Technische
Hochschule
Rosenheim

Analysis 1

Priv.-Doz. Dr. Sven-Joachim Kimmerle

Winter term 2021/22
Bachelor Applied Artificial Intelligence (AAI)

Outline

Lecturer: Sven-Joachim Kimmerle

- 2000: “Vordiplom” in Mathematics & “Vordiplom” in Physics (U Heidelberg)
- 2002: Maîtrise in Mathematics (U Paris 7, France)
- 2004: Diploma in Mathematics (U Heidelberg)
- 2004-2009: Research center MATHEON, Berlin
- 2009: PhD in Mathematics (HU Berlin)
- 2010: Toyota/U Ottawa, Ottawa, Canada
- 2011-2018: Postdoc & deputy professor, UniBw München, Neubiberg
- 2019: “Habilitation” in Mathematics (UniBw München, Neubiberg)
- Since 2018: Physical Software Solutions GmbH, Münsing & Ottobrunn
- Since 2021: Lecturer (part-time), TH Rosenheim

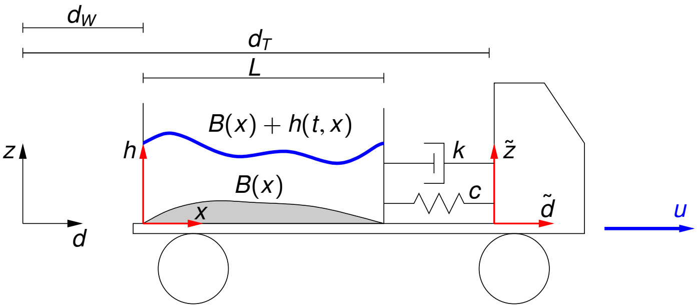
What is mathematics/analysis good for?

What is mathematics/analysis good for?

- Modelling real-world problems in terms of mathematics
  allowing for simulations/solutions
  and predictions of the future
- How do we formulate mathematics?
- A prerequisite for computer sciences and others ...
- The beauty within mathematics
- ...

Example: An optimal control problem [K., Gerdts 2015 -]

Truck with a fluid container: horizontal control (force) u, parameter driving time T



m_T: mass truck, m_W: mass fluid container, d_T: distance (travelled) truck, d_W: distance container, $B(x)$: floor profile container, $h(t, x)$: fluid level at time t at position x, $v(t, x)$: horizontal fluid velocity at time t at position x, L: container length, c : spring constant, k: damping constant

Mathematical optimal control problem

Optimal control problem

Minimize objective function $\mathcal{J}$ i.e. a linear combination of

- Driving time T ($\rightsquigarrow$ weight α_0);
- Deviation from a desirable (constant, e.g.) fluid level $(h(t,x) - h_d)^2$ ($\rightsquigarrow$ weight α_1);
- Control costs $\|u\|^2_{L^2([0,T])}$ ($\rightsquigarrow$ weight α_2);
- Deviation to terminal position $\|d_T(T) - d_T^T\|^2 + \|d_W(T) - d_W^T\|^2$ and velocity $\|d_T'(T) - (d_T')^T\|^2 + \|d_W'(T) - (d_W')^T\|^2$ ($\rightsquigarrow$ weights α_3, α_4)

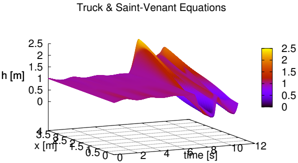
subject to the constraints

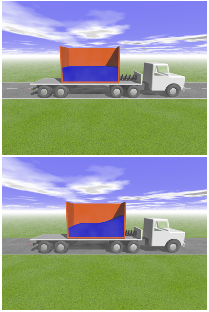
- coupled differential equations (ODE & PDE) together with initial and boundary conditions
- control constraints
  $u(t) \in \mathcal{U} := [u_{min}, u_{max}], \quad U_{ad} = \left\{ u \in L^2(0,T) \,\middle|\, u(t) \in \mathcal{U} \right\}.$
- state constraints ($0 < \underline{h} \leq h \leq \overline{h}$)

Numerical optimal control: driving time only as objective [Gerdts/K. 2015]

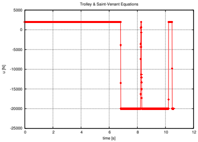
Ziel: Optimal braking

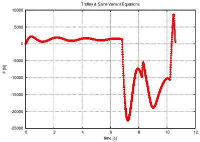
- within shortest driving time





Result: $T = 10.6$ s, $\epsilon = 0.453$, control u (l.h.s.), spring damper force (r.h.s.)



