/1 $u(x,y) = e^{2x} - 2x + y^2$

$$\frac{\partial u}{\partial x} = 2e^{2x} - 2 \qquad \nabla u(x,y) = \begin{pmatrix} 2e^{2x}-2 \\ 2y \end{pmatrix}$$

$$\frac{\partial u}{\partial x} = 2y$$

$$H_u(x,y) = \begin{array}{c|cc} & x & y \\ \hline x & 4e^{2x} & 0 \\ y & 0 & 2 \end{array}$$

$$\frac{\partial\, 2e^{2x}-2}{\partial x} = 4e^{2x} \qquad \frac{\partial\, 2y}{\partial x} = 0$$

$$\frac{\partial\, 2e^{2x}-2}{\partial y} = 0 \qquad \frac{\partial\, 2y}{\partial y} = 2$$

b) $f(x,y) = \ln(\sqrt{x} + \sqrt{y})$

ln defined for either $x > 0$ or $y > 0$ or both > 0

$$\frac{\partial f}{\partial x} = \frac{1}{\sqrt{x}+\sqrt{y}} \cdot \frac{\partial}{\partial x}(\sqrt{x}+\sqrt{y}) \rightarrow x^{\frac{1}{2}} + y^{\frac{1}{2}}$$

$$= \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\sqrt{x}+\sqrt{y}} = \frac{1}{2\sqrt{x}(\sqrt{x}+\sqrt{y})}$$

$$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{y}(\sqrt{x}+\sqrt{y})}$$

$$\frac{x}{2\sqrt{x}(\sqrt{x}+\sqrt{y})} + \frac{y}{2\sqrt{y}(\sqrt{x}+\sqrt{y})} \stackrel{!}{=} \frac{1}{2}$$

$$\frac{x}{2\sqrt{x}(\sqrt{x}+\sqrt{y})}\cdot\frac{2\sqrt{y}(\sqrt{x}+\sqrt{y})}{2\sqrt{y}(\sqrt{x}+\sqrt{y})}+\frac{y}{2\sqrt{y}(\sqrt{x}+\sqrt{y})}\cdot\frac{2\sqrt{x}(\sqrt{x}+\sqrt{y})}{2\sqrt{x}(\sqrt{x}+\sqrt{y})}=\frac{1}{2}$$

$$\frac{x(2y+2\sqrt{x}\sqrt{y})}{(2x+2\sqrt{x}\sqrt{y})(2y+2\sqrt{x}\sqrt{y})}+\frac{y(2x+2\sqrt{x}\sqrt{y})}{(2y+2\sqrt{x}\sqrt{y})(2x+2\sqrt{x}\sqrt{y})}=\frac{1}{2}$$

$$\frac{(2xy+2\sqrt{x}\sqrt{y}\,x)+(2xy+2\sqrt{x}\sqrt{y}\,y)}{(2x+2\sqrt{x}\sqrt{y})(2y+2\sqrt{x}\sqrt{y})}=\frac{1}{2}$$

$$\frac{4xy+2\sqrt{x}\sqrt{y}(x+y)}{4xy+4\sqrt{x}\sqrt{y}\,x+4\sqrt{x}\sqrt{y}\,y+4xy}=\frac{1}{2}\quad\Big|\cdot 2$$

$$\frac{8\cancel{x}y+4\cancel{\sqrt{x}\sqrt{y}}(\cancel{x}+\cancel{y})}{4\cancel{x}y+4\cancel{\sqrt{x}\sqrt{y}}\,x+4\cancel{\sqrt{x}\sqrt{y}}\,y+4\cancel{x}y}=1 \quad ✓$$

/2a it's a halved globe around the origin above/on the x1-x2 plane with radius 1.

