EE360C: Algorithms

Priority Queues

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Motivation: Stable Marriage

The stable marriage algorithm needs a data structure that maintains the dynamically changing set of all free men. The algorithm needs to be able to:

- add elements to the set
- delete elements from the set
- select an element from the set, based on some assigned *priority*

Motivation: Sort a List of Numbers

Sort

Instance: Nonempty list $x_1, x_2, \ldots, x_n$ of integers
Solution: A permutation $y_1, y_2, \ldots y_n$ of $x_1, x_2, \ldots, x_n$ such that $y_i \leq y_{i+1}$ for all $1 \leq i < n$

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Possible Algorithm

- Store all of the numbers in a data structure D
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To get $O(n \log n)$ running time, each "find minimum" step must take $O(\log n)$ time

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Sorted array Finding the minimum takes $O(1)$ time, but insertion and deletion take $\Omega(n)$ time in the worst case

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- Key update and element deletion require knowledge of the position of the element in the priority queue

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- we need to maintain a set of *active processes* with the ability to quickly extract the one with the highest priority so it can be scheduled
- using a priority queue keyed by process priority, scheduling the highest priority process entails simply finding the one with the lowest priority key

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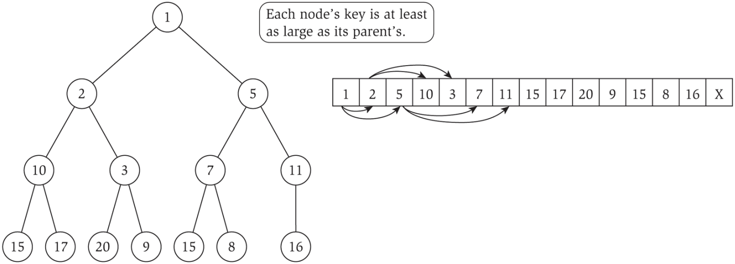
A Heap Example



INSERTING AN ELEMENT: Heapify-up

1. Insert a new element at $n+1$
2. Fix the heap order using Heapify-up$(H, n+1)$

```
Heapify-up(H,i):
  If i > 1 then
    let j = parent(i) = ⌊i/2⌋
    If key[H[i]]<key[H[j]] then
      swap the array entries H[i] and H[j]
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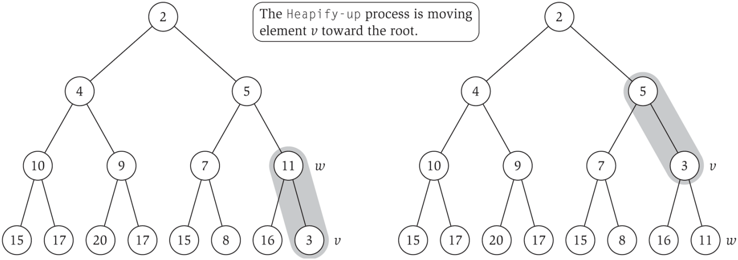
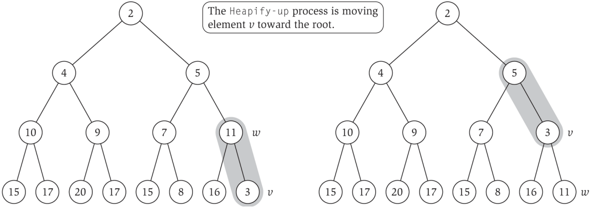
    Endif
  Endif
```

`Heapify-Up` Example



CORRECTNESS OF `Heapify-Up`



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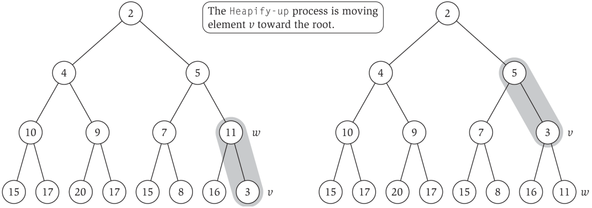

