Problem 4

a. There are 12 phases: selecting a fruit for each of the 12 children.
For each of the children, there are 3 possible fruit to be given.
The quantity that will give us the total number of possibilities is $3^{12} = 531{,}441$.
-All valid outcomes are accounted for: every possible permutation of the 12 children with their fruit.
-There are no invalid outcomes because there is no limitations.
-There is no overcounting because every outcome is non-permutable (the children are distinct)

b. There are 3 phases: selecting 4 children to give apples, 4 children to give oranges, and 4 children to give bananas.
For each phase, there are a decrementing amount of children to give a fruit.
Phase 1: 12 * 11 * 10 * 9
Phase 2: 8 * 7 * 6 * 5
Phase 3: 4 * 3 * 2 * 1
However with this method there is a source of overcounting: the children within the fruit-bound phases are permutable: by 4! to be exact. And because there are three phases where this overcounting is possible, we need to divide by 4! three times.
The number of possibilities is $\frac{12!}{4!^3}$ = 34650.
-All valid outcomes are accounted for: every child can be given a fruit, and the different apples/bananas/oranges that can given to the children do not matter.
-There are no invalid outcomes because every child has been placed into one of the three phases and been given a fruit.
-Overcounting has already been accounted for.