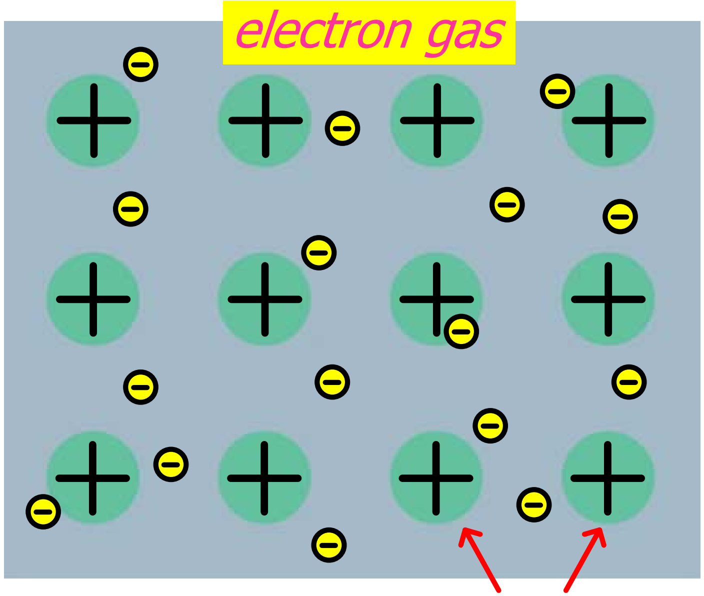
§ 5 Conductors in Electrostatic Equilibrium

(P473 § 19-9)

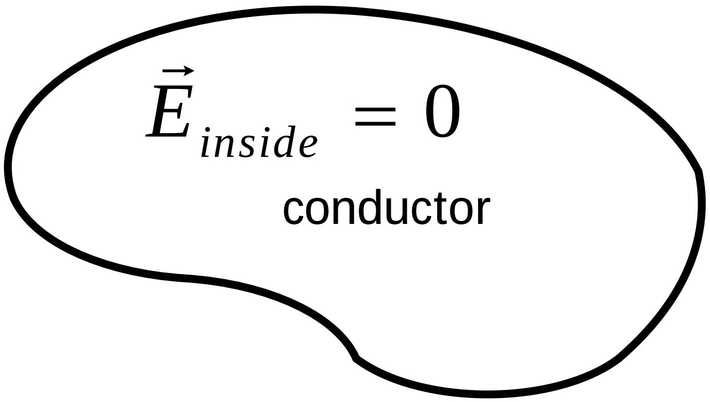
- **The characteristics of a electrical conductor.**
  - **A good electrical conductor contains charges that are not bound to any atom and free to move about within the conductor —— called free charge.**
  - **When no motion of charge occurs within the conductor, the conductor is in electrostatic equilibrium.**



The properties

- The properties that an isolated conductor in electrostatic equilibrium.

① The electric field is **zero** everywhere **inside** the conductor.



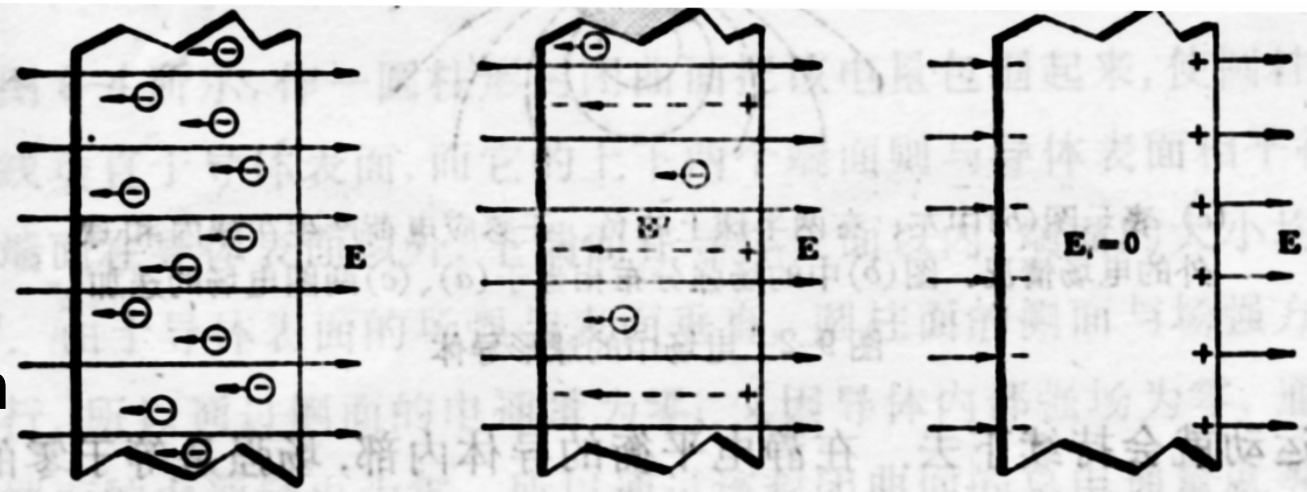
If the field were not zero, **free** charges in the conductor would accelerate under the action of the electric field —— not the case in electrostatic equilibrium

