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Exercise Sheet 6

Stochastics (AAI)

Exercise 6.1

Let X_1, X_2 be independent with $X_1 \sim \mathrm{Exp}(\lambda_1)$, $X_2 \sim \mathrm{Exp}(\lambda_2)$, and $\lambda_1, \lambda_2 > 0$.

a) Compute $P(\{\min(X_1, X_2) \leq x\})$ for $x \in \mathbb{R}$, and determine the distribution of the random variable $\min(X_1, X_2)$.
Hint: For $a, b \in \mathbb{R}$ we have

$$\min(a, b) > x \;\Leftrightarrow\; (a > x) \wedge (b > x).$$

b) Compute $\mathrm{E}(\min(X_1, X_2))$.

Exercise 6.2

Let $X \sim \mathrm{N}(2, 9)$.

a) Compute the following probabilities approximately using Table B.1:

$$P(\{X \leq 2.5\}), \quad P(\{2 \leq X \leq 3\}), \quad P(\{\sqrt{|X|} \leq 2\}).$$

b) Compute the p-quantile of X for $p = 0.95$, and determine $a \in \mathbb{R}$ such that

$$P(\{2 - a \leq X \leq 2 + a\}) = 0.95.$$

Exercise 6.3

Let F_X be the cumulative distribution function of a random variable $X\colon \Omega \to [1, \infty[$ attaining the following values:

x	1	2	3	4
$F_X(x)$	0	0.2	0.6	1

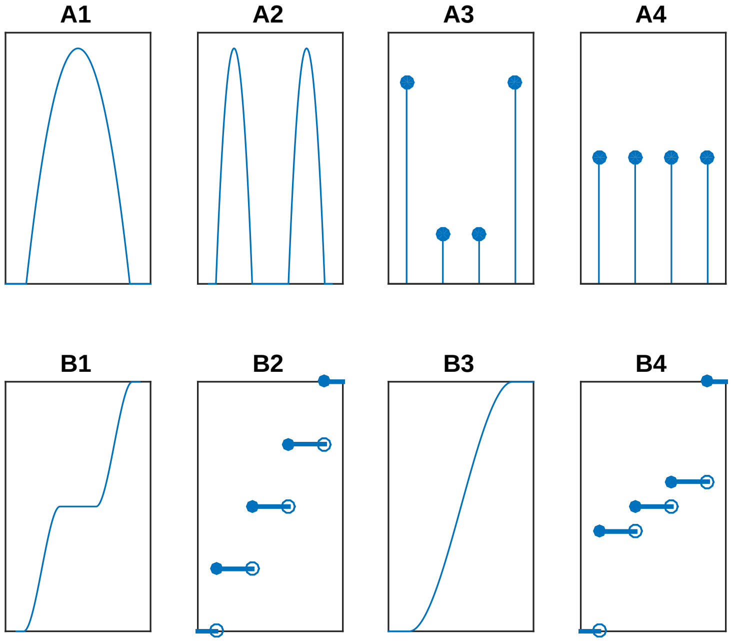
a) Prove or disprove:
   i) $F_X(2.5) = 0.8$,
   ii) $P(\{3 < X \leq 4\}) = 0.4$,
   iii) $\mathsf{m}(X) \in]2, 3]$,
   iv) $\mathrm{E}(X) = 0$.

b) Determine a probability mass function $p_X\colon \mathbb{N} \to \mathbb{R}$ such that the corresponding CDF satisfies the above table.

c) Determine a probability density function $f_X\colon \mathbb{R} \to \mathbb{R}$ such that the corresponding CDF satisfies the above table.

Exercise 6.4

The charts A1-A4 show probability density or probability mass functions. Determine the corresponding cumulative distribution functions given by charts B1-B4.



Exercise 6.5* (P)

Implement a Monte Carlo algorithm to compute

$$\int_0^2 \frac{1}{\sqrt{2\pi}} \exp(-x^2/2)\,\mathrm{d}x$$

and compare the numerical results with Table B.1.

Hint: `rand` (Matlab/Octave)