Exercise assignment for the course Introduction to AI (Part I) in the Bachelor of AAI at Rosenheim University of Applied Sciences

Assignment 08 - First-Order Logic

Task 1

a) In a propositional logic formula, what does each variable represent? In a first-order logic formula, what does each variable represent?

b) What is the difference between a predicate and a function?

Task 2

Define an appropriate language and formalize the following sentences using FOL formulas:

1. All Students are smart.
2. There exists a student.
3. There exists a smart student.
4. Every student loves some student.
5. Every student loves some other student.
6. There is a student who is loved by every other student.
7. Bill is a student.
8. Bill takes either Analysis or Geometry (but not both).
9. Bill takes Analysis and Geometry.
10. Bill doesn't take Analysis.
11. No students love Bill.

Task 3

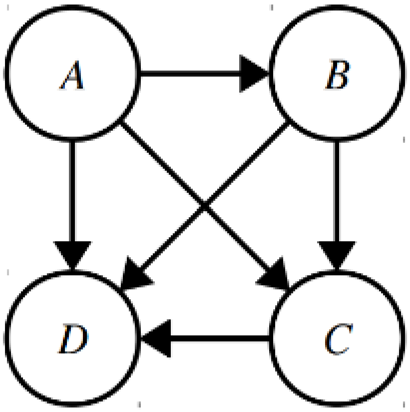
Define an appropriate language and formalize the following sentences in FOL:

1. "A is above C, D is on E and above F."
2. "A is green while C is not."
3. "Everything is on something."
4. "Everything that is free has nothing on it."
5. "Everything that is green is free."
6. "There is something that is red and is not free."
7. "Everything that is not green and is above B, is red."

- What are the constants?
- What are the predicates?
- Which axioms di exist?

Task 4

The following diagram represents a set of people named A, B, C, and D. If there's an arrow from a person x to a person y, then person x loves person y. We'll denote this by writing *Loves(x, y)*. Below is a list of formulas in first-order logic about the picture. In those formulas, the letter P represents the set of all the people. For each formula, determine whether that formula is true or false.



a) $\forall x \in P. \forall y \in P. (Loves(x, y) \lor Loves(y, x))$

b) $\forall x \in P. \forall y \in P. (x \neq y \to Loves(x, y) \lor Loves(y, x))$

c) $\forall x \in P. \forall y \in P. (x \neq y \to (Loves(x, y) \leftrightarrow \neg Loves(y, x)))$

d) $\exists x \in P. \forall y \in P. (Loves(x, y))$

e) $\exists x \in P. \forall y \in P. (x \neq y \to Loves(x, y))$

f) $\forall y \in P. \exists x \in P. (Loves(x, y))$

g) $\forall y \in P. \exists x \in P. (x \neq y \land Loves(x, y))$

h) $\exists x \in P. \forall y \in P. (\neg Loves(x, y))$