Example: Loss of continuity under pointwise convergance

$D = \mathbb{R}$ $\quad f_n : D \to \mathbb{R}$

$$f_n(x) = (\cos(\pi x))^{2n} = \underbrace{\cos^{2n}(\pi x)}_{\text{is a continuous function}} \xrightarrow{n\to\infty} f(x) = \begin{cases} 1 & ; x \in \mathbb{Z} \\ 0 & ; \text{otherwise} \end{cases}$$

$f(x)$ is discontinuous

The pointwise limit $f(x) = \lim_{n\to\infty} f_n(x)$ is a discontinuous function.

$\Rightarrow$ $\{f_n\}_{n\in\mathbb{N}}$ is <u>not</u> uniformly convergent on $\mathbb{R}$.