Problem Set #7

You should try to solve these problems by yourself. I recommend that you start early and get help in office hours if needed. If you find it helpful to discuss problems with other students, go for it. **You do not need to turn these problems in. The goal is to be ready for the in class quiz that will cover the same or similar problems.**

Problem 1: Fractional Knapsack

A thief is robbing a store finds n items; the i^{th} item is worth v_i dollars and weights w_i pounds (both integers). The thief wants to take as valuable a load as possible but can carry only W pounds. However, this thief can take fractions of any item he finds. In other words, the thief doesn't have to take all or none of any particular item.

(a) Explain why this problem displays the optimal substructure property.

(b) Develop an algorithm to maximize the value of thief's load.

Problem 2: Minimum Spanning Trees

Let G be a weighted undirected graph, where the edge weights are distinct. An edge is *dangerous* if it is the longest edge in some cycle, and an edge is $useful$ if it does not belong to any cycle in G.

(a) Prove that any MST of G contains every useful edge

(b) Prove that any MST of G contains no dangerous edge

Problem 3: Greedy Algorithms

Consider a long, quiet country road with houses scattered sparsely along it. (We can picture the road as a long line segment, with an eastern endpoint and a western endpoint.) Further, let's suppose that despite the bucolic setting, the residents of all these houses are avid cell phone users. You want to place cell phone base stations at certain points along the road, so that every house is within 4 miles of one of the base stations.

(a) Give an efficient algorithm that achieves this goal, using as few base stations as possible.

(b) Prove that the greedy choice that your algorithm makes is the optimal choice.

Problem 4: Modifying Dijkstra's Algorithm

We are given an directed graph $G = (V, E)$ on which each edge $(u, v) \in E$ has an associated value $r(u, v)$, which is a real number in the range $0 \leq r(u, v) \leq 1$ that represents the reliability of a communication channel from vertex u to vertex v. We interpret $r(u, v)$ as the probability that the channel from u to v will not fail, and we assume that these probabilities are independent. Give an efficient algorithm to find the most reliable path between two given vertices.