

Exercise Sheet 5 Stochastics (AAI)

Exercise 5.1 (H)

Compute the expected value and the variance of the following distributions:

- a) $X \sim U(a, b)$, $a < b$,
- b) $X \sim \text{Exp}(\lambda)$, $\lambda > 0$,
- c) $X \sim N(\mu, \sigma^2)$, $\mu \in \mathbb{R}$, $\sigma > 0$.

Exercise 5.2 (H)

For $a \in \mathbb{R}$ let $f_a: \mathbb{R} \rightarrow \mathbb{R}$ be given by

$$f_a(x) = \begin{cases} ax + a, & \text{if } x \in [-1, 0], \\ a - ax, & \text{if } x \in [0, 1], \\ 0, & \text{else.} \end{cases}$$

- a) Determine all $a \in \mathbb{R}$ such that f_a is a probability density.
- b) Let X be an absolutely continuous random variable with density f_1 . Compute $E(|X| - X^2)$ and $P(\{X \leq x\})$ for $x \in \mathbb{R}$.

Exercise 5.3 (H)

The mean lifetime of an electronic component is 2000 days.

- a) Specify an appropriate model for the lifetime.
- b) Compute the probability of the lifetime being longer than 500 days and of the lifetime being at most 300 days.

Exercise 5.4 (H)

Let X, Y be independent random variables with $X \sim \text{Exp}(1)$ and $Y \sim B(1, 3/4)$.

- a) Is $X \cdot Y$ discrete or absolutely continuous?
- b) Compute $P(\{X \cdot Y \leq x\})$ for $x \in \mathbb{R}$.