Homework 6: Cauchy product, continuity

To submit: on Thursday, 18.11.2021, 9:30 a.m., online by the learning campus

Exercise 1 (9 pts.)

a) Compute the Cauchy product of the series

$$\sum_{k=0}^{\infty} \frac{1}{9^k}$$

with the series

$$\sum_{k=0}^{\infty} \frac{1}{3^k}.$$

$c_n = \sum_{k=0}^{n} \frac{1}{9^k} \cdot \frac{1}{3^{n-k}}$

b) Consider the alternating series $\sum_{k=0}^{\infty} r_k$ with

$$\{r_k\}_{k\in\mathbb{N}_0} = \frac{(-1)^k}{\sqrt{k+1}}.$$

Show that the Cauchy product of $\sum_{k=0}^{\infty} r_k$ with itself is not absolutely convergent.

Please explain why this is no contradiction to the result derived in the lecture!

Exercise 2 (3 pts.)

Consider the fractional rational function

$$f : \mathbb{R} \setminus \{-1\} \to \mathbb{R}, x \mapsto f(x) = \frac{x^2-1}{x+1}.$$

Explain why f is not continuous !

How could you derive from f a new function g by complementing $g(-1)$ such that the function g becomes continuous in $x=-1$, too?

Exercise 3 (3 pts.)

Demonstrate that there exists a solution $x \in [0,1]$ for the equation

$$\exp(x) = \frac{1}{x}.$$