- **The Mechanism.**

  ➡ **If we apply a external field *E*, the free electrons inside the conductor will move under *E* and are accumulated on the surface of the conductor, and establish another field *E′* until the total field inside the conductor reach zero.**

$$\vec{E}_{inside} = \vec{E} + \vec{E}'$$
$$= 0$$

➡ **Now the conductor is in electrostatic equilibrium.**



(a) 导体中自由电子在外电场作用下作宏观的定向运动

(b) 导体的两个相对表面上出现的感应电荷在增加中

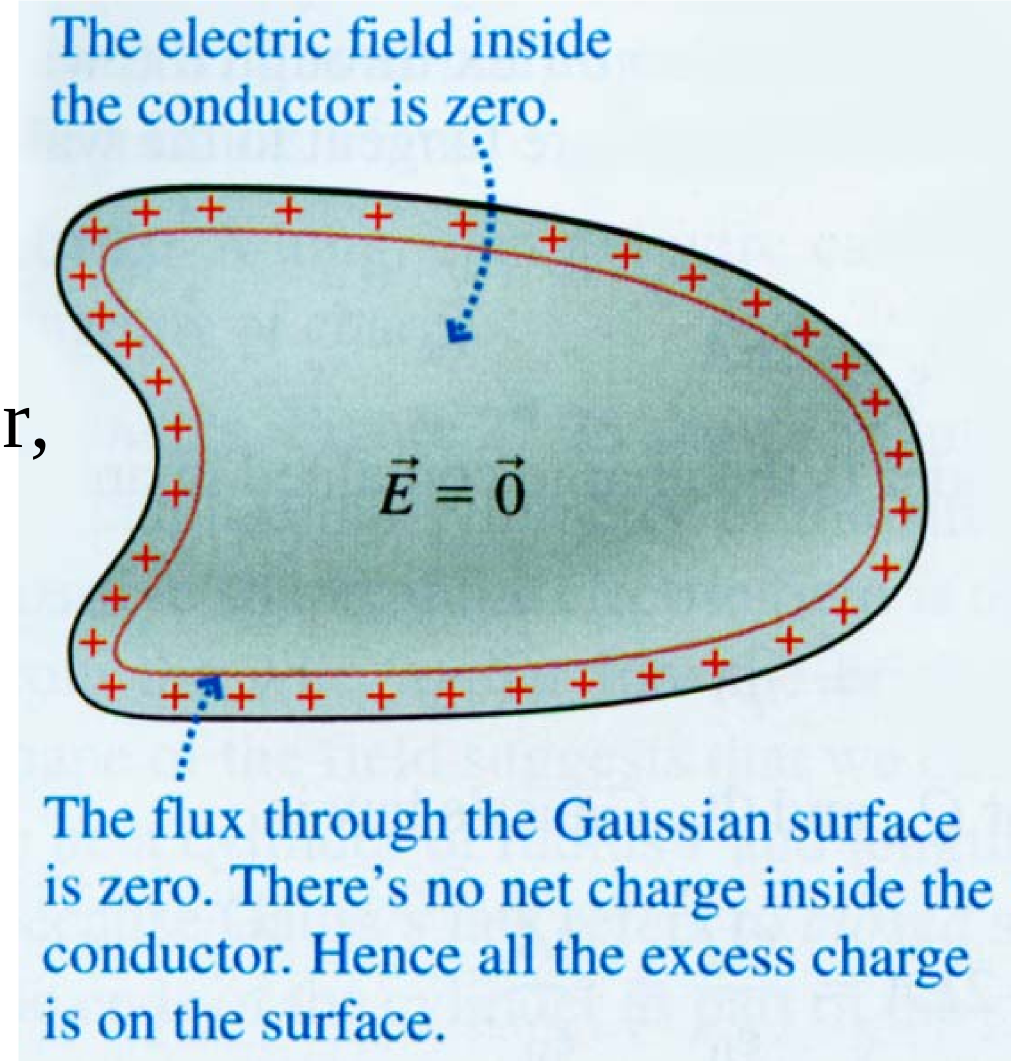
(c) 最后，导体处于静电平衡状态，感应电荷不再变化，导体内的总场强 $E_i=0$

