Technische Hochschule Rosenheim
Applied Artificial Intelligence - Bachelor
Analysis 2
Priv.-Doz. Dr. S.-J. Kimmerle

SoSe 2022

Friday, 06.05.2022

Homework 7: derivatives w.r.t. to several variables

To submit: on **Friday, 13.05.2022**, 9:00 a.m., online by the learning campus

Exercise 1 (6 pts.)
Consider the function

$$f : \mathbb{R}^3 \to \mathbb{R}^2,\ (x,y,z) \mapsto \begin{pmatrix} x^2 + 9y^2 + z\sin(x) \\ z^2 + z\sin(3y) \end{pmatrix}.$$

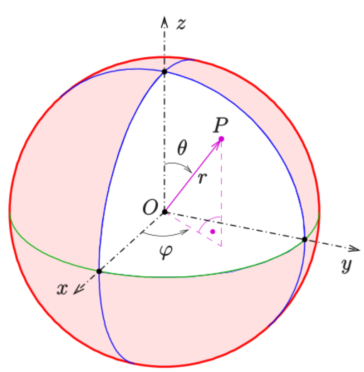
a) [3 pts.] Compute the Jacobian matrix J_f.

b) [3 pts.] Compute the Hessian matrix H_{f_1} of the first component of f, i.e. $f_1(x,y,z) = x^2 + 9y^2 + z\sin(x)$.

Exercise 2 (3 pts.)
Compute a tangent plan to the function $h : \mathbb{R}^2 \to \mathbb{R}$, $(x,y) \mapsto (x^3 + y^3)\exp(-y)$ at the point $(1,0)$.

Exercise 3 (6 pts.)
The spherical coordinates are given by $g : [0,\infty) \times (-\pi,\pi] \times [0,\pi) \to \mathbb{R}^3$,

$$(r,\varphi,\theta) \mapsto \begin{pmatrix} r\cos(\varphi)\sin(\theta) \\ r\sin(\varphi)\sin(\theta) \\ r\cos(\theta) \end{pmatrix} =: \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$



Spherical coordinates (r,φ,θ) of a point P and Cartesian coordinate system with axes x, y, and z. (Source: Wikipedia)

We consider a function $f : \mathbb{R}^3 \to \mathbb{R}, (x, y, z) \mapsto f(x, y, z)$ and transform it into a function

$$F : [0, \infty) \times (-\pi, \pi] \times [0, \pi) \to \mathbb{R}^3, F(r, \varphi, \theta) := f(r\cos(\varphi)\sin(\theta), r\sin(\varphi)\sin(\theta), r\cos(\theta))$$

depending on spherical coordinates.

Suppose the derivative of f exists, how does it transform (see lecture) into the derivative of F? To do that compute the Jacobians J_g, J_f, and J_F.