

Q1. Suppose that $Q = f(t)$ is an exponential function of t . If $f(19) = 89.2$ and $f(22) = 99.4$:

- (a) Find the base.
- (b) Find the growth rate.
- (c) Evaluate $f(19)$.

Solution

- (a) Let $Q = Q_0 a^t$.

Substituting $t = 19$, $Q = 89.2$ and $t = 22$, $Q = 99.4$ gives two equations for Q_0 and a :

$$89.2 = Q_0 a^{19} \text{ and } 99.4 = Q_0 a^{22}.$$

Dividing the two equations enables us to eliminate Q_0 :

$$\frac{99.4}{89.2} = \frac{Q_0 a^{22}}{Q_0 a^{19}} = a^3.$$

Solving for the base, a , gives

$$a = \left(\frac{99.4}{89.2} \right)^{1/3} = 1.037.$$

- (b) Since $a = 1.037$, the growth rate is $1.037 - 1 = 0.03675 = 3.675\%$.

- (c) We want to evaluate $f(19) = Q_0 a^{19} = Q_0 (1.037)^{19}$. First we need to find Q_0 from the equation

$$89.2 = Q_0 (1.037)^{19}$$

Solving gives $Q_0 = 44.93$. Thus,

$$f(19) = 44.93(1.037)^{19} = 89.2.$$

Answer

- (a) 1.037
- (b) 3.675%
- (c) 89.2