② **If the isolated conductor carries a net charge, the net charge resides entirely on its surface.**

$$\oiint_S \vec{E}\cdot d\vec{A} = \frac{q_{inside}}{\varepsilon_0},$$

$\vec{E} = 0$ inside the conductor,

$\Rightarrow\ q_{inside} = 0$



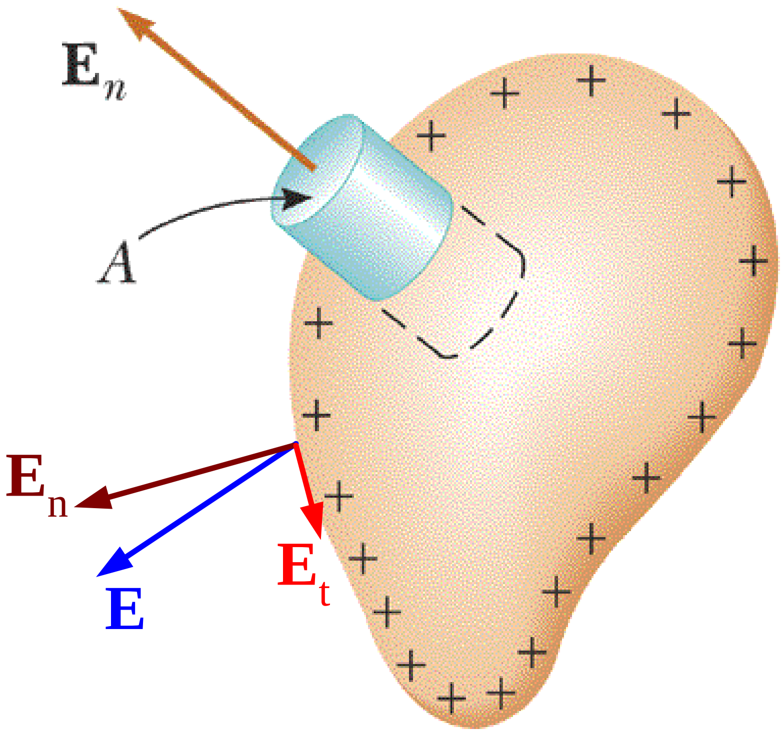
③ **The electric field just outside the charged conductor is perpendicular to the conductor surface and has a magnitude σ / ε_0, where σ is the surface charge density at that point.**

If *E* had a component parallel to the surface, the free charges would move along the surface, and so the conductor would not be in equilibrium.

Draw a small cylinder just containing the surface of the conductor.

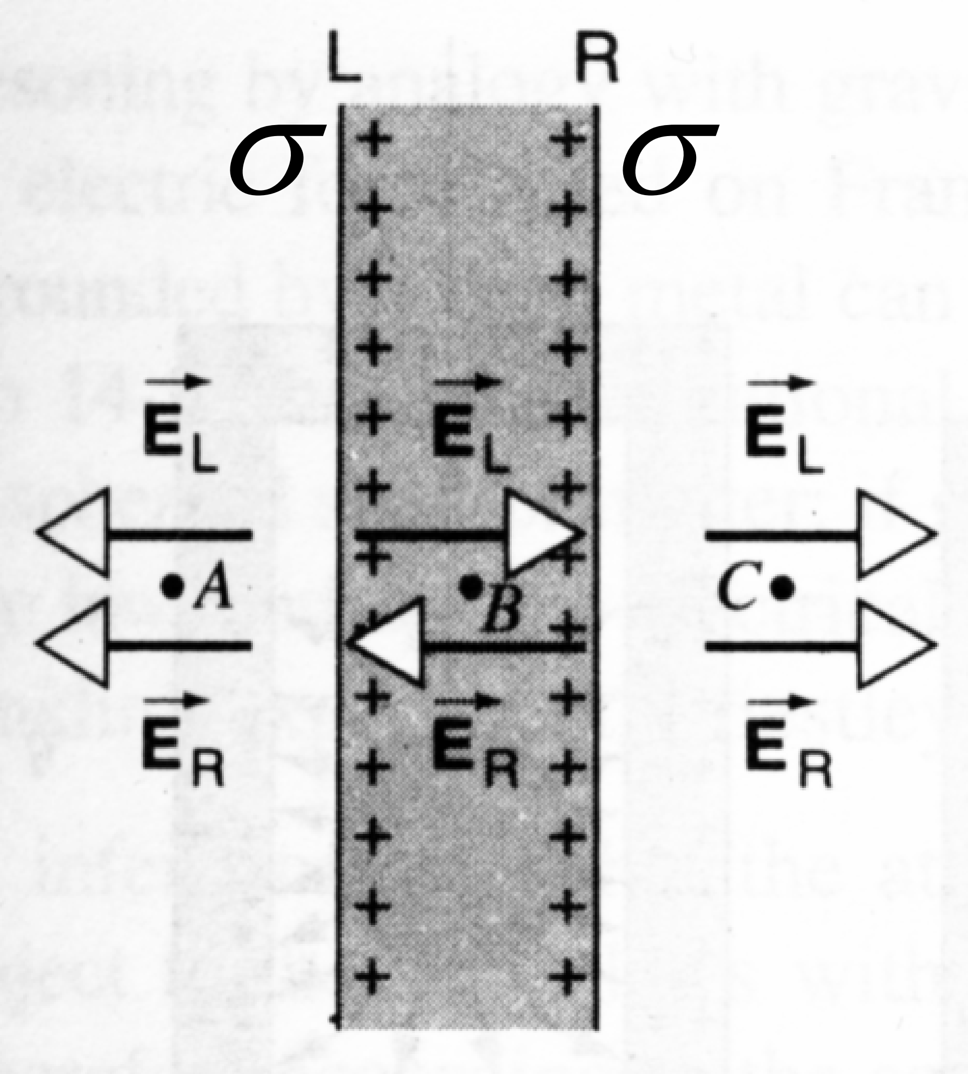
$$\oiint_S \vec{E} \cdot d\vec{A} = EA = \frac{q_{in}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$



