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Exercise Sheet 1

Linear Algebra (AAI)

Exercise 1.1 (H)

Prove Lemma I.2.4.

Exercise 1.2 (H)

true because all elements of the vector (x1,...,xn) are element of R

a) Verify that $V = \mathbb{R}^n$ is a $\mathbb{R}$-vector space.

b) Is $\mathbb{C}^n$ a $\mathbb{R}$-vector space (based on (1) from Example II.1.2)?

yes. R is a subset of C and the coordinates of a C vector are element of R.
C1 works out to be a R2 vector space and C2 -> R4?

Exercise 1.3 (H)

Let $V = \mathbb{R}^{[0,1]}$.

x1 of R1 is element of R.
they are also element of R if the vector space is being restricted to R0. x1 = 0.

a) Verify that V is a $\mathbb{R}$-vector space.

b) Show that $C([0,1])$ is a subspace of V.

C([0, 1]) is the space from x e [0-1] with continuous functions on [0-1]. so basically they're 1-dimensional lines which is a valid subspace if they go through the origin.

Exercise 1.4 (H)

Consider Example II.1.7.(i) with $a \in \mathbb{R}^n$, $b \in \mathbb{R}$, and

$$U = \left\{ x \in \mathbb{R}^n \colon \sum_{i=1}^{n} a_i \cdot x_i = b \right\}.$$

a) Show that U is a subspace of $\mathbb{R}^n$ if and only if $b = 0$.

b) Let $n = 2$. Determine and illustrate U for all combinations of $a \in \{(1,1), (-1,0)\}$ and $b \in \{0, 1, -2\}$.

c) Show Remark II.1.10 (regarding the union of subspaces).

a)
proof by contradiction:
R2 -> a1, a2, x1, x2, b
let b = 1

x e R2: a1*x1 + a2*x2 = 1
now ai would have to be the inverse of xi which would yield no valid subspace.

x e R2: a1*x1 + a2*x2 = 0
now the first sum has to be the negative of the second sum or both products need to be 0 which still allows 1 part to be chosen freely.

b)
U1: x1 + x2 = 0
-45° diagonal from (-1,1) to (1,-1)

U2: -x1 = 0
-only the point (0,0)

U3: x1 + x2 = 1
-45° diagonal from (-2,3) to (2,-1)
lifted by 1 unit compared to U1

U4: -x1 = 1
-only the point (-1,0)
shifted left by 1 unit compared to U2

U5: x1 + x2 = -2
-45° diagonal from (-2,0) to (2,-4)
shifted down by 2 units compared to U1

U6: -x1 = -2
only the point (2,0)

c)
the union of subspaces can agin be a subspace if one of the subspaces is already contained in the other.
otherwise the subspaces aren't closed under addition and form a whole new vector space that's in a higher dimension.