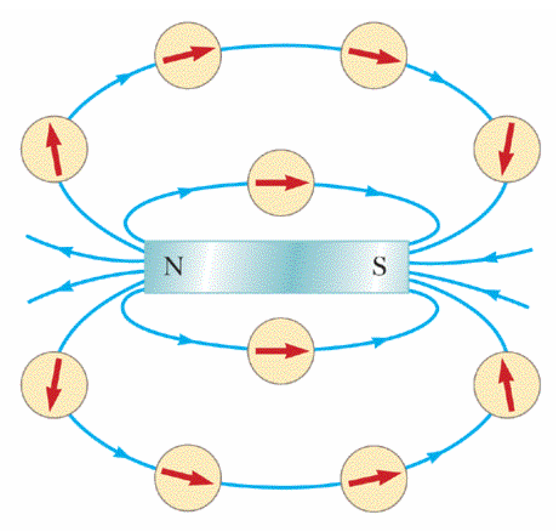
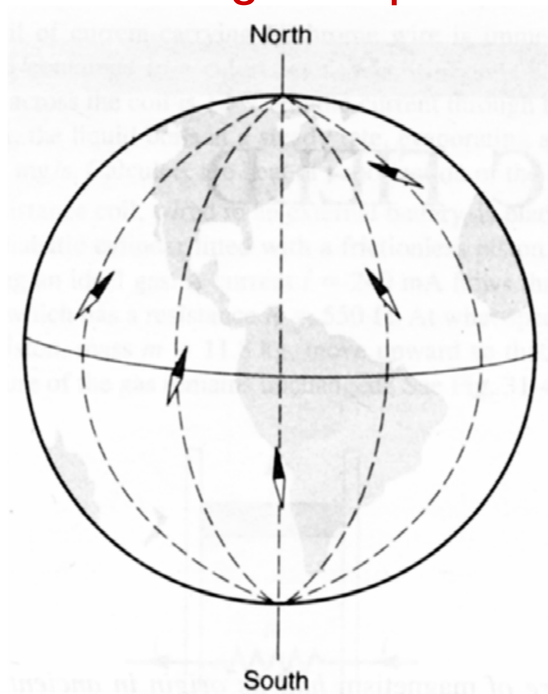
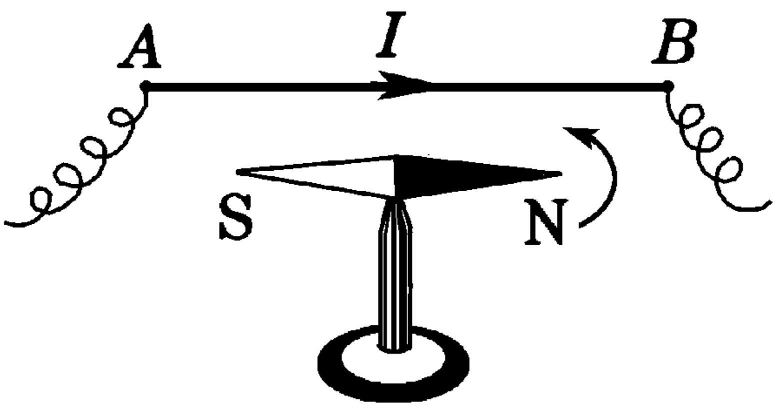
Chapter 25-26 Magnetic Forces and Magnetic fields

§ 1 Magnetic Fields and Magnetic Forces

Magnetic phenomena







The Comparison between Electronic and Magnetic Interaction Models

Electric interaction model

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r} \qquad \vec{F}_e = q\vec{E}$$

charge $\rightleftarrows$ electric field $\rightleftarrows$ charge

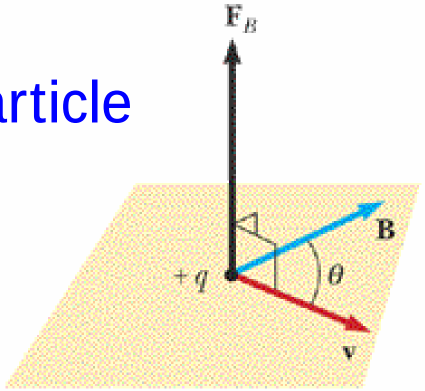
Magnetic interaction model

moving charge / current $\overset{?}{\rightleftarrows}$ magnetic field $\overset{?}{\rightleftarrows}$ moving charge / current

- How does a moving charge or a current create the magnetic field throughout the space?
- How does the magnetic field exert a force on any other moving charge or current that presents in the field?

The magnetic force on a moving charged particle

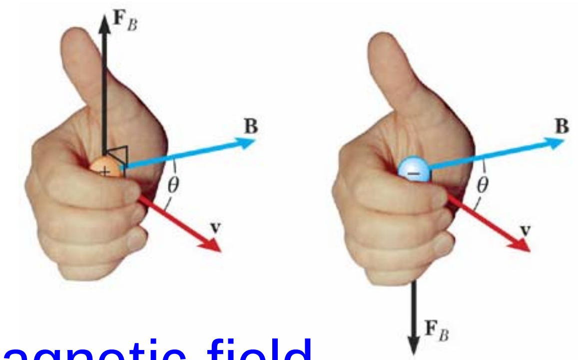
- The magnetic force on a moving charged particle

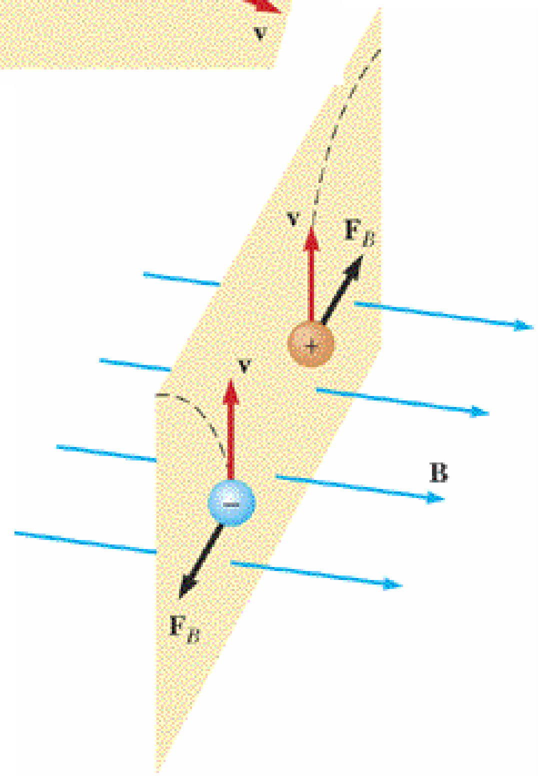


Lorentz force: $$\boxed{\vec{F}_B = q\vec{v} \times \vec{B}}$$

$$F_B = |q| vB \sin\theta$$

- Right hand rule for the magnetic force:





- The unit of magnetic field
  - SI unit: tesla or T. $1\ \mathrm{T} = 1\ \mathrm{N \cdot s/C \cdot m}$
  - cgs unit: gauss or G. $1\ \mathrm{G} = 10^{-4}\ \mathrm{T}$

    The magnetic field of the earth is of the order of 1G or 10^{-4}T.

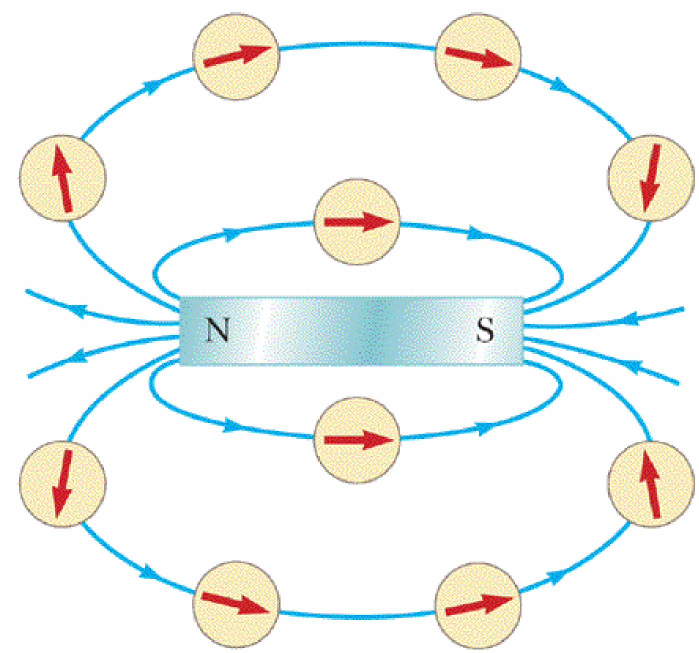
- The important differences between electric force and magnetic forces
  - The electric force is always parallel or anti-parallel to the direction of the electric field ($\vec{F}_e = q\vec{E}$), whereas the magnetic force is perpendicular to the magnetic field ($\vec{F}_B = q\vec{v} \times \vec{B}$).
  - The electric force acts on a charged particle is independent of the particle's velocity, whereas the magnetic force acts on a charged particle only when the particle is in motion.
  - The electric force does work in displacing a charge particle, whereas the magnetic force does no work when a charged particle is displaced (because the magnetic force is always perpendicular to its velocity $\vec{F}_B \perp \vec{v}$).

§ 2 Magnetic Field Lines and Magnetic Flux

- Magnetic field lines, a graphical way, are related to the magnetic field in the following manner:

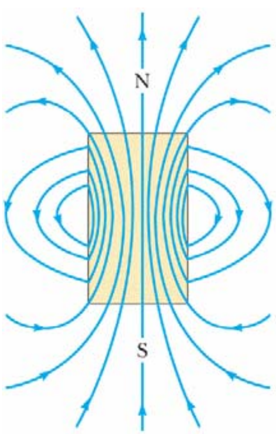
  Magnetic field in space:

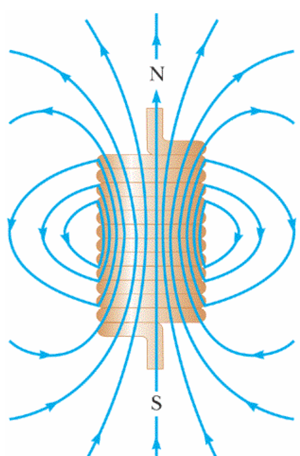
  - Direction —— is tangent to the magnetic field line at that point.
  - Magnitude —— is proportional to the number of magnetic field lines per unit area through the cross-sectional surface in that region. The magnitude of *B* is larger where the adjacent field lines are close together and small where they are far apart.

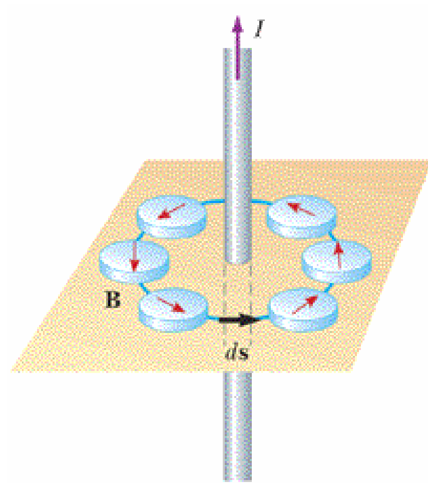


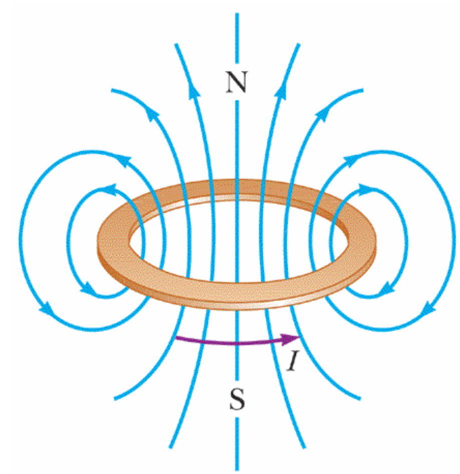
The Fundamental Properties for Magnetic Field Lines

- The fundamental properties for magnetic field lines
  - Unlike electric field lines that begin and end on electric charges, magnetic field lines never have end point, and always form closed loops; (If a magnetic field line had end point, such a point would indicate the existence of a magnetic monopole (磁单极)).
  - Because the direction of magnetic field at each point is unique, field lines never intersect.





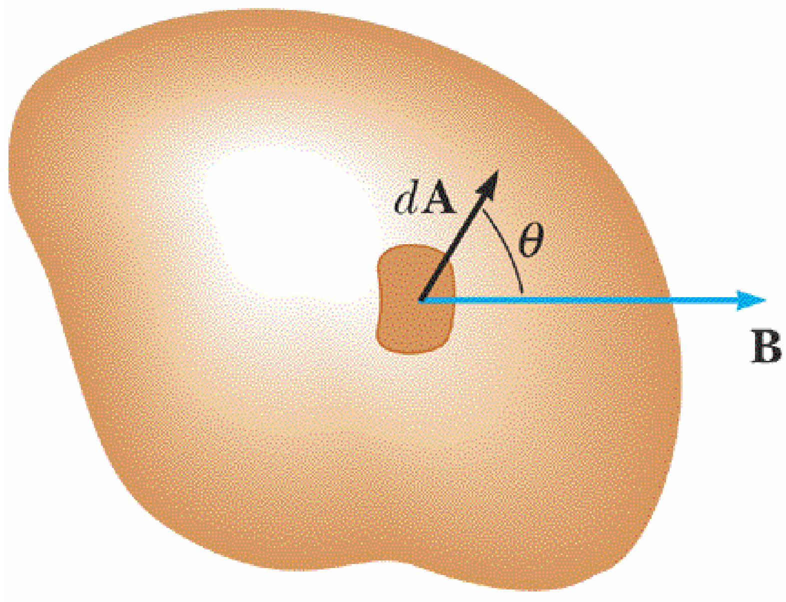


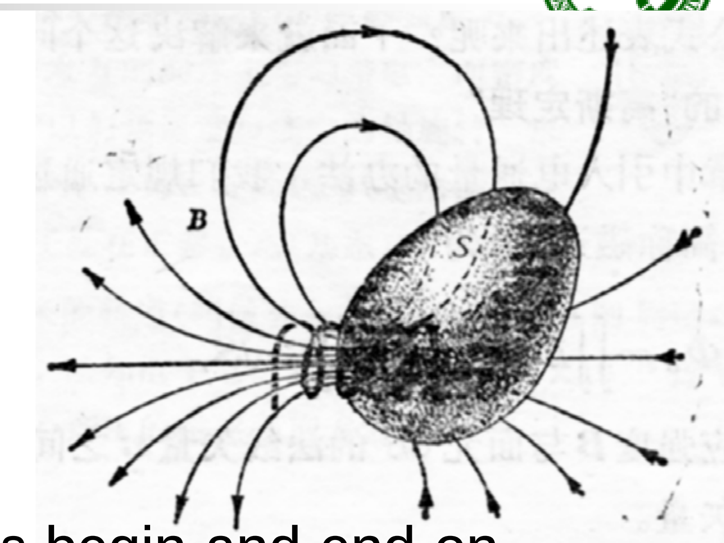


- Magnetic flux:
  - Magnetic flux through a surface:

$$\boxed{\Phi_B = \iint_{\text{surface}} \vec{B} \cdot d\vec{A}}$$

  - SI unit: weber(Wb)





Gauss's law for magnetism

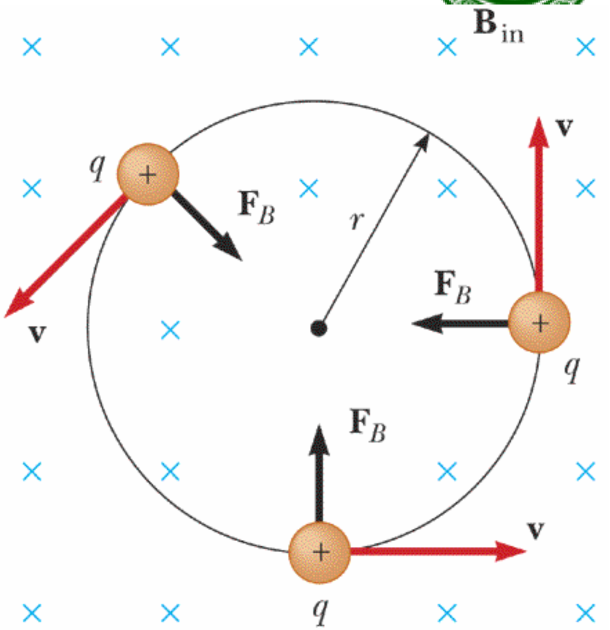
Review: In Gauss's law for electric field, the total electric flux through a closed surface is proportional to the total electric charge enclosed by the surface, partly for the reason of electric field lines begin and end on electric charges. By analogy, if we could separate the north and south poles of a magnet, or there were a single magnetic charge (magnetic monopole), the total magnetic flux through a closed surface would be proportional to the total magnetic charges enclosed. But no magnetic monopole has ever been observed.