Example

Find E_A, E_B, and E_C.

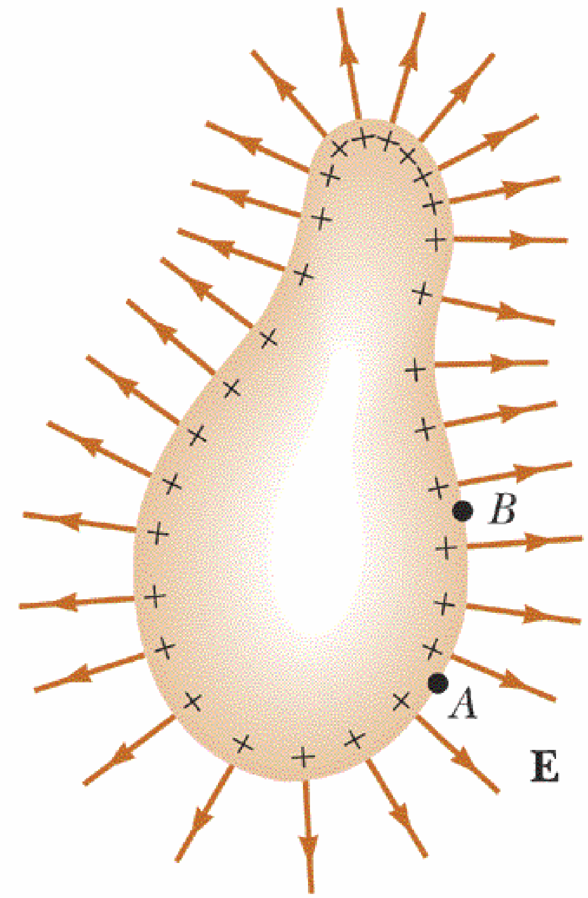


$$E_A = -\frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = -\frac{\sigma}{\varepsilon_0}$$

$$E_B = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

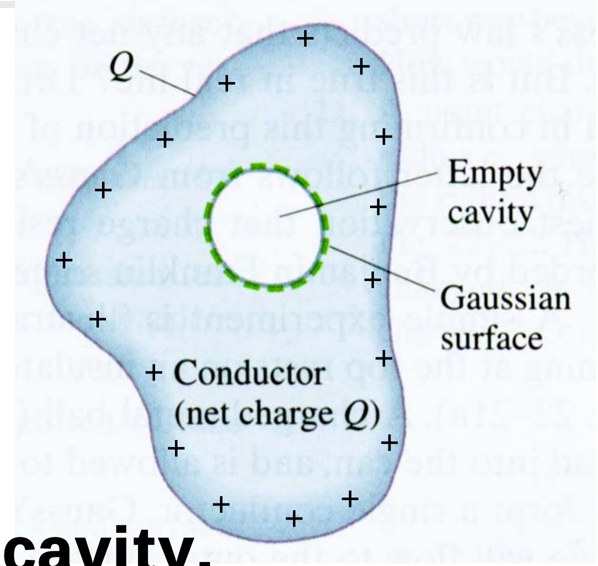
$$E_C = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

