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Exercise Sheet 9

Linear Algebra (AAI)

Exercise 9.1 (H)

Let $A_1 \in \mathbb{R}^{3\times 3}$ and $b_1 \in \mathbb{R}^3$ be given by

$$A_1 = \begin{pmatrix} 2 & 1 & 1 \\ -2 & 0 & 0 \\ -2 & -1 & 0 \end{pmatrix}, \qquad b_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Moreover, let $A_2 \in \mathbb{R}^{3\times 4}$ and $b_2 \in \mathbb{R}^3$ be given by

$$A_2 = \begin{pmatrix} 3 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}, \qquad b_2 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}.$$

Determine $\mathfrak{L}(A_1, b_1)$ and $\mathfrak{L}(A_2, b_2)$.

Exercise 9.2 (H)

Let $a \in \mathbb{R}$, and let $A \in \mathbb{R}^{3\times 3}$ and $b \in \mathbb{R}^3$ be given by

$$A = \begin{pmatrix} 1 & -a & 2 \\ -1 & 2a & 1 \\ 1 & 0 & 2 \end{pmatrix}$$

and $b = (1, 0, 2)^\top$. Determine $\mathfrak{L}(A, b)$ for all $a \in \mathbb{R}$.

Exercise 9.3 (H)

Consider the subspaces U_1, U_2 of $\mathbb{R}^3$ given by

$$\begin{aligned} U_1 &= \operatorname{span}(\{(1,0,2)^\top, (0,1,-1)^\top\}), \\ U_2 &= \{x \in \mathbb{R}^3 \colon x_1 + x_2 - 2x_3 = 0\}. \end{aligned}$$

Determine bases of U_2 and $U_1 \cap U_2$.

Hint: Find $A \in \mathbb{R}^{m\times n}$ such that $U_1 = \mathfrak{L}(A, 0)$. Cf. Proposition II.7.9 and Example II.7.10.

Exercise 9.4 (H)

Let Π_2 be the $\mathbb{R}$-vector space of polynomial functions where the degree is at most 2, see Example II.4.13. Determine all functions $f \in \Pi_2$ such that

$$\int_0^1 f(x)\,\mathrm{d}x = 1, \quad f'(0) = -2, \quad f(1) = 2.$$

Hint: Consider the linear maps $F_1, F_2, F_3 \colon \Pi_2 \to \mathbb{R}$ given by $F_1(f) = \int_0^1 f(x)\,\mathrm{d}x$, $F_2(f) = f'(0)$, and $F_3(f) = f(1)$. Use $f = a_0 v_0 + a_1 v_1 + a_2 v_2$ with $a_0, a_1, a_2 \in \mathbb{R}$.