- Gauss's law for magnetism: The total magnetic flux through a closed surface is always zero. In other word, the magnetic flux penetrating a closed surface is always equal to the flux leaving the closed surface.

$$\boxed{\oiint_S \vec{B} \cdot d\vec{A} = 0}$$

§ 3 Motion of a Charged Particle in a Uniform Magnetic Field

Magnetic force: $\vec{F}_B = q\vec{v} \times \vec{B}$



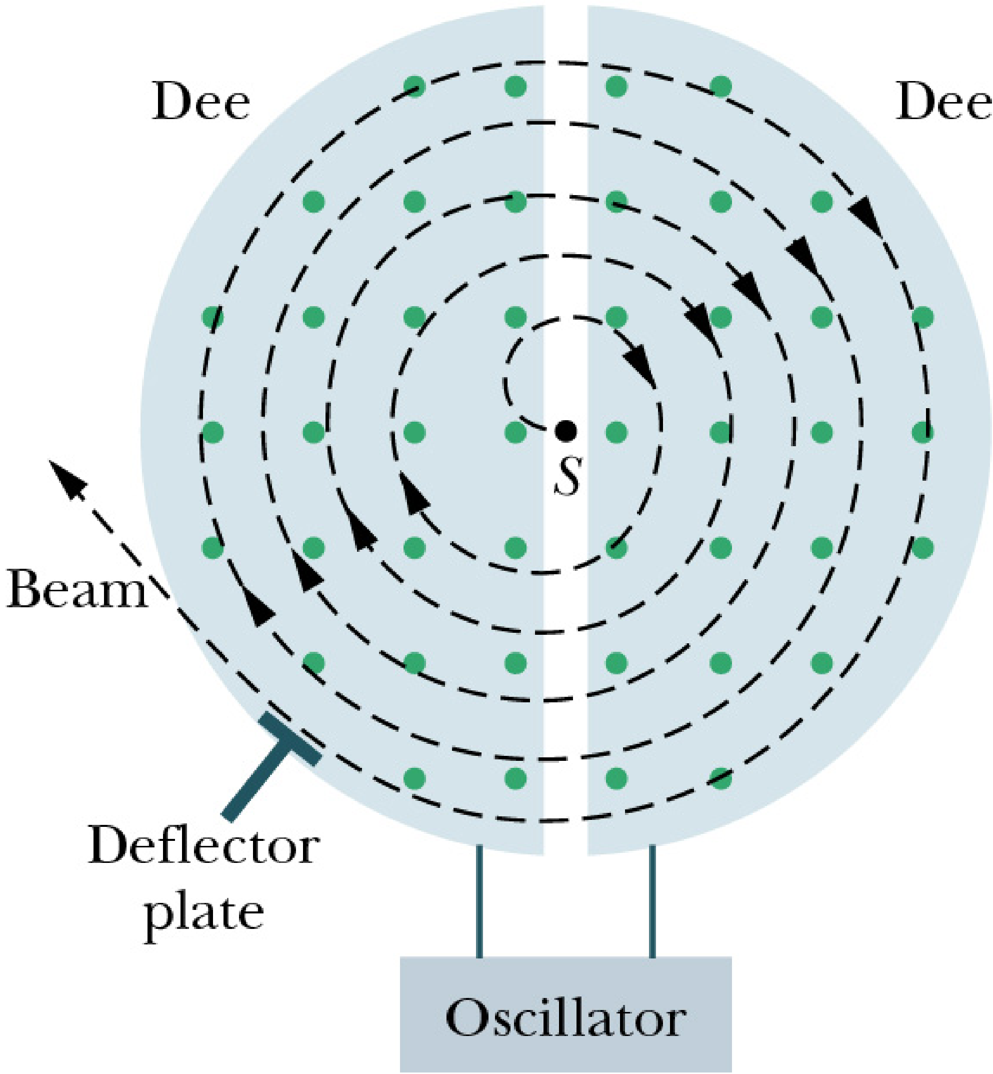
- For the case that the initial velocity of the particle is perpendicular to the magnetic field:
  - $\vec{v}_0 \perp \vec{F}_B$, the magnetic force provides the centripetal force. The particle is in uniform circular motion:

$$F_B = qvB = ma = m\frac{v^2}{r},$$

$$r = \frac{mv}{qB}, \qquad T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

$$T = \frac{2\pi m}{qB}$$

- The angular speed or the period of the circular motion do not depend on speed v or radius of the orbit r, which is the basis for the cyclotron. ω is often called cyclotron frequency.



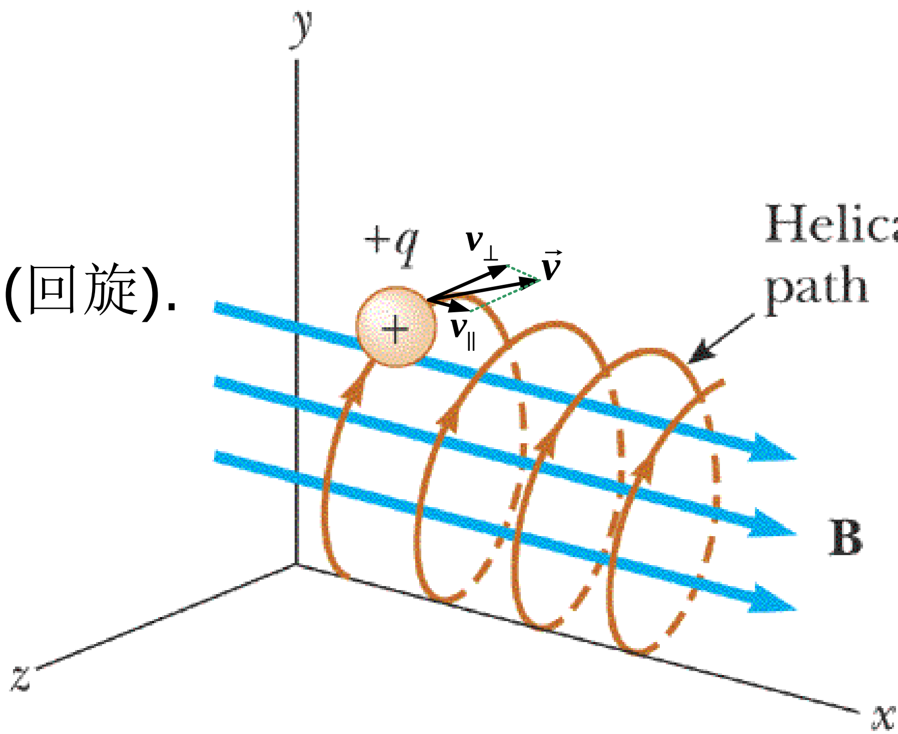
Cyclotron（回旋加速器）
[ˈsaiklәˌtrɔn]

Magnetic force Cont'd

- For the case the initial velocity of the particle is not perpendicular to the magnetic field:
  - The parallel component of acceleration $a_{\parallel} = 0$.
  - The perpendicular component of acceleration

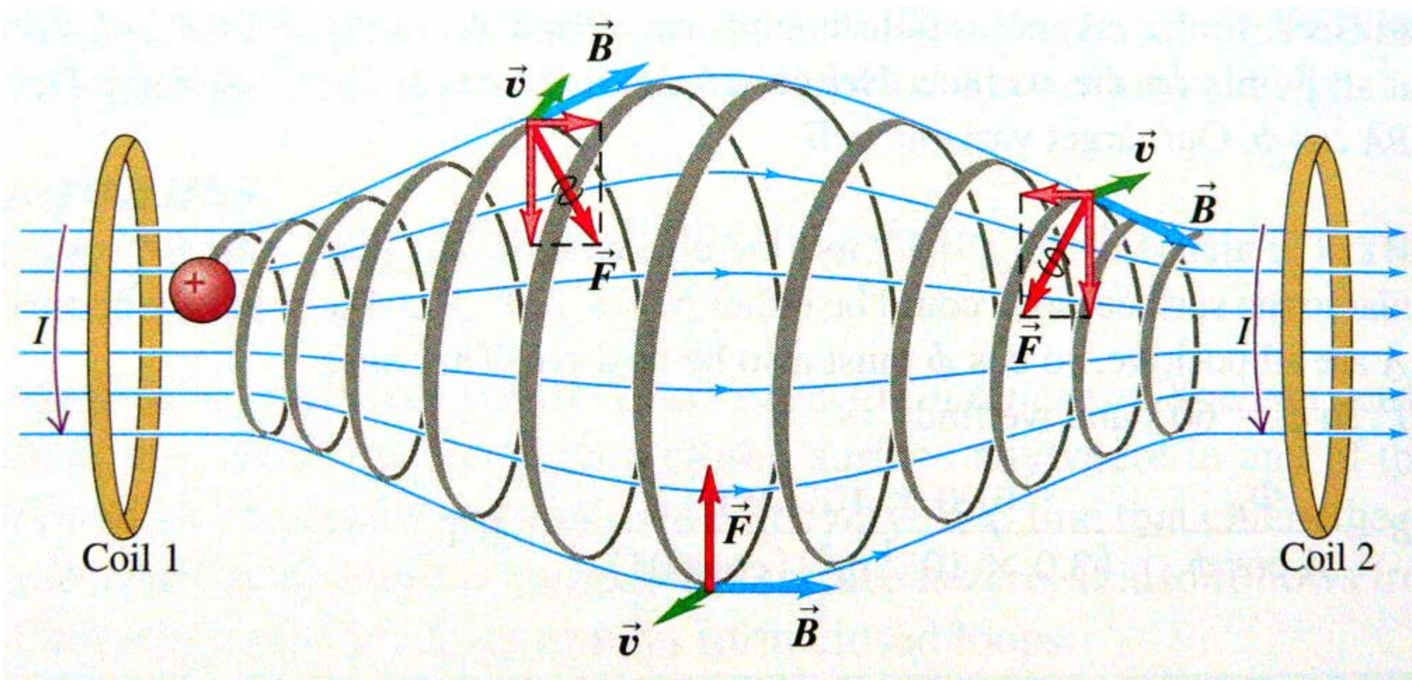
$$a_{\perp} = \frac{v_{\perp}^2}{r} \qquad r = \frac{mv_{\perp}}{qB}$$

  - The particle moves in a helix (回旋).



- The magnetic mirror or magnetic bottle
  - A non-uniform magnetic field can be used to trap a charged particle in a region of space.
  - A device made by two circular coils (线圈) can be used to trap charged particles

    The charged particles continue to spiral (盘旋) back and forth, confined to the space between two coils.

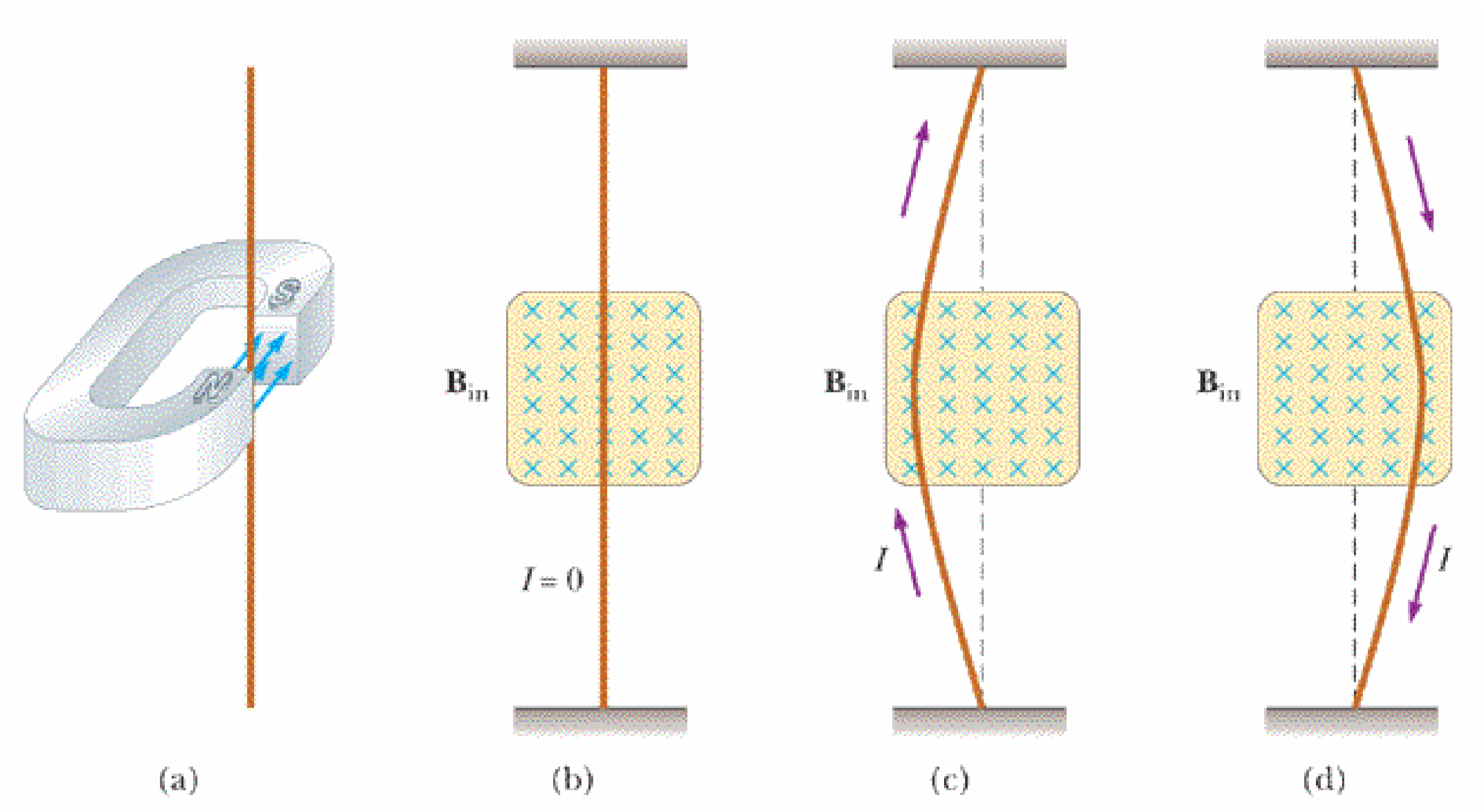


§ 4 Applications of the Motion of Charged Particle in a Magnetic Field

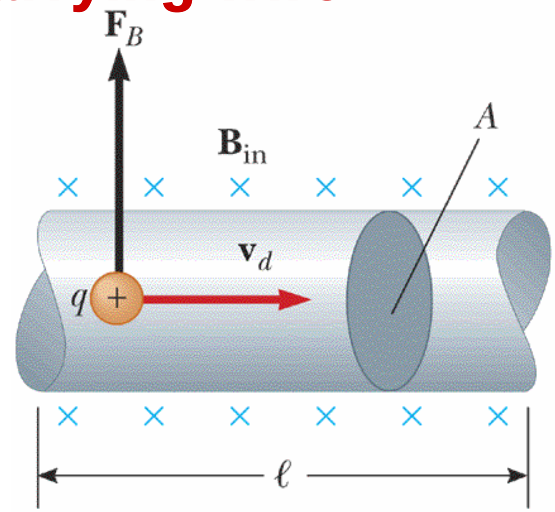
Self-Taught

§ 5 Magnetic Force on a Current-Carrying Conductor

The phenomena of the magnetic force on the current-carrying conductor acted by an external magnetic field.



- **The magnetic force on a straight current-carrying wire with segment of length *l* :**



  - The magnetic force on a charge q in the wire moving with drift velocity v_d is: $q\vec{v}_d \times \vec{B}$
  - The total magnetic force on the wire segment: the number of charges in the segment is nAl, where n is the number of charges per unit volume, A and l are the cross-sectional area and length of the wire.

$$\vec{F}_B = (q\vec{v}_d \times \vec{B})(nAl)$$

  - The current in the wire is $I = nqv_dA$. So the $\vec{F}_B$ can be expressed as

$$\boxed{\vec{F}_B = I\vec{l} \times \vec{B}}$$

$\vec{l}$ is the length vector in the direction of the current I.