b) $f_1(x,0)=\sqrt{1-x^2}$

$f_2(x,\frac{1}{2})=\sqrt{1-x^2-\frac{1}{4}}$

c) $\sqrt{1-x^2-y^2}=0 \quad |^2$

$1-x^2-y^2=0^2$

$y^2=1-x^2$

$y_1=\sqrt{1-x^2}$

$y_3=\sqrt{0.36-x^2}$

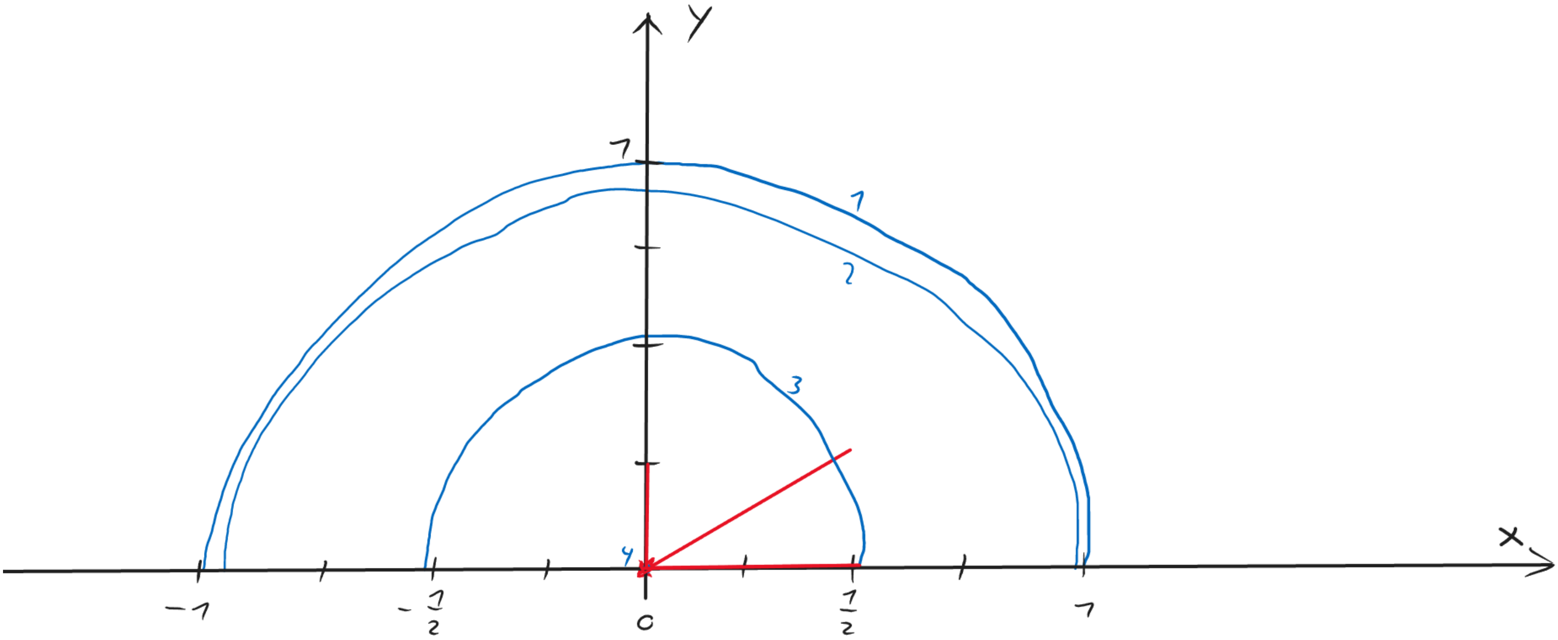
$\sqrt{1-x^2-y^2}=\frac{2}{5}$

$1-x^2-y^2=\left(\frac{2}{5}\right)^2$

$y^2+0.16=1-x^2$

$y_2=\sqrt{0.84-x^2}$

$y_4=\sqrt{0-x^2}$



d) $f(x,y) = \sqrt{1 - x^2 - y^2}$

$$= (1 - x^2 - y^2)^{\frac{1}{2}}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2}(1 - x^2 - y^2)^{-\frac{1}{2}} \cdot \frac{\partial\, 1 - x^2 - y^2}{\partial x} = \frac{-2x}{2\sqrt{1 - x^2 - y^2}}$$

$$\frac{\partial f}{\partial y} = \frac{-y}{\sqrt{1 - x^2 - y^2}} \qquad \nabla f(x,y) = \begin{pmatrix} \frac{-2x}{2\sqrt{1 - x^2 - y^2}} \\ \frac{-y}{\sqrt{1 - x^2 - y^2}} \end{pmatrix}$$

$$\nabla f(0,0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \nabla f(0.5, 0.5) = \begin{pmatrix} -0.577 \\ -0.577 \end{pmatrix}$$

$$\nabla f(\tfrac{1}{2}, 0) = \begin{pmatrix} -0.577 \\ 0 \end{pmatrix}$$

$$\nabla f(0, \tfrac{1}{4}) = \begin{pmatrix} 0 \\ -0.267 \end{pmatrix}$$

13a $f(0,0)$ is continuous if $\lim\limits_{\substack{x \to 0 \\ y \to 0}} f = 0$

but $\lim\limits_{\substack{x \to 0 \\ y \to 0}} f = 1$ Let $x = y$

$\to \frac{2xx}{x^2 + x^2} = \frac{2x^2}{2x^2} = 1$

b) $\frac{2xy}{x^2+y^2}$

$x = 0$: $\frac{0}{y^2}$ $\lim f(0, y) = 0$

$f(x, y) = 0$ at $(0,0)$

$y = 0$: $\frac{0}{x^2}$ $\lim f(x, 0) = 0$

$\to$ continuous