④ **On an irregularly shaped conductor, the surface charge density is highest at locations where the radius of curvature of the surface is smallest.**

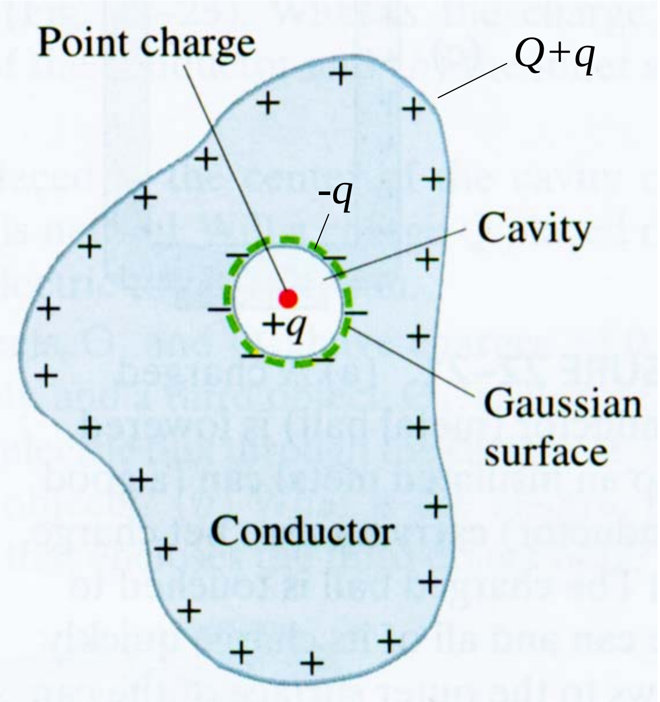


The charge distribution for a conductor cavity

- **No charge** in the internal cavity of the conductor.
  - There is no charge at the inner surface of the cavity.

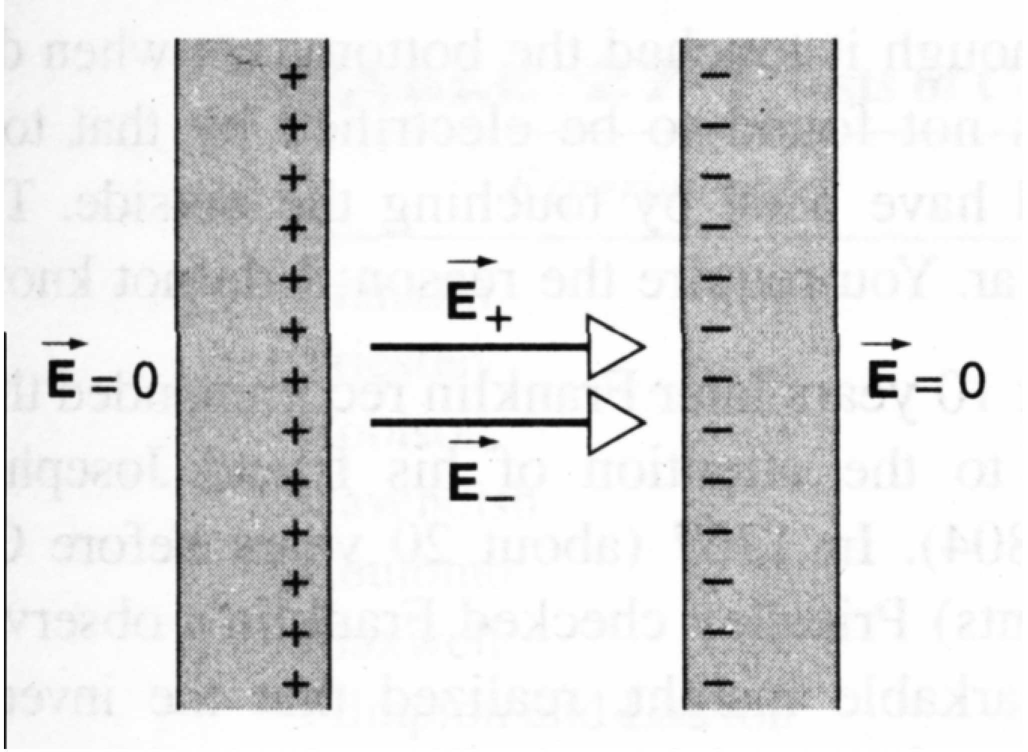


- A point charge **+q** is place inside the cavity.
  - A charge **–q** must be attracted to the inner surface of the cavity to keep the net charge zero within the Gaussian surface.
  - A charge of **Q+q** will appear on the outer surface of the cavity, so that the net charge of the conductor does not change.