The magnetic force on a non-straight current-carrying wire

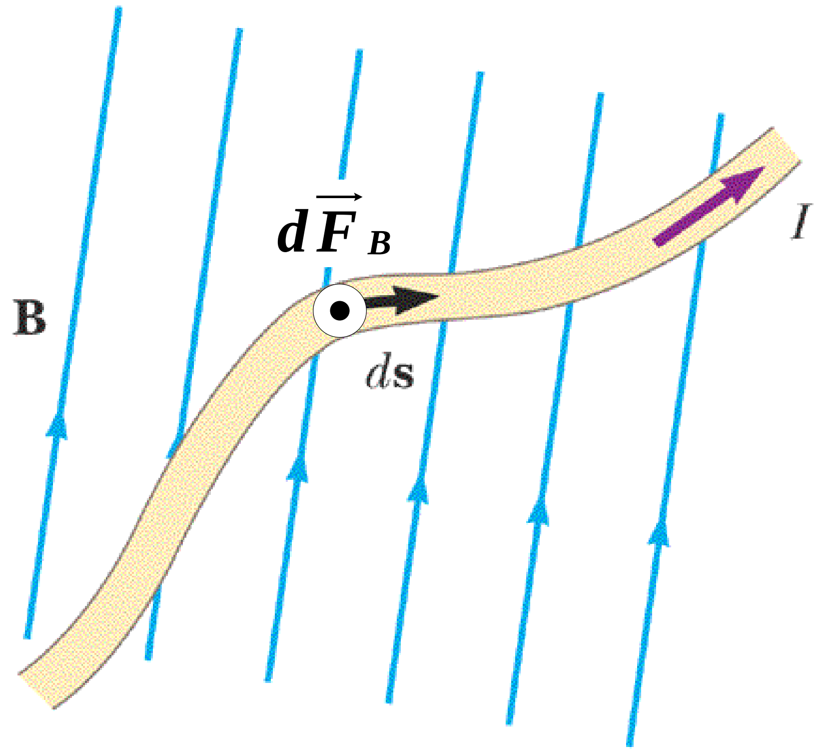
- **If the wire is not straight or the magnetic field is not uniform**
  - Imaging the wire to be broken into small segments of length $d\vec{s}$.

For each small segment:

$$\boxed{d\vec{F}_B = I d\vec{s} \times \vec{B}}$$

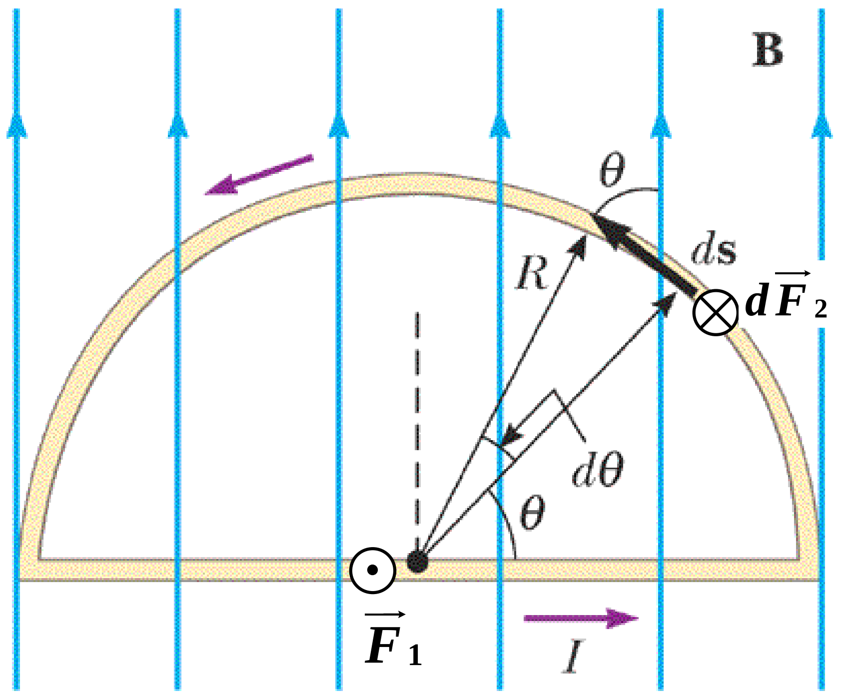
  - The total magnetic force on a length of the wire between arbitrary point ***a*** and ***b***:

$$\boxed{\vec{F}_B = I\int_a^b d\vec{s} \times \vec{B}}$$



Magnetic force on a semicircular conductor

A wire bent the shape of a semicircle of radius R forms a closed circuit and carries a current I. The circuit lies in the xy plane, and a uniform magnetic field is present along the positive y axis as in the figure. Find the magnetic force on the straight portion of wire and on the curved portion.



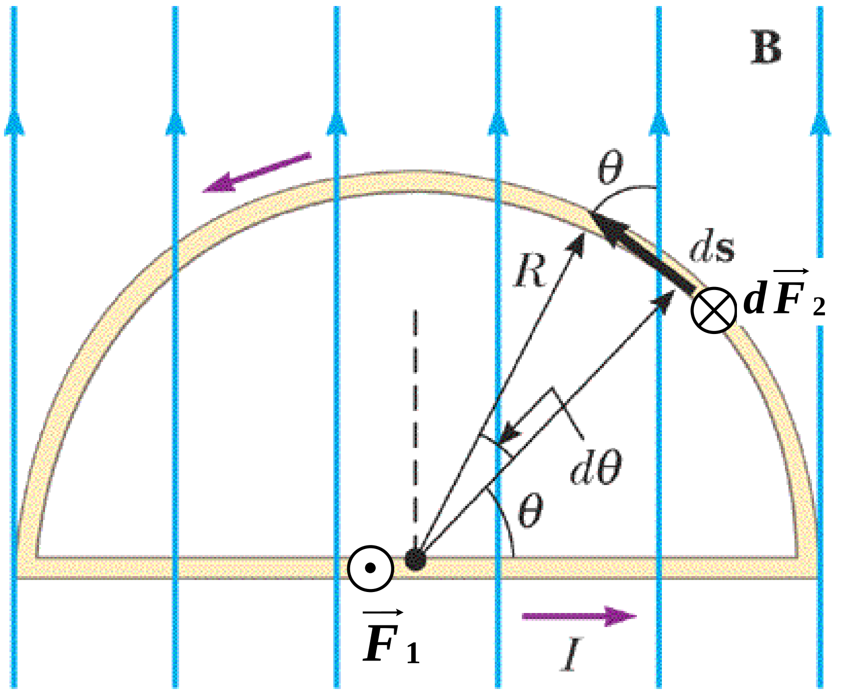
Example

Solution: (1) The force on the straight portion.

$$\vec{F}_B = I\vec{l} \times \vec{B}$$

$$F_1 = IlB = 2IRB$$

The direction of $\vec{F}_1$ is outward.



Example Cont'd

(2) The magnetic force $d\vec{F}_2$ on the element $d\vec{s}$

$$dF_2 = I\,|\,d\vec{s}\times\vec{B}\,| = IB\sin\theta\, ds$$

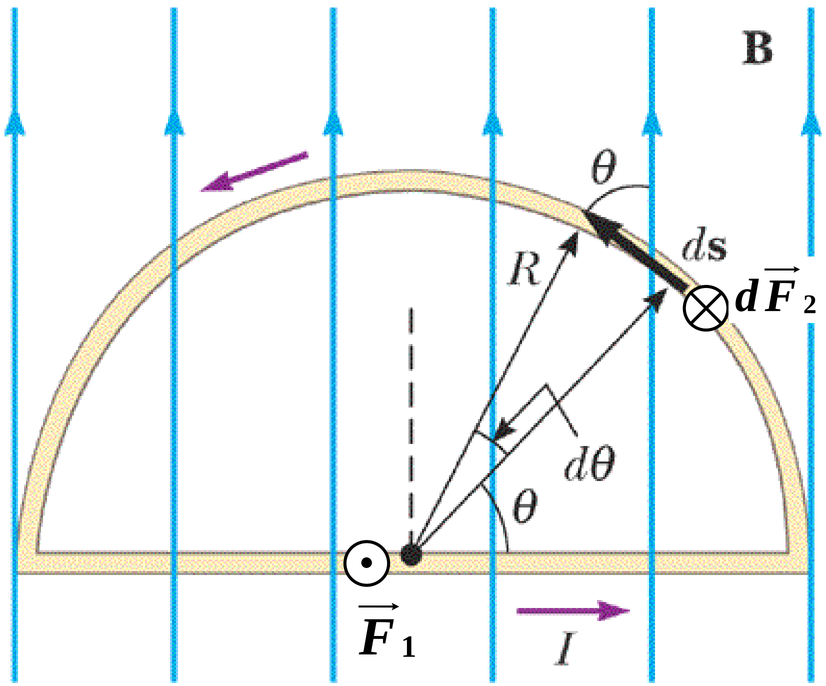
$ds = R\,d\theta$, $\quad dF_2 = IRB\sin\theta\, d\theta$

$$F_2 = IRB\int_0^{\pi}\sin\theta d\theta$$

$$= -IRB(\cos\pi - \cos 0) = 2IRB$$

The magnetic force $\vec{F}_2$ is inward.

We see that the net magnetic force on the closed loop is zero when the magnetic field is uniform.



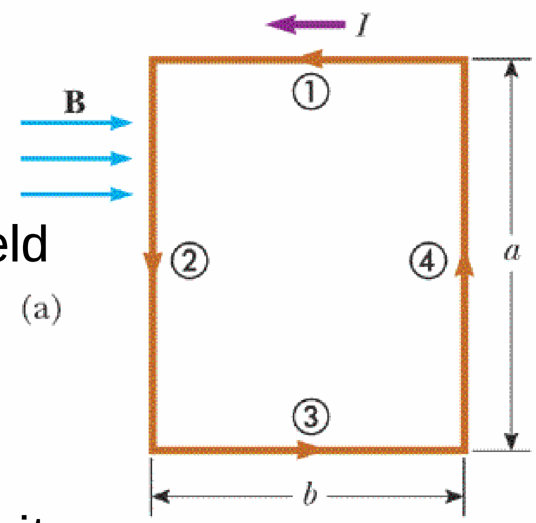
§ 6 Torque on a Current Loop

- The net force on a current loop in a uniform magnetic field is zero.
- However, the net torque is generally not zero.
  - Example: a rectangular current loop of wire, with side length a and b.
  - When the loop is oriented so that the magnetic field is in the plane of the loop, according to

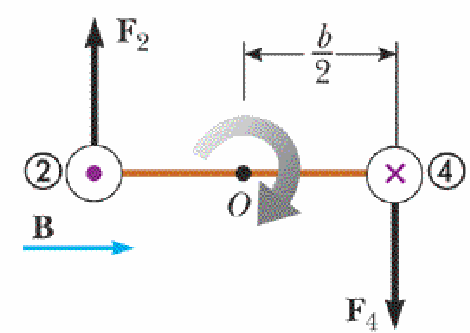
$$\vec{F}_B = I\vec{l} \times \vec{B}$$

    the magnetic forces on the short ends are zero.
    On the long ends, the force are equal but in opposite directions. $|\vec{F}_2| = |\vec{F}_4| = IaB$
    The net force on the loop is zero.
  - The net torque: tend to rotate the loop clockwise.

$$|\vec{\tau}| = \left(\frac{b}{2}\right)F_2 + \left(\frac{b}{2}\right)F_4 = 2\left(\frac{b}{2}\right)IaB = I(ab)B = (IA)B$$



(a)



(b)

Torque on A Current Loop

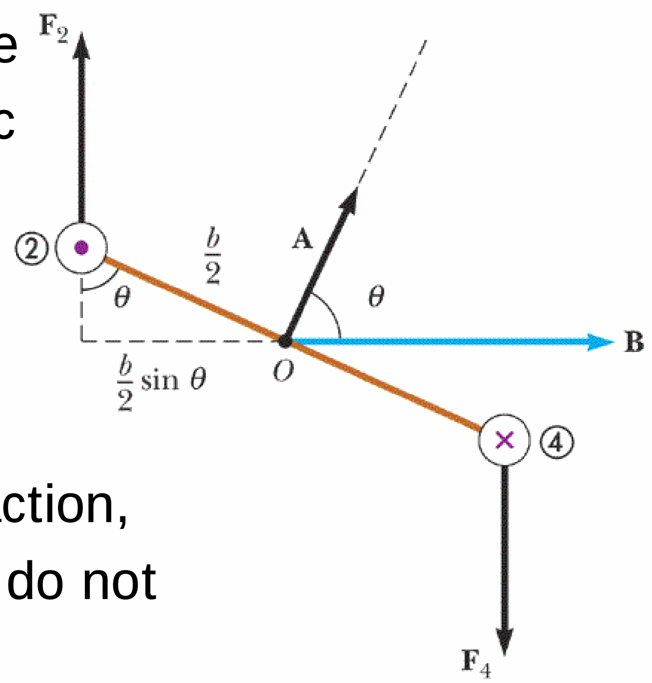
- When the loop is oriented so that the loop plane makes an angle θ with the direction of magnetic field, F_1 is inward and has a magnitude of

$$F_1 = IbB\sin(90° + \theta) = IbB\cos\theta$$

F_3 is outward and has a magnitude of

$$F_3 = IbB\sin(90° - \theta) = IbB\cos\theta$$

The forces of F_1 and F_3 have the same line of action, not only they create a total zero force, but also do not contribute to the net torque.



- The forces F_2 and F_4:

They also create a total zero force, but they create a torque:

$$|\vec{\tau}| = F_2\left(\frac{b}{2}\right)\sin\theta + F_4\left(\frac{b}{2}\right)\sin\theta$$

$$= 2\left(\frac{b}{2}\right)IaB\sin\theta = (IA)B\sin\theta$$

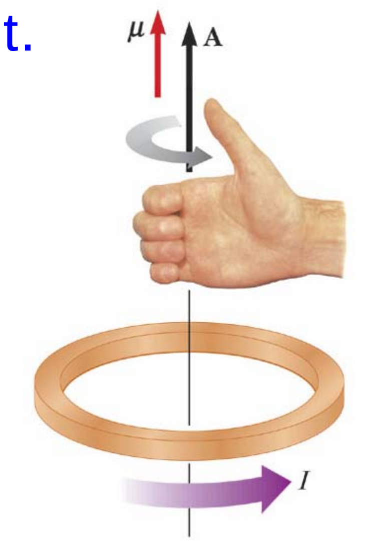
If we define the A as a vector $\vec{A}$ perpendicular to the plane of the loop

$$\boxed{\vec{\tau} = (I\vec{A}) \times \vec{B}}$$

Magnetic Dipole

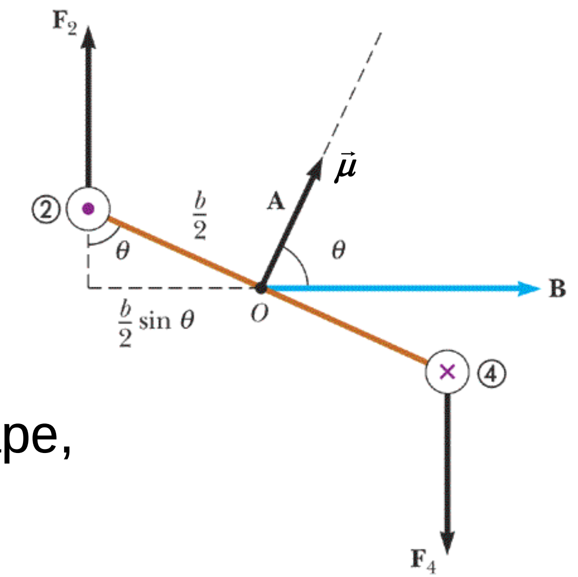
- Introducing the magnetic dipole and magnetic dipole moment.
  - For any current loop with any shape, we can defined a vector magnetic moment $\vec{\mu}$ with magnitude ***IA*.** The direction of $\vec{\mu}$ is determined by right-hand rule.

$$\boxed{\overrightarrow{\mu} \equiv I\overrightarrow{A}}$$



  - If a coil consists of *N* turns of wire, the total magnetic moment of the coil is: $\overrightarrow{\mu} = NI\overrightarrow{A}$
- Torque on the current loop in a magnetic field

