Technische Hochschule Rosenheim SoSe 2022
Applied Artificial Intelligence - Bachelor
Analysis 2

Priv.-Doz. Dr. S.-J. Kimmerle Wednesday, 25.05.2022

Homework 10: revision integration & curves

To submit: on **Friday, 03.06.2022**, 9:00 a.m., online by the learning campus

Exercise 1 (6 pts.)

Compute the following (proper and improper) integrals:

a)
$$\int_0^{\pi/2} \exp(x)\sin(x)\,dx$$

b)
$$\int_0^1 \frac{1}{\sqrt{1-x}}\,dx$$

c)
$$\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$$

Exercise 2 (4 pts.)

A mass point moves with time $t \in [0,1]$ on the curve $\mathbf{r}(t)$ through the space $\mathbb{R}^3$. The parametrization of the curve reads

$$\mathbf{r}(t) := \begin{pmatrix} R\sin(\omega t) \\ R\cos(\omega t) \\ \frac{1}{2}c_a t^2 \end{pmatrix},$$

where R is a length, ω is the angular frequency, and c_a is the acceleration constant.

a) [2 pts.] Compute the velocity $\mathbf{v}(t) = \dot{\mathbf{r}}(t)$ and the acceleration $\mathbf{a}(t)$ of the mass point for all $t \in [0,1]$.

b) [2 pts.] Consider the case $R = 1$, $\omega = 2\pi$ and $c_a = 1$. We continue at $t = 1$ with the constant acceleration $c_b = 0$, whereas the mass point continues (with constant velocity) tangentially to the previous curve from $t = 1$ on. Where is the mass point at $t = 2$?

Exercise 3 (5 pts.)

We consider the standard parabola $f(x) = x^2$.

Compute the arc length s between the points $(0,0)$ and $(1,1)$ (rounded to 2 decimals, in length units).

Hint: Proof and use that

$$\int \sqrt{x^2+a^2}\,dx = \frac{1}{2}\left(x\sqrt{x^2+a^2} + a^2\ln\left(x+\sqrt{x^2+a^2}\right)\right) + C, C \in \mathbb{R}$$