

Homework 11: simple integrals in \mathbb{R}^m

To submit: on **Friday, 10.06.2022**, 9:00 a.m., online by the learning campus

Exercise 1 (3 pts.)

For $a > 0$ we consider the curve

$$r(\phi) = a(1 + \cos(\phi)), \quad \phi \in [0, 2\pi)$$

in polar representation.

Plot the sector S that is enclosed by this curve and compute its area A in dependence of a . For the plot you may use $a = 1$.

Remark: This curve is called a cardioid.

Hint: You may use (w/o proof)

$$\int \cos^2(\phi) d\phi = \frac{1}{2}\phi + \frac{1}{2}\sin(\phi)\cos(\phi) + C, \quad C \in \mathbb{R}.$$

Exercise 2 (4 pts.)

Find the volume of the solid of revolution, that is generated by rotating the graph of

$$f(x) = a \cosh\left(\frac{x}{a}\right), \quad x \in [0, \ell], \quad a, \ell > 0$$

around the x -axis.

Remark: f is called a catenoid. It is relevant, for instance, for chains or suspension bridges.