EE 360C - ALGORITHMS
The University of Texas at Austin

Dr. Pedro Santacruz
September 18, 2014

Name: **UT EID:**

Decide whether the following statement is true or false. If it is true, give a short proof. If it is false, give a counter example.

Consider an instance of the Stable Marriage problem in which there exists a man m and a woman w such that m is ranked first on the preference list of w and w is ranked first on the preference list of m. Then in every stable matching S for this instance, the pair (m, w) belongs to S.

Solution

Suppose this is not the case. Then there exists a stable matching S such that (m, w) does not belong to the matching. Then the pairs (m, w') and (m', w) do belong to S, where w prefers m to m' and m prefers w to w' ($m \neq m'$ and $w \neq w'$). (By the definition of stability, S is not stable. It's ok to end the proof here. But let's look deeper...) Did m propose to w? He must have, since he preferred w over w' and he wouldn't have proposed to w' until after he'd proposed to everyone he preferred over her. When he proposed to w, she was either still free, engaged to m' already, or engaged to some other $m'' \neq m' \neq m$. If w was still free, she would have accepted. And she never would have dropped m, because it is impossible that a better choice would come along (since m is at the top of her list). If she were already engaged to m' when m proposed, she would have dropped m' in favor of m and kept m until the end (because it is impossible that a better choice would have come along). Finally, if w were engaged to some other m'', she would have dropped him in favor of m. The only way she could have ended up with m' was if he had then come along at a later time and she had dropped m for m'. But she wouldn't have done that, since m is at the top of her list.