

$$1a) \lim_{x \rightarrow \infty} \frac{\ln(x)}{e^x} \rightarrow \frac{\infty}{\infty}$$

L'Hôpital ✓

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$$\lim_{x \rightarrow \infty} \frac{1}{x} \neq \lim_{x \rightarrow \infty} \frac{e^x}{x} \rightarrow \frac{\infty}{\infty}$$

$$\lim_{x \rightarrow \infty} \frac{1}{x e^x} = 0$$

$$\lim_{x \rightarrow \infty} \frac{e^x}{1} = \infty$$

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$$1b) \lim_{x \rightarrow 1} \frac{1 + \cos(x\pi)}{x^2 - 2x + 1} \rightarrow \frac{1 + \cos(\pi)}{1 - 2 + 1} = \frac{0}{0}$$

$$\lim_{x \rightarrow 1} \frac{-\pi \sin(x\pi)}{2x - 2} \rightarrow \frac{-\pi \sin(\pi)}{2 - 2} = \frac{0}{0} \quad \text{L'Hôpital}$$

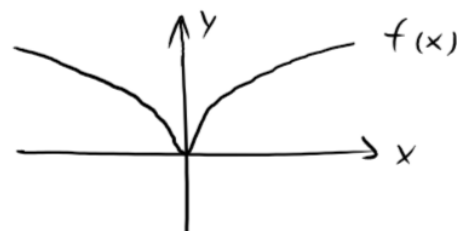
$$\lim_{x \rightarrow 1} \frac{-\pi^2 \cos(x\pi)}{2} = \frac{\pi^2}{2} \approx 4.935 \quad \checkmark$$

$$12 \quad f(x) = \ln(1 + x^2)$$

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$$f'(x) = \frac{x}{1+x^2}$$

Just going by the behaviour of graphs, we can see that $\ln(x)$ has no maxima and increases indefinitely. Using x^2 instead of x , only changes the fact that it works in the negative x -direction as well. By adding 1 we remove the singularity at $x = 0$ of any logarithm and, in this case, get a minima of $y = 0$.



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$$\begin{aligned} 13 \quad x(t) &= A \sin(\omega t + \phi_0) \\ \checkmark \quad x'(t) &= \omega A \cos(\omega t + \phi_0) \\ \checkmark \quad x''(t) &= -\omega^2 A \sin(\omega t + \phi_0) \end{aligned} \quad \left| \quad \begin{aligned} -\omega^2 A \sin(\omega t + \phi_0) + \omega^2 \cdot x(t) &= 0 \\ -A \sin(\omega t + \phi_0) + x(t) &= 0 \\ x(t) &= A \sin(\omega t + \phi_0) \quad \checkmark \end{aligned} \right.$$

$$x(0) = A \sin(\omega \cdot 0 + \phi_0) = A \sin(\phi_0) \quad \checkmark$$

$$x\left(\frac{2\pi}{\omega}\right) = A \sin\left(\omega \cdot \frac{2\pi}{\omega} + \phi_0\right) = A \sin(2\pi + \phi_0)$$

$$\text{since } \sin(k \cdot 2\pi + x) \quad k \in \mathbb{N} = \sin(x) \quad \checkmark$$

$$\text{we get } A \sin(0 + \phi_0) \quad \checkmark$$

1/4 assume $U(p) = p N(p) - k(x)$ $U(p)$ is already defined!

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$$U(p) = p(100 - 0.1p - 0.2p^2) - (100 + x) \quad x = N(p) \quad \checkmark$$

$$U(p) = -0.2p^3 - 0.1p^2 + 100p - 100 - (100 - 0.1p - 0.2p^2)$$

$$U(p) = -0.2p^3 - 0.1p^2 + 100p - 100 - 100 + 0.1p + 0.2p^2$$

$$U(p) = -0.2p^3 + 0.1p^2 + 100.1p - 200 \quad \checkmark$$

$$U'(p) = -0.6p^2 + 0.2p + 100.1$$

$$U''(p) = -1.2p + 0.2$$

$$U'(p) = 0$$

$$0 = -0.6p^2 + 0.2p + 100.1 \quad \checkmark$$

$$p_{1/2} = \frac{-0.2 \pm \sqrt{0.2^2 - 4 \cdot (-0.6) \cdot 100.1}}{-1.2}$$

~~$p_1 = -12.75$~~ not relevant

$p_2 = 13.08 \quad \checkmark$

Since p_1 is negative, which doesn't make sense for a profit-oriented price, p_2 is the max.