Technische Hochschule Rosenheim | WiSe 2021/22
Applied Artificial Intelligence - Bachelor
Analysis 1

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Homework 7: Complex numbers etc.

To submit: on Thursday, 25.11.2021, 9:30 a.m., online by the learning campus

Exercise 1 (4 pts.)

Which of the following functions $f : \mathbb{R} \to \mathbb{R}$ are continuous on $\mathbb{R}$? Please give a justification!

✓ a) $f(x) = 2x^5 + x - 1$

✓ b) $f(x) = \frac{x}{x^2+5}$

✓ c) $f(x) = 17^{-3x} + \sin(2-x)$

✗ d) $f(x) = \exp(x) + \frac{1}{\cos(x)}$

a) polynomials are continuous.
b) polynomials are continuous + the denominator never reaches zero. x² always >= 0, y = 5 lowest point.
c) exponential function is continuous + sin is continuous.
d) exponential function is continuous but 1/cos is not. so it has multiple "holes".

You may use that sin and cos are continuous on $\mathbb{R}$.

Exercise 2 (4 pts.)

Solve for x in $\mathbb{C}$ the equations:

a) $x^2 - 10x + 4 = 0$

$x_{1/2} = \frac{10 \pm \sqrt{100 - 4 \cdot 1 \cdot 4}}{2}$ $\quad x_1 = 9,583$ $\quad x_2 = 0,417$

b) $x^2 - 2\cos(a)x + 1 = 0$, a a fixed real number.

$x_{1/2} = \frac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot 1}}{2}$ $\quad x_1 = 1$ $\quad x_2 = 1$

1. $\cos(0) = 1$

2. $\cos(2) = -0,416$

$x_{3/4} = \frac{-0,832 \pm \sqrt{(-0,832)^2 - 4}}{2} = \frac{-0,832 \pm \sqrt{-3,308}}{2} = \frac{-0,832 \pm \sqrt{3,308 i^2}}{2}$

$x_3 = -0,416 + 0,909i$

$x_4 = -0,416 - 0,909i$

Exercise 3 (4 pts.)

Proof that in the field $\mathbb{C}$ the associative property holds for the multiplication.

Later

Exercise 4 (8 pts.)

a) Let $z = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$.
Compute: $|z| = \sqrt{z\bar{z}}$, $\frac{1}{z}$, $1 + z + z^2 + \ldots + z^7$.

b) Let $z = \frac{12+5i}{2+3i}$.
Compute: $\mathrm{Re}(z)$, $\mathrm{Im}(z)$.

c) Let $z = \sum_{n=3}^{13}(12 + 2ni)$.
Compute: $\mathrm{Re}(z)$, $\mathrm{Im}(z)$.

14a $\bar{z} = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i$, $\sqrt{z\bar{z}} = \sqrt{\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i\right) \cdot \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i\right)}$

$$= \mathrm{sqrt}\left(\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i \cdot \frac{\sqrt{2}}{2} i\right)$$

$$= \mathrm{sqrt}\left(0{,}5 - 0{,}5 i + 0{,}5 i - 0{,}5 i^2\right) = \sqrt{0{,}5 + 0{,}5} = 1$$

$$|z| = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2} = \sqrt{0{,}5 + 0{,}5} = 1$$

$$\frac{1}{z} = \frac{1}{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i} = \frac{1}{z} \cdot \frac{\bar{z}}{\bar{z}} = \frac{\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i}{\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i\right) \cdot \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i\right)}$$

$$= \frac{\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i}{\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} - \cancel{\frac{\sqrt{2}}{2} i \cdot \frac{\sqrt{2}}{2}} + \cancel{\frac{\sqrt{2}}{2} i \cdot \frac{\sqrt{2}}{2}} - \frac{\sqrt{2}}{2} i \cdot \frac{\sqrt{2}}{2} i}$$

$$= \bar{z} : (0{,}5 + 0{,}5) = \bar{z} : 1 = \bar{z}$$

$$\sum_{k=0}^{7} z^n = 1 + z + i - \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i - 1 - z - i + \bar{z} = 0$$

b) $\frac{12+5i}{2+3i} \cdot \frac{2-3i}{2-3i} = \frac{24 - 36i + 10i - 15i^2}{4 - 6i + 6i - 9i^2} = \frac{39 - 26i}{15} = \underbrace{2{,}6}_{Re(z)} - \underbrace{\frac{26}{15}}_{Im(z)} i$

c) $\sum_{n=3}^{13} (12 + 2in) = \underbrace{132}_{Re(z)} + \underbrace{176}_{Im(z)} i$

13 $(a+bi)\cdot(c+di) = (a+bi)\cdot c + (a+bi)\cdot di$

$$= ac + bic + adi + \underbrace{bidi}_{= bdi^2 = -bd}$$

$$= ac - bd + adi + bci$$