EE 360C - ALGORITHMS
The University of Texas at Austin

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Efficient Recruiting: Suppose you're helping to organize a summer sports camp, and the following problem comes up. The camp is supposed to have at least one counselor who is skilled at each of the n sports covered by the camp (baseball, volleyball, etc.). They have received job applications from m potential counselors. For each of the n sports, there is some subset of the m applicants qualified in that sport. The question is: For a given number $k < m$, is it possible to hire at most k of the counselors and have at least one counselor qualified in each of the n sports? We'll call this the *Efficient Recruiting* Problem. Show that *Efficient Recruiting* is NP-Complete by reducing from the Vertex Cover Problem.

The Vertex Cover Problem. *Given a graph G and a non-negative integer k, does G contain a vertex cover of size at most k? (Recall that a vertex cover $V' \subseteq V$ is a set of vertices such that every edge $e \in E$ has at least one of its endpoints in V'.)*

Solution

Efficient Recruiting is in NP, since given a set of k counselors, we can check that they cover all of the sports.

Suppose we had an algorithm A that solves *Efficient Recruiting*; here is how we would solve an instance of *Vertex Cover*. Given a graph $G = (V, E)$ and an integer k, we would define a sport S_e for each edge e and a counselor C_v for each vertex v. C_v is qualified in sport S_e if and only if e has an endpoint equal to v.

Now if there are k counselors that, together, are qualified in all sports, the corresponding vertices in G have the property that each edge has an end in at least one of them; so they define a vertex cover of size k. Conversely, if there is a vertex cover of size k, then this set of counselors has the property that each sport is contained in the list of qualifications of at least one of them.

Thus, G has a vertex cover of size at most k if and only if the instance of *Efficient Recruiting* that we create can be solved with at most k counselors. Moreover, the instance of *Efficient Recruiting* has size polynomial in the size of G. Thus if we could determine the answer to the *Efficient Recruiting* instance in polynomial time, we could also solve the instance of *Vertex Cover* in polynomial time.