# **Optics**

## Haskell and Cryptocurrencies

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### Goals

- Semigroup & Monoid
- · Identity
- Traversable
- Lenses
- Traversals

Semigroup & Monoid

# Semigroup

## class Semigroup a where

## Semigroup

### class Semigroup a where

#### Law

The operation (<>) should be associative:

$$x <> (y <> z) == (x <> y) <> z$$

## Monoid

```
class Semigroup m => Monoid m where
  mempty :: m
```

## Monoid

```
class Semigroup m => Monoid m where
  mempty :: m
```

#### Laws

```
mempty should be a neutral element for (<>):
x <> mempty == mempty <> x == x
```

## Example: Lists

```
instance Semigroup [a] where
  (<>) = (++)
```

```
instance Monoid [a] where
mempty = []
```

```
GHCi> "Haskell" <> mempty <> "Mongolia"
"HaskellMongolia"
```

## Example: Sum

```
newtype Sum a = Sum {getSum :: a}
```

```
instance Num a => Semigroup (Sum a) where
Sum a <> Sum b = Sum (a + b)
```

```
instance Num a => Monoid (Sum a) where
mempty = Sum 0
```

```
GHCi> getSum $ Sum 3 <> mempty <> Sum 7 10
```

## Example: **Product**

```
newtype Product a = Product {getProduct :: a}
```

```
instance Num a => Semigroup (Product a) where
  Product a <> Product b = Product (a * b)
```

```
instance Num a => Monoid (Product a) where
  mempty = Product 1
```

```
GHCi> getProduct $
  Product 3 <> mempty <> Product 7
21
```

## Example: First

```
newtype First a = First {getFirst :: Maybe a}
```

```
instance Semigroup (First a) where
First (Just a) <> _ = First (Just a)
First Nothing <> x = x
```

```
instance Monoid (First a) where
mempty = First Nothing
```

```
GHCi> getFirst $
  mempty <> First (Just 'x') <> First (Just 'y')
Just 'x'
```

## Example: Last

```
newtype Last a = Last {getLast :: Maybe a}
instance Semigroup (Last a) where
   _ <> Last (Just a) = Last (Just a)
   x <> Nothing = x
```

```
instance Monoid (Last a) where
mempty = Last Nothing
```

```
GHCi> getLast $
  mempty <> Last (Just 'x') <> Last (Just 'y')
Just 'y'
```

```
instance Monoid (Endo a) where
  mempty = Endo id

GHCi> (Endo succ <> mempty <> Endo (*2))
  `appEndo` 5
11
```

newtype Endo a = Endo {appEndo :: a -> a}

instance Semigroup (Endo a) where
Endo f <> Endo g = Endo (f . g)

# Identity

```
newtype Identity a = Identity {runIdentity a}
```

```
instance Monad Identity where
return = Identity
Identity a >>= cont = cont a
```

```
GHCi> runIdentity $ do
  x <- Identity 42
  let y = 8
  return $ x + y
50
```

## Traversable

```
GHCi> mapM print [1, 2, 3]

1

2

3
[(),(),()]
```

```
mapA :: Applicative f \Rightarrow (a \rightarrow fb) \rightarrow [a] \rightarrow f[b]
mapA [] = pure []
mapA f (a : as) = (:) < f a < * > mapA f as
GHCi> mapA print [1, 2, 3]
1
[(),(),()]
```

## Other uses of mapA

Can we use mapA in place of map? What Applicative f would we have to use?

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Can we use mapA in place of map? What Applicative f would we have to use?

Let's try Identity!

GHCi> runIdentity
  (mapA (Identity . succ) [1, 2, 3])
[2, 3, 4]
```

## Other uses of mapA

```
Can we use mapA in place of map? What Applicative f would we have to use?

Let's try Identity!

GHCi> runIdentity
  (mapA (Identity . succ) [1, 2, 3])

[2, 3, 4]
```

So we can define map in terms of mapA:

```
map :: (a -> b) -> [a] -> [b]
map f = runIdentity . mapA (Identity . f)
```

## Other uses of mapA (cntd.)

What about foldMap? Can we get that with mapA as well?

