EE360C: Algorithms
University of Texas at Austin — Problem Set #6
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Problem Set #6

You should try to solve these problems by yourself. I recommend that you start early and get help in office hours if needed. If you find it helpful to discuss problems with other students, go for it. **You do not need to turn these problems in. The goal is to be ready for the in class quiz that will cover the same or similar problems.**

Problem 1: Counting Shortest Paths

A number of art museums around the country have been featuring work by an artist named Mark Lombardi (1951-2000), consisting of a set of intricately rendered graphs. Building on a great deal of research, these graphs encode the relationships among people involved in major political scandals over the past several decades: the nodes correspond to participants, and each edge indicates some type of relationship between a pair of participants. And so, if you peer closely enough at the drawings, you can trace out ominous looking paths from a high-ranking U.S. government official to a former business partner to a bank in Switzerland to a shadowy arms dealer.

Such pictures form striking examples of *social networks*, which have nodes representing people and organizations and edges representing relationships of various kinds. And the short paths that abound in these networks have attracted considerable attention recently, as people ponder what they mean. In the case of Mark Lombardi's graphs, they hint at the short set of steps that can carry you from the reputable to the disreputable.

Of course, a single, spurious short path between nodes v and w in such a network may be more coincidental than anything else; a large number of short paths between v and w can be much more convincing. So in addition to the problem of computing a single shortest v-w path in a graph G, social networks researchers have looked at the problem of determining the *number* of shortest v-w paths.

This turns out to be a problem that can be solved efficiently. Suppose we are given an undirected graph $G = (V, E)$, and we identify two nodes v and w in G. Give an algorithm that computes the number of shortest v-w paths in G. (The algorithm should not list all the paths; just the number suffices.) The running time of your algorithm should be $O(m + n)$ for a graph with n nodes and m edges.

Problem 2: Greedy Choice

You are consulting for a trucking company that does a large amount of business shipping packages between New York and Boston. The volume is high enough that they have to send several trucks each day between the two locations. Trucks have a fixed limit W on the maximum amount of weight they are allowed to carry. Boxes arrive at the New York station one by one, and each package i has a weight w_i. The trucking station is quite small, so at most one truck can be in the station at any time. Company policy requires that boxes are shipped in the order they arrive; otherwise a customer might get upset. At the moment, the company is using a simple greedy algorithm for packing: they pack boxes in the order they arrive, and whenever the next box does not fit, they send the truck on its way.

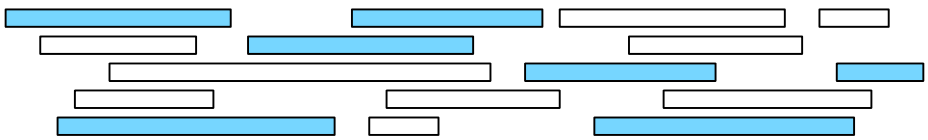
But they wonder if they might be using too many trucks, and they want your opinion on whether the situation can be improved. Here is how they are thinking. Maybe one could decrease the number of trucks needed by sometimes sending off a truck that was less full, and in this way allow the next few trucks to be better packed.

Prove that, for a given set of boxes with specified weights, the greedy algorithm currently in use actually minimizes the number of trucks that are needed.

Problem 3: Intervals

Let X be a set of n intervals on the real line. A subset of intervals $Y \subseteq X$ is called a *tiling path* if the intervals in Y cover the intervals in X, that is, any real value that is contained in some interval in X is also contained in some interval in Y. The *size* of a tiling cover is just the number of intervals.

Describe an algorithm to compute the smallest tiling path of X. Assume that your input consists of two arrays $X_L[1..n]$ and $X_R[1..n]$, representing the left and right endpoints of the intervals in X. Argue that your algorithm is correct (hint: remember that an argument of correctness for any greedy algorithm has two components). The figure below shows an example of a tiling path, though a non-optimal one.



A set of intervals. The seven shaded intervals form a tiling path.

Problem 4: Greedy Scheduling

Consider a situation where you have to find available classrooms for n different lectures. Of course, you must avoid scheduling two or more overlapping lecture in the same room. Each lecture i begins at s_i and ends at t_i.

(a) Find an algorithm that assigns the smallest number of rooms possible.

(b) Show that your algorithm is optimal. (Hint: Look at slides 10-12 in from Lecture 8)