A3 Simulative Engineering - Learning paper is NOT allowed in exam

Contents

1 **Definitions** 1

2 **ODEs** 4
- 2.1 How to solve an ODE? 4
- 2.2 How to get $G(s)$? 4
- 2.3 Solvers 4

3 **DFT** 4

1 Definitions

Simulation is the imitation of the operation of a real world process or system over time (e.g. via MATLAB/Simulink)

Hardware in the loop (HiL) means testing of software in combination with an existing hardware component OR: HiL is a technique for testing a embedded system by simulating the real environment around the embedded system. (Fig. 1)

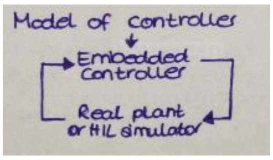

Figure 1: Hardware in the loop (HiL)

Software in the loop (SiL) is a simulation technique for software models only with a simulated hardware and not with a realworld existing hardware OR SiL is a technique for testing software by simulating the target hardware (Fig. 2).

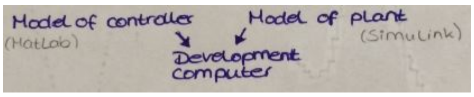

Figure 2: Software in the loop (SiL)

Model is an abstraction from realworld objects; mostly easier to display.

Fourier-Transform transforms time domain based signals into frequendy domain based signals; needed for calculation of signals

Time discrete means that a signal is only defined at specifiv time values (Fig. 3).

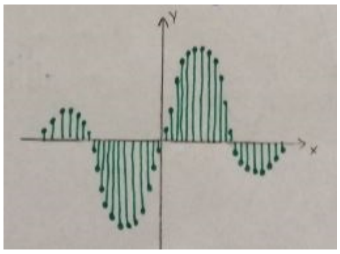

Figure 3: Time discrete signal: A y-value exists only for specific x-values.

Time continuos means that a signal is defined for the whole time or for an specific interval (Fig. 4).

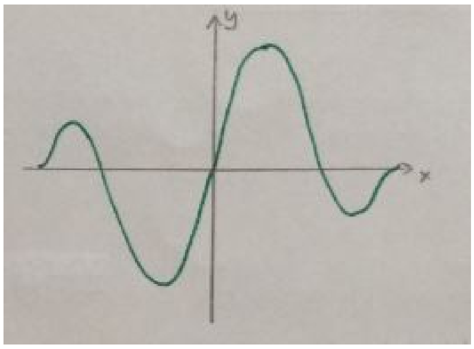

Figure 4: Time continuos signal: A y-value exists for every x-value (in the intervall).

Discrete values A signal can be measured in time ranges (time continuos) (Fig. 5) or at specific time values (time discrete)(Fig. 6). Each y-value in one time range has got the same value. Discrete values can be counted.

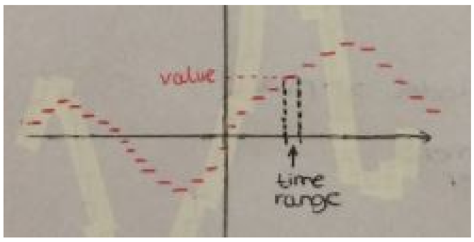

Figure 5: Continuos X, discrete Y

Continuos values can be measured and can take any

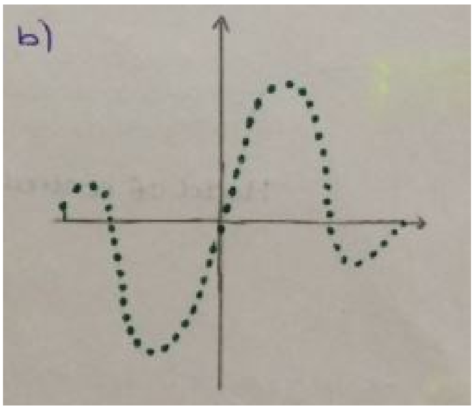

Figure 6: Discrete X, discrete Y

value ((Fig. 7) and (Fig. 8))