```
foldMap :: Monoid m => (a -> m) -> [a] -> m
```

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What about foldMap? Can we get that with mapA as well?

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foldMap :: Monoid m => (a -> m) -> [a] -> m
```

We need an Applicative f that can store a value of type m, independent of the type it is applied to.

## Other uses of mapA (cntd.)

What about foldMap? Can we get that with mapA as well?

```
foldMap :: Monoid m => (a -> m) -> [a] -> m
```

We need an Applicative f that can store a value of type m, independent of the type it is applied to.

```
data Const a b = Const {getConst :: a}
```

```
instance Functor (Const a) where
fmap _ (Const a) = Const a
```

# Making Const Applicative

But is **Const** an instance of **Applicative**?

```
But is Const an instance of Applicative?
```

Not in general, but we only need it to be when **a** is a **Monoid**:

```
instance Monoid m => Applicative (Const m) where
  pure _ = Const mempty
  Const m <*> Const n = Const (m <> n)
```

Now we can implement foldMap in terms of mapA:

```
foldMap :: Monoid m => (a -> m) -> [a] -> m
foldMap f = getConst . mapA (Const . f)

GHCi> getSum (foldMap Sum [1, 2, 3])
6
```

Now we can implement **foldMap** in terms of **mapA**:

```
foldMap :: Monoid m => (a -> m) -> [a] -> m
foldMap f = getConst . mapA (Const . f)
```

```
GHCi> getSum (foldMap Sum [1, 2, 3])
6
```

It seems mapA is very powerful, providing us with Functor and Foldable instances for [], in addition to doing effectful mappings.

Having seen the power of mapA for lists, what about other "container" types like trees?

```
data Tree a = Leaf a | Bin (Tree a) (Tree a)
  deriving Show
```

```
GHCi> mapT print (Bin (Leaf 1) (Leaf 2))

1

2

Bin (Leaf ()) (Leaf ())
```

Having seen the power of mapA for lists, what about other "container" types like trees?

```
data Tree a = Leaf a | Bin (Tree a) (Tree a)
  deriving Show
```

```
GHCi> runIdentity (mapT (Identity . succ))
  (Bin (Leaf 1) (Leaf 2))
Bin (Leaf 2) (Leaf 3)
```

Having seen the power of <a href="mapA">mapA</a> for lists, what about other "container" types like trees?

```
data Tree a = Leaf a | Bin (Tree a) (Tree a)
  deriving Show
```

```
GHCi> getSum (getConst (mapT (Const . Sum)))
  (Bin (Leaf 1) (Leaf 2))
3
```

## The **Traversable** class

### From Data.Traversable:

```
class (Functor t, Foldable t)
  => Traversable t where
  traverse :: Applicative f
    => (a -> f b) -> t a -> f (t b)
```

Similarly to how we can use liftM and ap to define

Functor and Applicative instances, once we have

defined return and (>>=), Data.Traversable provides

fmapDefault and foldMapDefault to implement

Functor and Foldable, once we have defined

traverse.

## Tree as Traversable

### instance Functor Tree where

fmap = fmapDefault

### instance Foldable Tree where

foldMap = foldMapDefault

### instance Traversable Tree where

traverse = mapT

#### Note

Intuitively, for a **Traversable** t, t a is like a "container" for a 's that you can inspect and manipulate.

## Composing traverse

Composing several **traverse** 's let's us traverse nested containers!

```
GHCi> (traverse . traverse) print
  (Bin (Leaf [1, 2]) (Leaf [3, 4]))
1
2
3
4
Bin (Leaf [(), ()]) (Leaf [(), ()])
```

## Lenses

### Record types

#### Record types

As a quick exercise, implement a function

```
goTo :: String -> Company -> Company
```

that takes the name of city and a **Company** and moves all company staff to that city!

### Record types

```
data Company = Company { _staff :: [Person]}
data Person = Person { name :: String
                      , address :: Address }
data Address = Address { city :: String }
goTo :: String -> Company -> Company
goTo there c = c { staff = map movePerson ( staff c)}
 where
   movePerson p = p { address = ( address p)
                               { city = there}}
```

# Taking stock

- · What have we learned in this exercise?
- While record accessors are fine for flat records, they become a pain for handling (deeply) nested records.
- If we were in a language like C, Java or Python, we would use the \_\_-accessor to navigate deeply into a nested data structure and manipulate it in place.
- In Haskell, we prefer to use immutable data structures when possible.
- · Does that mean we are doomed?

