Technische Hochschule Rosenheim
Applied Artificial Intelligence - Bachelor
Analysis 2
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SoSe 2022

Friday, 10.06.2022

Homework 12: advanced integrals in $\mathbb{R}^n$

To submit: on **Friday, 17.06.2022**, 9:00 a.m., online by the learning campus

Exercise 1 (4 pts.)

From a full sphere with radius $R > 0$ a cylinder with radius ρ, $0 < \rho < R$, is removed by a drilling machine. The axis of the cylinder passes through the center of the sphere.

Using cylinder coordinates, describe the resulting solid body S as a normal area. Then compute the volume V of S, i.e.

$$V = \iiint_S r\,dr\,d\phi\,dz.$$

Exercise 2 (4 pts.)

Let a domain B be bounded by $y = -x^2 + 4$ and $y = x + 2$. Both graphs intersect at $x_1 = -2$ and $x_2 = 1$.

Plot B.

Compute the centroid S (geometrical barycenter) of B. You may use without a proof that the area of B is $\frac{9}{2}$.

Draw S into the plot.

Exercise 3 (5 optional pts.)

Consider the normal distribution

$$f_{\mu,\sigma}(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right)$$

where $\mu \in \mathbb{R}$, $\sigma > 0$.

Compute the barycenter

$$\int_{-\infty}^{\infty} x\, f_{\mu,\sigma}(x)\,dx = \lim_{R_1,R_2\to\infty} \int_{-R_1}^{R_2} x\, f_{\mu,\sigma}(x)\,dx.$$