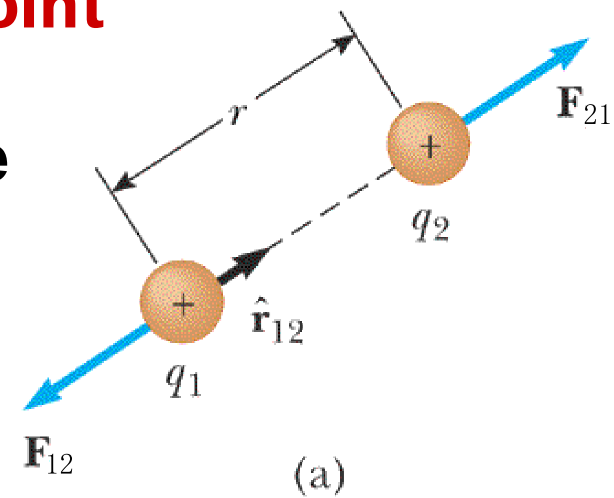
review

- Electric charge
  - positive, negative
  - Quantized: $e = 1.6 \times 10^{-19}$C
- Unlike charges attract; like charges repel.
- Law of conservation of electric charge
  - The net amount of electric charge produced in any process is zero.

§ 1 Coulomb's Law

- The electrostatic force exerted by **point charge** q_1 on q_2, written F_{12}, can be expressed in vector form as (inverse square law):

$$\boxed{\vec{F}_{21} = k_e \frac{q_1 q_2}{r^2}\hat{r}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}}$$

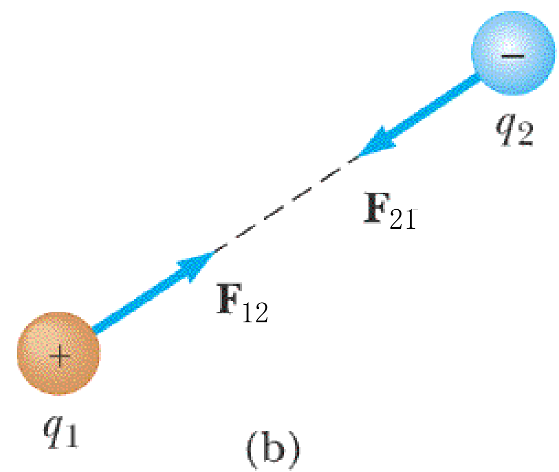


$$k_e = 8.99\times 10^9\,\mathrm{N\cdot m^2/C^2}$$

—— Coulomb constant

$$\varepsilon_0 = 8.8542\times 10^{-12}\,\mathrm{C^2/N\cdot m^2}$$

—— permittivity of free space (electric constant)

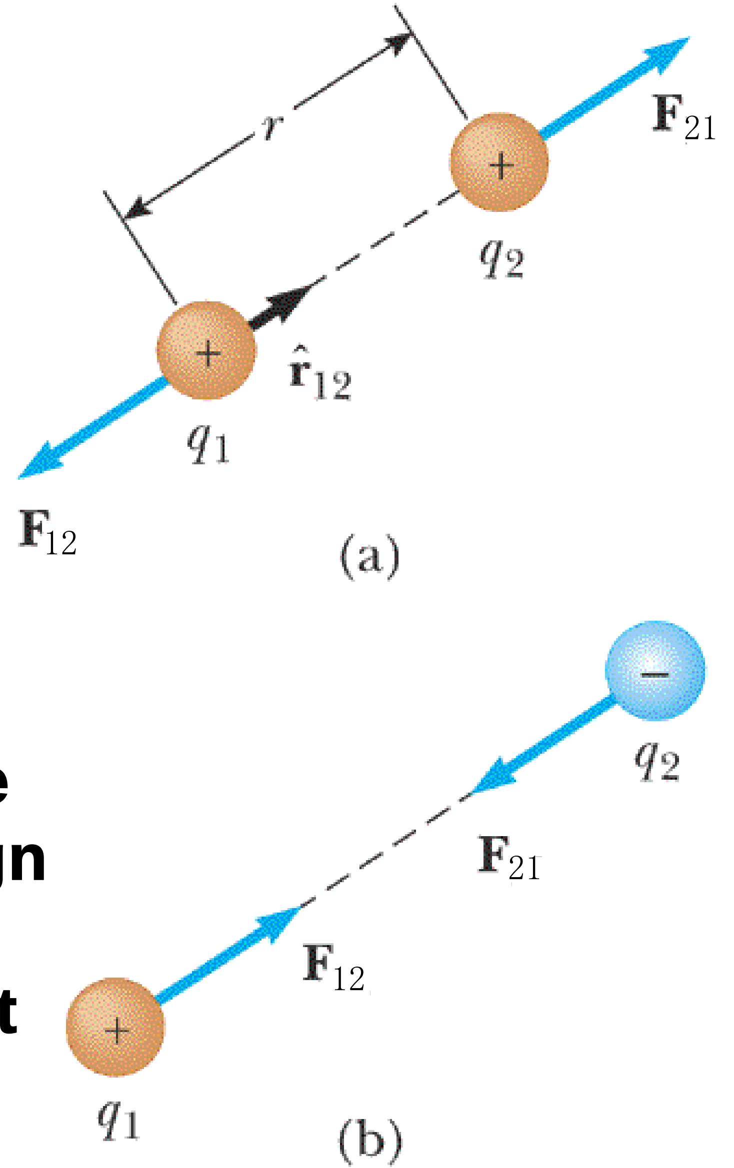


$\hat{r}_{12}$ is a unit vector directed from q_1 toward q_2

$$\boxed{\vec{F}_{21} = k_e \frac{q_1 q_2}{r^2}\hat{r}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}}$$

$$k_e = 8.99\times10^9\,\mathrm{N\cdot m^2/C^2}$$

$$\varepsilon_0 = 8.8542\times10^{-12}\,\mathrm{C^2/N\cdot m^2}$$



- **From Newton's third law**

$$\vec{F}_{21} = -\vec{F}_{12}$$

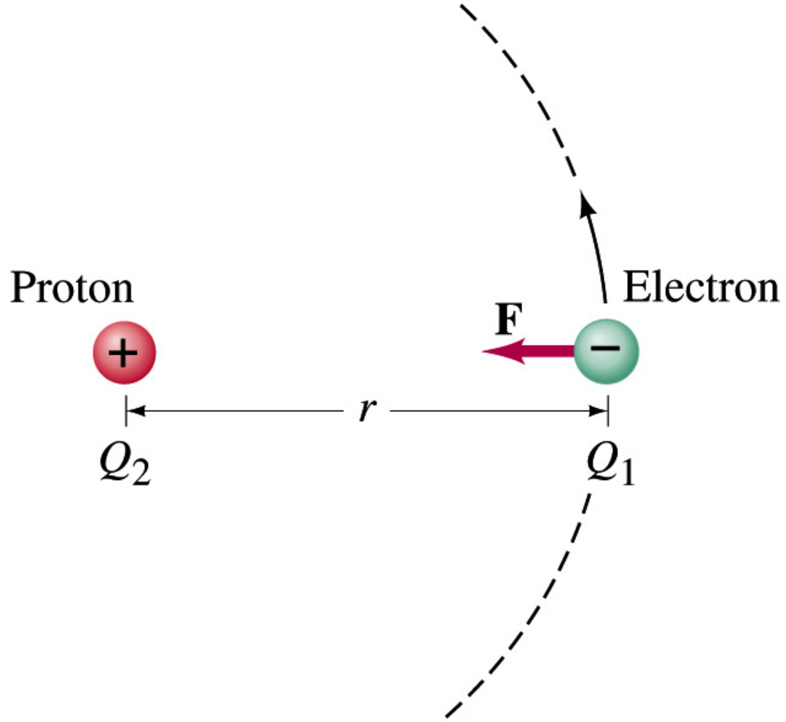
- **The Coulomb forces between the two charges having the same sign are repulsive, while the two charges with opposite sign result in attractive Coulomb forces.**

Coulomb's Law
vs. Newton's law of gravitation

$$\vec{F}_{21} = k_e \frac{q_1 q_2}{r^2} \hat{r}_{12}, \qquad \vec{F}_{21} = -G \frac{m_1 m_2}{r^2} \hat{r}_{12}$$

- **Both are inverse square laws, and charge q plays the same role in Coulomb's law as that the mass m plays in Newton's law of gravitation.**
- **One difference between the two laws is that gravitational forces are always attractive, whereas electrostatic forces can be either repulsive or attractive.**

Ex.19-1 (P462): the two forces in the hydrogen atom



$$r = 5.3\times10^{-11}\,\text{m}$$

Particle	Charge (C)	Mass (kg)
Electron (e)	$-1.602\,191\,7 \times 10^{-19}$	$9.109\,5 \times 10^{-31}$
Proton (p)	$+1.602\,191\,7 \times 10^{-19}$	$1.672\,61 \times 10^{-27}$
Neutron (n)	0	$1.674\,92 \times 10^{-27}$

the two forces in the hydrogen atom — the distance between the electron and proton: 5.3×10^{-11}m.

The electrostatic force:

$$F_e = k_e\frac{e^2}{r^2} = (8.99\times10^9\,\mathrm{N\cdot m^2/C^2})\frac{(1.60\times10^{-19}\,\mathrm{C})^2}{(5.3\times10^{-11}\,\mathrm{m})^2} = 8.2\times10^{-8}\,\mathrm{N}$$

The gravitational force:

$$F_g = G\frac{m_e m_p}{r^2} = (6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2})\frac{(9.11\times10^{-31}\,\mathrm{kg})(1.67\times10^{-27}\,\mathrm{kg})}{(5.3\times10^{-11}\,\mathrm{m})^2} = 3.6\times10^{-47}\,\mathrm{N}$$

- **The gravitational force is weaker than the electrostatic force by factor of about 10^{-39}.**

§ 2 Electric fields

- **Does Coulomb’s law means that the interaction between separated charges is an action-at-a-distance? Is the interaction direct and instantaneous?**
  - **Historical view: The interaction model for charges is the action-at-a-distance**

    $$\text{charge} \rightleftarrows \text{charge}$$

  - **The real interaction model for charges——The interaction between two charges is realized through the electric fields established around the charges.**

    $$\text{charge} \rightleftarrows \text{field} \rightleftarrows \text{charge}$$

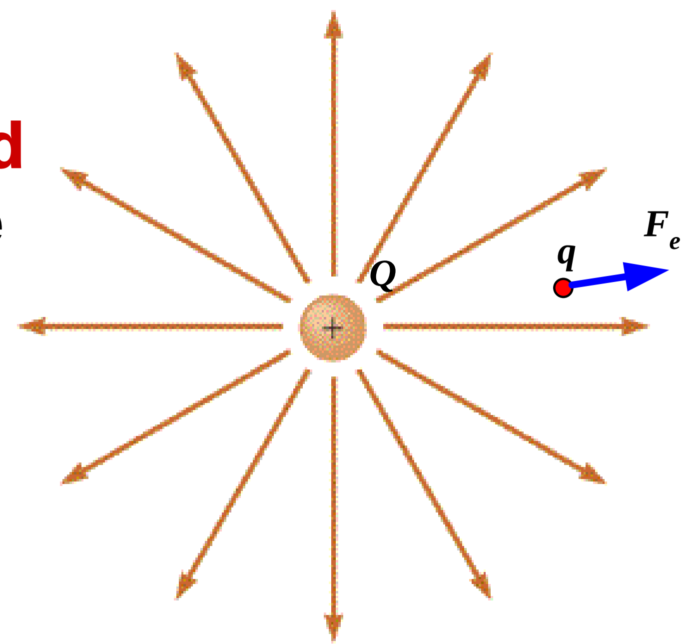
    The first charge sets up an electric field, and the second charge interacts with the electric field of the first charge.

The Electric Field

- The problem of determining the interaction between the charges is therefore reduced to two separate problem

charge ⇄ field ⇄ charge

  - Determine, by measurement or calculation, the **electric field** established by the first charge at every point in space.
  - Calculate the **force** that the field exerts on the second charge placed at a particular point in space.



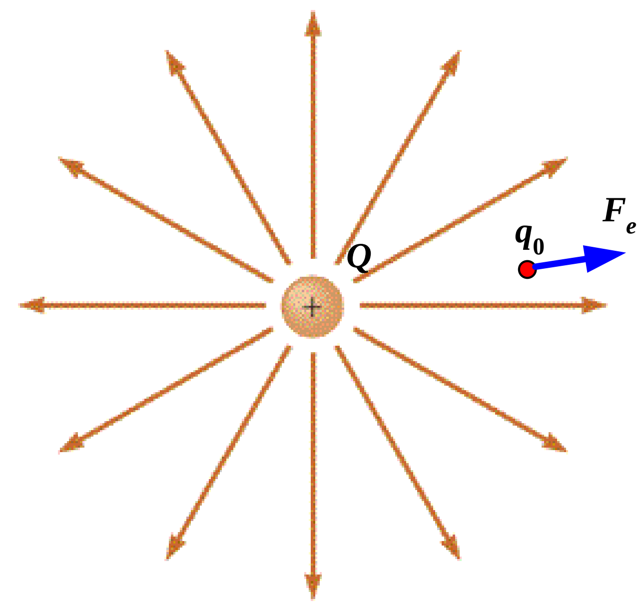
- The definition of electric field
  - The definition of the electric field $\vec{E}$ in terms of the electric force $\vec{F}_e$ exerted on a **positive** test charge q_0 placed at a particular point.

$$\boxed{\vec{E} \equiv \frac{\vec{F}_e}{q_0}}$$

SI unit: N/C or V/m

The direction of $\vec{E}$ is the same as the direction of $\vec{F}_e$.

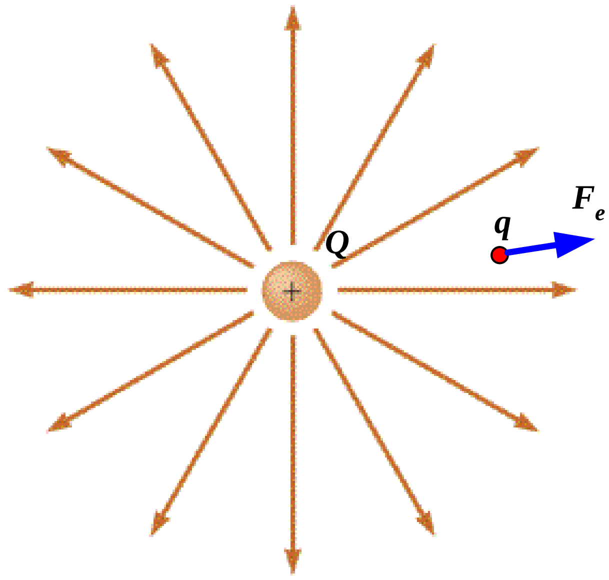
The test particle q_0 is used only to detect the existence of the field and evaluate its strength. The existence and strength of the electric field is feature of electric field itself, **not** dependent on the q_0.



