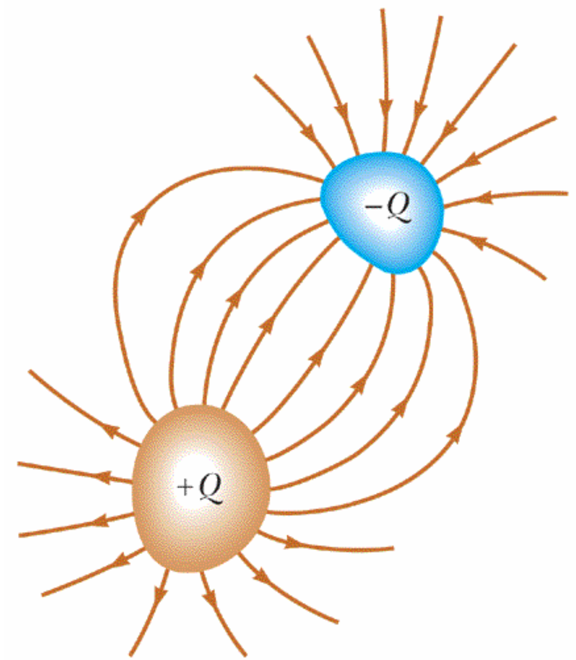
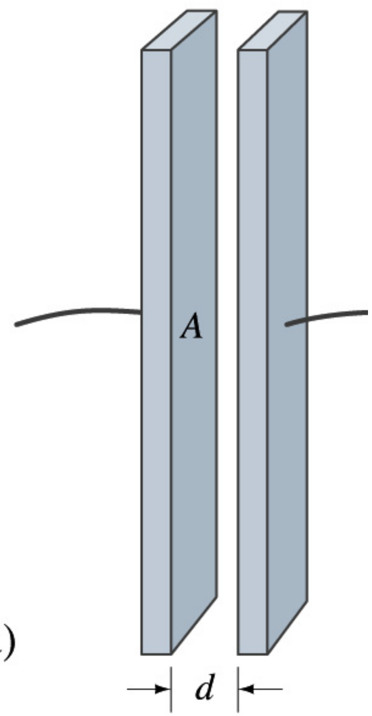
Chapter 22 Capacitance, Dielectrics, Electric Energy Storage

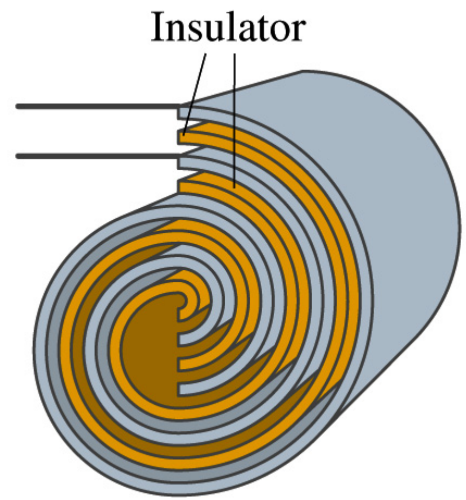
§ 22-1 Capacitance (P525)

- **Capacitors**
  - **Any two conductors separated by an insulator (or a vacuum) form a capacitor, which can store amount of charge.**





(a)



(b)

Capacitance of a capacitor

- **The capacitance *C* of a capacitor**

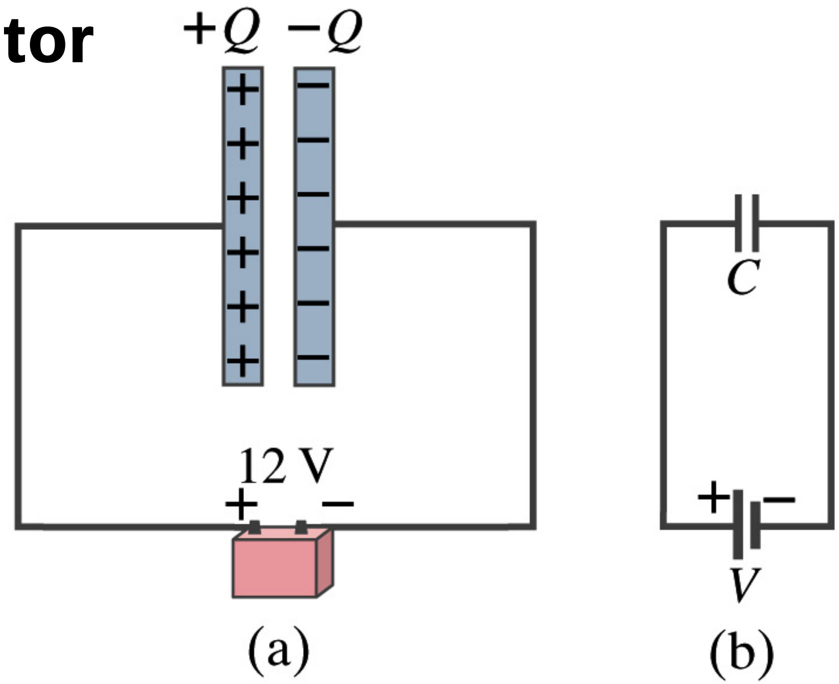
$$Q = C\Delta V$$

$$\boxed{C \equiv \frac{Q}{\Delta V}}$$

Farad

$p\mathrm{F}(10^{-12}\mathrm{F})$

$\mu\mathrm{F}(10^{-6}\mathrm{F})$

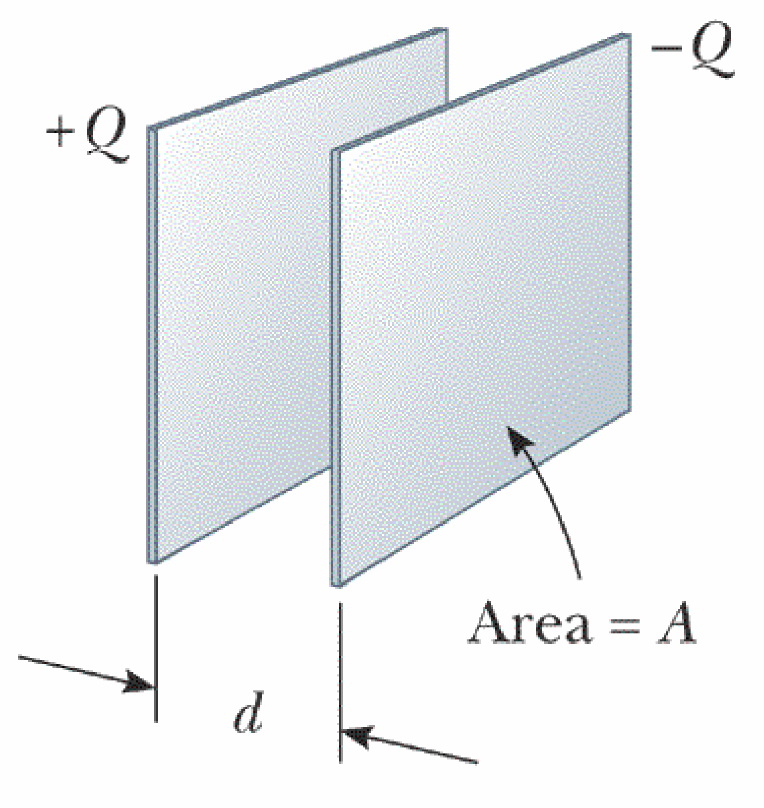


- **The capacitance of a capacitor depends on the geometric arrangement of the conductors, and is independent of the charge *Q* or the potential difference ΔV. Because the potential difference is proportional to the charge, the ratio $Q/\Delta V$ is constant for a given capacitor.**

§ 22-2 Determination of Capacitance (P526)

