

Introduction

Power series

**Differentiation in  
Higher Dimensions**

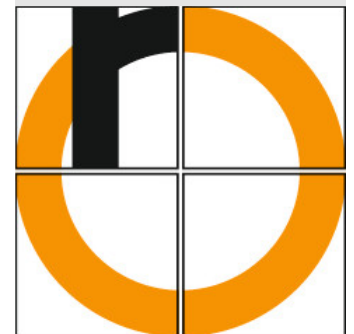
Derivatives

Integration in  
Higher Dimensions

Further Topics in  
Calculus

Summary -  
Outlook and  
Review

- 1 Introduction
- 2 Power series
- 3 Differentiation in Higher Dimensions**
  - Derivatives
- 4 Integration in Higher Dimensions
- 5 Further Topics in Calculus
- 6 Summary - Outlook and Review

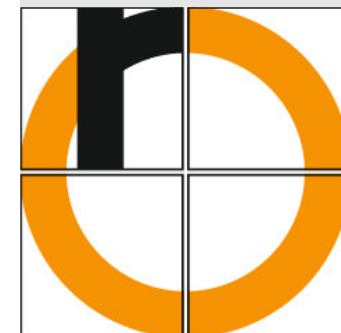


In Analysis 1 we have discussed differentiation of functions of 1 variable.

Now we consider a real-valued function in several variables

$$f : D \subseteq \mathbb{R}^n \rightarrow \mathbb{R}$$

$$x = (x_1, \dots, x_n)^\top \mapsto f(x_1, \dots, x_n) = f(x)$$



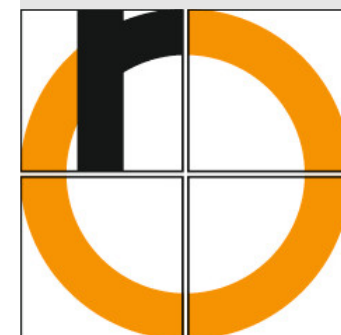
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Later we are going to extend the differential calculus to vector-valued functions (of several variables)

$$f : D \subseteq \mathbb{R}^n \rightarrow \mathbb{R}^m$$
$$x = (x_1, \dots, x_n)^\top \mapsto (f_1(x_1, \dots, x_n), \dots, f_m(x_1, \dots, x_n))^\top$$



# Special Case: $n = 2$

## Analysis 2

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Introduction

Power series

Differentiation in  
Higher Dimensions

Derivatives

Integration in  
Higher Dimensions

Further Topics in  
Calculus

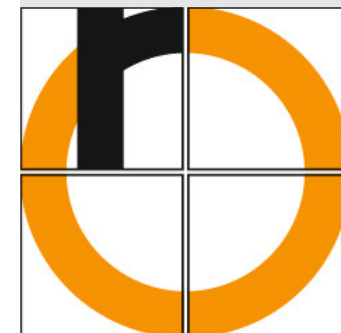
Summary -  
Outlook and  
Review

A real-valued function in 2 variables

$$f : D \subseteq \mathbb{R}^2 \rightarrow \mathbb{R}$$
$$x = (x_1, x_2)^\top \mapsto f(x_1, x_2)$$

We may plot the function value as 3rd coordinate over the real plane  $\mathbb{R}^2$ .

The graph of  $f$  is a subset of  $\mathbb{R}^3$ : a "landscape" or "mountains".



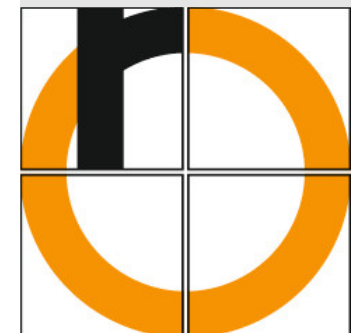
## Definition (Level set)

We define the **level set** of a function  $f : D \subseteq \mathbb{R}^n \rightarrow \mathbb{R}$ ,  $(x_1, \dots, x_n)^\top \mapsto f(x_1, \dots, x_n)$  for the function value  $c \in \mathbb{R}$  as the set

$$N_c := \{x \in D \mid f(x) = c\}.$$

The structure of  $N_c$  may be "complicated", it might also be the empty set.

For  $n = 2$  the level set is called a contour line (though it may be an area, e.g.),  
for  $n = 3$  the level set is called an equipotential surface (though it may be an solid, e.g.).

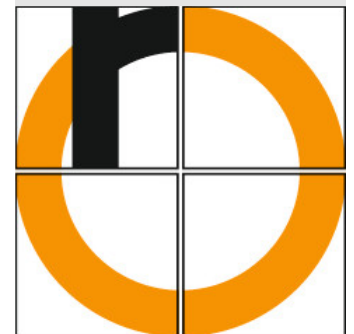


A partial function is a "cross section"-function that is obtained by freezing all but 1 variables, e.g.  $x_i$ :

$$g_i : D_i \subseteq \mathbb{R} \rightarrow \mathbb{R}$$

$$x_i \mapsto f(a_1, \dots, a_{i-1}, x_i, a_{i+1}, \dots, a_n),$$

with  $(a_1, \dots, a_{i-1}, x_i, a_{i+1}, \dots, a_n) \in D$  for all  $x_i \in D_i$ ,  
 $a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_n$  fixed.



# Partial Function: Example

Analysis 2

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Introduction

Power series

Differentiation in  
Higher Dimensions

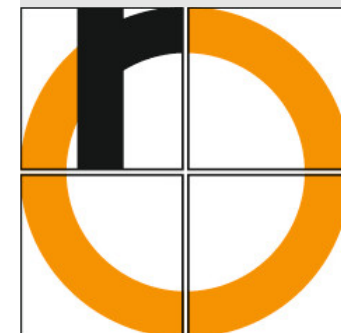
Derivatives

Integration in  
Higher Dimensions

Further Topics in  
Calculus

Summary -  
Outlook and  
Review

$$f(x_1, x_2) = x_1^2 + x_2^2$$



## Definition (Partial derivative)

Let  $f : D \subseteq \mathbb{R}^n \rightarrow \mathbb{R}$ ,  $x = (x_1, \dots, x_n)^\top \mapsto f(x_1, \dots, x_n)$  and  $a \in D$ ,  $D$  an open set.

If the derivative of the partial function

$$x_i \mapsto f(a_1, \dots, a_{i-1}, x_i, a_{i+1}, \dots, a_n)$$

exists at  $x_i = a_i$ ,

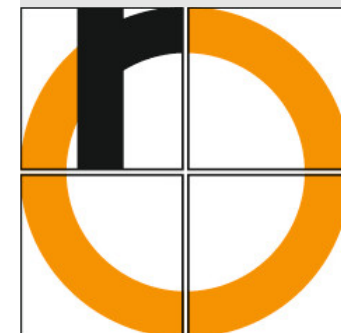
then we call it the **partial derivative** of  $f$  w.r.t.  $x_i$  at  $a$ .

We write:

$$\frac{\partial f}{\partial x_i}(a) \quad \text{or} \quad f'_{x_i}(a) \quad \text{or} \quad \dots$$

We say  $f$  is **partially differentiable** in  $a$ , if all  $\frac{\partial f}{\partial x_i}(a)$  exist.

We say  $f$  is partially differentiable in  $E \subseteq D$ , if  $f$  is partially differentiable at any  $a \in E$ .





# Partial Derivatives: Examples

Analysis 2

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Introduction

Power series

Differentiation in  
Higher Dimensions

Derivatives

Integration in  
Higher Dimensions

Further Topics in  
Calculus

Summary -  
Outlook and  
Review

