/7 1a

1NF - atomic elements

2NF - nothing depends on just a key subset

3NF - no transitives

BCNF - left side super key

R was a transitive $A,B \to C, D, E \to D \to F$

b) D->F is problematic

$R_1 = \{D\}^+ = \{D, F\}$

$R_2 = (R - \{D, F\}) \cup \{D\} = \{A, B, C, D, E\}$

Lossless ?

$(R_1 \cap R_2) = \{D\}$, $\{D\} \to \{D, F\} \in F^+$

dependency preserving ?

$F_{R_1} = \{D \to D, D \to F\}$

$F_{R_2} = \{A, B \to A,$
$A, B \to B,$
$A, B \to C,$
$A, B \to D,$
$A, B \to E\}$

$(F_{R_1} \cup F_{R_2})^+ = \sim\sim = F$ ✓

c) Cover(R) = R

$R_1 = \{A, B, C, D, E\}$

$R_2 = \{D, F\}$

key contained? yes, in R_1

cleanup? not necessary

synthesis is always lossless and dependency preserving

d) Y -> Z

not a super key

e) Y -> Z is problematic

$S_1 = \{Y\}^+ = \{Y, Z\}$

$S_2 = (S - S_1) \cup \{Y\} = \{V, W, X, Y\}$

Lossless ?

$(S_1 \cap S_2) = \{Y\}$, $\{Y\} \to S_1 \in F^+$ ✓

dependency preserving ?

$F_{S_1} = \{Y \to Y,\ Y \to Z\}$

$F_{S_2} = \{V \to W,\ W \to V,\ W \to X,\ W \to Y\}$

$F_{S_1} \cup F_{S_2} \neq F$, it's missing $V \to Z$

not dependency preserving. not useful

f) $S_1 = \{V, W, Z\}$

$S_2 = \{W, Z, V, X, Y\}$

$S_3 = \{Y, Z\}$

key contained? yes, all 3

cleanup -> S_1, S_3 removed

lossless? S_2 is the only relation, so yes

dep. pres. ? ~

$S = S_2$

12a

runtimes can get bad when looking for keys.

$R_1 = \{A, B, C\}$

$R_2 = \{D, E, F\}$

Key contained? no -> brute force search

b) no brute force search, only 1 relation as a result

c) $A \to B, C$

$D \to E, F$

$A, B, C, D, E, F \to \delta$

Cover:

no trivial FDs ✓

shorten left side

$A \to B, C$ single left side

$D \to E, F$

$A, B, C, D, E, F \to \delta$

no unnecessary FDs ✓

$R_1 = \{A, B, C\}$

$R_2 = \{D, E, F\}$

$R_3 = \{A, D, \delta\}$

contains key? yes $\{A, D\} \in R_3$

cleanup ✓

/3

$S_{1a} = \{R\}^+ = \{R, O, D, A, H, P\}$

$S_{1b} = (S - S_{1a}) \cup \{R\} = \{I, T, E, R\}$

lossless ?

$\{I, T, E, R\}^+ = S$, $\{R, O, D, A, H, P\}^+ = S$ ✓

dep. pres. ?

$R \to O, D \in S_{1a}$ ✓

$O \to A, H, P, R \in S_{1a}$

BCNF? left side super key?

S_{1a} $\{R\}^+ = S_{1a}$ ✓

S_{1b} $\{I, T, E, R\}^+ = S_{1b}$ ✓

b) Cover $(S_1) = S_1$

$S_{1a} = \{R, O, D\}$

$S_{1b} = \{O, A, H, P, R\}$

key contained? no -> new relation

$S_{1c} = \{I, T, E, R\}$

cleanup ✓

a) S_2

$S_{2a} = \{R\}^+ = \{A, P, H, R, O, D\}$

$S_{2b} = (S - S_{2a}) \cup \{R\} = \{I, T, E, R\}$

Lossless ?

$\{I, T, E, R\}^+ = S$ ✓

dep. pres. ?

$R \to O \in S_{1a}$ ✓

$O \to A, P, H, R, D \in S_{2a}$

BCNF ? yes

b) Cover $(S_2) = S_2$

$S_{2a} = \{R, O\}$

$S_{2b} = \{A, P, H, R, O, D\}$

key contained? no

$S_{2c} = \{I, T, E, R\}$

BCNF? yes

/4a

$S_1 = \{R, O, D\}$

$S_2 = \{A, P, H, R, O\}$

$S_3 = \{A, P, H, R, O, D, I, T, E, \delta\}$ <- key

Cover:

S_1 ✓

S_2 ✓

$S_3 = \{R/O, I, T, E, \delta\}$