Exercise assignment for the course Introduction to AI (Part I) in the Bachelor of AAI at Rosenheim University of Applied Sciences

Assignment 08 - First-Order Logic

Task 1

a) In a propositional logic formula, what does each variable represent? In a first-order logic formula, what does each variable represent?

Solution

In propositional logic, each variable (and formula) stands for a proposition, something that's either true or false. In a first-order logic formula, each variable stands for an object.

b) What is the difference between a predicate and a function?

Solution

Predicates produce propositions as output, and functions produce objects as output.

Task 2

Define an appropriate language and formalize the following sentences using FOL formulas:

1. All Students are smart.
2. There exists a student.
3. There exists a smart student.
4. Every student loves some student.
5. Every student loves some other student.
6. There is a student who is loved by every other student.
7. Bill is a student.
8. Bill takes either Analysis or Geometry (but not both).
9. Bill takes Analysis and Geometry.
10. Bill doesn’t take Analysis.
11. No students love Bill.

Solution

1. $\forall x.(Student(x) \rightarrow Smart(x))$
2. $\exists x.Student(x)$
3. $\exists x.(Student(x) \wedge Smart(x))$
4. $\forall x.(Student(x) \rightarrow \exists y.(Student(y) \wedge Loves(x, y)))$
5. $\forall x.(Student(x) \rightarrow \exists y.(Student(y) \wedge \neg(x = y) \wedge Loves(x, y)))$
6. $\exists x.(Student(x) \wedge \forall y.(Student(y) \wedge \neg(x = y) \rightarrow Loves(y, x)))$
7. $Student(Bill)$
8. $Takes(Bill, Analysis) \leftrightarrow \neg Takes(Bill, Geometry)$
9. $Takes(Bill, Analysis) \wedge Takes(Bill, Geometry)$
10. $\neg Takes(Bill, Analysis)$

11. ¬∃x.(Student(x) ∧ Loves(x, Bill))

Task 3

Define an appropriate language and formalize the following sentences in FOL:

1. "A is above C, D is on E and above F."
2. "A is green while C is not."
3. "Everything is on something."
4. "Everything that is free has nothing on it."
5. "Everything that is green is free."
6. "There is something that is red and is not free."
7. "Everything that is not green and is above B, is red."

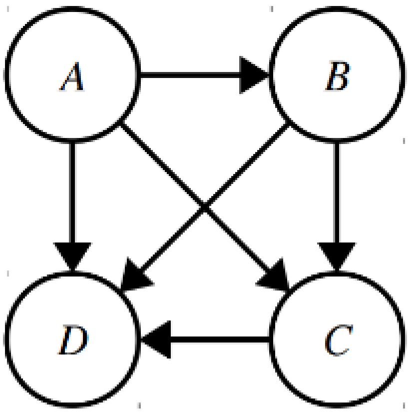
- What are the constants?
- What are the predicates?
- Which axioms di exist?

Solution

- Constants: A, B, C, D, E, F
- Predicates: On2, Above2, F ree1, Red1, Green1.
- Axioms
    1. "A is above C, D is above F and on E." φ1 : Above(A, C) ∧ Above(E, F) ∧ On(D, E)
    2. "A is green while C is not." φ2 : Green(A) ∧ ¬Green(C)
    3. "Everything is on something." φ3 : ∀x∃y.On(x, y)
    4. "Everything that is free has nothing on it." φ4 : ∀x.(F ree(x) → ¬∃y.On(y, x))
    5. "Everything that is green is free." φ5 : ∀x.(Green(x) → F ree(x))
    6. "There is something that is red and is not free." φ6 : ∃x.(Red(x) ∧ ¬F ree(x))
    7. "Everything that is not green and is above B, is red." φ7 : ∀x.(¬Green(x) ∧ Above(x, B) → Red(x))

Task 4

The following diagram represents a set of people named A, B, C, and D. If there's an arrow from a person x to a person y, then person x loves person y. We'll denote this by writing *Loves(x, y)*. Below is a list of formulas in first-order logic about the picture. In those formulas, the letter P represents the set of all the people. For each formula, determine whether that formula is true or false.



a) $\forall x \in P. \forall y \in P. (Loves(x, y) \lor Loves(y, x))$

> This statement is false. Pick x and y to be A. Then Loves(x, y) is false and Loves(y, x) is false. Remember that quantifiers can range over the same objects at the same time!

b) $\forall x \in P. \forall y \in P. (x \neq y \to Loves(x, y) \lor Loves(y, x))$

> This statement is true – given any pair of two people in this diagram, one of them loves the other.

c) $\forall x \in P. \forall y \in P. (x \neq y \to (Loves(x, y) \leftrightarrow \neg Loves(y, x)))$

> This statement is true. Given any pair of two people, exactly one of them loves the other, so either Loves(x, y) will be true, or Loves(y, x) will be true, but not both. The biconditional in this case will therefore always evaluate to true.

d) $\exists x \in P. \forall y \in P. (Loves(x, y))$

> This statement is false – no one loves everyone, because no one loves themselves.

e) $\exists x \in P. \forall y \in P. (x \neq y \to Loves(x, y))$

> This statement is true – pick x to be person A.

f) $\forall y \in P. \exists x \in P. (Loves(x, y))$

> This statement is false. No one loves person A.

g) $\forall y \in P. \exists x \in P. (x \neq y \land Loves(x, y))$

> This statement is still false – no one loves person A.

h) $\exists x \in P. \forall y \in P. (\neg Loves(x, y))$

> This statement is true – pick x to be person D.