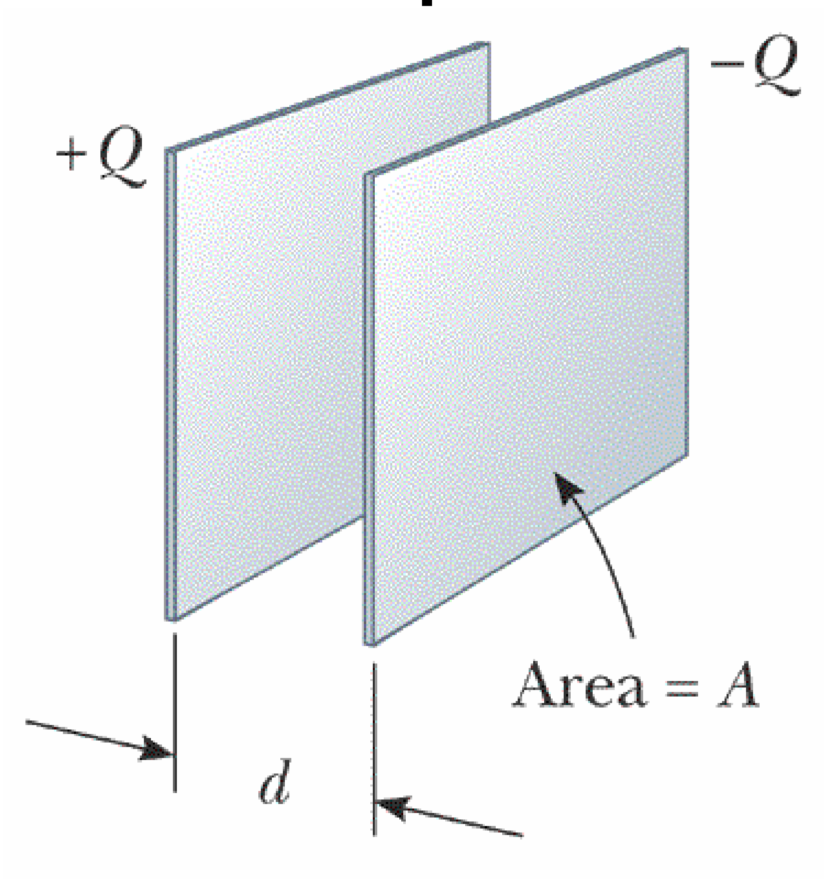
Problem-Solving Strategy :

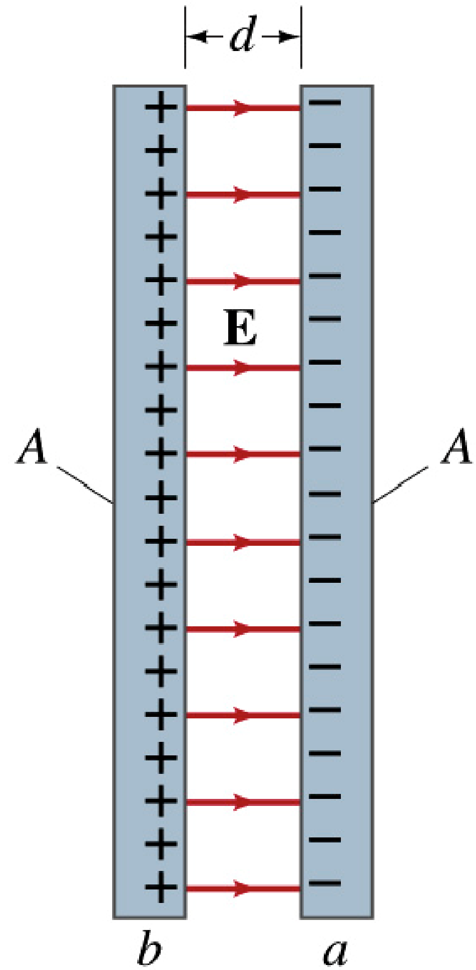
- **A convenient charge of magnitude Q is assumed.**
- **The potential difference ΔV is calculated.**
- **Use $C = Q/\Delta V$ to evaluate the capacitance.**



The parallel-plate capacitor (P527)

A parallel-plate capacitor consists of two parallel plates of equal area *A*, separated by a distance *d*. Find the capacitance.





The parallel-plate capacitor

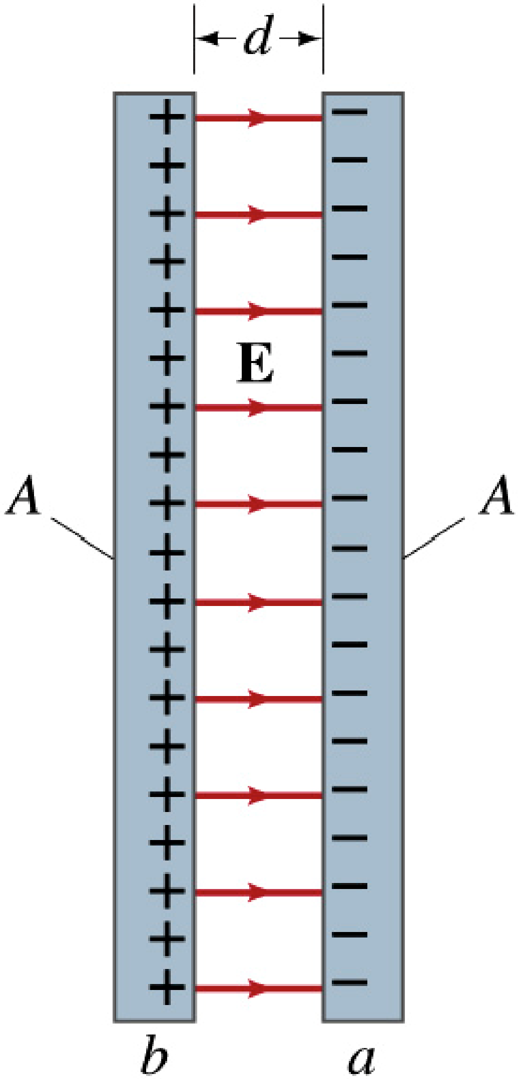
Solution: Assume the two plates have opposite charges +*Q* and −*Q*. An uniform electric field is:

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

The potential difference:

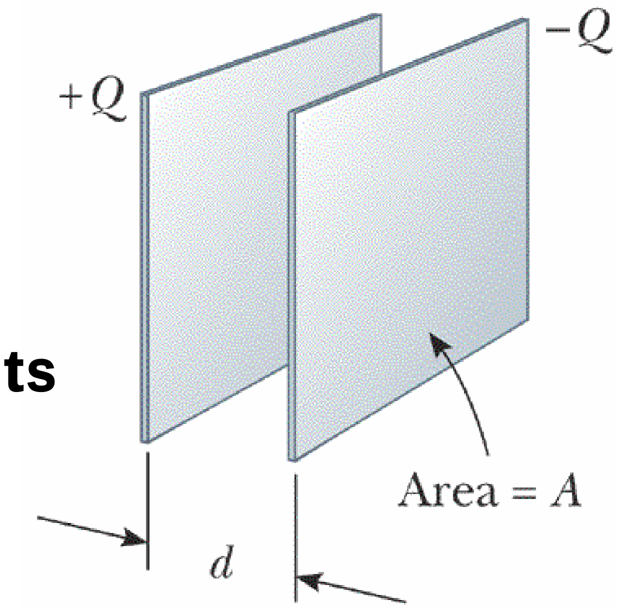
$$\Delta V = \int_{+}^{-} \vec{E} \cdot d\vec{l} = Ed = \frac{Qd}{\varepsilon_0 A}$$

