HW12

/1a $\int_3^4 \frac{x+10}{x^2+5x-14}\,dx$

$$x_{1/2} = \frac{-5 \pm \sqrt{25-4\cdot(-14)}}{2} = \frac{-5 \pm \sqrt{81}}{2} \qquad x_1 = 2 \quad x_2 = -7$$

$$x^2+5-14 = (x-2)\cdot(x+7)$$

$$\frac{x+10}{x^2+5x-14} = \frac{A}{(x-2)} + \frac{B}{(x+7)}$$

$$= \frac{A(x+7) + B(x-2)}{x^2+5x-14}$$

$$A(x+7) + B(x-2)$$
$$= x(A+B) + 7A - 2B$$

$$A + B = 1$$
$$7A - 2B = 10$$

$$A = 1 - B$$
$$7A - 2B = 10$$

$$7\cdot(1-B) - 2B = 10$$
$$7 - 7B - 2B = 10$$
$$-9B = 3$$
$$B = -\tfrac{1}{3}$$

$$A - \tfrac{1}{3} = 1$$
$$A = 1\tfrac{1}{3} = \tfrac{4}{3}$$

$$\frac{x+10}{x^2+5x-14} = \frac{\frac{4}{3}}{(x-2)} + \frac{-\frac{1}{3}}{(x+7)}$$

$$\int_3^4 \frac{x+10}{x^2+5x-14}\,dx = \int_3^4 \frac{\frac{4}{3}}{(x-2)} + \frac{-\frac{1}{3}}{(x+7)}\,dx$$

$$= \frac{4}{3}\int_3^4 \frac{1}{(x-2)}\,dx - \frac{1}{3}\int_3^4 \frac{1}{x+7}\,dx$$

$$= \frac{4}{3}\left(\ln(4-2) - \ln(3-2)\right) - \frac{1}{3}\left(\ln(4+7) - \ln(3+7)\right)$$

$$= \frac{4}{3}\ln(2) - \frac{1}{3}\left(\ln(11) - \ln(10)\right)$$

b) $\int_0^1 \frac{x-1}{(x-2)^3}\,dx \rightarrow x = 2$

$$(x-2)^2 = x^2 - 4x + 4$$
$$(x-2)^3 = x^3 - 6x^2 + 12x - 8$$

$$\frac{x-1}{(x-2)^3} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3}$$
$$= \frac{A(x-2)^2 + B(x-2)^1 + C(x-2)^0}{(x-2)^3}$$
$$= \frac{x^2(A) + x(-4A+B) + 1(4A-2B+C)}{(x-2)^3}$$

$$\int_0^1 \frac{x-1}{(x-2)^3}\,dx$$
$$= \int_0^1 \frac{1}{(x-2)^2} + \frac{1}{(x-2)^3}\,dx$$
$$= \frac{1}{1}\cdot\frac{1}{(1-2)^1} - \frac{1}{(-2)}$$
$$+ \frac{1}{2}\cdot\frac{1}{(1-2)^2} - \frac{1}{2}\cdot\frac{1}{(-2)^2}$$
$$= \frac{1}{8} = 0.125$$

$$A = 0$$
$$-4A + B = 1$$
$$4A - 2B + C = -1$$

$$A = 0$$
$$B = 1$$
$$-2B + C = -1$$

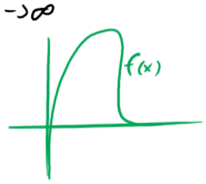
$$A = 0$$
$$B = 1$$
$$C = 1$$

$$\rightarrow \frac{(x-1)}{(x-2)^3} = \frac{0}{(x-2)} + \frac{1}{(x-2)^2} + \frac{1}{(x-2)^3}$$

/2a

$$\int_0^\infty \frac{1}{(1+x^2)}\,dx = \lim_{x\to\infty} \int_0^\infty \frac{1}{(1+x^2)}\,dx$$
$$= \lim_{x\to\infty} \arctan(x) - \arctan(0)$$
$$= \lim_{x\to\infty} \arctan(x) = \frac{\pi}{2}$$

b) $\int_0^R x e^{(-\lambda x)}\,dx$



$$\int_a^b f'(x)\,g(x)\,dx = [f(x)\cdot g(x)]_a^b - \int_a^b f(x)\cdot g'(x)\,dx$$

$$= \left[\frac{x}{-\lambda} e^{(-\lambda x)}\right]_0^R - \int_0^R \frac{x}{-\lambda} e^{(-\lambda x)}\,dx$$
$$= \left[\frac{x}{-\lambda} e^{(-\lambda x)}\right]_0^R - \frac{x}{-\lambda}\cdot\int_0^R e^{(-\lambda x)}\,dx$$
$$= \left[\frac{x}{-\lambda} e^{(-\lambda x)}\right]_0^R - \frac{x}{-\lambda}\left[\frac{x}{-\lambda} e^{(-\lambda x)}\right]_0^R$$

$$\lim_{R\to\infty} \int_0^R x e^{(-\lambda x)}\,dx = \lim_{R\to\infty}\left(\left[\frac{x}{-\lambda} e^{(-\lambda x)}\right]_0^R - \frac{x}{-\lambda}\left[\frac{x}{-\lambda} e^{(-\lambda x)}\right]_0^R\right)$$

c) $$\int_{-R}^{1} \frac{1}{\sqrt{1-x}}\, dx = \lim_{N \to 1} \int_{-R}^{N} \frac{1}{\sqrt{1-x}}\, dx$$

$$= \lim_{N \to 1} \int_{-R}^{N} (1-x)^{-0.5}$$

$$= \lim_{N \to 1} \left[-2(1-x)^{0.5}\right]_{-R}^{N}$$

$$= \lim_{N \to 1} \underbrace{-2\sqrt{1-N}}_{=0} - (-2)\cdot\sqrt{1+R}$$

$$= 2\sqrt{1+R}$$

$$\lim_{R \to \infty} 2\sqrt{1+R} = \infty \to \text{diverges}$$