Technische Hochschule Rosenheim WiSe 2021/22
Bsc. Applied Artificial Intelligence

Module: Analysis 1 **Exam number:** Mock exam **Examiner:** PD Dr. Kimmerle **Date of exam:** **Duration of exam:** 75 minutes **Authorised auxiliary means:** 1 hand written A4 page (written on both sides), no calculator	**Examination candidate** (Please in block letters.) **Surname:** __________ **First name:** __________ **Degree programme:** __________ **Matriculation no.:** __________ **Room and seating:** __________ **Signature:** __________

Question	1	2	3	4	5	$\sum$	grade
Points achieved							
Possible points	12	5	5	12	11	45	

Information:

- This exam has 5 questions.
- This exam may not be separated and has to be submitted completely.
- Please answer on the task sheets (also use the back page). If additional sheets are handed in, please note their number on the cover sheet.
- Unless otherwise stated, all results must be justified and proven by the corresponding calculation steps.
- Please write with a non-erasable pen (e.g. ballpoint pen, fountain pen). Do not use the color red.

Legal notice: Please note this a mock exam. No conclusions for the (final) exam can be guaranteed.

Question 1

In this question please give your answer (a number, t(rue)/f(alse) or a cross/checkmark in each of the given boxes. Justify your answers shortly. [a) - f) 1 pt. each, g) 4 pts., h) 2 pts.]

a) Determine $\inf\{1+(-1)^n - \frac{1}{n} | n \in \mathbb{N}\} =$ $= \boxed{}$.

b) Compute $\log_2\left(2^{10}\right) + \ln(e^{-1}) =$ $= \boxed{}$.

c) Let $w = 3+2i$ and $z = 2+i \in \mathbb{C}$. Compute $w \cdot z =$ $= \boxed{}$.

d) We consider the inverse multiplicative element of $z = 2+i \in \mathbb{C}$.

The real part of z^{-1} reads: $\boxed{}$

e) Consider the sets $A = \{1;2\}$ and $B = \{0;3\}$. What is the cardinality of the cartesian product of both sets, i.e. $|A \times B|$? $\boxed{}$.

f) How many permutations has a set with 5 distinct elements? $\boxed{}$.

g) Fill the gaps in the following table: space for, e.g., auxiliary columns:

A	B	C	$(A \vee B) \vee C$	$A \Leftrightarrow B$	$\overline{A \wedge C}$
w	w	f			
w	f	f			
f		f	f	w	

h) Which of the following terms is equivalent to $\exp(\ln(2x))$? (multiple choice, please encircle!)

a) $\ln\left(e^{2x}\right)$, b) $2x$, c) x^2,

d) $\ln(\exp(x^2))$, e) None of the answers a) - d).

Question 2
Show with complete induction over n that we have for any $a \in \mathbb{N}$:

$$\forall\, n \in \mathbb{N} : \quad (2a-1)^n \ \text{ is odd}$$

[5 pt.]

Question 3

a) [2 pts.] Proof by the root test that the series

$$\sum_{k=0}^{\infty} \frac{1}{100^k}$$

converges absolutely.

b) [3 pts.] By using the geometric series rewrite

$$0,\overline{12} = 0,12121212\dots$$

as a rational number, i.e. as a fraction $\frac{p}{q}$, $p \in \mathbb{Z}$, and $q \in \mathbb{N}$.

Question 4

a) [3 pts.] Determine the following limit

$$\lim_{x \downarrow 0} \frac{\sqrt{x}}{\sin(x)}.$$

b) [4 pts.] Let Arcosh be the inverse function of $\cosh : \mathbb{R} \to \mathbb{R}^+$. Compute the derivative

$$\text{Arcosh}'(x)$$

and determine its domain of definition.

c) [5 pts.] Determine all stationary points of the function

$$f(x) = x(\ln(x) - 1)$$

and classify w.r.t. minima or maxima.

Question 5

a) [3 pts.] Determine all primitives for

$$\int \cos(x)\sin(x)\,dx.$$

b) [3 pts.] Compute the integral

$$\int_0^{\pi} \cot(x)\,dx.$$

c) [5 pts.] Plot the set H enclosed between $f : [-1,1] \to \mathbb{R}, x \mapsto \sqrt{1-x^2}$ and the x-axis. Compute the area of H by integration.

Free page, e.g. for secondary calculations