- The electric force exerted on a charge.
  - Once the electric field is known at some point, the electric force on any particle with charge q placed at that point can be calculated by

$$\boxed{\overrightarrow{F}_e = q\overrightarrow{E}}$$

  - Here the electric field $\overrightarrow{E}$ is caused by other charges that may be present, not by the charge q.

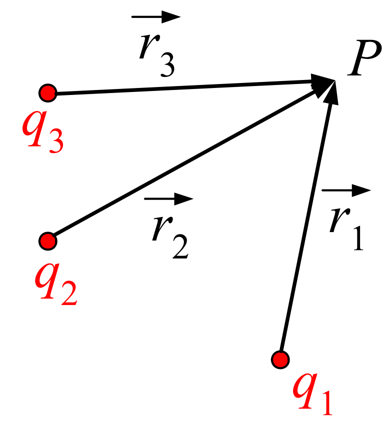


The electric field of point charge

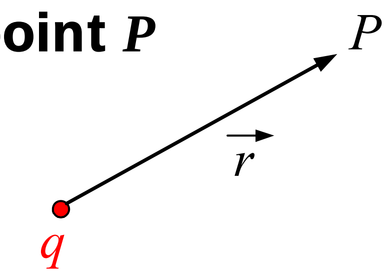
- The calculation of electric field due to the individual point charges
  - The electric field due to single point charge

    According to Coulomb's law, a test q_0 experience a electric force

$$\vec{F}_e = \frac{1}{4\pi\varepsilon_0}\frac{q\,q_0}{r^2}\hat{r}$$



The electric field created by q at point P



$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

  - The electric field due to a series of point charges distributed in space.

    According to the superposition principle, the total electric field at point P

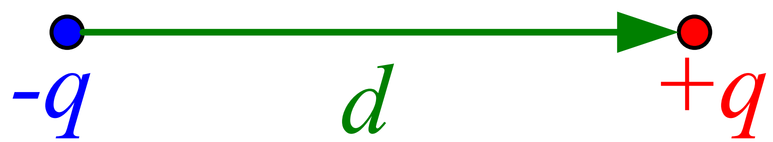
$$\vec{E} = \sum_i \vec{E}_i = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i^2}\hat{r}_i$$

Example —— The Electric Dipole (P475 § 19-11)

Equal positive and negative charges $+q$ and $-q$ separated by a fixed distance d.

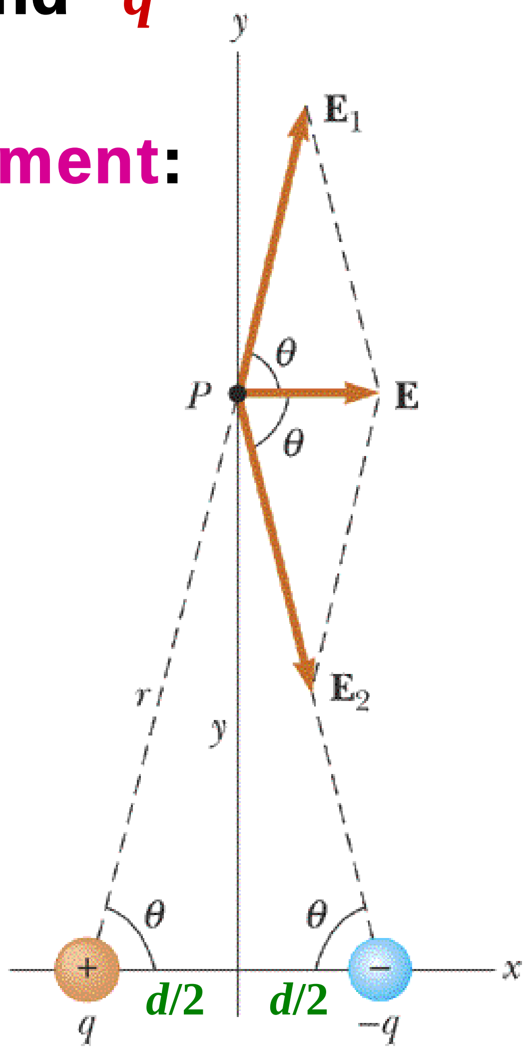
- Definition of the electric dipole moment:

$$\vec{p} = q\vec{d}$$



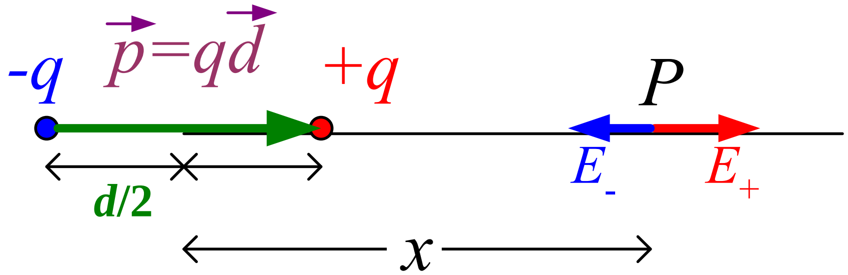
(a) Find the electric field E due to the dipole along the x axis at the point P;

(b) Find the electric field E due to the dipole along the y axis at the point P.



Example Cont'd

Solution: (a)



$$E = E_{+} - E_{-}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\left(x-(d/2)\right)^2} - \frac{1}{\left(x+(d/2)\right)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{2xd}{(x^2 - d^2/4)^2}$$

$$= \frac{2xp}{4\pi\varepsilon_0(x^2 - d^2/4)^2}, \qquad x >> d/2, \qquad \boxed{E = \frac{1}{2\pi\varepsilon_0}\frac{p}{x^3} \propto \frac{p}{x^3}}$$

Example Cont'd

Solution: (b)

$$\vec{E} = \vec{E}_{+} + \vec{E}_{-}$$

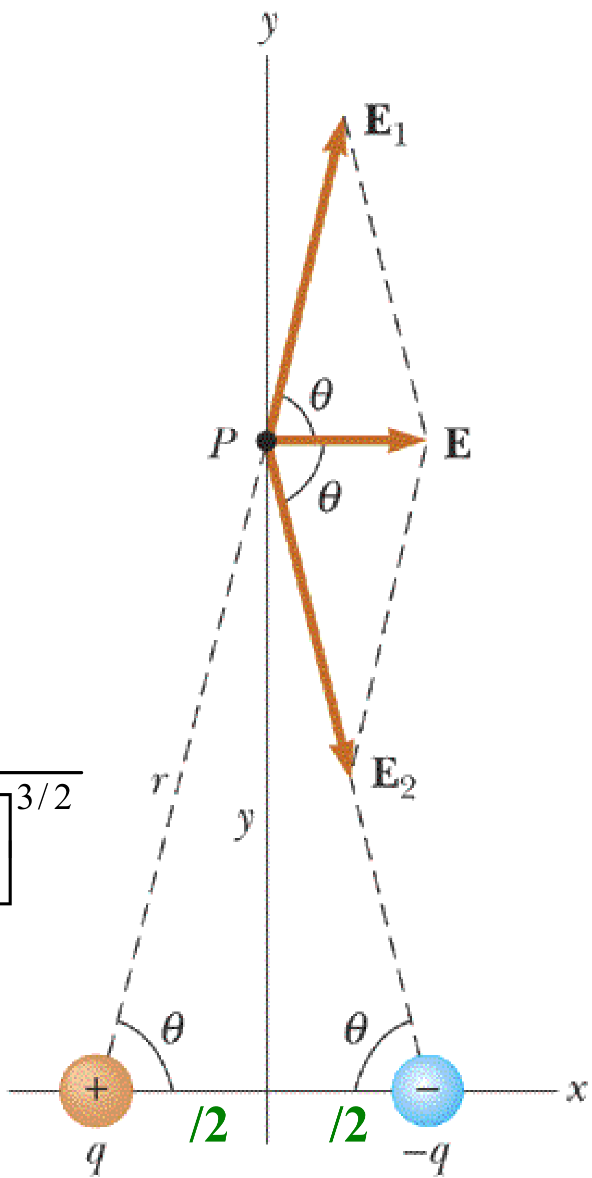
$$E_{+} = E_{-} = \frac{1}{4\pi\varepsilon_0}\frac{q}{y^2 + (d/2)^2}$$

$$E = 2E_{+}\cos\theta = \frac{2}{4\pi\varepsilon_0}\frac{q}{y^2+(d/2)^2}\frac{(d/2)}{\sqrt{y^2+(d/2)^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{qd}{\left[y^2+(d/2)^2\right]^{3/2}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{\left[y^2+(d/2)^2\right]^{3/2}}$$

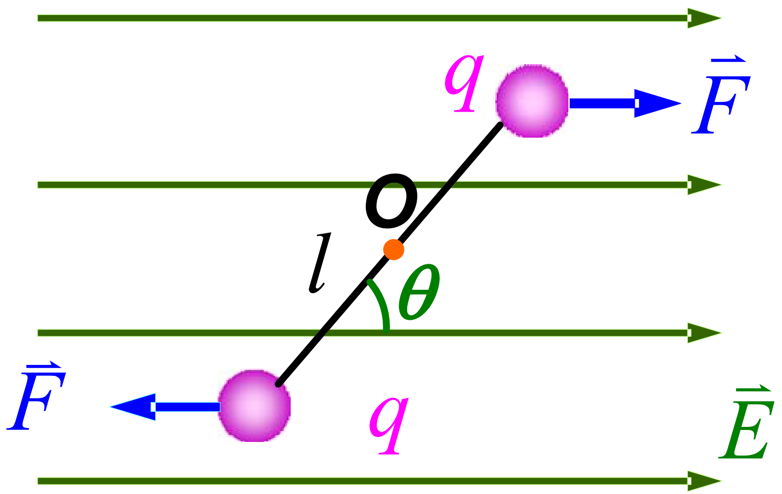
$y >> d/2,$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{p}{y^3} \propto \frac{p}{y^3}}$$



Example: Find the torque exert on an electric dipole.

Solution: to the origin point O



The magnitude:

$$\tau = \frac{l}{2}F_+ \sin\theta + \frac{l}{2}F_- \sin\theta$$

$$= qEl\sin\theta = pE\sin\theta$$

The vector description:

$$\boxed{\vec{\tau} = q\vec{l} \times \vec{E} = \vec{p} \times \vec{E}}$$

The effect of this torque is to try to turn the dipole so $\vec{p}$ is parallel to $\vec{E}$

The **electric field** due to **continuous** charge distributions (P468 § 19-7)

- The calculation method for electric field due to continuous charge distributions

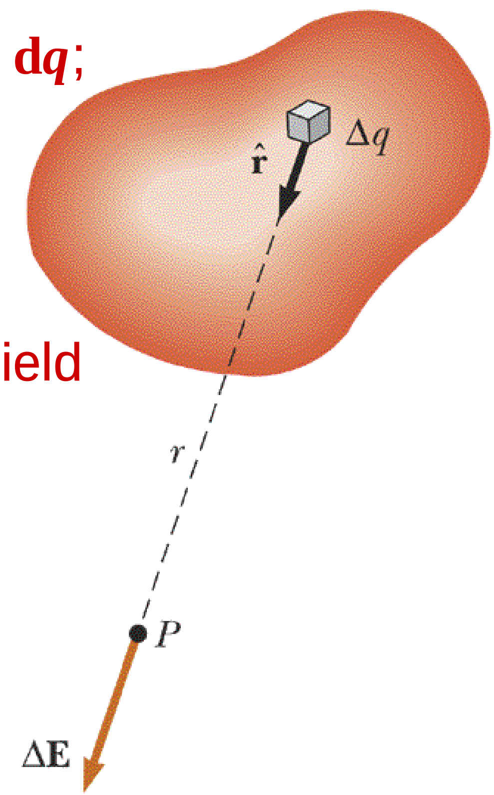
  The procedure:

  - Divide the charge distribution into small elements dq;
  - Model each element as a point charge;

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$$

  - Apply the superposition principle to get the total field at P

$$\vec{E} = \int d\vec{E} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\hat{r}$$



- Charge distribution manners

  The linear charge density: $dq = \lambda dl$

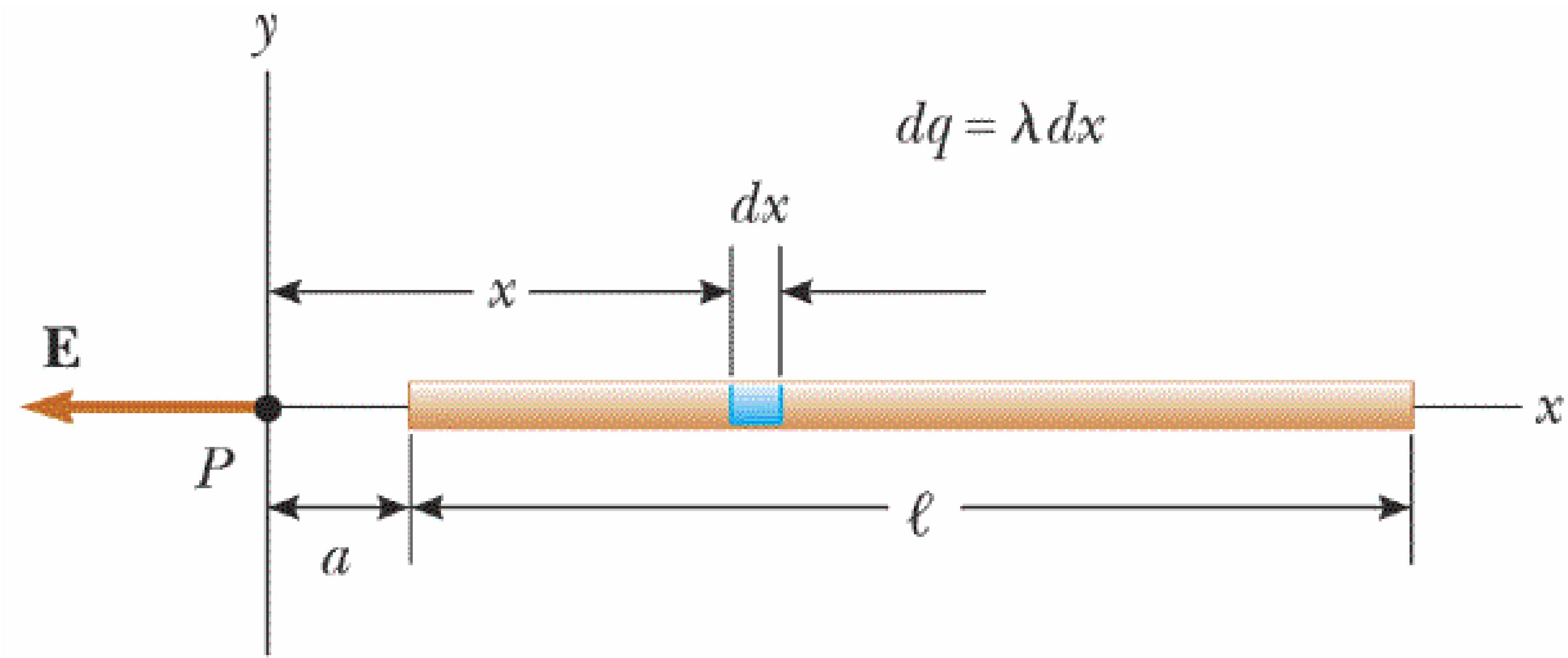
  The surface charge density: $dq = \sigma dA$

