Technische Hochschule Rosenheim
Applied Artificial Intelligence - Bachelor
Analysis 1

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Homework 7: Complex numbers etc.

To submit: on Thursday, 25.11.2021, 9:30 a.m., online by the learning campus

Exercise 1 (4 pts.)

Which of the following functions $f : \mathbb{R} \to \mathbb{R}$ are continuous on $\mathbb{R}$? Please give a justification!

a) $f(x) = 2x^5 + x - 1$

b) $f(x) = \frac{x}{x^2+5}$

c) $f(x) = 17^{-3x} + \sin(2-x)$

d) $f(x) = \exp(x) + \frac{1}{\cos(x)}$

a) polynomials are continuous.
b) polynomials are continuous + the denominator never reaches zero. x² always >= 0, y = 5 lowest point.
c) exponential function is continuous + sin is continuous.
d) exponential function is continuous but cos^-1 is not. so it has multiple "holes".

You may use that sin and cos are continuous on $\mathbb{R}$.

Exercise 2 (4 pts.)

Solve for x in $\mathbb{C}$ the equations:

a) $x^2 - 10x + 4 = 0$

$x_{1/2} = \frac{10 \pm \sqrt{100 - 4 \cdot 1 \cdot 4}}{2}$

$x_1 = 9{,}583$

$x_2 = 0{,}417$

b) $x^2 - 2\cos(a)x + 1 = 0$, a a fixed real number.

$x_{1/2} = \frac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot 1}}{2}$

$x_1 = 1$

$x_2 = 1$

1. $\cos(0) = 1$
2. $\cos(2) = -0{,}416$

$x_{3/4} = \frac{-0{,}832 \pm \sqrt{(-0{,}832)^2 - 4}}{2} = \frac{-0{,}832 \pm \sqrt{-3{,}308}}{2}$

$= \frac{-0{,}832 \pm \sqrt{3{,}308 i^2}}{2}$

Exercise 3 (4 pts.)

Proof that in the field $\mathbb{C}$ the associative property holds for the multiplication.

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Exercise 4 (8 pts.)

a) Let $z = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$.
Compute: $|z| = \sqrt{z\bar{z}}$, $\frac{1}{z}$, $1 + z + z^2 + \ldots + z^7$.

b) Let $z = \frac{12+5i}{2+3i}$.
Compute: $\mathrm{Re}(z)$, $\mathrm{Im}(z)$.

c) Let $z = \sum_{n=3}^{13} (12 + 2ni)$.
Compute: $\mathrm{Re}(z)$, $\mathrm{Im}(z)$.

$x_3 = -0{,}416 + 0{,}909 i$

$x_4 = -0{,}416 - 0{,}909 i$