Technische Hochschule Rosenheim
Applied Artificial Intelligence - Bachelor
Analysis 1

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Homework 3: induction

To submit: on Thursday, 28.10.2021, 9:30 a.m., online by the learning campus

Exercise 1

Show:

a) (4 pts.) for all $n \in \mathbb{N}$: $\sum_{k=1}^{n}(4k-1) = 2n^2+n$

b) (4 pts.) for all $n \in \mathbb{N}$: $\sum_{k=1}^{n-1}\frac{1}{k\cdot(k+1)} = 1-\frac{1}{n}$

Exercise 2 (4 pts.)

Proof the following statement by complete induction:

For all $n \in \mathbb{N}$: n^3+2n is divisible by 3

$\sum_{k=1}^{n}(4k-1) = 2n^2+n \quad n \in \mathbb{N}, n \geq 1$

Base $n=1$ $\quad \sum_{k=1}^{1}(4\cdot 1-1) = 2\cdot 1^2+1$

$3 = 3$ ✓

$\exists n \in \mathbb{N}: \sum_{k=1}^{n}(4k-1) = 2n^2+n$

$n \to (n+1)$ Proof $\quad \sum_{k=1}^{n+1}(4k-1) = \underbrace{2\cdot(n+1)^2+(n+1)}_{\text{goal}}$

$\sum_{k=1}^{n+1}(4k-1) = \sum_{k=1}^{n}(4k-1) + \overbrace{(4(n+1)-1)}^{k \to n}$

$= 2n^2+n + (4n+4-1)$

$= 2n^2+4n+2 + (n+1)$

$= 2(n+1)^2+(n+1)$

qed

Base $n=1$ $\quad \sum_{k=1}^{0}\frac{1}{1\cdot(1+1)} = 1-\frac{1}{1}$

$\left(\sum_{k=1}^{0} x = 0\right) \quad 0 = 0$ ✓

$\exists n \in \mathbb{N}: \sum_{k=1}^{n-1}\frac{1}{k(k+1)} = 1-\frac{1}{n}$

$n \to (n+1)$ Proof $\quad \sum_{k=1}^{n}\frac{1}{k(k+1)} = 1-\frac{1}{(n+1)}$ goal

$\sum_{k=1}^{n}\frac{1}{k(k+1)} = \sum_{k=}^{n-1}\frac{1}{(n-1)n} + \frac{1}{n(n+1)}$

$(k \to (n-1))$

$= 1-\frac{1}{n}+\frac{1}{n(n+1)}$

$=$?

$=$

$= 1-\frac{1\ n}{(n+1)n}$

$= 1-\frac{1}{(n+1)}$

maybe qed

$n^3 + 2n \% 3 = 0$ $\quad n \in \mathbb{N} \quad n \geq 1$

Base
$n=1 \quad 1^3 + 2 \cdot 1 = 3$ ✓

$\exists n \in \mathbb{N}: n^3 + 2$ divisible by 3

$n \to (n+1)$

Proof

$(n+1)^3 + 2(n+1)$

$= n^3 + 3n^2 \cdot 1 + 3n \cdot 1^2 + 1^3 + 2n + 2$

$= n^3 + 3n^2 + 5n + 3$

↳ always divisible

✓

3 times any number is again divisible ($\geq 1, \in \mathbb{N}$)

$= n^3 (+ 3n^2) + 5n (+ 3)$

$n^3 + 5n$ is divisible by 3 $\quad$ qed

$n=1$	$1 + 5 = 6$
$n=2$	$8 + 10 = 18$
$n=3$	$27 + 15 = 42$
$n=4$	$64 + 20 = 84$
$n=5$	$125 + 25 = 150$