  The volume charge density: $dq = \rho dV$

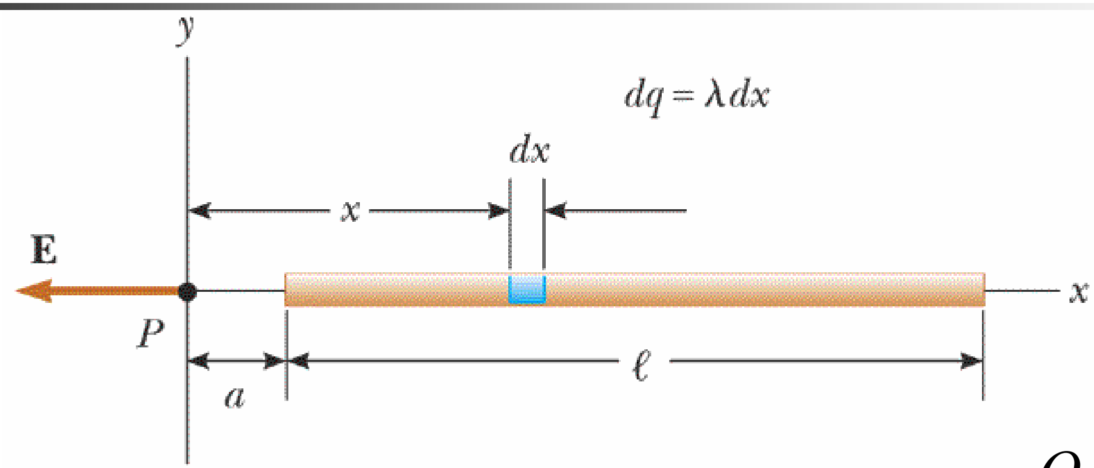
Example

The electric field due to a charged rod

A rod of length *l* has a uniform linear charge density and a total charge *Q*. Calculate the electric field at a point *P* along the axis of the rod, a distance *a* from one end.



Example



Solution:

Step 1: Choose the segment dq. $dq = \lambda\, dx = \dfrac{Q}{l} dx$

Step 2: Write the expression of $\vec{E}$ due to dq.

$$dE = -\frac{1}{4\pi\varepsilon_0}\frac{dq}{x^2} = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{x^2}$$

Step 3: Obtain the total field $\vec{E}$ by integration.

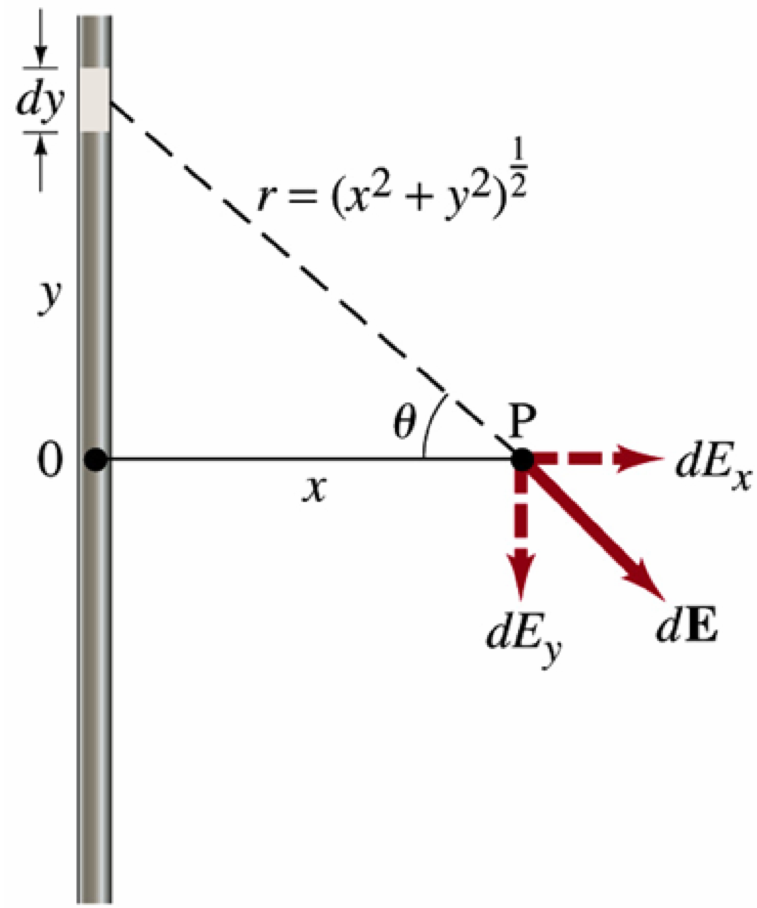
$$E = -\frac{\lambda}{4\pi\varepsilon_0}\int_a^{a+l}\frac{dx}{x^2} = \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{x}\right]_a^{a+l} = -\frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{a+l}\right)$$

$$= -\frac{Q}{4\pi\varepsilon_0 a(a+l)}$$

Example (P469 Ex.19-10)

The electric field of long line of charge

A **long** wire has a uniform linear charge density λ. Calculate the electric field at a point *P* a distance x from the wire.



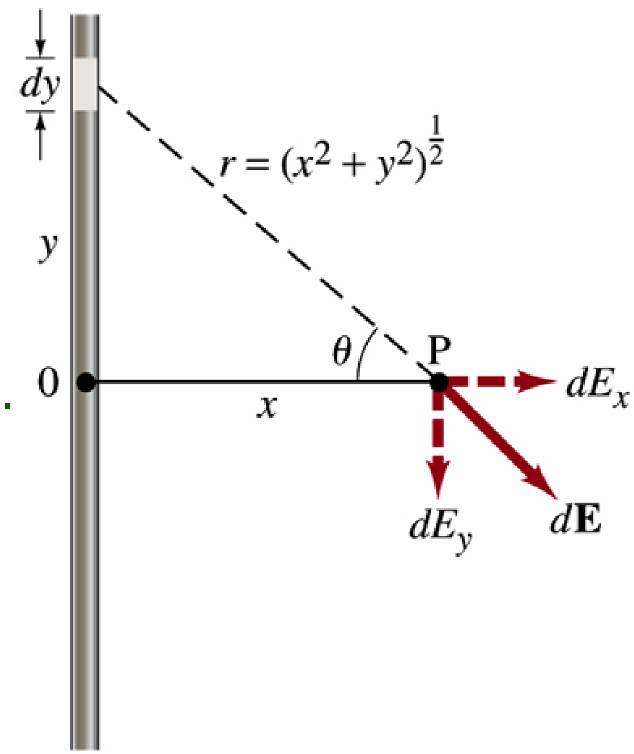
Example

Solution: Step 1: Choose the segment dq. $dQ = \lambda dy$

Step 2: Write the expression of $\vec{E}$ due to dq.

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dy}{(x^2+y^2)}$$

$$dE_x = dE\cos\theta, \quad dE_y = dE\sin\theta$$



Step 3: Obtain the total field $\vec{E}$ by integration.

for the symmetry: $E_y = \int dE\sin\theta = 0$

$$E_x = \int dE\cos\theta = \frac{\lambda}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{\cos\theta dy}{x^2+y^2}$$

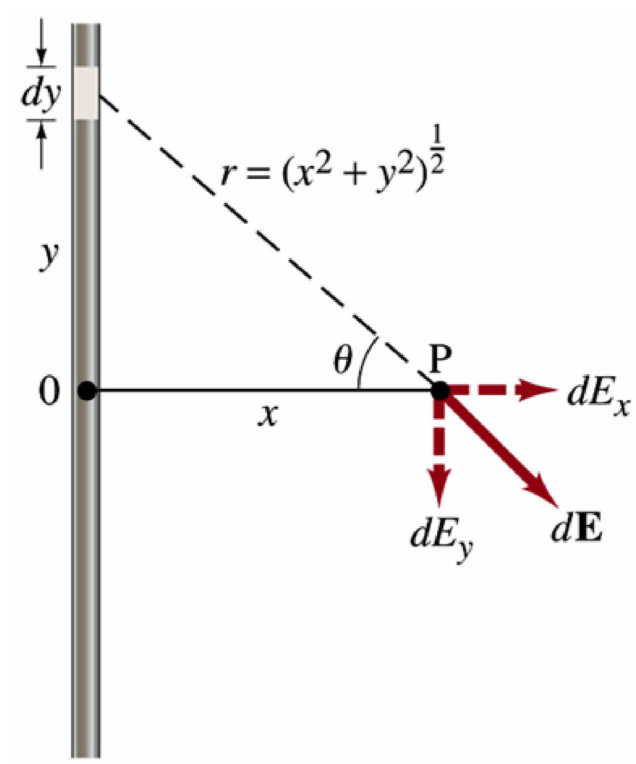
Example cont'd

$$E_x = \int dE \cos\theta = \frac{\lambda}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{\cos\theta dy}{x^2+y^2}$$

$$y = x\tan\theta, \qquad dy = x\,d\theta/\cos^2\theta, \qquad (x^2+y^2) = x^2/\cos^2\theta$$

$$E_x = \frac{\lambda}{4\pi\varepsilon_0}\frac{1}{x}\int_{-\pi/2}^{\pi/2}\cos\theta d\theta$$

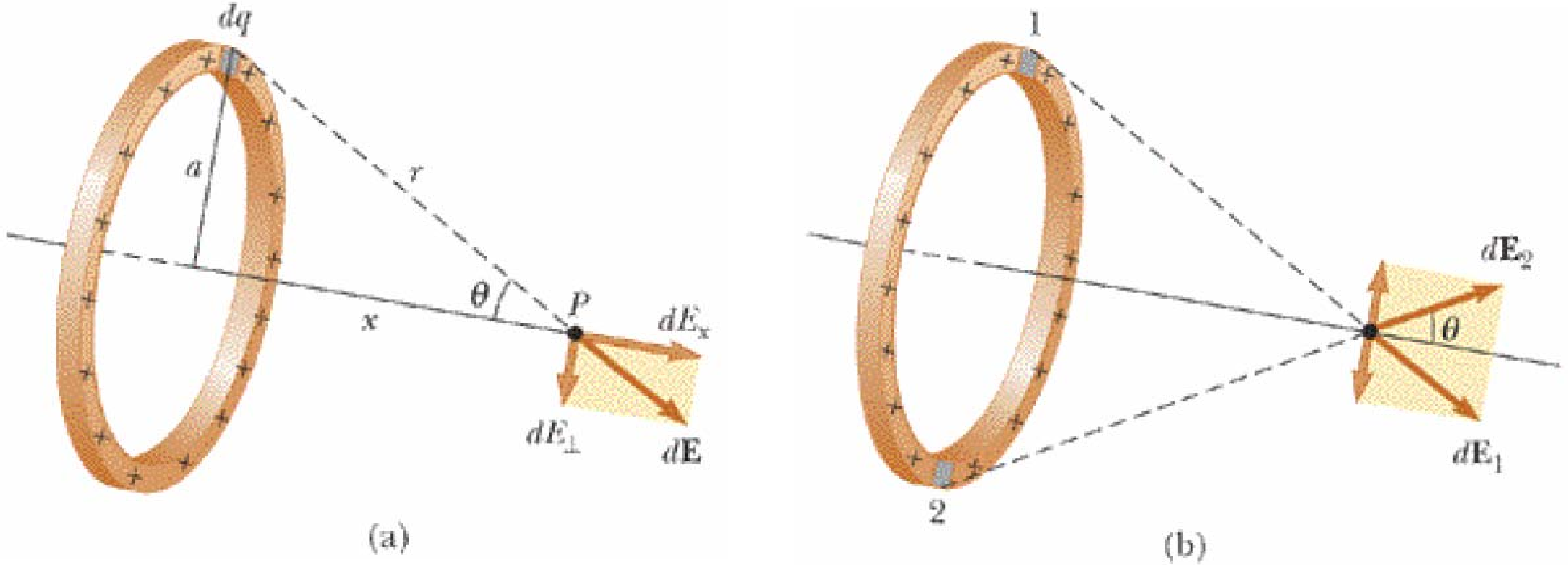
$$= \frac{\lambda}{4\pi\varepsilon_0 x}(\sin\theta)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x}$$



Example (P468 Ex.19-9)

The electric field of a uniform ring of charge

A ring of radius a has a uniform positive charge distribution, with a total charge Q. Calculate the electric field at a point P on the axis of the ring, at a distance x from the center of the ring.



Example

Solution: Choose the segment ***dq***.

