Exercise 6.1 *(Computing)***.** Determine the Cholesky decompositions of the following matrices. That is, a lower triangular matrix G with $A = GG^T$.

a) $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 8 & 12 \\ 3 & 12 & 27 \end{pmatrix}$ b) $A = \begin{pmatrix} 1 & -2 & 3 & -4 \\ -2 & 8 & -8 & 14 \\ 3 & -8 & 11 & -14 \\ -4 & 14 & -14 & 35 \end{pmatrix}$

Exercise 6.2 *(Octave)***.** Implement the Cholesky method in `Octave`, i.e. write a program that requires a matrix A as input and - if A is spd - returns the decomposition matrix G as output, so that $A = GG^T$ holds. Test the program with the matrices from Exercise 6.1.

Exercise 6.3. Show that for all regular $A \in \mathbb{R}^{n\times n}$

$$\mathrm{cond}_2(A) \le \mathrm{cond}_F(A) \le n\,\mathrm{cond}_2(A).$$

Show that both inequalities are sharp.

Exercise 6.4. Let $A \in \mathbb{R}^{2\times 2}$ be symmetric and assume that

$$\det(A) = 1 \quad \text{and} \quad \frac{\mathrm{trace}(A)}{2} = N,$$

where $N > 0$. Show the following:

(a) It is necessarily true that $N \ge 1$.

(b) It is

$$\mathrm{cond}_2(A) \ge 4N^2 - 2N - 1 \quad \text{for } N \ge 1. \tag{1}$$

Exercise 6.5 *(Octave)***.** Write an Octave program that calculates the largest and smallest eigenvalues of a matrix $A \in \mathrm{Gl}(n)$. Avoid the calculation of the inverse matrix A^{-1} and use the LU decomposition of A instead (without row permutations).

The exercises can be discussed in the Tuesday lecture next week.