EE 360C - ALGORITHMS
The University of Texas at Austin

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**Network Flow:** Network flow issues come up in dealing with natural disasters and other crises, since major unexpected events often require the movement and evacuation of large numbers of people in a short amount of time.

Consider the following scenario. Due to large-scale flooding in a region, paramedics have identified a set of $n$ injured people distributed across the region who need to be rushed to hospitals. There are $k$ hospitals in the region, and each of the $n$ people needs to be brought to a hospital that is within a half-hour's driving time of their current location(so different people will have different options for hospitals, depending on where they are right now).

At the same time, one doesn't want to overload any one of the hospitals by sending too many patients its way. The paramedics are in touch by cell phone, and they want to collectively workout whether they can choose a hospital for each of the injured people in such a way that the load on the hospital is balanced: Each hospital receives at most $\lceil n/k \rceil$ people.

Give a polynomial-time algorithm that takes the given information about the people's locations and determines whether this is possible. You may assume access to a polynomial-time algorithm for max flow.

**Solution**

We build the following flow network. There is a node $v_i$ for each patient $i$, a node $w_j$ for each hospital $j$, and an edge $(v_i, w_j)$ of capacity 1 if patient $i$ is within a half hour drive of hospital $j$. We then connect a super-source $s$ to each of the patient nodes by an edge capacity 1, and we connect each of the hospital nodes to a super-sink $t$ by an edge of capacity $\lceil n/k \rceil$.

We claim that there is a feasible way to send all patients to hospitals if and only if there is an $s - t$ flow of value $n$. If there is a feasible way to send patients, then we send one unit of flow from $s$ to $t$ along each of the paths $s, v_i, w_j, t$, where patient $i$ is sent to hospital $j$. This does not violate the capacity conditions, in particular on the edges $(w_j, t)$, due to the load constraints. Conversely, if there is a flow of value $n$, then there is one with integer values. We send patient $i$ to hospital $j$ if the edge $(v_i, w_j)$ carries one unit of flow, and we observe that the capacity condition ensures that no hospital is overloaded.

The running time is the time required to solve a max-flow problem on a graph with $O(n + k)$ nodes and $O(nk)$ edges.