Problem 1

a. There are two stages for placing a snake:
   1. Placing a head: (n) possible squares
   2. Placing a tail: $(\frac{n}{2})$ possible squares (because of the opposite-tiled rule; we don't need to subtract -1 because this rule already takes care of the outcome of placing a head and tail on the same square (unable because same color))

   So with this, the formula for the number of snakes possible is: $\frac{n(\frac{n}{2})}{2}$, dividing by two because the head and tail belong to the same snake.
   n = 64 on an 8x8 chess board, so therefore the total number of snakes possible is 1024.

b. There are four stages for placing a snake and a ladder:
   1. Placing a snake head: (n) possible squares
   2. Placing a snake tail: $(\frac{n}{2})$ possible squares
   3. Placing a ladder top: $(n-2)$ possible squares
   4. Placing a ladder bottom: $(n-3)$ possible squares

   The snake head/tail and ladder top/bottom are both permutable in the steps, so we must compensate by dividing by for $2^2 = 4$ to eliminate overcounting. The formula is:
   $$\frac{(n)(\frac{n}{2})(n-2)(n-3)}{4}$$
   Evaluating for n = 64, the possible number of combinations is 1,936,384.

c. There are four stages for placing two snakes:
   1. Placing the first snake head: (n) possible squares
   2. Placing the first snake tail: $(\frac{n}{2})$ possible squares
   3. Placing the second snake head: $(n-2)$ possible squares
   4. Placing the second snake tail: $(\frac{n}{2}-1)$ possible squares (one of the opposite color to be guaranteed to be already used)

   Not only are the snake heads/tails permutable with each other, but the snakes as a whole are also permutable; therefore we must divide by $2^3 = 8$ to eliminate overcounting.
   The formula is : $\frac{(n)(\frac{n}{2})(n-2)(\frac{n}{2}-1)}{8}$
   Evaluating for n = 64, we get 492,032 possible combinations.

Problem 2

a. Start with: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

Factor out the leading factors: $k\binom{n}{k} = n(\frac{(n-1)!}{(k-1)!(n-k)!})$

Adjust the second half of the denominator of the right side (the -1 terms are subtracted, and therefore canceled out; no quantity change, but preparing for compression):

$$k\binom{n}{k} = n(\frac{(n-1)!}{(k-1)!((n-1)-(k-1))!})$$

Final identity achieved after converting into binomial coefficient notation:

$$k\binom{n}{k} = n\binom{n-1}{k-1}$$

b. Start with: $\sum_{i=a}^{b}(ix+y)$

Expand: $(ax+y)+((a+1)x+y)+...+((b-1)x+y)+(bx+y)$

Looking at the expanded expression, there are two terms needed to condense it:

- an expression for the value of i (changing from a to b) to multiply x (k)
- an expression representing the number of terms being summated (n)

Rewrite: $(n)(kx+y)$

The number of terms (n) is simply the upper bound (b) subtracted by the lower bound (a), added by 1 (so that both bounds are inclusive): $(b-a+1)$.

The value of i over the course of the summation (k) can be expressed as an average of the upper and lower bounds ($(\frac{a+b}{2})$), only needed to be multiplied by n to reach the summation value.

Substituting the values of n and k with the equivalent expressions, we can obtain the final expression:

$$(b-a+1)(\frac{a+b}{2}x+y)$$

Problem 3

a. Stage 1: Choose one of three courses
   Stage 2: Choose one of six courses
   Stage 3: Choose one of five courses
   3 * 6 * 5 = 90 different combinations of classes

b. There are three combinations of two class times possible:
   1. Morning & Noon (3 * 6 class combinations)
   2. Morning & Afternoon (3 * 5 class combinations)
   3. Noon & Afternoon (6 * 5 class combinations)

   3 * 6 + 3 * 5 + 6 * 5 = 63 different combinations of classes