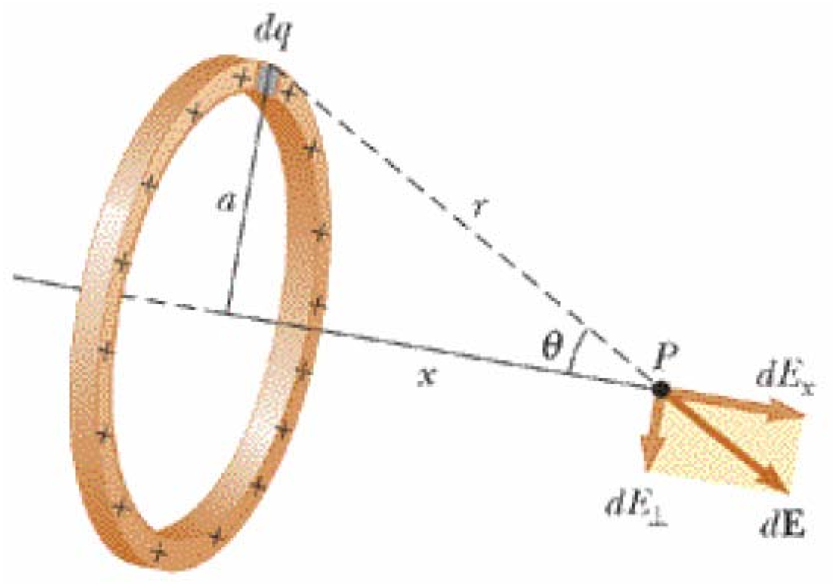
Write the expression of $d\vec{E}$ due to ***dq***. $$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

This field include the x component **d*E*$_x$=d*E*cos*θ***, and perpendicular component **d*E*$_\perp$**, which is canceled by another **d*E*$_\perp$** on the opposite side of the ring.

$$dE_x = dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\left(\frac{x}{r}\right),$$

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{x}{r^3}\int dq = \frac{x}{4\pi\varepsilon_0 r^3}Q$$

$$= \frac{x}{4\pi\varepsilon_0\left(x^2+a^2\right)^{3/2}}Q$$

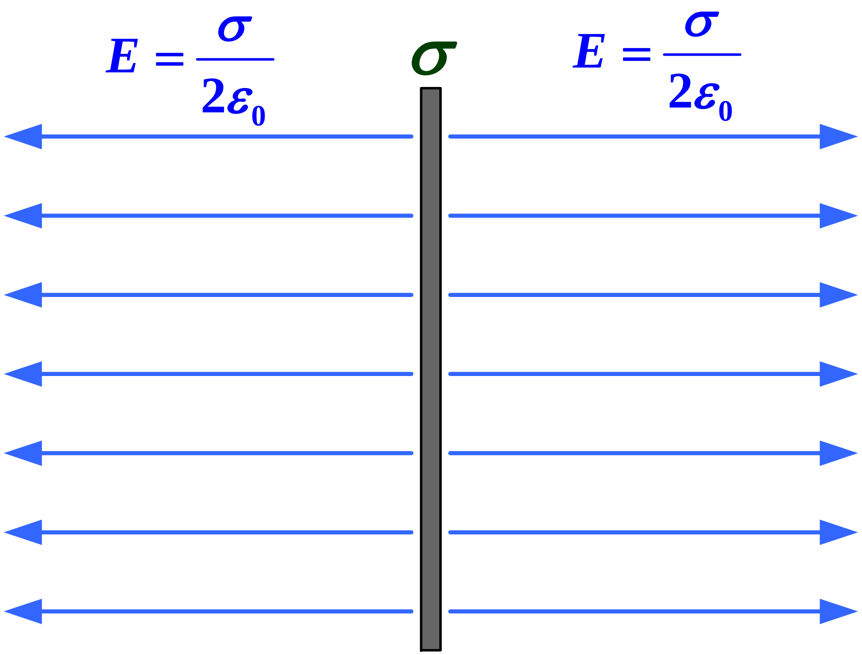


(a)

Example

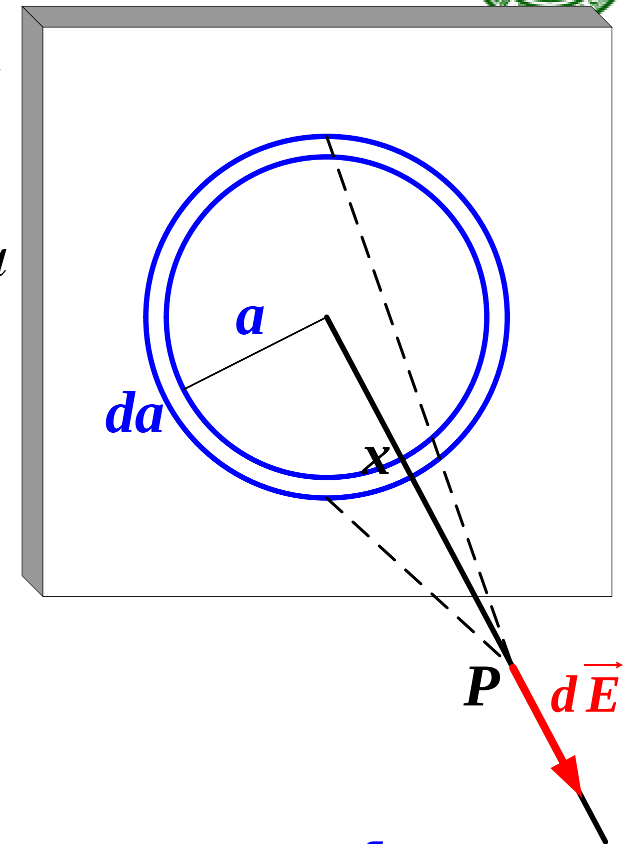
The electric field of an infinite plane sheet of charge

An infinite plane sheet with uniform surface charge density σ. Calculate the electric field at a point *P* on the axis perpendicular to the plane, at a distance x from the plane.



Example

Solution I: Choose the ring with radius a and width da as dq. Starting from:

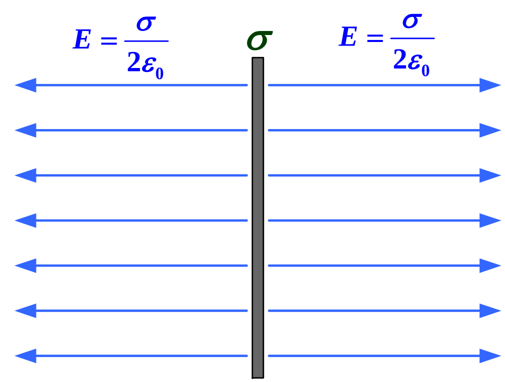


$$dE = \frac{x}{4\pi\varepsilon_0 \left(x^2 + a^2\right)^{3/2}} dq, \qquad dq = \sigma dA = \sigma\, 2\pi a\, da$$

$$E = \int_0^\infty \frac{\sigma x a da}{2\varepsilon_0 \left(x^2 + a^2\right)^{3/2}} = \frac{\sigma}{4\varepsilon_0} \int_0^\infty \frac{x d\left(x^2 + a^2\right)}{\left(x^2 + a^2\right)^{3/2}}$$

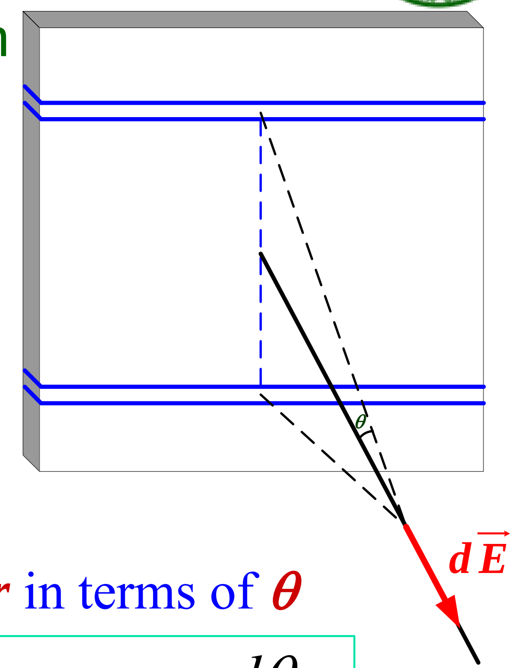
$$= \frac{\sigma}{4\varepsilon_0} \left[-2 \frac{x}{\sqrt{x^2 + a^2}} \right]_0^\infty = \frac{\sigma}{2\varepsilon_0}$$

The electric field keeps constant at any distance from the plane —— the filed is uniform everywhere.



Example cont'd

Solution II: Choose the infinite lengthy rod as ***dq*** , which is at the distance ***a*** from the center, and width ***da***. Using the conclusion of previous example:



$$E=\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$

where ***r*** is the distance from the rod along its perpendicular bisector.

$$\lambda=\sigma\, da$$

$$dE=\frac{\sigma}{2\pi\varepsilon_0}\frac{da}{r}$$

Find the ***a*** and ***r*** in terms of ***θ***

$$a=x\tan\theta,\quad da=x\frac{d\theta}{\cos^2\theta}$$

$$\frac{1}{r}=\frac{\cos\theta}{x}$$

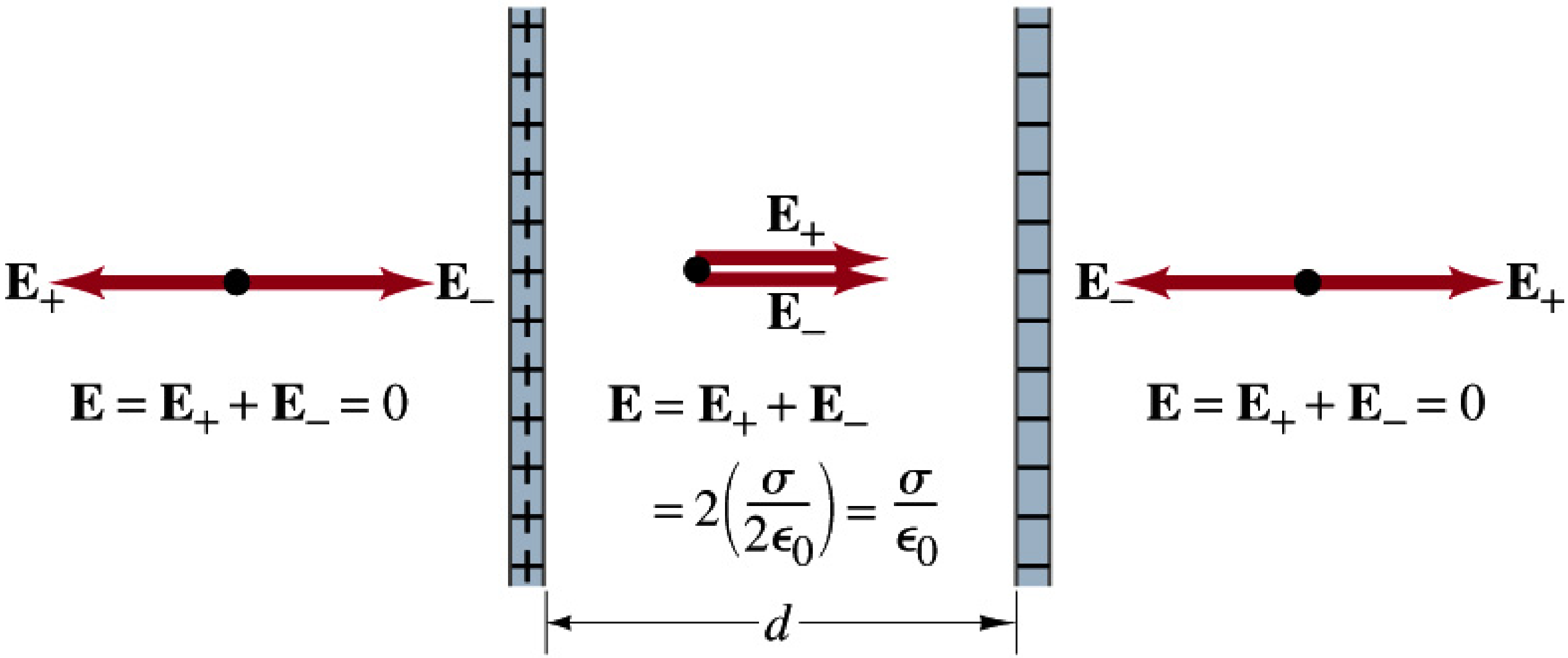
$$dE_x=2dE\cos\theta=\frac{\sigma}{\pi\varepsilon_0}\cos\theta\frac{da}{r}$$

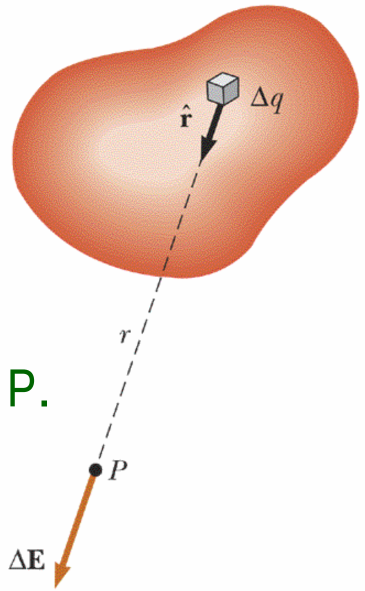
$$=\frac{\sigma}{\pi\varepsilon_0}d\theta$$

$$\boxed{E_x=\frac{\sigma}{\pi\varepsilon_0}\int_0^{\pi/2}d\theta=\frac{\sigma}{2\varepsilon_0}}$$

Example (P471 Ex. 19-12)

The electric field of two parallel plates





- Divided the charge distribution into small elements dq.
  - Model each element as a point charge.

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$$

  - Apply the superposition principle to get the total field at P.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\hat{r}$$

- Establish a convenient coordinate system to complete the integral.
  - Using component representations to solve the vector integral separately.
- Choose appropriate infinitesimal charge element to simplified the integral.
  - Generally choice: $dq = \lambda\, dl$ for line distribution, $dq = \sigma\, dA$ for surface distribution, and $dq = \rho\, dV$ for volume distribution.
  - Using some symmetry of charge distribution to canceling some field components.

Problem-Solving Strategy to Calculating Electric field Due to Continuous Charge Distributions cont'd

- Using the known low-dimensional results for calculating field due to high-dimensional charge distribution.
  - For example, using the result of line charge distribution in a rod or a ring as the bases for calculation of field in the case of surface charge distribution.
  - Using the symmetry as possible as you can. For a disk of charge, adopt the ring result as the base. For a plane of charge, choose the rod result as the base.

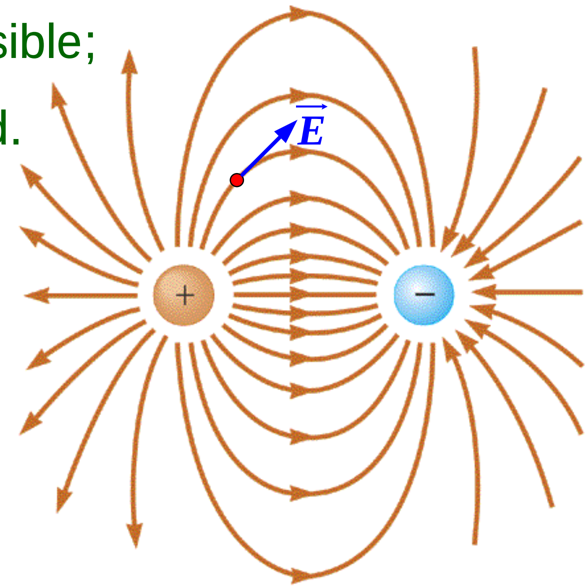
Problems

Ch19 Prob. 49, 50, 70 (P481, 483)

§ 3 Electric Field Lines (P471 § 19-8)

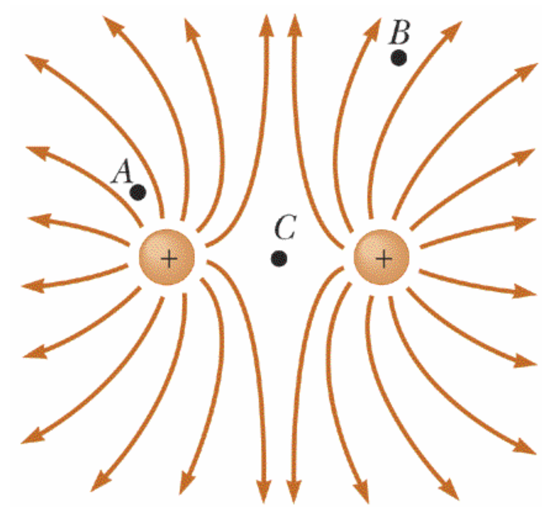
Why introduce electric field lines?

A graphic way for description of electric field:
- Visualize the electric field which is not visible;
- Clarified the characteristic of electric field.

