EE 360C - ALGORITHMS
The University of Texas at Austin

Dr. Pedro Santacruz
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Name: UT EID:

Problem 1: Summing Integers

Suppose you are given a collection $A = \{a_1, a_2, \ldots, a_n\}$ of n positive integers that add up to $2Z$. We want to design an $O(nZ)$ time algorithm to decide if the set can be partitioned into two groups B and $A - B$ such that:

$$\sum_{a_j \in B} a_j = \sum_{a_i \in (A-B)} a_i = Z,$$

in other words, there is a subset of A that adds up to Z.

(a) Define an $Z \times n$ array m, where $m[z, i]$ is 1 if there exists a subset of A, $\{a_1, a_2, ..., a_i\}$, that sums to z and 0 otherwise. Write the dynamic equation formula for computing $m[z, i]$.

Solution

First we consider the two possible cases: either the i^{th} member of the subset is included in the sum that add up to Z or it doesn't. If it is not included, then our problem reduces to the problem with using only the first $i - 1$ members of the subset. If the i^{th} member of the subset is included in the sum, then the problem reduces to the adding the first $i-1$ elements to the value of $z - a_i$. Therefore the dynamic programming equation is:

$$m[z, i] = \max\{m[z, i-1], m[z - a_i, i-1]\}$$

(b) Write pseudocode that uses your dynamic equation formula to fill in the table m.

Solution

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SUMMATION SUBSET(A, Z)
Initialize row 0 with value of 1, m[0, 0 : n] = 1
Initialize left column with value of 0, m[1 : Z, 0] = 0
for z ← 1 to Z
    for i ← 1 to n
        m[z, i] = max{m[z, i − 1], m[z − a_i, i − 1]}
return m[Z, n]
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(c) In English, describe how you would reconstruct the solution from the table m (describe how you would return the actual subset of numbers that sums to Z instead of just "yes" or "no").

Solution

If there is a solution, the entry to $m[Z, n]$ must be 1. Find the smallest i such that the entry $m[Z, i] = 1$. Then, the value a_i must be in the subset that adds up to Z. Compute a new value $Z' = Z - a_i$. Again, find the smallest value i' such that $m[Z', i] = 1$, this means $a_{i'}$ should also be part of the subset. Iterate this process until $Z' = 0$.