# Taking stock

- · What have we learned in this exercise?
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- In Haskell, we prefer to use immutable data structures when possible.
- · Does that mean we are doomed?

#### Plan

This is Haskell, after all! We have a programmable ; , so let's create a programmable .!

#### Teaser

By the end of this lecture, we will be able to write goTo like this:

```
goTo :: String -> Company -> Company
goTo s c = set (staff . each . address . city) c s
```

#### **Getters & setters**

```
_staff :: Company -> [Person]
```

Record accessors are just functions, and therefore composable, which is good. But if we want to *update* a record, we have to use record syntax, and composability breaks down.

Let's change that and make accessors "first-class citizens"!

#### **Getters & setters**

```
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```

Record accessors are just functions, and therefore composable, which is good. But if we want to *update* a record, we have to use record syntax, and composability breaks down.

Let's change that and make accessors "first-class citizens"!

# **Writing lenses**

```
staff :: Lens Company [Person]
staff = Lens staff (\c ps -> c { staff = ps})
name :: Lens Person String
name = Lens name (\p n -> p { name = n})
address :: Lens Person Address
address = Lens address (\pa -> p { address = a})
city :: Lens Address String
city = Lens city (\ac -> a { city = c})
```

# Writing lenses (cntd.)

This is easy, but fairly mechanical and boring.

So mechanical and boring, in fact, that it can be automated via Template Haskell or datatype-generic programming, topics we will hopefully learn about later in this course.

## Trying our lenses

Let's take our shiny new lenses for a spin!

```
GHCi> get name lars
"Lars"
```

```
GHCi> set name lars "Dr. Lars"
Person {_name = "Dr. Lars",
   _address = Address {_city = "Regensburg"}}
```

So far, so good. But not much better than plain old record syntax – yet.

#### Other lenses

Even though we motivated lenses with records, the concept applies to many more situations:

```
_1 :: Lens (a, b) a
_1 = Lens fst (\( (_, b) a -> (a, b))

_2 :: Lens (a, b) b
_2 = Lens snd (\( (a, _) b -> (a, b))
```

```
GHCi> set _2 ('x', False) True ('x', True)
```

#### Other lenses (cntd.)

We can even leave the realm of product types altogether:

```
data Sign = Plus | Zero | Minus
```

```
sign :: Lens Int Sign
sign = Lens gt st
 where
   gt n
      | n > 0 = Plus
      | n == 0 = Zero
      l otherwise = Minus
   st n Plus = if n == 0 then 1 else abs n
   st Zero = 0
   st n Minus = if n == 0 then (-1) else - (abs n)
```

### Other lenses (cntd.)

We can even leave the realm of product types altogether:

```
data Sign = Plus | Zero | Minus
```

```
Sign :: Lens Int Sign

GHCi> get sign (- 42)

Minus

GHCi> set sign (- 42) Plus

42

GHCi> set sign 111 Zero

0
```

#### Lawful lenses

There are lens laws, too; every lens should obey them, but some don't and might still be useful:

- get set: You get back what you set: get l (set l s a) = a
- set get: Setting what you got does not change anything:set l s (get l s) = s
- set set: Setting twice is the same as setting once: set l (set l s a') a = set l s a

The "lens" on the last slide actually violates one of the laws – can you spot which one?

### An Iso example

```
lazy :: Lens S.ByteString L.ByteString
lazy = Lens L.fromStrict (\s a -> L.toStrict a)
```

This lens obeys the laws and is quite useful. But it is even stronger than a "normal" lens, it is a so-called iso.

We will talk about isos in the next lecture.

#### Another useful lens

```
at :: Ord k => k -> Lens (Map k v) (Maybe v)
at k = Lens gt st
   where
   gt = lookup k
   st m Nothing = delete k m
   st m (Just v) = insert k v m
```

