/1a

$$\begin{array}{ccc|ccc} & & & 1 & 0 & 0 \\ & & & 0 & 1 & 0 \\ & A= & & 0 & 0 & 1 \\ \hline \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}$$

b) $A' = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 0 & \frac{2}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\dim(\ker(A)) = 3$, $\dim(\operatorname{im}(A)) = 0$

$F' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\dim(\ker(F)) = 3$, $\dim(\operatorname{im}(F)) = 0$

basis of $\operatorname{im}(A) = \left(\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}, \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right)$

basis of $\ker(A)$:

$$x_1 = \sqrt{2} \cdot x_3 - x_2$$
$$x_2 = \sqrt{2} \cdot x_3 - x_1 \quad \to \left(\begin{pmatrix} 0 \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}, \begin{pmatrix} \sqrt{2} \\ \sqrt{2} \\ 0 \end{pmatrix}\right)$$
$$\frac{1}{\sqrt{2}} x_1 + \frac{1}{\sqrt{2}} x_2 = x_3$$

$$\ker(A) = \left\{ \begin{pmatrix} \sqrt{2}\, x_3 - x_2 \\ \sqrt{2}\, x_3 - x_1 \\ \frac{1}{\sqrt{2}} x_1 + \frac{1}{\sqrt{2}} x_2 \end{pmatrix} \middle| \ x_{1/2/3} \in \mathbb{R} \right\}$$

basis of $\ker(F) = F = \left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right)$ = basis of $\operatorname{im}(F)$

kernel $(F) = \{0\}$ -> injective ✓

F is a generating set -> surjective ✓

c) $A=$

$$\left(\begin{array}{ccc|ccc} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 1 & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \cdot\sqrt{2} \\ \cdot\sqrt{2} \\ \ \end{array} \to \begin{pmatrix} 1 & -1 & 0 & \sqrt{2} & 0 & 0 \\ 1 & 1 & 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \ 1:2$$

$$\begin{pmatrix} 1 & -1 & 0 & \sqrt{2} & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} + \to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \sqrt{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 1 & 0 & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$A^{-1} =$

$F == \mathbb{R}^3$?

/2a

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 9 \\ 7 & 8 & 15 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$\ker(F) = \left(\begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}, \ x + y = z\right)$ dim = 1

$\operatorname{Im}(F) = \left(x \cdot \begin{pmatrix} 1 \\ 4 \\ 7 \end{pmatrix} + y \begin{pmatrix} 2 \\ 5 \\ 8 \end{pmatrix}\right)$ dim = 2

basis kernel $= x \begin{pmatrix} 1 \\ 4 \\ 7 \end{pmatrix} + y \begin{pmatrix} 2 \\ 5 \\ 8 \end{pmatrix} - z \begin{pmatrix} 3 \\ 9 \\ 15 \end{pmatrix}$

basis image $= x \cdot \begin{pmatrix} 1 \\ 4 \\ 7 \end{pmatrix} + y \begin{pmatrix} 2 \\ 5 \\ 8 \end{pmatrix}$

b)

$$\begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 4 & 5 & 9 & 0 & 1 & 0 \\ 7 & 8 & 15 & 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -3 & -3 & -4 & 1 & 0 \\ 0 & -6 & -6 & -7 & 0 & 1 \end{pmatrix}$$

not injective?

c)

$$\begin{array}{l|c|c} 1x + 2y + 3z & -1 & 1 \\ 4x + 5y + 9z & -4 & 0 \\ 7x + 8y + 15z & -7 & 0 \end{array}$$

$x = -1$ $\quad -1 + 0 + 0 = -1$

$y = 0$ $\quad -4 + 0 + 0 = -4$

$z = 0$ $\quad -7 + 0 + 0 = -7$

$x =$

$y =$ not on plane?

$z =$

/4a

$B = \left\{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right\}$ linearly independent ✓ spans Π_2 ✓

b) $v = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} \to$

$$\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{array}$$

$z = 2$ $\quad x - 1 + 2 = -3$

$y + 2 = 1$ $\quad x = -4$

$y = -1$

$$2 \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - 1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - 4 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{array}{l} = 2 - 1 - 4 = -3 \\ = 2 - 1 = 1 \\ = 2 \end{array}$$

$v = \to$