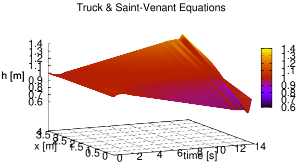


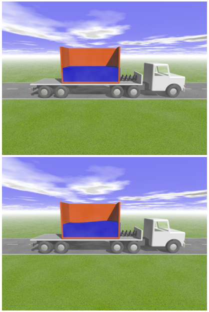
($\alpha_0 = 0.1$, $\alpha_1 = 0$, $\alpha_2 = 0$, $\alpha_3 = 0$, $\alpha_{4a} = 0$, $\alpha_{4b} = 100$, $d_T(T) = 100$, $d_W(T) = 95$, $d_T'(0) = 10$, $d_W'(0) = 10$, $N = 1500$, $M = 50$, $L = 4$, $m_T = 2000$, $m_W = 4000$, $h_0 = 1$, $c = 40000$, $k = 10000$, $g = 9.81$, $u_{min} = -20000$, $u_{max} = 2000$)

Numerical optimal control: comprise in the objective function [Gerdts/K. 2015]

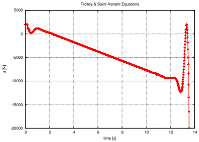
Objective: Optimal braking

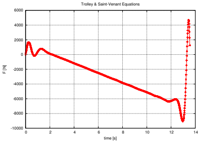
- within shortest time
- with minimal control costs and
- with minimal excitation of the fluid





Result: $T = 13.8$ s, $\epsilon = 0.644$, control u (l.h.s.), spring-damper-force (r.h.s.)





($\alpha_0 = 0.1$, $\alpha_1 = 1$, $\alpha_2 = 0.001$, $\alpha_3 = 0$, $\alpha_{4a} = 0$, $\alpha_{4b} = 100$, $d_T(T) = 100$, $d_W(T) = 95$, $d_T'(0) = 10 = d_W'(0)$, $N = 1000$, $M = 30$, $L = 4$, $m_T = 2000$, $m_W = 4000$, $h_0 = 1$, $c = 40000$, $k = 10000$, $g = 9.81$, $u_{min} = -20000$, $u_{max} = 2000$)

Other applications where analysis is crucial

- Analysis is the base for various mathematical disciplines:
  - AI, machine learning, etc.
  - numerics
  - optimization (operations research)
  - statistics, data analysis
  - ...
- Analysis has applications in, e.g.:
  - Computer science
  - Economics
  - Technology
  - Physics, (chemistry, biology,) astronomy, geography
  - ...

Aims of the course

Teaching mathematical basics:

- Review of mathematics from “school”
- Basics of calculus (mainly functions, differentiation & integration in 1d)
- Introduction to mathematical structures
- Introduction to mathematical methods
- Logical reasoning (proof techniques, structuring thoughts)

Next semester:

- Analysis 2: calculus (differentiation & integration in higher dimensions, power series)
- Linear Algebra: systems of equations, matrices & vectors, eigenvalues, vector spaces

Administrative & organisational matters 1

- 6 hours lecture with 2×2 hours exercise

  Monday, 11:45-15:15 (with break between about 13:15-13:45)
  Wednesday, 09:45-11:15

  As virtual teaching (in ZOOM)

  Please login directly & early!

  In case of technical issues, we wait for 20 minutes!

- 2×2 hours exercise

  Thursday, 09:45-11:15 in A2.11 (group 2)
  Thursday, 11:45-13:15 in A2.11 (group 1)

  In presence

  We start with 2 groups (each about half the people).

