\partial t} = \bigtriangledown \cdot (c(x,y,t) \bigtriangledown u) = \bigtriangledown c(x,y,t) \cdot \bigtriangledown u + c(x,y,t)\Delta u$$

- 简化代码(根据微分运算符$\bigtriangledown$ 和 Laplace运算符Δ进行拆解)

```
def g(s,k):
    return np.exp(-(s/k)**2)
def get_c(grad_u,k):
    return g(abs(grad_u),k)

def PM1(src ,k = 12,lambd = .01 ):

    Ker_x = np.array([[-1, 0, 1],
                      [-1, 0, 1],
                      [-1, 0, 1]])
    Ker_y = np.array([[1, 1, 1],
                      [0, 0, 0],
                      [-1,-1,-1]])
    # Laplace扩展算子
    laplace = np.array([[1, 1, 1],
                        [1,-8, 1],
                        [1, 1, 1]])
    # grad_u
    grad_x = cv2.filter2D(src,  -1 , Ker_x )
    grad_y = cv2.filter2D(src,  -1 , Ker_y )
    grad_u = np.abs(grad_x)  + np.abs(grad_y)
    # c
    c = get_c(grad_u,k = k)
    # grad_c
    grad_x_c = cv2.filter2D(src,  -1 , Ker_x )
    grad_y_c = cv2.filter2D(src,  -1 , Ker_y )
    grad_c = np.abs(grad_x_c)  + np.abs(grad_y_c)
    # Laplace_u
    Laplace_u = cv2.filter2D(src,  -1 , laplace )
    #
    im_out = (src + lambd*(grad_c * grad_u + c* Laplace_u ) )
    return im_out

# 迭代次数
step_num = 1
# copy
im_out1 = gray.astype(float)
for t in range(step_num):
    im_out1 = PM1( im_out1, k = 12,lambd = .002)
```

- 按书上逻辑代码(5个点(5个u)、涉及9个值的运算)

$$\begin{aligned} u\left(x,y,t_{n+1}\right) &\approx u\left(x,y,t_n\right) + \delta t \nabla \cdot \left(c\left(x,y,t_n\right) \nabla u\left(x,y,t_n\right)\right) \\ &= u\left(x,y,t_n\right) + \delta t\left[\nabla c\left(x,y,t_n\right) \cdot \nabla u\left(x,y,t_n\right) + c\left(x,y,t_n\right) \Delta u\left(x,y,t_n\right)\right] \\ &= u\left(x,y,t_n\right) + \delta t\left[I_1^n + I_2^n\right] \end{aligned}$$

$$I_1^n + I_2^n = \frac{1}{2}\left[c_{i+1,j}^n \nabla_{\mathrm{S}} u_{i,j}^n + c_{i,j+1}^n \nabla_{\mathrm{E}} u_{i,j}^n + c_{i-1,j}^n \nabla_{\mathrm{N}} u_{i,j}^n + c_{i,j-1}^n \nabla_{\mathrm{W}} u_{i,j}^n\right]$$

$$\begin{aligned} \nabla_{\mathrm{S}} u_{i,j}^n &= u_{i+1,j}^n - u_{i,j}^n \\ \nabla_{\mathrm{E}} u_{i,j}^n &= u_{i,j+1}^n - u_{i,j}^n \\ \nabla_{\mathrm{N}} u_{i,j}^n &= u_{i-1,j}^n - u_{i,j}^n \\ \nabla_{\mathrm{W}} u_{i,j}^n &= u_{i,j-1}^n - u_{i,j}^n \end{aligned}$$

