CSCI 150 Discrete Mathematics
Homework 2
Due 9/20/2018

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Problem 0

- Understand the connection between the product rule, $n!$, and permutations
- Understand the meaning of "n choose k", and in particular "n choose 2"
- Make yourself comfortable with summation and product notations, Σ and $\prod$
- Review power series and the definition of logarithm (chapter 0)
- Watch the videos posted on the course website

Problem 1: Round table (will not be graded)
We have seen in class the problem of seating n people on n chairs. This can be done in $n!$ ways because any seating can be represented by some ordering of the people, i.e. a permutation, and there are $n!$ permutations on n objects.

However, it is known that seating n people on a round table with n chairs can be done in $(n-1)!$ ways. Your job is to learn about this by searching the internet and understanding why $n!$ is an overcount (by n) for the round setting. Try to describe this in your own words.

Problem 2: An interesting product
Given a fixed number x where $0 \leq x \leq 1$, consider the following quantity:

$$f(x) = \prod_{i=0}^{\infty}(1-(1-x)x^i)$$

(a) Express this quantity by writing the first 4 terms of the product explicitly, followed by "...". Each of the 4 terms must be simplified as much as possible; for instance, there should be no nested parenthesis.

(b) What are $f(0)$ and $f(1)$?

(c) If $x \approx 1$ (x is approximately 1), then $(1-x)x^i \approx 0$. Consider the natural logarithm of the above quantity, and use the fact that $\ln(1-\epsilon) \approx -\epsilon$ when $\epsilon \approx 0$ to find $\ln f(x)$ and then $f(x)$ when $x \approx 1$. *Hint*: A useful tool here is power series. Also review what the log of a product is. Both of these concepts are in chapter 0.

Problem 3: Unique pairwise sums (will not be graded)
Consider a set of n positive integers $S = \{a_1, a_2, \ldots, a_n\}$. You should know by now that there are $n(n-1)/2$ pairs of integers in this set. However, if we compute $a_i + a_j$ for each of these pairs, we do not necessarily get different results. Here's an example: In $S = \{1, 2, 3, 4, \ldots, n\}$, $(1, 4)$ and $(2, 3)$ are different pairs, but their sums are equal, $1 + 4 = 2 + 3$. It is not hard to construct a set where each pair gives a different sum; for instance, $S = \{10^0, 10^1, 10^2, \ldots, 10^{n-1}\}$, but can you construct a set with unique pairwise sums while keeping the numbers as small as possible? Try...

Problem 4: Apples and Oranges and ...
For each of the following, design the generation procedure by explicitly describing the number of phases, what each phase does, and the number of ways each phase can be carried out. Then answer the following questions:

- does my procedure generate all valid outcomes?
- does my procedure generate any outcome that is not valid?
- does my procedure overcount (and by how much)?

Then use the product rule, with possible adjustment, to determine the final count.

There are 12 children. In how many ways can you give them fruits if:

(a) You have an infinite supply of apples, oranges, and bananas (you're lucky!), and you want to give each child one fruit.

(b) You have 4 of each kind, and you want to give each child one fruit.

Problem 5: A 3-letter "word"

Use any strategy to solve this problem, but explain your reasoning. Consider the following three-letter "word", where the first two letters are hidden:

$$\cdot \quad \cdot \quad Z$$

In how many ways can you specify the two hidden letters if:

(a) Letters cannot repeat.

(b) Letters can repeat.

(c) Letters cannot repeat, and the three letters must appear in alphabetical order.

(d) Letters can repeat, and the three letters must appear in alphabetical order.