Technische Hochschule Rosenheim
Applied Artificial Intelligence - Bachelor
Analysis 1

WiSe 2021/22

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Homework 9: complex roots and derivatives

To submit: on Thursday, 09.12.2021, 9:30 a.m., online by the learning campus

Exercise 1 (7 pts.)

a) Determine all $n \in \mathbb{N}$ s.t. $(-8-8\sqrt{3}i)^n$ is a real number. $n = 3k \ , \ k \in \mathbb{N}_0$

b) Find all solutions of the algebraic equation

$$z^4 + 8 + 8\sqrt{3}i = 0,$$

expressed in Cartesian representation.

Exercise 2 (4 pts.)

Let $x > 0$.

a) Compute

$$\lim_{x \to 0} (x^x).$$

any number to the power of 0 is 1. so lim(x^x) = 1
x -> 0

b) Compute the derivative of

$$f(x) = x^x \quad \text{for } x > 0.$$

$f'(x) = \frac{d}{dx} e^{(x \ln(x))}$

$f'(x) = e^{(x \ln(x))} \cdot \left(1 \ln(x) + x \cdot \frac{1}{x}\right) = e^{x \ln(x)} \cdot (\ln(x) + 1) = x^x \cdot (\ln(x) + 1)$

Exercise 3 (4 pts.)

Show for $x \in \mathbb{R}^+$ and $n \in \mathbb{N}$

$$\left(\frac{d}{dx}\right)^n \ln(x) = \frac{(-1)^{n-1}(n-1)!}{x^n}.$$

Base
$n = 1$

$$\frac{d}{dx} \ln(x) = \frac{(-1)^{1-1} \cdot (1-1)!}{x^1} = \frac{1 \cdot 0!}{x} = \frac{1}{x} \checkmark$$

$$\exists n \in \mathbb{N}: \left(\frac{d}{dx}\right)^n \ln(x) = \frac{(-1)^{n-1} \cdot (n-1)!}{x^n}$$

Proof
$n \to n+1$

$$\left(\frac{d}{dx}\right)^{n+1} \ln(x) = \frac{(-1)^{n+1-1} \cdot (n+1-1)!}{x^{n+1}} \leftarrow \text{goal}$$

$$= \left(\frac{d}{dx}\right)^1 \left(\frac{d}{dx}\right)^n \ln(x) \overset{IA}{=} \frac{d}{dx} \frac{(-1)^{n-1} \cdot (n-1)!}{x^n}$$