/1a

$$f_1 = x^2 + 9y^2 + z\cdot\sin(x)$$

$$f_2 = z^2 + z\cdot\sin(3y)$$

$$\frac{\partial f_1}{\partial x} = 2x + z\cdot\cos(x)$$

$$\frac{\partial f_1}{\partial y} = 18y$$

$$\frac{\partial f_1}{\partial z} = \sin(x)$$

$$\frac{\partial f_2}{\partial x} = 0$$

$$\frac{\partial f_2}{\partial y} = 3z\cdot\cos(3y)$$

$$\frac{\partial f_2}{\partial z} = 2z + \sin(3y)$$

$$J_f(x,y,z) = \begin{bmatrix} 2x + z\cdot\cos(x) & 18y & \sin(x) \\ 0 & 3z\cdot\cos(3y) & 2z + \sin(3y) \end{bmatrix}$$

b) $f_1 = x^2 + 9y^2 + z\cdot\sin(x)$

$$\frac{\partial f_1}{\partial x} = 2x + z\cdot\cos(x) \quad =: f_x$$

$$\frac{\partial f_1}{\partial y} = 18y \quad =: f_y$$

$$\frac{\partial f_1}{\partial z} = \sin(x) \quad =: f_z$$

$$\left[\begin{array}{lll} \frac{\partial f_x}{\partial x} = 2 - z\cdot\sin(x) & \frac{\partial f_y}{\partial x} = 0 & \frac{\partial f_z}{\partial x} = \cos(x) \\ \frac{\partial f_x}{\partial y} = 0 & \frac{\partial f_y}{\partial y} = 18 & \frac{\partial f_y}{\partial y} = 0 \\ \frac{\partial f_x}{\partial z} = \cos(x) & \frac{\partial f_y}{\partial z} = 0 & \frac{\partial f_z}{\partial z} = 0 \end{array}\right]$$

/2 $h = (x^3 + y^3) \cdot e^{-y}$

$h_0 = (1^3 + 0^3) \cdot e^{-0} = 1$

$\nabla h \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{\partial h}{\partial x} \\ \frac{\partial h}{\partial y} \end{pmatrix} = \begin{pmatrix} 3x^2 \cdot e^{-y} \\ (-x^3 + 3y^2 - y^3) \cdot e^{-y} \end{pmatrix}$

$x^3 \cdot e^{-y} + y^3 \cdot e^{-y}$

$x^3 \cdot e^{-y} \cdot -1 \qquad 3y^2 \cdot e^{-y} + y^3 \cdot e^{-y} \cdot -1$

$= -x^3 \cdot e^{-y} + 3y^2 \cdot e^{-y} - y^3 \cdot e^{-y}$

$z = h(x_0, y_0) + \nabla h \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \cdot \begin{pmatrix} x - x_0 \\ y - y_0 \end{pmatrix}$

$z = 1 + \begin{pmatrix} 3x_0^2 \cdot e^{-y_0} \\ (-x_0^3 + 3y_0^2 - y_0^3) \cdot e^{-y} \end{pmatrix} \cdot \begin{pmatrix} x - 1 \\ y \end{pmatrix}$

$= 1 + (3x_0^2 \cdot e^{-y_0})(x - 1) + ((-x_0^3 + 3y_0^2 - y_0^3) \cdot e^{-y_0}) \cdot y$

$= 1 + 3x - 3 - y = 3x - y - 2$

/3

$$J_g = \begin{bmatrix} \cos(\varphi) \cdot \sin(\theta) & -r \cdot \sin(\varphi) \cdot \sin(\theta) & r \cdot \cos(\varphi) \cdot \cos(\theta) \\ \sin(\varphi) \cdot \sin(\theta) & r \cdot \cos(\varphi) \cdot \sin(\theta) & r \cdot \sin(\varphi) \cdot \cos(\theta) \\ \cos(\theta) & 0 & -r \cdot \sin(\theta) \end{bmatrix}$$

$$J_f = \begin{bmatrix} \frac{\partial f}{\partial x} & r \cdot \cos(\varphi) \cdot \sin(\theta) \\ \frac{\partial f}{\partial y} & r \cdot \sin(\varphi) \cdot \sin(\theta) \\ \frac{\partial f}{\partial z} & r \cdot \cos(\theta) \end{bmatrix}^T$$

$J_F = J_g \cdot J_f$

$$= \begin{bmatrix} \frac{\partial f}{\partial x}(r \cdot \cos(\varphi) \cdot \sin(\theta)) \cdot \cos(\varphi) \cdot \sin(\theta) & \frac{\partial f}{\partial y}(r \cdot \sin(\varphi) \cdot \sin(\theta)) \cdot \cos(\varphi) \cdot \sin(\theta) & \frac{\partial f}{\partial z}(r \cdot \cos(\theta)) \cdot \cos(\varphi) \cdot \sin(\theta) \\ \frac{\partial f}{\partial x}(r \cdot \cos(\varphi) \cdot \sin(\theta)) \cdot \sin(\varphi) \cdot \sin(\theta) & \frac{\partial f}{\partial y}(r \cdot \sin(\varphi) \cdot \sin(\theta)) \cdot \sin(\varphi) \cdot \sin(\theta) & \frac{\partial f}{\partial z}(r \cdot \cos(\theta)) \cdot \sin(\varphi) \cdot \sin(\theta) \\ \frac{\partial f}{\partial x}(r \cdot \cos(\varphi) \cdot \sin(\theta)) \cdot \cos(\theta) & \frac{\partial f}{\partial y}(r \cdot \sin(\varphi) \cdot \sin(\theta)) \cdot \cos(\theta) & \frac{\partial f}{\partial z}(r \cdot \cos(\theta)) \cdot \cos(\theta) \end{bmatrix}$$