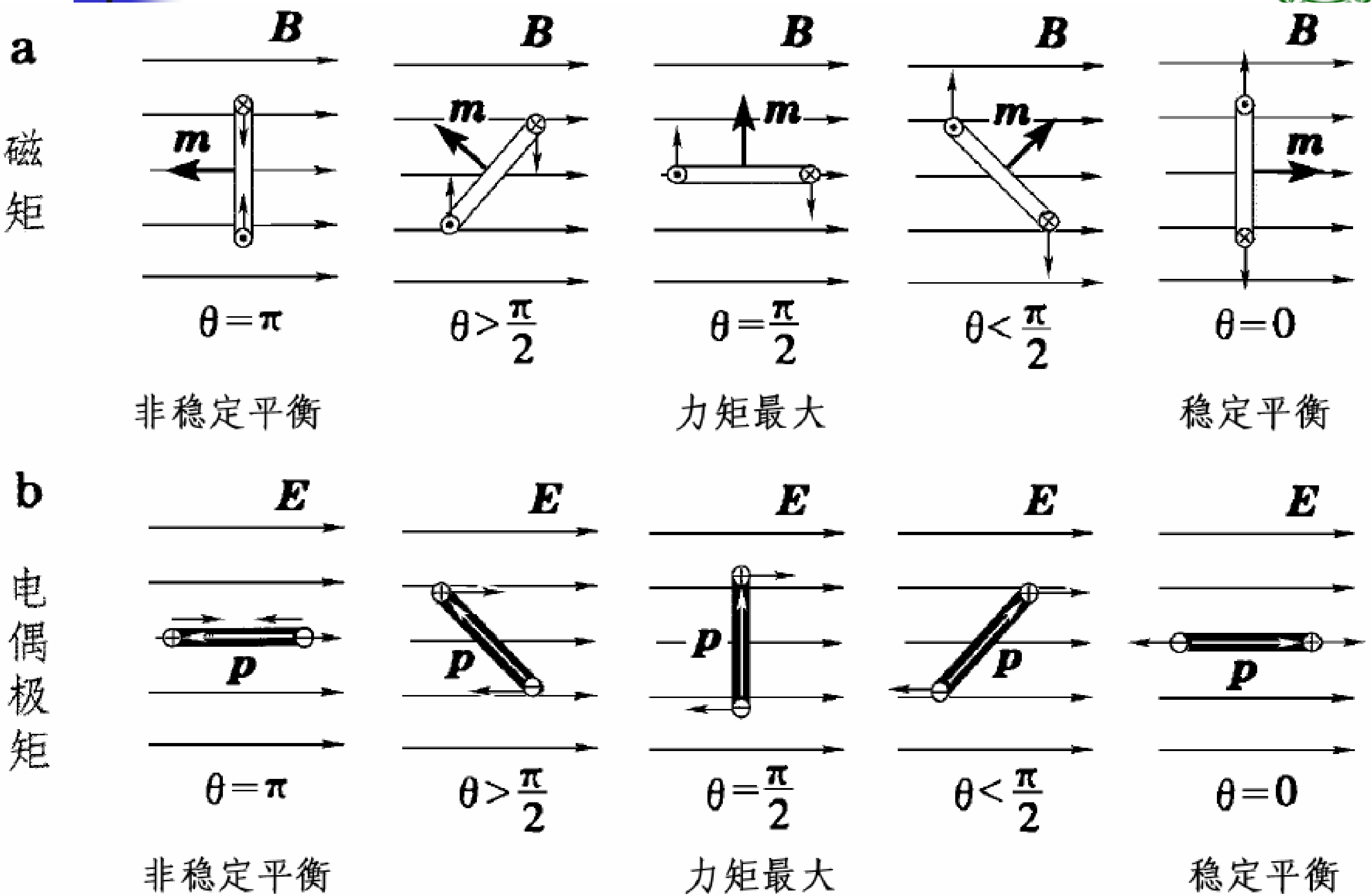
$$\boxed{\vec{\tau} = I\overrightarrow{A} \times \overrightarrow{B} = \overrightarrow{\mu} \times \overrightarrow{B}}$$



  - The torque tries to rotate the loop so that $\vec{\mu}$ is brought into alignment with $\overrightarrow{B}$.
  - The torque expression is valid for loop of any shape, although it was derived for a rectangular loop.

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$



Problems

Ch25 Prob. 27, 29 (P600)

Ch25 Prob. 12, 37 (P599)

§ 7 The Biot-Savart Law

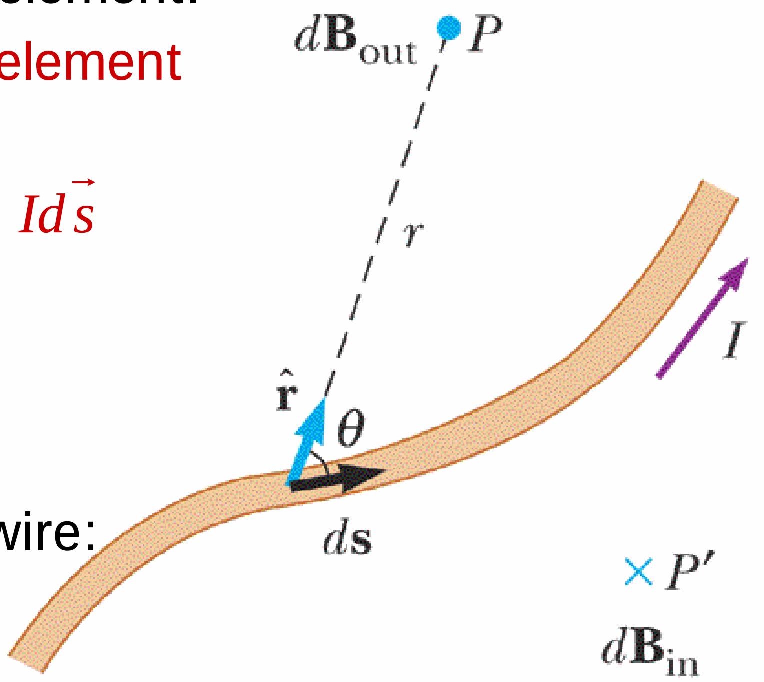
- If we will find the magnetic field due to a current in a wire, our strategy is first to find the field due to the current in a short element of the wire.
  - The total magnetic field caused by entire wire is the vector sum of the fields caused by individual current element.
- The magnetic field produced by a current element

  —— Biot-Savart Law
  - Definition of vector of current element $I d\vec{s}$
  - Biot-Savart law:

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{s} \times \hat{r}}{r^2}}$$

  - The total magnetic field due to entire wire:

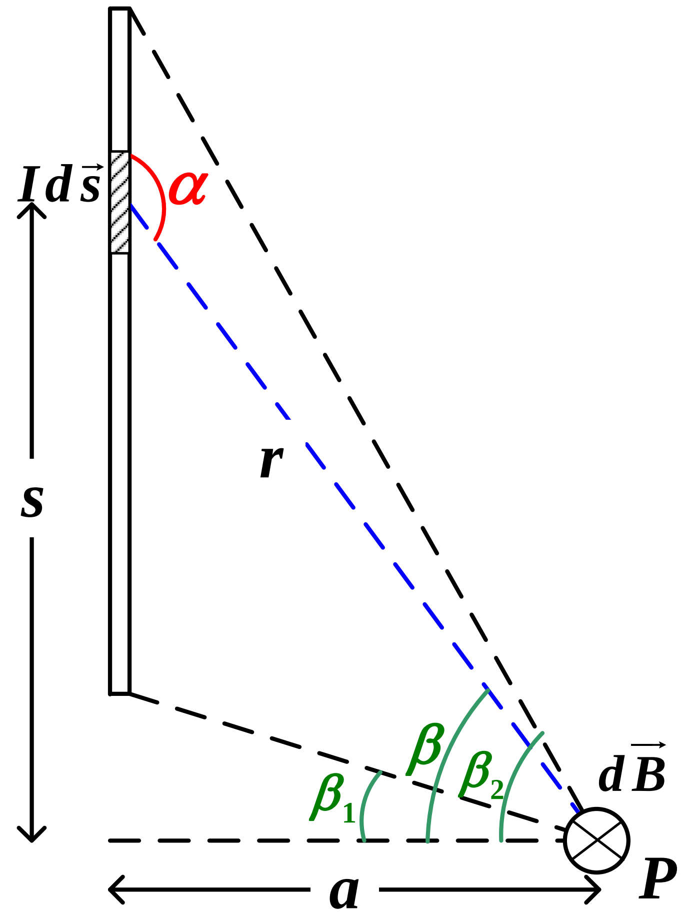
$$\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{s} \times \hat{r}}{r^2}$$



μ_0 is called the permeability of free space. $\frac{\mu_0}{4\pi} = 10^{-7}\ \text{T}\cdot\text{m/A}$

Magnetic field of a straight current wire segment

Find the magnetic field at the point P, located a distance a from the wire. The straight wire carries a constant current I. Assume the lines connecting two ends of wire and point P make the angles β_1 and β_2 with the horizontal line.



Magnetic field of a straight current wire segment

Solution: $d\vec{B}$ **produced by the current element** $Id\vec{s}$ **is always inward.**

$$dB = \frac{\mu_0}{4\pi}\frac{Ids\sin\alpha}{r^2}, \qquad B = \frac{\mu_0}{4\pi}\int\frac{Ids\sin\alpha}{r^2}$$

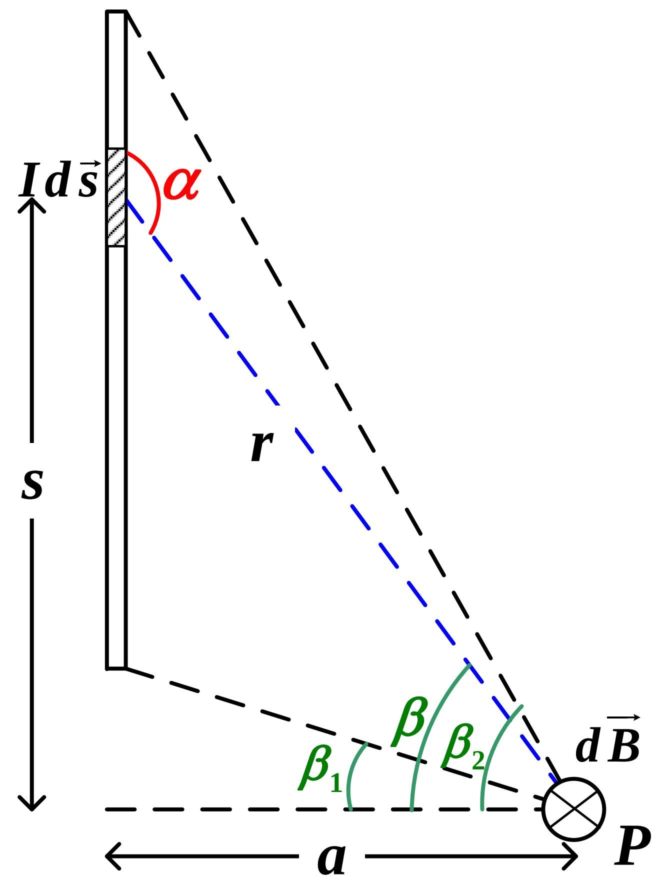
Find r, s, α **in terms of** β

$$\alpha = 90^\circ + \beta, \qquad \sin\alpha = \cos\beta$$

$$r = \frac{a}{\cos\beta} = a\sec\beta, \quad s = a\tan\beta, \quad ds = a\sec^2\beta d\beta$$

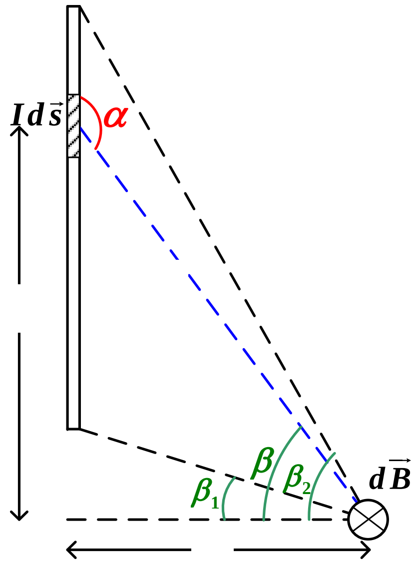
$$B = \frac{\mu_0 I}{4\pi}\int_{\beta_1}^{\beta_2}\frac{(\cos\beta)(a\sec^2\beta d\beta)}{a^2\sec^2\beta}$$

$$= \frac{\mu_0 I}{4\pi a}\int_{\beta_1}^{\beta_2}\cos\beta d\beta = \frac{\mu_0 I}{4\pi a}\left(\sin\beta_2 - \sin\beta_1\right)$$



Example Cont'd

$$B=\frac{\mu_0 I}{4\pi a}\left(\sin\beta_2-\sin\beta_1\right)$$

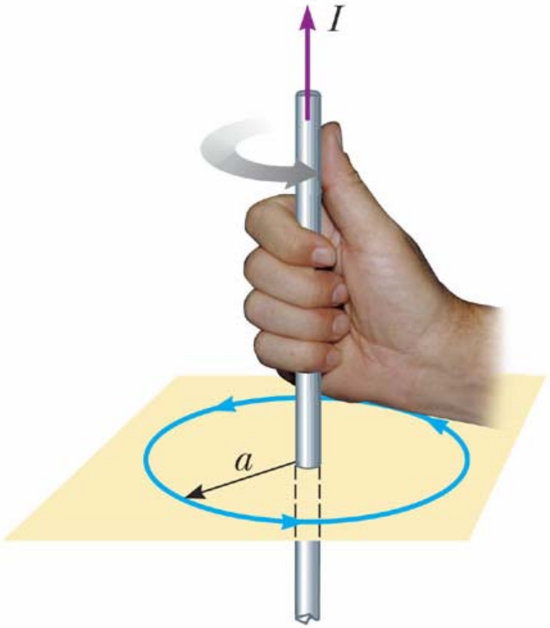


For a very long wire, s>>a

$$\beta_1\to-\frac{\pi}{2},\quad \beta_2\to\frac{\pi}{2}$$

$$B\to\frac{\mu_0 I}{2\pi a}\propto\frac{1}{a}$$

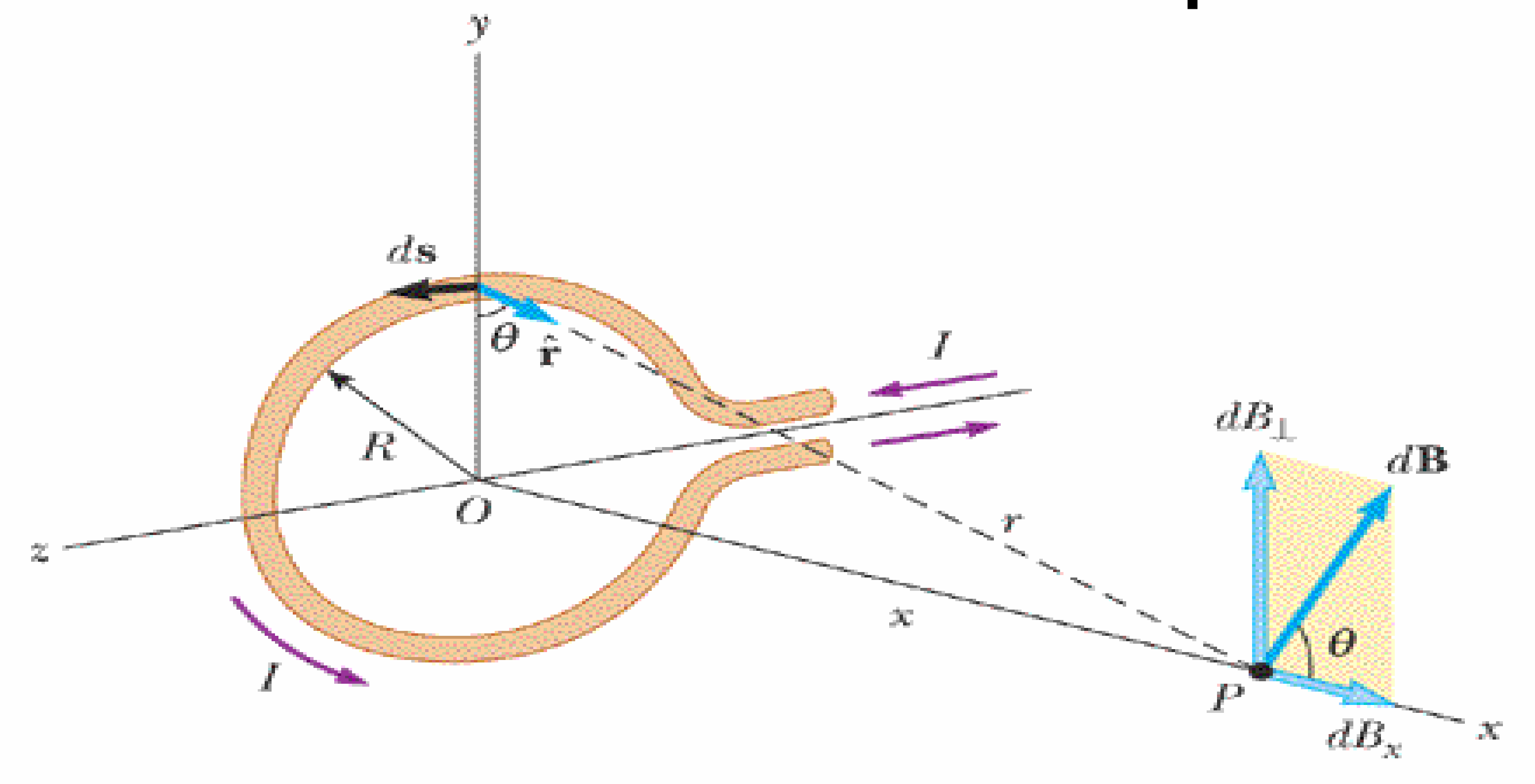
For a long, straight, current-carrying wire, a set of magnetic lines form concentric circles around the wire.



