Problem Set 0: Introduction to Numerical Problems

The purpose of this problem set is to begin solving numerical problems on a computer. You can use Mathematica, or any other suitable mathematical or programing environment to solve them. Part of this exercise is to find a copy of Mathematica (or another environment), start learning how to write an algorithm, plot your results, and submit your answers using the instructions on our course website.

1: Prime numbers

a) How many of the numbers between 1 and 100 are prime?

b) Many prime numbers are of the form 2^n-1. How many of the first such 10 numbers (2^n-1 for $n = [1..10]$) are prime?

2: Gaussian distribution

A normal (Gaussian) distribution is of the form $F(x) = c\, e^{-(x-\bar{x})^2/(2\Delta x^2)}$.

a) Use the values: $c = 1$, $\bar{x} = 0$, and $\Delta x = 1$, to plot the distribution.

b) Plot this function with a few other values $\bar{x}$ and Δx. How do these parameters change the distribution?

c) Find the value of c in terms of Δx so that the distribution is normalized. Take your normalization to be that the $\int_{-\infty}^{\infty} F(x)dx = 1$.

d) Find, by integrating your normalized distribution, what fraction of the distribution is within $\pm 2.5\Delta x$ of the mean.

3: Dice

Consider a game involving 1000 rolls of three dice.

a) Simulate a game by choosing 1000 triplets of random integers, each between 1 and 6. Plot the distribution of the sum of the dice, i.e. how many of the 1000 rolls produced a sum of 3, 4, 5, ... , 3*6.

b) Simulate many such games to find, on average, how many times you expect to roll a triple-6 in a game.

c) Show that the average number of triple-6 rolls found in (b) matches what you expect theoretically. To do this, determine the probability of rolling a triple-6, and multiply by the number of rolls in a game.