Technische Hochschule Rosenheim WiSe 2021/22
Applied Artificial Intelligence - Bachelor
Analysis 1
Priv.-Doz. Dr. S.-J. Kimmerle Thursday, 09.12.2021

Exercise 10: differential calculus II

Exercise 30
Differentiate the following functions

a) $a(x) = \sqrt[4]{x^3}$

b) $b(x) = a\cos(x) - x^2 + \exp(x) + 1, \quad a \in \mathbb{R}$

c) $c(x) = x^2 \arcsin(x)$

d) $d(x) = \frac{1+\cos(x)}{1-\sin(x)}$

e) $e(x) = \frac{5x^5-6x^2+1}{(x+1)^2}$

f) $f(t) = \sin\left(3t + \frac{\pi}{2}\right)$

g) $g(x) = \left(\frac{1+x}{x}\right)^n$

h) $h(x) = 2t\sqrt{t^2-1}$

i) $i(x) = \exp(-5t)\,(3\sin(2t) + 4\cos(2t))$

Moreover, give the maximal domain of definition, the set A, for the function and the corresponding set D for the derivative.

Exercise 31
Compute the following limits:

a) $\lim_{x\to 1} \frac{x^2+x-2}{x^2-1}$

b) $\lim_{x\to 0} \frac{\ln(\cos(x))}{x}$

c) $\lim_{x\to\infty} \left(\sqrt{x+\sqrt{x}} - \sqrt{x}\right)$

Exercise 32
Proof the generalized mean value theorem:

Let $a < b$ and $f, g : [a,b] \to \mathbb{R}$ continuous functions
that are continuously differentiable in (a,b),
and $g'(x) \neq 0$ for all $x \in (a,b)$,

then there exists a $\xi \in (a,b)$ such that

$$\frac{f'(\xi)}{g'(\xi)} = \frac{f(b)-f(a)}{g(b)-g(a)}.$$

Hint: Apply the Rolle theorem.

Exercise 33

Let $f:(x_1,x_2) \to \mathbb{R}$ with $x_1 < x_2$.

Suppose that

$$\frac{f(x)-f(x_1)}{(1-\lambda)(x_2-x_1)} \leq \frac{f(x_2)-f(x)}{\lambda(x_2-x_1)}$$

for a convex combination

$$x = \lambda x_1 + (1-\lambda)x_2 = x_2 + \lambda(x_1-x_2), \quad 0 < \lambda < 1.$$

Show that f is convex on $(x_1,x_2) \subseteq \mathbb{R}$.

Remark: This is the skipped intermediate step in the proof, that $f''(x) \geq 0$ on $x \in X$ implies that f is convex on $x \in X$, where $(x_1,x_2) \subseteq X \subseteq \mathbb{R}$.