Theoretical Computer Science – Exercise 4

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Please prepare the following exercises at home prior to the tutorial:

Exercise 1

Draw the transition diagram of a deterministic PDA with $\Sigma = \{a, b, c\}$, $\Gamma = \{\#, A\}$ that recognizes the language $L = \{ a^n b^{2n} c^m \mid n,m \in \mathbb{N} \}$ and accepts using end states (as opposed to acceptance by empty stack).

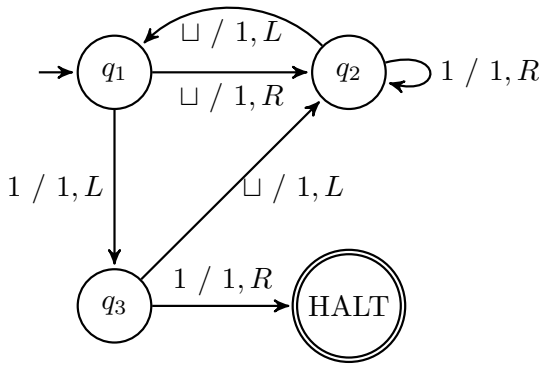
Exercise 2

Draw the transition diagram of a nondeterministic PDA with $\Sigma = \{a, b, c\}$, $\Gamma = \{\#, X\}$ that recognizes the language $L = \{ a^i b^j c^k \mid i = j \text{ or } j = k; i, j, k \in \mathbb{N}_0 \}$ and accepts using end states. Hints:

1. Distinguish between the three cases i = j = 0, i = j > 0, and j = k
2. and connect these parts with an initial state using ε-moves

Exercise 3

Consider the following Turing Machine with input Alphabet $\Sigma = \{1\}$, and tape alphabet $\Gamma = \{\sqcup, 1\}$:



a) Write down the instruction tables of this TM.
b) Let the read/write head be initially positioned anywhere on a tape filled with blank (⊔). What does this TM do?

We will do the following exercises together during the tutorial:

Exercise 4

Draw the transition diagram of a PDA with $\Sigma = \{[,], x\}$, $\Gamma = \Sigma \cup \{\#\}$ that recognizes strings built of brackets as follows:

1. The total number of opening and closing brackets must be the same, for each closing bracket there must be an opening one beforehand.
2. An accepted word may begin and end with any number of x (even zero), but it must contain at least one x.
3. A closing bracket must never immediately follow an opening bracket (i.e., not [ ]).
4. An opening bracket may be followed by any number of x (even zero), but rule 3 must be observed.

Start by writing down some examples for valid words.

Exercise 5

Draw the transition diagram of a Turing Machine with input Alphabet $\Sigma = \{0, 1\}$, and tape alphabet $\Gamma = \{␣, 0, 1\}$ that extends a contiguous bit sequence on an otherwise empty tape (= filled with blanks) with a parity bit to even parity:

- Initially, the read/write head is positioned anywhere to the right of the bit sequence.
- The parity bit is attached on the left of the sequence: A zero, if the total number of ones in the original sequence is even; a one if the total number of ones in the sequence is odd.

Examples: ␣␣␣␣10101␣␣␣␣ becomes ␣␣␣110101␣␣␣␣
␣␣␣␣1001␣␣␣␣ becomes ␣␣␣01001␣␣␣␣