Theoretical Computer Science – Exercise 9

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Please prepare the following exercises at home prior to the tutorial:

Exercise 1

In the lecture slides it has been shown that addition of two natural numbers as well as IF ... THEN belong to the class of LOOP-computable functions.
Show that the following functions are also LOOP-computable. If necessary, you may use the addition function and IF ... THEN; also, in all questions you can use the operations from the previous questions if required.
According to the convention for LOOP programs, the result of the calculation should be in the variable x_0 at the end.

a) (Modified) subtraction of two variables x_1 and x_2:
   $x_0 := x_1 - x_2$ if the resulting number is positive
   $x_0 := 0$ if the resulting number would be negative
b) Multiplication of two variables x_1 and x_2.
c) The exponential function $\exp(x_1, x_2) = x_1^{x_2}$
d) The factorial n!, defined by:
   $n! := n\,(n-1)!$
   $0! := 1$
e) Computation oft the Fibonacci sequence fib(n):
   $fib(n) := fib(n-1) + fib(n-2)$
   $fib(0) := 0$, $fib(1) := 1$
   The Fibonacci sequence starts with: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, …
   For better readability you can use more expressive variable names than x_1, x_2 etc., but make sure that the final result is contained in x_0.

Hint for (d) and (e): Do not calculate the functions based on their recursive definition, but by iterating from bottom to top. The parameter n is passed in the variable x_1.

Exercise 2

Show by mathematical induction (*vollständige Induktion*) that all polynomial functions *p*(*x*) of the following form are LOOP-computable:

$p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \ldots + a_n x^n$ where $a_0, a_1, \ldots, a_n \in \mathbb{N}$

Hints:

- First, show that a polynomial of degree 0 is LOOP-computable.
- Then show that based on the assumption that a polynomial of degree n is LOOP-computable by a program P, this also applies to a polynomial of degree n + 1.
- To improve readability, you can use any variable identifiers (and you can use the coefficients a_i as they are), but the end result should be in x_0.
- If necessary, you can use the addition and multiplication function, as well as IF ... THEN.