We can use the right-hand rule to determine the direction of the magnetic field surrounding a long, straight wire carrying a current.

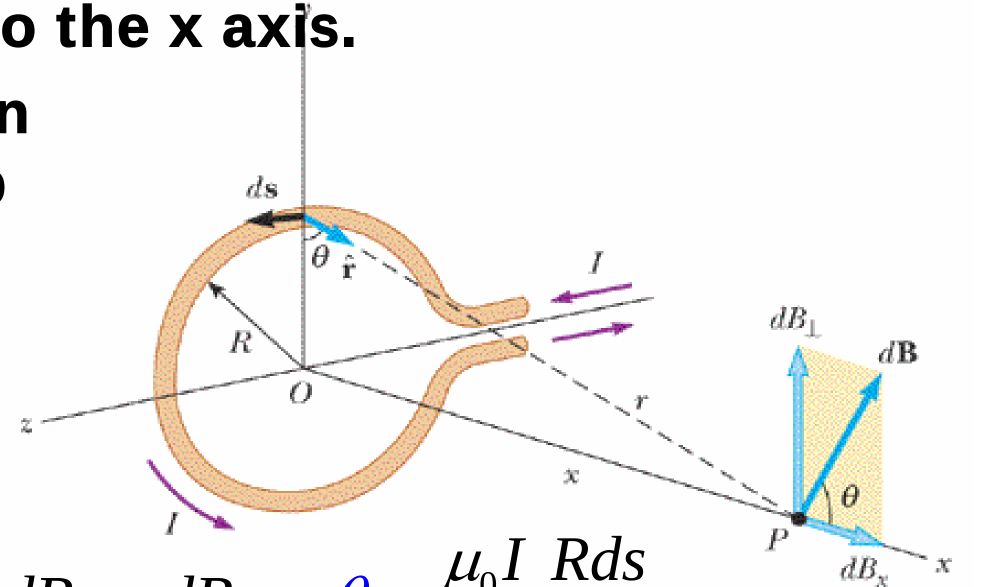
Magnetic field on the axis of a circular current loop

Consider a circular loop of wire of radius R located in the y-z plane and carrying a steady current I. Calculate the magnetic field at an axial point P a distance x from the center of the loop.



Solution: The $d\vec{B}$ due to the element $d\vec{s}$ can be resolved into a component dB_x, along the x axis, and a component $dB_\perp$, which perpendicular to the x axis.

By **symmetry**, any element on one side of the loop sets up a component $dB_\perp$ that cancels the component set up by an element diametrically opposite it.



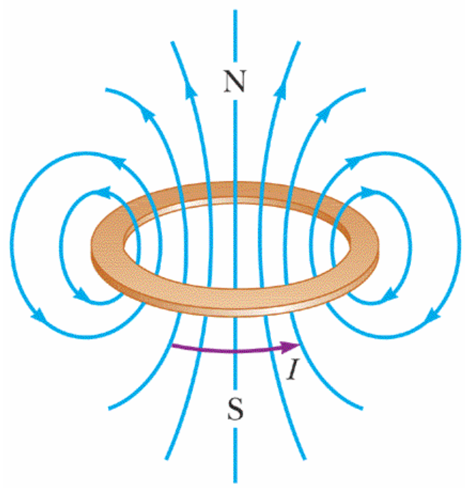
$$dB = \frac{\mu_0 I}{4\pi}\frac{|d\vec{s}\times\hat{r}|}{r^2} = \frac{\mu_0 I}{4\pi}\frac{ds}{r^2},\qquad dB_x = dB\cos\theta = \frac{\mu_0 I}{4\pi}\frac{Rds}{r^3}$$

$$B = \oint dB_x = \frac{\mu_0 I}{4\pi}\frac{R}{r^3}\oint ds = \frac{\mu_0 I}{4\pi}\frac{R}{\left(x^2+R^2\right)^{3/2}}(2\pi R) = \frac{\mu_0 I R^2}{2\left(x^2+R^2\right)^{3/2}}$$

At the center of the loop: $B = \dfrac{\mu_0 I}{2R}$ $(\text{at } x = 0)$

The direction is determined by the right-hand rule.

Example Cont'd



It is interesting to determine the behavior of the magnetic field far from the loop, x>>R

$$B = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}} \xrightarrow{x>>R} \frac{\mu_0 I R^2}{2x^3}$$

Consider the magnetic dipole moment of the loop $\mu = I(\pi R^2)$

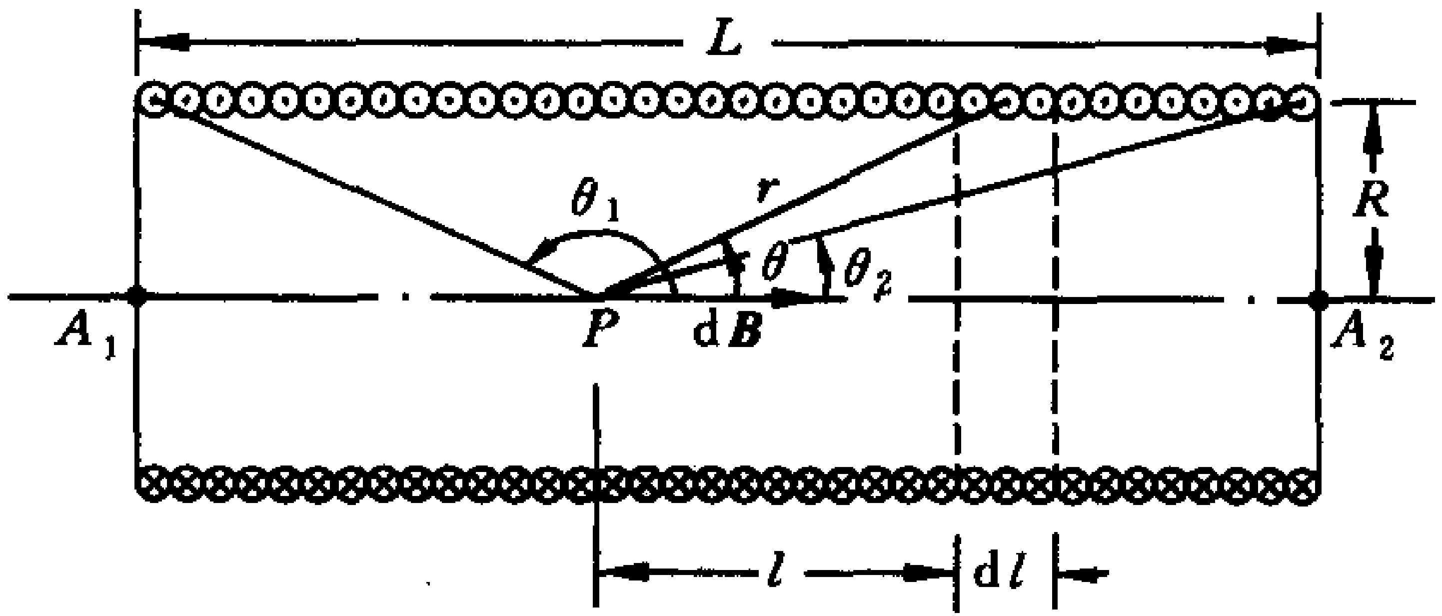
$$B = \frac{\mu_0}{2\pi}\frac{\mu}{x^3} \propto \frac{\mu}{x^3}$$

Compare the electric field due to a electric dipole: $E = \frac{1}{2\pi\varepsilon_0}\frac{p}{x^3} \propto \frac{p}{x^3}$

Example

Magnetic field on the axis of a solenoid

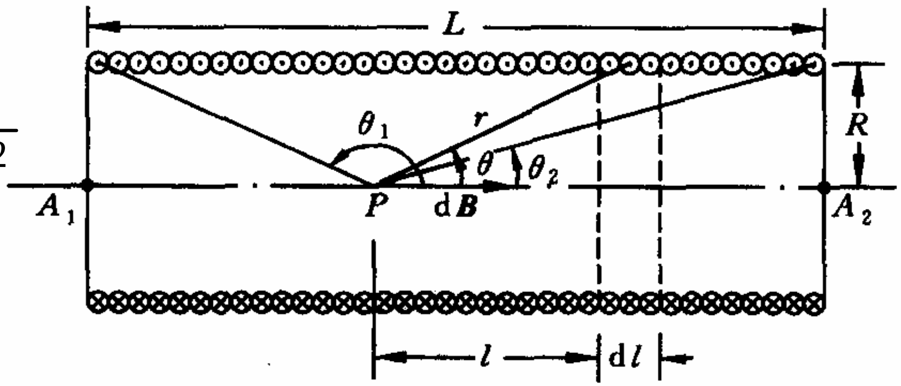
A solenoid is a helical winding of wire on a cylindrical core of radius R. The wire carries a current I. The number of the turns per unit length is $n = N/L$. Consider a point P on the central axis of the solenoid make the angles of θ_1 and θ_2 from axis up to the edges of two ends.



Magnetic field on the axis of a solenoid

Solution: Consider a thin ring of width dl. The number of turns in that ring is ndl, and so the total current carried by the ring is $nIdl$. The field at *P* due to this ring is:

$$dB = \frac{\mu_0 R^2 dI'}{2\left(l^2+R^2\right)^{3/2}} = \frac{\mu_0 R^2 (nIdl)}{2\left(l^2+R^2\right)^{3/2}}$$



Express the l in terms of θ:

$$l = R\cot\theta, \quad dl = -R\csc^2\theta d\theta, \quad l^2+R^2 = R^2\csc^2\theta$$

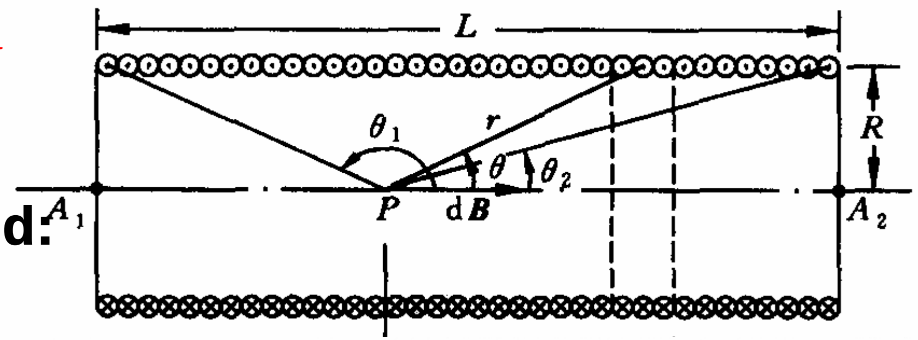
$$B = \frac{\mu_0 nI}{2}\int_{\theta_1}^{\theta_2} \frac{R^2(-R\csc^2)d\theta}{R^3\csc^3\theta} = -\frac{\mu_0 nI}{2}\int_{\theta_1}^{\theta_2}\sin\theta d\theta = \frac{\mu_0 nI}{2}(\cos\theta_2 - \cos\theta_1)$$

The direction of the field is determined using right-hand rule.

Example Cont'd

$$\boxed{B = \frac{\mu_0 n I}{2}(\cos\theta_2 - \cos\theta_1)}$$

For an ideal solenoid, whose length is very long, L>>R



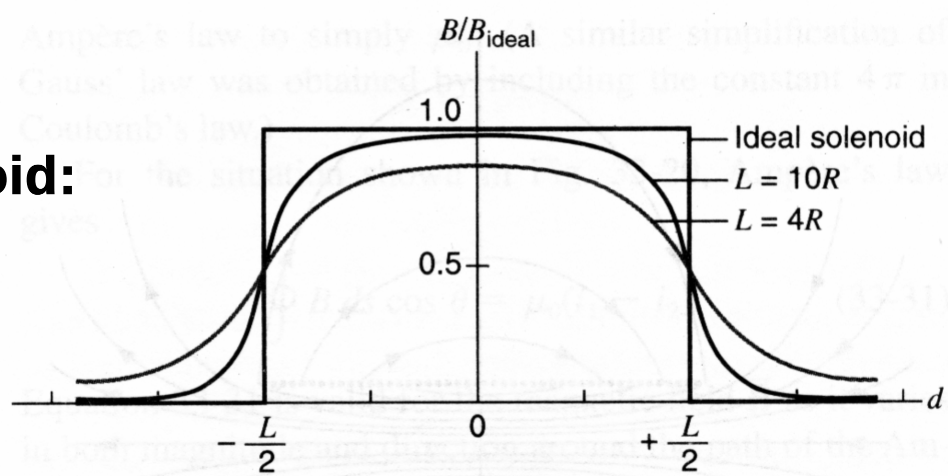
$$\theta_2 \to 0, \quad \theta_1 \to \pi, \quad B \xrightarrow{L>>R} \mu_0 n I$$

At the end at point A_1 of the solenoid:

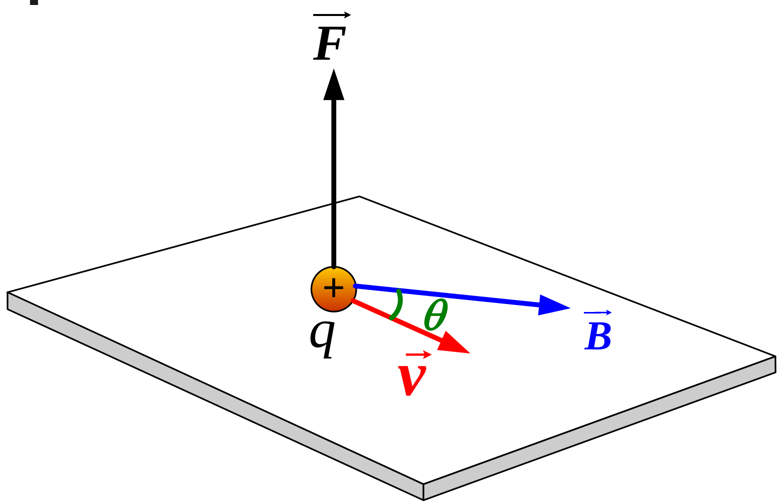
$$\theta_2 \to 0, \quad \theta_1 \to \frac{\pi}{2}, \quad B = \frac{1}{2}\mu_0 n I$$

At the end at point A_2 of the solenoid:

$$\theta_2 \to \frac{\pi}{2}, \quad \theta_1 \to \pi, \quad B = \frac{1}{2}\mu_0 n I$$



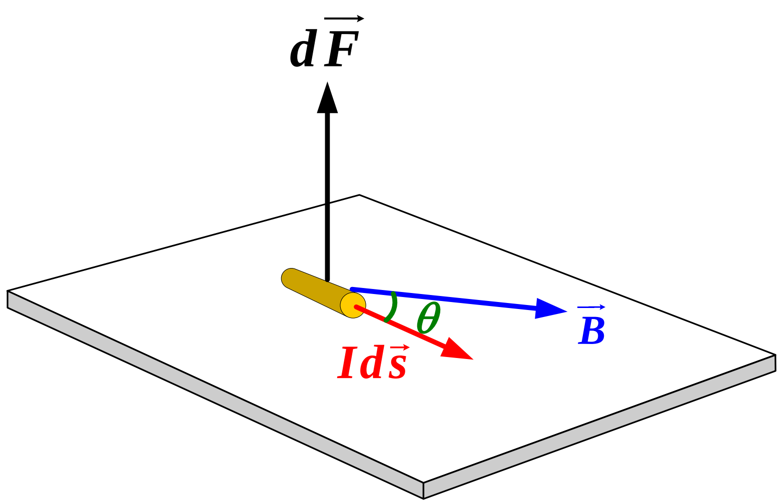
§ 8 Magnetic Field of a Moving Charge



Magnetic force on a moving charge

$$\boxed{\vec{F} = q\vec{v} \times \vec{B}}$$

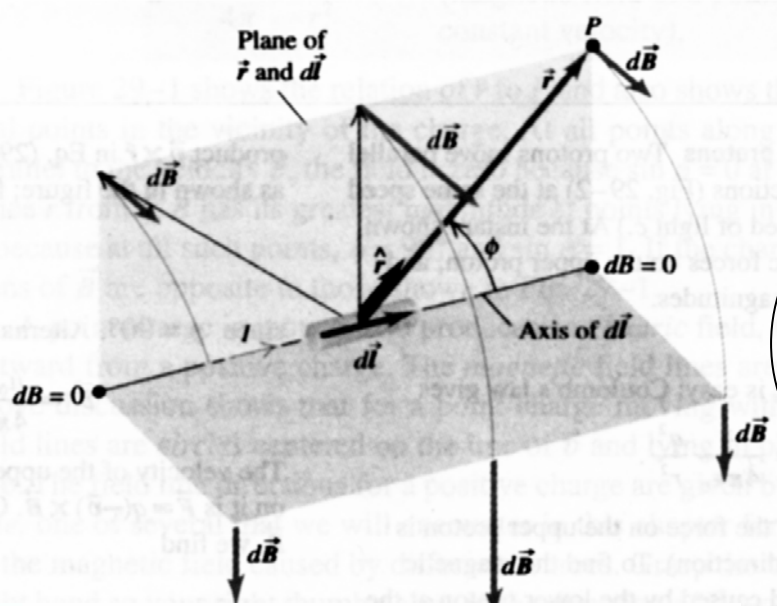
$$q\vec{v} \;\rightarrow\; I\,d\,\vec{s}$$

