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Exercise Sheet 3

Linear Algebra (AAI)

Exercise 3.1 (H)

a) Show that (v_1, v_2, v_3) given by
$$v_1 = (1, 0, 1), \quad v_2 = (1, 1, 0), \quad v_3 = (0, 1, 1)$$
is a basis of $\mathbb{R}^3$.

b) Let V be an $\mathbb{R}$-vector space with $\dim V = 3$, and let (v_1, v_2, v_3) be a basis of V. Show that
$$(v_1 - v_3, v_1 + v_2 - v_3, v_1 + v_2 + v_3)$$
is a basis of V, too.

Exercise 3.2 (H)

Determine a basis and the dimension of the following subspaces of $\mathbb{R}^3$:

a) $U_1 = \{(x, y, z) \in \mathbb{R}^3 \colon x = y = z = 0\}$,

b) $U_2 = \{(x, y, z) \in \mathbb{R}^3 \colon x + y - z = 0\}$.

Exercise 3.3 (H)

Let V be a K-vector space, let $(v_1, \ldots, v_n)$ be a basis of V, and let $(w_1, \ldots, w_m)$ be a generating set of V. Prove or disprove:

a) $(v_1, w_2, \ldots, w_m)$ is a generating set of V.

b) $(v_1 + w_1, \ldots, v_n + w_n)$ is a basis of V.

c) There exists $i \in \{1, \ldots, n\}$ such that $(v_1, \ldots, v_{i-1}, w_1, v_{i+1}, \ldots, v_n)$ is a basis of V.

Exercise 3.4 (H)

Let U_1, U_2 be subspaces of V, and let
$$U_1 + U_2 = \{u_1 + u_2 \colon u_1 \in U_1, u_2 \in U_2\}.$$

a) Show that $U_1 + U_2$ is a subspace of V.

b) Let $V = \mathbb{R}^n$ and $\dim U_1 = \dim U_2 = n-1$. Determine all possible cases of $\dim U_1 \cap U_2$ and provide an explicit example for each case with $n = 3$. Cf. Exercise 1.4.