Exercise 06: Relational database design and normal forms

Additional task 1: Normal forms

Given the following relations Y in the first normal form with functional dependencies F :

$$\begin{aligned} Y &= (A,P,H,R,O,D,I,T,E)\ with \\ R &\rightarrow O \\ O &\rightarrow A,H,P \\ O,P &\rightarrow D,R \\ H,P &\rightarrow P \\ H,P,R &\rightarrow D \end{aligned}$$

1. First, use the COVER algorithm to simplify the set of functional dependencies.
2. Determine the keys of the relations.
3. Do the relations correspond with the second normal form (2NF)?
4. Do the relations correspond with the third normal form (3NF)?
5. Do the relations correspond with the Boyce-Codd normal form (BCNF)?

COVER for R:

1. $F_C = F$
2. $F_C = SPLITTING(F_C) =$

$$\begin{aligned} R &\rightarrow O \\ O &\rightarrow A \\ O &\rightarrow H \\ O &\rightarrow P \\ O,P &\rightarrow D \\ O,P &\rightarrow R \\ H,P,R &\rightarrow D \end{aligned}$$

3. Minimise left sides

$$\begin{aligned} D &\in \{O\}^+_F = \{O,A,H,P,D,R\} \\ &\Rightarrow F_C = F_C - (O,P \rightarrow D) \cup (O \rightarrow D) \\ R &\in \{O\}^+_F = \{O,A,H,P,D,R\} \\ &\Rightarrow F_C = F_C - (O,P \rightarrow R) \cup (O \rightarrow R) \\ D &\in \{R\}^+_F = \{R,O,A,H,P,D\} \\ &\Rightarrow F_C = F_C - (H,P,R \rightarrow D) \cup (R \rightarrow D) \end{aligned}$$

4. So it is $F_C =$

$$\begin{aligned} R &\rightarrow O,D \\ O &\rightarrow A,P,H,R,D \end{aligned}$$

5. Remove unnecessary FDs

$$
\begin{array}{rcl}
O & \notin & \{R\}^+_{F_C-(R\to O)} = \{R, D\} \\
D & \in & \{R\}^+_{F_C-(R\to D)} = \{R, O, A, P, H, D\} \\
 & \Rightarrow & F_C = F_C - (R \to D) \\
A & \notin & \{O\}^+_{F_C-(O\to A)} = \{O, P, H, R, D\} \\
P & \notin & \{O\}^+_{F_C-(O\to P)} = \{A, H, R, D, O\} \\
H & \notin & \{O\}^+_{F_C-(O\to H)} = \{A, P, R, D, O\} \\
R & \notin & \{O\}^+_{F_C-(O\to R)} = \{A, P, H, D\} \\
D & \notin & \{O\}^+_{F_C-(O\to D)} = \{A, P, H, R, O\}
\end{array}
$$

6. Summarise left sides

$$
\begin{array}{rcl}
R & \to & O \\
O & \to & A, P, H, R, D
\end{array}
$$

Keys for Y based on heuristics:

1. Attributes that do not occur in any functional dependency: I, T, E: I, T, E
2. Attributes that do not occur on any right side of a functional dependency: none
3. Test single-element sets with I, T, E:

$$
\begin{array}{rcl}
\{I, T, E, R\}^+_F & = & \{A, P, H, R, O, D, I, T, E\} = Y, \ is\ Key \\
\{I, T, E, O\}^+_F & = & \{A, P, H, R, O, D, I, T, E\} = Y, \ is\ Key
\end{array}
$$

Others do not need to be checked since they will all contain either I, T, E, R or I, T, E, O.

2NF, 3NF und BCNF for R:

- Y does not correspond with the 2NF because every non-key attribute only depends on one part of the key.
- Since Y does not correspond with the 2NF, Y also does not correspond with the 3NF or the BCNF

Additional task 2: Normal forms

1. Given the following relations W' in the first normal form with functional dependencies W :

$$\begin{aligned} W' &= (A, B, C, D)\ with \\ A, B &\rightarrow C \\ B &\rightarrow D \end{aligned}$$

Show that W' does not correspond with the second normal form.

Key determination: all attributes occur, check whether the ones that do not occur on the right are keys:

$$\{A, B\}^+_W = \{A, B, C, D\} = W'$$

D does not depend on the whole key, it only depends on B. Therefore W does not correspond with the second normal form.

2. Given the following relations X' in the first normal form with functional dependencies X :

$$\begin{aligned} X' &= (A, B, C, D)\ with \\ A, B, C &\rightarrow D \\ B, C &\rightarrow A \end{aligned}$$

Show that X' corresponds with the third normal form.

Show that X' corresponds with the BCNF, i.e. all left sides are superkeys, instead:

$$\{A, B, C\}^+_X = \{A, B, C, D\} = X'$$
$$\{B, C\}^+_X = \{B, C, A, D\} = X'$$

3. Given the following relations V' in the first normal form with functional dependencies V :

$$\begin{aligned} V' &= (A, B, C, D, E)\ with \\ C, D, E &\rightarrow A, C \\ A, E &\rightarrow B, D \\ C, D &\rightarrow E \end{aligned}$$

Show that V' does not corresponds with the third normal form.

Key heuristics does not return attributes. Assume that $\{C, D\}$ is a key:

$$\{C, D\}^+_V = \{C, D, E, A, B\} = V'$$

Can there still be another key, only A, B, E is left. It is worth checking:

$$\{A, E\}^+_V = \{A, E, B, D\} \neq V'$$

So C, D is the only key of V'.
Now assume that B is transitively dependent on the key. Then the following must apply:

$$B \in \{C, D\}^+_V = \{C, D, E, A, B\}$$

of course, since $\{C, D\}$ is the key and thus $A, E \rightarrow B$ is a transitive dependency.

4. Given the following relations Y' in the first normal form with functional dependencies Y :

$$\begin{aligned} Y' &= (A, B, C, D, E, F)\ with \\ A &\rightarrow B, C \\ C &\rightarrow D \\ E &\rightarrow F \end{aligned}$$

Show that Y' does not correspond with the third normal form.

Key determination: all attributes occur, check whether the ones that do not occur on the right are keys:

$$\{A, E\}^+_Y = \{A, E, B, C, D, F\} = Y'$$

F does not depend on the whole key, so therefore Y' does not correspond with the second normal form and thus also not with the third normal form.

5. Given the following relations Z' in the first normal form with functional dependencies Z:

$$\begin{aligned} Z' &= (A, B, C, D)\ with \\ A, B, C &\rightarrow D \\ A, B &\rightarrow C, D \\ C &\rightarrow A \end{aligned}$$

Show that Z' does not correspond with the Boyes-Codd normal form.

Key determination: all attributes occur, check whether the ones that do not occur on the right are keys:

$$\{B\}^+_Z = \{B\} \neq Z'$$

This means that B is contained in every key. Now check the two-element sets:

$$\{B, A\}^+_Z = \{A, B, C, D\} = Z'\ ist\ Key$$
$$\{B, C\}^+_Z = \{B, C, A, D\} = Z'\ ist\ Key$$
$$\{B, D\}^+_Z = \{B, D\} \neq Z'$$

Finished, now there can be no three-element sets. Thus, Z' does not correspond with the Boyes-Codd normal form, since with the functional dependency $C \rightarrow A$ there is no superkey on the left.