Technische Hochschule Rosenheim | WiSe 2021/22
Applied Artificial Intelligence - Bachelor
Analysis 1

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Exercise 11: applications of differentiation, Riemann sum

Exercise 34

Show that among the ellipses, prescribed by

$$\frac{1}{a^2}x^2 + \frac{1}{b^2}y^2 = 1, \quad a,b > 0,$$

a circle minimizes the enclosed area

$$\pi ab$$

subject to the constraint that the perimeter $p > 0$ of the ellipse is given approximately by:

$$\pi(a+b) = p.$$

a) Formulate the corresponding **minimization** problem in 1d, where the variable b has been eliminated by the constraint.

b) Compute the minimizer of this optimization problem.

c) Check that for the minimizer $a = b =: r$ holds. Show that r corresponds to the radius of a circle.

Exercise 35

Calculate the square root of a positive real number a by the Newton method, i.e. find a zero of

$$f(x) = 1 - \frac{a}{x^2}.$$

As initial value we consider $x_0 = \frac{1+a}{2}$

a) Derive the abstract steps of the Newton method for arbitrary a.

b) Solve two steps of the Newton method for $a = 2$.

c) Solve two steps of the Newton method for $a = 5$.

In b) and c) use a calculator or a mathematical software.

Exercise 36

a) Let $t \in \mathbb{R}$ and t is not an integer multiple of 2π.
Show that for any $n \in \mathbb{N}$ we have

$$\frac{1}{2} + \sum_{k=1}^{n} \cos(kt) = \frac{\sin\left(\left(n+\frac{1}{2}\right)t\right)}{2\sin\left(\frac{1}{2}t\right)}.$$

Hint: Use the definition of the cosine by the complex exponential function and then exploit a geometric series.

b) Compute the integral

$$\int_0^a \cos(x)\,dx, \quad a > 0,$$

by means of a Riemann sum.

Consider an equidistant partition of $[0,a]$ with fineness a/n. We choose as intermediate points $t_k = x_k = ka/n$, $k = 0,1,\ldots,n$.