Example

Two thin conducting plates carry equal and opposite charges *+q* and *−q*. Find the electric fields between the two plates and at the two sides of the plates.



Example

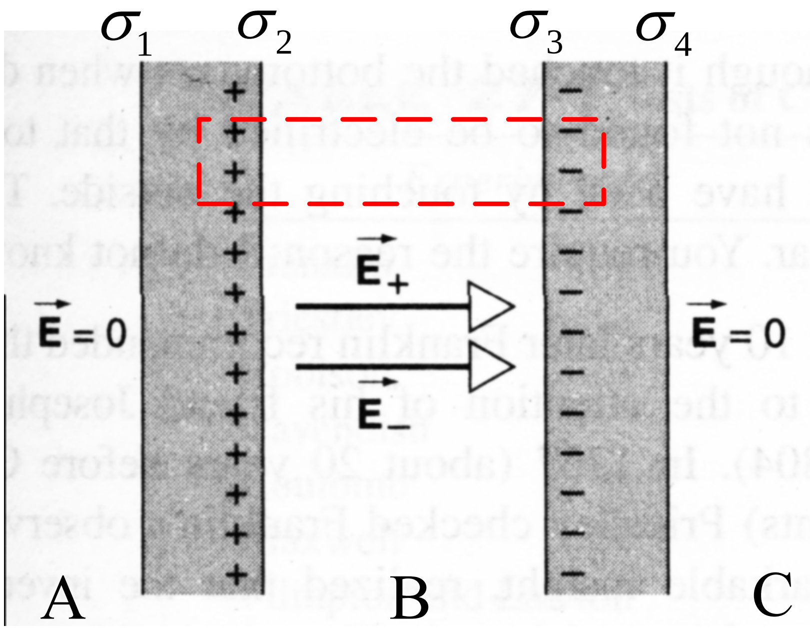
Solution: Conservation of net charge:

$$(\sigma_1+\sigma_2)S=+q$$

$$(\sigma_3+\sigma_4)S=-q$$

Gauss's law:

$$\oiint_S \vec{E}\cdot d\vec{A}=0=(\sigma_2+\sigma_3)A \Rightarrow \sigma_2=-\sigma_3$$



The field inside the plate 2 is zero:

$$E_{2in}=\frac{\sigma_1}{2\varepsilon_0}+\frac{\sigma_2}{2\varepsilon_0}+\frac{\sigma_3}{2\varepsilon_0}-\frac{\sigma_4}{2\varepsilon_0}=0$$

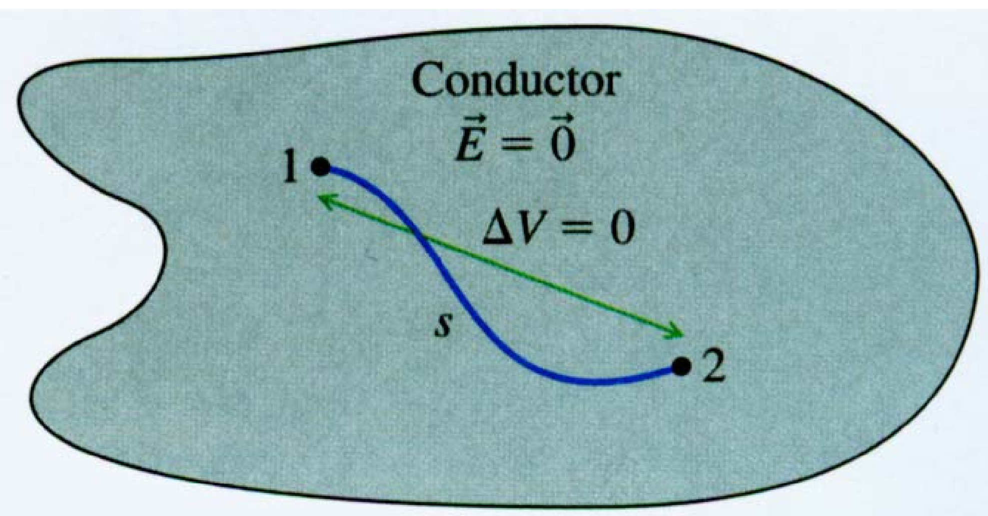
$$\Rightarrow \quad \sigma_1=\sigma_4=0$$