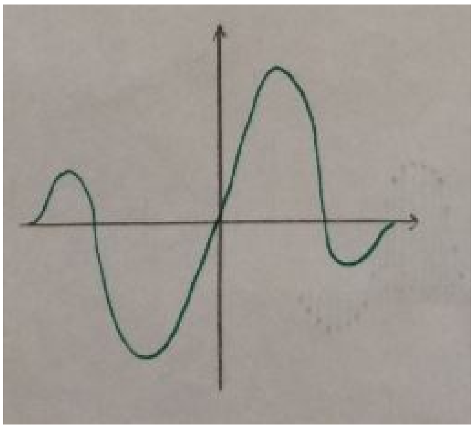

Figure 7: Continuos X, continuos Y

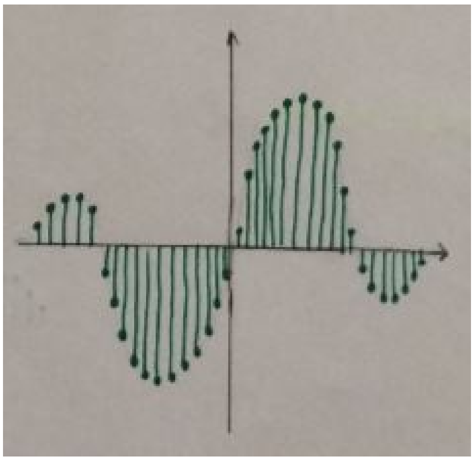

Figure 8: Discrete X, continuos Y

Discrete Fourier Transform (DFT) is a stand alone definition for time discrete signals of limited duration besides the Fourier Transform (Fig. 9).

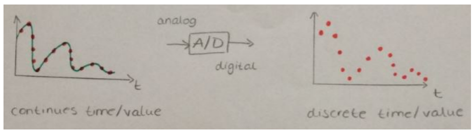

Figure 9: Analog-Digital-Converter

Dirac distribution $\delta(t)$ ist not function, but a distribution. It cannot be defined directly, but has to be described indirectly e.g. by intregration.

1. $\delta(t) = 0$ for all $t \neq 0$ and $\int_{-\infty}^{+\infty} \delta(t)dt = 1$

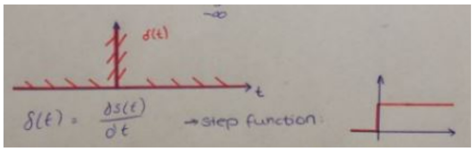

Figure 10: Dirac

2. Sifting property (similar to window function) $\int_{-\infty}^{+\infty} f(t) \cdot \delta(t)dt = f(0)$

3. $\delta(t) = \int_{-\infty}^{+\infty}(\lim\limits_{n\to\infty} \frac{1}{2\varepsilon}[s(t+\varepsilon) - s(t-\varepsilon)]); \varepsilon > 0$

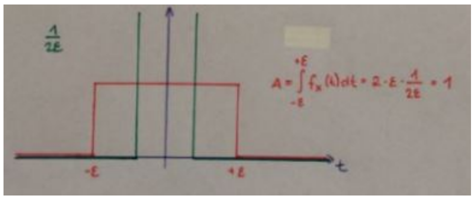

Figure 11: Dirac2

Dirac comb III_1 is a periodic funtion of the Dirac distribution. The ideal sampling can be described by Dirac distributions located with distances T_s which results in a spectrum of Dirac distributions having the distance $\frac{1}{T_s}$ (Fig. 12).

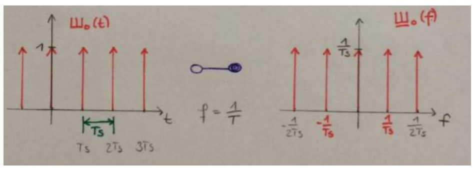

Figure 12: Dirac Comb

Algorithm is a self-contained step by step set of operations to be performed in order to get a solution. Algorithms perform calculations, data processing and/or automated reasoning tasks.

DFT/FFT The Fast Fourier Transform (FFT) is a special algorithm for calculating the DFT.

Sequence is a ordered collection of objects in which repetitions are allowed. The number of elements is called the length n of a sequence and can be infinite. The position of the elements is called index. E.g. prim numbers until 41:

prim numbers	2	3	5	7	...	37	41
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$		$\downarrow$	$\downarrow$
index	1	2	3	4	...	12	13

The Length of this sequence is $n = 13$.

