Exercise assignment for the course Introduction to AI (Part I) in the Bachelor of AAI at Rosenheim University of Applied Sciences

Assignment 09 - Uncertainty and Probability

Task 1

It is estimated that 50% of emails are spam emails. Some software has been applied to filter these spam emails before they reach your inbox. A certain brand of software claims that it can detect 99% of spam emails, and the probability for a false positive (a non-spam email detected as spam) is 5%.

Now if an email is detected as spam, then what is the probability that it is in fact a non-spam email?

a) Fist write down the confusion matrix.

b) Calculate the probability that it is in fact a non-spam email?

Solution

- A = event that an email is detected as spam
- B = event that an email is spam
- nB = event that an email is not spam.

We know P(B) = P(nB) =.5, P (A|B) = 0.99, P (A|nB) = 0.05.

Hence by the Bayes's formula we have

$$P(nB|A) = \frac{P(A|nB)P(Bc)}{P(A|B)P(B) + P(A|nB)P(nB)}$$

$$= \frac{0.05 * 0.5}{0.05 * 0.5 + 0.99 * 0.5} = \frac{5}{104}$$

// $P(nB|A) = \frac{P(A|nB)P(Bc)}{P(A|B)P(B) + P(A|nB)P(nB)}$ // $=\frac{0.05 × 0.5}{0.05 × 0.5 + 0.99 × 0.5}= \frac{5}{104}$

Task 2

In a study, physicians were asked what the odds of breast cancer would be in a woman who was initially thought to have a 1% risk of cancer but who ended up with a positive mammogram result (a mammogram accurately classifies about 80% of cancerous tumors and 90% of benign tumors.)

95 out of a hundred physicians estimated the probability of cancer to be about 75%. Do you agree?

Solution

Introduce the events:

- R = mammogram result is positive,
- B = tumor is benign,

- M = tumor is malignant.

We are given P(M) = .01, so P(B) = 1 – P (M) = .99.

We are also given the conditional probabilities P (R | M) = .80 and P (Q | B) = .90, where the event Q is the complement of R, thus P (R | B) = .10

Bayes' formula in this case is

$$P(M|R) = \frac{P(R|M) * P(M)}{(P(R|M)P(M) + P(R|B)P(B))}$$

$$= \frac{0.80 * 0.01}{(0.80 * 0.01 + 0.10 * 0.99)} = 0.075$$

//$$P (M|R) = \frac{P(R|M)*P(M)}{(P (R|M)P(M) + P (R|B)P(B))}$$ //$$= \frac{0.80 × 0.01}{(0.80 × 0.01 + 0.10 × 0.99)}= 0.075$$

So the chance would be 7.5%. A far cry from a common estimate of 75

Task 3

Suppose we have 3 cards identical in form except that both sides of the first card are colored red (RR), both sides of the second card are colored black (BB), and one side of the third card is colored red and the other side is colored black (RB).

The 3 cards are mixed up in a hat, and 1 card is randomly selected and put down on the ground.

If the upper side of the chosen card is colored red, what is the probability that the other side is colored black?

Solution

Let RR, BB, and RB denote, respectively, the events that the chosen cars is the red-red, the black-black, or the red-black card. Letting R be the event that the upturned side of the chosen card is red, we have that the desired probability is obtained by

P(RB|R) = P(RB∩R)\P(R)}

using P(A∩B) =P(A)P(B|A)

$$= \frac{P(R|RB)P(RB)}{P(R|RR)P(RR) + P(R|RB)P(RB) + P(R|BB)P(BB)}$$

$$= \frac{1/2 * 1/3}{1 * 1/3 + 1/2 * 1/3 + 0 * 1/3} = 1/3$$

//$$=\frac{P(R|RB)P(RB)}{P(R|RR)P(RR)+P(R|RB)P(RB)+P(R|BB)P(BB)}$$

//$$= \frac{1/2 * 1/3}{1 *1/3 + 1/2* 1/3 + 0 * 1/3} = 1/3$$