$$\boxed{C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}}$$



The parallel-plate capacitor

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

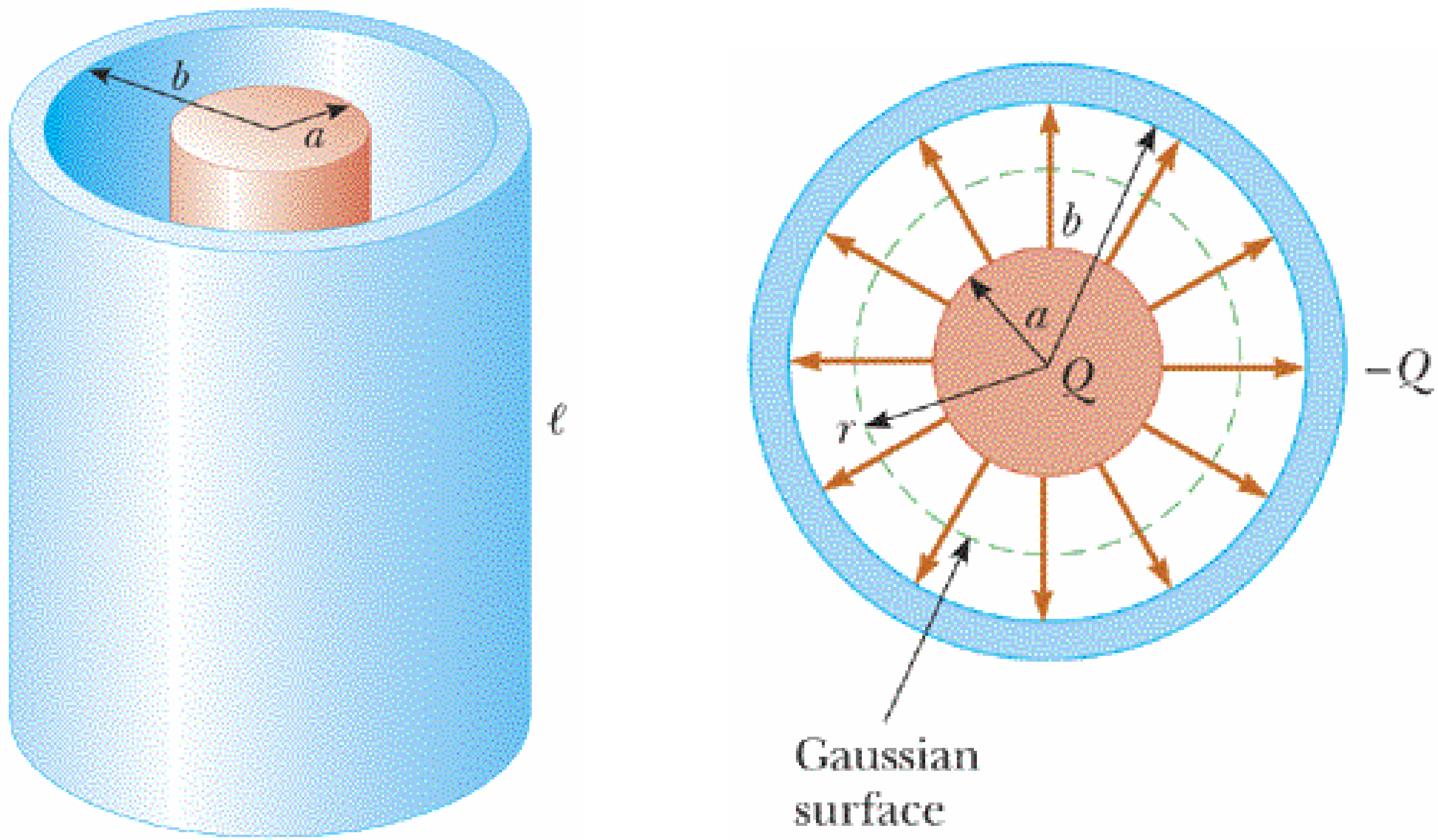


- The capacitance of a parallel-plate capacitor is proportional to the **area** of its plates and inversely proportional to the plate **separation**, which are the geometrical factors.
- The capacitance does **not** depend on the potential difference or the charge carried by the plates.
- The capacitance has form of ε_0 times a quantity with the dimension of length (A/d), which is essential form for all the capacitors.

$$\varepsilon_0 = 8.85\times10^{-12}\ \mathrm{F/m} = 8.85\mathrm{pF/m}$$

The Cylindrical Capacitor (P528 Ex.22-2)

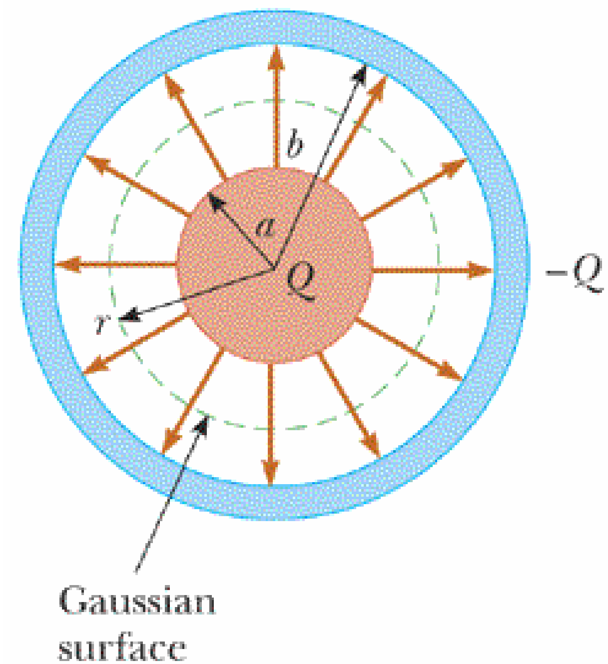
A cylindrical capacitor consists of a cylindrical conductor of radius *a* coaxial with a larger cylindrical shell of radius *b*. Find the capacitance of this device if its length is *l*.



The Cylindrical Capacitor

Solution: Assume the inner and outer conductors have opposite charges $+Q$ and $-Q$. In the region $a<r<b$, we can use Gauss' law to determine:

$$\oint_S \vec{E}\cdot d\vec{A} = E2\pi rl = \frac{\lambda l}{\varepsilon_0} \qquad E = \frac{\lambda}{2\pi\varepsilon_0 r}$$



The potential difference:

$$\Delta V = \int_+^- \vec{E}\cdot d\vec{s} = \int_a^b \frac{\lambda}{2\pi\varepsilon_0}\frac{dr}{r} = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

$$Q = \lambda l$$

$$\boxed{C = \frac{Q}{\Delta V} = \frac{2\pi\varepsilon_0 l}{\ln(b/a)}}$$

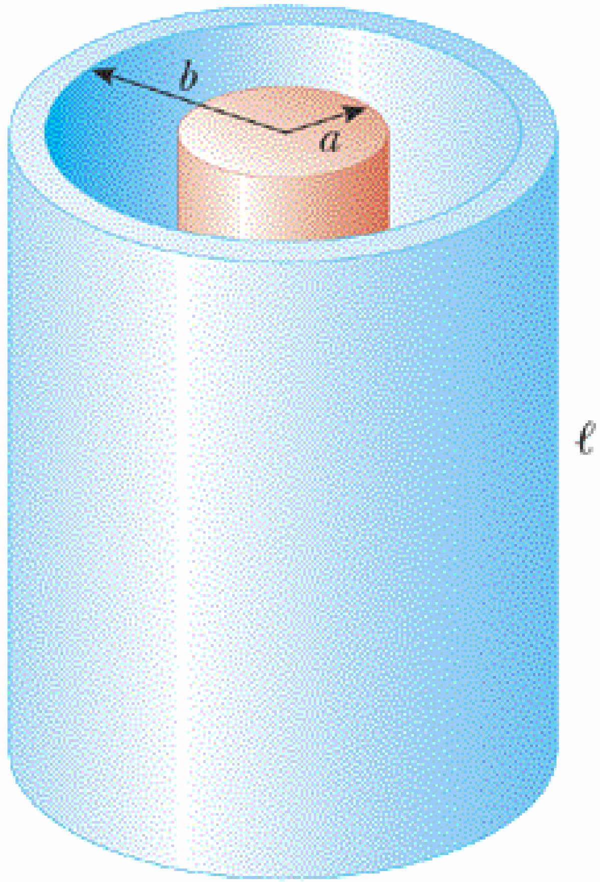
The Cylindrical Capacitor

$$\boxed{C = \frac{2\pi\varepsilon_0 l}{\ln\left(b/a\right)}}$$

- **Has the form of ε_0 times a quantity with dimension of length.**
- **When $d=b-a<<a$**