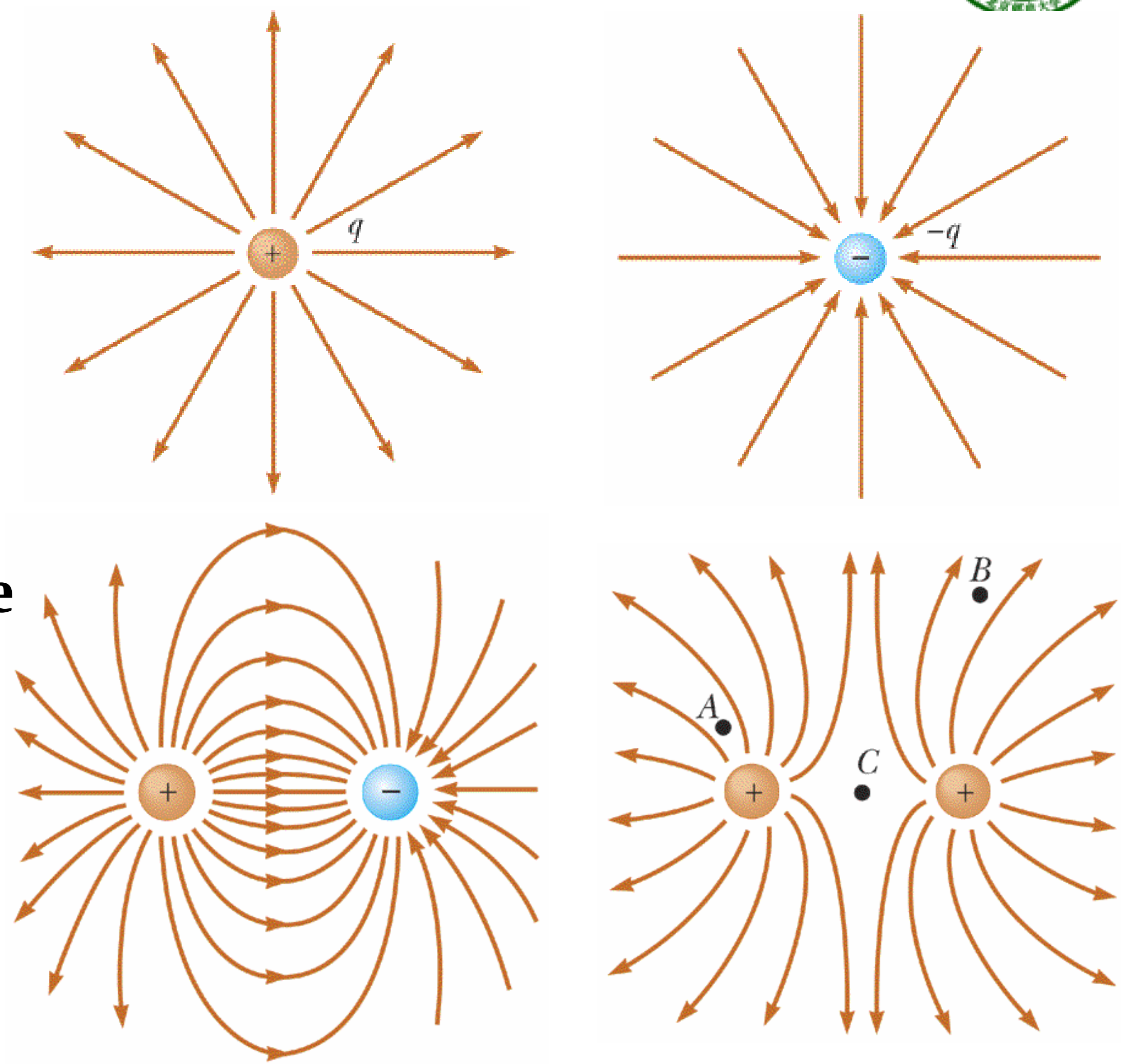


- Electric field lines are related to the electric field in the following manner:
  - Direction —— is tangent to the electric field line at that point.
  - Magnitude —— is proportional to the number of electric field lines per unit area through the cross-sectional surface in that region. E is larger where the field lines are close together and smaller where they are far apart.

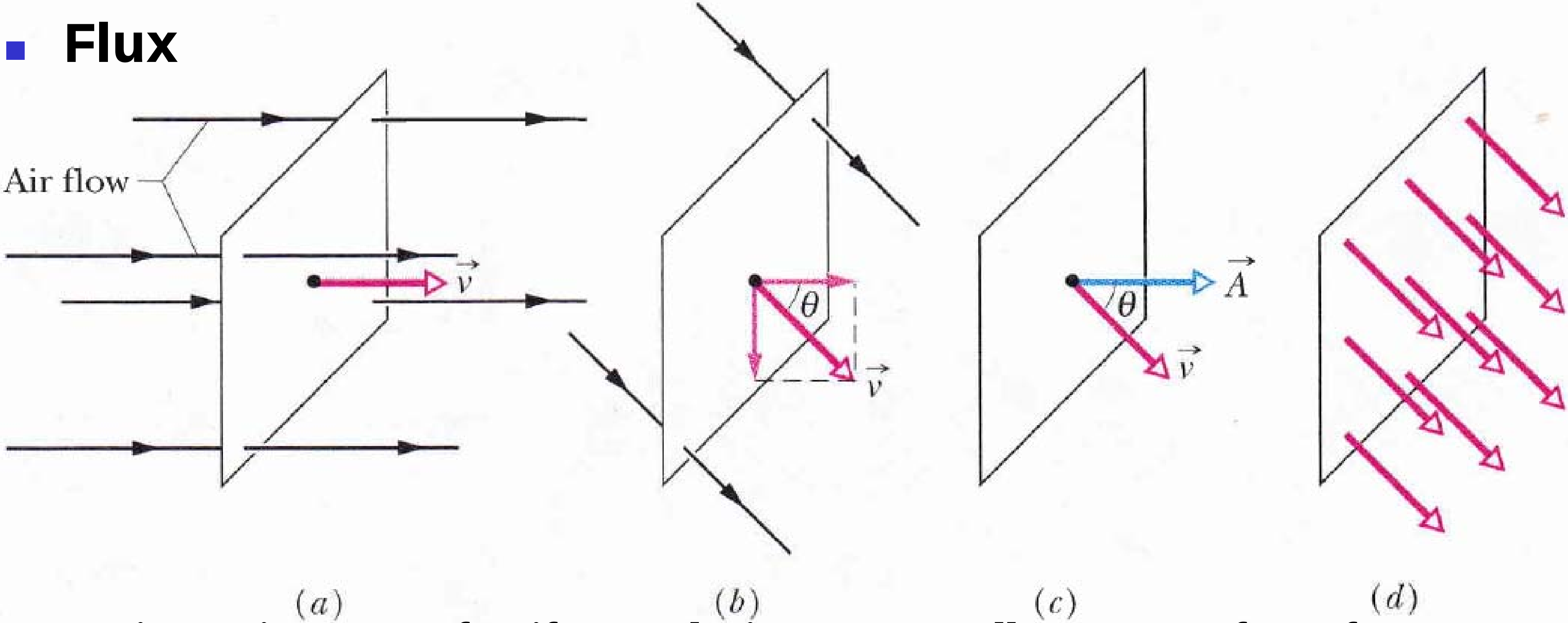
$$E = \frac{\mathrm{d}N}{\mathrm{d}A_{\perp}}$$



- No two field lines can cross each other.
- **Begin** on positive charges (or infinite far away) and **end** on negative charges (or infinite far away). In the case of an excess of one type of charge, some lines will begin or end infinitely far away.



- **Flux**



Imagine a airstream of uniform velocity v at a small square surface of area A.

The flow of air volume (incompressible) through the surface per unit time:

$$\Phi = vA\cos\theta = \vec{v} \bullet \vec{A}$$

(a volume flux)

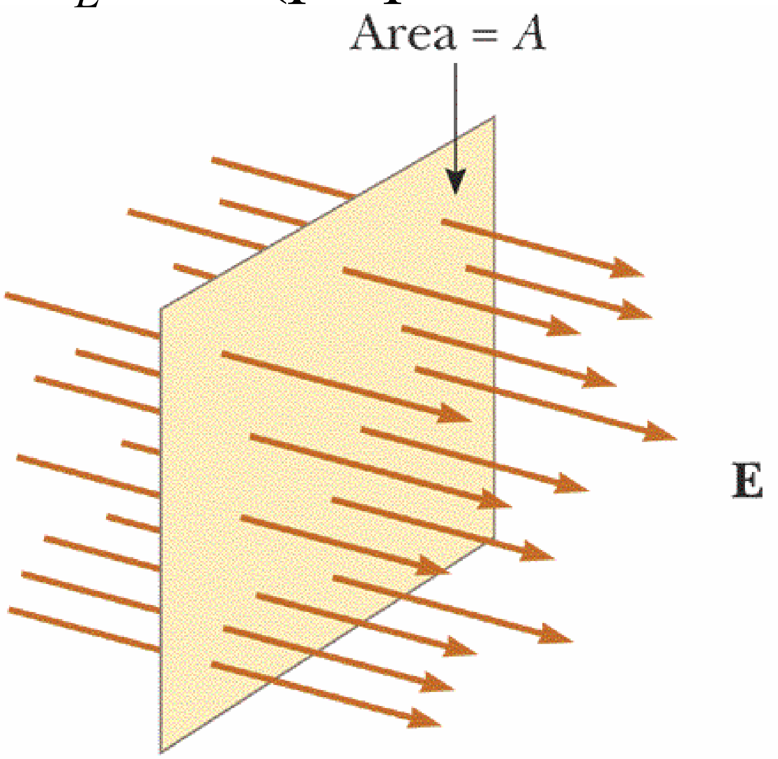
Velocity field: $\vec{v}(\vec{r})$

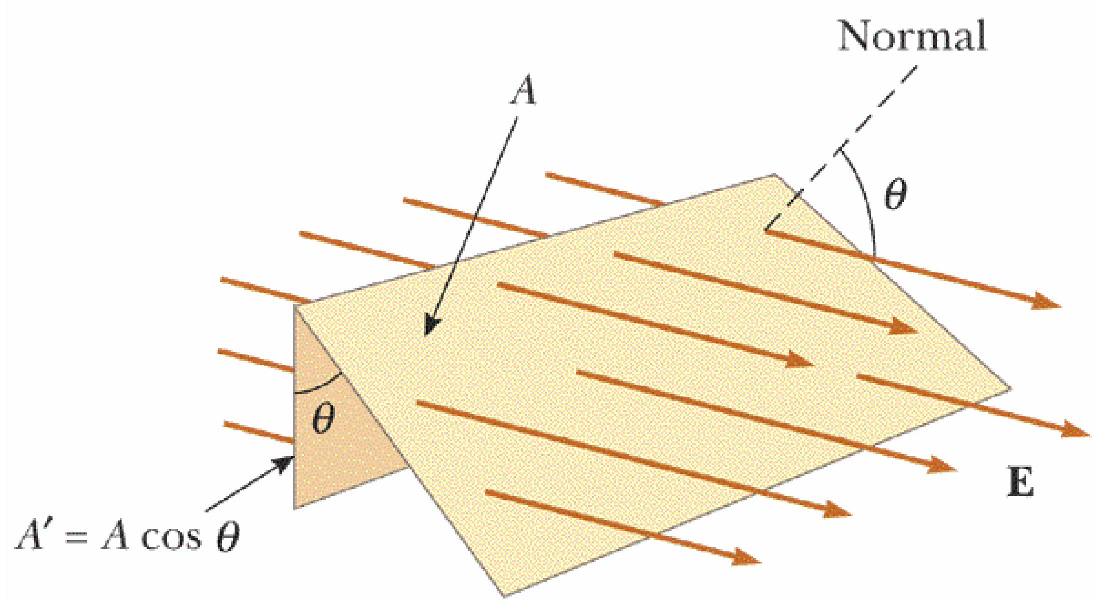
Φ —— Flux of the velocity field through the surface

Electric Flux Φ_E

For uniform electric field

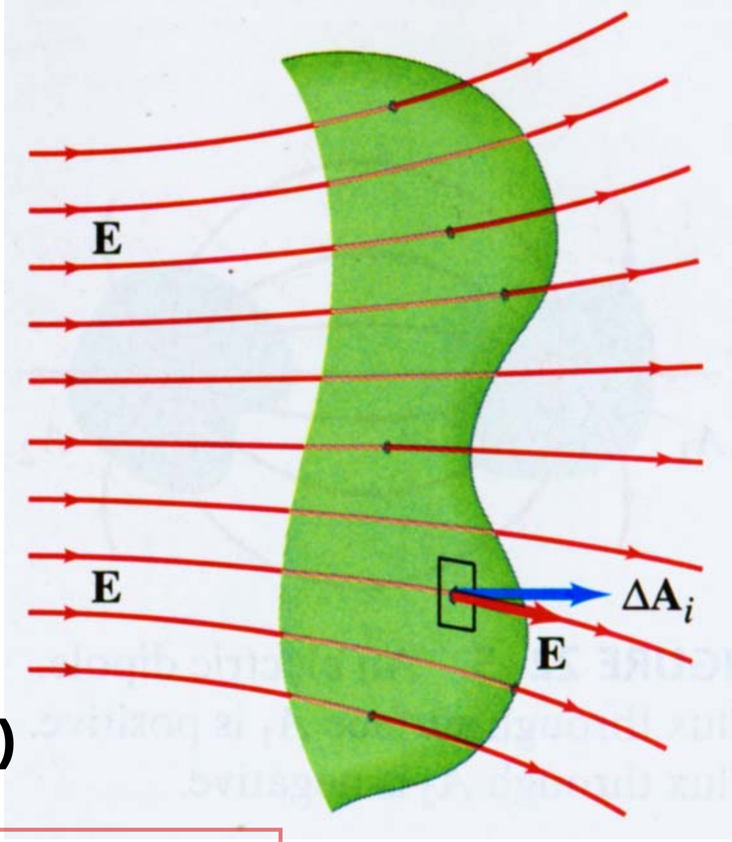
$\Phi_E = EA$ (perpendicular area) $\quad \Phi_E = EA' = EA\cos\theta = \vec{E}\cdot\vec{A}$





Electric Flux

- For **general** electric field that may vary in both magnitude and direction, **curved** surface —— be divided into a large number of a small element of area: ΔA_i.



The flux through a small element:

$$\Delta\Phi_E = E_i \Delta A_i \cos\theta_i = \vec{E}_i \cdot \Delta\vec{A}_i$$

The total electric flux:

$$\Phi_E = \lim_{\Delta A_i \to 0} \sum \vec{E}_i \cdot \Delta\vec{A}_i = \iint_{surface} \vec{E} \cdot d\vec{A}$$

(surface integral)

$$E = \frac{\mathrm{d}N}{\mathrm{d}A_\perp}$$

$$\boxed{\mathrm{d}\Phi_{\mathrm{e}} = \vec{E} \cdot d\vec{A} = E\mathrm{d}A_\perp = \mathrm{d}N}$$

the number of field lines

Electric flux through a closed surface

For a closed surface, **outward** direction is defined to be **positive**.

