Exercise assignment for the course Introduction to AI (Part I) in the Bachelor of AAI at Rosenheim University of Applied Sciences

Assignment 07 - Propositional Logic (Solution)

Task 1

Three boxes are presented to you. One contains gold, the other two are empty.

Each box has imprinted on it a clue as to its contents; the clues are:

- Box 1 “The gold is not here”
- Box 2 “The gold is not here”
- Box 3 “The gold is in Box 2”

Only one message is true; the other two are false. Which box has the gold?

Formalize the puzzle in Propositional Logic and find the solution using a truth table.

Solution

Let Bi with i ∈ {1,2,3} stand for “gold is in the i-th box”. We can formalize the statements of the problem as follows:

1. One box contains gold, the other two are empty.

```
(B1 ∧¬B2 ∧¬B3) ∨(¬B1 ∧B2 ∧¬B3) ∨(¬B1 ∧¬B2 ∧B3) (2.1)
```

2. Only one message is true; the other two are false.

```
(¬B1 ∧¬¬B2 ∧¬B2) ∨(¬¬B1 ∧¬B2 ∧¬B2) ∨(¬¬B1 ∧¬¬B2 ∧B2) (2.2)
```

(2.2) is equivalent to:

```
(B1 ∧¬B2) ∨(B1 ∧B2) (2.3)
```

Let us compute the truth table for (2.1) and (2.3)

B1	B2	B3	(2.1)	(2.3)
T	T	T	F	T
T	T	F	F	T
T	F	T	F	T

B1	B2	B3	(2.1)	(2.3)
T	F	F	T	T
F	T	T	F	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

Task 2

The Labyrinth Guardians

You are walking in a labyrinth and all of a sudden you find yourself in front of three possible roads: the road on your left is paved with gold, the one in front of you is paved with marble, while the one on your right is made of small stones. Each street is protected by a guardian. You talk to the guardians and this is what they tell you:

- *The guardian of the gold street*: "This road will bring you straight to the center. Moreover, if the stones take you to the center, then also the marble takes you to the center."
- *The guardian of the marble street*: "Neither the gold nor the stones will take you to the center."
- *The guardian of the stone street*: "Follow the gold and you'll reach the center, follow the marble and you will be lost."

Given that you know that all the guardians are liars, can you choose a road being sure that it will lead you to the center of the labyrinth?

If this is the case, which road you choose?

Provide a propositional language and a set of axioms that formalize the problem and show whether you can choose a road being sure it will lead to the center.

Solution

Language

- g: "the gold road brings to the center"
- m: "the marble road brings to the center"
- s: "the stone road brings to the center"

Axioms

1. "The guardian of the gold street is a liar"

```
¬(g ∧(s →m))   (2.7)
```

which can be simplified to obtain

```
¬g ∨(s ∧¬m)
```

2. “The guardian of the marble street is a liar”

```
¬(¬g ∧¬s)      (2.8)
```

which can be simplified to obtain

```
g ∨ s
```

3. “The guardian of the stone street is a liar”

```
¬(g ∧¬m)       (2.9)
```

which can be simplified to obtain

```
¬g ∨ m
```

Truth Table

g	m	s	2.7	2.8	2.9	2.7 ∧2.8 ∧2.9
1	1	1	0	1	1	0
1	1	0	0	1	1	0
1	0	1	1	1	0	0
1	0	0	0	1	0	0
0	1	1	1	1	1	1
0	1	0	1	0	1	0
0	0	1	1	1	1	1
0	0	0	1	0	1	0

We have two possible interpretations that satisfy the axioms,and in both of them the stone street brings to the center. Thus I can choose the stone street being sure that it leads to the center.

Task 3

Define a propositional language which allows to describe the state of a traffic light on different instants.

With the language defined above provide a (set of) formulas which expresses the following facts:

1. the traffic light is either green, or red or orange;
2. the traffic light switches from green to orange, from orange to red, and from red to green;
3. it can keep the same color over at most 3 successive states.

Solution

Language

- gk="traffic light is green at instant k"
- rk="traffic light is red at instant k"
- ok="traffic light is orange at instant k"

Axioms

1. "the traffic light is either green, or red or orange"

```
(gk ↔(¬rk ∧¬ok)) ∧(rk ↔(¬gk ∧¬ok)) ∧(ok ↔(¬rk ∧¬gk))
```

2. "the traffic light switches from green to orange, from orange to red, and from red to green"

```
(gk−1 →(gk ∨ok)) ∧(ok−1 →(ok ∨rk)) ∧(rk−1 →(rk ∨gk))
```

3. "it can keep the same color over at most 3 successive states"

```
(gk−3 ∧gk−2 ∧gk−1 →¬gk) ∧(rk−3 ∧rk−2 ∧rk−1 →¬rk) ∧(ok−3 ∧ok−2 ∧ ok−1
→¬ok)
```

Task 4

a) Reduce to Conjunctive Normal Form (CNF) the formula $\neg(\neg p \lor q) \lor (r \Rightarrow \neg s)$

Solution

```
¬(¬p ∨q) ∨(¬r ∨¬s)

(¬¬p ∧¬q) ∨(¬r ∨¬s)

(p ∧¬q) ∨(¬r ∨¬s) NNF

(p ∨¬r ∨¬s) ∧(¬q ∨¬r ∨¬s)
```

b) Reduce to CNF the formula (¬p ⇒ q) ⇒ (q ⇒ ¬r)

Solution

```
(¬p →q) ∨(q →¬r)

¬(p ∨q) ∨(¬q ∨¬r)

(¬p ∧¬q) ∨(¬q ∨¬r)

(¬p ∨¬q ∨¬r) ∧(¬q ∨¬r)
```