Homework 5: series

To submit: on Thursday, 11.11.2021, 9:30 a.m., online by the learning campus

Exercise 1 (4 pts.)

Prove:

If $\{a_n\}_{n\geq n_0}$ is definitely divergent, then $\{1/a_n\}_{n\geq n_0}$ is a zero sequence.

$\rightarrow \lim\limits_{n\to\infty} a_n = \infty$ $\qquad \left\{\frac{1}{\lim\limits_{n\to\infty} a_n}\right\} \rightarrow \lim\limits_{n\to\infty}\left(\frac{1}{a_n}\right) = 0$

Exercise 2 (3 pts.)

Show that the series

$$\sum_{n=1}^{\infty} \frac{1}{(n+1)n}$$

$$\Rightarrow \frac{1+n-n}{(n+1)n} = \frac{1+n}{(n+1)n} - \frac{n}{(n+1)n}$$

$$= \underbrace{\frac{1}{n}}_{\lim\limits_{n\to\infty}=0} - \underbrace{\frac{1}{n+1}}_{\lim\limits_{n\to\infty}=0} \qquad \lim_{n\to\infty}(0-0) \rightarrow \text{conv.}$$

converges by computing its limit.

Hint: rewrite the fraction s.t. you obtain a telescopic sum.

Exercise 3 (8 pts.)

Discuss the (absolute) convergence of the following series:

a) any series with a^k where a is positive but smaller than 1, converges absolutely. $\quad \displaystyle\sum_{k=0}^{\infty} \Big(\underbrace{\frac{1}{7}}_{=:a}\Big)^k$ [2 pt.]

b) (-1)^k only changes the sign for k element N. ((-1)^k*k)/(k^2+1) has an alternating sign and the sum is absolutely convergent but the limit is different when adding absolute values. $\quad \displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k k}{k^2+1}$ [3 pt.] $\qquad \left|\frac{a_3}{a_2}\right| = 0{,}75$

c) the ratio test only results in a number < 1 for k >= 4 which doesn't show the whole behaviour but the series still converges absolutely. $\quad \displaystyle\sum_{k=1}^{\infty} \frac{1+k^4}{1+3^k}$ [3 pt.]

Hint: in c) you may use the ratio test.

Remark: It is not required to compute the limits.