15周上课补充

$$x \sim N(0,\sigma_1^2) \quad y \sim N(0,\sigma_2^2) \quad \left(egin{array}{c} x \\ y \end{array}
ight) \sim N(\left(egin{array}{c} 0 \\ 0 \end{array}
ight), \Sigma)$$
 $g_{\sigma_x,\sigma_y}(x,y) = rac{1}{2\pi |\Sigma|^{rac{1}{2}}} \mathrm{exp} \Biggl(-rac{(x-y)\,\Sigma^{-1}\,\left(x \\ y \end{matrix} \Biggr)}{2} \Biggr)$ $\Sigma = \left[egin{array}{c} E(x-\mu_x)^2 & E(x-\mu_x)(y-\mu_y) \\ E(x-\mu_x)(y-\mu_y) & E(y-\mu_y)^2 \end{array}
ight] = \left[egin{array}{c} \sigma_1^2 &
ho\sigma_1\sigma_2 \\
ho\sigma_1\sigma_2 & \sigma_2^2 \end{array}
ight]$ $ho = rac{E(x-\mu_x)(y-\mu_y)}{\sqrt{E(x-\mu_x)^2E(y-\mu_y)^2}} \quad , \quad |
ho| \leq 1$

$$\begin{split} g(x,y) &= \frac{1}{\left(2\pi\sigma_{1}\sigma_{2}\sqrt{1-\rho^{2}}\right)} \exp\left[-\frac{1}{2\left(1-\rho^{2}\right)} \left(\frac{\left(x-\mu_{1}\right)^{2}}{\sigma_{1}^{2}} - \frac{2\rho\left(x-\mu_{1}\right)\left(y-\mu_{2}\right)}{\sigma_{1}\sigma_{2}} + \frac{\left(y-\mu_{2}\right)^{2}}{\sigma_{2}^{2}}\right)\right] \\ &= \frac{1}{\left(2\pi\sigma_{1}\sigma_{2}\sqrt{1-\rho^{2}}\right)} \exp\left[-\frac{\left(\sigma_{2}^{2}\left(x-\mu_{1}\right)^{2} - 2\sigma_{1}\sigma_{2}\rho\left(x-\mu_{1}\right)\left(y-\mu_{2}\right) + \sigma_{1}^{2}\left(y-\mu_{2}\right)^{2}\right)}{2\left(1-\rho^{2}\right)\sigma_{1}^{2}\sigma_{2}^{2}}\right] \end{split}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\Sigma^{-1} = \begin{bmatrix} \sigma_1^2 & \rho \sigma_1 \sigma_2 \\ \rho \sigma_1 \sigma_2 & \sigma_2^2 \end{bmatrix}^{-1} = \frac{1}{(1 - \rho^2)\sigma_1^2 \sigma_2^2} \begin{bmatrix} \sigma_2^2 & -\rho \sigma_1 \sigma_2 \\ -\rho \sigma_1 \sigma_2 & \sigma_1^2 \end{bmatrix}$$

空间旋转

$$A(\Sigma^{-1}) = \left[egin{matrix} a & c \ c & b \end{matrix}
ight]$$

A的二次型为 $L(x,y) = ax^2 + by^2 + 2cxy$

$$egin{bmatrix} x' \ y' \end{bmatrix} = egin{bmatrix} \cos heta & \sin heta \ -\sin heta & \cos heta \end{bmatrix} egin{bmatrix} x \ y \end{bmatrix}$$
 (负角表示逆时针旋转)

我们用的是这个,(x,y)旋转到(x',y')

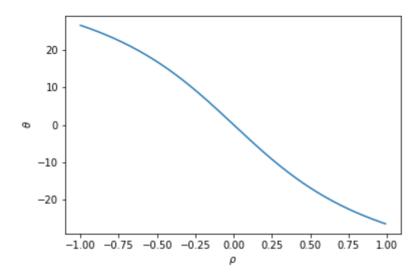
$$egin{aligned} egin{aligned} x \ y \end{bmatrix} &= egin{bmatrix} \cos heta & -\sin heta \ \sin heta & \cos heta \end{bmatrix} egin{bmatrix} x' \ y' \end{bmatrix} egin{bmatrix} \mathbb{E} \, \mathbb{A} \, \mathbb{R} \, \mathbb{$$

交叉项

$$-2a\cos\theta\sin\theta + 2b\sin\theta\cos\theta + 2c\cos^2\theta - 2c\sin^2\theta$$
$$= -a\sin 2\theta + b\sin 2\theta + 2c\cos 2\theta$$

让L(x',y') 没有交叉项,得到

$$-a\sin 2 heta + b\sin 2 heta + 2c\cos 2 heta = 0 \Rightarrow an 2 heta = rac{2c}{a-b} = rac{2
ho\sigma_1\sigma_2}{\sigma_1^2 - \sigma_2^2}, \quad a
eq b$$
 $heta = egin{cases} rac{1}{2} \mathrm{arctan}(rac{2
ho\sigma_1\sigma_2}{\sigma_1^2 - \sigma_2^2}) & ext{if } a
eq b \end{cases}$ $heta = b$



ρ	-1 ightarrow +1	
θ	$+ \rightarrow -$	
图	顺时针 $ ightarrow$ 逆时针	

Gabor

$$\int_{-\infty}^{+\infty} exp\left[-ax^2
ight] exp\left[-2\pi ix\xi
ight] dx = \sqrt{rac{\pi}{a}}exp\left[-rac{\pi^2}{a}\xi^2
ight]$$

 f_0 频点调制

$$egin{split} \int_{-\infty}^{+\infty} exp\left[-ax^2
ight] exp\left[2\pi ixf_0
ight] exp\left[-2\pi ix\xi
ight] dx \ &= \int_{-\infty}^{+\infty} exp\left[-ax^2
ight] exp\left[-2\pi ix(\xi-f_0)
ight] dx \ &= \sqrt{rac{\pi}{a}} exp\left[-rac{\pi^2}{a}(\xi-f_0)^2
ight] \end{split}$$

空间旋转	频点调制	频率旋转
$exp\left[-rac{1}{2}(rac{x'^2}{a^2}+rac{y'^2}{b^2}) ight]$	$exp\left[2\pi i(x'f_0+y'f_1)\right]$	$exp\left[-2\pi i(x'\xi'+y'\eta') ight]$