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Exercise Sheet 2

Stochastics (AAI)

Exercise 2.4 (H) – Solution Hints

For $Y_1 = X_1 + X_2$ and $Y_2 = \exp(X_3)$ we have

$$Y_1 \colon \Omega \to \{0,1,2\} \quad \text{and} \quad Y_2 \colon \Omega \to \{1, e\}.$$

For $y_1 \in \{0,1,2\}$ and $y_2 \in \{1, e\}$ we have

$$Y_2 = y_2 \;\Leftrightarrow\; X_3 = \ln(y_2)$$

and

$$\begin{aligned}
&P(\{Y_1 = y_1\} \cap \{Y_2 = y_2\}) \\
&\quad = P(\{X_1 + X_2 = y_1\} \cap \{X_3 = \ln(y_2)\}) \\
&\quad = P\Big(\bigcup_{i=0}^{1}\big(\{X_1 = i\} \cap \{X_2 = y_1 - i\}\big) \cap \{X_3 = \ln(y_2)\}\Big) \\
&\quad = P\Big(\bigcup_{i=0}^{1}\big(\{X_1 = i\} \cap \{X_2 = y_1 - i\} \cap \{X_3 = \ln(y_2)\}\big)\Big) \\
&\quad = \sum_{i=0}^{1} P\big(\{X_1 = i\} \cap \{X_2 = y_1 - i\} \cap \{X_3 = \ln(y_2)\}\big) \\
&\quad = \sum_{i=0}^{1}\big(P(\{X_1 = i\}) \cdot P(\{X_2 = y_1 - i\}) \cdot P(\{X_3 = \ln(y_2)\})\big) \\
&\quad = P(\{X_3 = \ln(y_2)\}) \cdot \sum_{i=0}^{1}\big(P(\{X_1 = i\}) \cdot P(\{X_2 = y_1 - i\})\big) \\
&\quad \overset{(1)}{=} P(\{X_3 = \ln(y_2)\}) \cdot \sum_{i=0}^{1} P(\{X_1 = i\} \cap \{X_2 = y_1 - i\}) \\
&\quad = P(\{X_3 = \ln(y_2)\}) \cdot P\Big(\bigcup_{i=0}^{1}\big(\{X_1 = i\} \cap \{X_2 = y_1 - i\}\big)\Big) \\
&\quad = P(\{X_3 = \ln(y_2)\}) \cdot P(\{X_1 + X_2 = y_1\}) \\
&\quad = P(\{Y_2 = y_2\}) \cdot P(\{Y_1 = y_1\}).
\end{aligned}$$

Note that

$$\begin{aligned}
&P(\{X_1 = i\}) \cdot P(\{X_2 = y_1 - i\}) \\
&\quad = P(\{X_1 = i\}) \cdot P(\{X_2 = y_1 - i\}) \cdot P(\underbrace{\{X_3 \in \{0,1\}\}}_{\Omega}) \\
&\quad = P(\{X_1 = i\} \cap \{X_2 = y_1 - i\} \cap \underbrace{\{X_3 \in \{0,1\}\}}_{\Omega}) \\
&\quad = P(\{X_1 = i\} \cap \{X_2 = y_1 - i\}).
\end{aligned}$$

(1)