# Uge 46

## Ex 1

Which of the following formulas are satisfiable (give a satisfying assignment)? Which are not (give reasons)?

a) $A \wedge B$

b) $A \vee B$

c) $A \rightarrow B$

d) $A \wedge -A$

e) $A \vee -A$

f) $(A \rightarrow B) \wedge (B \rightarrow A)$

g) $(A \rightarrow B) \wedge (B \rightarrow A) \wedge A$

h) $(A \rightarrow B) \wedge (B \rightarrow A) \wedge -A$

i) $(A \rightarrow B) \wedge (B \rightarrow -A) \wedge (-A \rightarrow -B) \wedge (-B \rightarrow A)$

a. $A = 1, B = 1$
b. $A = 1, B = 0$
c. $A = 0, B = 1$ (if A then B, remember same as $\neg A \vee B$
d. Nope not satisfiable
e. Yep, always true. $A = 1$
f. $A = 1, B = 1$
g. $A = 1, B = 1$
h. $A = 0, B = 0$
i. Nope, clearer when written out:

$$(\neg A \vee B) \wedge (\neg B \vee \neg A) \wedge (A \vee \neg B) \wedge (B \vee A)$$

led 2 og 4 indicates they cannot have same truth value.

led 1 og 3 indicates they cannot have different truth value.

No options left.

**Ex 2**

Two formulas are equivalent, if the same assignments satisfy both of them.

Which of the following formulas are equivalent?

a) –A ∧ B

b) –A ∨ B

c) A ➔ B

d) (A ➔ B) ∧ (–B ➔ A)

e) (–A ➔ B) ∧ (–B ➔ –A)

making truthtables:

| $\neg A \wedge B$ | | |
|---|---|---|
| **A** | **B** | **O** |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

| $\neg A \vee B$ | | |
|---|---|---|
| **A** | **B** | **O** |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

| $A \rightarrow B$ | | |
|---|---|---|
| **A** | **B** | **O** |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

| $(A \rightarrow B) \wedge (\neg B \rightarrow A)$ | | |
|---|---|---|
| **A** | **B** | **O** |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

| $(\neg A \rightarrow B) \wedge (\neg B \rightarrow \neg A)$ | | |
|---|---|---|
| **A** | **B** | **O** |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

Also seen when writing out:

$$\neg A \wedge B$$

$$\neg A \vee B$$

$$\neg A \vee B$$

$$(\neg A \vee B) \wedge (B \vee A)$$

$$(A \vee B) \wedge (B \vee \neg A)$$

We find that the following pairs are equal:

- $b \text{ and } c$
- $d \text{ and } e$

## Ex 3

We rembember:

**Conversion to CNF**

- Implications can be replaced by disjunction:
  - A ➔ B converted to $-A \vee B$
- DeMorgan's rules specify how to move negation "inwards":
  - $-(A \wedge B) = -A \vee -B$
  - $-(A \vee B) = -A \wedge -B$
- Double negations can be eliminated:
  - $-(-A) = A$
- Conjunction can be distributed over disjunction:
  - $A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C)$

A and B are literals.

Convert the following formulas into CNF:

a) $-A \wedge B$

b) $-A \vee B$

c) A ➔ B

d) (A ➔ B) $\wedge$ (–B ➔ A)

e) (–A ➔ B) $\wedge$ (–B ➔ –A)

f) A ➔ (– (B $\wedge$ D))

g) A ➔ (– (B $\vee$ D))

h) A ➔ (– (B ➔ (C $\wedge$ D)))

a. is in CNF
b. is in CNF
c. $\neg A \vee B$
d. $(\neg A \vee B) \wedge (B \vee A)$
e. $(A \vee B) \wedge (B \vee \neg A)$
f.

$$\neg A \vee \left(\neg(B \wedge D)\right)$$
$$\neg A \vee \neg B \vee \neg D$$

g.

$$\neg A \vee \left(\neg(B \vee D)\right)$$
$$\neg A \vee (\neg B \wedge \neg D)$$

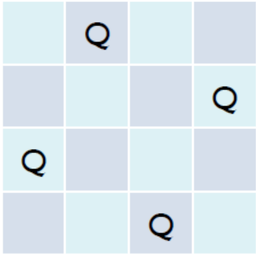
$$(\neg A \vee \neg B) \wedge (\neg A \vee \neg D)$$

h.

