Technische Hochschule Rosenheim
Applied Artificial Intelligence - Bachelor
Analysis 1

WiSe 2021/22

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Homework 4: sequences and convergence

To submit: on Thursday, 04.11.2021, 9:30 a.m., online by the learning campus

Exercise 1 (5 pts.)
In the lecture we encountered the sequence of Fibonacci numbers that serve as a simple population model:

$$a_{n+1} := a_n + a_{n-1} \quad \text{for } n \geq 3, \quad a_1 = 1, a_0 = 1.$$

List a_n for $n = 0, \ldots, 10$. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144

Prove that the following formula (related to the Golden ratio) is equivalent to the recursive definition:

$a_n = a_{n+1} - a_{n-1}$ |

$$a_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right)$$

Base: $n=3$ $a_3 = a_4 - a_2 = 5 - 2 = 3 \mid a_3 = \frac{1}{\sqrt{5}} \cdot \left(\left(\frac{1+\sqrt{5}}{2}\right)^4 - \left(\frac{1-\sqrt{5}}{2}\right)^4\right) = 3$ ✓

Exercise 2 (6 pts.)
Decide whether the following sequences $\{a_n\}_{n\in\mathbb{N}}$ converge or diverge.
If possible determine the value of the limit.

a) $a_n = \frac{2021n^2+10n+28}{n}$ $2021n^2 + 10n + 28 > n \Rightarrow \lim_{n\to\infty} a_n = +\infty$ div.

b) $a_n = \frac{1}{3n^2}\left(1 - \frac{1}{2^n}\right)$ $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{3n^2} = 0^+$ conv.

c) $a_n = -3^n - \underbrace{\frac{1}{n}\cos(n)}_{\lim_{n\to\infty} = 0}$ $\lim_{n\to\infty} -(3^n) = -\infty \Rightarrow \lim_{n\to\infty} a_n = -\infty$ div.

Please give an argument for each of your answers.

Exercise 3 (3 pts.)
Show that the product of a zero sequence and a bounded sequence is again a zero sequence.

$\{a_n \cdot b_n\}_{n\geq 0}$ converges to $a \cdot b$

$\lim_{n\to\infty}(a_n b_n) = \lim_{n\to\infty} a_n \cdot \lim_{n\to\infty} b_n = a \cdot b$

so if a or b is a zero sequence and the other is bounded
$\rightarrow$ $0 \cdot b = 0$
$a \cdot 0 = 0$