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Exercise Sheet 11

Linear Algebra (AAI)

Exercise 11.1 (H)

Let $A_1, A_2, A_3, A_4 \in \mathbb{R}^{3\times 3}$ be according to Exercise 10.1.

a) Determine for every $i = 1, 2, 3, 4$ whether A_i is diagonalizable.

b) If A_i is diagonalizable, determine $S_i \in \mathrm{Gl}(3, \mathbb{R})$ and a diagonal matrix $D_i \in \mathbb{R}^{3\times 3}$ such that $A_i = S_i \cdot D_i \cdot S_i^{-1}$ (for $i = 1, 2, 3, 4$).

Exercise 11.2 (H)

a) Let $A \in \mathbb{R}^{3\times 3}$ with $P_A(t) = -t^3 + 4t^2 - 3t$ for $t \in \mathbb{R}$. Determine P_{A^2}.
Hint: Is A similar to a diagonal matrix?

b) Show that every matrix $B \in \mathbb{R}^{3\times 3}$ has an eigenvalue, i.e., $\sigma(B) \neq \emptyset$.
Hint: Intermediate value theorem.

c) Let $C \in \mathbb{R}^{2\times 2}$ be given by

$$C = \begin{pmatrix} -1 & 3 \\ 0 & 2 \end{pmatrix}.$$

Compute C^{100} (without using a computer).

Exercise 11.3 (H)

A vector $p = (p_1, \ldots, p_n)^\top \in \mathbb{R}^n$ is called *stochastic* if $p_i \geq 0$ for all $i = 1, \ldots, n$ and $\sum_{i=1}^n p_i = 1$.

Let $n = 3$ and $M \in \mathbb{R}^{3\times 3}$ be given by

$$M = \frac{1}{2} \cdot \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

Show that $(M^n p)_{n\in\mathbb{N}}$ converges (componentwise) to $(1/3, 1/3, 1/3)^\top$ for every stochastic vector $p \in \mathbb{R}^3$.

Hint: Compute M^n for $n \in \mathbb{N}$, cf. Exercise 11.2 c).

Exercise 11.4 (H)

Let Π_2 be the $\mathbb{R}$-vector space of polynomial functions where the degree is at most 2 and let $F\colon \Pi_2 \to \Pi_2$ be given by $F(f)(x) = 2x \cdot f'(x)$ for $f \in \Pi_2$ and $x \in \mathbb{R}$.

a) Show that F is linear.

b) Determine whether F is diagonalizable. If so, determine a basis consisting of eigenvectors and determine the corresponding eigenvalues.