$F(v_1) = v_1 = \lambda v_1 + 0 v_2$ $\quad T$

$F(v_2) = -v_2 = 0 v_1 - \lambda v_2$

$\begin{matrix} \lambda & 0 \\ 0 & -\lambda \end{matrix} \to \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

9.1

$$A_1 = \begin{pmatrix} 2 & 1 & 1 \\ -2 & 0 & 0 \\ -2 & -1 & 0 \end{pmatrix}, \quad b_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ -2 & 0 & 0 & 1 \\ -2 & -1 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right) \qquad \mathcal{L}(A_1, b_1) = \left\{\begin{pmatrix} -\frac{1}{2} \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}\right\}$$

$$\begin{array}{c} A_2 \quad\quad b_2 \\ \left(\begin{array}{cccc|c} 3 & 0 & -2 & 0 & 1 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & -2 & 0 \end{array}\right) \end{array} \to \left(\begin{array}{cccc|c} 3 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & -2 & 0 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

$$\mathcal{L}(A_2, b_2) = \left\{\begin{pmatrix} 0 \\ 0 \\ -\frac{1}{2} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -3 \\ \frac{3}{2} \\ 0 \end{pmatrix}\right\}$$

$0 = x_1 + \frac{1}{3} x_2$

$-1 = x_2 + 2x_3$

$0 = x_4$

$\underline{x_1} = \lambda \to 0 = \lambda + \frac{1}{3} x_2$

$\underline{x_2} = -3\lambda$

$-1 = -3\lambda + 2x_3 \to x_3 = \frac{3\lambda - 1}{2} = \frac{3}{2}\lambda - \frac{1}{2}$

$0 = \underline{x_4}$

$$\mathcal{B} = \left\{ \underset{e_1}{\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}}, \underset{e_1}{\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}} \right\} \qquad v = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$F\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \underbrace{\left\langle \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \right\rangle}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} + \underbrace{\left\langle \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \right\rangle}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \sim \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

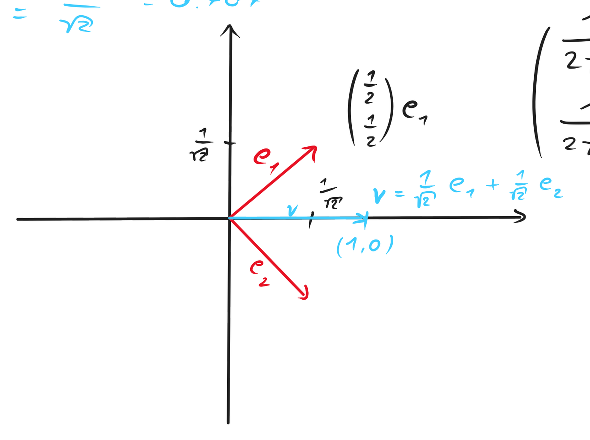
$$= \underset{\lambda_1}{\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}} + \underset{\lambda_2}{\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \checkmark$$

$$\begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} e_1 + \begin{pmatrix} 0.5 \\ -0.5 \end{pmatrix} e_2$$

$$\begin{pmatrix} \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \end{pmatrix} + \begin{pmatrix} \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \end{pmatrix} = \vec{V}$$

$\sqrt{0.5} = \frac{1}{\sqrt{2}} = 0.707$

$=$



$\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} e_1$

$$F\begin{pmatrix} 2 \\ 0 \end{pmatrix} = \left\langle \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \right\rangle \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} + \left\langle \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \right\rangle \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$= \frac{2}{\sqrt{2}} \cdot \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} + \frac{2}{\sqrt{2}} \cdot \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2}{2} \\ \frac{2}{2} \end{pmatrix} + \begin{pmatrix} \frac{2}{2} \\ -\frac{2}{2} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$\left\| \begin{matrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{matrix} \right\| = 1$$

$$\mathcal{B} = \left\{ \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$F\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} \right\rangle \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} + \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} \right\rangle \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} + \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \lambda_1 \frac{1}{\sqrt{2}} + \lambda_2 \frac{1}{\sqrt{2}} + \lambda_3 0$$

$$F\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

a)

$$\begin{array}{c|ccc} & 1 & 0 & 0 \\ & 0 & 1 & 0 \\ & 0 & 0 & 1 \\ \hline A = & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ & 0 & 0 & 1 \end{array}$$

b)

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{2}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\dim(A) = 3$

$\dim(\text{im}(F)) = 3 \qquad B = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

$\dim(\ker(F)) = 0 \qquad B = \{0\}$

c) since rank is full → bijective

$$\left(\begin{array}{ccc|ccc} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 1 & 0 & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)^{1} \to {}^{2}\left(\begin{array}{ccc|ccc} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 1 & 0 & 0 \\ 0 & \frac{2}{\sqrt{2}} & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|ccc} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)^{3} \to {}^{4}\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & \sqrt{2} & 0 & 0 \\ 0 & \frac{\sqrt{2}}{\sqrt{2}} & 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 1 & 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)^{5} \qquad \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = 0.5\sqrt{2}$$

F^{-1}

6.2

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 9 \\ 7 & 8 & 15 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 \\ 0 & -3 & -3 \\ 0 & -6 & -6 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$\dim(\text{im}(A)) = 2$

$\dim(A) = 2$

$B = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}$

$x = -z$

$y = -z \quad \to \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$

$\dim(\ker(A)) = 1$

$B_k = \left\{ \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \right\}$

b) since A is not full rank → not bijective

$$\mathcal{L}(A, b_1): \left(\begin{array}{ccc|c} 1 & 2 & 3 & -1 \\ 4 & 5 & 9 & -4 \\ 7 & 8 & 15 & -7 \end{array}\right) \to \begin{array}{c} x \;\; y \;\; z \quad b \\ \left(\begin{array}{ccc|c} 1 & 2 & 3 & -1 \\ 0 & -3 & -3 & 0 \\ 0 & -6 & -6 & 0 \end{array}\right) \end{array}$$

$-1 = x + 2y + 3z$

$-x = 1 + 2y + 3z$

$x = -1 - 2y - 3z$

$x = -1 - 2 \cdot (-z) - 3z$

$x = -1 + 2z - 3z$

$x = -1 - z$

$$\mathcal{L}(A, b_1) = \left\{ \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}, z\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \right\}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 4 & 5 & 9 & 0 \\ 7 & 8 & 15 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -3 & -3 & -4 \\ 0 & -6 & -6 & -7 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & \frac{4}{3} \\ 0 & 1 & 1 & \frac{7}{6} \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & 4/3 \\ 0 & 0 & 0 & -\frac{1}{6} \end{array}\right)$$

no

7.1a)

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \mathcal{M}^{\varepsilon}_{\varepsilon}(F) \,\Big|\, \mathcal{M}^{\varepsilon}_{\varepsilon}(F^{-1}) \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

b) $\mathcal{M}^{A}_{\varepsilon}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \frac{a_2 - a_3}{2} \qquad \mathcal{M}^{A}_{\varepsilon}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \frac{a_2 + a_3}{2} \qquad \mathcal{M}^{A}_{\varepsilon}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = -a_1$

$\mathcal{M}^{A}_{\varepsilon}\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = -e_3 \qquad \mathcal{M}^{A}_{\varepsilon}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = e_1 + e_2 \qquad \mathcal{M}^{A}_{\varepsilon}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} = e_2 - e_1$

$$\mathcal{M}^{A}_{\varepsilon}(F) = \begin{pmatrix} 0 & 1 & -1 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$

$\mathcal{M}^{A}_{\varepsilon}(F)$