Th. (<u>Powers of complex numbers</u>) $\varphi := \phi$

For any $z \in \mathbb{C}$ with polar representation $z = r e^{i\varphi}$, we have:

$$z^n = r^n (\cos(n\varphi) + i \sin(n\varphi))$$

Special case: $r = |z| = 1$: $z^n = \cos(n\varphi) + i \sin(n\varphi)$ $\leftarrow$ Moivre formula

Proof

$$z^n = (r e^{i\varphi})^n = r^n (e^{i\varphi})^n \overset{\text{funct. eq.}}{=} r^n e^{in\varphi}$$

$$\overset{\text{Euler formula}}{=} r^n (\mathrm{Re}(e^{in\varphi}) + i\, \mathrm{Im}(e^{in\varphi}))$$

$$= r^n (\cos(n\varphi) + i \sin(n\varphi)) \quad \square$$

Application: $z^n - a = 0$ for a given complex number $a = r e^{i\varphi}$, $\varphi \in (-\pi, \pi]$

$$\Rightarrow z_k = \sqrt[n]{r}\, e^{i \frac{\varphi + 2k\pi}{n}} \qquad k = 0, \dots, n-1$$

n roots!