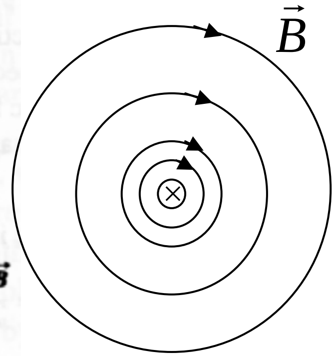


Magnetic force on a current element

$$\boxed{d\vec{F} = Id\vec{s} \times \vec{B}}$$

Magnetic field of a moving charge

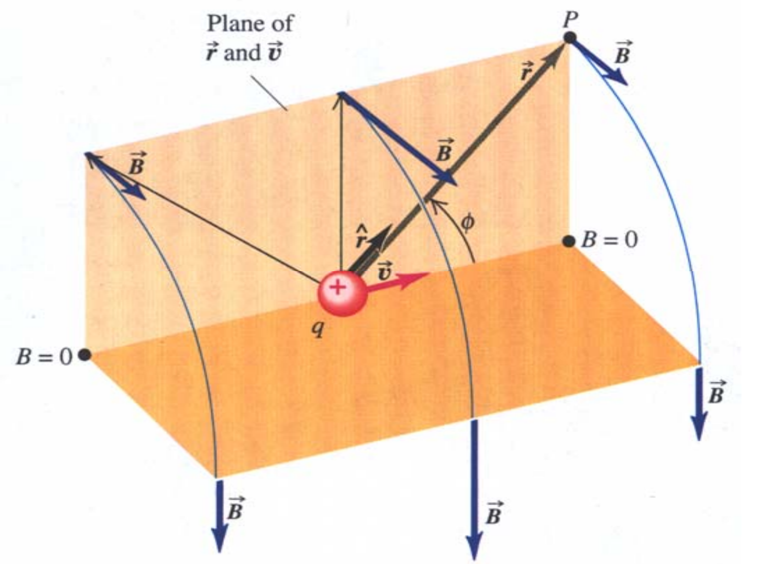


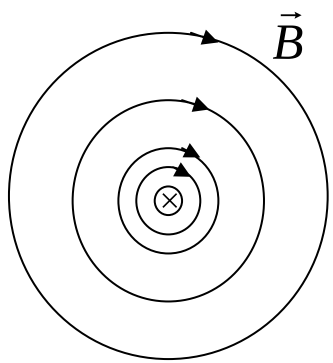


Magnetic field of a current element

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{s}\times\hat{r}}{r^2}$$

$$Id\vec{s} \rightarrow q\vec{v}$$

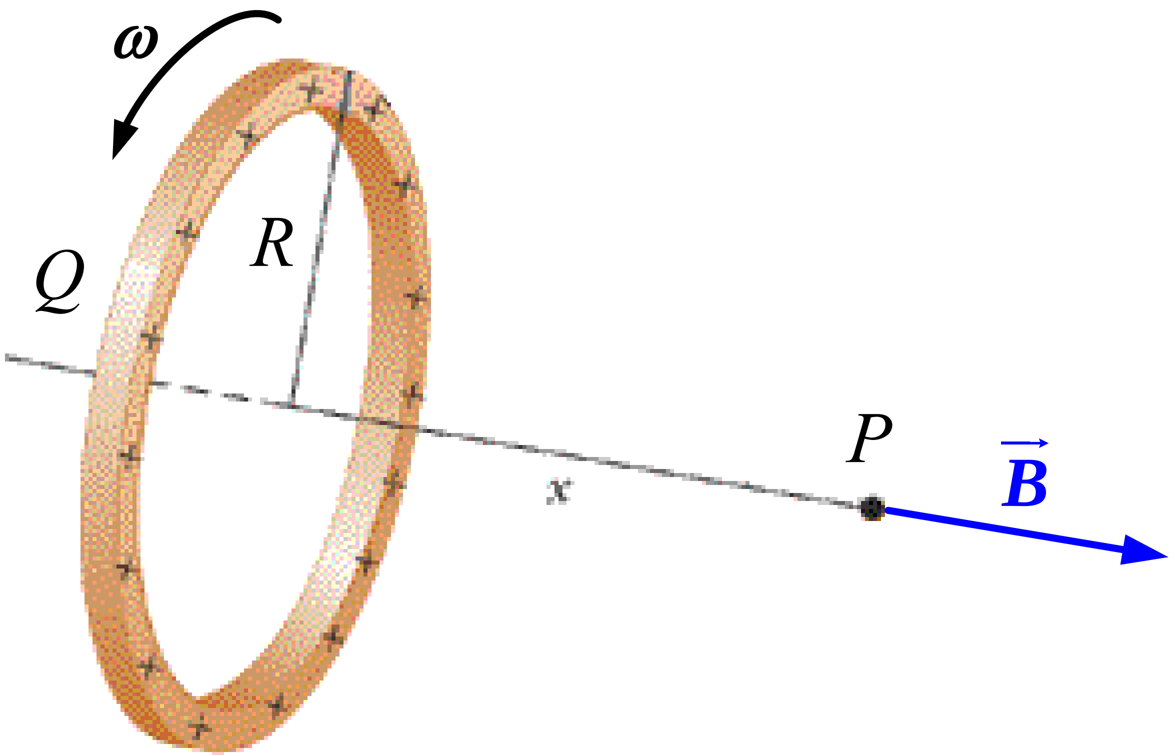




Magnetic field of a moving charge

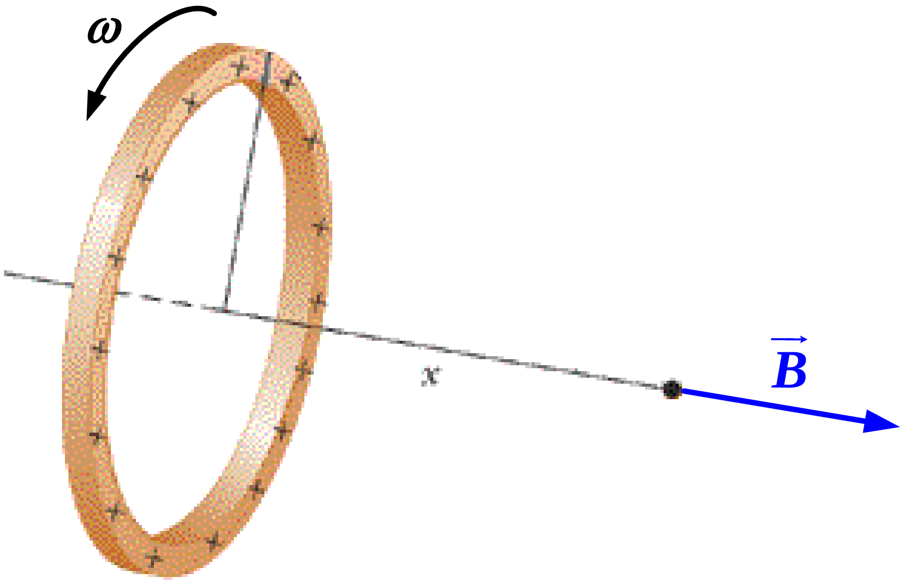
$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

A ring of radius R has a uniform positive charge distribution, with a total charge Q. Now the ring rotates anti-clockwise with ω about its central axis. Calculate the magnetic field at the point P located on the axis a distance x from the center of the ring.



Solution I: **The a rotating charge ring is equivalent to a circular current loop. For a circular current loop:**

$$B = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}$$

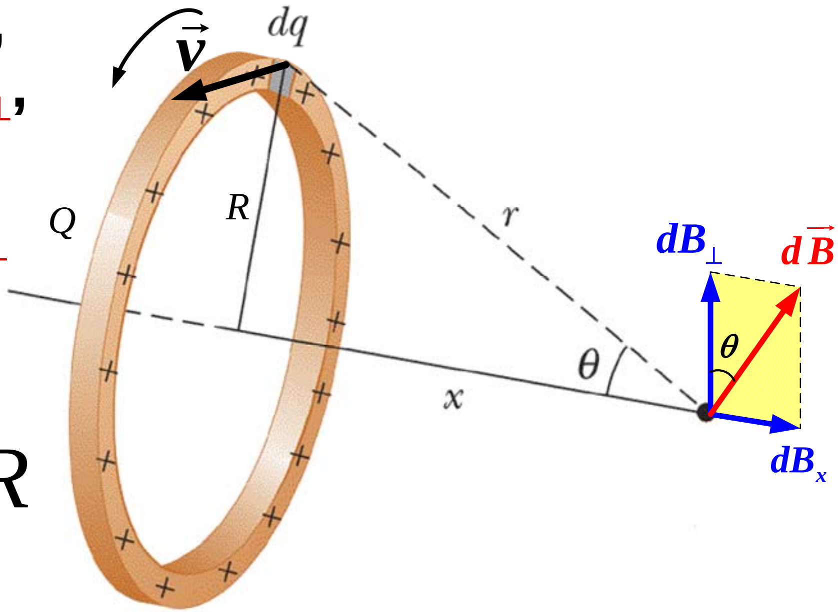


where $$I = \frac{Q}{T} = \frac{Q\omega}{2\pi}$$

So $$B = \frac{\mu_0 Q\omega R^2}{4\pi\left(x^2 + R^2\right)^{3/2}}$$

$$\mu = IA = \frac{Q\omega}{2\pi}\pi R^2 = \frac{Q\omega R^2}{2}, \quad B = \frac{\mu_0}{2\pi}\frac{\mu}{\left(x^2 + R^2\right)^{3/2}}$$

Solution II: Dividing the ring into small segment of charge dq.
$d\vec{B}$ **is the field due to the charge dq, which can be resolved into a component dB_x, along the x axis, and a component $dB_\perp$, which is perpendicular to the x axis.**



By symmetry, the vector sum of all $dB_\perp$ vanishes. The total field is only contributed by the sum of dB_x.

$$dB = \frac{\mu_0 dq}{4\pi}\frac{|\vec{v}\times\hat{r}|}{r^2} = \frac{\mu_0 v}{4\pi}\frac{dq}{r^2}, \quad v = \omega R$$

$$dB_x = dB\sin\theta = \frac{\mu_0 \omega R}{4\pi}\frac{R}{r}\frac{dq}{r^2} = \frac{\mu_0 \omega}{4\pi}\frac{R^2 dq}{r^3}$$

$$B = \oint dB_x = \frac{\mu_0 \omega}{4\pi}\frac{R^2}{r^3}\oint dq = \frac{\mu_0 \omega Q}{4\pi}\frac{R^2}{\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0}{2\pi}\frac{\mu}{\left(x^2+R^2\right)^{3/2}}$$

Problems

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§ 9 Ampère's Law

- **Review: Calculation of electric field.**
  - **Find the total electric field by summing all the $d\vec{E}$**
  - **For a symmetric charge distribution, it is often easier to use Gauss's law to find E.**

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$$

$$\oiint_S \vec{E}\cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0}$$

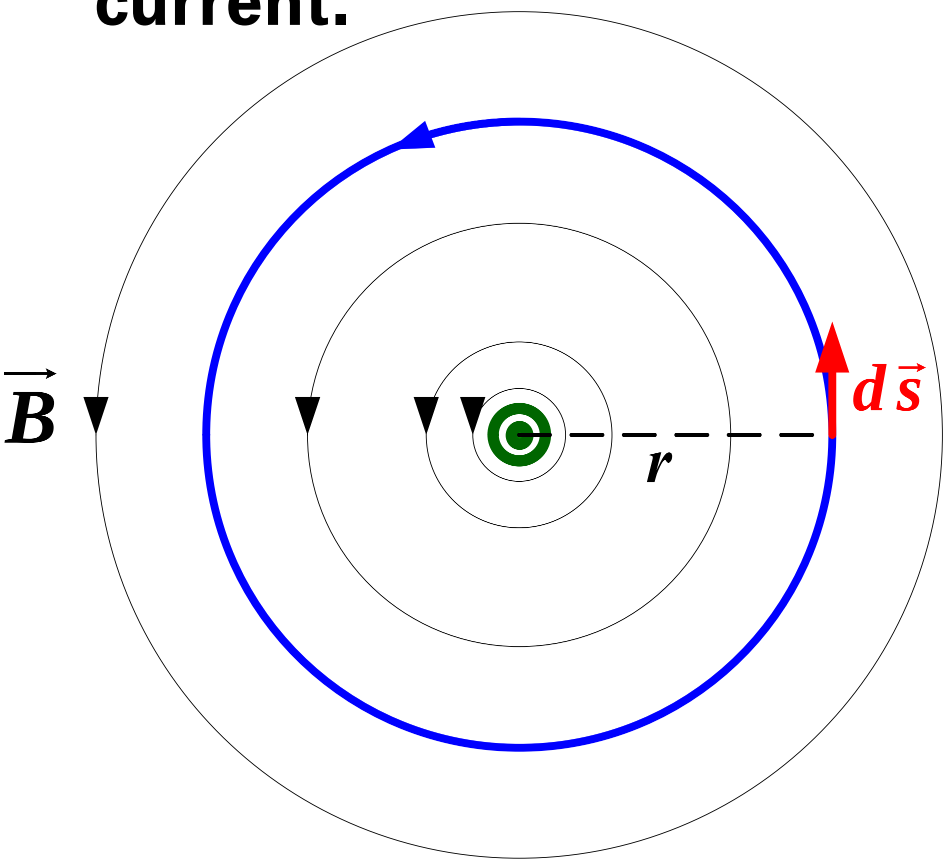
- **Calculation of magnetic field:**
  - **Find the total magnetic field by summing all the**

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{s}\times\hat{r}}{r^2}$$

  - **Gauss' law for magnetism can't be used to determine the magnetic field produced by a particular current distribution. Is there a law which plays the similar role in magnetism as Gauss's law in electrics?**

Ampère's Law

- **The line integral around a loop near a long, straight current-carrying wire.**
  - **The circle loop is centered on the wire, the direction of loop is right-hand related to the direction of the current.**



