CSCI 150 Discrete Mathematics
Homework 4
Due 10/4/2018

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Problem 1: Counting with binary patterns

(a) How many bit strings contain exactly seven 0s and nine 1s such that every 0 is immediately followed by a 1?

Amy wrote nine happy stories and seven sad ones. To collect them in a book, she decided that each sad story must be immediately followed by a happy one. Her publisher argues that there is a one to one correspondence between any order and the binary strings of part (a).

(b) Explain why the publisher is wrong.

(c) In how many ways can Amy order the stories?

Problem 2: Integer solutions
Find the number of solutions of the equation $x+y+z+w=15$ in the following cases:

(a) $x, y, z, w \geq 0$.

(b) $x, y, z, w > 0$.

(c) $x > 2$, $y > -2$, $z > 0$, $w > -3$.

Problem 3: Another truncated sum (will not be graded)

Consider the following product

$$(1+x)^n(1-x+x^2-x^3+\ldots)$$

(a) Use the binomial theorem to expand the parenthesis on the left.

(b) Using your answer for (a), apply the distributive law to find the coefficient of x^0, x^1, x^2, and x^3. This should help you express the coefficient of x^k in general using a $\sum$ notation involving binomial coefficients. Do it.

(c) Using the fact that $1 - x + x^2 - x^+ \ldots = (1+x)^{-1}$ (a technical condition here is that $|x| < 1$ but this is not relevant to the problem), figure out which binomial coefficient must be equal to the sum you found in (b).

Problem 4: Balls and bins

For this problem, just provide the answer, but make sure you can explain how you obtained it. Find the number of ways of placing 4 balls in 10 labeled bins, when:

(a) balls are labeled, and no bin can hold more than one ball.

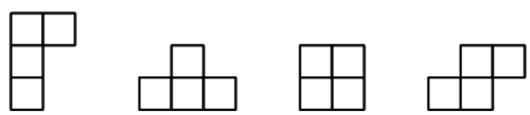
(b) balls are not labeled, and no bin can hold more than one ball.

(c) balls are labeled, and each bin can hold any number of balls.

(d) balls are not labeled, and each bin can hold any number of balls.

Problem 5: Proofs

(a) Consider the following tiles.



Show by contradiction that these tiles cannot be put together to make a square (rotations of tiles are allowed). *Hint*: use ideas from the mouse trap example in the notes.

(b) Show that if the product of two numbers a and b is irrational, then at least one of the two numbers must be irrational. *Hint*: use the contrapositive.

(c) Let P, Q, and R be three statements. Consider the following logical expressions and suppose they are all true:

$$P \vee (Q \Rightarrow R) \qquad Q \vee R \qquad R \Rightarrow P$$

Prove that P must be true by contradiction. (will not be graded)