Problem 1

1.1 Inference on T2

Combining rule (1), with fact (11) and fact(19) with substitution {ha → H3, hb → H2, hc → H1, tx → T1, ty → T2} infer (21) P(H1, T2).
Combining rule (2), with fact (11) and fact(19) with substitution {ha → H3, hb → H2, hc → H1, tx → T1, ty → T2} infer (22) E(H3, T2).
Combining rule (3), with fact (11) and fact(19) with substitution {ha → H3, hb → H2, hc → H1, tx → T1, ty → T2} infer (23) E(H2, T2).
Combining rule (6), with fact (15) with substitution {ha → H1, hb → H3} infer (24) U(H4, H1).
Combining rule (6), with fact (17) with substitution {ha → H1, hb → H3} infer (25) U(H4, H2).
Combining rule (6), with fact (18) with substitution {ha → H1, hb → H3} infer (26) U(H4, H3).
Combining rule (4), with fact (11), fact(24), fact(25), fact(26), fact(10), and fact(19) with substitution {ha → H3, hb → H2, hc → H1, tx → T1, ty → T2, hd → H4} infer (27) P(H4, T2).

1.2 Inference on T3

Combining rule (1), with fact (12) and fact(20) with substitution {ha → H4, hb → H1, hc → H2, tx → T2, ty → T3, } infer (28) P(H2, T3).
Combining rule (2), with fact (12) and fact(20) with substitution {ha → H4, hb → H1, hc → H2, tx → T2, ty → T3, } infer (29) E(H4, T3).
Combining rule (3), with fact (12) and fact(20) with substitution {ha → H4, hb → H1, hc → H2, tx → T2, ty → T3, } infer (30) E(H1, T3).
Combining rule (6), with fact (14) with substitution {ha → H1, hb → H3} infer (31) U(H3, H1).
Combining rule (6), with fact (16) with substitution {ha → H2, hb → H3} infer (32) U(H3, H2).
Combining rule (5), with fact (12), fact(18), fact(31), fact(32), fact(22), and fact(20) with substitution {ha → H4, hb → H1, hc → H2, tx → T2, ty → T3, hd → H3} infer (33) E(H3, T3).

Problem 2

Let Ω be the set of holes and points in time. Let L be a Datalog language with the following predicates:
P(h,p,t). Hole h has peg p in it at time t.
E(h,t). Hole h is empty at time t.
J(ha,hb,hc,t,p). At time t the peg p in ha is jumped to hc over hb.
U(ha, hb). Holes ha and hb are unequal.
S(tx, ty). Time instant tx and ty occur in sequence.
UP(pa,pb). Pegs pa and pb are unequal.

Causal axioms

1. J(ha,hb,hc,tx,p) ∧ S(tx,ty) ⇒ P(hc,p,ty).
2. J(ha,hb,hc,tx,p) ∧ S(tx,ty) ⇒ E(ha,ty).
3. J(ha,hb,hc,tx,p) ∧ S(tx,ty) ⇒ E(hb,ty).

Frame axioms
4. $J(ha,hb,hc,tx,pa) \land U(hd,ha) \land U(hd,hb) \land U(hd,hc) \land P(hd,pb,tx) \land S(tx,ty) \Rightarrow P(hd,pb,ty)$.
5. $J(ha,hb,hc,tx,pa) \land U(hd,ha) \land U(hd,hb) \land U(hd,hc) \land E(hd,tx) \land S(tx,ty) \Rightarrow E(hd,ty)$.

Inequality is symmetric
6.$U(ha,hb) \Rightarrow U(hb,ha)$.
21. $UP(pa,pb) \Rightarrow UP(pb,pa)$

Starting state
7. E(H1,T1).
8. P(H2,PR,T1).
9. P(H3,PW,T1).
10. P(H4,PB,T1).

Jumps executed
11. J(H3,H2,H1,T1,PW).
12. J(H4,H1,H2,T2,PB).

Unique names
13. U(H1,H2).
14. U(H1,H3).
15. U(H1,H4).
16. U(H2,H3).
17. U(H2,H4).
18. U(H3,H4).
22. UP(PR, PW).
23. UP(PR, PB).
24. UP(PW, PB).

Time Sequence
19. S(T1.T2).
20. S(T2.T3).

Problem 3
A. Denote “pick a coin out of the box at random and flip it, resulting in heads up” as A
$P(A) = \frac{1}{5}*(1*0.1+2*0.3+2*0.8) = 0.46$

B. Denote “pick a coin out of the box and flip it twice, resulting in two heads” as B
$P(B) = \frac{1}{5}*(1*0.1^2+2*0.3^2+2*0.8^2) = 0.294$

C. Denote “Pick two coins out of the box together (at the same time) and flip each of them once, resulting in two heads” as C

$P(C) =$
$(C(1,1)*C(2,1)*0.1*0.3+C(1,1)*C(2,1)*0.1*0.8+C(2,1)*C(2,1)*0.3*0.8+C(2,2)*0.3^2+C(2,2)*0.8^2)/C(5,2) = 0.191$

D. Denote "Pick a coin of the box at random, flip it, put it back, again pick a coin at random, and flip it, resulting in two heads" as D
Given "Two flips of two different coins are conditionally independent given the categories of the two coins."
$P(D) = (P(A))^2 = 0.2116$

E. Denote, "the coin is in the category 1" as E1, and "the coin is in the category 2" as E2, knowing "pick a coin out of the box at random and flip it. It comes up heads." as A
$P(E1|A) = P(E1, A)/P(A) = P(A, E1)*P(E1)/P(A) = (0.1*1/5) / 0.46 = 1/23$
$P(E2|A) = P(E2, A)/P(A) = P(A, E2)*P(E2)/P(A) = (0.3*2/5) / 0.46 = 6/23$

F. Denote "the coin is in the category 3" as E3 and knowing the denotations above
$P(E1|-A) = P(E1, -A)/P(-A) = P(-A, E1)*P(E1)/P(-A) = (0.9*1/5) / 0.54 = 1/3$
$P(E2|-A) = P(E2, -A)/P(-A) = P(-A, E2)*P(E2)/P(-A) = (0.7*2/5) / 0.54 = 14/27$
$P(E3|-A) = P(E3, -A)/P(-A) = P(-A, E3)*P(E3)/P(-A) = (0.2*2/5) / 0.54 = 4/27$

G. Denote "second flip is a head" as G and knowing the above denotations
Given that "Two flips of a coin of unknown category are not absolutely independent."
$P(G|A) = P(G,A)/P(A) = P(B)/P(A) = 0.294/0.46 = 0.639$

H. With the above denotations
$P(E1|B) = P(E1,B)/P(B) = P(B|E1)*P(E1)/P(B) = 1/147$
$P(E2|B) = P(E2,B)/P(B) = P(B|E2)*P(E2)/P(B) = 6/49$
$P(E3|B) = P(E3,B)/P(B) = P(B|E3)*P(E3)/P(B) = 128/147$