Technische Hochschule Rosenheim — SoSe 2022
Applied Artificial Intelligence - Bachelor
Analysis 2

Priv.-Doz. Dr. S.-J. Kimmerle — Friday, 29.04.2022

Homework 6: functions of several variables

To submit: on **Friday, 06.05.2022**, 9:00 a.m., online by the learning campus

Exercise 1 (6 pts.)

a) Consider the function $u : \mathbb{R}^2 \to \mathbb{R}$, $(x,y) \mapsto \exp(2x) - 2x + y^2$.
Compute the gradient and the Hesse matrix of u. [3 pts.]

b) Consider the function

$$f : D \subseteq \mathbb{R}^2 \to \mathbb{R},\ (x,y) \mapsto \ln\left(\sqrt{x} + \sqrt{y}\right).$$

Determine the maximal domain of definition D. [1 pt.]

Check that

$$x \cdot \frac{\partial f}{\partial x}(x,y) + y \cdot \frac{\partial f}{\partial y}(x,y) - \frac{1}{2} = 0 \quad \text{[2 pts.]}.$$

Exercise 2 (8 pts.)

Consider the function

$$f : D := \{(x,y) \in \mathbb{R}^2 \,|\, x^2 + y^2 \leq 1\} \to \mathbb{R},\ (x,y) \mapsto z := \sqrt{1 - x^2 - y^2}.$$

a) [2 pts.] Which geometric figure is represented by f (more precisely by the graph of f) and its domain D ?

b) [1 pt.] For f compute the partial functions f_1 for $y = 0$ and f_2 for $y = \frac{1}{2}$.

c) [2 pts.] Compute the level curves for $f(x,y) = 0$, $f(x,y) = \frac{2}{5}$, $f(x,y) = \frac{4}{5}$, and $f(x,y) = 1$ and sketch them in a diagram.

d) [3 pts.] Compute on $D \setminus \partial D$ the gradient $\nabla f(x,y)$. Sketch the gradients at the points $(0,0)$, $(\frac{1}{2},0)$, $(0,\frac{1}{4})$, and $(\frac{1}{2},\frac{1}{2})$ as arrows in the diagrams.

Exercise 3 (6 pts.)

Consider $f : \mathbb{R}^2 \to \mathbb{R}$,

$$f(x,y) := \begin{cases} \frac{2xy}{x^2+y^2}, & \text{if } x \neq 0 \text{ or } y \neq 0, \\ 0, & \text{if } (x,y)^\top = (0,0)^\top. \end{cases}$$

a) [2 pts.] Show that f is not continuous in the point $(0,0)$.

b) [4 pts.] Examine the continuity of the partial functions related to f, namely

$$f_1(x) := f(x, y_0),$$
$$f_2(y) := f(x_0, y),$$

with $x_0, y_0 \in \mathbb{R}$.