Exercise Sheet 4

Stochastics (AAI)

Exercise 4.1 (H)

Let $(\Omega, \mathcal{P}(\Omega), P)$ be a discrete probability space and let $X, Y : \Omega \to \mathfrak{X}$.

a) For $a \in \mathbb{R}$ let

$$f(a) = \mathrm{E}\big((X - a)^2\big).$$

Determine the global minima of f.

b) Let $X \sim \mathrm{B}(1, 1/2)$. For $a \in \mathbb{R}$ let

$$g(a) = \mathrm{E}\big(|X - a|\big).$$

Determine the global minima of g.

c) Prove or disprove:

    i) If $\mathrm{E}(X) \le \mathrm{E}(Y)$, then $X(\omega) \le Y(\omega)$ for all $\omega \in \Omega$.

    ii) $\mathrm{Var}(X + Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$.

Exercise 4.2 (H)

Let $(\Omega, \mathcal{P}(\Omega), P)$ be a discrete probability space and let $X, Y, Z : \Omega \to \mathfrak{X}$ be independent with $X \sim \mathrm{B}(2, 3/4)$, $Y \sim \mathrm{Poi}(5)$, and $Z \sim \mathrm{Geo}(1/2)$. Compute

    a) $\mathrm{Var}(2Y - 3Z)$,   b) $\mathrm{E}\big((X - Y) \cdot Z\big)$,   c) $\mathrm{E}\big(\sin(\pi X) + Z\big)$.

Exercise 4.3 (H)

A company sends vouchers to $n \in \mathbb{N}$ distinct persons. For each redeemed voucher there is cost of 10 euro. Based on experience persons act independently and redeem a voucher with probability 60%.

a) Specify an appropriate model for the total cost and compute its expected value. What is the maximum number of vouchers that can be sent if the expected total cost is at most 5000 euro.

b) Compute the variance of the total cost for $n = 2000$.

Exercise 4.4 (H)

Let the following sample with parameter $a \in \mathbb{R}$ be given:

l	1	2	3	4	5
x_l	2	0	-1	1	a

a) Compute the sample mean as a function of a.

b) Determine all $a \in \mathbb{R}$ such that the sample variance is minimal.