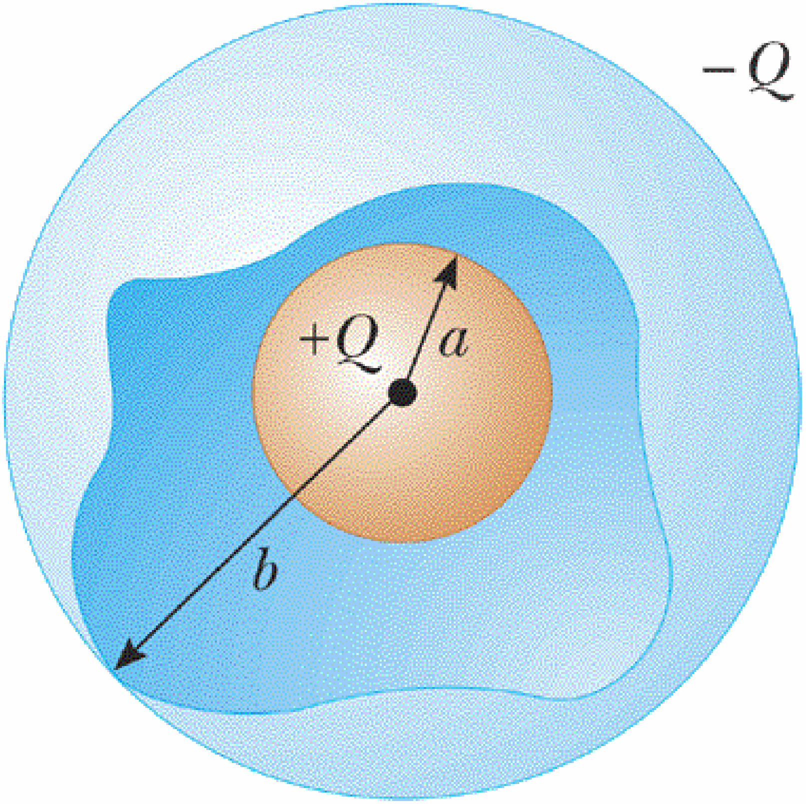
$$\ln\left(\frac{b}{a}\right) = \ln\left(\frac{a+d}{a}\right) = \ln\left(1+\frac{d}{a}\right) \approx \frac{d}{a}$$

$$\boxed{C = \frac{\varepsilon_0 A}{d}} \qquad A = 2\pi a l$$

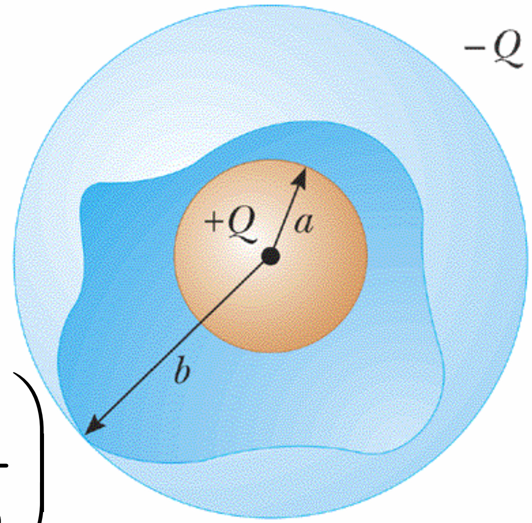


The Spherical Capacitor (P529 Ex. 22-3)

A spherical capacitor in which the inner conductor is a solid sphere of radius a, and outer conductor is a hollow spherical shell of inner radius b. Find the capacitance.



Solution: Assume the inner and outer sphere have opposite charges +*Q* and −*Q*. In the region *a*< *r*< *b*, we can use Gauss' law to determine:



$$E=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

The potential difference:

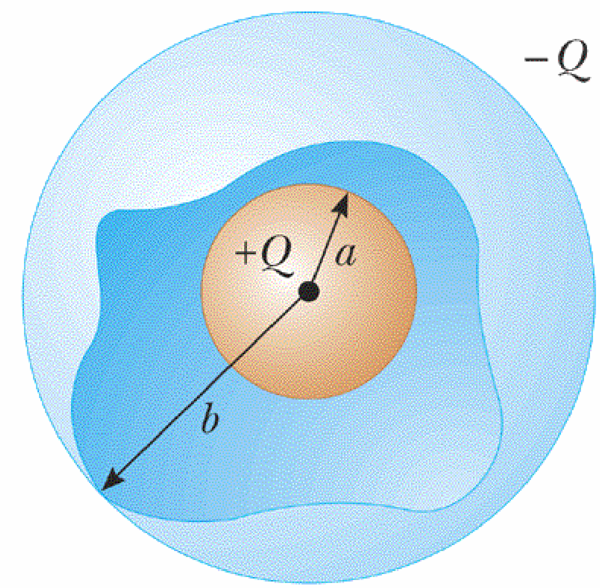
$$\Delta V=\int_{+}^{-}\vec{E}\cdot d\vec{s}=\frac{Q}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{dr}{r^2}=\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$=\frac{Q}{4\pi\varepsilon_0}\frac{b-a}{ab}$$

$$\boxed{C=\frac{Q}{\Delta V}=4\pi\varepsilon_0\frac{ab}{b-a}}$$

The Spherical Capacitor

$$\boxed{C = 4\pi\varepsilon_0 \frac{ab}{b-a}}$$



- **Has the form of ε_0 times a quantity with dimension of length.**
- **When $b \to \infty$, $C = 4\pi\varepsilon_0 a$**
- **When $b-a \ll a$, $ab \approx a^2$, $d=b-a$, $A=4\pi a^2$, $C= \varepsilon_0 A/d$**

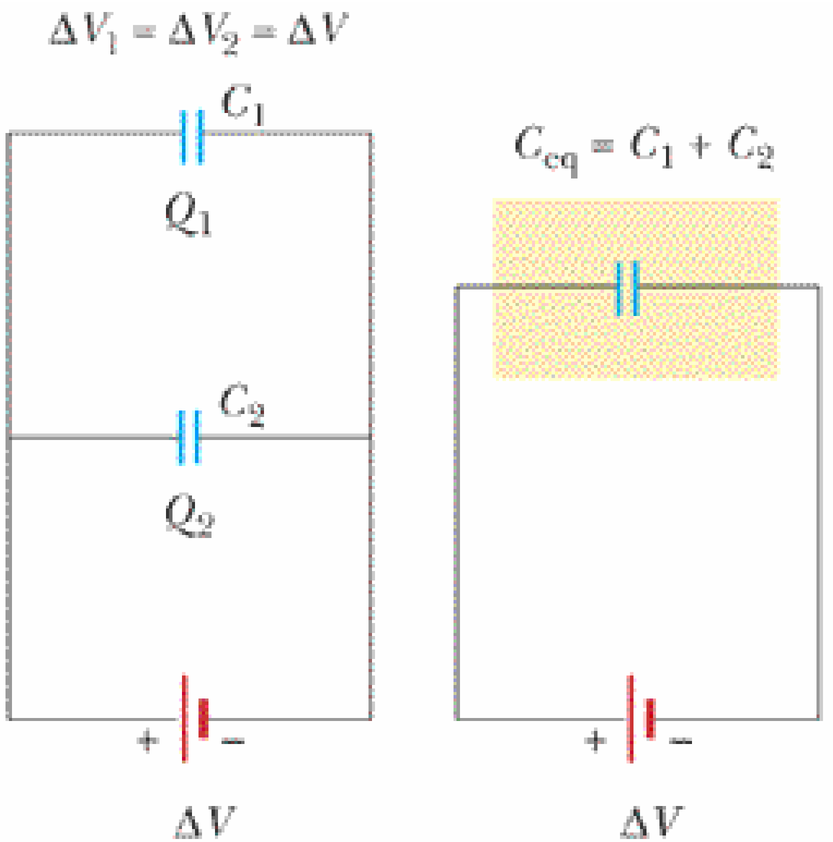
§ 22-3 Combinations of Capacitor (P529)

- **Parallel Combination**

$$C_1 = \frac{Q_1}{\Delta V_1} \qquad C_2 = \frac{Q_2}{\Delta V_2}$$

$$Q = Q_1 + Q_2 \qquad \Delta V = \Delta V_1 = \Delta V_2$$

$$\boxed{C_{eq} = \frac{Q}{\Delta V} = C_1 + C_2}$$



- **The equivalent capacitance of a parallel combination of capacitors is the algebraic sum of the individual capacitances.**

