CSCI 150 Discrete Mathematics
Homework 1
Due 9/11/2018

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Problem 1: Snakes and Ladders
Consider a regular 8×8 chessboard. The chessboard is to be used to play snakes and ladders, but there is an extra condition on how to place snakes on the chessboard: The head and tail of a snake must not occupy squares of the same color. Ladders remain unconstrained. In how many ways can we:

(a) Place one snake.

(b) Place a ladder and a snake.

(c) Place two snakes.

In answering the questions above, show the reasoning using the product rule, and explicitly describe the different stages and in how many ways they can be carried out. In addition, justify any adjustment you make due to possible over-counting.

Problem 2: Mathematical identities
We have seen in class that $\left(\begin{array}{c} n \\ 2 \end{array} \right) = n(n-1)/2$. It is tempting, based on this notation, to conclude that:

$$\left(\begin{array}{c} n \\ k \end{array} \right) = n(n-1)/k \quad \text{or} \quad \left(\begin{array}{c} n \\ k \end{array} \right) = n(n-k/2)/k$$

or make any kind of generalization based on the pattern. Making this kind of hasty generalizations based on patterns is similar to saying that if $5^2 = 25$ and $6^2 = 36$, then 7^2 must be 47. It turns out that

$$\left(\begin{array}{c} n \\ k \end{array} \right) = \frac{n!}{k!(n-k)!}$$

(a) Show using algebraic manipulation that

$$k \left(\begin{array}{c} n \\ k \end{array} \right) = n \left(\begin{array}{c} n-1 \\ k-1 \end{array} \right)$$

Consider the sum

$$\sum_{i=a}^{b}(ix+y)$$

where a, b, x, and y are given.

(b) Show using algebraic manipulations that this sum is equal to

$$(b-a+1)\Big(\frac{a+b}{2}x+y\Big)$$

Problem 3: Schedule
A department offers three courses at 10:00 AM, six courses at 12:00 PM, and five courses at 2:00 PM.

(a) Use the multiplication rule to figure out the number of ways you can register for three courses.

(b) Use the addition rule and the multiplication rule to figure out the number of ways you can register for two courses.