Technische Hochschule Rosenheim SoSe 2022
Applied Artificial Intelligence - Bachelor
Analysis 2

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Homework 8: Taylor series w.r.t. to several variables

To submit: on **Friday, 20.05.2022**, 9:00 a.m., online by the learning campus

Exercise 1 (4 pts.)
By using a Taylor expansion, compute a) a linear and b) a quadratic approximation of the function $f : \mathbb{R}^2 \to \mathbb{R}$, $(x_1, x_2) \mapsto x_1 \sin(x_2)$ at the point $\left(1, \frac{\pi}{2}\right)$. (A remainder is not required.)

Exercise 2 (7 + 1 pts.)
The so-called Rosenbrock (or banana) function is given by

$$g : \mathbb{R}^2 \to \mathbb{R},\ g(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2.$$

Compute

a) [2 pts.] the approximation of g by a linear function in $(-1, 2)$.

b) [2 pts.] the approximation of g by a quadratic function in $(-1, 2)$.

c) [1 pt.] all stationary points.

d) [2 pts.] all minima. Are they local or global minima?

Optional [+ 1 pt.]: Plot the Rosenbrock function and its quadratic approximation for the domain $x \in [-2, 0]$ and $y \in [1, 3]$.

Exercise 3 (5 pts.)
Let $x, a \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n\times n}$ a symmetric matrix. Compute the gradient and the Hessian of:

a) $f_1(x) = a^T x$

b) $f_2(x) = x^T A x$

Hint: Consider at first $x, a \in \mathbb{R}^2$ and $A \in \mathbb{R}^{2\times 2}$.

Exercise 4 (4 pts.)
A position vector $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ [cm] has been drawn a bit sloppy, i.e. $\begin{pmatrix} 3 \pm 0.1 \\ 4 \pm 0.2 \end{pmatrix}$ [cm].

Give a (linear) estimate for the resulting maximal error, if we compute the angle

$$\alpha := \sphericalangle\left(\mathbf{x}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right) = \arctan\left(\frac{y}{x}\right) \quad \text{in [degree]}.$$

Your answer should include the angle with an upper/lower tolerance and have the precision of 1 decimal.