EE 360C - ALGORITHMS
The University of Texas at Austin

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An undirected graph is connected if each pair of vertices is connected by a path. Prove that the following two statements are equivalent:

1. A graph $G = (V, E)$ is disconnected
2. The vertices of the graph $G = (V, E)$ can be partitioned into two sets V_1, V_2 such that there are no edges between any member of V_1 and any member of V_2. A Partition means that $V_1 \cup V_2 = V$ and $V_1 \cap V_2 = \emptyset$.

Solution: To prove that two statements are equivalent, we must prove implication in both directions. First we prove $1 \implies 2$ and then $2 \implies 1$.

$1 \implies 2$:
Assume G is disconnected, therefore there exists a pair of vertices a and b for which there is not a path between them. Let V_1 be the connected component that contains a. Let V_2 contain all the remaining nodes, including b (which makes it not empty). There is no edge between any member of V_2 and any member of V_1, otherwise there would be a path between that member of V_2 and a, which would mean it should have been in V_1 originally. Therefore, if G is disconnected, such partition exists.
$2 \implies 1$:
Assume there is a partition V_1, V_2 such that there are no edges between V_1 and V_2. Choose $u \in V_1$ and $v \in V_2$. There cannot be a path between u and v since, if there was, there would be and edge between a member of V_1 and a member of V_2, keeping in mind that $V_1 \cap V_2 = \emptyset$ in the partition. Therefore, G must be disconnected.
Since $1 \implies 2$ and $2 \implies 1$, it follows that 1 and 2 are equivalent.