```
GHCi> let m = set (at "Mongolia") empty
  (Just "Ulaanbaatar")
GHCi> get (at "Mongolia") m
Just "Ulaanbaatar"
GHCi> get (at "USA") m
Nothing
```

#### Another useful lens

```
at :: Ord k => k -> Lens (Map k v) (Maybe v)
at k = Lens gt st
  where
  gt = lookup k
  st m Nothing = delete k m
  st m (Just v) = insert k v m
```

```
GHCi> set (at "USA") m
  (Just "Washington DC")
fromList [("Mongolia", "Ulaanbaatar"),
  ("USA", "Washington DC")]
GHCi> set (at "Mongolia") m Nothing
fromList []
```

# Composing lenses

Having city:: Lens Address String and address:: Lens Person Address, we would like to compose those two to get a Lens Person String. Let's do that next!

```
compose :: Lens a x -> Lens s a -> Lens s x
compose ax sa = Lens
  { get = get ax . get sa
  , set = \ s x -> set sa s (set ax (get sa s) x)
  }
}
```

# Composing lenses (cntd.)

```
GHCi> let ca = compose city address
GHCi> get ca alejandro
"Zacatecas"
```

```
GHCi> set ca alejandro "Ulaanbaatar"
Person {_name = "Alejandro",
   _address = Address {_city = "Ulaanbaatar"}}
```

### From Control.Category:

```
class Category (cat :: k -> k -> *) where
  id :: forall (a :: k) . cat a a
  (.) :: forall (b :: k) (c :: k) (a :: k) .
      cat b c -> cat a b -> cat a c
```

In order to use it, you have to hide (.) and id from the **Prelude**:

```
import Prelude hiding ((.), id)
```

# Turning lenses into a category

Before we can write a **Category** instance for our **Lens** type, we need an *identity lens*, i.e. one that zooms in into itself, but that is easy.

```
instance Category Lens where
id = Lens id (\ _ a -> a)
(.) = compose
```

```
GHCi> get (city . address) lars
"Regensburg"
```

## Taking stock again

What does goTo look like now?

```
goTo :: String -> Company -> Company
goTo there c = set staff c
  (map movePerson (get staff c))
where
  movePerson p = set (city . address) p there
```

The **movePerson** -part is much nicer now, but overall composability still leaves room for improvement.

### Updating

Instead of just getting and setting, we would like to be able to update parts of data, too. We can of course do that by combining getting and setting:

```
over :: Lens s a -> (a -> a) -> s -> s
over sa f s = set sa s (f (get sa s))
```

```
GHCi> over name (map toUpper) lars
Person {_name = "LARS",
   _address = Address {_city = "Regensburg"}}
```

# Changing the **Lens** type

Update works, but it is not very efficient for composed lenses to descend deep into a data structure, grab the value, apply a function, then descend all the way down again to put in the new value.

Seeing as setting is just a special form of updating, why don't we promote **over** to constructor status?

```
set :: Lens s a -> s -> a -> s
set sa s a = over sa (const a) s
```

We can still construct a lens from getter and setter:

```
lens :: (s -> a) -> (s -> a -> s) -> Lens s a
lens gt st = Lens gt (\ f s -> st s (f (gt s)))
```

Then we only have to slightly change the implementation of our sample lenses.

# Rewriting our lenses

```
staff :: Lens Company [Person]
staff = lens staff (\c ps -> c { staff = ps})
name :: Lens Person String
name = lens name (\p n -> p { name = n})
address :: Lens Person Address
address = lens address (\pa -> p { address = a})
city :: Lens Address String
city = lens city (\ac -> a { city = c})
```

And we have to adapt our **Category** instance:

```
compose :: Lens a x -> Lens s a -> Lens s x
compose ax sa = Lens
  { get = get ax . get sa
  , over = over sa . over ax
  }
```

```
instance Category Lens where
id = Lens id ($)
(.) = compose
```

#### Note

Note how nicely **over** composes!

### Effectful updates

What if we want effectful updates of parts of our data structures?

