Technische Hochschule Rosenheim | WiSe 2021/22
Applied Artificial Intelligence - Bachelor
Analysis 1

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Homework 11: applications of differentiation, integration theory

To submit: on **Wednesday, 29.12.2021**, 9:30 a.m., online by the learning campus

Exercise 1 (7 pts.)
In the production of canned food, different materials are used for the base and lid, and for the curved surface of the can. The material for base and lid cost $c_1 = 0,02$ and for the curved surface $c_2 = 0,04$ cents per cm^2. For a given volume $V = 1000\text{cm}^3$ we would like to manufacture a suitable can with material costs as less as possible.

Let $r > 0$ the radius and the height $h > 0$ of the can. For the volume of a cylinder we have

$$V = \pi r^2 h$$

and for the area

$$A = A_1 + A_2 = 2\pi r^2 + 2\pi r h,$$

where A_1 is the area of base and lide together and A_2 the area of the curved surface of the can, i.e. the jacket.

a) [2 pt.] Formulate the optimization problem for the radius r of the can as the only variable.

b) [5 pt.] Minimize $f(r) = 0,04\pi r^2 + \frac{0,8}{r}$ over all radii $r > 0$. What is the corresponding optimal height?

Exercise 2 (8 pts.)
Compute for the following functions all points where the tangent line is horizontal.

Classify the shape of the curve (convex or concave) at these (stationary) points.

a) [4 pts.] $f(x) = x^3 - 6x^2 + 9x + 1513$

b) [4 pts.] $f(x) = \sin\left(\frac{1}{x}\right), \quad x \neq 0$

Exercise 3 (5 pts.) (“Langsam rieselt der Schnee” (Slowly trickles the snow))
We consider a snowflake on its way (from a sufficiently large height) down to earth.

The question is: What is the maximal velocity of a snowflake when it hits the surface of the earth?

The following model is given: a snowflake is subject to gravity and that its aerodynamic drag and lift is not neglectable. This yields as differential equation for the velocity v (as a function of time t, in direction of the surface) of the snowflake

$$\frac{d}{dt}v(t) = -bv^2(t) - lv(t) + g,$$

where g is the well-known gravity acceleartion on earth (being positive) and l as well as b are strictly positive fraction constants.

In order to determine the maximal velocity $v_\infty \in [0,\infty)$ that may be reached by the snowflake we consider the following iterative method: We start with an initial velocity $v^0 = v(t=0) = 0$ at time $t = 0$ and compute the velocity v_{n+1} after $n+1$ time intervals of duration Δt according to the following iteration:

$$\begin{aligned} v_{n+1} &= -(\Delta t)bv_n^2 + (1-(\Delta t)l)v_n + (\Delta t)g \qquad \forall n \in \mathbb{N}_0, & (1) \\ v_0 &= v^0. & (2) \end{aligned}$$

These iteration is well-posed, if, for instance, $\Delta t \in (0, \frac{-l+2\sqrt{l^2+3bg}}{l^2+4bg}]$.

Which value is obtained for v_∞ by the iteration scheme (1) and (2)? Check the correct answer below (one and only one is true).

Hint: Find fixed points, i.e. $v_* = v_n$ such that $v_{n+1} = v_n$, for the given iteration. No assumptions on Δt are required.

Answering options:

a) $v_\infty = \frac{l}{2b}$

b) $v_n \to +\infty$ as $n \to \infty$

c) If M is the mass of the snowflake, then we have: $v_\infty = \frac{1}{2}Mg^2$.

d) $v_\infty = \frac{g}{l}$

e) $v_\infty = -\frac{l}{2b} - \sqrt{\frac{l^2}{4b^2} + \frac{g}{b}}$

f) The answer depends on the height of fall h: $v_\infty = \sqrt{2gh}$.

g) $v_\infty = \frac{l}{2b}(\sqrt{1+\frac{4gb}{l^2}} - 1)$

h) $v_\infty = -\frac{l}{2b} + \sqrt{\frac{l^2+4g}{4b^2}}$

i) $v_\infty = 0$

j) $v_\infty = \arctan(-2\pi bg)$

Justify shortly, why your answer is correct and why the others are wrong.

Exercise 4 (5 bonus pts.)
Go through the two slides from the lectures on the golden-section search.

Write down an algorithm in pseudo code **or** implement the algorithm in a programming language of your choice.

(The solution sketches will be in MATLAB, the algorithm or the written code will be marked, not the running code.)