

$$\int_0^1 \int_0^{2\pi} \int_p^R \int_{-\sqrt{R^2-r^2}}^{\sqrt{R^2-r^2}} r dz dr d\theta = \int_0^{2\pi} \int_p^R \underbrace{r(\sqrt{R^2-r^2} + \sqrt{R^2-r^2})}_{= 2r\sqrt{R^2-r^2}} dr d\theta$$

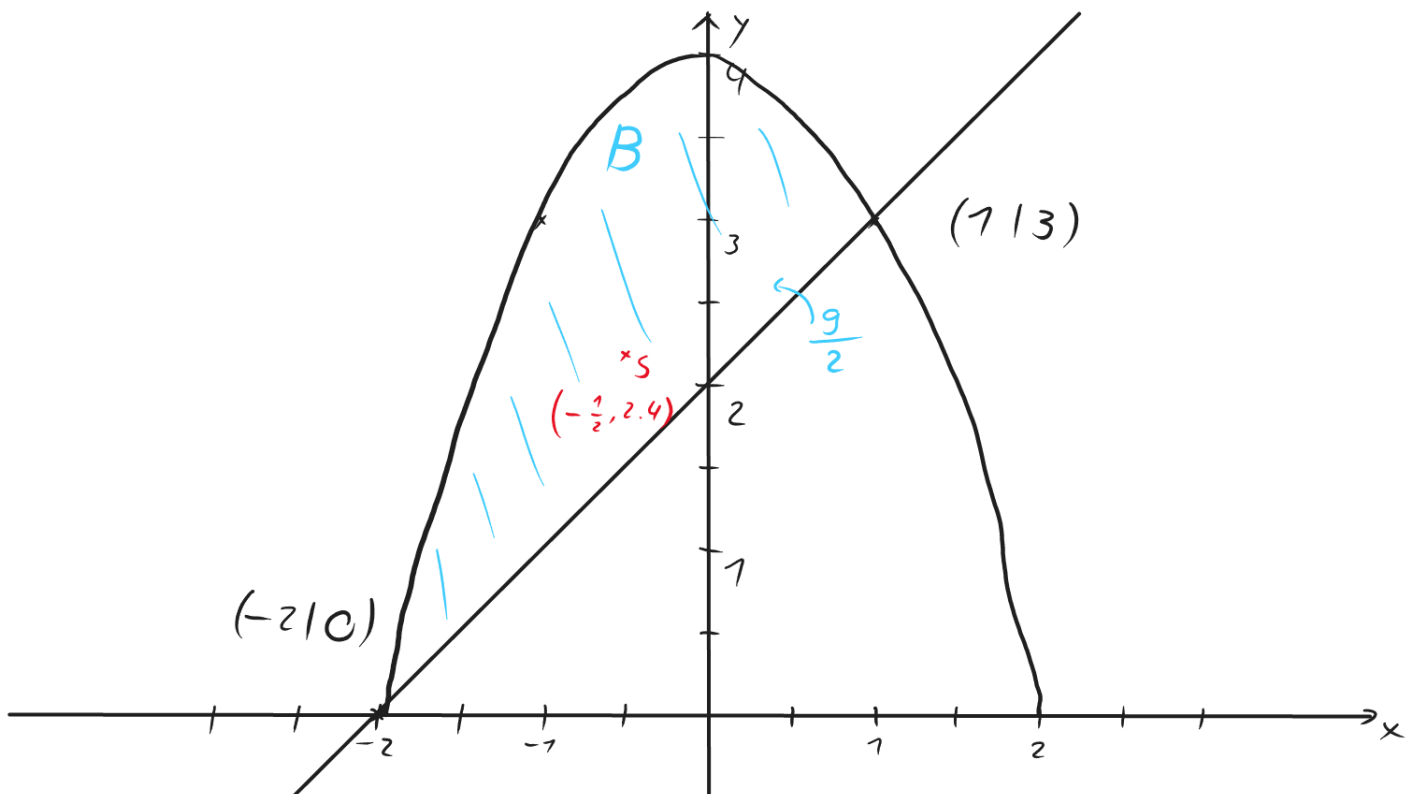
$$= \int_0^{2\pi} 2r \int_p^R \sqrt{u} - \frac{1}{2r} du d\theta \quad \text{where } R^2 - r^2 = u$$

$$= \int_0^{2\pi} -1 \cdot \int_p^R \frac{u^{1.5}}{1.5} du d\theta$$

$$= \int_0^{2\pi} \left[\frac{2}{3} (R^2 - r^2)^{\frac{3}{2}} \right]_p^R d\theta = \int_0^{2\pi} -\frac{2}{3} (R^2 - p^2)^{\frac{3}{2}} d\theta$$

$$V = -\frac{2}{3} \left[\theta (R^2 - p^2)^{\frac{3}{2}} \right]_0^{2\pi} = -\frac{4\pi (R^2 - p^2)^{\frac{3}{2}}}{3}$$

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$$M_x = \frac{1}{2} \int_{-2}^1 ((-x^2 + 4)^2 - (x+2)^2) dx$$

$$= \frac{1}{2} \int_{-2}^1 (x^4 - 8x^2 + 16 - (x^2 + 4x + 4)) dx$$

$$= \frac{1}{2} \int_{-2}^1 (x^4 - 9x^2 - 4x - 12) dx = \frac{1}{2} \left[\frac{x^5}{5} - 3x^3 - 2x^2 + 12x \right]_{-2}^1$$

$$= \frac{1}{2} \left(\frac{1}{5} - 3 - 2 + 12 - \left(-\frac{32}{5} + 24 - 8 - 24 \right) \right) = 10,8$$

$$M_y = \int_{-2}^1 x(-x^2 + 4 - (x+2)) dx = \int_{-2}^1 x(-x^2 - x + 2) dx = \int_{-2}^1 -x^3 - x^2 + 2x dx$$

$$= \left[-\frac{x^4}{4} - \frac{x^3}{3} + x^2 \right]_{-2}^1 = -\frac{1}{4} - \frac{1}{3} + 1 - \left(-\frac{16}{4} - \frac{8}{3} + 4 \right) = -2,25$$

$$(\bar{x}, \bar{y}) = \left(\frac{-2,25}{4,5}, \frac{10,8}{4,5} \right) \quad S\left(-\frac{1}{2}, 2, 4\right)$$

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$$\lim_{R_1, R_2 \rightarrow \infty} \int_{-R_1}^{R_2} x \left(\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} \right) dx$$

$$\lim_{R_1, R_2 \rightarrow \infty} \int_{-R_1}^{R_2} x \left(\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(x^2 - 2x\mu + \mu^2)} \right) dx$$

$$\lim_{R_1, R_2 \rightarrow \infty} \frac{1}{\sqrt{2\pi}\sigma} \cdot \int_{-R_1}^{R_2} x e^{-\frac{1}{2\sigma^2}(x^2 - 2x\mu + \mu^2)} dx$$

$$\lim_{R_1, R_2 \rightarrow \infty} \frac{1}{\sqrt{2\pi}\sigma} \cdot \int_{-R_1}^{R_2} x e^{-\frac{x^2 - 2x\mu + \mu^2}{2\sigma^2}} dx$$