Technische Hochschule Rosenheim — WiSe 2021/22
Applied Artificial Intelligence - Bachelor
Analysis 1

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Exercise 6: functions: limits and continuity

Exercise 17
We consider a connection of n identical voltage sources in series (e.g., electrical batteries) in a circuit with one consumer.
Each of the voltage sources has an interior ohmic resistance R_i and yields a source voltage U_q. Hence the total voltage is $U_0 = nU_q$.
The consumer has an ohmic resistance R_a.

Compute the resulting current $I(n)$ in the circuit as function of the number of voltage sources. Plot $I(n)$.
What is the limit $I(n)$ as n tends to infinity? Remark: The latter is the so-called short-circuit current I_{sc}.

Note that according to the Kirchhoff laws, the total ohmic resistance is

$$R_g = nR_i + R_a$$

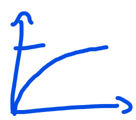
$$I(n) = \frac{nU_q}{nR_i + R_a} \quad \Big| \cdot \frac{\frac{1}{n}}{\frac{1}{n}}$$

and by Ohm's law

$$I = \frac{U_0}{R_g}.$$

$$= \frac{\frac{nU_q}{n}}{\frac{nR_i}{n} + \frac{R_a}{n}}$$

$$\frac{U_q}{R_i + \frac{R_a}{n}} \qquad \lim I(n) = \frac{U_q}{R_i}$$



Exercise 18
Compute the limits

c)

a) $\lim_{x\to 2} \frac{x^3-x^2-2x}{x^3+x^2-7x+2}$

b) $\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}$

c) $\lim_{x\downarrow 1} \frac{x-3}{x^2+x-2}$

Exercise 19
Let

$$f : [0,1] \to \mathbb{R}, x \mapsto f(x) := x^2 - 2x + 1,$$
$$g : [0,1] \to \mathbb{R}, x \mapsto g(x) := -x + 1.$$

a) Justify that f and g are continuous.
Show that f and g attain its maximum and minumum.
Determine the images of f and g.

b) Compute $h_1(x) = f(g(x))$ and $h_2(x) = g(f(x))$.

c) Show that h_1 and h_2 are strictly monotone increasing.