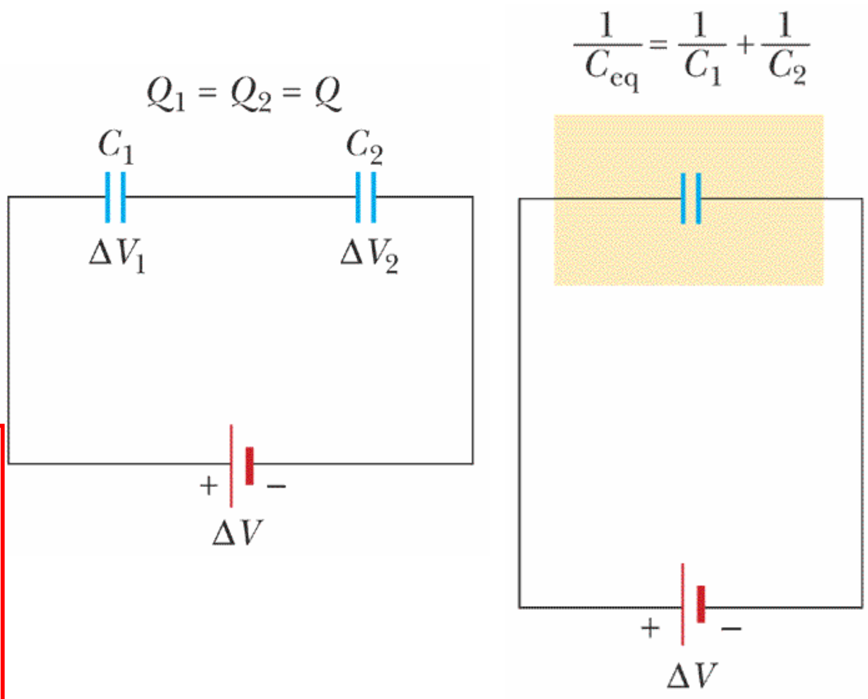
Combinations of Capacitor

Series Combination

$$C_1 = \frac{Q_1}{\Delta V_1} \qquad C_2 = \frac{Q_2}{\Delta V_2}$$

$$Q = Q_1 = Q_2 \qquad \Delta V = \Delta V_1 + \Delta V_2$$

$$\boxed{\frac{1}{C_{eq}} = \frac{\Delta V}{Q} = \frac{1}{C_1} + \frac{1}{C_2}}$$

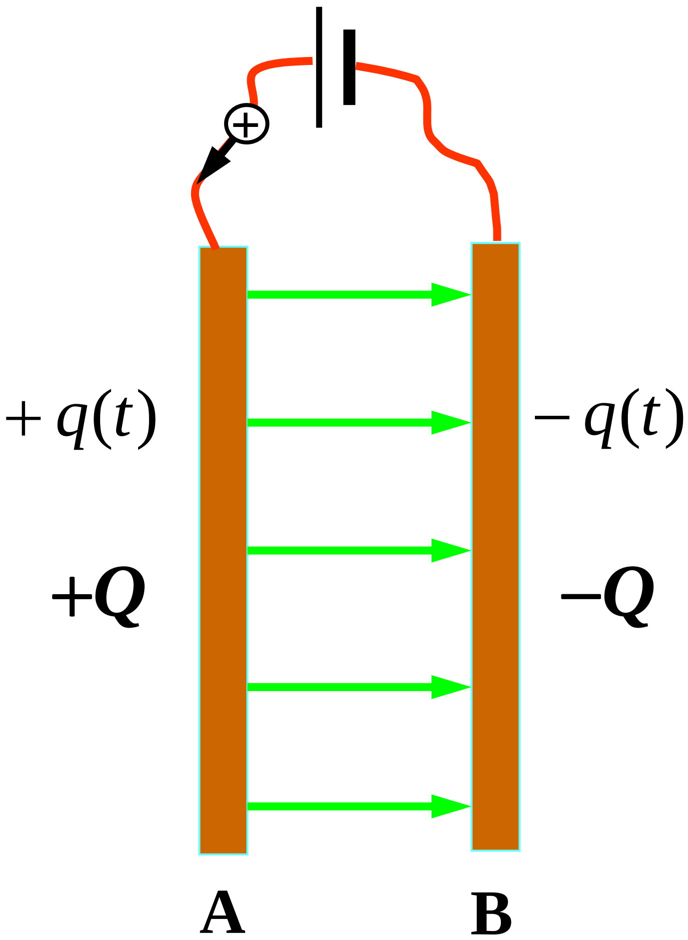


- The inverse of the equivalent capacitance is the algebraic sum of the inverse of the individual capacitances.

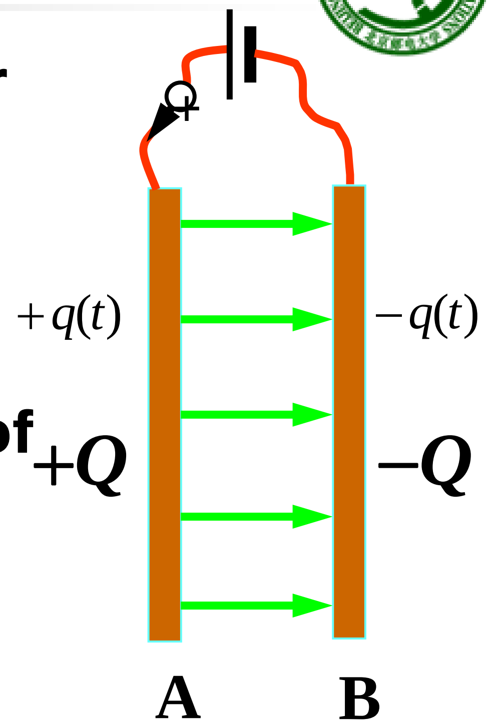
§ 22-4 Electric Energy Storage (P532)

A capacitor can store charge, and can also store energy!

- **The potential energy of a charged capacitor**
  - **The energy stored in a capacitor will be equal to the work done to charge it.**
  - **We evaluate the work of charging that an external agent continuously pulls charge *dq* from negative plate to positive plate until the capacitor has the opposite charge of $\pm Q$.**



The potential energy of a charged capacitor



- Suppose that q is the charge on the capacitor at some instant during this charging process, the potential difference across the capacitor is $\Delta V = q/C$. Imaging that the external agent transfers an additional increment of charge dq from the plate of charge $-q$ to the plate of charge q, the resulting small change dU in the electric potential energy is:

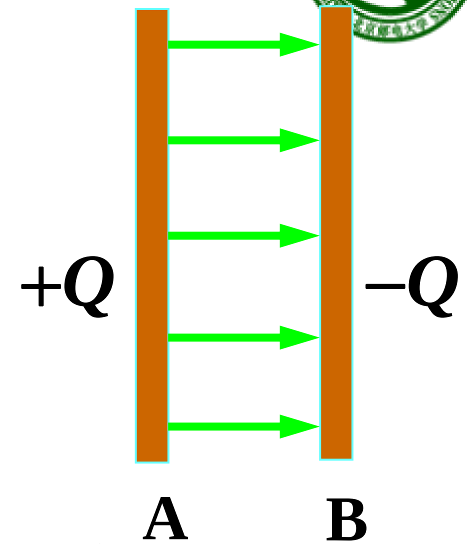
$$dU = \Delta V dq = \frac{q}{C} dq$$

- If this process is continued until to charge the capacitor from $q = 0$ to the final charge $q = Q$, the total potential energy is:

$$U = \int dU = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}$$

$$\boxed{U = \frac{Q^2}{2C} = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2}$$

Where does the potential energy reside?



Question: Which one is the storehouse of the energy, the charge or the electric field itself?

- **From the equation U=$Q^2/2C$, we conclude that the energy relates to the charging.**

- **Another point of view:**

$$C = \varepsilon_0 \frac{A}{d},$$

$$\Delta V = Ed,$$

$$U = \frac{1}{2}C(\Delta V)^2 = \left(\frac{1}{2}\varepsilon_0 E^2\right)(Ad)$$

U **is proportional to the volume** ***Ad*** **between the two plates.**

- **Because the electric field is present in the space between the two plates, the energy is stored in the electric field that is present in this region.**

Where does the potential energy reside?

$$U = \left(\frac{1}{2}\varepsilon_0 E^2\right)(Ad)$$

- **The energy density:** $$u = \frac{U}{Ad} = \frac{1}{2}\varepsilon_0 E^2$$

- **If an electric field $\vec{E}$ exists at any point in empty space, we can think of that point as the site of stored energy in amount of $\frac{1}{2}\varepsilon_0 E^2$.**

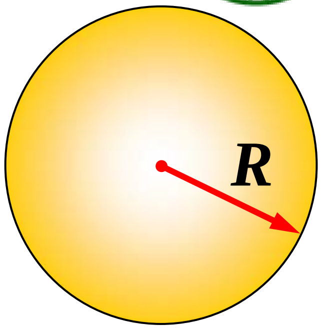
$$U = \int dU = \iiint\limits_V u dV = \iiint\limits_V \left(\frac{1}{2}\varepsilon_0 E^2\right) dV$$

Where does the potential energy reside?

