Technische Hochschule Rosenheim | SoSe 2022
Applied Artificial Intelligence - Bachelor
Analysis 2

Priv.-Doz. Dr. S.-J. Kimmerle | Monday, 11.04.2022

# Homework 4: Taylor & Fourier series

To submit: on **Friday, 22.04.2022**, 9:00 a.m., online by the learning campus

**Exercise 1 (12 pts.)**

a) Compute the Maclaurin series of $f(x) = \ln(1+x^2)$ up to order 5. [7 pts.]

b) Expand $g(x) = \frac{1}{\sqrt{1-x^3}}$ into a Maclaurin series up to order 3.
Compute an approximation of $g(x = 0.2)$ and give an estimate of the error. [5 pts.]

**Exercise 2 (5 pts.)**

Approximate the sine function near its maximum in the interval $[0, 2\pi)$ by a parabola. (Hint: Compute the Taylor series at $\frac{\pi}{2}$ up to second order.)

**Exercise 3 (12 pts.)**

We consider the $4\pi$-periodic function

$$u(x) = \begin{cases} \frac{x+2\pi}{\pi}, & \text{for} \quad -2\pi \leq x \leq -\pi, \\ 1, & \text{for} \quad -\pi \leq x \leq \pi, \\ -\frac{x-2\pi}{\pi}, & \text{for} \quad \pi \leq x \leq 2\pi. \end{cases}$$

a) Plot the function $u$ for at least 2 periods and draw in the period in your sketch. What is the angular frequency $\omega$? [3 pts.]

b) Discuss the symmetry of the function. What are the implications for the Fourier coefficients? [2 pts.]

c) Compute the Fourier series $U(x)$ of the function $u(x)$. Is $u$ represented by its Fourier series $U$? [7 pts.]

**Exercise 4 (4 pts.)**

Proof for $m, n \in \mathbb{N}$:

$$\begin{aligned} \int_0^T \sin(m\omega x)\cos(n\omega x)dx &= 0, \\ \int_0^T \sin(m\omega x)\sin(n\omega x)dx &= \begin{cases} 0, & \text{for } m \neq n, \\ \frac{T}{2}, & \text{for } m = n. \end{cases} \end{aligned}$$

**Exercise 5 (7 pts.)**

Let $T > 0$. Compute the complex Fourier series of

$$f(x) = \begin{cases} 1 - \frac{2}{T}x, & \text{for} \quad 0 \leq x \leq T/2, \\ 0, & \text{for} \quad T/2 \leq x < T. \end{cases}$$

Plot at first the function $f(x)$, as next compute the Fourier coefficients, and then rewrite the complex Fourier series as a real Fourier series.