Remark w.r.t. the chain rule

Only as a rule of thumb $\quad (h(f(x)))' = \frac{dh}{df}\,\frac{df}{dx}$

$$\frac{dh}{dx} = \lim_{\substack{\Delta x\to 0\\ \Delta x\neq 0}} \frac{\Delta h}{\Delta x} = \lim_{\substack{\Delta x\to 0\\ \Delta x\neq 0}} \frac{\Delta h}{\Delta f}\,\frac{\Delta f}{\Delta x} \overset{h'\,\&\,f'}{\underset{\text{exist}}{=}} \lim_{\substack{\Delta x\to 0\\ \Delta x\neq 0}} \frac{\Delta h}{\Delta f} \cdot \lim_{\substack{\Delta x\to 0\\ \Delta x\neq 0}} \frac{\Delta f}{\Delta x}$$

f cont.: $\Delta f \to 0 \;\not\Leftarrow\; \Delta x \to 0$

$$= \lim_{\substack{\Delta f\to 0\\ \Delta f\neq 0}} \frac{\Delta h}{\Delta f} \cdot \lim_{\substack{\Delta x\to 0\\ \Delta x\neq 0}} \frac{\Delta f}{\Delta x} = h'(f(x)) \cdot f'(x)$$

Problem: $\Delta f = 0$ for infinitely many points $\Longrightarrow$ case „$\frac{0}{0}$"

Proof of $\exp' = \exp$

$$\exp'(x) = \lim_{\substack{h\to 0\\ h\neq 0}} \frac{\exp(x+h) - \exp(x)}{h} = \lim_{\substack{h\to 0\\ h\neq 0}} \frac{\exp(x)\exp(h) - \exp(x)}{h}$$

$$= \exp(x) \lim_{\substack{h\to 0\\ h\neq 0}} \frac{\exp(h)-1}{h} = \exp(x) \lim_{\substack{h\to 0\\ h\neq 0}} \frac{\cancel{1} + \cancel{h} + \frac{h^{\cancel{2}}}{2} + \ldots \cancel{-1}}{\cancel{h}} = \exp(x) \cdot 1 = \exp(x) \qquad \square$$

does not depend on h

$$\exp(h) = 1 + h + \frac{h^2}{2} + \frac{h^3}{6} + \ldots$$