- In the case of electrostatic field, we can not answer which one is the storehouse of the energy.
  - Because in the case of electrostatic field, the electric field is always accompanied with the charge.
- In the case of time-varying electromagnetism field
  - The electromagnetic wave can exists in the vacuum, whether the charge exists or not.

Example

Example: How much energy is stored in the electric field of an isolated conducting sphere of radius *R* and charge *Q*.



Solution (I):

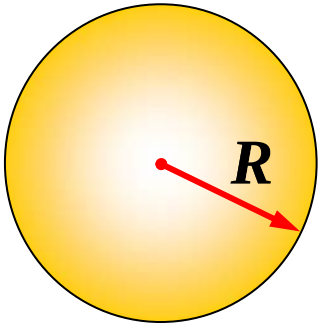
The electric field distribution:
$$E = \begin{cases} 0 & \text{if} \quad r < R \\ \dfrac{Q}{4\pi\varepsilon_0 r^2} & \text{if} \quad r > R \end{cases}$$

$$U = \iiint \left[\frac{1}{2}\varepsilon_0 \left(\frac{Q}{4\pi\varepsilon_0 r^2} \right)^2 \right] dV = \int_R^\infty \left[\frac{1}{2}\varepsilon_0 \left(\frac{Q}{4\pi\varepsilon_0 r^2} \right)^2 \right] \left(4\pi r^2 dr \right)$$

$$= \frac{Q^2}{8\pi\varepsilon_0} \int_R^\infty \frac{dr}{r^2} = \frac{Q^2}{8\pi\varepsilon_0 R}$$

Solution (II):

$$U = \frac{Q^2}{2C},$$



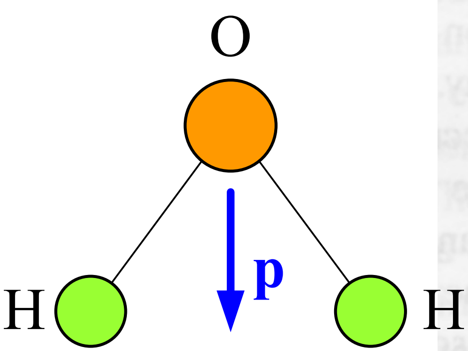
$$C = 4\pi\varepsilon_0 R,$$

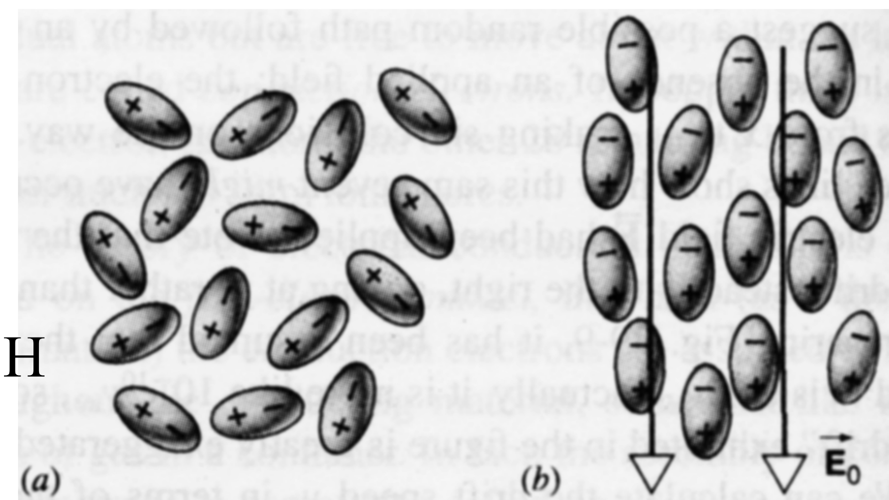
$$U = \frac{Q^2}{8\pi\varepsilon_0 R}$$

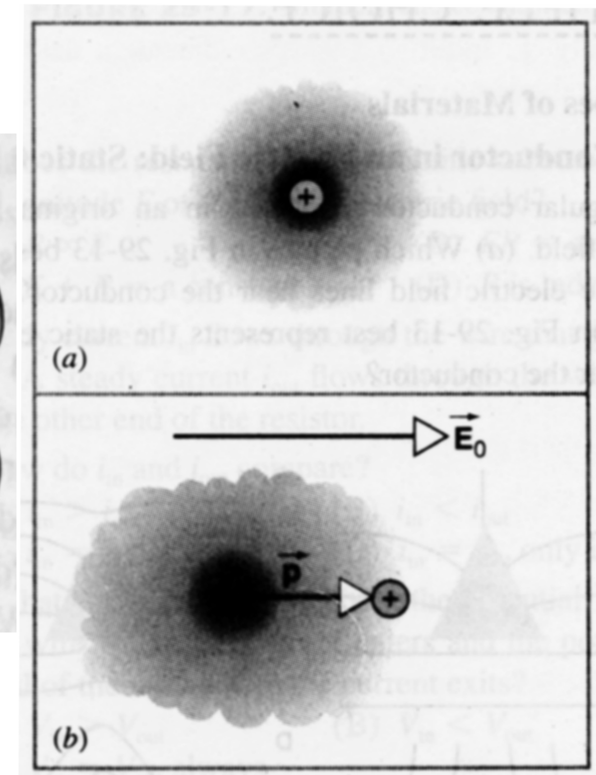
§ 22-5, 22-6 Dielectric Materials (P533, P536)

- **Dielectrics vs. conductors**

- **Polar vs. nonpolar dielectric materials**



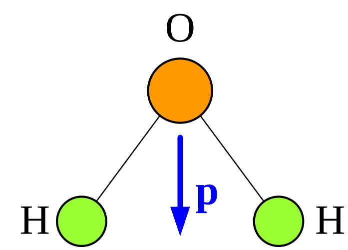




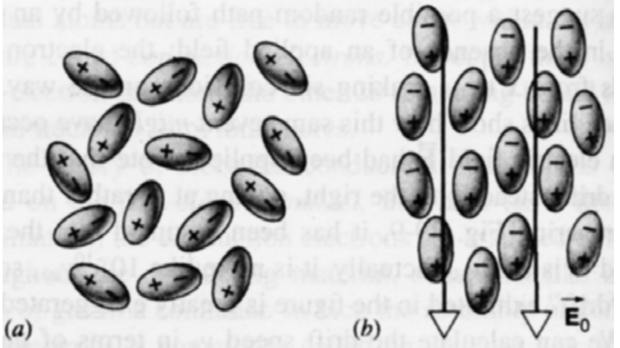
Polar and nonpolar dielectric materials

- **Polar dielectric material —— its molecule has a permanent electric dipole moment, such as water.**

  The external electric field exerts a torque on the dipole that tries to align it with the field.

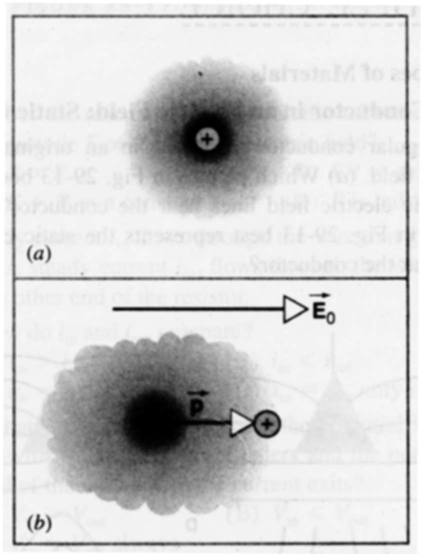


$$\vec{\tau} = \vec{p} \times \vec{E}_0$$



- **Nonpolar dielectric material —— its molecule has no permanent electric dipole.**

  The atom acquires an induced dipole moment when the atom is placed in an external electric field.