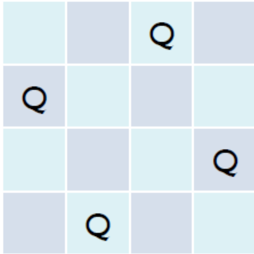
$$\neg A \vee \left(\neg\left(B \rightarrow (C \wedge D)\right)\right)$$
$$\neg A \vee \left(\neg\left(\neg B \vee (C \wedge D)\right)\right)$$
$$\neg A \vee \left(\left(B \wedge \neg(C \wedge D)\right)\right)$$
$$\neg A \vee \left(B \wedge (\neg C \vee \neg D)\right)$$
$$\neg A \vee \left((B \wedge \neg C) \vee (B \wedge \neg D)\right)$$
$$\left(\neg A \vee (B \wedge \neg C)\right) \wedge \left(\neg A \vee (B \wedge \neg D)\right)$$
$$\left(\neg A \vee (B \wedge \neg C)\right) \wedge \left(\neg A \vee (B \wedge \neg D)\right)$$
$$(\neg A \vee B) \wedge (\neg A \vee \neg C) \wedge (\neg A \vee B) \wedge (\neg A \vee \neg D)$$
$$(\neg A \vee \neg C) \wedge (\neg A \vee B) \wedge (\neg A \vee \neg D)$$

**Ex 4**

Solutions to N-Towers and N-Queens are symmetric:



and



a) Write two clauses that forbid solutions where there is a queen in the right half of the first row.

b) Instead of adding two clauses, change an existing clause.

| | | | |
|---|---|---|---|
| $X_{1,1}$ | $X_{1,2}$ | $X_{1,3}$ | $X_{1,4}$ |
| $X_{2,1}$ | $X_{2,2}$ | $X_{2,3}$ | $X_{2,4}$ |
| $X_{3,1}$ | $X_{3,2}$ | $X_{3,3}$ | $X_{3,4}$ |
| $X_{4,1}$ | $X_{4,2}$ | $X_{4,3}$ | $X_{4,4}$ |

---

a. clauses:

$$\neg x_{1,3}$$

$$\neg x_{1,4}$$

b. changing where the tower can be:

from:

$$X_{1,1} \lor X_{1,2} \lor X_{1,3} \lor X_{1,4}$$

to:

$$X_{1,1} \lor X_{1,2}$$

**Ex 5**

- Install **lingeling** or another compatible SAT solver
- Alternatively, use a Javascript SAT solver, e.g.:
  - https://www.msoos.org/2013/09/minisat-in-your-browser/
- Test it using the following input saved as `test.cnf`

```
p cnf 4 6
-1 -2 0
-1 -3 0
-2 -4 0
-3 -4 0
1 2 0
3 4 0
```

**Ex 6**

The formula from Slide 11 contains redundant information. For example, $X_{1,1} \rightarrow -X_{1,2}$ and $X_{1,2} \rightarrow -X_{1,1}$ are equivalent. Understand and remove these redundancies:

a) Why do these redundancies occur?

b) Identify all such redundancies!

c) Write down a simplified formula without redundancies!

d) Convert the simplified formula into CNF!

e) Write the formula in DIMACS format!

f) Run the lingeling solver on it and interpret the result!

