

CSCI 150 Discrete Mathematics

Homework 3

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Problem 0

- Review sets and the connection between the binomial coefficient and number of subsets of a given size
- Understand one-to-one, onto, and bijections

Problem 1: Prove in two ways

Prove the following:

$$n + \binom{n}{2} = \binom{n+1}{2}$$

- (a) Algebraically
- (b) By a combinatorial argument (count in two ways)

Problem 2: A truncated sum

In class, we saw that

$$\sum_{k=0}^n \binom{n}{k} = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n} = 2^n$$

Assume that $n > 1$ is odd, and consider this truncated sum:

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{\frac{n-1}{2}}$$

- (a) Write this sum using \sum notation.
- (b) How many terms does this sum have?
- (c) Show that the sum has an even number of terms that are odd. *Hint:* use the fact that $\binom{n}{k} = \binom{n}{n-k}$.

Problem 3

For each of the following functions, indicate which ones are one-to-one, which ones are onto, and which ones are a bijection (one-to-one correspondence).

(a) $f : \mathbb{N} \rightarrow \mathbb{R}, f(x) = 1/x$.

(b) $g : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{Q}^+, g(x, y) = x/y$, where \mathbb{Q}^+ is the set of all rational numbers greater than zero (the positive rational numbers).

(c) $h : \mathbb{Z} \rightarrow \mathbb{Z}, h(x) = x^2$.

(d) $w : \mathbb{R} \rightarrow \mathbb{R}, w(x) = 3x + 1$.

Problem 4: Itsy Bitsy spider

Itsy Bitsy has one sock and one shoe for each of its 8 legs. On each leg, the sock must be put on before the shoe.

For instance, let a_i be the i^{th} sock, and b_i be the i^{th} shoe. An order is good for leg i if a_i appears before b_i . An order is good if it is good for all legs. The following two orders are good (but there are many others):

$$a_1 \ a_2 \ a_3 \ a_4 \ a_5 \ a_6 \ a_7 \ a_8 \ b_1 \ b_2 \ b_3 \ b_4 \ b_5 \ b_6 \ b_7 \ b_8$$

$$a_1 \ b_1 \ a_2 \ b_2 \ a_3 \ b_3 \ a_4 \ b_4 \ a_5 \ b_5 \ a_6 \ b_6 \ a_7 \ b_7 \ a_8 \ b_8$$

An order is bad, if b_i appears before a_i for some i . For instance, below (and there are many others):

$$b_1 \ a_1 \ a_2 \ b_2 \ a_3 \ b_3 \ a_4 \ b_4 \ a_5 \ b_5 \ a_6 \ b_6 \ a_7 \ b_7 \ a_8 \ b_8$$

How many good orders are there? We will solve this problem in multiple steps:

(a) Consider the set of orders that are good for leg 1 and the set of orders that are bad for leg 1. Prove that the two sets have the same size. *Hint:* define a one-to-one correspondence.

(b) Consider the set of orders that are good for leg 1 and divide it into two sets of orders: those that are good for leg 2 and those that are bad for leg 2. Prove that these two sets have the same size.

(c) Can you now figure out how many good orders Itsy Bitsy has (answer by more than YES or NO)?

Hint: Try your strategy on a human being (two legs) instead of a spider, and see if your answer makes sense. What if we add two gloves? (not graded)

Problem 5 (will not be graded)

Do exercises 1-6 in:

<http://www2.math.ou.edu/~amiller/2513/review/CountingProblemsAnswer.pdf>