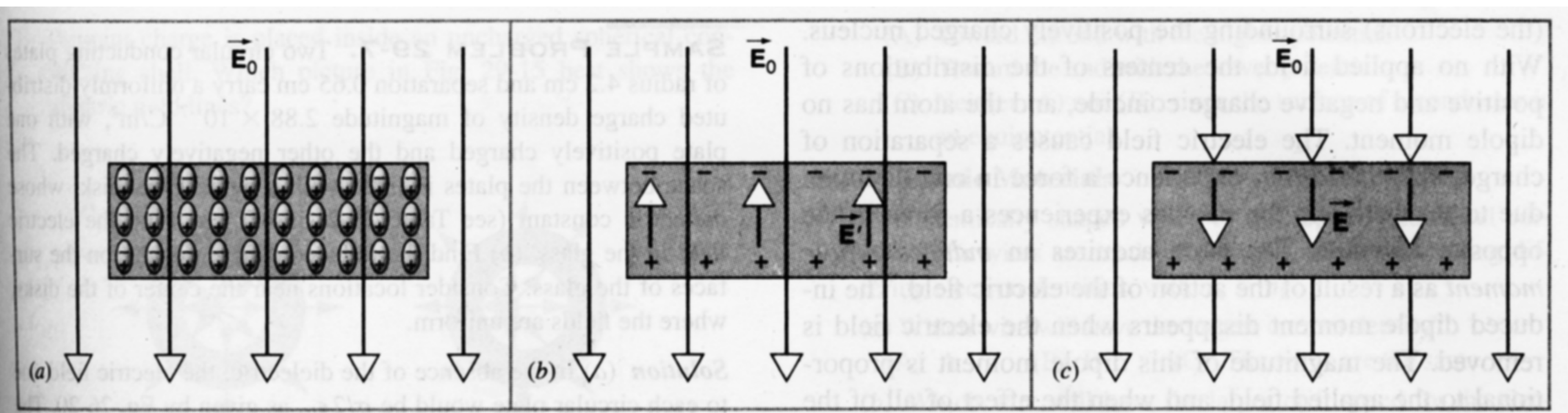


The induced surface charge and induced polarization field

- **When a dielectric material is placed in an external applied field E_0, induced surface charges q' appear that tend to weaken the original field E_0 by a polarization field E' within the material. For a linear material, the net field inside the material**

$$\vec{E} = \vec{E}_0 + \vec{E'}$$

$\vec{E'}$ **is called polarization field.**

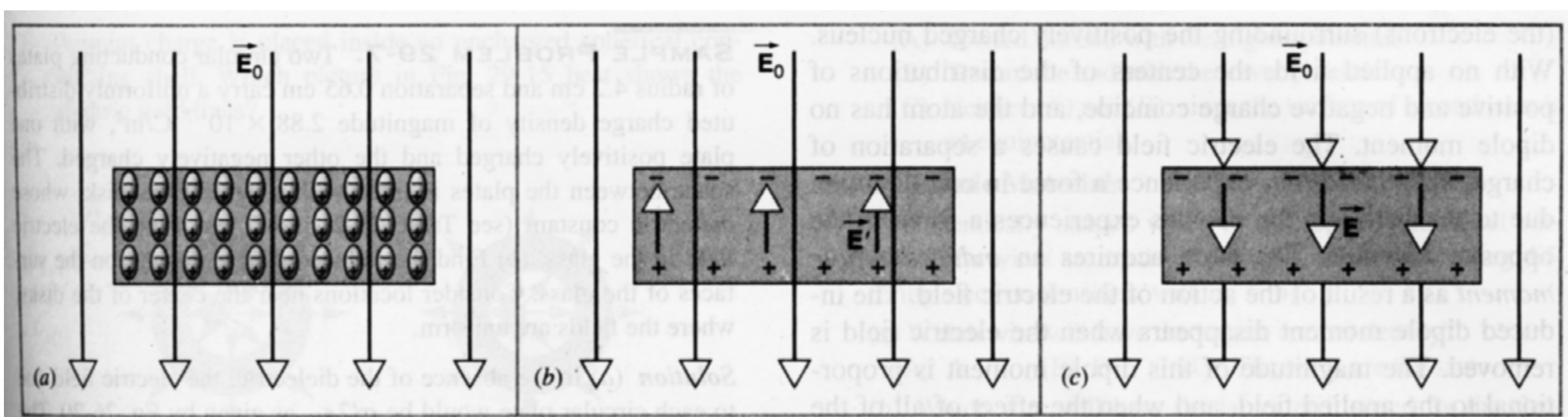


The induced surface charge and induced polarization field

$$\vec{E} = \vec{E}_0 + \vec{E}'$$

$$\boxed{E = \frac{1}{\kappa} E_0}$$

- κ is called the dielectric constant, which is greater than 1.
- The charge q_0, the origin of E_0, that resides in the conductors is called free charge, and induced charge q' that resides in the surface of dielectric materials, that not free to move and bound to a molecule, is called bound charge.



The polarization and the dielectric strength

- When either polar or nonpolar materials are put in an external field, the materials are said to be **polarized**.
- The dielectric strength: E_{break}

  If we apply a large enough electric field to an insulator, we can ionize atoms or molecules of the insulator and thus create a condition for electric charge to flow, as in a conductor. The field necessary for the breakdown of the insulator is called the **dielectric strength**.

TABLE 20.1 Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (V/m)
Vacuum	1.00000	—
Air (dry)	1.00059	3×10^6
Bakelite	4.9	24×10^6
Fused quartz	3.78	8×10^6
Pyrex glass	5.6	14×10^6
Polystyrene	2.56	24×10^6
Teflon	2.1	60×10^6
Neoprene rubber	6.7	12×10^6
Nylon	3.4	14×10^6
Paper	3.7	16×10^6
Strontium titanate	233	8×10^6
Water	80	—
Silicone oil	2.5	15×10^6

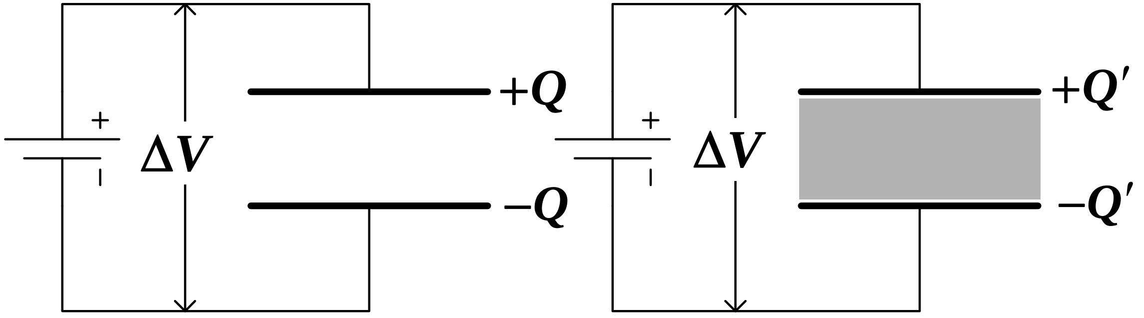
[a] The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown.

§ 7 Capacitors with Dielectrics (P534)

Two identical capacitors, filling one with a dielectric material and leaving the other with air between its plates

- **When both capacitors are connected to batteries with the same potential difference.**

$$\left.\begin{array}{l} \Delta V = \Delta V' \Rightarrow E = E' \\ E = \dfrac{Q}{\varepsilon_0 A}, \quad E' = \dfrac{1}{\kappa}\dfrac{Q'}{\varepsilon_0 A} \end{array}\right\}$$

