MAPK Networks: Compute Numerical Simulation

In [1]:

```
import scipy as sp
import scipy.integrate as integrate
import numpy as np
import matplotlib.pyplot as plt
import os
import subprocess
%config InlineBackend.figure format = 'svg'
```

In [4]:

```
def systemOfEquations(variables, time):
    x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11 = variables
    k1 = 0.02
    k2 = 1
    k3 = 0.01
    k4 = 0.032
    k5 = 1
    k6 = 15
    k7 = 0.045
    k8 = 1
    k9 = 0.092
    k10 = 1
    k11 = 0.01
    k12 = 0.01
    k13 = 1
    k14 = 0.5
    k15 = 0.086
    k16 = 0.0011
    a1 = (k2*x6) + (k15*x11) - (k1*x1*x4) - (k16*x1*x5)
    a2 = (k3*x6) + (k5*x7) + (k10*x9) + (k13*x10) - (x2*x5*(k11 + k12)) - (k4*x2*x4)
    a3 = (k6*x7) + (k8*x8) - (k7*x3*x5)
    a4 = (x6*(k2 + k3)) + (x7*(k5 + k6)) - (k1*x1*x4) - (k4*x2*x4)
    a5 = (k8*x8) + (k10*x9) + (k13*x10) + (k15*x11) - (x2*x5*(k11 + k12)) - (k7*x3*)
    a6 = (k1*x1*x4) - (x6*(k2 + k3))
    a7 = (k4*x2*x4) - (x7*(k5 + k6))
    a8 = (k7*x3*x5) - (x8*(k8 + k9))
    a9 = (k9*x8) - (k10*x9) + (k11*x2*x5)
    a10 = (k12*x2*x5) - (x10*(k13 + k14))
    a11 = (k14*x10) - (k15*x11) + (k16*x1*x5)
    return [a1, a2, a3, a4, a5, a6, a7, a8, a9, a10, a11]
```

In [5]:

```
def criteriaCheck1(variables):
    x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11 = variables
    if (x5 + x8 + x9 + x10 + x11 == 100) and (x4 + x6 + x7 == 50) and (x1 + x2 + x3)
        return True
    else:
        return False
```

In [6]:

```
def criteriaCheck2(variables):
    x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11 = variables
    if (x5 + x8 + x9 + x10 + x11 == 100) and (x4 + x6 + x7 == 50) and (x1 + x2 + x3)
        return True
    else:
        return False
```

In [7]:

```
def createPlotsTask7N8(solution, t):
    fig, axes = plt.subplots(nrows=1, ncols=1, figsize=(5, 5), dpi=360)
    axes.plot(t, solution[:, 0], "black",
              t, solution[:, 1],
                                 "green",
              t, solution[:, 2], "red",
              t, solution[:, 3], "cyan"
              t, solution[:, 4],
                                  "magenta",
              t, solution[:, 5], "yellow",
              t, solution[:, 6], "blue",
              t, solution[:, 7], "silver"
              t, solution[:, 8],
                                  "gold",
              t, solution[:, 9], "indigo"
              t, solution[:, 10], "brown")
    plt.title("MAPK Cascade")
    plt.xlabel('Time')
    plt.ylabel('Concentration')
    plt.show()
```

In [8]:

```
def evaluateTrajectories(solution, t):
    eval1 = solution[:, 0] + solution[:, 1] + solution[:, 2] + solution[:, 3] + sol
    eval2 = solution[:, 4] + solution[:, 7] + solution[:, 8] + solution[:, 9] + sol
    eval3 = solution[:, 3] + solution[:, 5] + solution[:, 6]
    eval4 = solution[:, 0] + solution[:, 1] + solution[:, 2] + solution[:, 5] + sol
    eval5 = solution[:, 0] + solution[:, 1] + solution[:, 2] + solution[:, 3] + sol
    figeval, axeseval = plt.subplots(nrows=1, ncols=1, figsize=(5, 5), dpi=360)
    axeseval.plot(t, eval1, "b",t, eval2, "g",t, eval3, "r",t, eval4, "c",t, eval5,
    plt.title("MAPK Cascade")
    plt.xlabel('Time')
    plt.ylabel('Concentration')
    plt.show()
```

In [9]:

```
def plotGraphsForTask7(initialCondition):
    if criteriaCheck1(initialCondition) == True:
        t start = 0
        t end = 7000
        t step = 100000
        t = np.linspace(t start, t end, t step)
        solution = integrate.odeint(systemOfEquations, initialCondition, t)
        createPlotsTask7N8(solution, t)
        evaluateTrajectories(solution,t)
    else:
        print("Wrong set of inputs")
```

