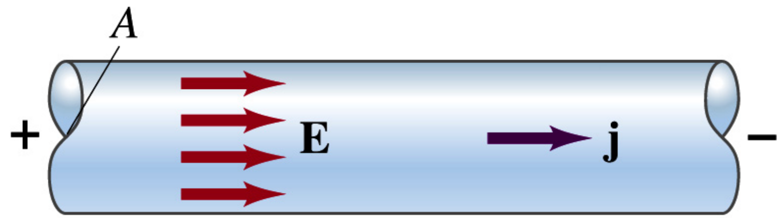
# Chapter 23, 24 Current Behaviors in Metallic Conductors

## § 1 Current Density and Drift Velocity (P556 § 23-7)

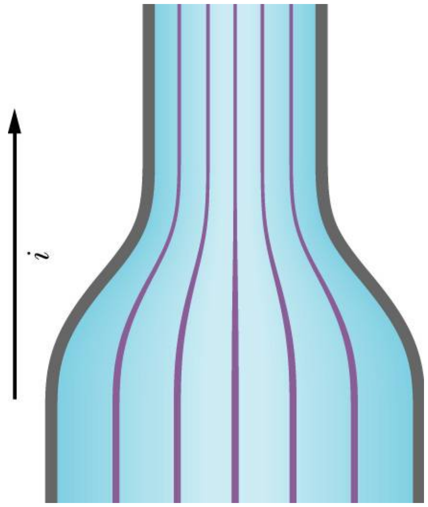
- **Current density**

  - **Electric current per unit cross-sectional area at any point in space**

(uniform) $$j = \frac{I}{A}$$



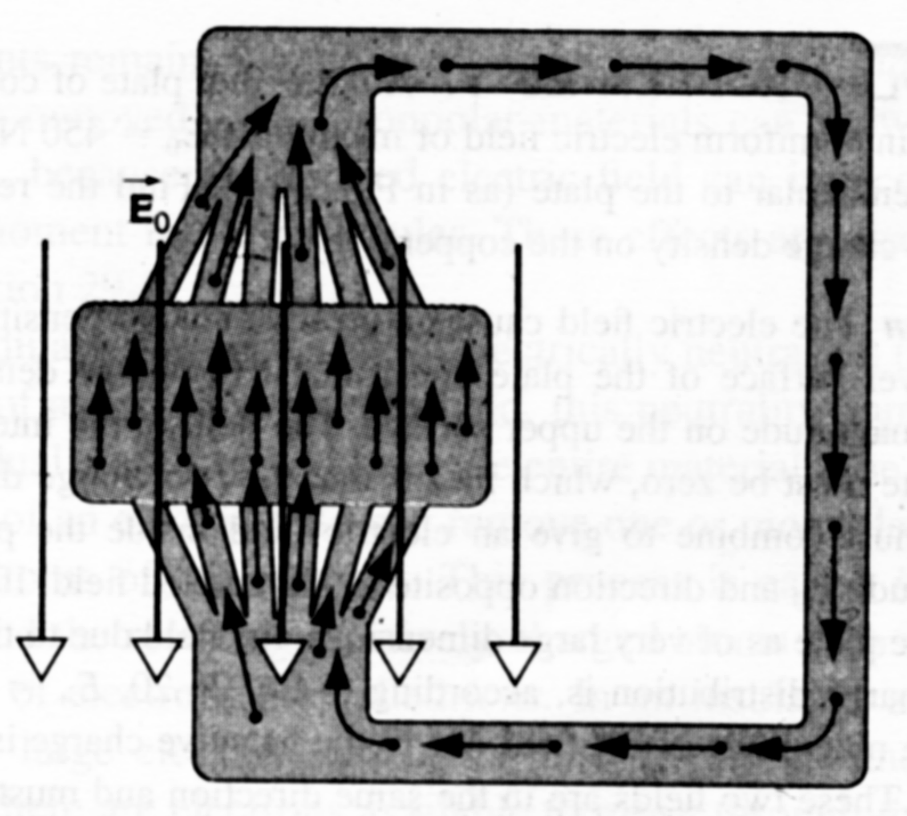

  - **The current density is a vector. The direction of $\vec{j}$ is defined to be the direction of the flow of positive charge.**