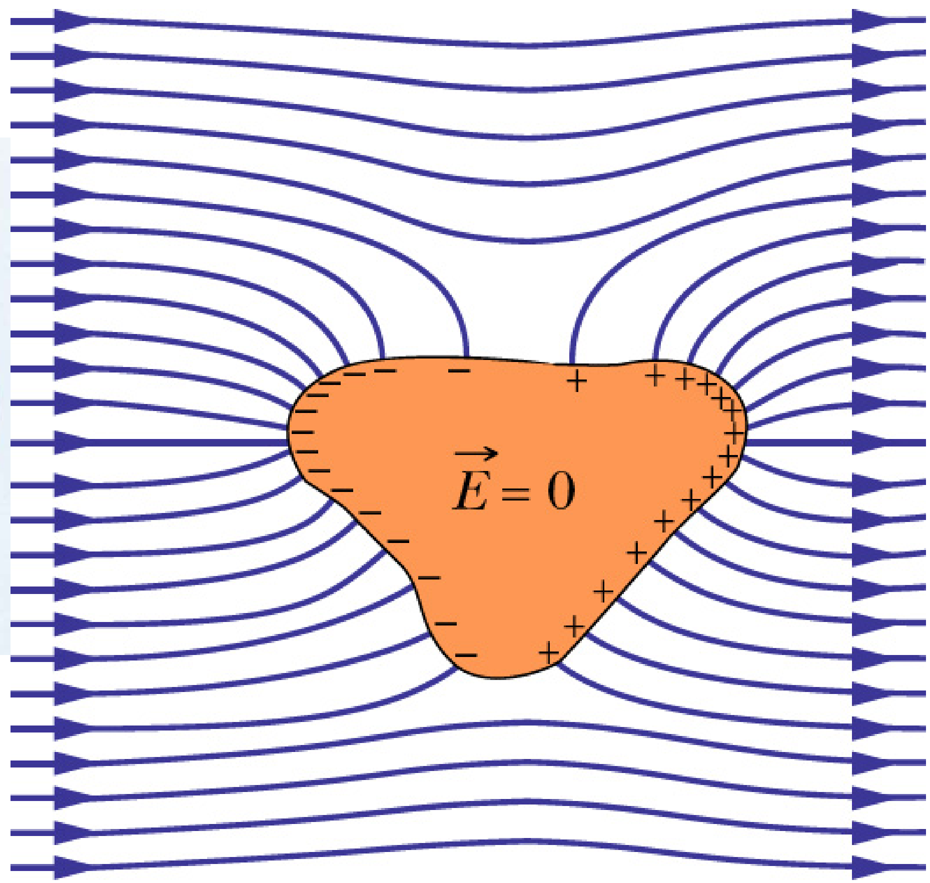
$$\sigma_2=-\sigma_3=\sigma=\frac{q}{S}$$

$$\boxed{E_A=\frac{\sigma_1}{\varepsilon_0}=0,\quad E_B=\frac{\sigma_2}{\varepsilon_0}=\frac{q}{S\varepsilon_0},\quad E_C=\frac{\sigma_4}{\varepsilon_0}=0}$$

§ 6 Electric Potential of a Charged Conductor

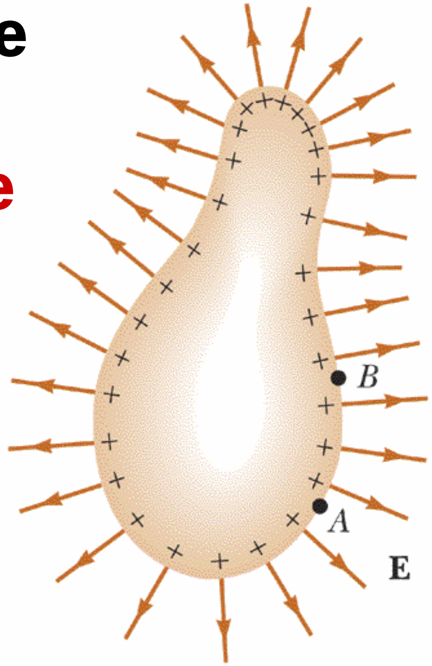
- **The properties that an isolated conductor in electrostatic equilibrium**
  - ⑤ **The entire conductor is at the same potential. The surface of a conductor is always an equipotential surface.**





The validity of property ④

④ **On an irregularly shaped conductor, the surface charge density is highest at locations where the radius of curvature of the surface is smallest.**

