/1a $f(x_1, x_2) = x_1 \sin(x_2)$

$$f\left(1, \frac{\pi}{2}\right) = 1$$

$$\frac{\partial f}{\partial x_1} = 1 \cdot \sin(x_2) \quad =: f_{x_1}$$

$$\frac{\partial f}{\partial x_2} = x_1 \cdot 1 \cdot \cos(x_2) =: f_{x_2}$$

$$t_1(x,y) = 1 + \sin\left(\frac{\pi}{2}\right)(x-1) + 1 \cdot \cos\left(\frac{\pi}{2}\right)\left(y - \frac{\pi}{2}\right)$$

b) $\frac{\partial f_{x_1}}{\partial x_1} = 0 \qquad \frac{\partial f_{x_2}}{\partial x_2} = -x_1 \sin(x_2)$

$\frac{\partial f_{x_1}}{\partial x_2} = 1 \cdot \cos(x_2) \qquad \frac{\partial f_{x_2}}{\partial x_1} = \cos(x_2)$

$$t_2(x,y) = 1 + \sin\left(\frac{\pi}{2}\right)(x-1) + \cos\left(\frac{\pi}{2}\right)\left(y - \frac{\pi}{2}\right) + \frac{0}{2}(x-1)^2$$
$$+ \cos\left(\frac{\pi}{2}\right)(x-1)\left(y - \frac{\pi}{2}\right) - \frac{1 \cdot \sin\left(\frac{\pi}{2}\right)}{2}\left(y - \frac{\pi}{2}\right)^2$$

/2a

$$g(x,y) = 100(y-x^2)^2 + (1-x)^2 = 100(y^2 - 2yx^2 + x^4) + (1 - 2x + x^2)$$
$$= 100x^4 + x^2 + 100y^2 - 200yx^2 - 2x + 1$$

$$g(-1,2) = 100(2-(-1)^2)^2 + (1+1)^2 = 104$$

$$\frac{\partial g}{\partial x} = 400x^3 + 2x - 400yx - 2 \quad =: g_x$$

$$\frac{\partial g}{\partial y} = 200y - 200x^2 =: g_y$$

$$t_1(x,y) = 104 + (400(-1)^3 - 2 + 800 - 2)(x+1) + (400 - 200)(y-2)$$
$$= 104 + 396(x+1) + 200(y-2)$$

b)

$$\frac{\partial g_x}{\partial x} = 1200x^2 + 2 - 400y$$

$$\frac{\partial g_x}{\partial y} = -400x$$

$$\frac{\partial g_y}{\partial x} = -400x$$

$$\frac{\partial g_y}{\partial y} = 200$$

$$t_2(x,y) = t_1(x,y) + \frac{1200 \cdot 1 + 2 - 800}{2}(x+1)^2 + 400(x+1)(y-2)$$
$$+ 200(y-2)^2$$

c) $g_x = 0$

$g_y = 0 \quad \rightarrow \quad y = x^2$

$400x^3 + 2x - 400x^2x - 2 = 0$

$0 = 2x - 2$

$x = 1$

$y = 1^2$

$y = 1$

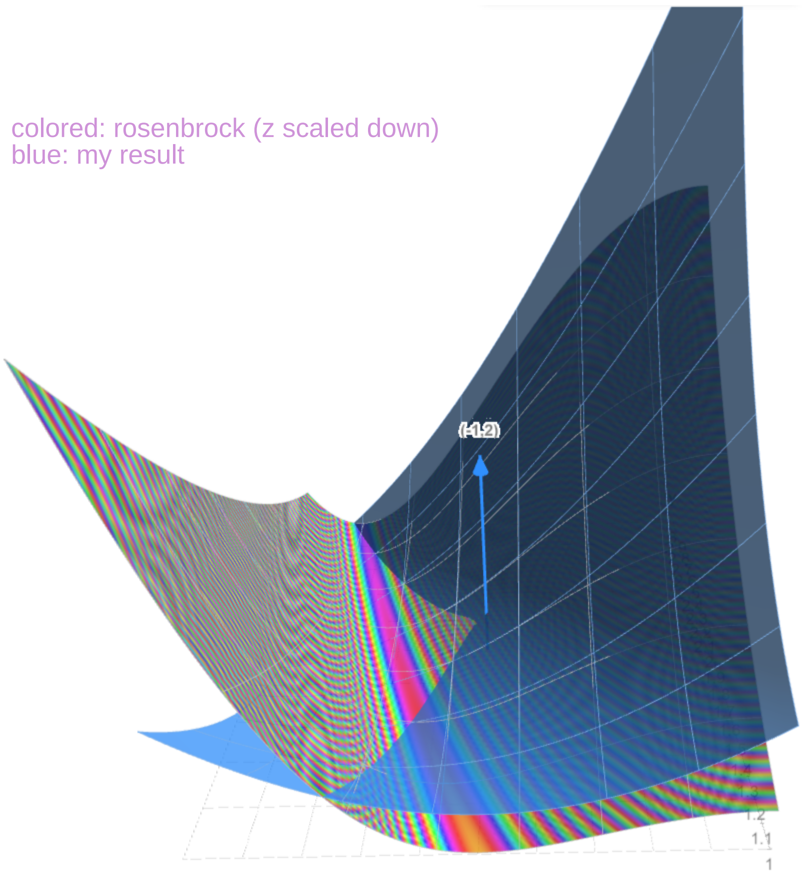
$P(1,1)$

d)

