

## Homework 10: derivatives

To submit: on Thursday, 16.12.2021, 9:30 a.m., online by the learning campus

### Exercise 1 (5 pts.)

Compute the following limits:

a)  $\lim_{x \rightarrow 0} \left( \frac{1}{\sin(x)} - \frac{1}{x} \right)$

b)  $\lim_{x \rightarrow 0} x \cot(x)$

### Exercise 2 (6 pts.)

We consider the following functions  $f : \mathbb{R} \rightarrow \mathbb{R}$ :

a)  $f(x) = \frac{1}{6}x^3 - \frac{5}{4}x^2 + 2x + 3$

b)  $f(x) = x \exp(-x)$

Decide for both functions:

In which interval are the functions (strictly) monotone increasing/decreasing?

In which interval are the functions convex or concave?

### Exercise 3 (4 pts.)

Proof the so-called inequality of geometric and arithmetic mean:

$$\sqrt{xy} \leq \frac{x+y}{2} \quad \text{for all } x, y \in \mathbb{R}^+$$

Hint: Show and use that the logarithm is concave.