| | |
|---|---|
| $X_{1,1} \rightarrow -X_{1,2}$ | "Tower at (1,1) attacks to the right" |
| $X_{1,1} \rightarrow -X_{2,1}$ | "Tower at (1,1) attacks downwards" |
| $X_{1,2} \rightarrow -X_{1,1}$ | "Tower at (1,2) attacks to the left" |
| $X_{1,2} \rightarrow -X_{2,2}$ | "Tower at (1,2) attacks downwards" |
| $X_{2,1} \rightarrow -X_{2,2}$ | "Tower at (2,1) attacks to the right" |
| $X_{2,1} \rightarrow -X_{1,1}$ | "Tower at (2,1) attacks upwards" |
| $X_{2,2} \rightarrow -X_{1,2}$ | "Tower at (2,2) attacks to the left" |
| $X_{2,2} \rightarrow -X_{2,1}$ | "Tower at (2,2) attacks upwards" |
| $X_{1,1} \vee X_{1,2}$ | "Tower in first row" |
| $X_{2,1} \vee X_{2,2}$ | "Tower in second row" |

a. Because of symmetry
b. Rendundant pairs:
   - 1 and 3
   - 2 and 6
   - 4 and 7
   - 5 and 8
c. without reundancies:
   - $X_{1,1} \rightarrow \neg X_{2,1}$
   - $X_{1,2} \rightarrow \neg X_{1,1}$
   - $X_{1,2} \rightarrow \neg X_{2,2}$
   - $X_{2,1} \rightarrow \neg X_{2,2}$
   - $X_{1,1} \vee X_{1,2}$
   - $X_{2,1} \vee X_{1,2}$

d. Convert into CNF

$$(X_{1,1} \rightarrow \neg X_{2,1}) \wedge (X_{1,2} \rightarrow \neg X_{1,1}) \wedge (X_{1,2} \rightarrow \neg X_{2,2}) \wedge (X_{2,1} \rightarrow \neg X_{2,2}) \wedge (X_{1,1} \vee X_{1,2}) \wedge (X_{2,1} \vee X_{1,2})$$

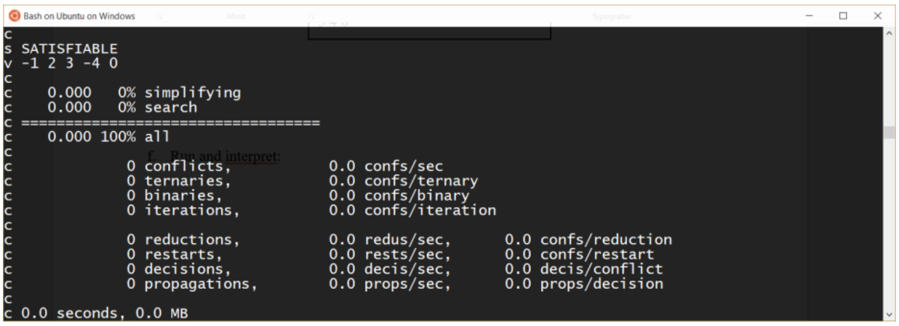
$$(\neg X_{1,1} \vee \neg X_{2,1}) \wedge (\neg X_{1,2} \vee \neg X_{1,1}) \wedge (\neg X_{1,2} \vee \neg X_{2,2}) \wedge (\neg X_{2,1} \vee \neg X_{2,2}) \wedge (X_{1,1} \vee X_{1,2}) \wedge (X_{2,1} \vee X_{2,2})$$

e. Write in DIMACS:

| | |
|---|---|
| $X_{1,1}$ | $X_{1,2}$ |
| $X_{2,1}$ | $X_{2,2}$ |

```
p cnf 4 8
-1 -3 0
-2 -1 0
-2 -4 0
-3 -4 0
1 2 0
3 4 0
```

f. Run and interpret:



```
Bash on Ubuntu on Windows
c
s SATISFIABLE
v -1 2 3 -4 0
c
c    0.000   0% simplifying
c    0.000   0% search
c ==================================
c    0.000 100% all
c
c            0 conflicts,        0.0 confs/sec
c            0 ternaries,        0.0 confs/ternary
c            0 binaries,         0.0 confs/binary
c            0 iterations,       0.0 confs/iteration
c
c            0 reductions,       0.0 redus/sec,      0.0 confs/reduction
c            0 restarts,         0.0 rests/sec,      0.0 confs/restart
c            0 decisions,        0.0 decis/sec,      0.0 decis/conflict
c            0 propagations,     0.0 props/sec,      0.0 props/decision
c
c 0.0 seconds, 0.0 MB
```