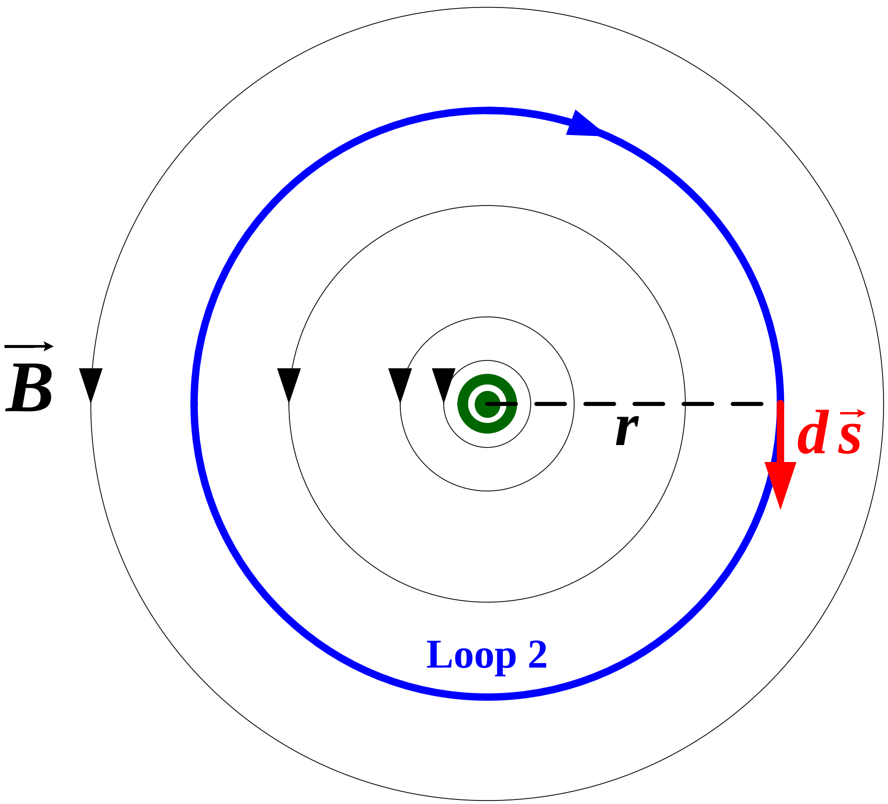
$$\oint_{L_1} \vec{B}\cdot d\vec{s} = \oint_{L_1} B ds$$

$$= B\oint_{L_1} ds$$

$$= \frac{\mu_0 I}{2\pi r_1}(2\pi r_1)$$

$$= \mu_0 I$$

- **The same circle loop but in opposite direction.**

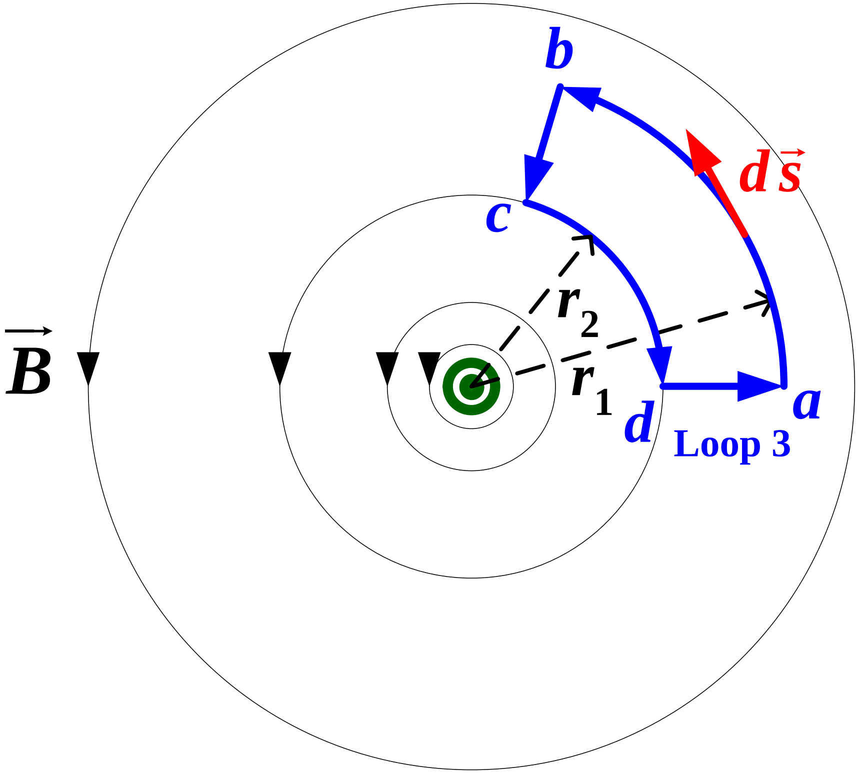


$$\oint_{L_2} \vec{B}\cdot d\vec{s} = -B\oint_{L_2} ds$$

$$= -\frac{\mu_0 I}{2\pi r_1}(2\pi r_1)$$

$$= -\mu_0 I$$

- **An integration loop does not enclose the wire.**

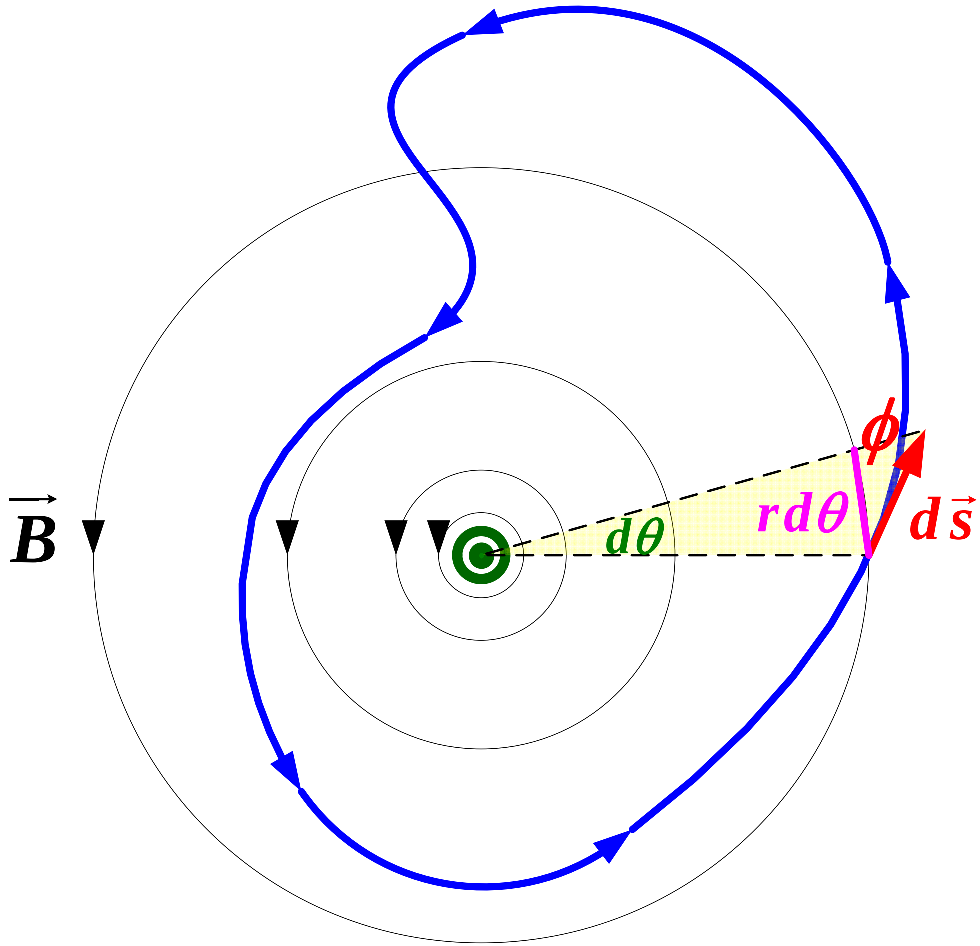


$$\vec{B}_1 \cdot d\vec{s}_1 = \frac{\mu_0 I}{2\pi r_1}(r_1 d\theta) = \frac{\mu_0 I}{2\pi} d\theta$$

$$\vec{B}_2 \cdot d\vec{s}_2 = -\frac{\mu_0 I}{2\pi r_2}(r_2 d\theta) = -\frac{\mu_0 I}{2\pi} d\theta$$

$$\oint_{L_3} \vec{B} \cdot d\vec{s} = \int_a^b B_1 ds + \int_b^c B ds \cos\frac{\pi}{2} + \int_c^d (-B_2) ds + \int_d^a B ds \cos\frac{\pi}{2}$$
$$= 0$$

- A more general loop that **encloses** the wire.



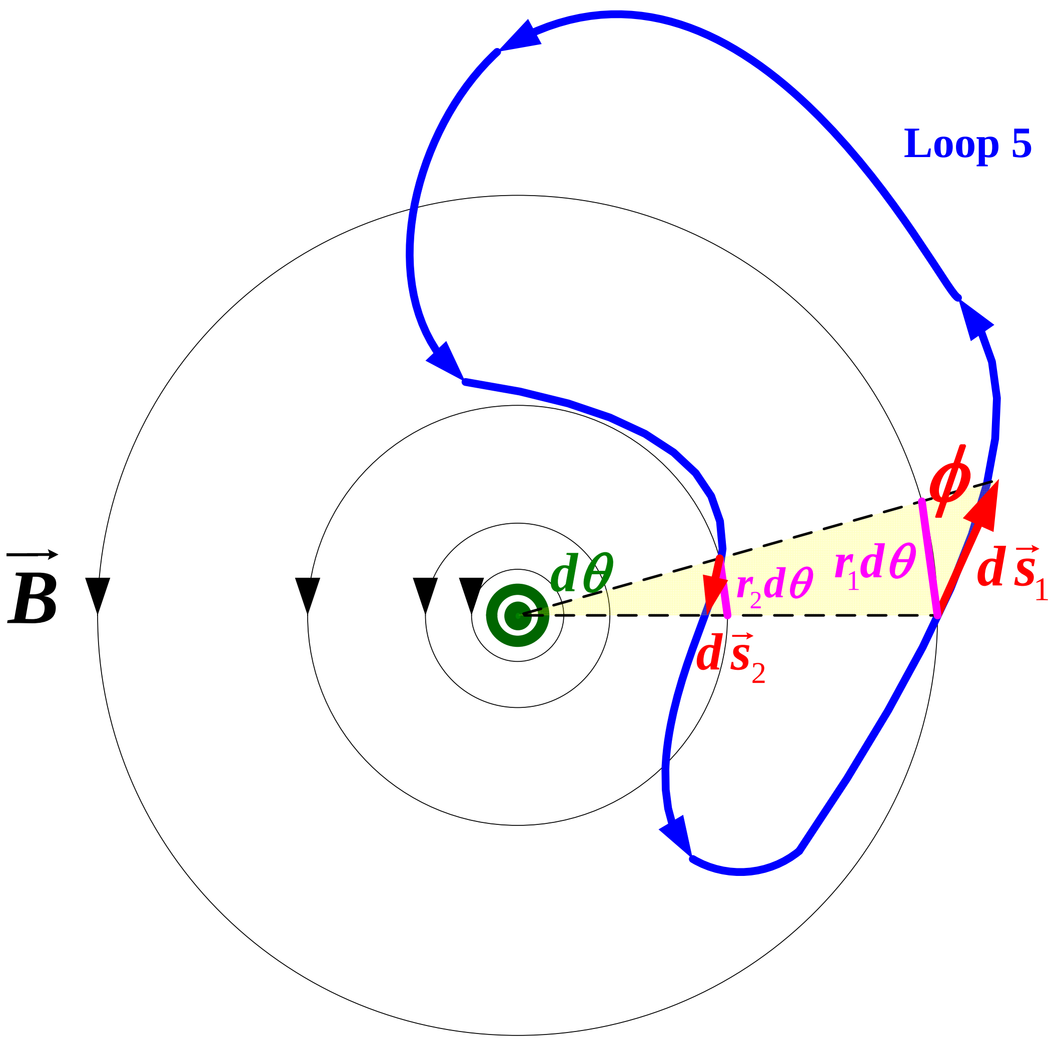
$$ds\cos\phi = r\,d\theta$$

$$\oint_{L_4} \vec{B}\cdot d\vec{s} = \oint_{L_4} B\,ds\cos\phi$$

$$= \oint_{L_4} \frac{\mu_0 I}{2\pi r}(r\,d\theta)$$

$$= \mu_0 I$$

- A more general loop that does not enclose the wire.



$$\vec{B}_1 \cdot d\vec{s}_1 = B_1 ds_1 \cos\phi_1$$

$$= \frac{\mu_0 I}{2\pi r_1}(r_1 d\theta)$$

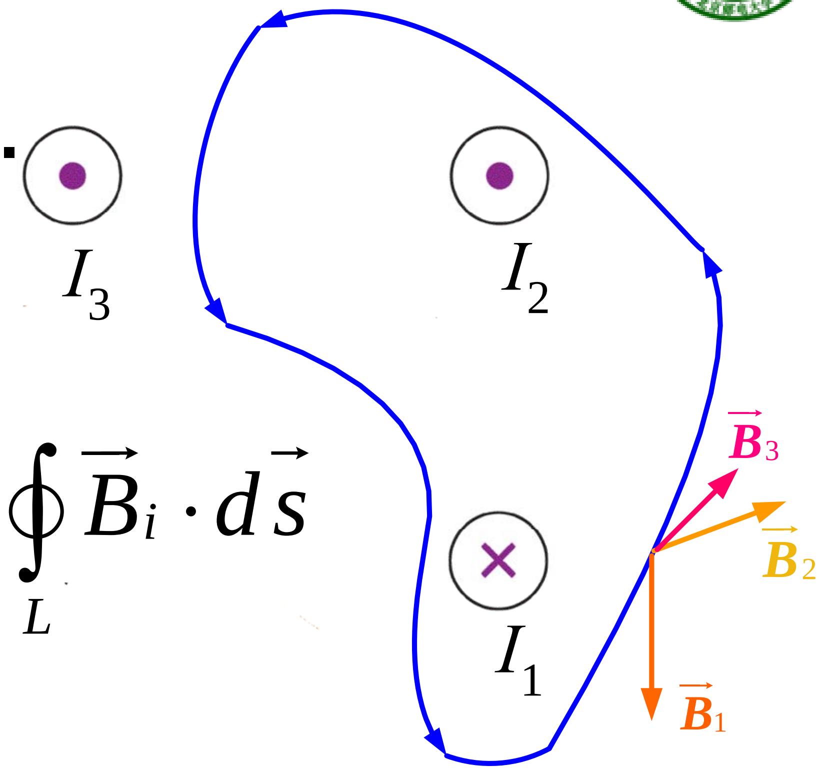
$$= \frac{\mu_0 I}{2\pi r_2}(r_2 d\theta)$$

$$= -\vec{B}_2 \cdot d\vec{s}_2$$

$$\oint_{L_5} \vec{B} \cdot d\vec{s} = 0$$

Ampère's Law

- **Ampère's Law**
  - **For any loop with any shape.**



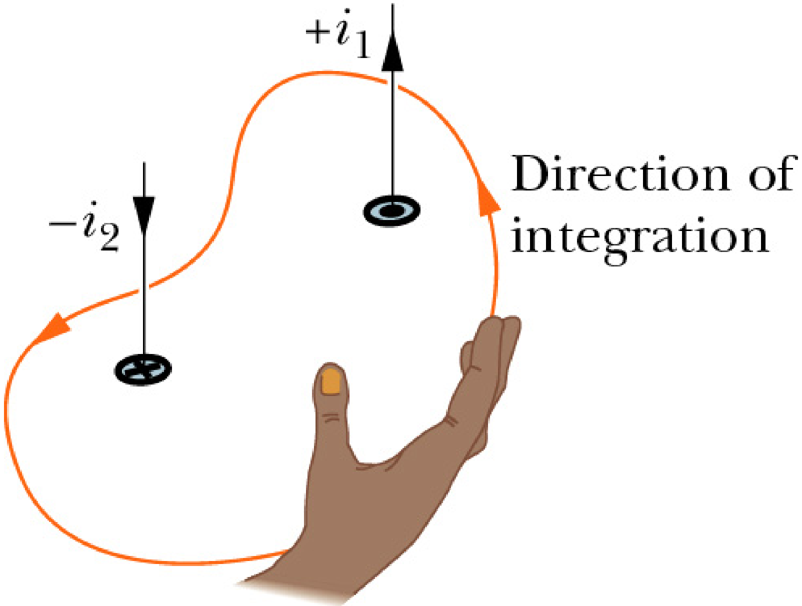
$$\vec{B} = \sum_i \vec{B}_i$$

$$\oint_L \vec{B} \cdot d\vec{s} = \oint_L \sum_i \vec{B}_i \cdot d\vec{s} = \sum_i \oint_L \vec{B}_i \cdot d\vec{s}$$

$$\oint_L \vec{B}_i \cdot d\vec{s} = \begin{cases} \mu_0 I & I \text{ within the loop, right-hand rule direction} \\ -\mu_0 I & I \text{ within the loop, left-hand rule direction} \\ 0 & I \text{ not within the loop} \end{cases}$$

Ampère's Law:

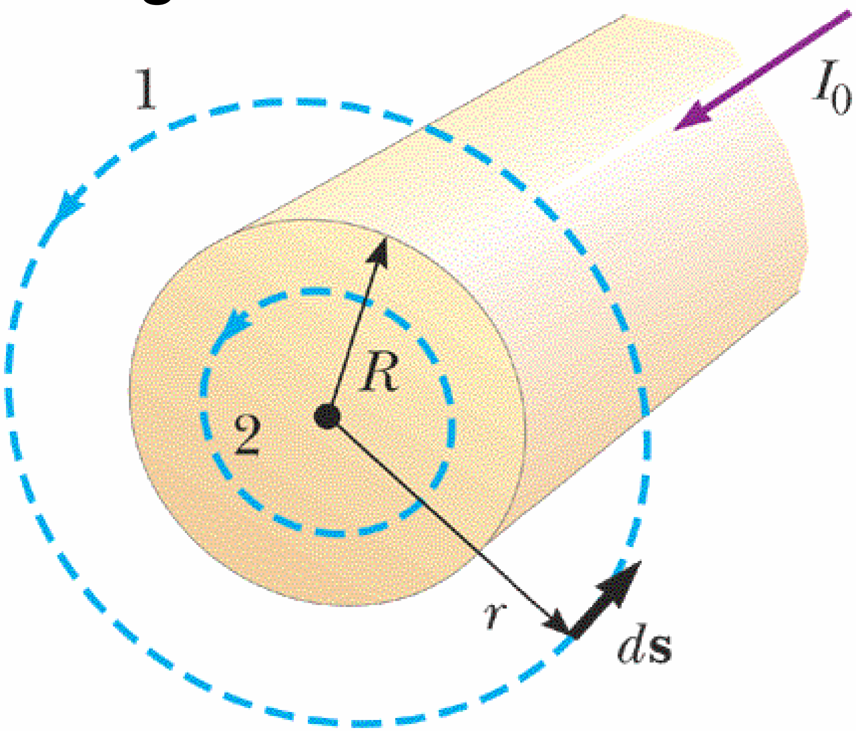
$$\boxed{\oint_L \vec{B} \cdot d\vec{s} = \mu_0 I_{\mathrm{encl}}}$$



The line integral of magnetic field along a loop equals μ_0 times the algebra sum of the currents enclosed or linked by the loop.

