Theoretical Computer Science – Exercise 8

SS 2022
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Please prepare the following exercises at home prior to the tutorial:

Exercise 1

Use the pumping lemma to show that the following language is not regular:
$L = \{u^k v^i a^j v^p \mid j > k; p > k; i, j, k, p \in \mathbb{N}_0\}$.

Exercise 2

Consider the context-free grammar with V = {Z, A, B}, Σ = {a, b, c, =, +, *, (,), ;}, start symbol Z, and production rules:

$Z \rightarrow B = A;$
$A \rightarrow B \mid A + A \mid A * A \mid (A)$
$B \rightarrow aB \mid bB \mid cB \mid a \mid b \mid c$

a) Describe in words which language is generated by the grammar. What is the meaning of the non-terminal symbols S, A, and B?
b) Convert the grammar to Chomsky normal form.
c) Use the CYK algorithm to check whether the following words are part of the language generated by the grammar (empty tables are provided at the end of this document):
   - b = c
   - b = c;
   - a = b * c;
   - a = (b * c;
   - c = a * ((b * c) + ba);

We will do the following exercises together during the tutorial:

Exercise 3

Consider the grammar with V = { S, A, B, C, Y, Z }, Σ = {x, y, z}, start symbol S, and production rules:

S → AS | AY
A → x, B→ y, C → z, Z → z
Y → BY | BZ
Z → CZ

Using the CYK algorithm, check whether the word xyyyzzyz is part of the language defined by the grammar. Fill in the table below:

x	y	y	y	z	z	y	z
A	B	B	B	C, Z	C, Z	B	C, Z
			Y	Z		Y	
		Y	Y				
	Y	Y					
S	Y						
S							

Exercise 4

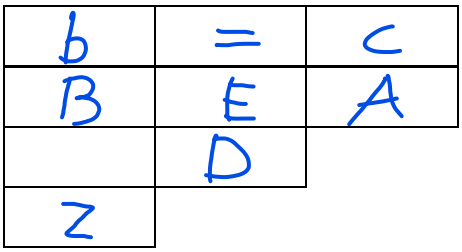
Consider the grammar with V = { S, Y, Z }, Σ = {x, y, z}, start symbol S, and production rules:

S → xS | xY
Y → yY | Zy
Z → Zz | z

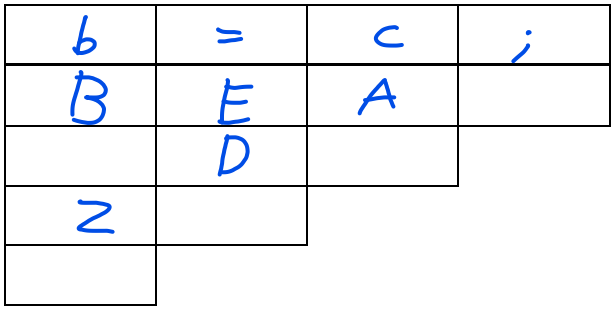
a) What type of the Chomsky hierarchy is this grammar? Restrict the type as much as possible, justify your answer.
b) Specify the associated language in set notation. What type is the language of?
c) Construct a Turing machine that accepts this language. Trap states may be omitted.
d) Convert the grammar to Chomsky normal form. Show the required steps.

Tables for exercise 2

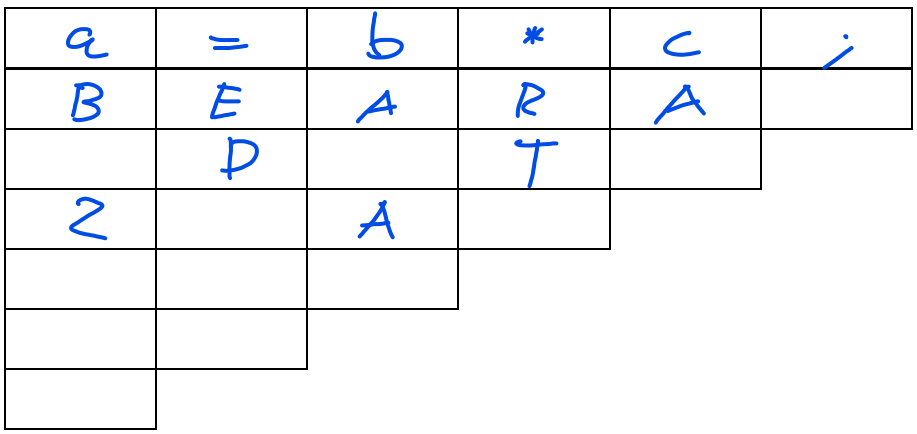
b = c



b = c;



a = b * c;



a = (b * c;

a	=	(	b	*	c	;
BE	Ö	A	R	A		
				T		

c = a * ((b * c) + ba);

C	=	a	*	(	(	b	*	c	)	+	b	a	)	;
B	E	A	R	Ö	Ö	A	R	A	Ü	Q	G	B	Ü	
											G	B	Ü	
							R	A	Ü	Q	A	Ü		
						A	T	Ü	Q	A	Ü			
						A	Ü	Q	A	Ü				
				Ö	Ö	Ä	Q	A	Ü					
				Ö	A	Q	A	Ü						
				Ö	A	P	Ü							
B	E	A	R	Ö	A	Ü								
			R	Ö	Ä									
			R	A										
B	E	A	T											
B	E	A												
B	D													
Z														