1a $(-8-8\sqrt{3}\,i) = -8(1+\sqrt{3}\,i) = -8\left(\cos(2\pi) + i\cdot\sin\left(\frac{\pi}{3}\right)\right)$

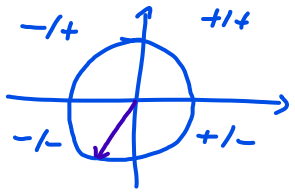
$z := (-8-8\sqrt{3}\,i)^n = (-8)^n\left(\cos(2\pi n) + i\cdot\sin\left(\frac{\pi n}{3}\right)\right)$

$\mathrm{Im}(z) = 0$ if $i\sin\left(\frac{\pi n}{3}\right) = 0$

$\mathrm{Im}(z) = 0$ for $0, 3, 6, 9\ldots$

$\mathrm{Im}(z) = 0$ for $3k$, $k \in \mathbb{N}_0$

b) $z^4 + 8 + 8\sqrt{3}\,i = 0 \quad z^4 = -8-8\sqrt{3}\,i$



$\theta_1 = \arctan\left(\frac{-8\sqrt{3}}{-8}\right)$

$= 60°$

$r = \sqrt{(-8)^2 + (-8\sqrt{3})^2}$

$= \sqrt{256} = 16$

$\theta_2 = 60° + 180° = \underline{240°}$

$\alpha_1 = \theta_2$

$\alpha_2 = \theta_2 + \frac{360°}{4} = \theta_2 + 90° = \underline{330°}$

$\alpha_3 = \alpha_2 + 90° = 420° \triangleq \underline{60°}$

$\alpha_4 = \alpha_1 - 90° = \underline{150°}$

$\sqrt[n]{z} = \sqrt[n]{r}\left[\cos\left(\frac{\theta+360k}{n}\right) + i\sin\left(\frac{\theta+360k}{n}\right)\right]$

$\sqrt[4]{z} = \sqrt[4]{16}$ "

$z_1 = 2\cdot\left(\cos\left(\frac{240°+0°}{4}\right) + i\sin\left(\frac{240°+0°}{4}\right)\right) = 2\cdot\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = 1+\sqrt{3}\,i$

$z_2 = 2\cdot(\cos(150°) + i\sin(150°)) = -\sqrt{3} + i$

$z_3 = 2\cdot(\cos(240°) + i\sin(240°)) = -1 - \sqrt{3}\,i$

$z_4 = 2\cdot(\cos(330°) + i\sin(330°)) = \sqrt{3} - i$

13 $\frac{d}{dx} \frac{(-1)^{n-1} \cdot (n-1)!}{x^n} = \frac{d}{dx}\left((-1)^{n-1} \cdot (n-1)! \cdot x^{-n}\right)$

$= (-1)^{n-1} \cdot (n-1)! \cdot \frac{d}{dx} x^{-n}$

$= (-1)^{n-1} \cdot (n-1)! \cdot -n \cdot x^{-n-1} = (-1)^{n-1} \cdot (n-1)! \cdot (-1) \cdot n \cdot x^{-n-1}$

$= (-1) \cdot (-1)^{n-1} \cdot n \cdot (n-1)! \cdot x^{-n-1}$

$= \frac{(-1)^{n+1-1} \cdot (n+1-1)!}{x^{n+1}}$ qed