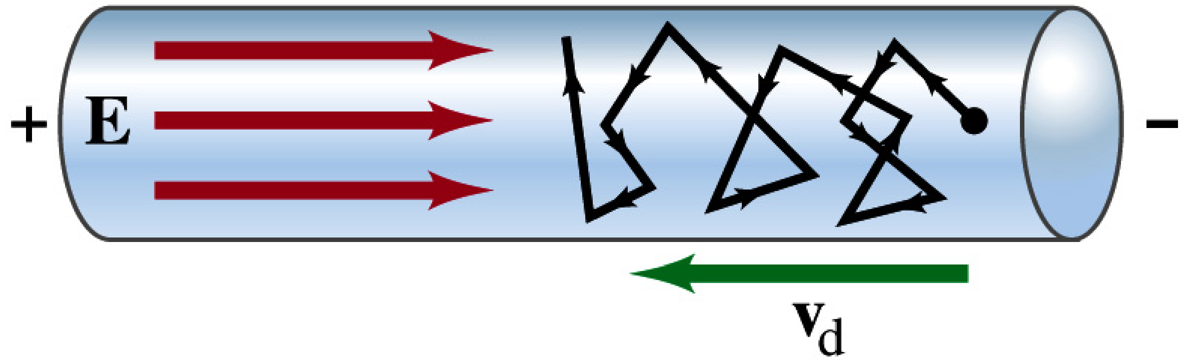
# Current Density

- **The relationship between $\vec{j}$ and $I$ :**

$$I = \iint_S \vec{j} \cdot d\vec{A}$$



- **Drift velocity**
  - **The electrons collide with the ions of the lattice. On the average, electrons can be described as moving with a constant *drift speed* $v_d$ in a direction opposite to the electric field.**



# The relationship between $j$ and $v_d$

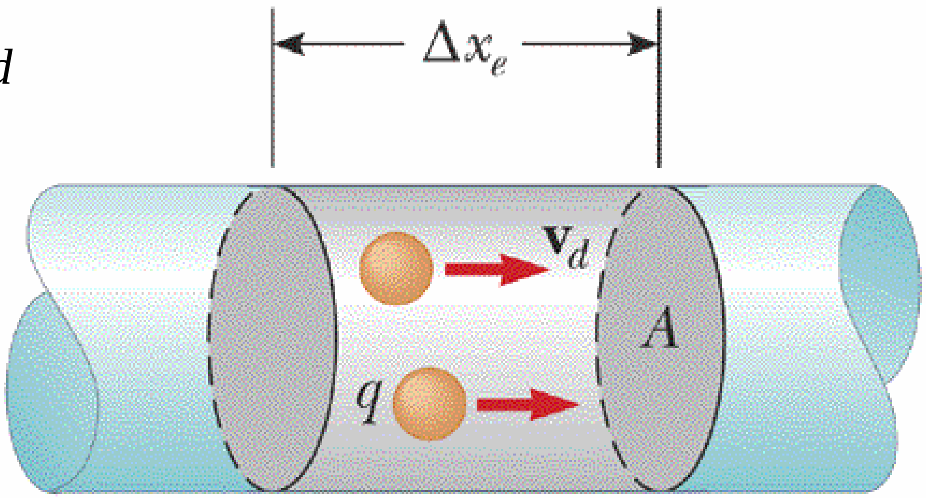
- In the interval $\Delta t$, the magnitude of net charge passing though the surface $A$ is

$$\Delta Q = n(Av_d \Delta t)q$$

***n***: the number of carrier per unit volume.

$$j = \frac{\Delta Q}{A\Delta t} = nqv_d = -nev_d$$

$$\boxed{\vec{j} = -en\vec{v}_d}$$



## Drift speed in a copper wire.

**Example: A copper wire of cross-sectional area $3.00\times10^{-6}$ m$^2$ carries a current of 10.0 A. Find the drift speed of the conduction electrons in this copper wire. The density of copper is 8.95 g/cm$^3$. the molar mass of copper is 63.5 g/mol.**

