integral

definisi dan notasi

Integral → lawan dari turunan

notasi : $\boxed{\int \_\_ \, dx}$ → yang diintegralkan variabelnya

rumus dasar

- $\int k \, dx = kx + C$
- $\int x^n dx = \frac{1}{n+1} x^{n+1} + C$
- $\int (ax+b)^n dx = \frac{1}{a(n+1)} \cdot (ax+b)^{n+1} + C$

$$\int f \;\downarrow\uparrow\; f \;\downarrow\uparrow\; f'$$

contoh :

① $\int 4x^3 + \frac{2}{x^2} - 3\sqrt{x} + 5 \, dx$

$= \int 4x^3 + 2x^{-2} - 3x^{\frac{1}{2}} + 5 \, dx$

$= x^4 - 2x^{-1} - 2x^{\frac{3}{2}} + 5x + C$

$= x^4 - \frac{2}{x} - 2x\sqrt{x} + 5x + C$

② $\int \frac{4}{\sqrt[3]{2x-5}} \, dx$

$= \int 4(2x-5)^{-\frac{1}{3}}$

$= \frac{4}{2 \cdot \frac{2}{3}} (2x-5)^{\frac{2}{3}} + C$

$= 3(2x-5)^{\frac{2}{3}} + C$

contoh halaman 12

① $\int \frac{(2\sqrt{x}+3)^2}{3\sqrt{x}} \, dx$

$= \int \frac{4x + 12\sqrt{x} + 9}{3\sqrt{x}} \, dx$

$= \int \frac{4}{3} x^{\frac{1}{2}} + 4 + 3x^{-\frac{1}{2}} \, dx$

$= \frac{4}{3} \cdot \frac{2}{3} x^{\frac{3}{2}} + 4x + 6x^{\frac{1}{2}}$

$= \frac{8}{9} x\sqrt{x} + 4\sqrt{x}\sqrt{x} + 6\sqrt{x} + C$

$= 2\sqrt{x} \left(\frac{4}{9}x + 2\sqrt{x} + 3\right) + C$

② $\int \frac{(x\sqrt{x}+2)^2}{2\sqrt{x}} \, dx$

$= \int \frac{x^3 + 4x\sqrt{x} + 4}{2\sqrt{x}} \, dx$

$= \int \frac{1}{2} x^{\frac{5}{2}} + 2x + 2x^{-\frac{1}{2}} \, dx$

$= \frac{1}{7} x^{\frac{7}{2}} + x^2 + 4x^{\frac{1}{2}} + C$

$= \frac{1}{7} x^3\sqrt{x} + x^2 + 4\sqrt{x} + C$

③ Jika $\int f(x) dx = \sqrt[3]{g(x)} + C$ dan $g(1) = g'(1) = 8$

maka $f(1) = \dots$

diturunkan

$$\int f \;\downarrow\; f \;\downarrow\; f'$$

$\int f(x) dx = (g(x))^{\frac{1}{3}} + C$

diturunkan $f(x) = \frac{1}{3} (g(x))^{-\frac{2}{3}} \cdot g'(x)$

$f(1) = \frac{1}{3} (8)^{-\frac{2}{3}} \cdot 8$

$= \frac{1}{3} \cdot \frac{1}{4} \cdot 8$

$= \frac{2}{3}$

Halaman 13

① Jika $\int g(x) \, dx = 3\sqrt{f(x)} + C$ dan $f(1) = f'(1) = 9$

maka $g(1) = \dots$

diturunkan

$\int g(x) = 3(f(x))^{\frac{1}{2}} + C$

$g(x) = \frac{3}{2} (f(x))^{-\frac{1}{2}} f'(x)$

$g(1) = \frac{3}{2} (9)^{-\frac{1}{2}} \cdot 9$

$= \frac{3}{2} \cdot 3 \cdot 9$

$= \frac{9}{2}$ (E)

