EE 360C - ALGORITHMS
The University of Texas at Austin

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Name: **UT EID:**

Let $f_1(n) = O(g_1(n))$ and $f_2(n) = O(g_2(n))$. Prove or disprove that $f_1(n) + f_2(n) = O(|g_1(n)| + |g_2(n)|)$.

Solution

We know that $f_1(n) = O(g_1(n))$ and $f_2(n) = O(g_2(n))$, therefore there exist $c_1, c_2, n_{0,1}, n_{0,2}$ such that:

$$f_1(n) \leq c_1 \cdot g_1(n) \text{ for } n > n_{0,1}$$
$$f_2(n) \leq c_2 \cdot g_2(n) \text{ for } n > n_{0,2}$$

Furthermore, by properties of absolute values, it is also true that

$$f_1(n) \leq c_1 \cdot |g_1(n)| \text{ for } n > n_{0,1}$$
$$f_2(n) \leq c_2 \cdot |g_2(n)| \text{ for } n > n_{0,2}$$

Because the values to the right of the inequality are all positive, it holds that

$$f_1(n) + f_2(n) \leq c_1 \cdot |g_1(n)| + c_2 \cdot |g_2(n)| \text{ for } n > \max(n_{0,1}, n_{0,2})$$

Notice our new n_0 is now $\max(n_{0,1}, n_{0,2})$. Now we simply need to find a correct c. Consider $c = \max(c_1, c_2)$, then

$$f_1(n) + f_2(n) \leq c_1 \cdot |g_1(n)| + c_2 \cdot |g_2(n)| \leq c(|g_1(n)| + |g_2(n)|) \text{ for } n > \max(n_{0,1}, n_{0,2})$$

Therefore, $f_1(n) + f_2(n) \leq c(|g_1(n)| + |g_2(n)|)$ for $n > n_0$, where $c = \max(c_1, c_2)$ and $n_0 = \max(n_{0,1}, n_{0,2})$, and so $f_1(n) + f_2(n) = O(|g_1(n)| + |g_2(n)|)$