Preliminary remarks: This exercise sheet is about the Simplex Algorithm. So that you do not have to do all the Gauss steps yourself, two octave scripts are provided (see Learning Campus): `sheet13_pivotstep.m` and `sheet13_sample_script.m`. You can use them to carry out the individual Gauss steps as soon as you know the corresponding data (the matrix representing the tableau, the pivot element) of the respective tableau.

Explanation of the variables used in the script `sheet13_sample_script.m`:

Assume your current tableau is

z	x_1	x_2	x_3	s_1	s_2	a_1	rhs	basis
1	2	-2	3	0	0	0	5	z
0	3	½	0	-1	0	1	3	a_1
0	4	-2	1	5	1	0	2	s_2

First, you have to define the rowvector

```
zrow=[1 2 -2 3 0 0 0 5];
```

representing the z-row and then the matrix M representing the rows below:

```
M = [0 3 1/2 0 -1 0 1 3;
0 4  -2 1  5 1 0 2];
```

To perform the pivot with pivot in row r and column c, enter:

```
[M,zrow]=sheet13_pivotstep(M,zrow,r,c,colnumber);
```

The return values of this function are the updated matrices `M` and `zrow`. For example,

```
[M,zrow]=sheet13_pivotstep(M,zrow,2,3);
```

would give the updated tableau

z	x_1	x_2	x_3	s_1	s_2	a_1	rhs	basis
1	14	0	3	-4	0	4	17	z
0	6	1	0	-2	0	2	6	x_1
0	16	0	1	1	1	4	14	s_2

or

```
zrow =
1 14 0 3 -4 0 4 17
```

for the vector `zrow`, and for `M`:

```
M =
0  6 1 0 -2 0 2  6
0 16 0 1  1 1 4 14
```

The next simplex step would now be

```
[M,zrow]=sheet13_pivotstep(M,zrow,3,5);
```

Exercise 13.1 *(Formulating LP models; graphical solution).*
Romeo Winery produces two types of wines, Bordeaux and Romerlot, by blending Merlot and Cabernet Sauvignon grapes. Making one barrel of Bordeaux blend requires 250 pounds of Merlot and 250 pounds of Cabernet Sauvignon, whereas making one barrel of Romerlot requires 450 pounds of Merlot and 50 pounds of Cabernet Sauvignon. The profit received from selling Bordeaux is £800 per barrel, and from selling Romerlot, £600 per barrel. Romeo Winery has 9000 pounds of Merlot and 5000 pounds of Cabernet Sauvignon available. Formulate an LP model aiming to maximize the winery's profit. Solve the LP graphically.

Exercise 13.2 *(beginners).* Solve the following LP using the simplex method (with Dantzig's rule):

$$\begin{aligned} \text{maximize} \quad & x_1 + x_2 \\ \text{subject to} \quad & 3x_1 + 7x_2 \leq 10 \\ & 2x_1 + x_2 \leq 3 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Exercise 13.3. Solve the following LP using the simplex method:

$$\begin{aligned} \text{maximize} \quad & x_1 - x_2 \\ \text{subject to} \quad & -2x_1 + x_2 \leq -1 \\ & -x_1 - 2x_2 \leq -2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Exercise 13.4. Solve the following LP using the simplex method:

$$\begin{aligned} \text{maximize} \quad & 4x_1 - 2x_2 \\ \text{subject to} \quad & 2x_1 + x_2 \leq 7 \\ & x_1 + 2x_2 \leq 4 \\ & -x_1 + 2x_2 = -2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Exercise 13.5. Solve the following LP using the simplex method:

$$\begin{aligned} \text{maximize} \quad & 2x_1 + x_2 \\ \text{subject to} \quad & 2x_1 + x_2 \leq 7 \\ & x_1 + 2x_2 \geq 5 \\ & -x_1 + 2x_2 = -2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

The exercises can be discussed in the Tuesday lecture next week.