Halaman 12

⑥ Diketahui $\int \frac{dx}{f(x)} = 5x^2 - 2x + c$

Jika $h(x) = f'(x)$, maka nilai $h(1) = \dots$

diturunkan

$$\frac{1}{f(x)} = 10x - 2$$

$$f(x) = \frac{1}{10x - 2}$$

$$f(x) = (10x-2)^{-1}$$

$$f'(x) = -(10x-2)^{-2} \cdot 10$$

$$f'(x) = \frac{-10}{(10x-2)^2} \rightarrow f'(x) = h(x) \rightarrow h(1) = f'(1) = \frac{-10}{8^2} = \frac{-5}{32}$$

integral ~~tak~~ tentu

misal $\int f(x)\,dx = F(x)$ maka $\boxed{\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)}$

sifat-sifat :

① $\int_a^b k \cdot f(x)\,dx = k \int_a^b f(x)\,dx$

② $\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

③ $\int_a^a f(x)\,dx = 0$

④ $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

⑤ $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ a c b

Halaman 13

⑩ Nilai dari $\int_{-1}^{1} (7x^3 - 2x^2 - 5)\,dx$ adalah ...

$$\int_{-1}^{1} (7x^3 - 2x^2 - 5)\,dx$$

$$= \underbrace{\left(\tfrac{7}{4}x^4 - \tfrac{2}{3}x^3 - 5x\right)}_{x=1} - \underbrace{\left(\tfrac{7}{4}x^4 - \tfrac{2}{3}x^3 - 5x\right)}_{x=-1}$$

$$= \left(\cancel{\tfrac{7}{4}} - \tfrac{2}{3} - 5\right) - \left(\cancel{\tfrac{7}{4}} + \tfrac{2}{3} + 5\right)$$

$$= -\frac{4}{3} - 10 = \frac{-34}{3}$$

Halaman 14

④ Jika $\int_{-1}^{3} (m+4)x^2 + 4x - 5\,dx = 52$, maka nilai $2m^2 + 1 = \dots$

$$52 = \int_{-1}^{3} (m+4)x^2 + 4x - 5\,dx$$

$$52 = \underbrace{\left(\tfrac{1}{3}(m+4)x^3 + 2x^2 - 5x\right)}_{3} - \underbrace{\left(\tfrac{1}{3}(m+4)x^3 + 2x^2 - 5x\right)}_{-1}$$

$$52 = 9(m+4) + 18 - 15 - \left(-\tfrac{1}{3}(m+4) + 2 + 5\right)$$

$$52 = 9(m+4) + 3 - 7 + \tfrac{1}{3}(m+4)$$

$$52 = \frac{28}{3}(m+4) - 4$$

$$\cancel{56}^{\,2} \cdot 3 = \cancel{28}(m+4)$$

$$m + 4 = 6$$

$$m = 2$$

maka

$$2m^2 + 1$$

$$= 8 + 1$$

$$= 9 \text{ (A)}$$

soal lain :

$f(x) = x^3 + 3x^2 - 5x + \boxed{\int_{-1}^{1} f(x)\,dx}$

$f(1) = \dots$

diintegralkan

$F(1) - F(-1)$

$\boxed{\int f(x) = F(x)}$

$$\int_{-1}^{1} f(x)\,dx = \left(\tfrac{1}{4}x^4 + x^3 - \tfrac{5}{2}x^2 + F(1)x - F(-1)x\right]_{-1}^{1}$$

$$= \left(\cancel{\tfrac{1}{4}} + 1 - \cancel{\tfrac{5}{2}} + F(1) - F(-1)\right) - \left(\cancel{\tfrac{1}{4}} - 1 - \cancel{\tfrac{5}{2}} - F(1) - F(-1)\right)$$

$$F(1) - F(-1) = 2 + 2F(1) - 2F(-1)$$

$$= 2 + 2(F(1) - F(-1))$$

$$\int_{-1}^{1} f(x)\,dx = -2$$

$$f(x) = x^3 + 3x^2 - 5x - 2$$

$$f(1) = 1 + 3 - 5 - 2$$

$$= -3$$