CSCI 150 Discrete Mathematics
Practice 2

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Problem 1: Taxi
Taxis in New York city have numbers like 7A31; in other words, a digit, followed by a letter, followed by a digit, followed by a digit. Using the product rule, find the maximum number of Taxis. Specify the procedure and determine whether it overcounts or not.

Problem 2: Couple
We have m boys and n girls. Let's call a couple homogeneous if the two people in the couple have the same gender. Otherwise, it's heterogeneous.

(a) In how many ways can we make a heterogeneous couple?

(b) In how many ways can we make a homogeneous couple?

In both cases above, explain your generation procedure and determine if there is overcounting or not.

Problem 3: Anagrams
(a) Read the section in Note 2 about Anagrams and understand it. Then figure out how many anagrams you can make from the word composed of your first name and last name.

(b) Assume a word of length n contains k_i occurrences of i^{th} letter in some alphabet S. For instance, the word "mathematics" contains 2 occurrence of "m", 2 occurrences of "a", 2 occurrences of "t", 1 occurrence of "h", 1 occurrence of "e", 1 occurrence of "i", 1 occurrence of "c", 1 occurrence of "s", and 0 occurrences of the other letters. So observe that it is always the case that $\sum_{i \in S} k_i = n$. Derive the formula

$$\frac{n!}{\prod_{i \in S}(k_i!)}$$

for the number of anagrams, by performing the following procedure and using the product rule: choose k_1 positions for the first letter, choose k_2 positions for the second letter, etc...

Note: The notation $\sum_{i \in S} E_i$ means to evaluate E_i for every i in the set S and add them up. A similar definition applies for $\prod_{i \in S} E_i$. This means i goes over all elements of S.

Problem 4: Subsets

Consider a set S such that $|S| \geq 1$, i.e. $S \neq \phi$. Call a subset of S even if is has an even number of elements. Similarly, call a subset of S odd if it has an odd number of elements. For example, if $S = \{1, 2, 3\}$, then

$$E_S = \{\emptyset, \{1,2\}, \{1,3\}, \{2,3\}\}$$

are the even sets, and

$$O_S = \{\{1\}, \{2\}, \{3\}, \{1,2,3\}\}$$

are the odd sets.

Show that the number of even sets and the number of odd sets are always equal when $S \neq \emptyset$. *Hint*: Assume that some element $x \in S$. If $T \in E_S$, consider this function $f : E_S \to O_S$

$$f(T) = R = \begin{cases} T \cup \{x\} & x \notin T \\ T - \{x\} & x \in T \end{cases}$$

and show that f is a bijection (one-to-one correspondence). This means you need to show:

- f is onto
- f is one-to-one

Problem 5: Pascal triangle

(a) Using a property of binomial coefficients we have seen in the Pascal triangle, show the following identity for $1 < k < n - 1$:

$$\binom{n}{k} = \binom{n-2}{k} + 2\binom{n-2}{k-1} + \binom{n-2}{k-2}$$

(b) Using the binomial theorem, show that the alternating sum of terms of the form $\binom{n}{k} 2^k$ starting with $k = n$ is equal to 1. For instance, if $n = 3$

$$\binom{3}{3} 2^3 - \binom{3}{2} 2^2 + \binom{3}{1} 2^1 - \binom{3}{0} 2^0 = 1$$