he Hochschule Rosenheim WiSe 2021/22

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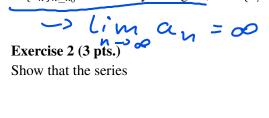
## Homework 5: series

To submit: on Thursday, 11.11.2021, 9:30 a.m., online by the learning campus

## Exercise 1 (4 pts.)

Prove:

If  $\{a_n\}_{n>n_0}$  is definitely divergent, then  $\{1/a_n\}_{n>n_0}$  is a zero sequence.



 $\begin{cases} \frac{1}{(\ln a_n)} = 0 \\ \frac{1}{(n-1)n} = 0 \end{cases}$   $\begin{cases} \frac{1}{(n+1)n} = \frac{1+n-h}{(n+1)n} = \frac{1}{(n+1)n} - \frac{h}{(n+1)n} \end{cases}$ 

converges by computing its limit.

Hint: rewrite the fraction s.t. you obtain a telescopic sum.

## Exercise 3 (8 pts.)

Discuss the (absolute) convergence of the following series:

- a) any series with a^k where a is positive but smaller than 1, converges absolutely.
- $\sum_{k=0}^{\infty} \left(\frac{1}{7}\right)^k \qquad [2 \text{ pt.}]$
- b) (-1)^k only changes the sign for k element N. ((-1)^k\*k)/(k^2+1) has an alternating sign and the sum is absolutely convergent but the limit is different when adding absolute values.  $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2+1} = \frac{1}{k^2+1}$
- [3 pt.]  $\left| \frac{\alpha_3}{\alpha_e} \right| = 0,75$

 c) the ratio test only results in a number < 1 for k >= 4 which doesn't show the whole behaviour but the series still converges absolutely.

$$\sum_{k=1}^{\infty} \frac{1+k^4}{1+3^k}$$
 [3 pt.]

Hint: in c) you may use the ratio test.

Remark: It is not required to compute the limits.