Technische Hochschule Rosenheim
Applied Artificial Intelligence - Bachelor
Analysis 1

WiSe 2021/22

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Exercise 7: functions II

$$\sqrt[11]{e^{\exp(\ln(11)^5):11^2} \cdot e^{11x}} = \sqrt[11]{e^{(11^5:11^2)} \cdot e^{11x}}$$

$$\sqrt[11]{e^{11^3} \cdot e^{11x}} = \sqrt[11]{e^{11^3+11x}} = e^{11^2+x}$$

Exercise 20

Let $x \in \mathbb{R}$. Simplify:

a) $\sqrt[11]{\exp\left(\frac{\exp(5\ln(11))}{121}\right)\exp(11x)}$

11root(exp(11^5/11^3)*exp(11x))
=exp(121 + x)

b) $\log_2 4^5 + \log_4 8$

log2(2^2)^5 + log(4)^3/2
=23/2

c) $\log_3 12 + \log_{10}(\log_{10} 10000000000)$

log3(3*4) + log10(10)^10
=2+log3(4)

$$\log_2(4)^5 = \log_2 (2^2)^5$$
$$= \log_2(2^{10}) = 1 \cdot 10 = 10$$
$$\log_4(8) = \log_4(4^{1,5}) = 1,5$$
$$10 + 1,5 = 11,5$$
$$\log_3(3 \cdot 4) = \log_3(3) + \log_3(4)$$
$$= +1 + \log_3(4)$$
$$\log_{10}(10^{10}) = 10$$
$$\log_{10} \quad = 1$$
$$\to \log_3(4) + 2$$

Exercise 21

Solve in $\mathbb{C}$:

a) $z^2 + 8z + 25 = 0$

b) $z^2 + 5 = 0$

c) $z^3 + 2z - 3 = 0$

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a) z1/2 = (-8+-sqrt((-8)^2-4*1*25))/2*1
z1/2 = (-8+-sqrt(-36))/2
//sqrt(-36) = -6*i
z1 = (-8 + 6i)/2
z2 = (-8 - 6i)/2

b) z^2=-5
z1/2 = sqrt(-5)
//sqrt(-5) = -sqrt(5)*i
z1 = -sqrt(5)*i
z2 = +sqrt(5)*i

c) z^3 + 2z = 3 (nullstelle erraten)
z = 1 (1^3 + 2*1 = 3)
 (z^3 + 2z - 3) / (z - 1) = z^2 + z + 3
-(z^3 - z^2)
  z^2 + 2z
-(z^2 - z)
        3z - 3
       -(3z - 3)
             0
```

Exercise 22

Consider the complex number $z = -3 + 3i$ and the following statements:

(1) $|z| := \sqrt{z\bar{z}} = 3\sqrt{2}$

(2) $\bar{z} = 3 + 3i$

(3) $\mathrm{Re}(z) = 3$

(4) $\mathrm{Re}(z) - i\,\mathrm{Im}(z) = -3(1+i)$

(5) $\dfrac{1}{z} = \dfrac{\bar{z}}{|z|^2}$.

Which are true?

Exercise 23

Due to our first version of the intermediate value theorem (Bolzano theorem on zeros), the following algorithm allows to find zeros:

(0) Given: function f, interval $[a,b]$ with $a<b$, *maxit* maximal number of iterations, *eps* given tolerance. Set $i=0$ (counter of iterations).

(1) If $f(a)f(b)\geq 0$ then print "method not applicable", STOP.

(2) While $i\leq maxit$ do
$c:=\frac{a+b}{2}$
If $|f(c)|<tol$ or $(b-a)/2<tol$ then print "solution:" c, STOP.
$i:=i+1$
If $f(c)f(a)\geq 0$ then $a:=c$ else $b:=c$.
End while

(3) Print "method failed (increase maxit)", STOP.

It is called **bisection search**.

Implement the bisection search, e.g., in MATLAB. The MATLAB function receives the function f and a start interval $[a,b]$ as input and returns as output a zero of the function within the interval, if successful. As stopping criterion we consider a sensitive number *maxit* of iterations.

By this means find a zero of the function $f(x)=-\exp(-x)+x$ in $[0,1]$. Alternatively, you may use a calculator.

Please state all intervals $[a_i,b_i]$ and the corresponding function values for each iteration i.

f(0) = -1
f(1) = 0.632
f(0.5) = -0.107
-> 0 is in right half

f(0.75) = 0.278
-> 0 is below

f(0.625) = 0.0897
-> 0 is below

f(0.6) = 0.0512
-> 0 is below

f(0.55) = -0.027
-> 0 is above

f(0.57) = 0.005
-> 0 is below

f(0.56) = -0.011
-> 0 is above

f(0.565) = -0.003
-> 0 is above

f(0.5675) = 0.001
-> 0 is below

close enough