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**Exercise Sheet 1**

# Stochastics (AAI)

## Exercise 1.1 (H)

Prove Proposition II.1.10.

## Exercise 1.2 (H)

Let $\Omega = \{0,1\}^2$ and let $A, B \subseteq \Omega$ be given by $A = \{(1,0),(1,1)\}$ and $B = \{(0,1),(1,1)\}$. Determine all probability measures $P$ on $\Omega$ with

$$P(A) = P(B) = \frac{1}{2}.$$

## Exercise 1.3 (H)

Let $(\Omega, \mathcal{P}(\Omega), P)$ be a discrete probability space and let $A, B, C \subseteq \Omega$ with

$$P(A^c) = \frac{7}{10}, \quad P(B) = \frac{3}{10}, \quad P(C) = \frac{7}{20}, \quad P(A^c \cap B) = \frac{1}{4},$$
$$P(A \cap C) = \frac{1}{10}, \quad P(A \cap B \cap C) = \frac{1}{20}, \quad P((A \cup C) \cap B) = \frac{3}{20}.$$

Compute

$$P(A \cap B), \quad P(A^c \cup B), \quad P(A \cup C), \quad P(B \cap C), \quad P(A \cap B \cap C^c), \quad P(A \cup B \cup C).$$

## Exercise 1.4 (H)

Let $(\Omega, \mathcal{P}(\Omega), P)$ be a discrete probability space and let $A, B \subseteq \Omega$ with

$$P(A) = 0.7, \quad P(B \mid A) = 0.8, \quad P(B \mid A^c) = 0.4.$$

Compute $P(B)$ and $P(A \mid B)$.

## Exercise 1.5 (H)

Let $(\Omega, \mathcal{P}(\Omega), P)$ be a discrete probability space.

a) Let $A, B \subseteq \Omega$. Prove or disprove:

   i) If $A$ and $B$ are independent, then $A$ and $B^c$ are independent.

   ii) If $P(A) \in \{0,1\}$, then $A$ and $B$ are independent.

   iii) If $A$ and $B$ are disjoint, then $A$ and $B$ are independent.

b) Let $P$ be the discrete uniform distribution on $\Omega = \{1,2,3,4\}$ and let

$$A = \{1,2\}, \quad B = \{1,3\}, \quad C = \{2,3\}.$$

   Are $A, B, C$ pairwise independent? Do we have

$$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)?$$

**Exercise 1.6 (H)**

a) Specify a discrete probability space that serves as a model for rolling two fair dice.
b) Determine the following events and compute their probabilities:
   i) "number of pips of first die is two higher than number of pips of second die",
   ii) "both dice show one",
   iii) "sum of pips is at least ten".
c) Are the events i) and iii) from part b) independent?