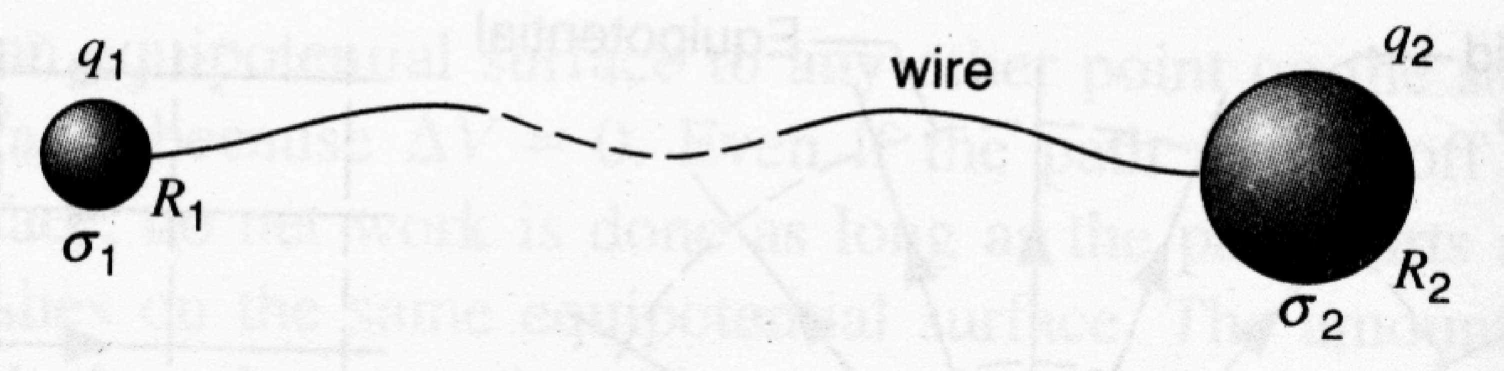


Consider two conducting spheres of different radii connected by a fine wire, let the entire assembly be raised to same arbitrary potential *V*.

$$V=\frac{1}{4\pi\varepsilon_0}\frac{q_1}{R_1}=\frac{1}{4\pi\varepsilon_0}\frac{q_2}{R_2},\quad \text{which yields}\quad \frac{q_2}{q_1}=\frac{R_2}{R_1}$$

$$\frac{\sigma_2}{\sigma_1}=\frac{q_2/4\pi R_2^2}{q_1/4\pi R_1^2}=\frac{q_2}{q_1}\frac{R_1^2}{R_2^2}$$

$$\boxed{\frac{\sigma_2}{\sigma_1}=\frac{R_1}{R_2}}$$

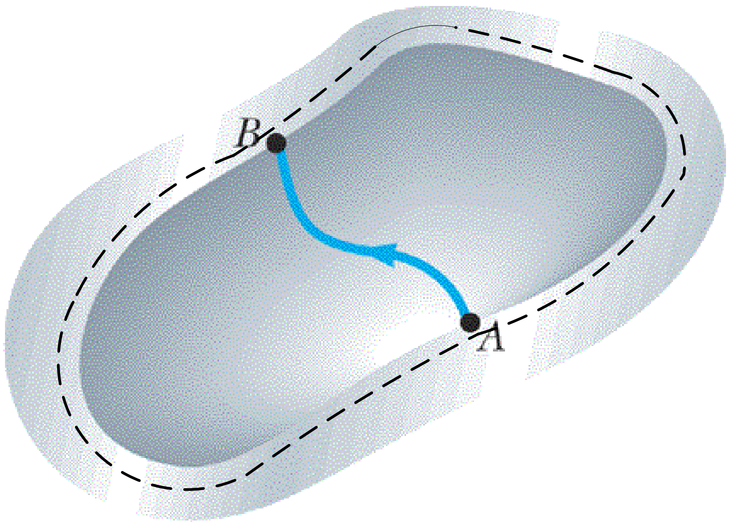


The property of an internal cavity in the conductor

- The validity of the statement "there is no charge at the surface of the cavity".

Draw a Gaussian surface just inside the inner surface.

$$\oint_S \vec{E}\cdot d\vec{A} = \frac{\sum q_{in}}{\varepsilon_0} = 0 \quad \Rightarrow \quad \sum q_{in} = 0$$



Is zero charge every where?

If not, then

$$V_A - V_B = \int_A^B \vec{E}\cdot d\vec{s} > 0$$

It is **contradictory** to the fact that

the surface of a conductor is an equipotential surface.

Summary

Electric properties of a conductor in electrostatic equilibrium