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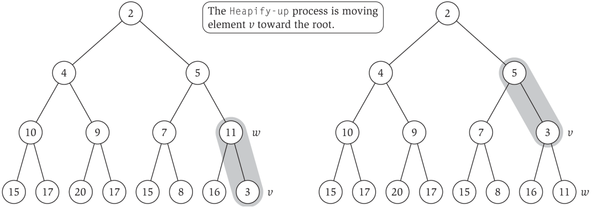

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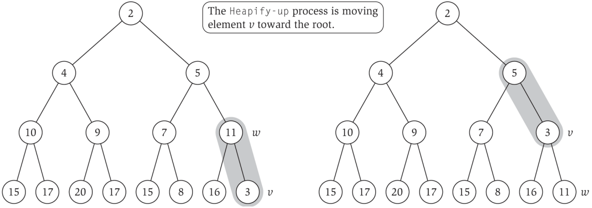

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- Prove by induction on i
  - Base case: $i = 1$
  - Inductive step: if H is almost a heap with key of $H[i]$ too small, after `Heapify-up`(H, i), H is a heap or almost a heap with the key of $H[j]$ too small.

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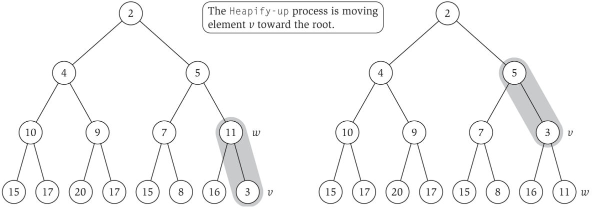

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- The running time of `Heapify-up` is $O(\log i)$