  "Corona tracking" within learning campus

Administrative & organisational matters 2

- Presentations, exercises and other material can be found in the Learning Campus
    - learning-campus.th-rosenheim.de
    - → Department ANG
      → Mathematics
      → Analysis 1 (AAI - B1), WS 2021/22
    - shortly: “Analysis 1 AAI, 21/22”
    - login: Thisisthekey!

- Office hours & contact
    - After each exercise group or
    - by appointment by email:
      `sven-joachim.kimmerle@th-rosenheim.de`

Marking

- **Examination type**
  - Written exam: 90 min.
  - Auxiliary tools: 1 sheet (DIN A4) both sides, hand written with formulas, e.g.
  - No calculators (or smartphones etc.) will be permitted.
- **Homework and bonus system**
  - Marked homework (bonus up to 15%), sometimes in groups up to 2
  - To hand-in each Thursday evening, discussion next Thursday

Workload

According to the module handbook for 10 ECTS we expect a workload of about 300 hours:

- 120 hours contact (virtual or in presence): 90 hours lecture, 30 hours exercise
- 180 hours independent study

Preliminaries:

- Good math skills from school
- Sound understanding of English
- Perseverance and endurance

Recommended literature

Stewart, James: *Calculus*. Brooks/Cole, 2002.

Hass, Heil, Weir: *Thomas' Calculus: Early transcendentals*. Pearsons, 1999.

In German only:

Forster, Otto: *Analysis 1*. Vieweg-Verlag, 1985.

Forster, Otto: *Analysis 2*. Vieweg-Verlag, 2. Aufl., 1986.

Further literature and material (software, e.g.) will be given during the course

Copying ban

All materials made available in this lecture have been protected by me with a password, which has only been made available to the registered participants of this course.
Any form of distribution is prohibited!

Sven-Joachim Kimmerle

Outline

Colloquial description of a set

A set is a collection of distinguishable, different objects (real or imaginary) to a whole.

Standard notation: $A, B, C, \ldots$

The objects in a set are called **elements.**

Elements of a set

We write shortly:

$$a \in A \text{ means: a is an element of A,}$$
$$a \notin A \text{ means: a is no element of A}$$

An element a is

- either an element of A
- or no element of A.

Remark: The elements of a set may be heterogeneous.

Representations of sets

- A list of (all) elements.
- A description of properties of all elements.

There is a unique set with no element:
the **empty set** $\emptyset$.

$$|M| = \text{“number” of elements of } M$$
$$= \textbf{cardinality of } M$$

Sets with a finite number of elements are called **finite sets**.

Subset and superset

Definition ((Proper) subset)

A set B is a **subset** of A, i.e.

$$B \subseteq A$$

if any element of B is an element of A.

A set B is a **proper subset** of A, i.e.

$$B \subset A,$$

if $B \subseteq A$ and
if there is an element $y \in A$ such that $y \notin B$.

Definition ((Proper) superset)

If B is a (proper) subset of A,
then A is a (proper) superset of B:

$$A \supseteq B \quad (A \supset B)$$

Set operations

Let A, B two sets. We define:

- Average of sets $A \cap B := \{x \,|\, x \in A \text{ and } x \in B\}$
- Union of sets $A \cup B := \{x \,|\, x \in A \text{ or } x \in B\}$
- Difference of sets $A \setminus B := \{x \,|\, x \in A \text{ or } x \notin B\}$

Let B a subset of a basic set A,
then $A \setminus B$ is called the **complement** of B w.r.t. A.
We write

$$\overline{B} := A \setminus B.$$

Cartesian or direct product

Definition

A cartesian (or direct) product of n (non-empty) sets $A_1, A_2, \ldots, A_n$ designates the set

$$A_1 \times A_2 \times \ldots \times A_n := \{(a_1, a_2, \ldots, a_n) \,|\, a_i \in A_i \text{ for all } i = 1, \ldots, n\}.$$

The elements $(a_1, a_2, \ldots, a_n)$ of this set are called **ordered n-tuples.**

In the special case $A_i = A$ for all i, we write A^n for this cartesian product.