

P1:

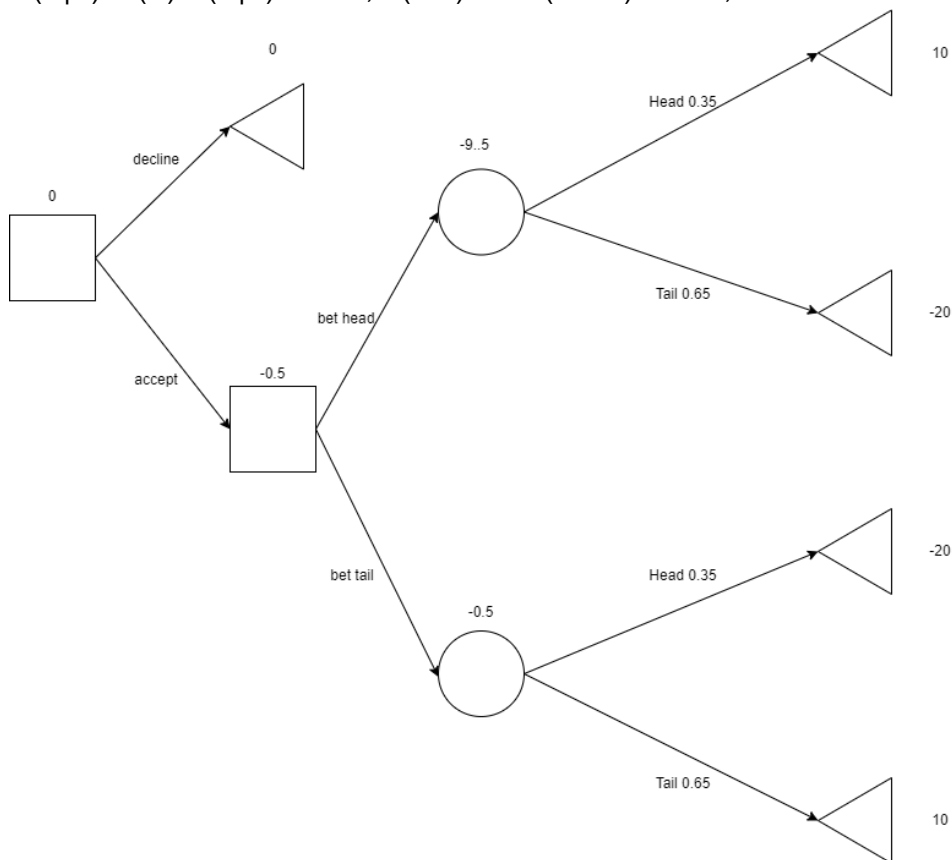
- A.
- a. $P(X=2) = P(X=2, Y=1) + P(X=2, Y=2) + P(X=2, Y=3) = 0.38$
 - b. $P(Y=2) = P(X=0, Y=2) + P(X=1, Y=2) + P(X=2, Y=2) = 0.54$
 - c. $P(X=Y) = P(X=1, Y=1) + P(X=2, Y=2) = 0.3$
 - d. $P(Y=2|X=2) = P(X=2, Y=2) / P(X=2) = 0.526$
 - e. $P(X=1 | X+Y=3) = P(X=1, X+Y=3) / P(X+Y=3) = 0.667$
 - f. $\text{Exp}(X) = 0 \cdot P(X=0) + 1 \cdot P(X=1) + 2 \cdot P(X=2) = 1.18$
 - g. $\text{Exp}(Y) = 1 \cdot P(Y=1) + 2 \cdot P(Y=2) + 3 \cdot P(Y=3) = 1.98$
 - h. $\text{Exp}(2X+Y) = 2\text{Exp}(X) + \text{Exp}(Y) = 4.34$

B. $X+Y$:

1	2	3	4	5
0.06	0.20	0.36	0.28	0.10

$$\text{Exp}(X+Y) = \text{Exp}(X) + \text{Exp}(Y) = 3.16$$

P2. A. Denote coin with 0.1 as head as coin A, and the other one as coin B; $P(\text{Head}) = P(A) \cdot P(H|A) + P(B) \cdot P(H|B) = 0.35$; $P(\text{Tail}) = 1 - P(\text{Head}) = 0.65$;



The expected loss is -0.5 if we bet, hence we shouldn't take the bet.

