$$\left(\begin{array}{ccc|c} 1 & -a & 2 & 1 \\ -1 & 2a & 1 & 0 \\ 1 & 0 & 2 & 2 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & -a & 2 & 1 \\ 0 & a & 3 & 1 \\ 0 & a & 0 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & -a & 2 & 1 \\ 0 & a & 0 & 1 \\ 0 & 0 & 3 & 0 \end{array}\right)$$

$3_2 = 0$

$a_y = 1 \mid y = \frac{1}{a} \rightarrow \begin{pmatrix} 2 \\ \frac{1}{a} \\ 0 \end{pmatrix}$ $a \neq 0$ if $a = 0 \rightarrow$ no solution

$x - ay = 1$

$\rightarrow x = ay + 1$

$x = \frac{a}{a} + 1$

if $a \neq 0$ $\mathcal{L}(A, b) = \left\{ \begin{pmatrix} 2 \\ \frac{1}{a} \\ 0 \end{pmatrix} \right\}$

9.3

$$U_1 = \left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} x_1 + \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} x_2 \right\} \qquad \left(\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right) \quad \begin{array}{l} x = -2y \\ z = y \end{array}$$

$x \; y \; z \; b$

$$U_1 = \begin{pmatrix} x_1 \\ 0 \\ 2x_1 \end{pmatrix} + \begin{pmatrix} 0 \\ x_2 \\ -x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ 2x_1 - x_2 \end{pmatrix} \quad = x_3$$

$x_3 = \frac{x_1 + x_2}{2}$

$\frac{x_1}{2} + \frac{x}{2} - x_3 = 0$

$\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$

$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 \end{array}\right)$ $\ker = \{0\}$

$2x_1 - x_2 - x_3 = 0$

$U_2 = x_1 + x_2 - 2x_3$

$\frac{x_1 + x_2}{2} = 2x_1 - x_2$

$$U_2 = x_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$$

$$U_2 = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} \right\}$$

$\left(\begin{array}{ccc|c} 2 & -1 & -1 & 0 \\ \frac{1}{2} & \frac{1}{2} & -1 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 2 & -1 & -1 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right)$

$\rightarrow \left(\begin{array}{ccc|c} 2 & -1 & -1 & 0 \\ 0 & \frac{3}{2} & -\frac{3}{2} & 0 \end{array}\right)$

$0x_1 + x_2 + x_3 = 0$

$\frac{3}{2} x_2 - \frac{3}{2} x_3 = 0$

$x_2 = x_3 \mid \begin{array}{l} 2x_1 - x_2 - x_3 = 0 \\ 2x_1 - x_2 - x_2 = 0 \\ 2x_1 - 2x_2 = 0 \\ x_1 = x_2 = x_3 \end{array} \quad \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$$\Pi_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$x^2 \quad x \quad 1$

	0	0	0	F_1, F_2, F_3
	0	0	0	
	1	-2	2	
0 1 0	0	0	0	
0 0 0	0	0	0	
0 0 1	1	-2	2	
Input	Output			

$\left(\begin{array}{ccc|c} 1 & \frac{1}{2} & \frac{1}{3} & 1 \\ 0 & 1 & 0 & -2 \\ 1 & 1 & 1 & 2 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & \frac{1}{2} & \frac{1}{3} & 1 \\ 0 & 1 & 0 & -2 \\ 0 & \frac{1}{2} & \frac{2}{3} & 1 \end{array}\right)$

$y = -2$

$\frac{2}{3} z = -0.5y + 1$

$\frac{2}{3} z = 2$

$z = 3$

$x = -0.5y - \frac{1}{3} z + 1$

$x = 1 - 1 + 1 = 1$

$\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$

$a x^2 - 2ax + 2a \rightarrow \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$