

Homework 1: revision differential calculus

To submit: on **Friday, 25.03.2022**, 9:00 a.m., online by the learning campus

Exercise 1 (4 pt.)

Determine

a)

$$\lim_{x \rightarrow \infty} \frac{\ln(x)}{e^x}$$

b)

$$\lim_{x \rightarrow 1} \frac{1 + \cos(\pi x)}{x^2 - 2x + 1}$$

Exercise 2 (3 pts.)

Consider $f(x) = \ln(1 + x^2)$. Determine all minima and maxima, without using a second derivative.

Exercise 3 (4 pts.)

Let $A, \omega \in \mathbb{R}, \phi_0 \in [0, 2\pi)$. Compute the first and the second derivative (w.r.t. time t) of the harmonic oscillation described by

$$x(t) = A \sin(\omega t + \phi_0).$$

Check that $x(t)$ is a solution of the following (differential) equation

$$x''(t) + \omega^2 x(t) = 0$$

and that there holds

$$x(0) = x(2\pi/\omega) = A \sin(\phi_0).$$

Exercise 4 (5 pts.)

Assume the demand N of a product as a function of its price p is given by

$$x = N(p) = 100 - 0,1p - 0,2p^2$$

(the so-called demand function).

The costs for the production of x units of the product shall be given by

$$K(x) = 100 + x.$$

The profit as a function of the price is thus given as the difference between the sales revenue

$$U(p) = pN(p)$$

and the costs K .

Which price \hat{p} maximizes the profit function $U(p)$?