1) $\max\ z = 2x_1 + 3x_2$

s.to a) $x_1 + x_2 \le 2$, $x_1, x_2 \ge 0$ b) $\underbrace{x_1 + x_2 \le -2}_{\text{infeasible}}$, $x_1, x_2 \ge 0$ c) $x_1 + x_2 \ge 2$, $x_1, x_2 \ge 0$ d) $x_1 + x_2 \ge -2$, $x_1, x_2 \ge 0$

unbounded

c) <u>with usual technique:</u>

$x_1 + x_2 \ge 2 \rightsquigarrow -x_1 - x_2 \le -2 \rightsquigarrow -x_1 - x_2 + s_1 = -2 \rightsquigarrow x_1 + x_2 - s_1 = 2 \rightsquigarrow x_1 + x_2 - s_1 + a_1 = 2$

→ intro. art. var.

<u>with excess var.s.</u>

$x_1 + x_2 \ge 2 \rightsquigarrow x_1 + x_2 - e_1 = 2 \rightsquigarrow x_1 + x_2 - e_1 + a_1 = 2$

d) <u>with usual technique:</u>

$x_1 + x_2 \ge -2 \rightsquigarrow -x_1 - x_2 \le 2 \rightsquigarrow -x_1 - x_2 + s_1 = 2$ ✓

<u>with excess var.s.</u>

$x_1 + x_2 \ge -2 \rightsquigarrow x_1 + x_2 - e_1 = -2 \rightsquigarrow -x_1 - x_2 + e_1 = 2$ ✓

a) <u>with usual technique:</u>

$x_1 + x_2 \le 2 \rightsquigarrow x_1 + x_2 + s_1 = 2$ ✓

<u>with excess var.s.</u>

$x_1 + x_2 \le 2 \rightsquigarrow -x_1 - x_2 \ge -2 \rightsquigarrow -x_1 - x_2 - e_1 = -2 \rightsquigarrow x_1 + x_2 + e_1 = 2$

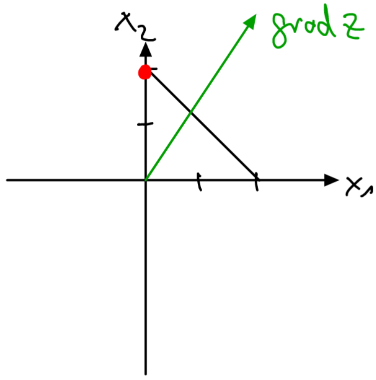
b) <u>with usual technique:</u> → intro. art. var.

$x_1 + x_2 \le -2 \rightsquigarrow x_1 + x_2 + s_1 = -2 \rightsquigarrow -x_1 - x_2 - s_1 = 2$

<u>with excess var:</u> $\rightsquigarrow -x_1 - x_2 - s_1 + a_1 = 2$

$x_1 + x_2 \le -2 \rightsquigarrow -x_1 - x_2 \ge 2 \rightsquigarrow -x_1 - x_2 - e_1 = 2 \rightsquigarrow -x_1 - x_2 - e_1 + a_1 = 2$

a)



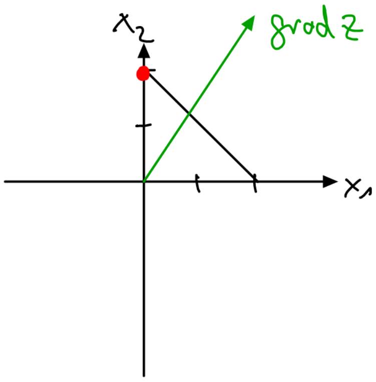
$\max \quad 2x_1 + 3x_2$

s.to $\quad x_1 + x_2 \leq 2 \quad \rightsquigarrow \quad x_1 + x_2 + s_1 = 2$

z	x_1	x_2	s_1	rhs	basis
1	-2	-3	0	0	z
0	1	1	1	2	s_1
1	1	0	3	6	z
0	1	1	1	2	x_2

$\rightsquigarrow \quad x_1 = 0, \quad x_2 = 2, \quad z = 6$

b)



$\max \quad 2x_1 + 3x_2$

s.to $\quad x_1 + x_2 \leq -2 \quad \rightsquigarrow$ art. var.