$$u_{i,j}^{n+1} = u_{i,j}^n + \lambda\left[c_{i+1,j}^n \nabla_{\mathrm{S}} u_{i,j}^n + c_{i,j+1}^n \nabla_{\mathrm{E}} u_{i,j}^n + c_{i-1,j}^n \nabla_{\mathrm{N}} u_{i,j}^n + c_{i,j-1}^n \nabla_{\mathrm{W}} u_{i,j}^n\right] (\text{代码根据这句})$$

```
def g(s,k):
    return np.exp(-(s/k)**2)
def f(s,k):
    return 1.0 / (1.0 + (s / k) ** 2)

def Perona_Malik(src , times =30,dt=.01 ,kappa =12, option = 1 ):
    ny,nx = src.shape
    src = src.astype('float')
    # copy
    res = src
    tmp = src
    # 迭代次数
    for t in range(times):
        # 一次迭代
        for i in range(ny):
            for j in range(nx):
                # 位置信息 边界处理
                iUp   =  max(0,i - 1)
                iDown =  min(ny-1,i + 1)
                jLeft =  max(0,j - 1)
                jRight = min(nx-1,j + 1)

                # 书本page216  先计算  deta_u
                deltaN = tmp[iUp,j] - tmp[i,j]
                deltaS = tmp[iDown,j] - tmp[i,j]
                deltaE = tmp[i,jRight] - tmp[i,j]
                deltaW = tmp[i,jLeft] - tmp[i,j]
                delta_u = np.array([deltaN , deltaS , deltaE , deltaW ])

                # 计算  c
                if (option == 1):
                    c = g(np.abs(delta_u),kappa)
                elif (option == 2):
                    c = f(np.abs(delta_u),kappa)
                # 相乘相加 加权赋值
                res[i,j] += dt * (sum(c * delta_u) )
        tmp = res
    return res
```

卷积信号估计(补充)

从$g(x) = f(x) * h(x)$ 中得到 $f(x)$的估计$\tilde{f}$,即:$min_{\tilde{f}} \left\|\widetilde{f(x)} - f(x)\right\|$

$$\hat{g}=\widehat{f}\times\widehat{h}\Longrightarrow\hat{g}\times\widehat{h}^{*}=\widehat{f}(\widehat{h}\times\widehat{h}^{*})\overset{\text{引入}\lambda\text{调整分母}}{\Longrightarrow}\widehat{f}=\frac{\hat{g}\times\widehat{h}^{*}}{\widehat{h}\times\widehat{h}^{*}+\lambda}$$

$$f\text{的估计}\tilde{f}=\mathscr{F}^{-1}(\widehat{f})$$

图像矫正(补充)

dst:(x,y) , src:(u,v) [对于dst的点,去src找源点像素对应]

仿射变换

$$\begin{bmatrix}x\\y\\1\end{bmatrix}=M\begin{bmatrix}u\\v\\1\end{bmatrix}\qquad M^{-1}\begin{bmatrix}x\\y\\1\end{bmatrix}=\begin{bmatrix}u\\v\\1\end{bmatrix}$$

透视变换

$$(x,y)=\left(\frac{M_{11}u+M_{12}v+M_{13}}{M_{31}u+M_{32}v+M_{33}},\frac{M_{21}u+M_{22}v+M_{23}}{M_{31}u+M_{32}v+M_{33}}\right)$$

$$x=\frac{M_{11}u+M_{12}v+M_{13}}{M_{31}u+M_{32}v+M_{33}}=\frac{\alpha T}{\gamma T}\qquad y=\frac{M_{21}u+M_{22}v+M_{23}}{M_{31}u+M_{32}v+M_{33}}=\frac{\beta T}{\gamma T}$$

$$\text{求解}:\begin{bmatrix}\alpha-x\times\gamma\\\beta-y\times\gamma\end{bmatrix}T=0$$

其中 $M=\begin{bmatrix}\alpha\\\beta\\\gamma\end{bmatrix}\quad T=\begin{bmatrix}u\\v\\1\end{bmatrix}$