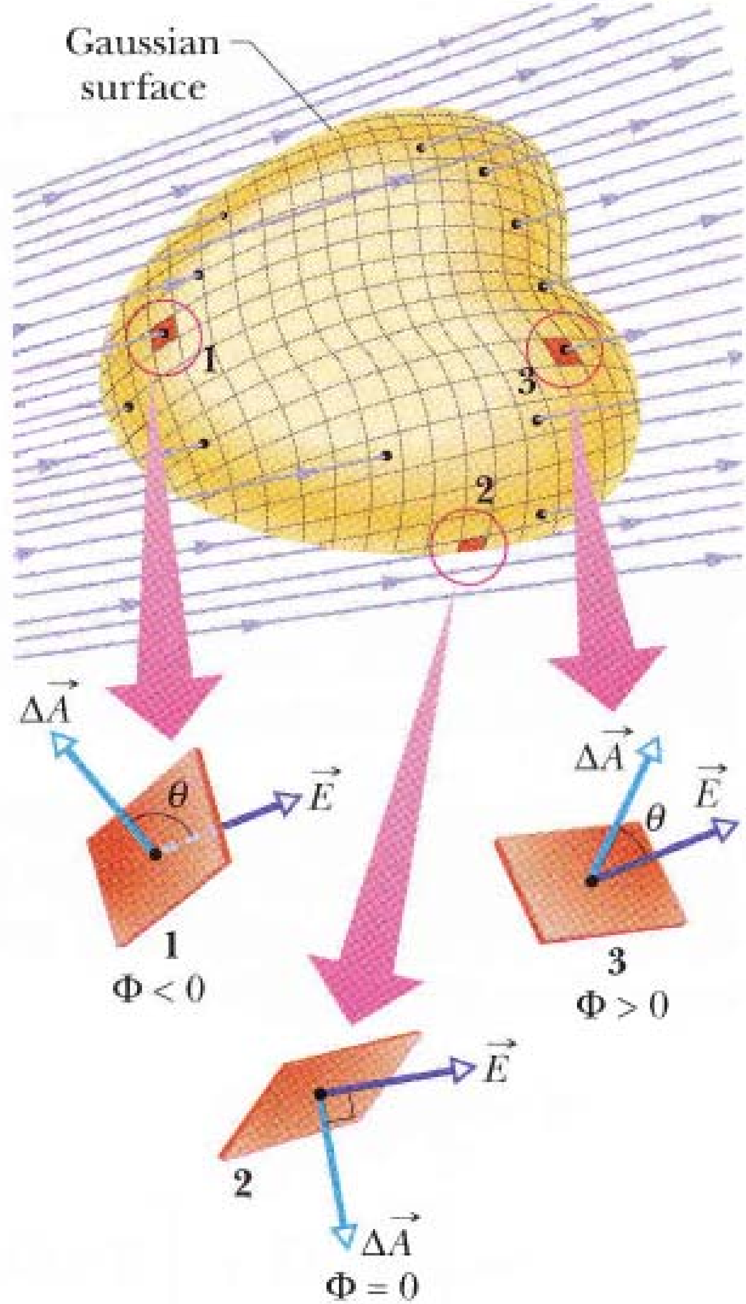
At point ①, $\theta_i > 90°$, $\Phi_E < 0$.

At point ②, $\theta_i = 90°$, $\Phi_E = 0$.

At point ③, $\theta_i < 90°$, $\Phi_E > 0$.

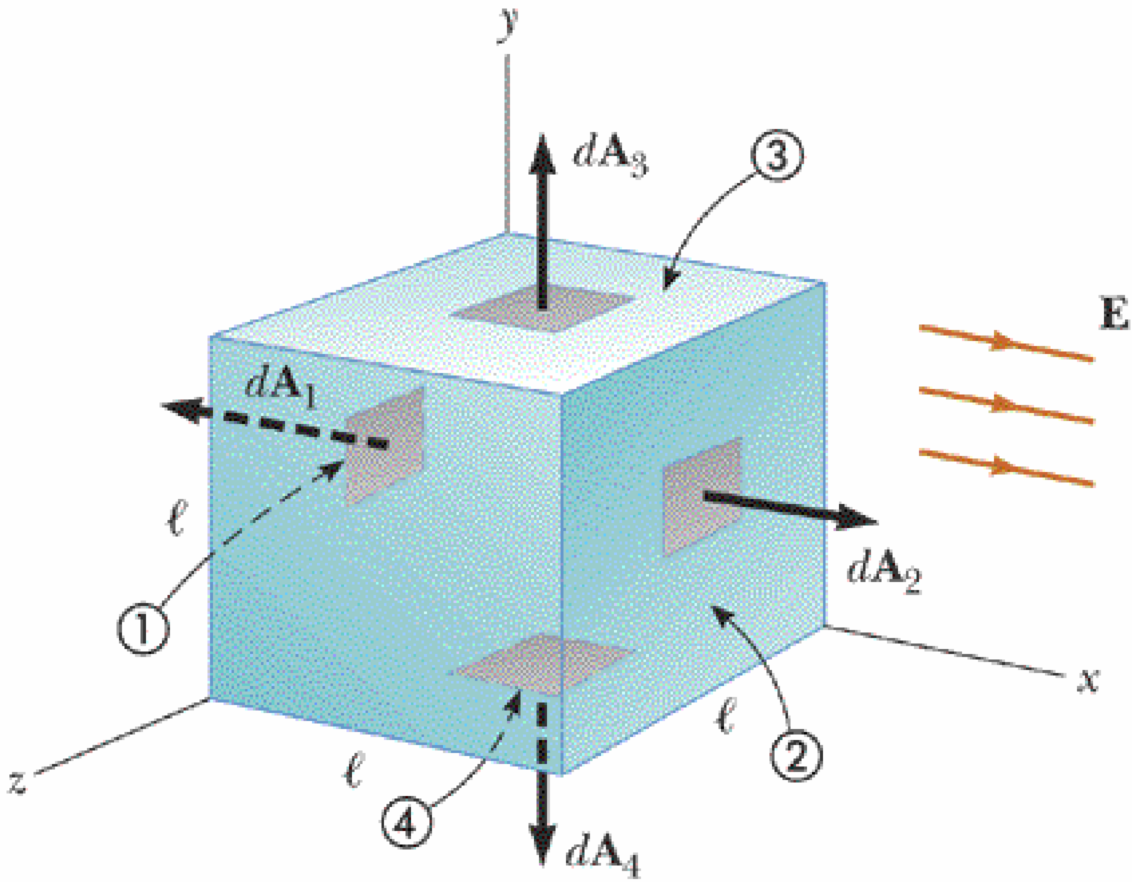
- The net electric flux through a closed surface:

$$\Phi_E = \oiint \vec{E} \cdot d\vec{A}$$



Example

Consider a uniform electric field E directed along the +x axis. Find the net electric flux through the surface of a cube of edges *l* shown in the figure.



Example

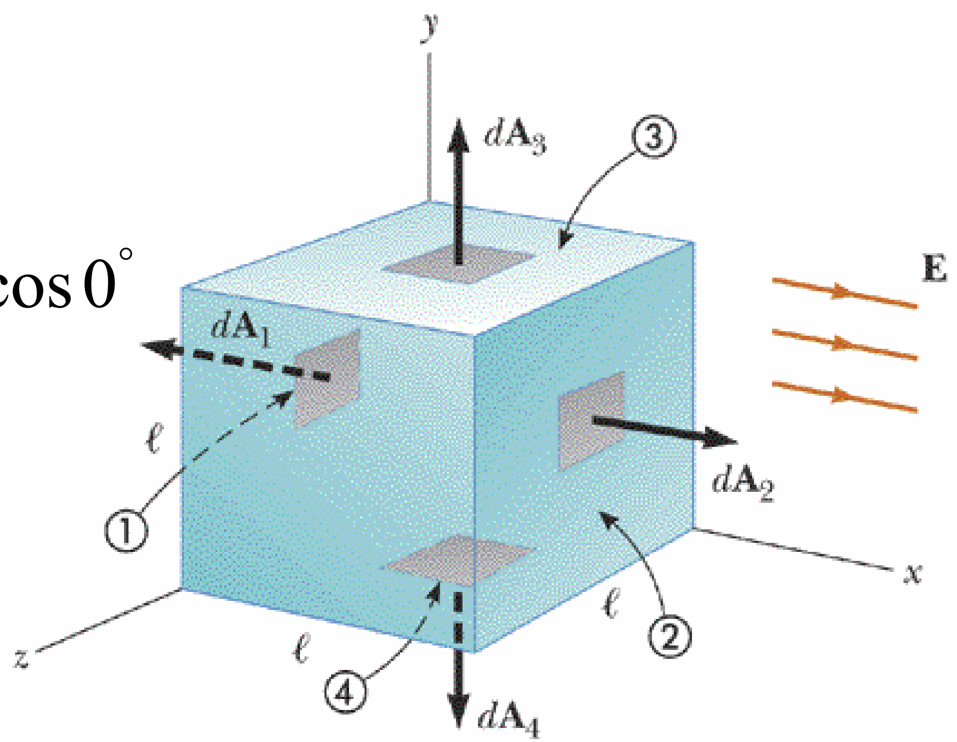
Solution: For the faces labeled ③ and ④ (⑤ and ⑥), the orientation of $d\vec{A}$ is perpendicular to $\vec{E}$.

The net flux through the surface of cube.

$$\Phi_E = \iint_1 \vec{E}\cdot d\vec{A} + \iint_2 \vec{E}\cdot d\vec{A}$$

$$= \iint_1 EdA\cos 180^\circ + \iint_2 EdA\cos 0^\circ$$

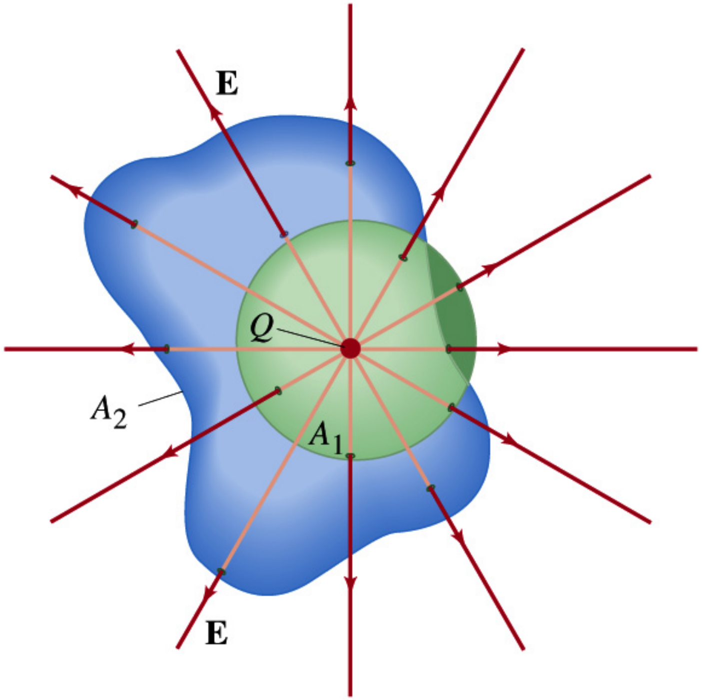
$$= -EA + EA = 0$$



§ 5 Gauss's Law

- **Gauss's Law:**

$$\Phi_E = \oiint_S \vec{E} \cdot d\vec{A} = \frac{q_{inside}}{\varepsilon_0}$$

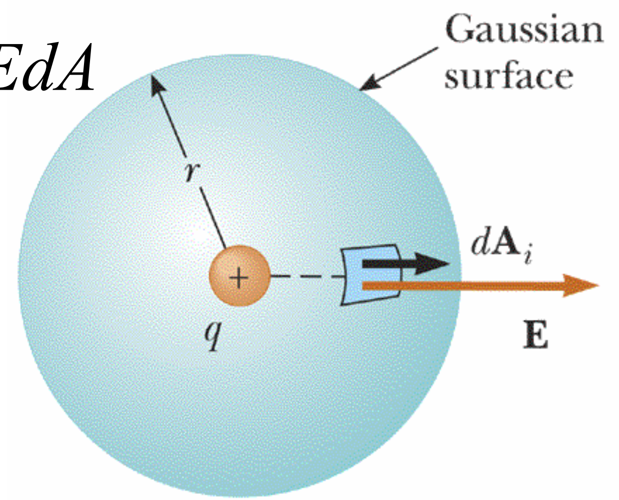


The net electric flux through any closed surface is equal to the net charge inside the surface divided by ε_0.

