EE 360C - ALGORITHMS
The University of Texas at Austin
Dr. Pedro Santacruz
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Name: **UT EID:**

Depth First Search: Suppose G is a connected undirected graph. An edge whose removal disconnects the graph is called a *bridge*. Either prove the following statement or provide a counter-example: every bridge e must be an edge in a depth-first search tree of G.

Solution

This is true. Suppose it were not true. Consider the edge e that connects vertices u and v, where e is a bridge. That is, if e were removed from the graph G, then we would have a partition of the vertices of G such that no edge crosses the partition. Start a depth first search in the partition that includes u. Eventually, since G is connected, this DFS must reach vertex u. When it does, it will examine all of the vertices adjacent to u and incorporate any who have not yet been "touched" into the DFS-tree. This includes v. We are guaranteed that v has not yet been examined since the edge (u, v) is the only edge that crosses our partition of vertices, and we started our DFS in the partition that included u. Therefore, the edge (u, v) must be included in the DFS, giving us our contradiction.