## Drift speed in a copper wire.

**Solution:**

$$v_d = \frac{j}{nq} = \frac{I}{nqA}$$

**Evaluate the number of electrons per unit volume: *n***

**In copper, there is nearly one conduction electron per atom on average. The number of atoms per unit volume:**

$$\frac{\text{atoms/m}^3}{\text{atoms/mol}} = \frac{\text{mass/m}^3}{\text{mass/mol}} \Longleftrightarrow \frac{n}{N_A} = \frac{\rho_m}{M}$$

$$n = \frac{N_A \rho_m}{M} = \frac{(6.02\times10^{23}\,\text{electrons/mol})(8.95\times10^{3}\,\text{kg/m}^3)}{63.5\times10^{-3}\,\text{kg/mol}}$$

$$= 8.48\times10^{28}\,\text{electrons/m}^3$$

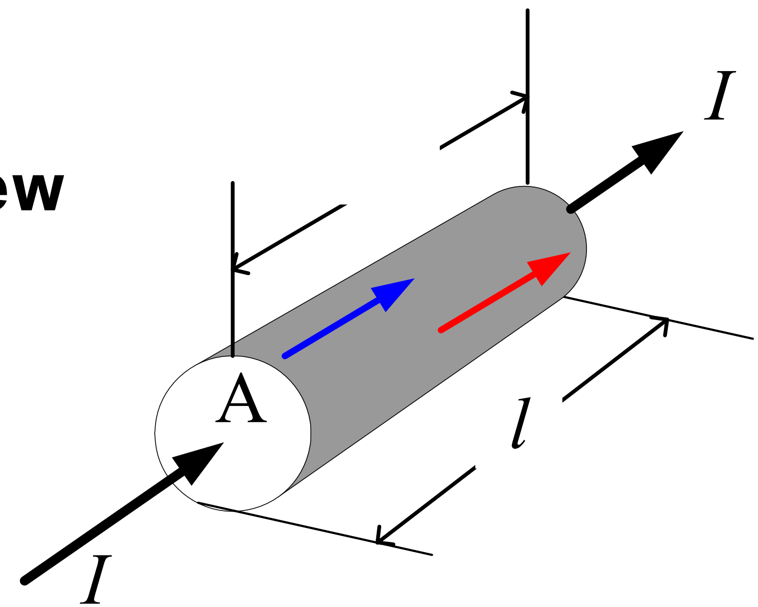
$$v_d = \frac{10.0\ \text{C/s}}{(8.48\times10^{28}\text{m}^{-3})(1.60\times10^{-19}\text{C})(3.00\times10^{-6}\text{m}^2)} = 2.46\times10^{-4}\,\text{m/s}$$

**Dependence $\vec{j}$ of $\vec{E}$ on**

- **In general, quite complex. Nonohmic material.**
- $j \propto E$, **for ohmic material.**

## Ohm's Law in microscopic view

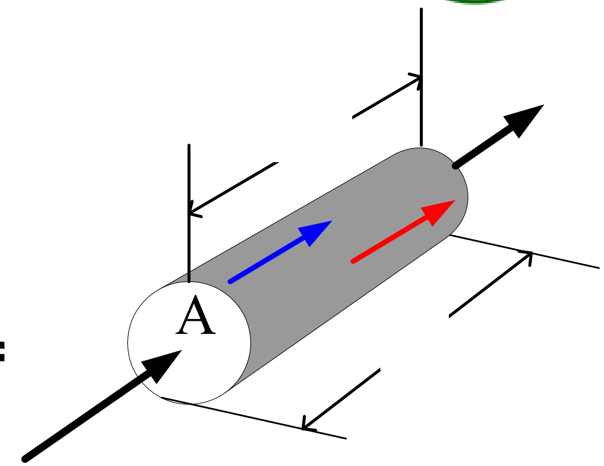
$$\boxed{\vec{j} = \sigma \vec{E} = \frac{\vec{E}}{\rho}}$$




- $\sigma$ **is conductivity.**
- $\rho$ **is resistivity which has the unit ohm-meter ($\Omega \cdot$m).**

