Suppose that $Q = f(t)$ is an exponential function of t. If $f(20) = 88.2$ and $f(23) = 91.4$

(a) Find the base.
(b) Find the growth rate.
(c) Evaluate $f(25)$.

SOLUTION
(a) Let $Q = Q_0 a^t$.
Substituting $t = 20$, $Q = 88.2$ and $t = 23$, $Q = 91.4$ gives two equations for Q_0 and a:

$$88.2 = Q_0 a^{20} \text{ and } 91.4 = Q_0 a^{23} \tag{1}$$

Dividing the two equations enables us to eliminate Q_0:

$$\frac{91.4}{88.2} = \frac{Q_0 a^{23}}{Q_0 a^{20}} = a^3 \tag{2}$$

Solving for the base, a, gives

$$a = \left(\frac{91.4}{88.2}\right)^{1/3} = 1.012 \tag{3}$$

(b) Since $a = 1.012$, the growth rate is $1.012 - 1 = 0.012 = 1.195\%$

(c) We want to evaluate $f(25) = Q_0 a^{25} = Q_0(1.012)^{25}$. First we need to find Q_0 from the equation

$$88.2 = Q_0(1.012)^{20} \tag{4}$$

Solving gives

$$Q_0 = \frac{88.2}{(1.012)^{20}} = 69.548 \tag{5}$$

Thus,

$$f(25) = Q_0 a^{25} = 93.598 \tag{6}$$

ANSWER
(a) 1.012
(b) 1.195%
(c) 93.598