```
overIO :: Lens s a -> (a -> IO a) -> s -> IO s
```

One solution would be to add another constructor to our **Lens** type:

```
data Lens s a = Lens
  {get     :: s -> a
  , over     :: (a -> a) -> s -> s
  , overIO :: (a -> IO a) -> s -> IO s
  }
```

## Effectful updates

What if we want effectful updates of parts of our data structures?

But we don't have to! Instead, we can implement over10 just using get and set:

```
overIO :: Lens s a -> (a -> IO a) -> s -> IO s
overIO sa g s = set sa s <$> g (get sa s)
```

## Effectful updates

What if we want effectful updates of parts of our data structures?

We have used no special properties of 10, not even that it is a monad – we only used fmap. So we can generalize:

This looks a lot like the signature of traverse!

## Effectful update example

#### Let's try this!

```
askName :: String -> IO String
askName n = do
  putStrLn ("old name was: " ++ n)
  getLine
```

```
GHCi> overF name askName lars
old name was: Lars
LARS
Person {_name = "LARS",
   _address = Address {_city = "Regensburg"}}
```

We can replace over with overF in our definition of lens ...

 $\dots$  and recover  $\begin{tabular}{ll} over using & Identity & for & f : \\ \end{tabular}$ 

```
over :: Lens s a -> (a -> a) -> s -> s
over sa f s =
  runIdentity (overF sa (Identity . f) s)
```

## overF is enough

We can drop **get** from the definition ...

 $\dots$  and still define  $\ensuremath{\mbox{get}}$  using  $\ensuremath{\mbox{Const}}$  a for  $\ensuremath{\mbox{f}}$  :

```
get :: Lens s a -> s -> a
get sa s = getConst (overF sa Const s)
```

#### van Laarhoven lenses

At this point, we see that we do not even *need* a **data** declaration any longer. We can just define a type synonym:

```
type Lens s a = forall f . Functor f
=> (a -> f a) -> s -> f s
```

These lenses are called van Laarhoven lenses.

Changing from a data type to a type synonym is a double edged sword. There are clear advantages to keeping a data type abstract. In this case though, we will see that we gain two important advantages:

- · Easy composability and
- A form of "subtyping" (which we'll understand better once we'll have learned about traversals, prisms and isos later today and in the next lecture).

#### van Laarhoven lenses

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```
type Lens s a = forall f . Functor f
=> (a -> f a) -> s -> f s
```

These lenses are called van Laarhoven lenses.

We can recover over and get:

```
over :: Lens s a -> (a -> a) -> s -> s
over sa f s =
  runIdentity (sa (Identity . f) s)
```

```
get :: Lens s a -> s -> a
get sa s = getConst (sa Const s)
```

#### van Laarhoven lenses

At this point, we see that we do not even *need* a **data** declaration any longer. We can just define a type synonym:

```
type Lens s a = forall f . Functor f
=> (a -> f a) -> s -> f s
```

These lenses are called van Laarhoven lenses.

And we can still construct a lens from a getter and a setter:

```
lens :: (s -> a) -> (s -> a -> s) -> Lens s a
lens gt st f s = st s <$> f (gt s)
```

This means that the definition of all our example lenses remains literally the same!

#### Composing van Laarhoven lenses

One of the nicest features of our previous attempts was composability. Now we do not even have a data type definition anymore, so we can't define a Category instance for our new lenses ...

### Composing van Laarhoven lenses

One of the nicest features of our previous attempts was composability. Now we do not even have a data type definition anymore, so we can't define a Category instance for our new lenses ...

...But we don't have to! Van Laarhoven lenses are just functions, and we know how to compose functions!

```
GHCi> get (address . city) lars
"Regensburg"
```

#### Note

Note that the order of composition has swapped! Now it looks like object accessors in languages like Java.

#### Where are we?

By using van Laarhoven lenses, we have drastically simplified our **Lens** type.

Composition of lenses is just function composition now.

We also have "built in" effectful updates, in addition to the more basic features of getting, setting and updating.

However, **goTo** still looks the same (except for the swapped order of composition).

But now we are in a position to fix that!

Traversals

#### Motivation

Lenses allow us to "zoom in" on one part of a structure.

They are naturally composable, because they are just functions.

Given a structure with many parts (of the same type), we would like to zoom in on those "simultaneously".

We also want to compose such Traversals with lenses and with eachother, so they should have a similar shape.