# Ohm's Law in macroscopic view

$$\boxed{\vec{j} = \frac{\vec{E}}{\rho}}$$




- **A potential difference $\Delta V$ is applied across a wire conductor of length $l$ and cross-section area $A$.**

$$I = jA = \frac{EA}{\rho} = \frac{El}{\rho \frac{l}{A}} = \frac{\Delta V}{R}$$

$$\boxed{I = \frac{\Delta V}{R}}$$

- **R is the resistance of the wire.**
- **For ohmic materials:**

$$\boxed{R = \rho \frac{l}{A}}$$

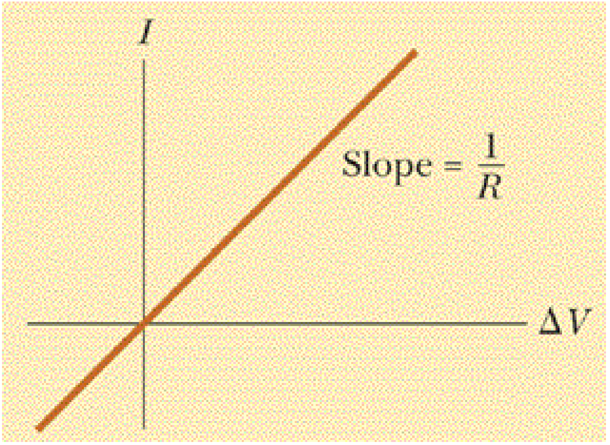
# Ohmic materials and nonohmic materails (P550 §23-3)

- The resistance of an ohmic material is independent of the magnitude or sign of applied potential difference.

$$I \propto \Delta V$$

$$R = \frac{\Delta V}{I}$$

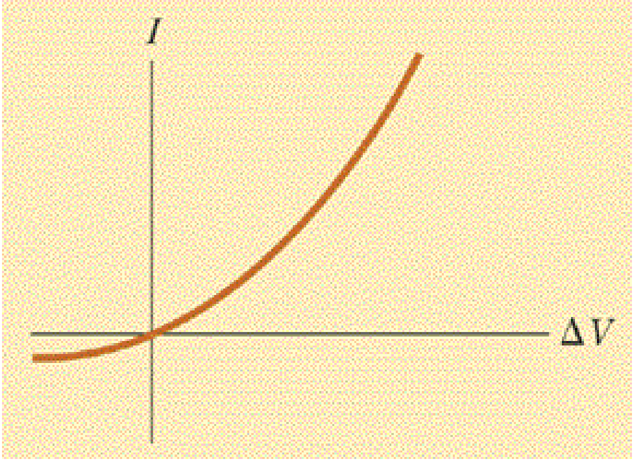
The slope of the *I*-Δ*V* graph




Ohmic material

- The semiconductor is not a nonohmic material.