B. Denote the event that the coin comes up head in the first flip as $H1$, head in the second flip as $H2$, tail in the first flip as $T1$, and tail in the second flip as $T2$.

Given that the first flip is a head, the probability that the coin is coin A: $P(A|H1) =$

$$P(H1|A) \cdot P(A) / P(H1) = 1/7$$

Given the first flip is a tail, the probability that the coin is coin B: $P(B|H1) = 1 - P(A|H1) = 6/7$

Therefore, the probability that the coin comes up head in the second flip is: $P(H2|H1) =$

$$P(H2|A) \cdot P(A|H1) + P(H2|B) \cdot P(B|H1) = 37/70$$

The probability that the coin comes up tail in the second flip is $P(T2|H1) = 1 - P(H2|H1) = 33/70$

Given that the first flip is a tail, the probability that the coin is coin A: $P(A|T1) =$

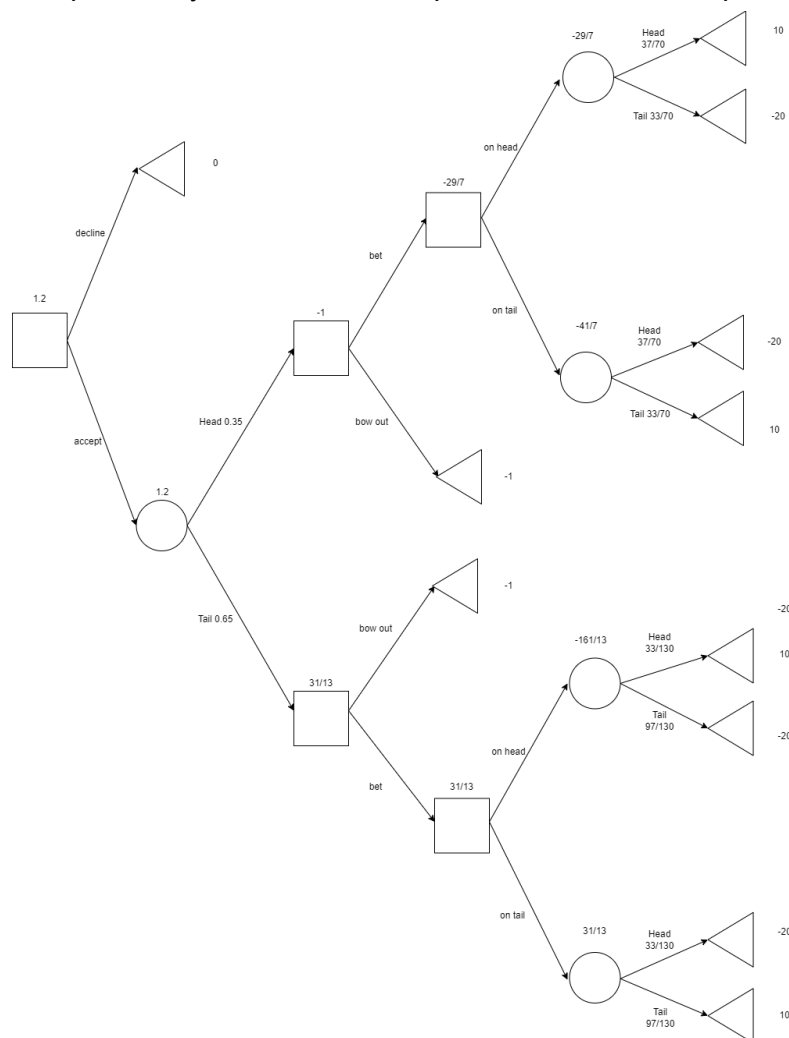
$$P(T1|A) \cdot P(A) / P(T1) = 9/13$$

Given the first flip is a tail, the probability that the coin is coin B: $P(B|T1) = 1 - P(A|T1) = 4/13$

Therefore, the probability that the coin comes up head in the second flip is: $P(H2|T1) =$

$$P(H2|A) \cdot P(A|T1) + P(H2|B) \cdot P(B|T1) = 33/130$$

The probability the coin comes up tails in the second flip is $P(T2|T1) = 1 - P(H2|T1) = 97/130$.



We should bet and if the first coin is head then bow out, but if it's tail then bet on tail. The expected profit is 1.2 following the above strategy.