透视变换(改进)

$$\begin{bmatrix}x'\\y'\\w\end{bmatrix}=M\begin{bmatrix}u\\v\\1\end{bmatrix}\qquad\frac{1}{w}\begin{bmatrix}x'\\y'\\w\end{bmatrix}=\begin{bmatrix}x\\y\\1\end{bmatrix}=\frac{1}{w}M\begin{bmatrix}u\\v\\1\end{bmatrix}$$

$$M^{-1}\begin{bmatrix}x\\y\\1\end{bmatrix}=\begin{bmatrix}u'\\v'\\t'\end{bmatrix}=\frac{1}{w}\begin{bmatrix}u\\v\\1\end{bmatrix}\qquad\begin{bmatrix}u\\v\end{bmatrix}=\begin{bmatrix}u'/t'\\v'/t'\end{bmatrix}(t'=\frac{1}{w})$$

微局部分析(补充)

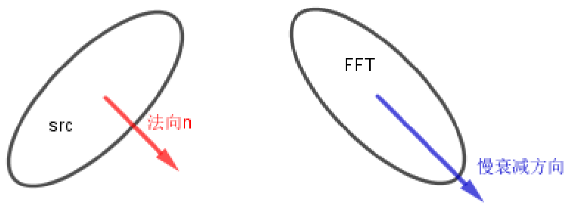
要获得图像$f(x,y)$在(a,b)附近的信息. 先在空间域做局部化处理,找一个窗函数$h(x,y)$,使其窗口中心落在$(0,0)$,用这个窗函数对图像做局部化

$$f(x,y)h(x-a,y-b)$$

对局部化的图像做Fourier变换,计算:

$$F_h(a,b;\omega,\eta)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}f(x,y)h(x-a,y-b)\mathrm{e}^{-\mathrm{j}(x\omega+y\eta)}\mathrm{d}x\mathrm{d}y$$

以幅度谱图的原点为中心,沿各个方向判断,哪个方向上Fourier变换的幅度衰减较慢: 慢衰减的方向实际就是原图中边缘曲线的法向. ==(与FFT变换的模值梯度与图像(二维函数值)的梯度方向相垂直==对应理解)



微局部分析算法: 对图像$f(x,y)$在任意指定的位置(a,b),寻找其变化剧烈的方向.

- 利用窗函数$h(x,y)$对图像做局部化.
- 对局部化图像$f(x,y)h(x-a,y-b)$做Fourier变换,算出$F(a,b;w,\eta)$.
- 在(w,η)平面上以$(0,0)$为中心,检查$|F(a,b;w,\eta)|$是否沿某个方向慢衰减。把所有的慢衰减方向$\theta_1,\theta_1,\cdots,\theta_K$找到,这些就是图像在点$(a,b)$位置的奇异性传播方向.

结论:

- ***边缘越清晰的图,其对应的局部Fourier变换衰减越慢***
- ***边缘越模糊的图,其对应的局部Fourier 变换衰减越快,以至于难以判断出边缘的方向信息.***

二元Gauss函数的调控(伸缩、旋转)及其Fourier变换(微局部的滤波函数)

图像FFT

相位重构:相位信息在图像认知中起主导作用

FFT变换的模值梯度与图像(二维函数值)的梯度方向相垂直

二元Gauss函数的调控

$$x \sim N(0,\sigma_1^2) \quad y \sim N(0,\sigma_2^2) \quad \begin{pmatrix} x \\ y \end{pmatrix} \sim N(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \Sigma)$$

$$g_{\sigma_x,\sigma_y}(x,y) = \frac{1}{2\pi|\Sigma|^{\frac{1}{2}}}\exp\left(-\frac{\begin{pmatrix} x & y \end{pmatrix}\Sigma^{-1}\begin{pmatrix} x \\ y \end{pmatrix}}{2}\right)$$

$$\Sigma = \begin{bmatrix} E(x-\mu_x)^2 & E(x-\mu_x)(y-\mu_y) \\ E(x-\mu_x)(y-\mu_y) & E(y-\mu_y)^2 \end{bmatrix} = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}$$

$$\rho = \frac{E(x-\mu_x)(y-\mu_y)}{\sqrt{E(x-\mu_x)^2E(y-\mu_y)^2}} \quad , \quad |\rho| \le 1$$