- **A point charge *q* locates at the center of a spherical surface.**

$$\Phi_E = \underset{\text{spherical surface}}{\oiint} \vec{E}\cdot d\vec{A} = \underset{\text{spherical surface}}{\oiint} E(dA)\cos\theta = \underset{\text{spherical surface}}{\oiint} EdA$$

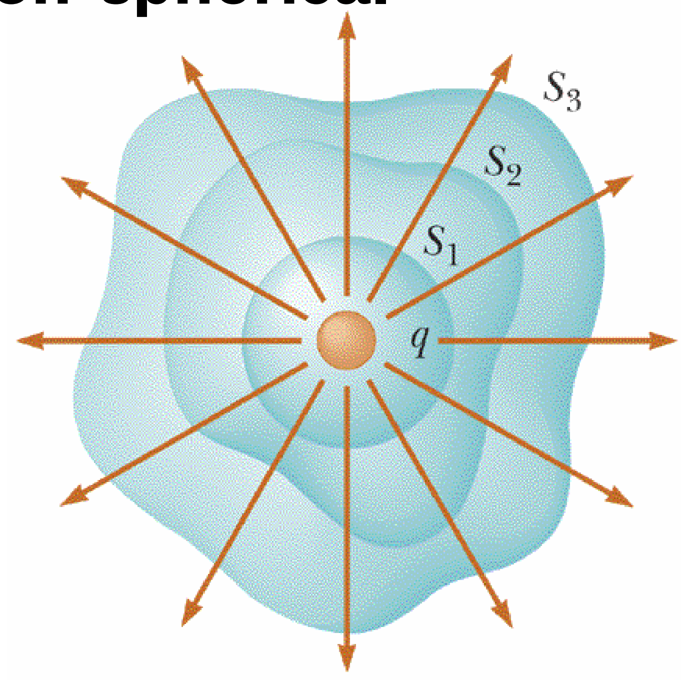
$$= E \underset{\text{spherical surface}}{\oiint} dA = \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)\left(4\pi r^2\right) = \frac{q}{\varepsilon_0}$$



- **The net flux is proportional to the charge inside the surface;**
- **The net flux is independent of the radius *r* —— every field line from the charge must pass through the surface**
- **The fact that the net flux is independent of the radius is consequence of inverse-square dependence of the electric field according to Coulomb's law.**

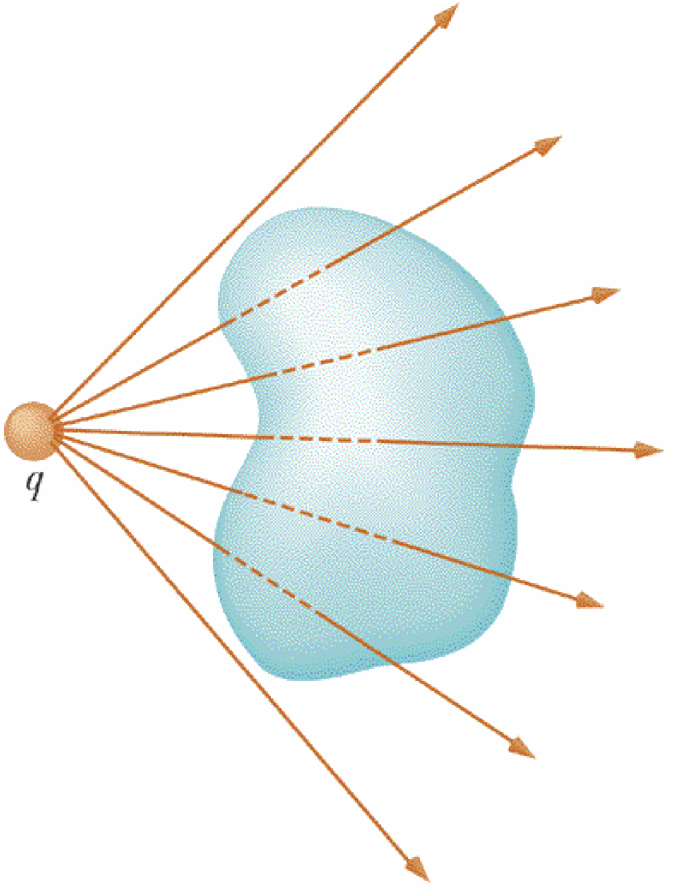
- The charge q inside, the closed surface **not spherical**.
  - The flux that passes through spherical surface S_1 has the value q / ε_0.
  - The number of electric field lines through the spherical surface S_1 is equal to the number of electric field lines through the non-spherical surfaces S_2 and S_3.

$$\oiint_{S_1} \vec{E} \cdot d\vec{A} = \oiint_{S_2} \vec{E} \cdot d\vec{A} = \oiint_{S_3} \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

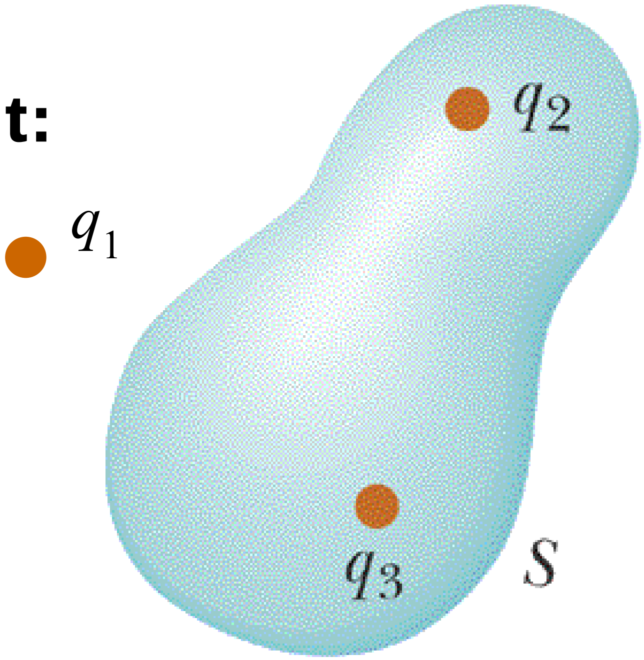


- A point charge locates **outside** a closed surface of arbitrary shape.
  - The number of electric field lines entering the surface equals the number leaving the surface.

$$\Phi_E = \Phi_E^{in} + \Phi_E^{out} = 0$$



- The **series** of charges, some inside, some outside the closed surface.
  - The total electric field at any point:



$$\vec{E} = \sum_i \vec{E}_i$$

$$\Phi_E = \oiint_S \vec{E} \cdot d\vec{A} = \oiint_S \sum_i \vec{E}_i \cdot d\vec{A}$$

$$= \sum_i \oiint_S \vec{E}_i \cdot d\vec{A} = \sum_i \Phi_{Ei}$$

$$\Phi_{Ei} = \begin{cases} \dfrac{q_i}{\varepsilon_0} & \text{if } q_i \text{ inside the } S \\ 0 & \text{if } q_i \text{ outside the } S \end{cases}$$

$$\Rightarrow \boxed{\Phi_E = \sum_i \Phi_{Ei} = \frac{q_{inside}}{\varepsilon_0}}$$

- **Gauss's Law:**

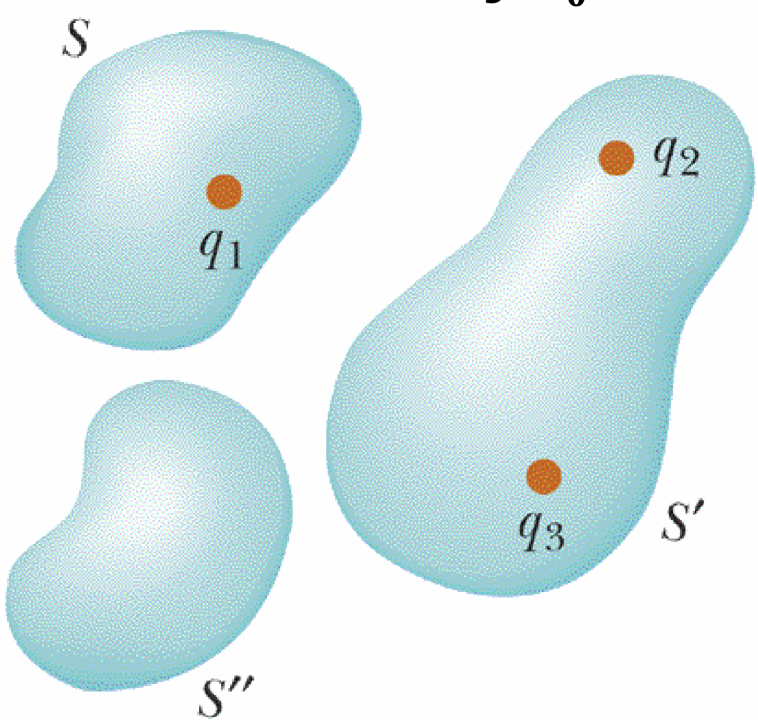
$$\boxed{\Phi_E = \oint\!\!\!\oint_S \vec{E}\cdot d\vec{A} = \frac{q_{inside}}{\varepsilon_0}}$$

- **The net electric flux through any closed surface is equal to the net charge inside the surface divided by ε_0.**

$$\oint\!\!\!\oint_S \vec{E}\cdot d\vec{A} = \frac{q_1}{\varepsilon_0},$$

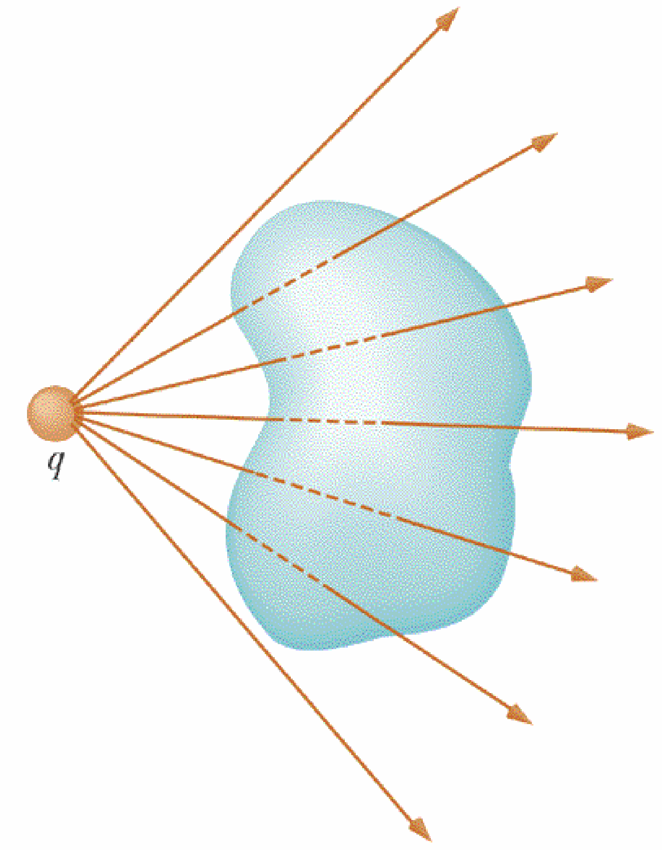
$$\oint\!\!\!\oint_{S'} \vec{E}\cdot d\vec{A} = \frac{q_2+q_3}{\varepsilon_0},$$

$$\oint\!\!\!\oint_{S''} \vec{E}\cdot d\vec{A} = 0$$



Some comments on Gauss's Law

$$\Phi_E = \oiint_S \vec{E} \cdot d\vec{A} = \frac{q_{inside}}{\varepsilon_0}$$



- **The flux only depends on the charges inside (enclosed).**
- ***E* on the left side of Gauss's law is the *E* in Gaussian surface and it is not necessarily due to the charge inside the surface. It is produced by all the charges in the space.**
- **Zero flux doesn't mean the zero field.**

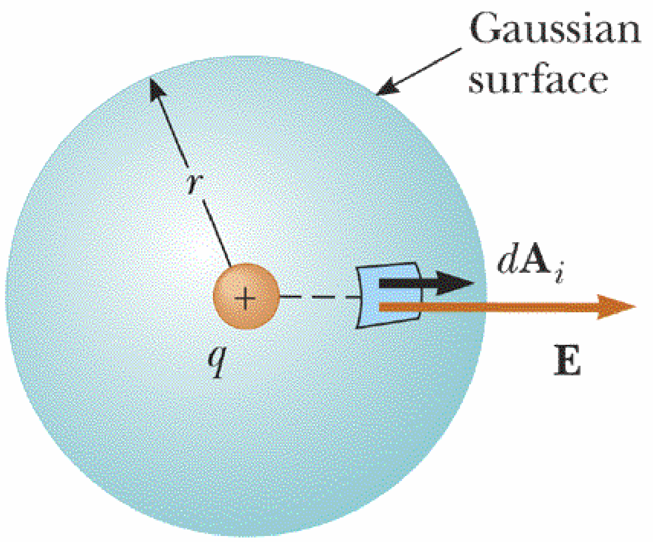
Gauss's Law and Coulomb's Law

- **Gauss's law is deduced from Coulomb's law. Coulomb's law can also be deduced from Gauss's law.**

For an isolated charge q locates inside a spherical surface

$$\oiint_S \vec{E}\cdot d\vec{A} = \oiint_S E\,dA = E\oiint_S dA = E\left(4\pi r^2\right) = \frac{q}{\varepsilon_0} \Longrightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

- **These two laws can regarded as equivalent in the situation of electrostatics, Gauss's law is found to hold also for electric fields generated by changing magnetic field.**
- **Gauss's law is a more general law than Coulomb's law, and so is regarded as a more fundamental equation of electromagnetisms.**



§ 6 Application of Gauss's Law to Symmetric Charge Distributions

Generally

The charge distribution is known $\xrightarrow{\text{Coulomb's Law}}$ The electric field

The electric field is known $\xrightarrow{\text{Gauss's Law}}$ The charge distribution

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\hat{r}$$

$$\Phi_E = \oiint_S \vec{E}\cdot d\vec{A} = \frac{q_{inside}}{\varepsilon_0}$$

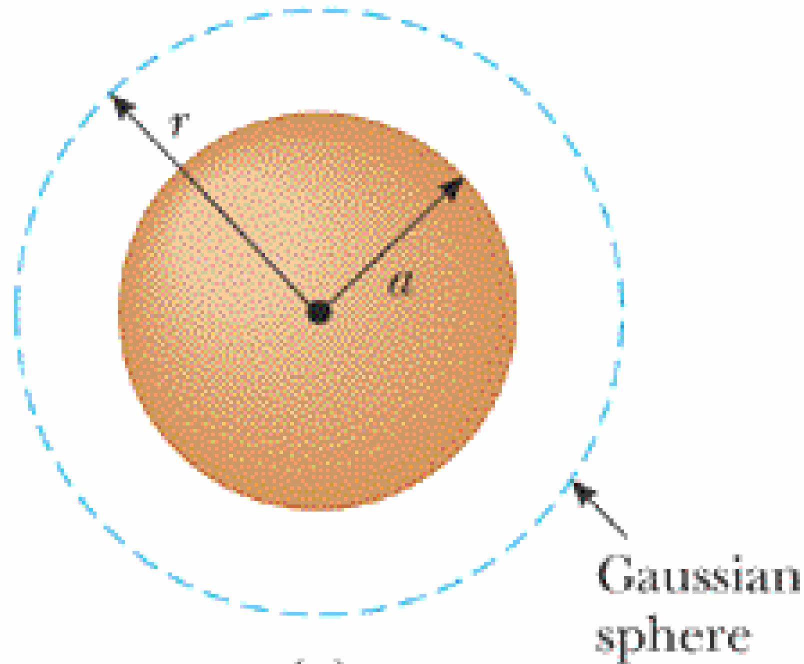
- **For special case where the charge distribution possesses a high degree of symmetry, Gauss's law can be used to calculate the electric field.**

A spherical symmetric charge distribution

An insulating solid sphere of radius a has a uniform volume charge density ρ and carries a total positive charge Q.

(1) Calculate the magnitude of the electric field at a point **outside** the sphere.

(2) Find the magnitude of the electric field at a point **inside** the sphere.