Sampling converts a continuos time signal into a discrete time signal. Sampling is done by a multiplication of the signal $u(t)$ with the dirac comb. A multiplication in the time domain results in a convolution in frequency domain. It is performed over a limited time.

Time domain and frequency domain The function values of a signal in time domain depend on time values. In the frequency domain the function values depend n frequency values.

$f(t) \circ\!\!-\!\!\bullet F(f)$ with $f = \frac{1}{T}$

Periodic funtion repeats itself after a period of time T. Examples: $\sin(x), \cos(x)$

Limited function is only defined for a specific time range. A limited function in time domain contains an unlimited spectrum of frequencies in frequency domain.

Convolution is a mathematical operation on two functions, similar to cross-correlation (system theory). The slution is a third function being the first function modified by the second one. Multiplication in time domain $\rightarrow$ Convolution in frequency domain.

Frequency spectrum is a composition of different frequencies.

Amplitude spectrum is the absolut value of a frequency spectrum OR is a composition of different amplitudes.

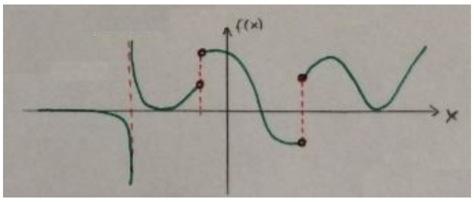

Figure 13: Example of a function with discontinuities

Discontinuities appear where a function is not continuous (Fig. 13).

Sifting property of Dirac distribution means that the function value $f(t = 0)$ can be extracted with the mulitplication of the function with the Dirac distribution.

$\int_{-\infty}^{+\infty} f(t) \cdot \delta(t) dt = f(0)$ because of $\delta(t) = \begin{cases} 0, & t \neq 0 \\ \infty, & t = 0 \end{cases}$ and $\int_{-\infty}^{+\infty} \delta(t) dt = 1$

Window function is a (e.g. rectangular) function used as a window for calculating a DFT. When a function is multiplied by a window function, the product is zero-valued outside of the window's interval; all that is left ist the part where they overlap, the "view though the window" (Fig. 14).

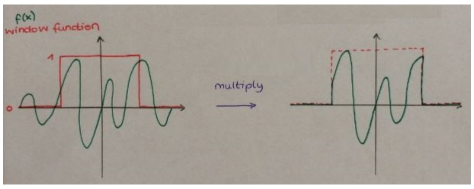

Figure 14: Use of a window function

Zero padding adds zeros to the end of signals to compress the signal (Fig. 15).

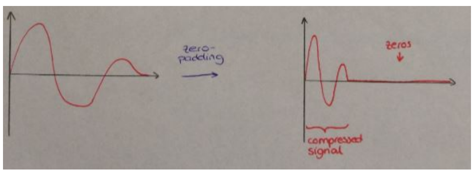

Figure 15: Zero padding

2 ODEs

2.1 How to solve an ODE?

An ODE of n-th order is given. Change it to n ODEs of 1st order via substitution. Now you can use MATLAB to solve it.

Example:
$y^{(3)} + a_2 y'' + a_1 y' + a_0 y = b$

Subsitution	Derivation
$y_1 = y$	$y_1' = y' = y_2$
$y_2 = y'$	$y_2' = y'' = y_3$
$y_3 = y''$	$y_3' = y^{(3)} = b - a_2 y_3 - a_1 y_2 - a_0 y_1$

2.2 How to get $G(s)$?

Transform the ODE from the time into the frequency domain.

Derivatives	Integrals
$y(t) \circ\!\!-\!\!\bullet Y(s)$	$\int y(t)dt \circ\!\!-\!\!\bullet \frac{1}{s}Y(s)$
$y'(t) \circ\!\!-\!\!\bullet sY(s)$	$\int\int y(t)dt \circ\!\!-\!\!\bullet \frac{1}{s^2}Y(s)$
$y''(t) \circ\!\!-\!\!\bullet s^2Y(s)$	$\int\int\int y(t)dt \circ\!\!-\!\!\bullet \frac{1}{s^3}Y(s)$
and so on...	and so on...

Exclude the output signal and calculate $G(s) = \frac{output}{input}$.