$\quad\quad x_1, x_2 \geq 0$

$\rightsquigarrow \quad x_1 + x_2 + s_1 = -2$

$\rightsquigarrow \quad -x_1 - x_2 - s_1 + a_1 = 2$

I

z	x_1	x_2	s_1	a_1	rhs	basis_1
1	0	0	0	1	0	z
0	-1	-1	-1	1	2	a_1
0	1	1	1	0	-2	z
0	-1	-1	-1	1	2	a_1

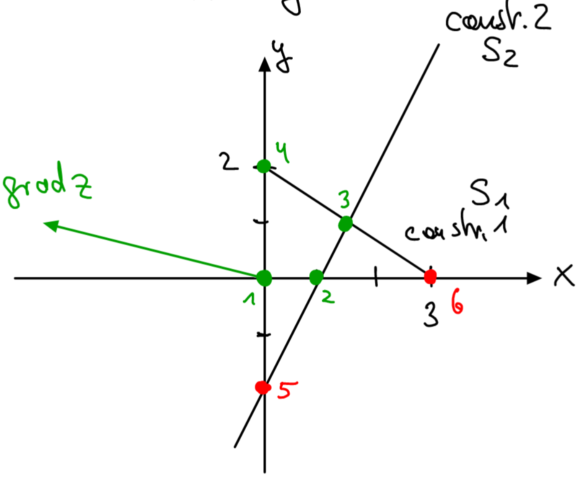
optimal with $z = -2$

Ex Consider

$\min \quad z = -4x + y \quad\quad \Leftrightarrow \quad\quad \max \bar{z} = 4x - y \quad\quad \bar{z} = -z$

s.t. $\quad 2x + 3y \leq 6 \quad\quad\quad\quad 2x + 3y + s_1 = 6$

$\quad\quad -2x + y \geq -2 \quad\quad\quad\quad 2x - y + s_2 = 2$



$\bar{z}$	x	y	s_1	s_2	rhs	basis	ratio
1	-4	1	0	0	0	z	
0	2	3	1	0	6	s_1	3
0	2	-1	0	1	2	s_2	1
1	0	-1	0	2	4	z	
0	0	4	1	-1	4	s_1	
0	1	-1/2	0	1/2	1	x	

vertex 1: $x=y=0 \Rightarrow s_1=6,\ s_2=2$

vertex 2: $s_2=0,\ y=0 \Rightarrow x=1,\ s_1=4$

vertex 3: $s_1=s_2=0 \Rightarrow$ opt. sol. $x=\frac{3}{2}, y=1$

vertex 4: $x=0,\ y=2 \Rightarrow s_1=0,\ s_2=4$

vertex 5: $s_2=0,\ y=-2$ ↯

vertex 6: $s_1=0,\ y=0 \Rightarrow x=3$ & $s_2=-4$ ↯

How many potential basic solutions?

$$\binom{m+n}{m} = \binom{4}{2} = \frac{4\cdot 3}{2} = 6.$$

1	0	0	1/4	7/4	5	$\bar{z}$
0	0	1	1/4	-1/4	1	y
0	1	0	1/8	3/8	3/2	x

$x=\frac{3}{2},\ y=1,\ \bar{z}=5 \Leftrightarrow z=-5$

$s_1=s_2=0$ corr. to vertex 3.

Last $\bar{z}$-row: $\bar{z} = -\frac{1}{4}s_1 - \frac{7}{4}s_2 + 5$

$\Leftrightarrow \quad z = \frac{1}{4}s_1 + \frac{7}{4}s_2 - 5$

$s_1=s_2=0 \quad (\Rightarrow x=\frac{3}{2}, y=1)$

$\Leftrightarrow$ $\bar{z}$ maximal

$\Leftrightarrow$ z minimal