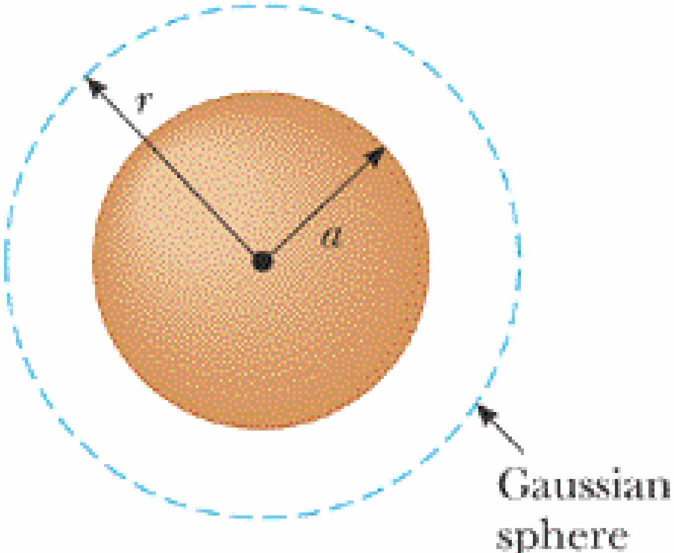
(a)

Solution:

(1) Select a spherical gaussian surface of radius $r > a$

$$\oiint_S \vec{E} \cdot d\vec{A} = \oiint_S E(dA) = E \oiint_S dA = E\left(4\pi r^2\right) = \frac{q_{in}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \qquad (\text{for } r > a)}$$



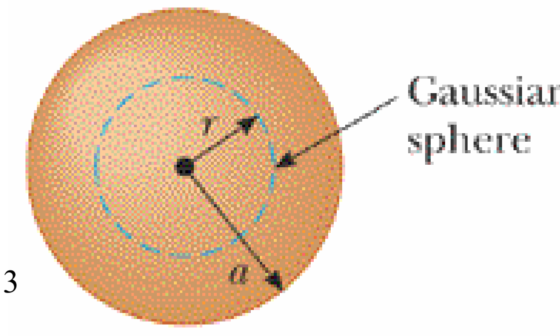
(a)

This result is identical to that obtained for a point charge.

Solution: (2) Select a spherical gaussian surface of radius *r* < *a*

$$\oiint_S \vec{E}\cdot d\vec{A} = \oiint_S E(dA) = E\oiint_S dA = E\left(4\pi r^2\right) = \frac{q_{in}}{\varepsilon_0}$$

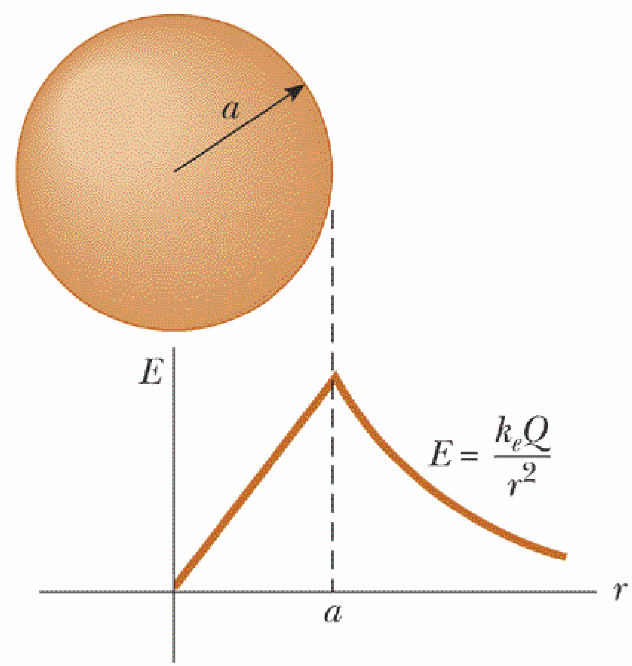
$$= \frac{1}{\varepsilon_0}\rho\left(\frac{4}{3}\pi r^3\right) = \frac{1}{\varepsilon_0}\left(\frac{Q}{\frac{4}{3}\pi a^3}\right)\left(\frac{4}{3}\pi r^3\right) = \frac{Q}{\varepsilon_0}\frac{r^3}{a^3}$$



(b)

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a^3}r \quad (\text{for } r < a)}$$

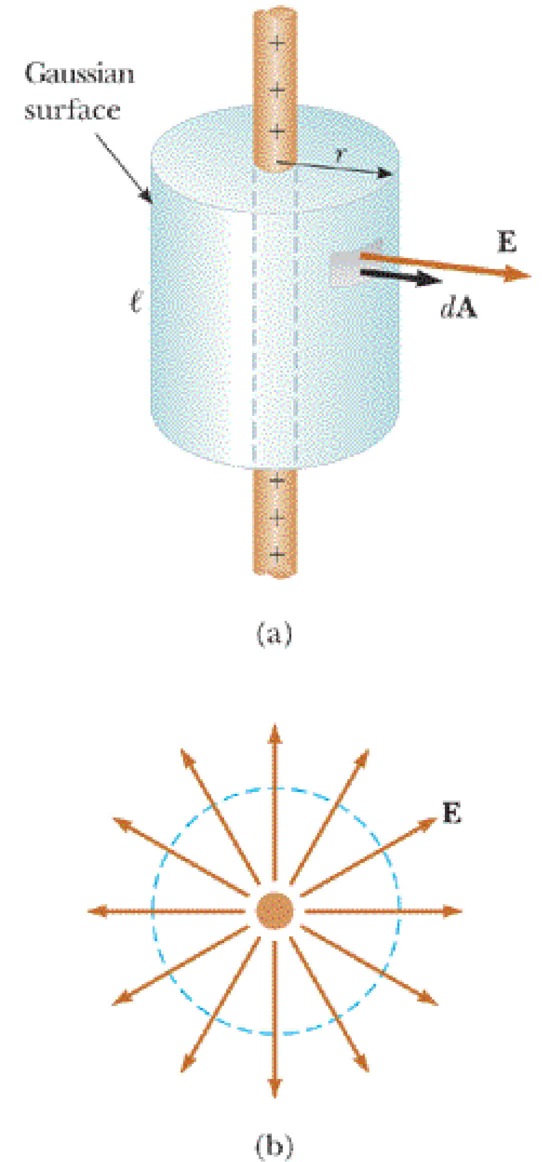
The expressions of electric fields inside and outside the sphere match when *r*=*a*.



Example

A cylindrically symmetric charge distribution

Find the electric field a distance r from a line of positive charge of infinite length and constant charge per unit length λ.



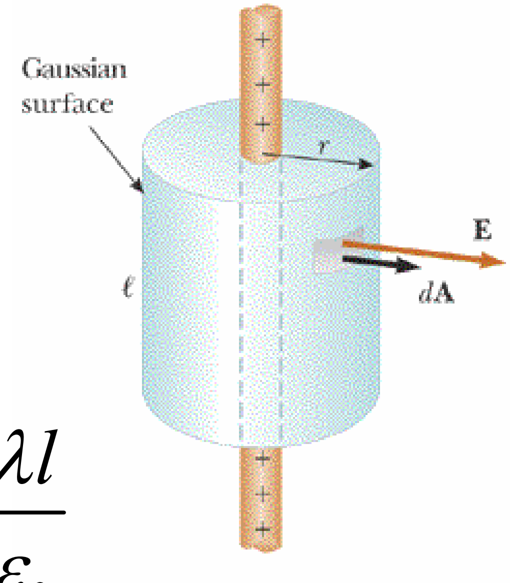
A cylindrically symmetric charge distribution

Solution: Select a cylindrical Gaussian surface of radius *r* and length *l* that is coaxial with the line charge.

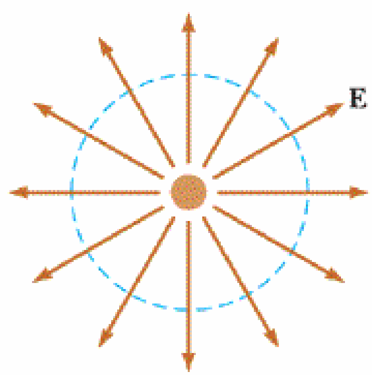
$$\oiint_S \vec{E}\cdot d\vec{A} = \iint_{\text{side surface}} \vec{E}\cdot d\vec{A} + \iint_{\text{top and bottom}} \vec{E}\cdot d\vec{A}$$

$$= \iint_{\text{side surface}} \vec{E}\cdot d\vec{A} = E\left(2\pi r l\right) = \frac{q_{in}}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$



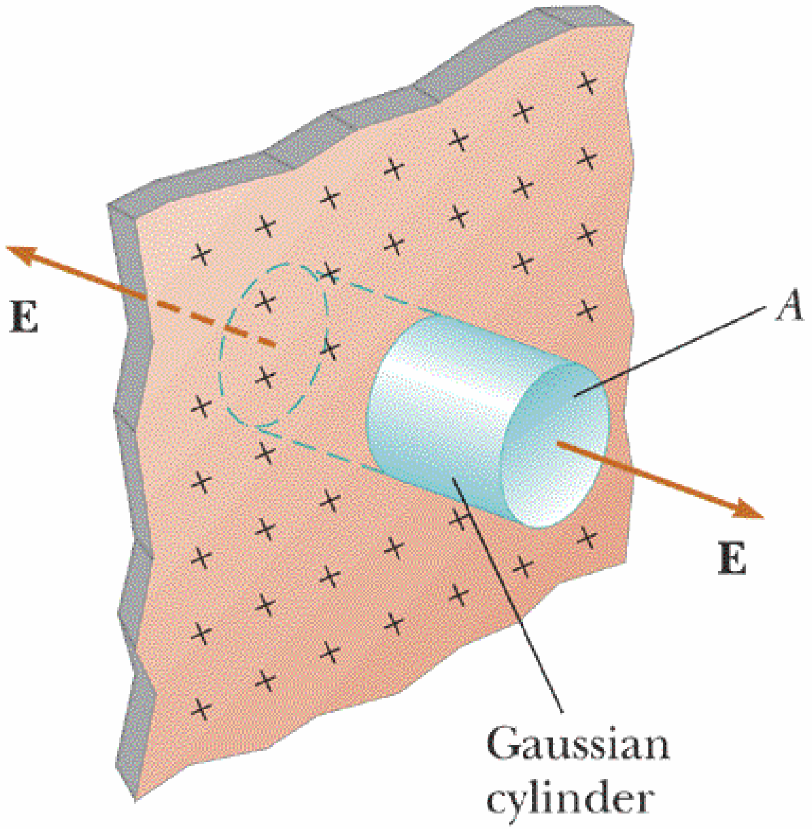
(a)



(b)

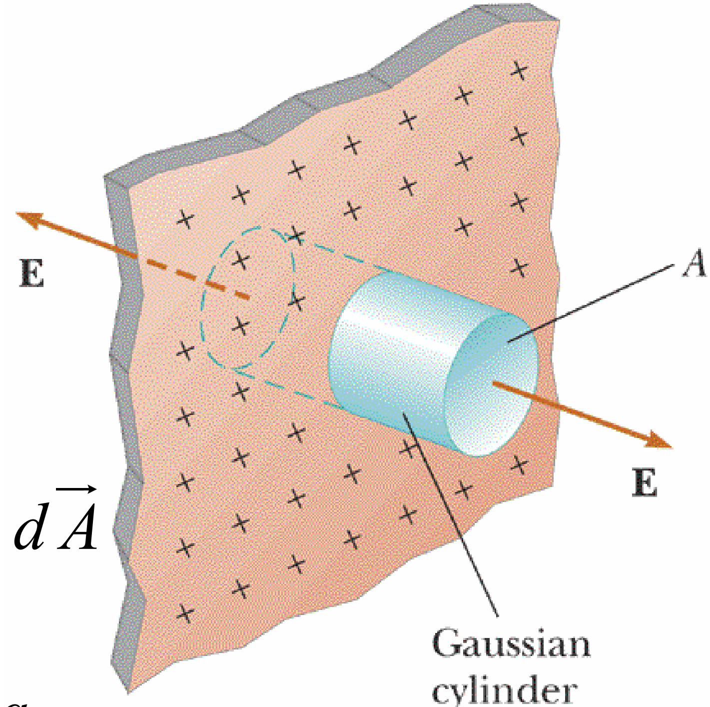
A Nonconducting plane sheet of charge

Find the electric field due to a non-conducting, infinite plane with uniform surface charge density σ.



A Nonconducting plane sheet of charge

Solution: Select the Gaussian surface to be a cylinder whose axis is perpendicular to the plane and whose ends each have an area *A* and are equidistant from the plane.



$$\oiint_S \vec{E}\cdot d\vec{A} = \iint_{\text{side surface}} \vec{E}\cdot d\vec{A} + \iint_{\text{two ends of the cylinder}} \vec{E}\cdot d\vec{A}$$

$$= \iint_{\text{two ends of the cylinder}} \vec{E}\cdot d\vec{A} = E(2A) = \frac{q_{in}}{\varepsilon_0}$$

$$= \frac{\sigma A}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

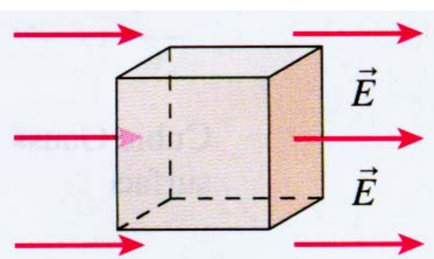
How to Choose the Gaussian Surfaces?

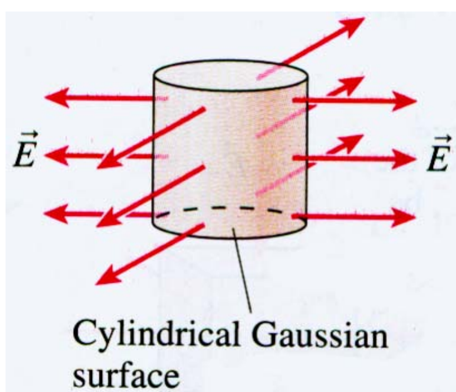
$$\Phi_E = \oiint_S \vec{E} \cdot d\vec{A} = \frac{q_{inside}}{\varepsilon_0}$$

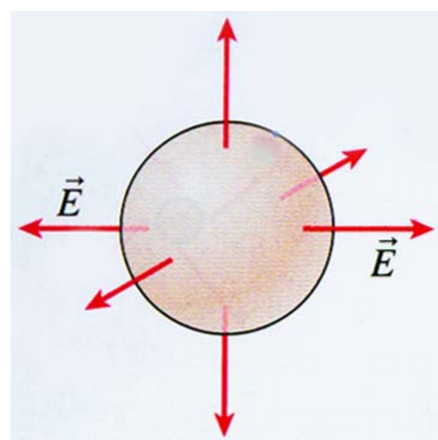
- **How to choose the gaussian surfaces?**
  - **The choice of appropriate gaussian surface that allows *E* to be removed from the integral in Gauss's law is the key problem.**

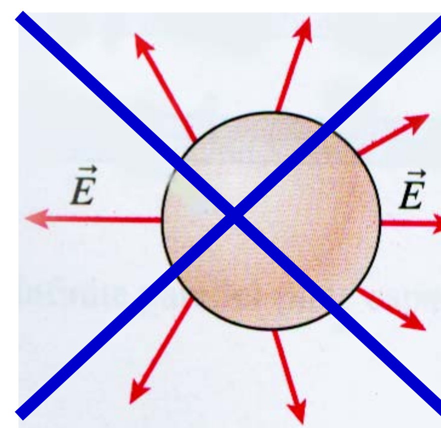
$$\vec{E} \cdot d\vec{A}$$

  - **With the appropriate gaussian surface, the dot product should be zero or equal to $E\,dA$, with the magnitude of E being constant.**









Problems

Ch20 Prob. 26, 33,34 (P499, 500)