Technische Hochschule Rosenheim
Applied Artificial Intelligence - Bachelor
Analysis 1

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Exercise 13: improper integrals and revision

Proof : $f^{(n+1)}(x) = 2^{n+1}\cos\left(2x + \frac{(n+1)\pi}{2}\right)$

$n \to n+1$

Exercise 40

We consider the function

$$f(x) = \cos(2x), \quad x \in \mathbb{R}.$$

$f^{(n+1)}(x) = 2^n \cdot 2\cos\left(2x + \frac{n\pi + \pi}{2}\right)$

Show that

$$f^{(n)}(x) = 2^n \cos\left(2x + n\frac{\pi}{2}\right) \quad \text{for all } n \in \mathbb{N}.$$

$= 2^n \cdot 2\cos\left(2x + \frac{n\pi}{2} + \frac{\pi}{2}\right)$

$= 2^n \cdot (-2)\sin\left(2x + \frac{n\pi}{2}\right)$

Hint: We have $\cos(x + \frac{\pi}{2}) = -\sin(x)$.

qed

Exercise 41

For each of the following improper integrals, determine whether it converges, and determine its value if it does.

div.

a) $\int_1^5 \frac{1}{t\ln t}\,dt$ — $\lim \frac{1}{t\ln(t)} = 0^+$ — $F(x) = \left[\ln(t \cdot \ln(t))\right]_1^5 = \ln(5\ln(5)) - -\infty$

b) $\int_0^\infty \frac{5}{32+2t^2}\,dt$ — $\lim \frac{5}{32+2t^2} = 0^+$ — $F(x) = 5\left[\ln(32+2t^2)\right]_0^\infty = 5\infty - 25$ div.

c) $\int_3^\infty \frac{\ln x}{x^2}\,dx$ — $\lim \frac{\ln(x)}{x^2} = 0^+$ — $F(x) = \frac{\frac{1}{x}\cdot x^2 - 2x\ln(x)}{{x^2}^2} = \frac{x - 2x\ln(x)}{x^4}$

$= \frac{1-2\ln(x)}{x^3} \;\Big|\; \left[\frac{1-2\ln(x)}{x^3}\right]_3^\infty =$

$0^- - \frac{1-2\ln(3)}{27}$ conv.

Exercise 42

Determine the mean value of

$$f(x) = x\ln(x)$$

over the interval $0 \le x \le 4$.

Hint: to calculate an indeterminate limit, rearrange and use L'Hôpital's rule.

Exercise 43

Complete the following truth table (f for false, t for true):

A	B	C	D	$(A \wedge B) \vee C$	$\overline{(A \wedge C)} \wedge D$	$(C \wedge D) \Rightarrow (A \vee B)$
		f	t	t		
	f		t		f	
						f