Example:
$y^{(3)} + a_2 y'' + a_1 y' + a_0 y = b$
with b as input signal and y as output signal.
Laplace Transform:
$s^3Y(s) + s^2 a_2 Y(s) + s a_1 Y(s) + a_0 Y(s) = B(s)$
$Y(s)(s^3 + s^2 a_2 + s a_1 + a_0) = B(s)$
$G(s) = \frac{Y(s)}{B(s)} = \frac{1}{s^3+s^2a_2+sa_1+a_0}$

If the input signal is 0, then integrate and solve the equation for the output variable.

2.3 Solvers

To learn the different solvers, please go to 3.3 - 3.9 in your notes of the lecture.

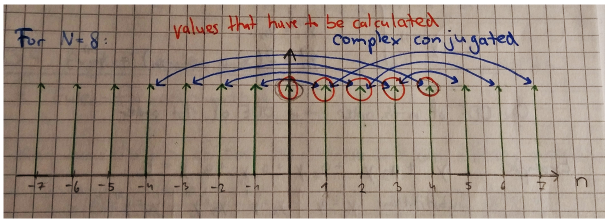

Figure 16: Complex conjugated values

3 DFT

Fourier Transform transforms time continuos values in time domain into values in frequency domain.

$u(t) \circ\!\!-\!\!\bullet \underline{U}(f) = \int_{-\infty}^{+\infty} u(t) \cdot e^{-j2\pi f t} dt$

Discrete Fourier Transform is the Fourier Transform for time discrete values.

$u(kT_s) \circ\!\!-\!\!\bullet \tilde{\underline{U}}(\frac{n}{N \cdot T_s}) = \sum_{k=0}^{N-1} u(kT_s) \cdot e^{-j2\pi n \frac{k}{N}}$

with N: number of sampled values
and $n = 0, 1, 2, ..., N-1$.

Quantity of must-calculate-values: $\frac{N}{2} + 1$ (Fig. 16)

Solution vector for $N = 8$:

$$\tilde{\underline{U}}(\tfrac{n}{N\cdot T_s}) = \begin{bmatrix} \tilde{\underline{U}}(\frac{0}{N\cdot T_s}) \\ \tilde{\underline{U}}(\frac{1}{N\cdot T_s}) \\ \tilde{\underline{U}}(\frac{2}{N\cdot T_s}) \\ \tilde{\underline{U}}(\frac{3}{N\cdot T_s}) \\ \tilde{\underline{U}}(\frac{4}{N\cdot T_s}) \\ \tilde{\underline{U}}(\frac{5}{N\cdot T_s}) \\ \tilde{\underline{U}}(\frac{6}{N\cdot T_s}) \\ \tilde{\underline{U}}(\frac{7}{N\cdot T_s}) \\ \tilde{\underline{U}}(\frac{8}{N\cdot T_s}) \\ \tilde{\underline{U}}(\frac{9}{N\cdot T_s}) \\ \vdots \end{bmatrix} = \begin{array}{c} \text{just calculate} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \text{complex conjugated of } \tilde{\underline{U}}(\frac{3}{N\cdot T_s}) \\ \text{complex conjugated of } \tilde{\underline{U}}(\frac{2}{N\cdot T_s}) \\ \text{complex conjugated of } \tilde{\underline{U}}(\frac{1}{N\cdot T_s}) \\ \tilde{\underline{U}}(\frac{0}{N\cdot T_s}) \\ \tilde{\underline{U}}(\frac{1}{N\cdot T_s}) \\ \vdots \end{array}$$

Here you have to calculate $\frac{8}{2} + 1 = 5$ values (namely $n = 0, 1, 2, 3, 4$). from who the other ones (namley $n = 5, 6, 7$) can be deduced. The N-th value is the same as $n = 0$ because of the periodic repitition.

Inverse discrete Fourier Transform calculates the values in time domain out of the DFT.

$u(kT_s) = \frac{1}{N}\sum_{n=0}^{N-1} \underline{\tilde{U}}(\frac{n}{N \cdot T_s}) \cdot \mathrm{e}^{j2\pi n \frac{k}{N}}$

with $k = 0, 1, 2, ..., N-1$.