Exercise 1 (live tutorial): sets and numbers

Exercise 1
Consider the sets $A_1 := \{1,2,3,4,5\}, B_1 := \{1,2,3\}, A_2 := \{3,4,5\}, B_2 := \{1,2\}$ that are subsets of the basic set $M := \{1,2,3,4,5,6,7,8,9,10\}$.

a) Determine $A_1 \setminus B_1$ and $B_1 \setminus A_1$, or $A_2 \setminus B_2$ and $B_2 \setminus A_2$, resp.
When is the statement $A \setminus B = B \setminus A$ true?
Please characterize all cases where the last statement holds.

b) Check the statements $A_1 \setminus B_1 = A_1 \cap \overline{B}_1$ or $A_2 \setminus B_2 = A_2 \cap \overline{B}_2$, resp.
Does this statement hold in general?

Solution for exercise 1

a) i)

$$\begin{aligned}
&A_1 \setminus B_1 = \{1,2,3,4,5\} \setminus \{1,2,3\} = \{4,5\} \\
&B_1 \setminus A_1 = \{1,2,3\} \setminus \{1,2,3,4,5\} = \emptyset \\
&A_1 \setminus B_1 \neq B_1 \setminus A_1
\end{aligned}$$

ii)

$$\begin{aligned}
&A_2 \setminus B_2 = \{3,4,5\} \setminus \{1,2\} = \{3,4,5\} \\
&B_2 \setminus A_2 = \{1,2\} \setminus \{3,4,5\} = \{1,2\} \\
&A_2 \setminus B_2 \neq B_2 \setminus A_2
\end{aligned}$$

The statement $A \setminus B = B \setminus A$ holds, if and only if $A = B$.

b) i)

$$\begin{aligned}
&A_1 \setminus B_1 = \{1,2,3,4,5\} \setminus \{1,2,3\} = \{4,5\} \\
&A_1 \cap \overline{B_1} = \{1,2,3,4,5\} \cap \{4,5,6,7,8,9,10\} = \{4,5\} \\
&A_1 \setminus B_1 = A_1 \cap \overline{B_1}
\end{aligned}$$

ii)

$$\begin{aligned}
&A_2 \setminus B_2 = \{3,4,5\} \setminus \{1,2\} = \{3,4,5\} \\
&A_2 \cap \overline{B_2} = \{3,4,5\} \cap \{3,4,5,6,7,8,9,10\} = \{3,4,5\} \\
&A_2 \setminus B_2 = A_2 \cap \overline{B_2}
\end{aligned}$$

The statement $A \setminus B = A \cap \overline{B}$ is true in general, as we may check by a diagram (a so-called Venn diagram).

Exercise 2
Let $\mathbb{Z} = \{\ldots,-2,-1,0,1,2,\ldots\}$ be the set of integers and consider its following subsets:
$D := \{1,2,3,\ldots,10\}$,
$S := \{n \text{ is the sum of two squares of integers}\}$, for instance $1 \in S$, since $0^2+1^2 = 1$ and $2 \in S$, since $1^2+1^2 = 2$,
$\mathbb{P} := \{2,3,5,7,11,13,17,19\}$ and
$U := \{n \text{ is odd}\}$.

Determine:

a) $\{2,3,5,7,11,13,17,19\} \cap \overline{U}$

b) $U \cup \overline{U}$

c) $S \cap D$

d) $\overline{S} \cap D$

e) $(\overline{S} \cap D) \setminus \overline{\mathbb{P}}$

f) $(\mathbb{P} \cap \overline{U}) \times (\overline{S} \cap D)$

Solution for exercise 2
We abbreviate $\mathbb{P} = \{2,3,5,7,11,13,17,19\}$.
We have the following table:

	0^2	1^2	2^2	3^2	4^2
0^2	0	1	4	9	16
1^2	1	2	5	10	17
2^2	4	5	8	13	20
3^2	9	10	13	18	25
4^2	16	17	20	25	32

Thus: $S = \{0,1,2,4,5,8,9,10,13,\ldots\}$

a) $\mathbb{P} \cap \overline{U} = \{2,3,5,7,11,13,17,19\} \cap \overline{U} = \{2\}$

b) $U \cup \overline{U} = \mathbb{Z}$

c) $S \cap D = \{1,2,4,5,8,9,10\}$

d) $\overline{S} \cap D = \{3,6,7\}$

e) $(\overline{S} \cap D) \setminus \overline{\mathbb{P}} = \{3,6,7\} \setminus \overline{\mathbb{P}} = \{3,7\}$

f) $(\mathbb{P} \cap \overline{U}) \times (\overline{S} \cap D) = \{2\} \times \{3,6,7\} = \Big\{(2,3),(2,6),(2,7)\Big\}$

Exercise 3
Which of the following statements hold for the sets $A := \{1,2,3,4,5\}$, $B := \{1,2,3,\ldots,100\}$ and $M := \{1,A\}$?

a) $1 \subset A$	d) $B \subset A$	g) $B \subset B$	j) $A \in M$
b) $1 \in B$	e) $A \subset B$	h) $\emptyset \in M$	k) $M \in M$
c) $\{1\} \in A$	f) $A \in B$	i) $\{1\} \in M$	ℓ) $M \subset B$

Solution for exercise 3

a) false, $1 \in A$ (1 is not set and is thus **never** a subset of anything)

b) true

c) false, $\{1\} \subset A$ or $1 \in A$

d) false, for instance $100 \in B$, but $100 \notin A$

e) true, since all elements of A belong to B and B contains elements, that are not in A, e.g. 100; thus A is a proper subset of B

f) false, $A \subset B$.

g) false, $B \subseteq B$ holds, but B is no proper subset of itself, i.e. $B \subset B$ is always false

h) false, $\emptyset \subset M$

i) false, $\{1\}$ is a set and no element

j) true

k) false, M is no element

ℓ) false, A is no element of B