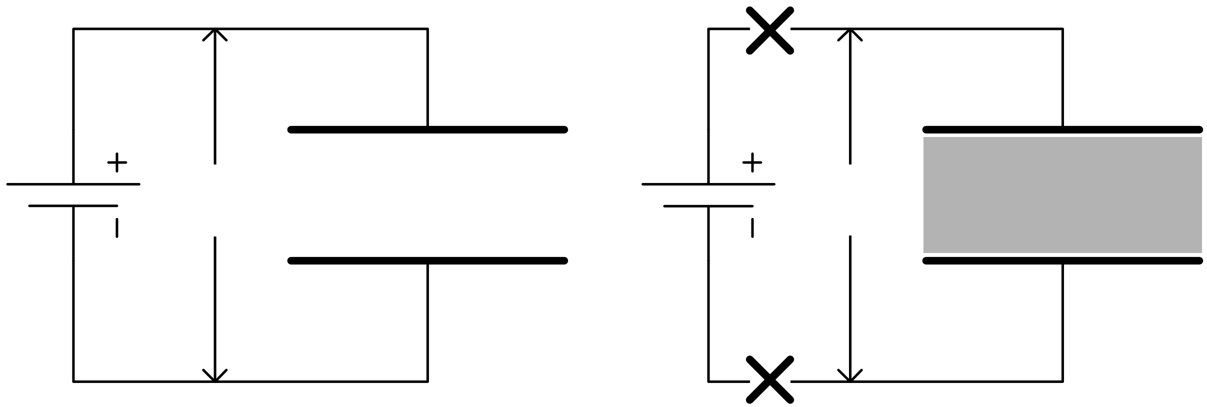


$$Q' = \kappa Q$$

$$C' = \frac{Q'}{\Delta V'} = \frac{\kappa Q}{\Delta V} \Rightarrow C' = \kappa C \Rightarrow C' = \frac{\kappa \varepsilon_0 A}{d} = \frac{\varepsilon A}{d}$$

$\varepsilon = \kappa \varepsilon_0$ **permittivity**

- When both are disconnected the batteries with the **same charge**



$$E = \frac{Q}{\varepsilon_0 A}, \quad E' = \frac{E}{\kappa} = \frac{1}{\kappa}\frac{Q}{\varepsilon_0 A},$$

$$\Delta V' = E'd = \frac{Ed}{\kappa} \Rightarrow \Delta V' = \frac{\Delta V}{\kappa} = \frac{Qd}{\kappa\varepsilon_0 A}$$

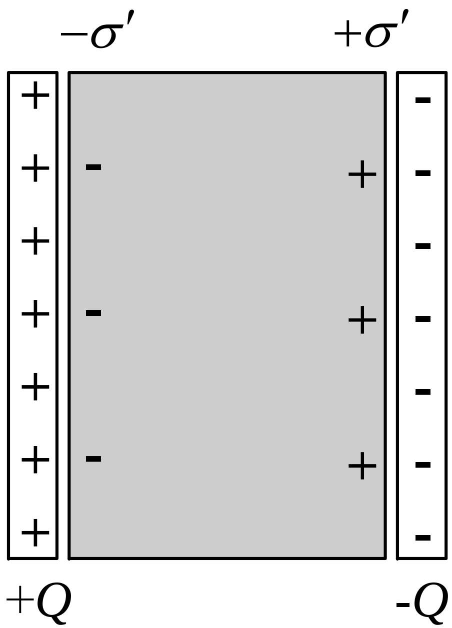
ΔV

$$C' = \frac{Q}{\Delta V'} = \frac{\kappa\varepsilon_0 A}{d} = \kappa C = \frac{\varepsilon A}{d}$$

- **The electric field energy stored in a capacitor with dielectric**

$$U=\frac{Q^2}{2C}=\frac{Q^2 d}{2\kappa\varepsilon_0 A}=\frac{1}{2}\kappa\varepsilon_0\left(\frac{Q}{\kappa\varepsilon_0 A}\right)^2(Ad)$$

$$E=\frac{E_0}{\kappa}=\frac{1}{\kappa}\frac{\sigma}{\varepsilon_0}=\frac{Q}{\kappa\varepsilon_0 A},\qquad \boxed{U=\frac{1}{2}\kappa\varepsilon_0 E^2(Ad)}$$



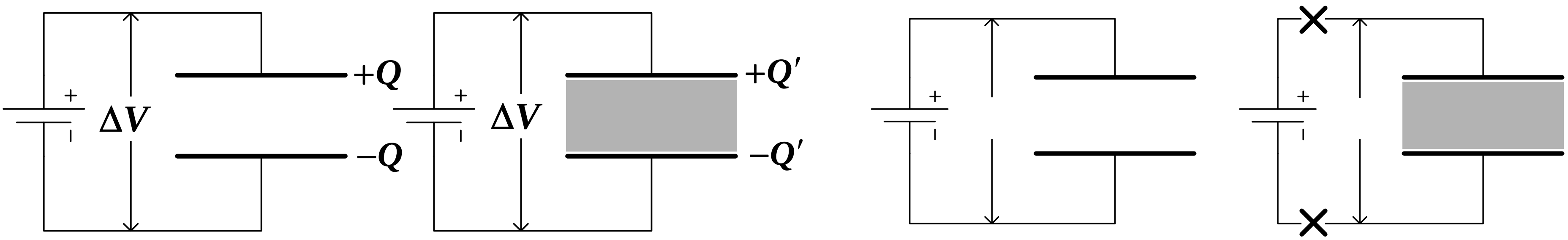
- **The electric field energy density in dielectric materials**

$$\boxed{u=\frac{1}{2}\kappa\varepsilon_0 E^2=\frac{1}{2}\varepsilon E^2}$$

In following two cases, find the electric field energy stored in a parallel-plate capacitor before and after the dielectric is inserted. The capacitor without dielectric is C_0, and dielectric material has dielectric constant κ.

(1) From beginning to end, the capacitor is always connected to the battery of voltage ΔV;

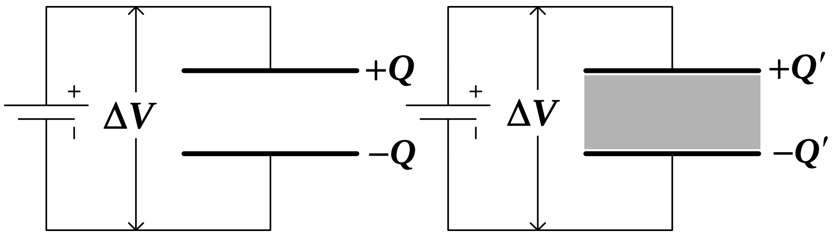
(2) At beginning, the capacitor, with empty, is connected to the battery of voltage ΔV. The battery is then removed, and the capacitor is fill with the dielectric material.



Solution:

(1) Before inserting the dielectric:

$$U_{before} = \frac{1}{2} C_0 \left(\Delta V\right)^2$$



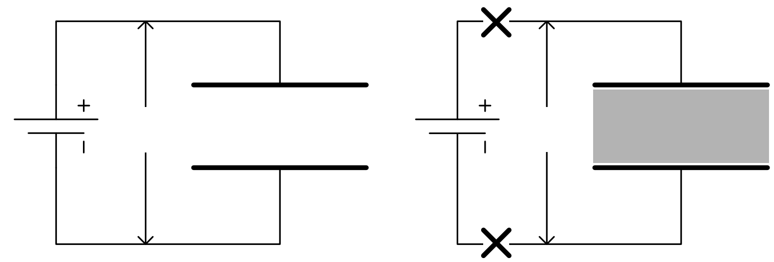
After inserting the dielectric:

$$U_{after} = \frac{1}{2} C \left(\Delta V\right)^2 = \frac{1}{2} \kappa C_0 \left(\Delta V\right)^2 = \kappa U_{before}$$

$$\Delta U = U_{after} - U_{before} = (\kappa - 1) U_{before} > 0$$

Example cont'd

(2) Before and after removing the battery, the charges in the capacitor are the same and the voltage across the plates decreases after removing the battery $\Delta V' = \Delta V/\kappa$.



$$\left.\begin{aligned} &E = \frac{Q}{\varepsilon_0 A}, \quad E' = \frac{Q}{\kappa \varepsilon_0 A} = \frac{E}{\kappa} \\ &\Delta V = Ed, \quad \Delta V' = E'd \end{aligned}\right\} \quad \Delta V' = \frac{\Delta V}{\kappa}$$

Before inserting the dielectric: $U_{before} = \frac{1}{2} C_0 (\Delta V)^2$

After inserting the dielectric:

$$U_{after} = \frac{1}{2} C (\Delta V')^2 = \frac{1}{2} \kappa C_0 \left(\frac{\Delta V}{\kappa} \right)^2 = \frac{U_{before}}{\kappa}$$

$$\Delta U = U_{after} - U_{before} = (1 - \kappa) U_{before} < 0$$

Problems

Ch22 Prob. 49, 50, 85 (P543)

Ch22 Prob. 65, 87 (P544)