Technische Hochschule Rosenheim
Fakultät ANG
Prof. Dr. André Herzwurm

SoSe 2022
29.03.2022

Exercise Sheet 2

Linear Algebra (AAI)

Exercise 2.1 (H)

Determine whether U is a subspace of $\mathbb{R}^3$:

a) $U = \{(x, y, z) \in \mathbb{R}^3 \colon z = 3x - y\}$,
b) $U = \{(x, y, z) \in \mathbb{R}^3 \colon x \cdot y \cdot z = 0\}$,
c) $U = \{(x, y, z) \in \mathbb{R}^3 \colon x \cdot (\exp(y) + z) = 0\}$,
d) $U = \{(x, y, z) \in \mathbb{R}^3 \colon x, y, z \geq 0\}$,
e) $U = \{(\lambda, 2\lambda, 4\lambda) \in \mathbb{R}^3 \colon \lambda \in \mathbb{R}\}$.

Exercise 2.2 (H)

Consider $v_1, v_2, v_3, v_4, v_5 \in \mathbb{R}^4$ given by

$$v_1 = (1, 2, 1, 2),\ v_2 = (1, 1, 1, 1),\ v_3 = (0, 1, 1, 0),\ v_4 = (0, 1, 0, 1),\ v_5 = (1, 0, 0, 1).$$

a) Express each vector v_i as a linear combination of the remaining vectors v_j with $j \neq i$ (if possible).
b) Prove or disprove:
   i) $\operatorname{span}(\{v_2, v_3, v_5\}) = \operatorname{span}(\{v_3, v_5\})$,
   ii) $\operatorname{span}(\{v_2, v_3, v_5\}) = \operatorname{span}(\{v_1, v_3, v_5\})$,
   iii) $\operatorname{span}(\{v_2, v_5\}) = \operatorname{span}(\{v_3, v_5\})$,
   iv) $\operatorname{span}(\{v_1, v_5\}) = \operatorname{span}(\{v_1, v_2, v_3, v_4, v_5\})$,
   v) $\operatorname{span}(\{v_1, v_2, v_4\}) = \operatorname{span}(\{v_2, v_3, v_5\})$.
c) Determine all linearly independent families $(v_i)_{i \in I}$ with $I \subseteq \{1, \ldots, 5\}$ and $|I| \leq 3$.

Exercise 2.3 (H)

Let $U \subseteq V$ be a subspace of V and let $x, y \in V$. Show that

$$y \in \operatorname{span}(U \cup \{x\})\ \wedge\ y \notin U\ \Rightarrow\ x \in \operatorname{span}(U \cup \{y\}).$$

Exercise 2.4 (H)

a) Let $v_1, v_2, v_3 \in \mathbb{R}^{[0,\infty)}$ be given by

$$v_1(x) = 1, \quad v_2(x) = \sqrt{x}, \quad v_3(x) = \sin(x)$$

for $x \in [0, \infty)$. Show that (v_1, v_2, v_3) is linearly independent.

b) Let V be a K-vector space and let (v_1, v_2) be linearly independent for $v_1, v_2 \in V$. Show that $(v_1 - v_2, v_1 + v_2)$ is linearly independent.