Technische Hochschule Rosenheim WiSe 2021/22
Applied Artificial Intelligence - Bachelor
Analysis 1

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Exercise 2 (live tutorial): bounds, inequalities, and logic

Exercise 5
Let A,B,C be three statements. Please check, whether the following linked statements are always true.

a) $(A \Longleftrightarrow C) \Longrightarrow ((A \Longleftrightarrow B) \wedge (B \Longleftrightarrow C))$

b) $((A \Longleftrightarrow B) \wedge (B \Longleftrightarrow C)) \Longrightarrow (A \Longleftrightarrow C)$

c) $((A \Longleftrightarrow B) \wedge (B \Longleftrightarrow C)) \Longleftrightarrow (A \Longleftrightarrow C)$

Solution for exercise 5
$A_1 := (A \Longleftrightarrow C) \Longrightarrow ((A \Longleftrightarrow B) \wedge (B \Longleftrightarrow C))$
$A_2 := ((A \Longleftrightarrow B) \wedge (B \Longleftrightarrow C)) \Longrightarrow (A \Longleftrightarrow C)$
$A_3 := ((A \Longleftrightarrow B) \wedge (B \Longleftrightarrow C)) \Longleftrightarrow (A \Longleftrightarrow C)$

A	B	C	$A \Leftrightarrow B$	$B \Leftrightarrow C$	$((A \Longleftrightarrow B) \wedge (B \Longleftrightarrow C))$	$A \Leftrightarrow C$	A_1	A_2	A_3
w	w	w	w	w	w	w	w	w	w
w	w	f	w	f	f	f	w	w	w
w	f	w	f	f	f	w	f	w	f
w	f	f	f	w	f	f	w	w	w
f	w	w	f	w	f	f	w	w	w
f	w	f	f	f	f	w	f	w	f
f	f	w	w	f	f	f	w	w	w
f	f	f	w	w	w	w	w	w	w

Thus only statement A_2, i.e. part b), is always true.

Exercise 6
Determine the solution sets of the following inequalities and represent the solution sets graphically:

a) $|x-5|-|x| \geq 4$

b) $2|x-1|+|y-2| \geq 1$

Solution for exercise 6

a) $|x-5|-|x| \geq 4$

Case-by-case analysis:

$$\begin{aligned} x<0:&\ 5-x+x\geq 4 \Leftrightarrow 5\geq 4 \text{ always true}\\ 0\leq x<5:&\ 5-x-x\geq 4 \Leftrightarrow 1\geq 2x \Leftrightarrow x\leq \frac{1}{2}\\ 5\leq x:&\ x-5-x\geq 4 \Leftrightarrow -5\geq 4 \text{ being unsolvable}\end{aligned}$$

The solution set

$$\mathbb{L}=\{x\in\mathbb{R}\,|\,x\leq 1/2\}$$

is the part of number line from the left until $x=1/2$ (inclusive).

b) $2|x-1|+|y-2|\geq 1$

Case-by-case analysis:

$$\begin{aligned} x<1\wedge y<2:&\ 2(1-x)+2-y\geq 1 \Leftrightarrow -2x-y\geq -3 \Leftrightarrow y\leq 3-2x\\ x<1\wedge 2\leq y:&\ 2(1-x)+y-2\geq 1 \Leftrightarrow -2x+y\geq 1 \Leftrightarrow y\geq 1+2x\\ 1\leq x\wedge y<2:&\ 2(x-1)+2-y\geq 1 \Leftrightarrow 2x-y\geq 1 \Leftrightarrow y\leq -1+2x\\ 1\leq x\wedge 2\leq y:&\ 2(x-1)+y-2\geq 1 \Leftrightarrow 2x+y\geq 5 \Leftrightarrow y\geq 5-2x\end{aligned}$$

The solution set

$$\mathbb{L}=\overline{\left\{(x,y)\in\mathbb{R}^2\,\middle|\,\left(\frac{1}{2}<x<1,\,3-2x<y<1+2x\right)\vee\left(1<x<\frac{3}{2},\,-1+2x<y<5-2x\right)\right\}}$$

is the subset (with border lines) of $\mathbb{R}^2$ excluding an **open** rhombus with corners $(0.5,2)$, $(1,1)$, $(1.5,2)$, and $(1,3)$.

Exercise 7

Determine the supremum in $\mathbb{R}$ of the following sets:

a)

$$A=\left\{1-\frac{1}{n}\,\middle|\,n\in\mathbb{N}\right\}$$

b)

$$B=\left\{-\frac{1}{2^n}\,\middle|\,n\in\mathbb{N}\right\}$$

c)

$$C=\left\{1+(-1)^n-\frac{1}{n}\,\middle|\,n\in\mathbb{N}\right\}$$

Solution for exercise 7

a) Since $1/n$ tends to 0 for large n, we see that

$$\sup(A)=1.$$

b) Since 2^n tends to 0 for large n, we see that

$$\sup(B)=0.$$

c) We see that $(-1)^n$ "oscillates" between 1 for n even and -1 for n odd.
Using the result from A, we get ("convergence" is not required!)

$$\sup(C) = 2.$$