Exercise Sheet 8
Stochastics (AAI)

Exercise 8.1

Let $X_1, \ldots, X_{10}$ be i.i.d. with $X_1 \sim \mathrm{N}(\mu, \sigma^2)$, $\mu \in \mathbb{R}$, and $\sigma \in \,]0, \infty[$. Moreover, let a sample be given with sample mean $\bar{x}_{10} = 5$ and sample variance $s_{10}^2 = 4$.

a) Assume that $\sigma = 2$ is known. Determine a confidence interval for μ with confidence level 0.95.

b) Assume that σ is unknown. Determine a confidence interval for μ with confidence level 0.95.

c) Determine a confidence interval for σ^2 with confidence level 0.95.

Exercise 8.2

The expected value $\mu \in \mathbb{R}$ of a normal distribution with known variance $\sigma^2 = 9$ shall be estimated.

a) Let a sample of size $n = 100$ be given with sample mean 53.97. Determine a 95% confidence interval for μ.

b) What sample size is required to obtain a 95% confidence interval of length less than 0.4 if the sample mean is given by 53.97?

c) What sample mean is required to obtain a 95% confidence interval of length less than 0.4 if the sample size is given by $n = 1000$?

Exercise 8.3

A sample yields the following weights of cement bags [in kg]:

$$
\begin{array}{cccccc}
50.100 & 49.980 & 50.040 & 50.050 & 50.000 & 50.150 \\
50.030 & 50.090 & 50.120 & 49.990 & 50.010 & 50.110 \\
50.120 & 50.050 & 50.000 & 50.050 & 49.980 & 50.080 \\
50.080 & 50.050 & 49.990 & 50.030 & 50.000 & 50.090 \\
49.980 & 50.000 & 50.070 & 50.100 & 50.050 & 50.120
\end{array}
$$

Determine an approximate 95% confidence interval for the average weight μ.

Exercise 8.4

Let $X_1, \ldots, X_n$ be i.i.d. with $X_1 \sim \mathrm{N}(\mu, \sigma^2)$ where $\sigma \in \,]0, \infty[$ is known and

$$\mu = \vartheta \in \Theta = \mathbb{R}$$

is unknown. For $0 < \alpha < 1$ construct a function $b_n \colon \mathbb{R}^n \to \mathbb{R}$ such that

$$\forall \vartheta \in \Theta \colon P^{\vartheta}\left(\{\vartheta \leq b_n(X_1, \ldots, X_n)\}\right) \geq 1 - \alpha.$$