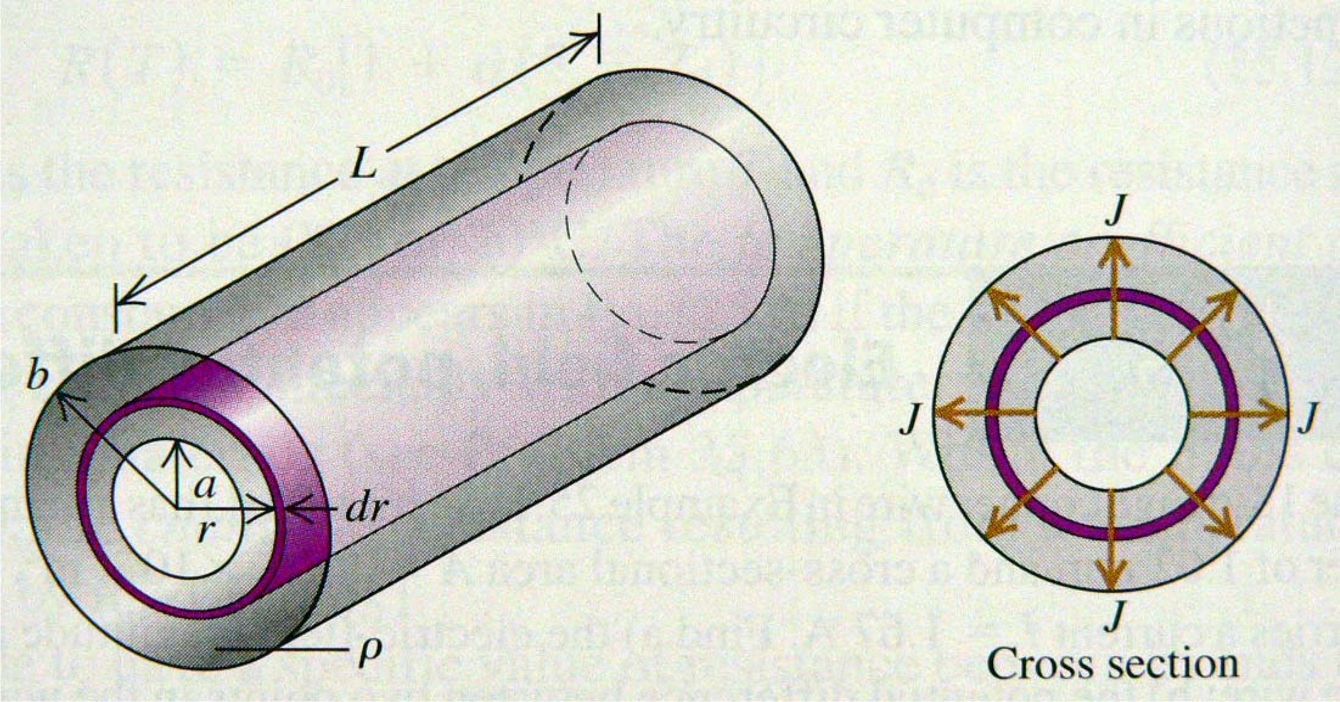
$$\cancel{I \propto \Delta V}$$




A semiconductor diode

# Example

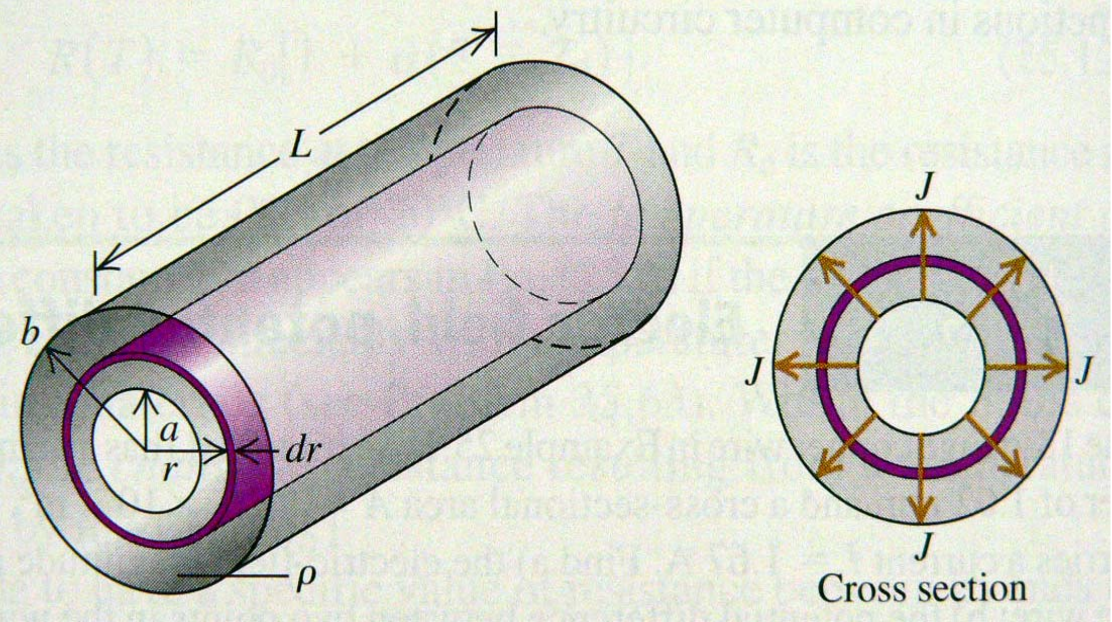
A hollow cylinder has length *L* and inner and outer radii *a* and *b*. It is made of a material with resistivity $\rho$. A potential difference is set up between the inner and outer surface of the cylinder so that current flows radically through the cylinder. What is the resistance to this radial current flow?



# Example

**Solution: We consider a thin cylindrical shell of inner radius *r* and thickness *dr*. The resistance *dR* of this shell is that of a conductor with length *dr* and area 2π*rL*, that is:**

$$dR = \frac{\rho dr}{2\pi rL}$$




**The total resistance:**

$$R = \int dR = \frac{\rho}{2\pi L}\int_a^b \frac{dr}{r}$$

$$= \frac{\rho}{2\pi L}\ln\frac{b}{a}$$

# Problems

Ch23 Prob. 20, 25 (P563)