Deleting an Element: `Heapify-down`

Suppose H has $n+1$ elements

1. Delete element at $H[i]$ by moving element at $H[n+1]$ to $H[i]$
2. If element at $H[i]$ is too small, fix heap order using `Heapify-up`(H, i)
3. If element at $H[i]$ is too large, fix heap order using `Heapify-down`(H, i)

```
Heapify-down(H,i):
  Let n = length(H)
  If 2i > n then
    Terminate with H unchanged
  Else if 2i < n then
    Let left = 2i, and right = 2i + 1
    Let j be the index that minimizes key[H[left]] and key[H[right]]
  Else if 2i = n then
    Let j = 2i
  Endif
  If key[H[j]] < key[H[i]] then
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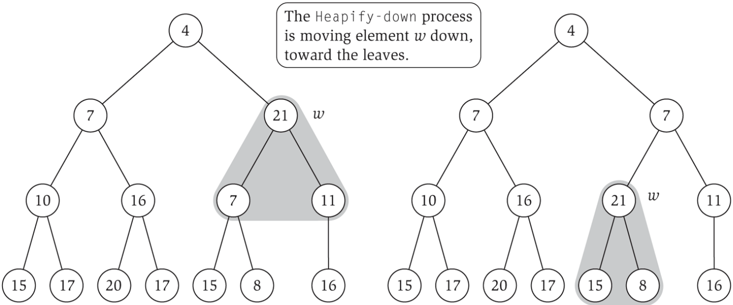
Heapify-down EXAMPLE



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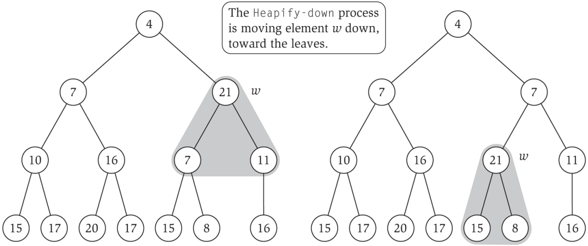

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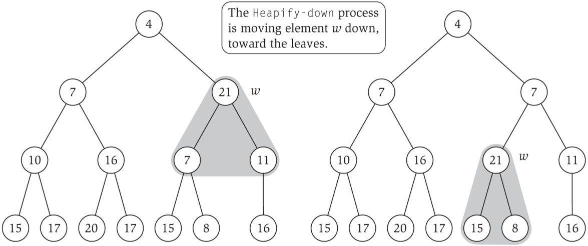

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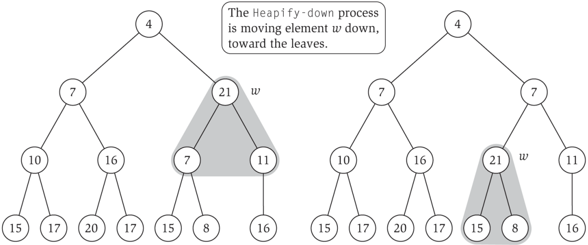

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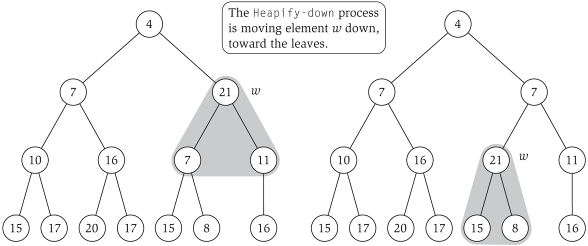

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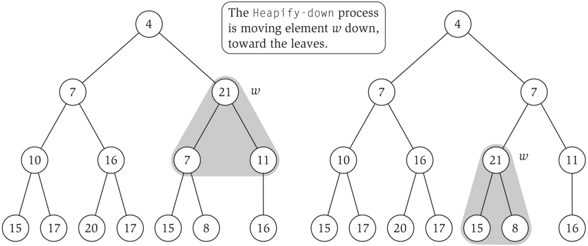

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- The running time of `Heapify-down`(H, i) is $O(\log n)$

Motivation
Priority Queue
Heaps
HeapSort
Questions

In Class Exercise 1

Problem

Naively, we can build a heap out of an arbitrary array using successive calls to HEAPIFY-DOWN, starting at element $\lfloor \text{length}[H]/2 \rfloor$ and going down to 1. If each call to HEAPIFY-DOWN takes $O(\log n)$ time and we have $O(n/2)$ such calls, we can build a heap in $O(n \log n)$ time. Prove that this process is actually faster than $O(n \log n)$ (i.e., provide a *tighter* bound on the process's running time).

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$O(\log n)$ (it's a (nearly) complete binary tree).

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$N_h = N'_{h-1} = \lceil n'/2^h \rceil = \lceil \lfloor n/2 \rfloor / 2^h \rceil \leq \lceil (n/2)/2^h \rceil = \lceil n/2^{h+1} \rceil$.

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In Class Exercise 1: Solution

Problem

Naively, we can build a heap out of an arbitrary array using successive calls to HEAPIFY-DOWN, starting at element $\lfloor \text{length}[H]/2 \rfloor$ and going down to 1. If each call to HEAPIFY-DOWN takes $O(\log n)$ time and we have $O(n/2)$ such calls, we can build a heap in $O(n \log n)$ time. Prove that this process is actually faster than $O(n \log n)$ (i.e., provide a *tighter* bound on the process's running time).

Solution

The time required by HEAPIFY-DOWN, when called on a node at height h is $O(h)$. The total cost of building a heap is bounded above by:

$$\sum_{h=0}^{\lfloor \log n \rfloor} \lceil \frac{n}{2^{h+1}} \rceil O(h) = O(n \sum_{h=1}^{\lfloor \log n \rfloor} \frac{h}{2^h}) = O(n)$$

The last step is because (looking up the summation):

$$\sum_{h=0}^{\infty} \frac{h}{2^h} = \frac{1/2}{(1-1/2)^2} = 2$$

Sorting with a Priority Queue

Sort

Instance: Nonempty list $x_1, x_2, \ldots, x_n$ of integers
Solution: A permutation $y_1, y_2, \ldots y_n$ of $x_1, x_2, \ldots, x_n$ such that $y_i \leq y_{i+1}$ for all $1 \leq i < n$

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Final Algorithm

- Insert each number in a priority queue H
- Repeatedly find the smallest number in H, output it, and delete it from H

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Each insertion and deletion takes $O(\log n)$ time for a total running time of $O(n \log n)$

In Class Exercise 2

Problem

One of your classmates claims that he built an alternative data structure (other than a heap) for representing a priority queue. He claims that, using his new data structure, Insert, Max, and ExtractMax all take constant ($O(1)$) time in the worst case. Give a very simple proof that he is mistaken.

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Solution

If this were true, we could comparison sort in $O(n)$ time. But we've already proven that this is not possible.

Questions

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