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Exercise Sheet 2

Stochastics (AAI)

Exercise 2.1 (H)

a) Specify a discrete probability space that serves as a model for tossing a fair coin independently three times. Model the number of "heads" and "tails" as random variables X_1 and X_2, respectively, and determine their probability mass functions.

b) Consider the situation of Example II.4.3 (fair die rolled twice independently).

   i) Let $Y = \max(X_1, X_2)$ and $Z = \min(X_1, X_2)$. Determine the probability mass functions p_Y and p_Z.

   ii) Compute $P_Y(\{1, 2\})$, $P(\{Z = 5\})$, and $P_{(Y,Z)}(\{1, 2\} \times \{5\})$.

Exercise 2.2 (H)

Let $X_1, X_2 \colon \Omega \to \{0, 1, 2\}$ be random variables satisfying the following table (cf. Remark II.4.16):

$X_1 \backslash X_2$				
	$p_{0,0}$	0.2	0.3	0.6
	0.2	0.1	$p_{1,2}$	$p_{1,\bullet}$
	$p_{2,0}$	$p_{2,1}$	$p_{2,2}$	0
	$p_{\bullet,0}$	$p_{\bullet,1}$	$p_{\bullet,2}$	

a) Complete the table.

b) Compute $P(\{X_2 = 0\} \mid \{X_1 \leq 1\})$.

c) Determine the probability mass function p_{X_2}.

d) Prove or disprove: X_1 and X_2 are independent.

Exercise 2.3 (H)

Let $X_1, X_2, X_3 \colon \Omega \to \{1, 2, 3\}$ be independent random variables with

$$\begin{aligned} p_{X_1}(1) &= 1/3, \quad & p_{X_1}(2) &= 1/3, \quad & p_{X_1}(3) &= 1/3, \\ p_{X_2}(1) &= 1/4, \quad & p_{X_2}(2) &= 1/4, \quad & p_{X_2}(3) &= 1/2, \\ p_{X_3}(1) &= 1/5, \quad & p_{X_3}(2) &= 1/5, \quad & p_{X_3}(3) &= 3/5. \end{aligned}$$

Compute $P(\{X_1 + X_2 + X_3 = 8\})$ and $P(\{\min(X_1, X_2, X_3) = 2\})$.

Exercise 2.4 (H)

Let $X_1, X_2, X_3 \colon \Omega \to \{0, 1\}$ be independent random variables. Moreover, we define the random variables $Y_1 = X_1 + X_2$ and $Y_2 = \exp(X_3)$. Show that Y_1 and Y_2 are independent.