Remarks:

a) $$\lim_{x\to 0+} f(x) = \lim_{t\to\infty} f\left(\frac{1}{t}\right)$$

b) $$\lim_{x\to 0-} f(x) = \lim_{t\to -\infty} f\left(\frac{1}{t}\right)$$

$$a_k \in \mathbb{R},\quad k = 0, 1, \dots, n$$

Claim: $$f(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n = \sum_{k=0}^{n} a_k x^k \Leftarrow \text{a } \underline{\text{polynomial}}$$

is continuous at every point $x_0 \in \mathbb{R}$.

Proof:

$$\lim_{x\to x_0} f(x) = \lim_{x\to x_0}\left(\sum_{k=0}^{n} a_k x^k\right) \overset{\text{rules for limits of sums}}{=} \sum_{k=0}^{n} \lim_{x\to x_0} (a_k x^k) =$$

$$\overset{\text{rules for limits}}{=} \sum_{k=0}^{n} a_k \lim_{x\to x_0} x^k \overset{\text{rules for limits}}{=} \sum_{k=0}^{n} a_k \big(\underbrace{\lim_{x\to x_0} x}_{= x_0}\big)^k$$

$$= \sum_{k=0}^{n} a_k x_0^k = f(x_0)$$

$\Rightarrow$ Thus any polynomial is continuous at all $x_0 \in \mathbb{R}$.

$\square$