Example: The magnetic field created by a long, straight cylindrical wire

A long, straight cylindrical wire of radius R carries a steady current I_0 that is uniformly distributed through the cross-section of the wire. Calculate the magnetic field a distance r from the center of the wire in the regions $r \geq R$ and $r < R$.

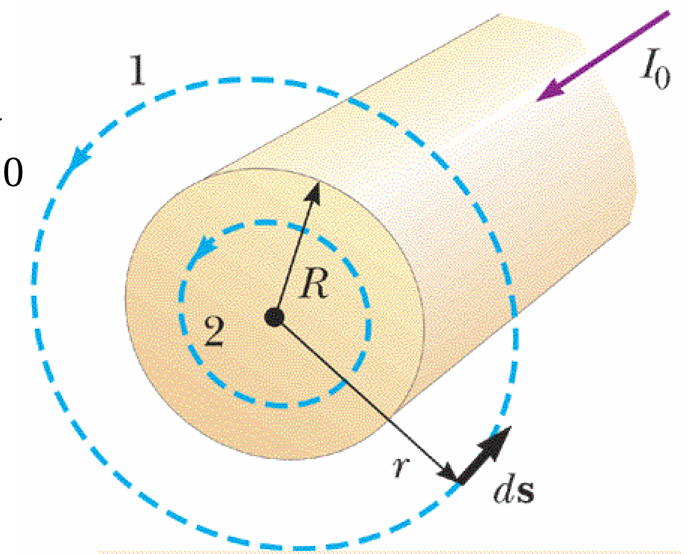


The magnetic field created by a long, straight cylindrical wire

Solution: **For $r \geq R$, we choose loop 1, a circle of radius r centered at wire.**

$$\oint_1 \vec{B}\cdot d\vec{s} = \oint_1 B ds = B\oint_1 ds = B(2\pi r) = \mu_0 I_0$$

$$B = \frac{\mu_0 I_0}{2\pi r} \qquad (\text{for } r \geq R)$$

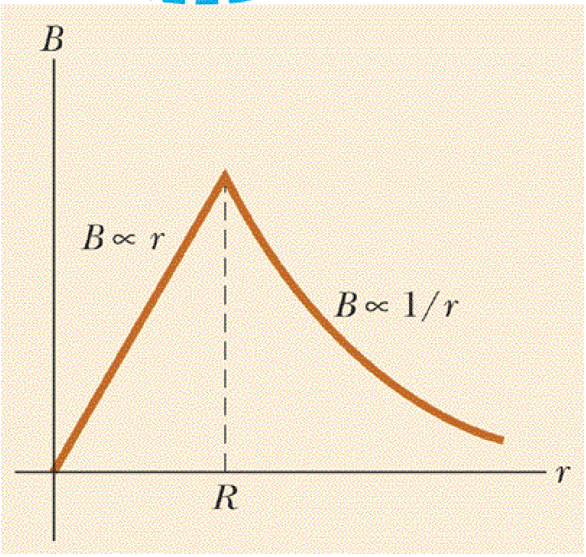


For $r < R$, we choose circular loop 2.

$$I_{encl} = \frac{r^2}{R^2} I_0$$

$$\oint_2 \vec{B}\cdot d\vec{s} = B(2\pi r) = \mu_0 I_{encl} = \mu_0\left(\frac{r^2}{R^2} I_0\right)$$

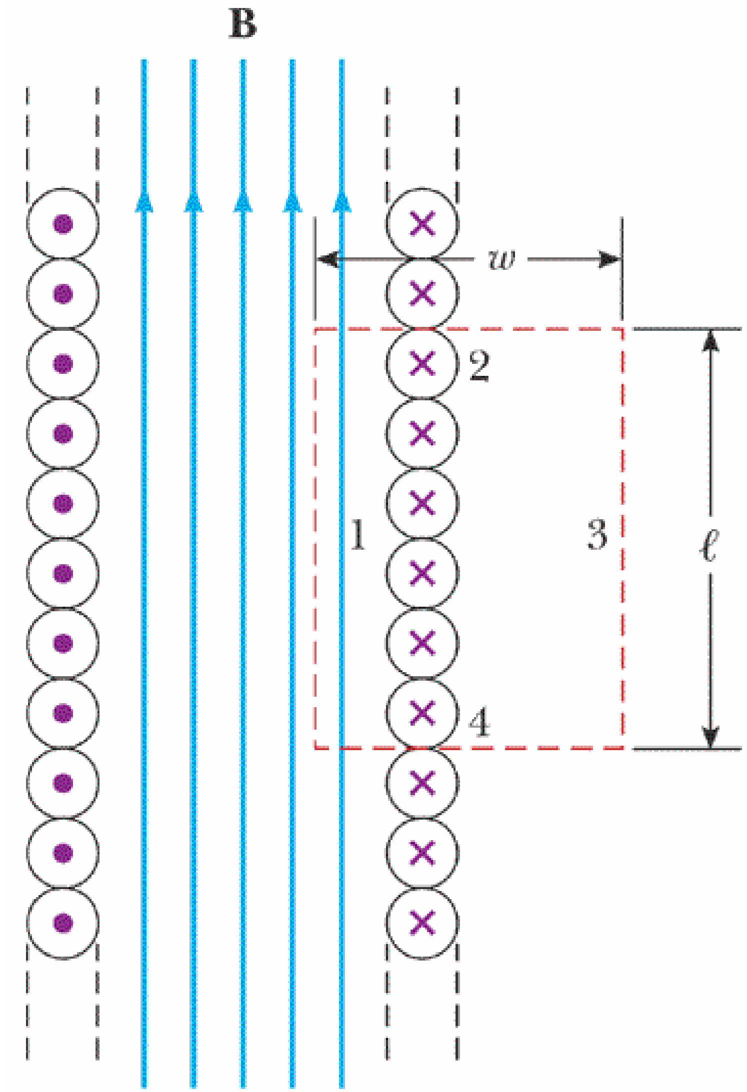
$$B = \frac{\mu_0 I_0}{2\pi R^2} r \qquad (\text{for } r < R)$$



Example

The magnetic field created by a solenoid

A **ideal** solenoid: its turns are closely spaced and its length is large compared with its radius. For an ideal solenoid, the field outside the solenoid is zero, and the field inside is uniform. Calculate the field inside an ideal solenoid carrying a current I. The number of turns per unit length is n.



The magnetic field created by a solenoid

Solution: **Choose a rectangular loop of length *l* and width *w*.**

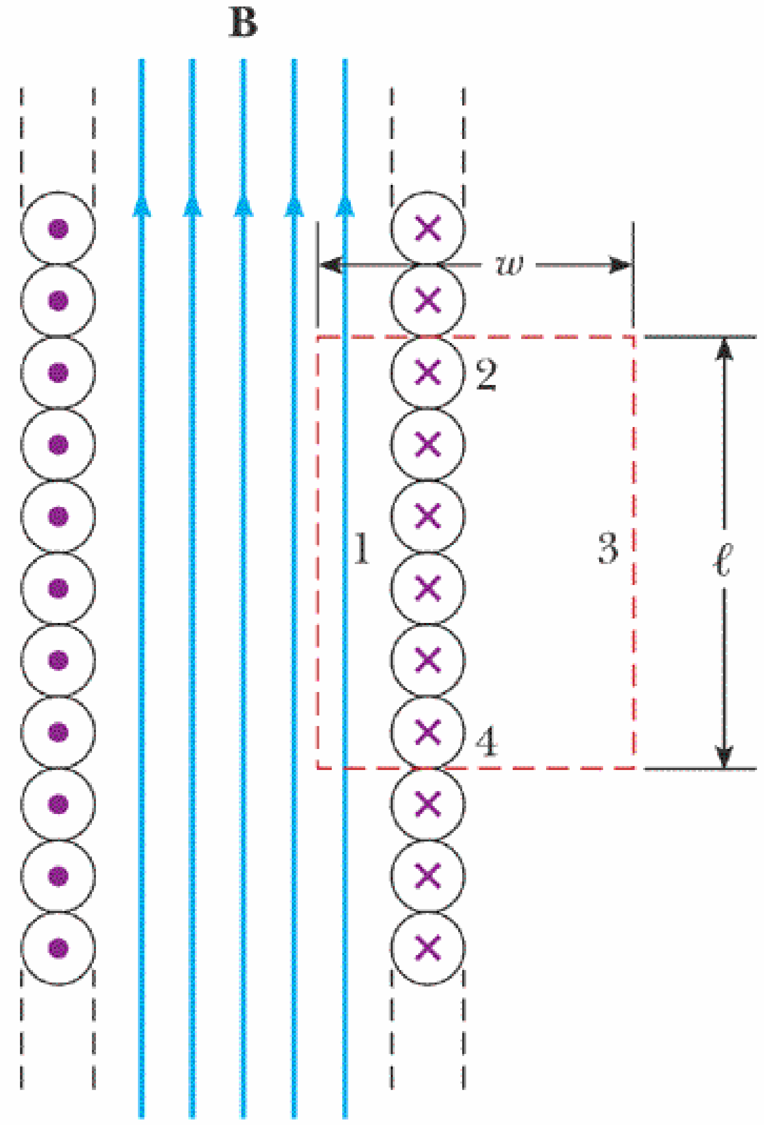
$$\oint_L \vec{B}\cdot d\vec{s} = \int_1 \vec{B}\cdot d\vec{s} + \int_2 \vec{B}\cdot d\vec{s}$$

$$+\int_3 \vec{B}\cdot d\vec{s} + \int_4 \vec{B}\cdot d\vec{s}$$

$$= \int_1 \vec{B}\cdot d\vec{s} = B\int_1 ds = Bl$$

$$= \mu_0 NI$$

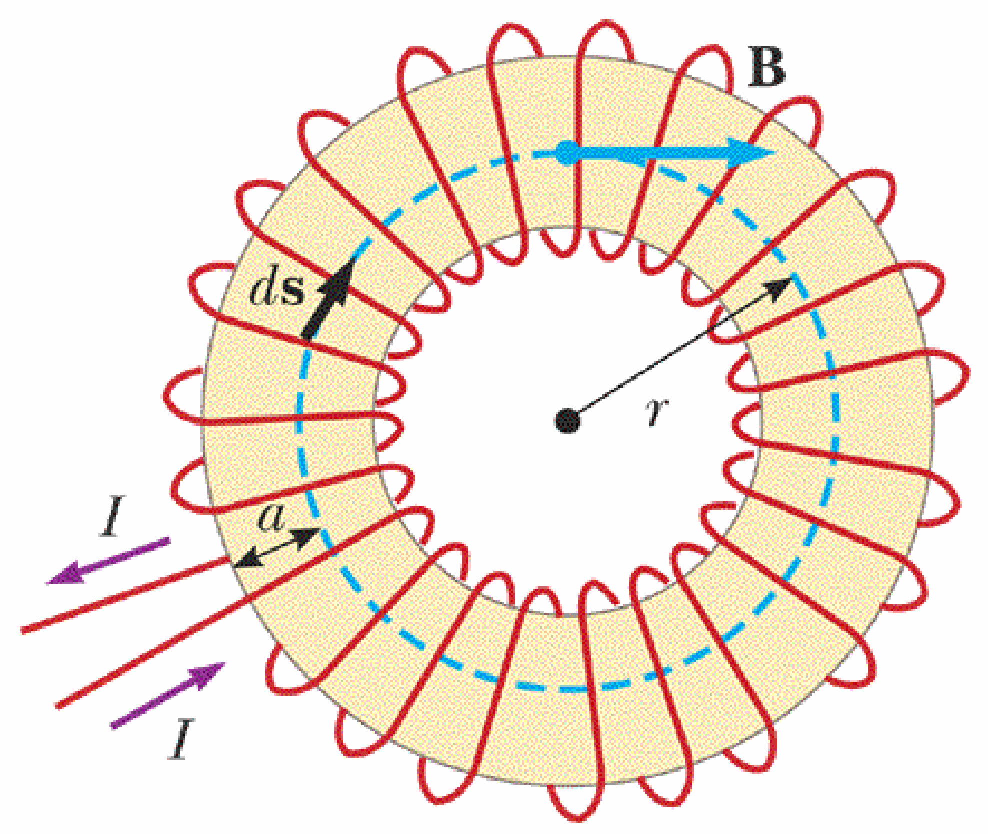
$$B = \mu_0 \frac{N}{l} I = \mu_0 nI$$



Example

The magnetic field created by a toroid solenoid (螺绕环)

A toroid has N closely spaced turns of wire carrying a current I. Calculate the magnetic field in the region occupied by the torus (圆环体), a distance r from the center.



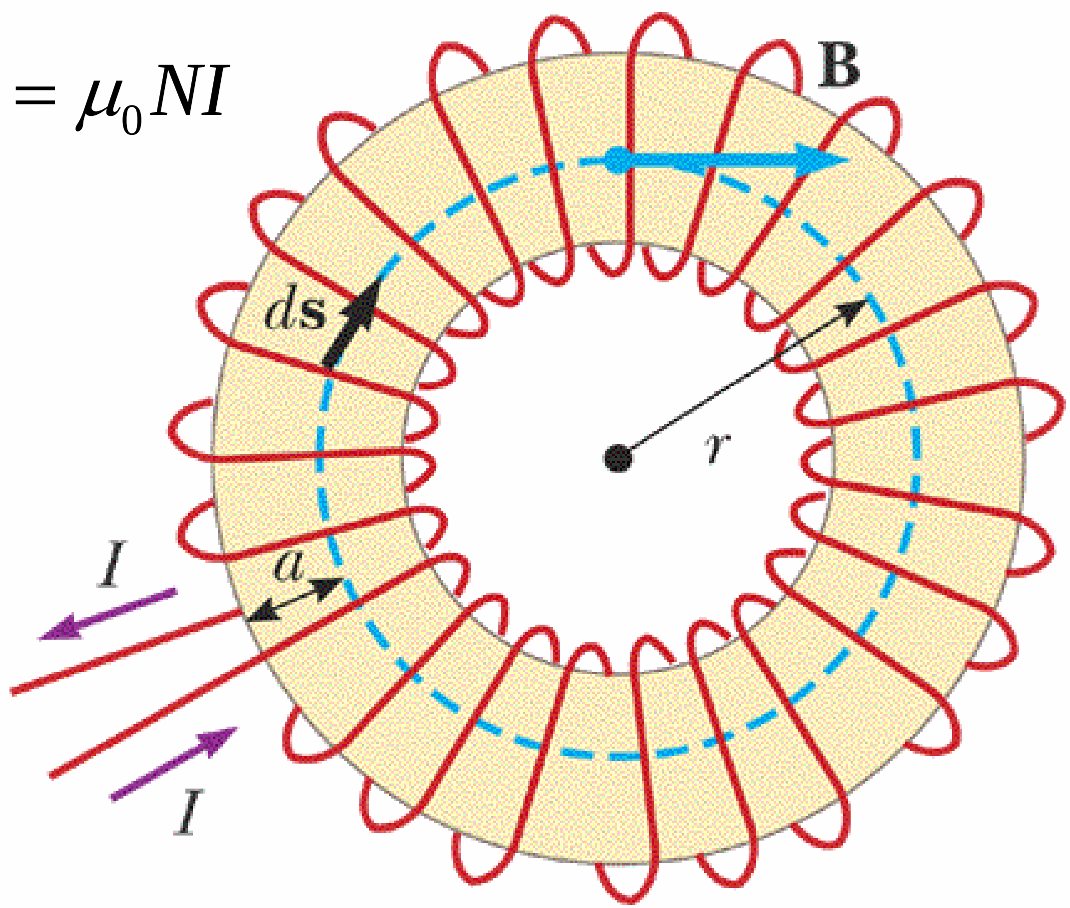
Solution: **Choose a circular loop of radius of r.**

$$\oint_L \vec{B}\cdot d\vec{s} = B\oint_L ds = B(2\pi r) = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi r}$$

$$\xrightarrow{a<<r}$$

$$\frac{\mu_0 NI}{2\pi r_{mid}} = \mu_0 nI$$



Problems

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