

## Exercise 8: functions III

### Exercise 24

Let

$$z_1 = 1 + i, \quad z_2 = 2 + i, \quad z_3 = 3 + 4i, \quad z_4 = 4 - 3i, \quad z_5 = i.$$

$$\hookrightarrow 90^\circ \quad 1e^{i\frac{\pi}{2}}$$

a) Express  $z_1 + z_3$ ,  $z_1 z_2$ ,  $z_1/z_2$ ,  $z_1^2$ , and  $z_4/z_3$  in cartesian coordinates and calculate their modulus.

b) Plot  $z_1$ ,  $z_4/z_3$ , and  $z_5$  and express these complex numbers in polar coordinates.

$$\sqrt{1+1} \cdot \exp(i \cdot \arctan(\frac{1}{1})) = \sqrt{2} e^{i\frac{\pi}{4}} \\ \hat{=} 45^\circ$$

### Exercise 25

Solve for  $x \in \mathbb{R}^+$

$$1 \exp(i \cdot 270^\circ) = e^{i\frac{\pi}{2}}$$

a)  $\ln(\sqrt{x}) + \frac{3}{2} \ln(x) = \ln(21x)$

b)  $\exp(x^2 - 2x) = 2$

c)  $\ln^2(x) - \ln(x) = 2 + \frac{1}{4} \ln(x^2)$

### Exercise 26

Show for all  $x \in \mathbb{R}$

a)  $\cosh(-x) = \cosh(x)$ ,  $\sinh(-x) = -\sinh(x)$

b)  $\cosh^2(x) - \sinh^2(x) = 1$

c)  $\cosh : \mathbb{R} \rightarrow \mathbb{R}$  and  $\sinh : \mathbb{R} \rightarrow \mathbb{R}$  are continuous on  $\mathbb{R}$ .

d) Addition theorems of  $\cosh$  and  $\sinh$ :

For all  $x, y \in \mathbb{R}$  there holds:

$$\cosh(x+y) = \cosh(x) \cosh(y) + \sinh(x) \sinh(y),$$

$$\sinh(x+y) = \sinh(x) \cosh(y) + \cosh(x) \sinh(y).$$