HW 10
Ana 1

/1a

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\left(g(x) - h(x)\right)$$

$$\overset{\text{"0/0"}}{=} \lim_{x\to 0}\left(\frac{1}{\sin(x)}\right) - \lim_{x\to 0}\overset{\text{"0/0"}}{\left(\frac{1}{x}\right)}$$

L'Hôpital $$= \lim_{x\to 0}\left(\frac{0}{\cos(x)}\right) - \lim_{x\to 0}\left(\frac{0}{1}\right)$$

$$= \frac{0}{1} - \frac{0}{1} = 0$$

b)

$$\lim_{x\to 0} x\cot(x) = \lim_{x\to 0} x\cdot\frac{1}{\tan(x)} = \lim_{x\to 0}\left(\frac{x}{\tan(x)}\right) \text{"0/0"}$$

L'Hôpital $$= \lim_{x\to 0}\frac{1}{\frac{1}{\cos^2(x)}} = \lim_{x\to 0}\frac{\cos^2(x)}{1} = 1^2 = 1$$

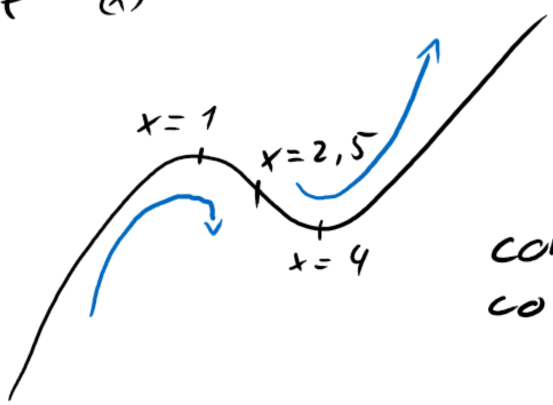
/2a $$f(x) = \frac{1}{6}x^3 - \frac{5}{4}x^2 + 2x + 3$$

$$f'(x) = \frac{1}{2}x^2 - \frac{10}{4}x + 2 \quad \text{—} \quad \frac{2{,}5 \pm \sqrt{6{,}25 - 4\cdot\frac{1}{2}\cdot 2}}{1} \quad x_1 = 4,\ x_2 = 1$$

$$f''(x) = x - 2{,}5 \quad \text{—} \quad x = 2{,}5$$

$$f'''(x) = 1$$

$$f^{(4)}(x) = 0$$



smi for $x \in]-\infty; 1]$
smd for $x \in [1; 4]$
smi for $x \in [4; \infty[$
concave for $x \in]-\infty; 2{,}5]$
convex for $x \in [2{,}5; \infty[$

b)

$f(x) = x e^{-x}$

$f'(x) = 1 \cdot e^{-x} + x \cdot - e^{-x}$

$= -xe^{-x} + e^{-x}$ $\quad$ $x = 1$

$f''(x) = -1 \cdot e^{-x} + -x \cdot - e^{-x} + - e^{-x}$

$= -e^{-x} + xe^{-x} - e^{-x}$

$= -2e^{-x} + xe^{-x}$ $\quad$ $x = 2$

smi for $x \in]-\infty; 1]$
smd for $x \in [1; \infty[$

concave for $x \in]-\infty; 2]$
convex for $x \in [2; \infty[$

13 goal: $\sqrt{xy} \leq \frac{x+y}{2}$ for all $x, y \in \mathbb{R}^+$

show that logarithm is concave:

$\frac{d}{dx} \ln(x) = \frac{1}{x}$

$\frac{d}{dx} \frac{1}{x} = \frac{0_x - 1 \cdot 1}{x^2} = \frac{-1}{x^2}$

$\frac{-1}{x^2} = 0 \rightarrow -1 = x^2 \rightarrow x = \sqrt{-1}$

no $\ln(x)'' = 0$ in $\mathbb{R}$,
so it's either strictly convex or strictly concave.
its graph shows that logarithm is concave.

$$f(x+\lambda\cdot(y-x)) \geq f(x) + \lambda\cdot(f(y)-f(x)) \qquad \lambda \in [0;1]$$

with $f(x) = \ln(x)$:

$$\ln(x+\lambda\cdot(y-x)) \geq \ln(x) + \lambda(\ln(y)-\ln(x)) \qquad | e^{\,}$$

$$e^{\ln(x+\lambda\cdot(y-x))} \geq e^{\ln(x)+\lambda\cdot(\ln(y)-\ln(x))}$$

$$x+\lambda\cdot(y-x) \geq x \cdot y^{\lambda} \cdot x^{-\lambda}$$

$$x+\lambda y-\lambda x \geq x^{1-\lambda}\cdot y^{\lambda}$$

for lambda $= \frac{1}{2}$ we get:

$$x + \frac{y}{2} - \frac{x}{2} \geq x^{\frac{1}{2}} \cdot y^{\frac{1}{2}}$$

$$\frac{y}{2} + \frac{x}{2} \geq (xy)^{\frac{1}{2}}$$

$$\frac{y+x}{2} \geq \sqrt{xy} \quad \text{qed}$$