Ana2 5/1a

$$F(x) = \int_{-\infty}^{x} f(\tilde{x})\, d\tilde{x} = \int_{-\infty}^{a} 0\, dx + \underbrace{\int_{a}^{c} f(x)\, dx}_{[a;c]} + \underbrace{\int_{c}^{b} f(x)\, dx}_{]c;b]} + \int_{b}^{\infty} 0\, dx$$

$$F_1(x) = \int_a^c 2 \frac{x-a}{(b-a)(c-a)}\, dx$$

$$= \frac{1}{(b-a)(c-a)} \int_a^c 2x - 2a\ dx$$

$$= \frac{1}{(b-a)(c-a)} \cdot \left[x^2 - 2ax\right]_a^c$$

$$= \frac{(c^2 - 2ac) - (a^2 - 2a^2)}{(b-a)(c-a)}$$

$$= \frac{(c-a)^2}{(b-a)(c-a)} = \frac{c-a}{b-a}$$

$$F_2(x) = \int_c^b 2 \frac{b-x}{(b-a)(b-c)}\, dx$$

$$= \frac{1}{(b-a)(b-c)} \cdot \int_c^b 2b - 2x\ dx$$

$$= \frac{1}{(b-a)(b-c)} \cdot \left[2bx - x^2\right]_c^b$$

$$= \frac{(2b^2 - b^2) - (2bc - c^2)}{(b-a)(b-c)}$$

$$= \frac{b^2 - 2bc + c^2}{(b-a)(b-c)}$$

$$= \frac{(b-c)^2}{(b-a)(b-c)} = \frac{b-c}{b-a}$$

$$F(x) = 0 + \frac{c-a}{b-a} + \frac{b-c}{b-a} + 0$$

b)
due to the limitation of "a < c < b", every part of f(x) is a continuous line.
the first part defines a monotonically increasing line from x = a to x = c.
the second part defines a monotonically decreasing line from x = c to x = b.
everything else is 0 (= x-achsis) and therefore also continuous.

c) $\lim_{x \to \infty} F(x) = F(x)$

for $a < c < b \qquad a, c, b \in \mathbb{R}$

$$\frac{c-a}{b-a} + \frac{b-c}{b-a} = 1$$

checked experimentally with desmos

/2 $$f(x) = \frac{x^3 - 6x^2 + 11x - 6}{x^2 + 3x - 2}$$

$\frac{x^3}{x^2} \to$ long division

$$(x^3 - 6x^2 + 11x - 6) : (x^2 + 3x - 2) = x - 9 + \frac{40x - 24}{x^2 + 3x - 2}$$
$$-(x^3 + 3x^2 - 2x)$$
$$-9x^2 + 13x - 6$$
$$-(-9x^2 - 27x + 18)$$
$$40x - 24$$

$$x^2 + 3x - 2 = 0$$

$$x_{1/2} = \frac{-3 \pm \sqrt{9 - 4 \cdot (-2)}}{2} = \frac{-3 \pm \sqrt{17}}{2} \qquad x_1 \approx -3.56 \quad x_2 \approx 0.56$$

$$\frac{40x - 24}{x^2 + 3x - 2} = \frac{A}{(x - 0.56)} + \frac{B}{(x + 3.56)} \quad \Big| \cdot (x - 0.56)(x + 3.56)$$

$$40x - 24 = (x + 3.56)A + (x - 0.56)B$$

$x = 0.56$

$$40 \cdot 0.56 - 24 = 4.12A + 0 \cdot B$$

$$A \approx -0.39$$

$x = -3.56$

$-40 \cdot 3.56 - 24 = 0 \cdot A - 4.12\, B$

$B \approx 40.39$

$$f(x) = x - 9 + \frac{-0.39}{(x - 0.59)} + \frac{40.39}{(x + 3.59)}$$

$$F(x) = \frac{x^2}{2} - 9x - 0.39 \int x - 0.59 \, dx + 40.39 \int x + 3.59 \, dx$$

$$= \frac{x^2}{2} - 9x - 0.39\left(\frac{x^2}{2} - 0.59x\right) + 40.39\left(\frac{x^2}{2} + 3.59x\right)$$

$$= 20.5x^2 + 134.83x$$

13a $\tilde{f}(x) = -\frac{(\pi - x)^2}{4}$ is an even function if extended periodically.

$$\tilde{F}(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cdot \cos(k\omega x) + 0$$

$$= \frac{a_0}{2} + \sum_{k=1}^{\infty} \frac{4}{2\pi} \cdot \left(-\frac{1}{4}\right) \cdot \int (\pi - x)^2 \cdot \cos(k \cdot 1 \cdot x)\, dx \cdot \cos(k \cdot 1 \cdot x)$$

$$= \frac{a_0}{2} + \sum_{k=1}^{\infty} -\frac{1}{2\pi} \cdot \left((\pi - x)^2 \cdot \frac{\sin(kx)}{k} - \int (-2\pi + 2x) \cdot \frac{\sin(kx)}{k} dx\right) \cdot \cos(kx)$$

$$= \frac{a_0}{2} + \sum_{k=1}^{\infty} -\frac{1}{2\pi} \cdot \left((\pi - x)^2 \cdot \frac{\sin(kx)}{k} - \left((-2\pi + 2x) \cdot \frac{-\cos(kx)}{k^2} - \int 2 \cdot \frac{-\cos(kx)}{k^2} dx\right)\right) \cdot \cos(kx)$$

$$= \frac{a_0}{2} + \sum_{k=1}^{\infty} -\frac{1}{2\pi}\left(\frac{(\pi - x)^2 \cdot \sin(kx)}{k} - \frac{2\pi \cos(kx)}{k^2} + \frac{2x\cos(kx)}{k^2} + \frac{2\sin(kx)}{k^3}\right) \cdot \cos(kx)$$

$$\frac{a_0}{2} = \frac{1}{2\pi} \int_0^{2\pi} -\frac{(\pi - x)^2}{4} \, dx = \frac{1}{2\pi} \cdot -\frac{1}{4} \cdot \left[-\frac{(\pi - x)^3}{3}\right]_0^{2\pi}$$

$$= -\frac{1}{8\pi} \cdot \left(\frac{\pi^3}{3} + \frac{\pi^3}{3}\right) \approx -0.82$$

b) $\frac{\pi}{6} = -2 \cdot a_0$

c) $F'(x) = \sum \frac{x \cdot \cos(kx) \cdot 1 - \sin(kx) \cdot 1}{x^2}$

on slide 66 it shows the complex representation while the F(x) above is real.