11

$$A = \frac{1}{2}\int_{\alpha}^{\beta} (r(\phi))^2 \, d\phi$$

$$(a + a\cos(\phi))^2 = a^2 + 2a^2\cos(\phi) + a^2\cos(\phi)^2$$

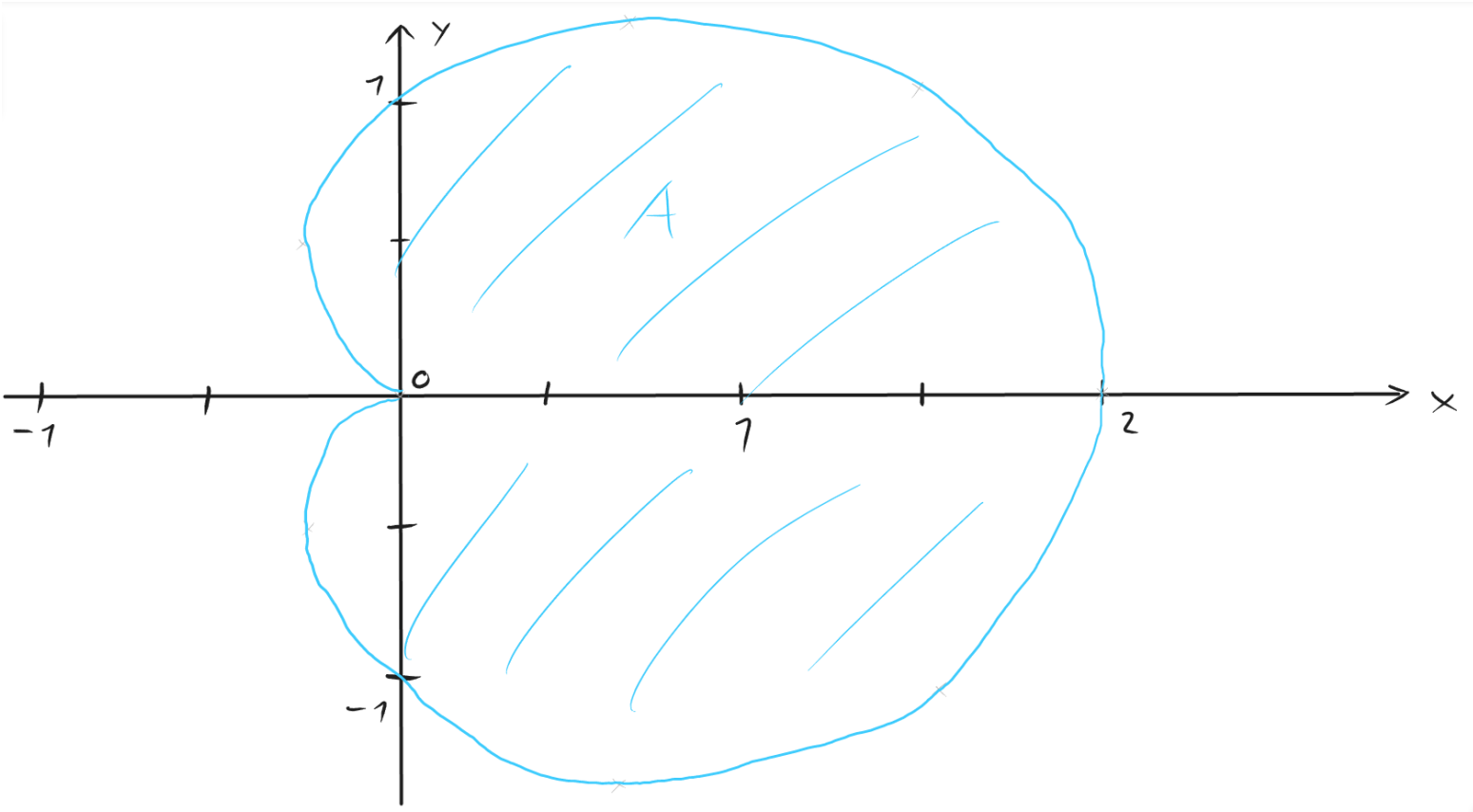
$$A(a) = \frac{1}{2}\int_{0}^{2\pi} (a(1+\cos(\phi)))^2 \, d\phi$$

$$= \frac{a^2}{2}\int_{0}^{2\pi} 1 + 2\cos(\phi) + \cos(\phi)^2 \, d\phi$$

$$= \frac{a^2}{2}\left[\phi + \sin(\phi)^2 + \frac{\phi}{2} + \frac{1}{2}\sin(\phi)\cos(\phi)\right]_0^{2\pi}$$

$$= \frac{a^2}{2}\left[2\pi + 0 + \pi + \frac{1}{2}\cdot 0\cdot 1 - 0 - 0 - \frac{0}{2} - \frac{1}{2}\cdot 0\cdot 1\right]$$

$$A(a) = \frac{a^2 3\pi}{2}$$



12

$$V = \pi \int_a^b (f(x))^2 \, dx$$

$\left(\frac{x}{a}\right) = u$

$$V(a) = \pi \int_0^l \left(a \cdot \cosh\left(\frac{x}{a}\right)\right)^2 dx = \pi a^2 \int_0^l \cosh\left(\frac{x}{a}\right)^2 dx \qquad dx = a \cdot du$$

$$= \pi a^3 \int_0^l \cosh(u)^2 \, du = \pi a^3 \int_0^l \frac{1}{2}(\cosh(2u) + 1) \, du$$

$$= \frac{\pi a^3}{2} \int_0^l \cosh(2u) + 1 \, du \qquad 2u = w \,, \; du = \frac{1}{2} \cdot dw$$

$$= \frac{\pi a^3}{2} \left(\int_0^l \cosh(w) \, dw + [u]_{x=0}^{x=l} = \frac{\pi a^3}{2} \left[\sinh(w) \frac{1}{2} + u \right]_{x=0}^{x=(} \right. \qquad w \to u$$

$$= \frac{\pi a^3}{2} \left[\sinh(2u) \frac{1}{2} + u \right]_{x=0}^{x=l} = \frac{\pi a^3}{2} \left[\sinh\left(2 \frac{x}{a}\right) \frac{1}{2} + \frac{x}{a} \right]_0^l \qquad u \to x$$

$$= \frac{\pi a^3}{2} \left[\sinh\left(\frac{2l}{a}\right) \frac{1}{2} + \frac{l}{a} - \sinh(0) \frac{1}{2} - 0 \right]$$

$$V(a) = \frac{\pi a^3}{4} \left(\sinh\left(\frac{2l}{a}\right) + 2 \frac{l}{a} \right)$$

$$= \frac{\pi}{4} a^3 \sinh\left(\frac{2l}{a}\right) + \frac{\pi a^2 l}{2}$$