Harmonic series

then it converges

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = 1 + \frac{1}{2} + \underbrace{\frac{1}{3} + \frac{1}{4}}_{\geq \frac{1}{2}} + \underbrace{\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}}_{\geq \frac{1}{2}} + \dots$$

$$\geq 1 + \underbrace{\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \dots}_{\text{infinite number of times}}$$

$$\geq 1 + \lim_{\ell \to \infty} \frac{\ell}{2}$$

$\Rightarrow$ harmonic series definitely diverges to $+\infty$

Geometric series

$$\underbrace{(1-q)}_{\neq 0} \cdot \sum_{k=0}^{n} q^k = (1-q)(\underset{q^0}{1} + \underset{q^1}{q} + q^2 + \dots + q^n)$$

$$= 1 + \cancel{q} + \cancel{q^2} + \dots + \cancel{q^n}$$

telescopic sum

$$- \cancel{q} - \cancel{q^2} + \dots - \cancel{q^n} - q^{n+1}$$

$$= 1 - q^{n+1}$$

$$\Leftrightarrow \sum_{k=0}^{n} q^k = \frac{1-q^{n+1}}{1-q} \xrightarrow{n \to \infty} \sum_{k=0}^{\infty} q^k = \frac{1}{1-q}$$

if $|q| < 1$ because we need: $\lim_{n \to \infty} q^{n+1} = 0$