$$D = \begin{vmatrix} 1200 + 2 \cdot 400 & -400 \\ -400 & 200 \end{vmatrix} = 160400 - 160000 = 400$$

$$D > 0 \text{ and } \frac{\partial^2 g}{\partial x^2} > 0 \rightarrow \text{minimum at } (1,1)$$

$$g(1,1) = 0$$



/4

$$\frac{\partial \arctan\left(\frac{x}{y}\right)}{\partial x} = \frac{1}{\frac{x^2}{y^2} + 1} \cdot \frac{\partial \frac{x}{y}}{\partial x} = \frac{1}{\frac{x^2}{y^2} + 1} \cdot 1 \cdot \frac{1}{y}$$

$$= \frac{1}{y\left(\frac{x^2}{y^2} + 1\right)}$$

$$\left(\frac{1}{y}\right)' = \frac{0 \cdot y - 1 \cdot 1}{y^2}$$

$$\frac{\partial \arctan\left(\frac{x}{y}\right)}{\partial y} = \frac{1 \cdot x}{\frac{x^2}{y^2} + 1} \cdot \frac{\partial \frac{x}{y} \rightarrow 1}{\partial y} = \frac{-1 \cdot x}{x^2 + y^2}$$

$$S = \frac{0.1}{4\left(\frac{9}{16}+1\right)} + \frac{0.2\cdot 3}{9+16} = 0.04$$

$$\frac{0.04}{\arctan\left(\frac{4}{3}\right)} = 4.3\%$$

$$\arctan\left(\frac{3\pm 0.1}{4\pm 0.2}\right) \in \left[50{,}8^\circ ;\ 55{,}4^\circ\right]$$

13a

$n=2$

$$\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix} \qquad f_1(\bar{x}) = (x, y)\begin{pmatrix} a \\ b \end{pmatrix} = ax + by \qquad \nabla f_n(\bar{x}) = \begin{pmatrix} \frac{\partial f_1}{\partial a} \\ \frac{\partial f_1}{\partial b} \\ \vdots \\ \frac{\partial f_1}{\partial n} \end{pmatrix}$$

$$\vec{a} = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$H_n(\bar{x}) = \begin{bmatrix} \frac{\partial^2 f_1}{\partial^2 a} & \frac{\partial^2 f_1}{\partial a \partial b} & \cdots & \frac{\partial^2 f_1}{\partial a \partial n} \\ \frac{\partial^2 f_1}{\partial b \partial a} & \frac{\partial^2 f_1}{\partial^2 b} & \cdots & \frac{\partial^2 f_1}{\partial b \partial n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f_1}{\partial n \partial a} & \frac{\partial^2 f_1}{\partial n \partial b} & \cdots & \frac{\partial^2 f_1}{\partial^2 n} \end{bmatrix}$$

b)

$$\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix} \qquad f_2(\bar{x}) = (x, y)\cdot\begin{bmatrix} a & b \\ c & d \end{bmatrix}\cdot\begin{pmatrix} x \\ y \end{pmatrix} = (ax + cy,\ bx + dy)\cdot\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\vec{x}^T = (x, y) \qquad = \left[ax^2 + cyx + bxy + dy^2\right]$$

$$\nabla f_n(\bar{x}) = \begin{pmatrix} \frac{\partial f_2}{\partial x} \\ \frac{\partial f_2}{\partial y} \\ \vdots \\ \frac{\partial f_2}{\partial n} \end{pmatrix}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$H_n(\bar{x}) = \begin{bmatrix} \frac{\partial^2 f_2}{\partial^2 x} & \frac{\partial^2 f_2}{\partial x \partial y} & \cdots & \frac{\partial^2 f_2}{\partial x \partial n} \\ \frac{\partial^2 f_2}{\partial y \partial x} & \frac{\partial^2 f_2}{\partial^2 y} & \cdots & \frac{\partial^2 f_2}{\partial y \partial n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f_2}{\partial n \partial x} & \frac{\partial^2 f_2}{\partial n \partial y} & \cdots & \frac{\partial^2 f_2}{\partial^2 n} \end{bmatrix}$$