Technische Hochschule Rosenheim
Applied Artificial Intelligence - Bachelor
Analysis 1

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Homework 8: logarithms, trigonometric and hyperbolic functions

To submit: on Thursday, 02.12.2021, 9:30 a.m., online by the learning campus

$\log_a(x) = n \quad x = a^n \quad \Big| \quad \frac{x}{y} = \frac{a^n}{a^m} = a^{n-m} \; \Big| \log_a$

$\log_a(y) = m \quad y = a^m \quad \Big| \quad \log_a\left(\frac{x}{y}\right) = n - m$

Exercise 1 (4 pts.)

a) Show for $a > 0, a \neq 1$ and $x, y > 0$

$$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y).$$

$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$

b) Simplify

$$\log_{a^p} b = \frac{1}{p}\log_a b \qquad \log_{a^2}(t) + \log_a(t) \qquad a \in \mathbb{R}^+/\{1\}$$

and determine for which a and t the last term is defined.

$t \in \mathbb{R}^+$

$= \frac{1}{2}\log_a(t) + \log_a(t) = \frac{3}{2}\log_a(t)$

Exercise 2 (4 pts.)

Show for all $x, y \in \mathbb{R}$:

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right),$$

$$\sin(x) - \sin(y) = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right).$$

$x = a + b \qquad a = \frac{x+y}{2}$

$y = a - b \qquad b = \frac{x-y}{2}$

$x = \frac{x+y+x-y}{2} = \frac{2x}{2} = x$

$y = \frac{x+y-x+y}{2} = \frac{2y}{2} = y$

Hint: consider $x = u + v$ and $y = u - v$.

$\cos(x) = \cos(a+b)$

$= \cos(a)\cos(b) - \sin(a)\sin(b)$

$\cos(y) = \cos(a-b)$

$\quad // \qquad + \qquad //$

$*$

Exercise 3 (7 pt.)

Show that for any $x \in \mathbb{R}$ and $n \in \mathbb{N}$, n **odd**,

$$\cos(nx) = \sum_{j=0}^{(n-1)/2} (-1)^j \binom{n}{2j} (\cos(x))^{n-2j} (\sin(x))^{2j},$$

$$\sin(nx) = \sum_{j=0}^{(n-1)/2} (-1)^j \binom{n}{2j+1} (\cos(x))^{n-2j-1} (\sin(x))^{2j+1}.$$

Hint: consider $(\exp(ix))^n = \exp(inx)$ and apply the binomial theorem.

Calculate by the given formulas:

$$\cos(3x), \sin(5x).$$

Exercise 4 (5 pt.)
Show for all $x \in \mathbb{R}$

$$\cosh(x) = \sum_{j=0}^{\infty} \frac{1}{(2j)!} x^{2j},$$

Justify that the series converges absolutely.

since (2j)! > x^2j:
(x^2j)/(2j)! is going to zero
zero sequence -> convergence of series

Just as a remark (not to show here!): For all $x \in \mathbb{R}$

$$\sinh(x) = \sum_{j=0}^{\infty} \frac{1}{(2j+1)!} x^{2j+1}$$

converges absolutely.

$$* \sin(x) = \sin(a+b)$$
$$= \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\sin(y) = \sin(a-b)$$
$$= \quad // \quad - \quad //$$

$$\cos(x) - \cos(y) = \cos(a)\cos(b) - \sin(a)\sin(b) - \cos(a)\cos(b) - \sin(a)\sin(b)$$
$$= -2\sin(a)\sin(b)$$
$$= -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\sin(x) - \sin(y) = \sin(a+b) - \sin(a-b)$$
$$= \sin(a)\cos(b) + \cos(a)\sin(b) - \sin(a)\cos(b) + \cos(a)\sin(b)$$
$$= 2\cos(a)\sin(b)$$
$$= 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

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$$\cos(3x) = 1 \cdot 1(\cos(x))^3 \cdot (\sin(x))^0 + (-1) \cdot 3(\cos(x))^1 \cdot (\sin(x))^2$$
$$= (\cos(x))^3 - 3\cos(x) \cdot (\sin(x))^2$$

$$\sin(5x) = 1 \cdot 5(\cos(x))^4 \cdot (\sin(x))^1 + (-1) \cdot 10(\cos(x))^2 \cdot (\sin(x))^3 + 1 \cdot 1(\cos(x))^0 \cdot (\sin(x))^5$$
$$= 5(\cos(x))^4 \cdot \sin(x) - 10(\cos(x))^2 \cdot (\sin(x))^3 + 0$$