$$\begin{aligned} g(x,y) &= \frac{1}{\left(2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}\right)}\exp\left[-\frac{1}{2\left(1-\rho^2\right)}\left(\frac{(x-\mu_1)^2}{\sigma_1^2}-\frac{2\rho\left(x-\mu_1\right)\left(y-\mu_2\right)}{\sigma_1\sigma_2}+\frac{(y-\mu_2)^2}{\sigma_2^2}\right)\right] \\ &= \frac{1}{\left(2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}\right)}\exp\left[-\frac{\left(\sigma_2^2(x-\mu_1)^2-2\sigma_1\sigma_2\rho\left(x-\mu_1\right)\left(y-\mu_2\right)+\sigma_1^2(y-\mu_2)^2\right)}{2\left(1-\rho^2\right)\sigma_1^2\sigma_2^2}\right] \end{aligned}$$

$$\text{二阶矩阵的逆：}\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\Sigma^{-1} = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}^{-1} = \frac{1}{(1-\rho^2)\sigma_1^2\sigma_2^2}\begin{bmatrix} \sigma_2^2 & -\rho\sigma_1\sigma_2 \\ -\rho\sigma_1\sigma_2 & \sigma_1^2 \end{bmatrix}$$

- ***伸缩*** σ_1, σ_2 大小值代表椭圆的形状,差距越大,椭圆越扁

- ***空间旋转***

```
def rotate(image, angle, center=None, scale=1.0):
    ##  h,w 尺寸
    (h, w) = image.shape[:2]
    if center is None:
        center = (w // 2, h // 2)

    M = cv2.getRotationMatrix2D(center, angle, scale)
    # print(M) ## 旋转阵  +  旋转中心
    # 旋转
    rotated = cv2.warpAffine(image, M, (w, h))
    return rotated
```

$$A(\Sigma^{-1}) = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$$

A的二次型为$L(x,y) = ax^2 + by^2 + 2cxy$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}(\text{负角表示逆时针旋转})$$

我们用的是这个,(x,y)旋转到$(x‘,y‘)$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} x' \\ y' \end{bmatrix}(\text{正角表示逆时针旋转})$$

$$L(x',y') = a(x'\cos\theta - y'\sin\theta)^2 + b(x'\sin\theta + y'\cos\theta)^2 + \\ 2c(x'\cos\theta - y'\sin\theta)(x'\sin\theta + y'\cos\theta)$$

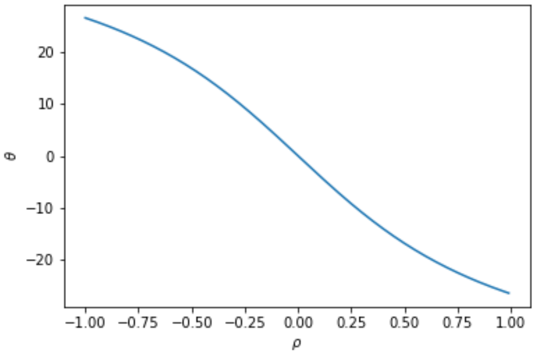
交叉项

$$-2a\cos\theta\sin\theta + 2b\sin\theta\cos\theta + 2c\cos^2\theta - 2c\sin^2\theta \\ = -a\sin 2\theta + b\sin 2\theta + 2c\cos 2\theta$$

让$L(x',y')$ 没有交叉项,得到

$$-a\sin 2\theta + b\sin 2\theta + 2c\cos 2\theta = 0 \Rightarrow \tan 2\theta = \frac{2c}{a-b} = \frac{2\rho\sigma_1\sigma_2}{\sigma_1^2 - \sigma_2^2}, \quad a \neq b$$

$$\theta = \begin{cases} \frac{1}{2}\arctan(\frac{2\rho\sigma_1\sigma_2}{\sigma_1^2-\sigma_2^2}) & \text{if } a \neq b \\ \frac{\pi}{4} & \text{if } a = b \end{cases}$$



