L'Hôpital

1a) $\lim\limits_{x \to \infty} \frac{\ln(x)}{e^x} \to$ „$\frac{\infty}{\infty}$"

$\lim\limits_{x \to \infty} \frac{\frac{1}{x}}{e^x} = \lim\limits_{x \to \infty} \frac{e^x}{x} \to$ „$\frac{\infty}{\infty}$"

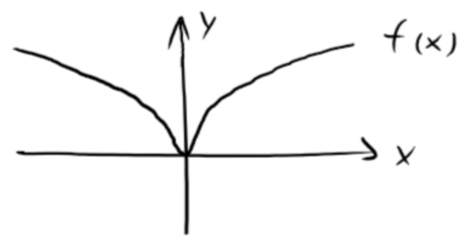
$\lim\limits_{x \to \infty} \frac{e^x}{1} = \infty$

1b) $\lim\limits_{x \to 1} \frac{1 + \cos(x\pi)}{x^2 - 2x + 1} \to \frac{1 + \cos(\pi)}{1 - 2 + 1} =$ „$\frac{0}{0}$"

$\lim\limits_{x \to 1} \frac{-\pi \sin(x\pi)}{2x - 2} \to \frac{-\pi \sin(\pi)}{2 - 2} =$ „$\frac{0}{0}$"

$\lim\limits_{x \to 1} \frac{-\pi^2 \cos(x\pi)}{2} = \frac{\pi^2}{2} \approx 4.935$

12 $f(x) = \ln(1 + x^2)$ $\quad f'(x) = \frac{1}{1 + x^2}$



Just going by the behaviour of graphs, we can see that ln(x) has no maxima and increases indefinitely. Using x² instead of x, only changes the fact that it works in the negative x-direction as well. By adding 1 we remove the singularity at x = 0 of any logarithm and, in this case, get a minima of y = 0.

13 $x(t) = A \sin(\omega t + \phi_0)$

$x'(t) = \omega A \cos(\omega t + \phi_0)$

$x''(t) = -\omega^2 A \sin(\omega t + \phi_0)$

$-\omega^2 A \sin(\omega t + \phi_0) + \omega^2 \cdot x(t) = 0$

$-A \sin(\omega t + \phi_0) + x(t) = 0$

$x(t) = A \sin(\omega t + \phi_0)$

$x(0) = A \sin(\omega \cdot 0 + \phi_0) = A \sin(\phi_0)$ ✓

$x\left(\frac{2\pi}{\omega}\right) = A \sin\left(\omega \cdot \frac{2\pi}{\omega} + \phi_0\right) = A \sin(2\pi + \phi_0)$

since $\sin(k \cdot 2\pi x)$ $k \in \mathbb{N}$ $= \sin(x)$

we get $A \sin(0 + \phi_0)$ ✓

14 assume $U(p) = p\,N(p) - k(x)$

$$U(p) = p(100 - 0.1p - 0.2p^2) - (100 + x) \qquad x = N(p)$$

$$U(p) = -0.2p^3 - 0.1p^2 + 100p - 100 - (100 - 0.1p - 0.2p^2)$$

$$U(p) = -0.2p^3 - 0.1p^2 + 100p - 100 - 100 + 0.1p + 0.2p^2$$

$$U(p) = -0.2p^3 + 0.1p^2 + 100.1p - 200$$

$$U'(p) = -0.6p^2 + 0.2p + 100.1$$

$$U''(p) = -1.2p + 0.2$$

$$U'(p) = 0$$

$$0 = -0.6p^2 + 0.2p + 100.1$$

$$p_{1/2} = \frac{-0.2 \pm \sqrt{0.2^2 - 4 \cdot (-0.6) \cdot 100.1}}{-1.2}$$

$$p_1 = -12{,}75$$

$$p_2 = 13{,}08$$

Since p1 is negative, which doesn't make sense for a profit-oriented price, p2 is the max.