Method **traverse** of class **Traversable** has a very promising signature. This leads us to our definition of **Traversal**.

#### Traversal

```
type Traversal s a = forall f . Applicative f
=> (a -> f a) -> (s -> f s)
```

A Traversable functor t gives us a Traversal via traverse:

```
each :: Traversable t => Traversal (t a) a
each = traverse
```

We defined **over** for lenses, but looking back at the definition, we don't actually *need* the full power of a lens. We only need the special case **f = Identity**. So let's change the signature of **over**, the implementation can stay exactly as it was:

We defined **over** for lenses, but looking back at the definition, we don't actually *need* the full power of a lens. We only need the special case **f = Identity**. So let's change the signature of **over**, the implementation can stay exactly as it was:

And we do the same for **set**:

```
set :: ((a -> Identity a) -> s -> Identity s)
    -> s -> a -> s
set sa s a = over sa (const a) s
```

### over and set for Traversal's

We defined **over** for lenses, but looking back at the definition, we don't actually *need* the full power of a lens. We only need the special case **f = Identity**. So let's change the signature of **over**, the implementation can stay exactly as it was:

#### Let's try it!

```
GHCi> set each [1, 2, 3] 0 [0, 0, 0]
```

#### both

As another example for a **Traversal**, we can traverse over both components of a pair if both have the same type:

```
both :: Traversal (a, a) a
both f (a, b) = (, ) <$> f a <*> f b
```

```
GHCi> set both (1, 2) 0 (0, 0)
```

We can do the same we did for **over** and **set** for **get** – but for historical reasons, we call the resulting function **view**.

We can do the same we did for **over** and **set** for **get** – but for historical reasons, we call the resulting function **view**.

For lenses, this works as expected:

```
GHCi> view name lars
"Lars"
```

We can do the same we did for **over** and **set** for **get** – but for historical reasons, we call the resulting function **view**.

For traversals, however, we seem to be out of luck...

```
GHCi> view both (True, False)
< interactive >: 6 : 6 : error :
   No instance for (Monoid Bool) arising from a use of both
   In the first argument of view, namely both
   In the expression : view both (True, False)
   In an equation for it : it = view both (True, False)
```

...and we remember that Const a is only Applicative if a is a Monoid.

We can do the same we did for **over** and **set** for **get** – but for historical reasons, we call the resulting function **view**.

```
GHCi> view both ([True], [False])
[True, False]
```

To make viewing **Traversal** s easier, we define:

```
GHCi> toListOf both (True, False)
[True, False]
```

## Composing Lens es & Traversal s

Because Lens es and Traversal s are just functions of a compatible shape, we can compose them freely:

Composing two Lens es gives a Lens.

```
GHCi> set (address . city) lars "Ulaanbaatar"
Person {_name = "Lars",
   _address = Address {_city = "Ulaanbaatar"}}
```

## Composing Lens es & Traversal s

Because Lens es and Traversal s are just functions of a compatible shape, we can compose them freely:

Composing two Traversal s gives a Traversal:

```
GHCi> set (each . each) [[1], [2, 3]] 0 [[0], [0, 0]]
```

## Composing Lens es & Traversal s

Because Lens es and Traversal s are just functions of a compatible shape, we can compose them freely:

Composing a Traversal with a Lens gives a Traversal ...

```
GHCi> set (each . _1) [(1, 'x'), (2, 'y')] 0 [(0, 'x'), (0, 'y')]
```

## Composing Lens es & Traversals

Because Lens es and Traversal s are just functions of a compatible shape, we can compose them freely:

```
...and so does composing a Lens with a Traversal:
```

```
GHCi> set (_2 . each) (True, "Ulaanbaatar") 'x'
(True, "xxxxxxxxxxxx")
```

## goTo again

Now we are finally in a position to write **goTo** in a very nice and compact way:

```
goTo :: String -> Company -> Company
goTo s c = set (staff . each . address . city) c s
```

```
GHCi> goTo "Ulaanbaatar" iohk
Company
 { staff =
   [ Person { name = "Alejandro"
            , _address = Address {_city = "Ulaanbaatar"}
   , Person { name = "Lars"
            , _address = Address {_city = "Ulaanbaatar"}
```