ρ	$-1 \rightarrow +1$
θ	$+ \rightarrow -$
图	顺时针 $\rightarrow$ 逆时针

- ***Gabor***

$$\int_{-\infty}^{+\infty} exp\left[-ax^2\right] exp\left[-2\pi i x \xi\right] dx = \sqrt{\frac{\pi}{a}} exp\left[-\frac{\pi^2}{a}\xi^2\right]$$

证明:

$$\begin{aligned}
&\int_{-\infty}^{+\infty} exp\left[-ax^2\right] exp\left[-2\pi i x\xi\right] dx \\
=&\int_{-\infty}^{+\infty} exp\left[-ax^2 - 2\pi i x\xi\right] dx \\
=&\int_{-\infty}^{+\infty} exp\left[-a(x^2 + \frac{2\pi i\xi}{a}x)\right] dx \\
=&\int_{-\infty}^{+\infty} exp\left[-a(x + \frac{\pi i\xi}{a})^2\right] exp\left[a\frac{\pi^2 i^2}{a^2}\xi^2\right] dx \\
=&exp\left[-\frac{\pi^2}{a}\xi^2\right] \int_{-\infty+\frac{\pi i\xi}{a}}^{+\infty+\frac{\pi i\xi}{a}} exp\left[-ax^2\right] dx \\
=&exp\left[-\frac{\pi^2}{a}\xi^2\right] \int_{-\infty}^{+\infty} exp\left[-ax^2\right] dx(\text{插一步见下面}) \\
=&\sqrt{\frac{\pi}{a}} exp\left[-\frac{\pi^2}{a}\xi^2\right]
\end{aligned}$$

补:求 $I = \int_{-\infty}^{+\infty} \mathrm{e}^{-ax^2} \mathrm{d}x$

方法一(二元积分):

$$\begin{aligned}
I^2 &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mathrm{e}^{-a(x^2+y^2)} \mathrm{d}x\mathrm{d}y \\
&= \int_0^{2\pi} \mathrm{d}\theta \int_0^{+\infty} \mathrm{e}^{-a\rho^2} \rho \mathrm{d}\rho \\
&= \frac{-\pi}{a}\int_0^{+\infty} e^{-a\rho^2} d(-a\rho^2) \\
&= \frac{-\pi}{a} e^{-a\rho^2}\Big|_0^{+\infty} = \frac{\pi}{a}
\end{aligned}$$

方法二(正太分布概率密度):

$$
\begin{aligned}
&\frac{1}{\sqrt{2\pi}\sigma}\int_R e^{-\frac{x^2}{2\sigma^2}}\,dx = 1 \\
\Rightarrow &I = \int_R e^{-\frac{x^2}{2\sigma^2}}\,dx = \sqrt{2\pi}\sigma(\frac{1}{2\sigma^2} = a, \sigma = \sqrt{\frac{1}{2a}}) \\
\Rightarrow &I = \int_R e^{-ax^2}\,dx = \sqrt{2\pi}\sigma = \sqrt{2\pi}\sqrt{\frac{1}{2a}} = \sqrt{\frac{\pi}{a}}
\end{aligned}
$$

---

f_0频点调制

$$
\begin{aligned}
&\int_{-\infty}^{+\infty} exp\left[-ax^2\right] exp\left[2\pi i x f_0\right] exp\left[-2\pi i x \xi\right] dx \\
&= \int_{-\infty}^{+\infty} exp\left[-ax^2\right] exp\left[-2\pi i x(\xi - f_0)\right] dx \\
&= \sqrt{\frac{\pi}{a}} exp\left[-\frac{\pi^2}{a}(\xi - f_0)^2\right]
\end{aligned}
$$

空间旋转	频点调制	频率旋转
$exp\left[-\frac{1}{2}(\frac{x'^2}{a^2}+\frac{y'^2}{b^2})\right]$	$exp\left[2\pi i(x' f_0 + y' f_1)\right]$	$exp\left[-2\pi i(x'\xi' + y'\eta')\right]$

对课程作业要多思考

这个得看命

王老师万岁!!!