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Exercise Sheet 3

Stochastics (AAI)

Exercise 3.1 (H)

a) Prove or disprove:

   i) For random variables $X \sim B(n,p)$ and $Y \sim B(m,p)$ with $m,n \in \mathbb{N}$ and $p \in [0,1]$ we have $X+Y \sim B(n+m,p)$.

   ii) For independent random variables $X \sim B(n,p)$ and $Y \sim B(m,q)$ with $m,n \in \mathbb{N}$ and $p,q \in [0,1]$ we have $X+Y \sim B(n+m,(p+q)/2)$.

b) Let $n \in \mathbb{N}$ and $k \in \{0,\dots,n\}$. For $p \in [0,1]$ and $X \sim B(n,p)$ let

$$f(p) = P\left(\{X = k\}\right).$$

Determine the global maximum of f (w.r.t. p).

Exercise 3.2 (H)

a) Let $k \in \mathbb{N}$. For $\lambda > 0$ and $X \sim \mathrm{Poi}(\lambda)$ let

$$f(\lambda) = P\left(\{X = k\}\right).$$

Determine the global maximum of f (w.r.t. λ).

b) Let $X \sim \mathrm{B}(500, 1/1000)$. Compute $P(\{X \leq 2\})$ exactly and approximately using the Poisson limit theorem.

c) Let $X \sim \mathrm{H}(2000, 400, 5)$. Compute $P(\{X > 1\})$ exactly and approximately using a binomial approximation (see Proposition II.6.11).

Exercise 3.3 (H)

a) Since not all airline passengers show up for their reserved seats, an airline sells 102 tickets for a flight that only holds 100 passengers (overbooking). The probability that a passenger shows up is 95%. The passengers are assumed to behave independently.

   i) Specify an appropriate model for the number of passengers who show up.

   ii) Compute the probability that all passengers who show up can take the flight.

b) Specify a discrete probability space that serves as a model for rolling a fair die five times independently and compute the probability of the number pattern "even–odd–even–odd–even".

c) Players of the dice game "Yatzy" (Kniffel) take turns rolling five dice simultaneously. The events "all five dice show the same number" and "three of a kind and two of a (different) kind" are called "Yatzy" and "Full House", respectively.

   i) Specify an appropriate model for the counts of each number of pips within a Yatzy turn.

   ii) Compute the probability of a "Yatzy" and a "Full House".

d) There are 8 job applicants (3 female, 5 male) for 4 indistinguishable jobs. Assume that all selections of 4 applicants are equally likely.

   i) Specify an appropriate model for the number of men that are recruited.

   ii) Compute the probability that solely men are recruited.

e) A consignment of 50 computers is classified as "good" if the number of faulty computers is at most one, and it is classified as "inadequate" if the number of faulty computers is greater than three. Customer and supplier agreed to check a consignment based on four randomly chosen computers. The consignment is accepted by the customer if and only if all these four computers are not faulty.

   a) Specify an appropriate model for the number of faulty computers. (Are there unknown model parameters?)

   b) Assume that a "good" consignment is refused. Determine the corresponding probability.

   c) Assume that the consignment is "inadequate". Determine the number of faulty computers such that the probability of acceptance of the consigment is maximal. Compute the corresponding probability.