Deep Learning – Backpropagation

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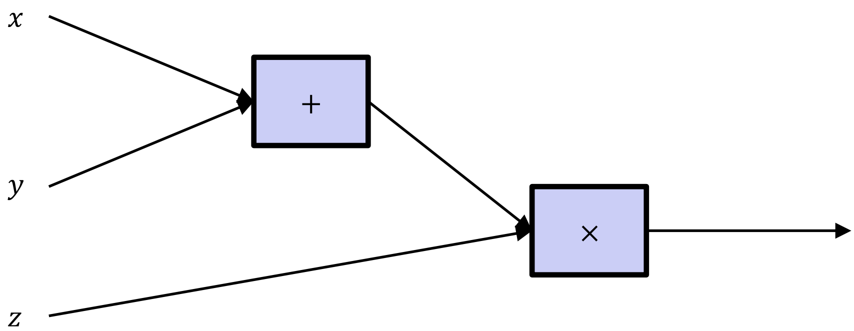
Prof. Dr. Jochen Schmidt

Technische Hochschule **Rosenheim**

1) Backpropagation – Simple Function

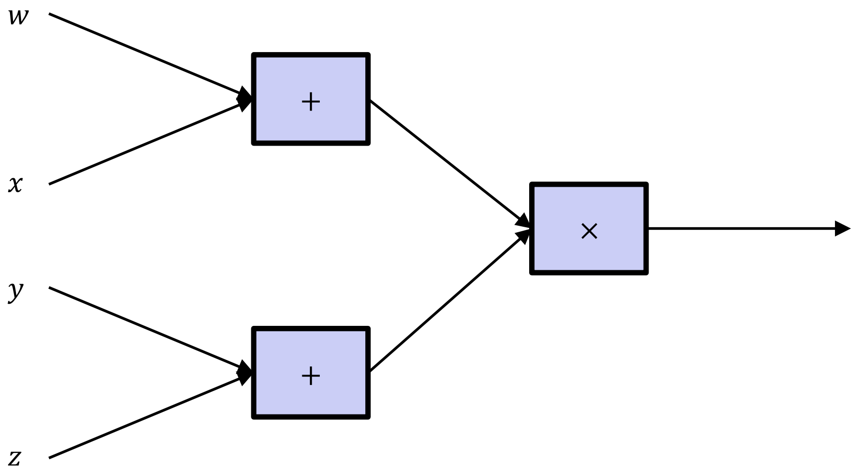
The following computational graph represents a function $f(x, y, z)$:



a) Write the function as an equation (× denotes multiplication).
b) Compute the value of $f(-2, 5, -4)$ from the graph using forward propagation.
c) Compute the value of $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$ for $x = -2$, $y = 5$, $z = -4$ from the graph using backward propagation.
d) Compute $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$ analytically. What are values of the partial derivatives when you plug in $x = -2$, $y = 5$, $z = -4$?

2) Backpropagation – A Slightly More Complex Function

The following computational graph represents a function $f(w, x, y, z)$.



a) Write the function as an equation (× denotes multiplication).
b) Compute the value of $f(4, 3, 2, 1)$ from the graph using forward propagation.
c) Compute the values of all partial derivatives at $(4, 3, 2, 1)$ from the graph using backward propagation.
d) Compute all partial derivatives analytically. What are values of the partial derivatives when you plug in $(4, 3, 2, 1)$?

3) Backpropagation – A Single Neuron

Let's now consider a single neuron with sigmoid activation and two inputs. Such a neuron computes the function

$$f(\boldsymbol{x}, \boldsymbol{w}) = \frac{1}{1 + e^{-(x_1 w_1 + x_2 w_2 - w_0)}}$$

a) Draw the computational graph of the function.

From here on, we consider the function at $x_1 = 2$, $x_2 = 4$, $w_1 = -3$, $w_2 = 2$, $w_0 = 1$.

b) Compute the function value from the graph using forward propagation.
c) Compute the values of all partial derivatives from the graph using backward propagation.
d) Calculate all partial derivatives analytically. Compare the result to the values of the analytical derivatives. Are they the same?