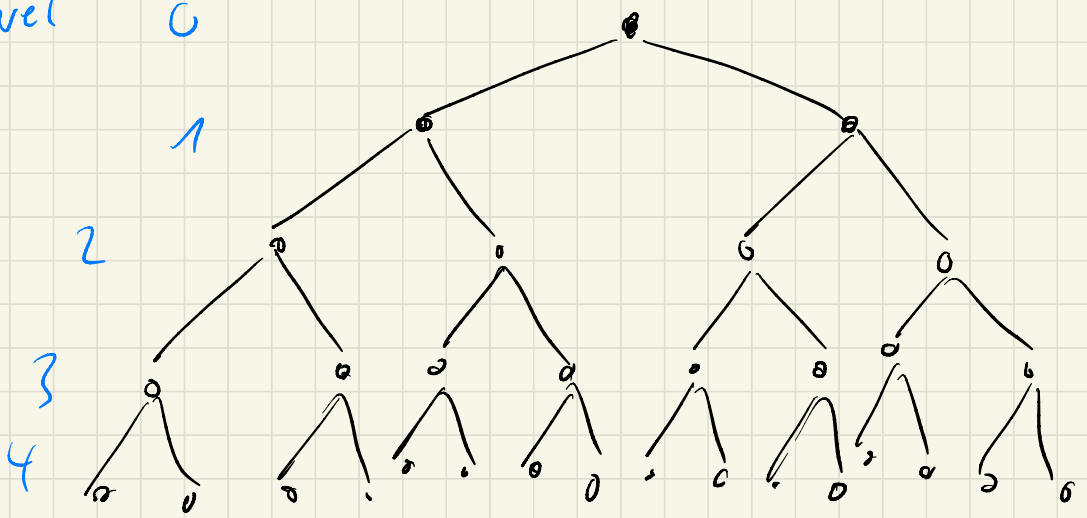
13/17) Level



nodes: $2^0 = 1$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

2) degree

$n-1$

$n = 1$: •

$n = 2$: •—•

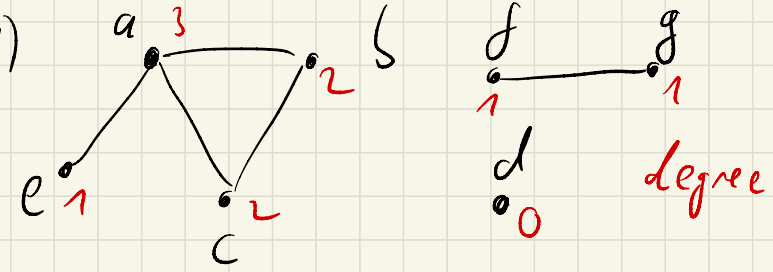
$n = 3$:

$\binom{3}{2} = \frac{3!}{2!1!} = 3$

$n = 4$:

edges $\binom{n}{2} = \frac{1}{2} n(n-1)$

3)



3 connected components

cycle: $a - b - c - a$

separating vertices: a

4) top left $\simeq$ top right

possible mappings:

$a - a$	$a - a$
$e - d$	$e - c$
$b - c$	$b - d$
$d - b$	$d - e$
$c - e$	$c - b$

bottom left $\simeq$ bottom right

mapping, e.g. (there are many):

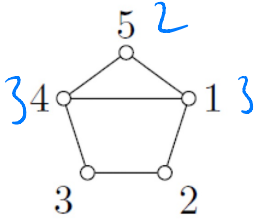
$a - a$	$a - b$
$b - c$	$b - e$
$c - e$	$c - c$
$d - b$	$d - a$
$e - d$	$e - d$

(mapping of edges is induced by mapping of nodes)

5)

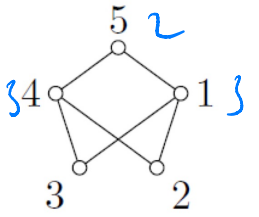
a) Determine the sequence of degrees for the following six graphs.

b) Give an isomorphism for every graph to one of the other five graphs.



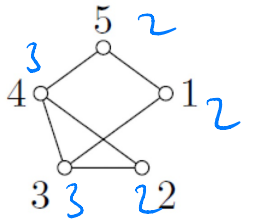
P_1

(2, 2, 2, 3, 3)



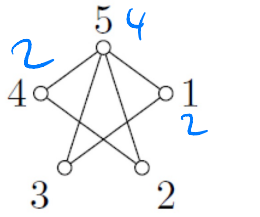
P_2

(2, 2, 2, 3, 3)



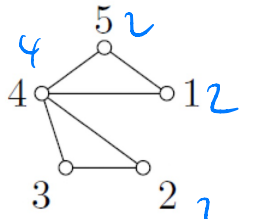
P_3

(2, 2, 2, 3, 3)



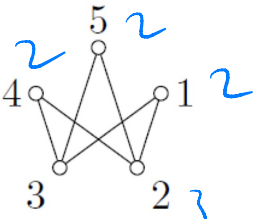
P_4

(2, 2, 2, 2, 4)



P_5

(2, 2, 2, 2, 4)



P_6

(2, 2, 2, 3, 3)

$P_4 \simeq P_5$

e.g.:

5 – 4

1 – 1

3 – 5

2 – 2

4 – 3

$P_1 \simeq P_3$

5 – 2

4 – 4

1 – 3

3 – 5

2 – 1

$P_2 \simeq P_6$

5 – 5

4 – 3

1 – 2

2 – 4

3 – 1

$$A = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

to: x_1, x_2, x_3, x_4 — from: x_1, x_2, x_3, x_4

$$A = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

to: x_1, x_2, x_3, x_4 — from: x_1, x_2, x_3, x_4

$$A = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

$$A^2 = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

→ there are paths of length 2
from x_1 to x_4
from x_3 to x_2

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$A^3 = A^4 = 0$

→ there are no paths of length 3 or more

→ there are no cycles