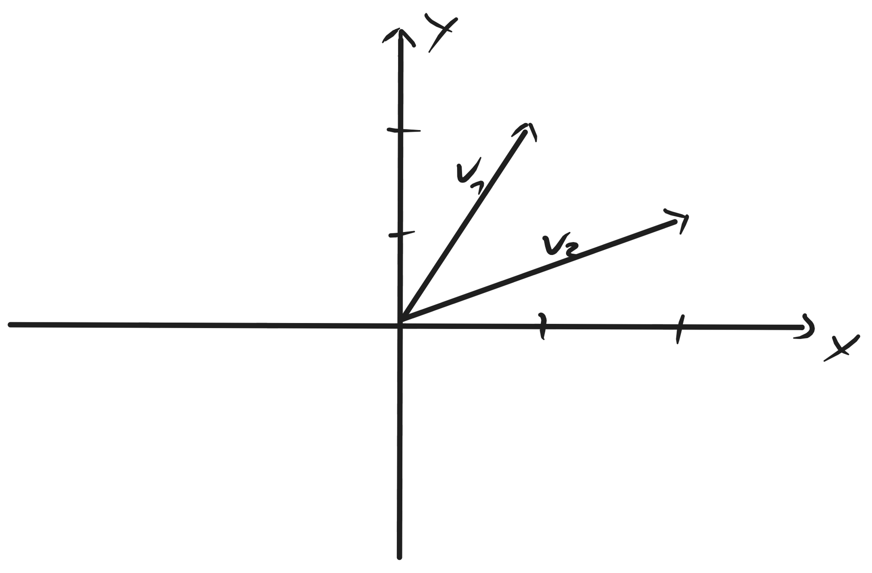
/2b

$v_1 \neq v_2 \rightarrow \lambda v_1 + \lambda v_2 = 0 \rightarrow x - x = 0$

$F(v_1) = F(v_2) \neq 0$

$F(\lambda_1 v_1) + F(\lambda_2 v_2) = F(0)$



$\lambda_1 F(v_1) + \lambda_2 F(v_2) = 0$

$0 \cdot F(v_1) + 0 \cdot F(v_2) = 0$

/3a

$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{1} \end{bmatrix}$

$F(x) = Ax = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$= \begin{bmatrix} x_1 a_{11} + x_2 a_{12} + x_3 a_{13} \\ x_1 a_{21} + x_2 a_{22} + x_3 a_{23} \\ x_1 a_{31} + x_2 a_{32} + x_3 a_{33} \end{bmatrix}$

b) i)

A =				1	0	0
				0	1	0
				0	0	1
1	2	0		1	2	0
1	0	1		1	0	1
1	0	0		1	0	0

ii)

A =				1	1	0
				0	0	1
				0	1	1
1	-1	1		1	2	0
1	2	-1		1	0	1
1	1	-1		1	0	0

1) $\begin{Bmatrix} 1 + 0 + 1 \\ 0 + 2 - 1 \end{Bmatrix}$

2) $\begin{matrix} 2 + -1 \\ 2 \cdot 3 - -1 \end{matrix} = \begin{matrix} 1 \\ 7 \end{matrix}$

$\begin{matrix} 5 + 2 \\ 2 - 2 \end{matrix} = \begin{matrix} 7 \\ 0 \end{matrix}$

/4a

$F(x) = Ax = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} x_1 a_{11} + x_2 a_{12} \\ x_1 a_{21} + x_2 a_{22} \end{bmatrix}$

$G(x) = Bx = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{bmatrix} y_1 b_{11} + y_2 b_{12} \\ y_1 b_{21} + y_2 b_{22} \end{bmatrix}$

$F(x) + G(x) = \begin{bmatrix} x_1 a_{11} + x_2 a_{12} \\ x_1 a_{21} + x_2 a_{22} \end{bmatrix} + \begin{bmatrix} y_1 b_{11} + y_2 b_{12} \\ y_1 b_{21} + y_2 b_{22} \end{bmatrix}$

$= \begin{bmatrix} x_1 a_{11} + x_2 a_{12} + y_1 b_{11} + y_2 b_{12} \\ x_1 a_{21} + x_2 a_{22} + y_1 b_{21} + y_2 b_{22} \end{bmatrix}$

$F(v + x) + G(x) = F(v) + F(x) + G(x)$

$= \begin{bmatrix} a_{11} + a_{12} + b_{11} + c_{12} \ldots \\ \ldots \quad \ldots \end{bmatrix}$

$\lambda F(x) \cdot G(x) = \begin{bmatrix} \lambda x_1 a_{11} + \lambda x_2 a_{12} \\ \lambda x_1 a_{21} + \lambda x_2 a_{22} \end{bmatrix} \cdot \begin{bmatrix} y_1 b_{11} + y_2 b_{12} \\ y_1 b_{21} + y_2 b_{22} \end{bmatrix}$

$= \begin{bmatrix} (x_1 a_{11} + x_2 a_{12})\lambda \cdot (y_1 b_{11} + y_2 b_{12}) \\ (x_1 a_{21} + x_2 a_{22})\lambda \cdot (y_1 b_{21} + y_2 b_{22}) \end{bmatrix}$

b)

i)

A =				1	0	0
				0	1	0
				0	0	1
1	1	0		1	1	0
0	0	1		0	0	1
0	1	1		0	1	1

ii)

$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$

$B^{-1} = \begin{pmatrix} 1 & 1 & -1 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

iii)

				1	1	0
				0	0	1
				0	1	1
1	1	-1		1	0	0
0	-1	0		0	0	-1
0	1	0		0	0	1

iv)

				1	1	-1
				0	-1	0
				0	1	0
1	1	0		1	0	-1
0	0	1		0	1	0
0	1	1		0	0	0

2