Idea of proof for L'Hôpital's rule

just for the case „$\frac{0}{0}$", i.e. $\lim\limits_{x \downarrow a} f(x) = \lim\limits_{x \downarrow a} g(x) = 0$

both functions may be extended continuously $f(a) = g(a) = 0$ $(*)$

by the generalized mean value theorem applied to (a, x)

$\xi \in (a, x)$

$$\frac{f'(\xi)}{g'(\xi)} = \frac{f(x) - f(a)}{g(x) - g(a)} \overset{(*)}{=} \frac{f(x)}{g(x)}$$

$$\Big\downarrow x \downarrow a$$

$$\frac{f'(x)}{g'(x)}$$

for the case „$\frac{\infty}{\infty}$" we could consider the variable

transformation $x \longmapsto \frac{1}{x - a}$