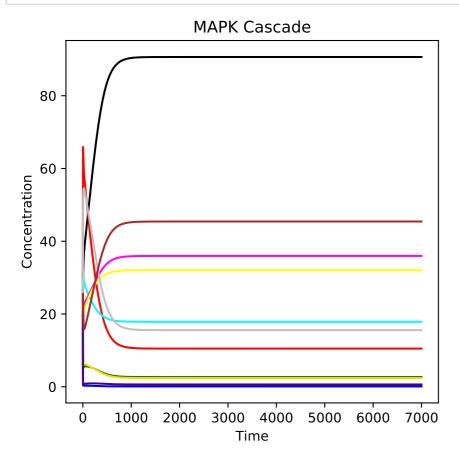
In [10]:

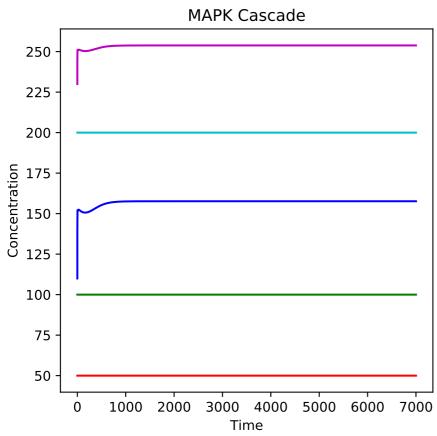
```
def plotGraphsForTask8(initialCondition):
    if criteriaCheck2(initialCondition) == True:
        t start = 0
        t end = 7000
        t step = 100000
        t = np.linspace(t start, t end, t step)
        solution = integrate.odeint(systemOfEquations, initialCondition, t)
        createPlotsTask7N8(solution, t)
        evaluateTrajectories(solution,t)
    else:
        print("Wrong set of inputs")
```

Task 7

In [11]:

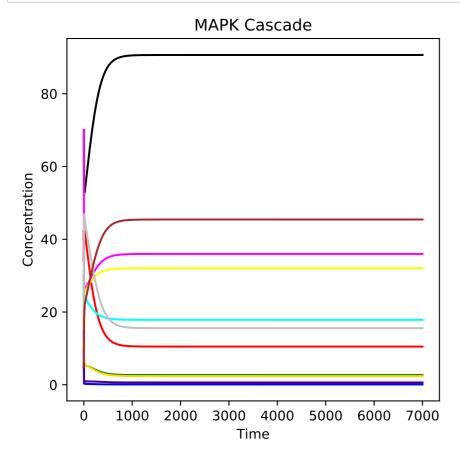
```
#Initial condition
initialCondition = [20, 30, 30, 10, 20, 20, 20, 20, 20, 20, 20]
plotGraphsForTask7(initialCondition)
```

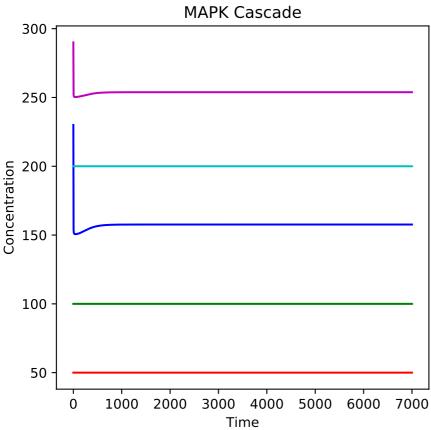




In [12]:

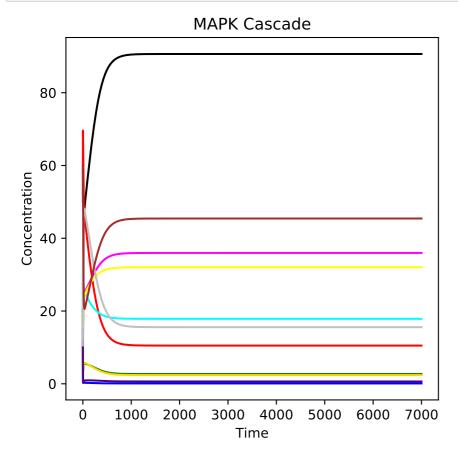
```
#Initial condition
initialCondition = [70, 30, 40, 20, 70, 15, 15, 10, 10, 5, 5]
plotGraphsForTask7(initialCondition)
```

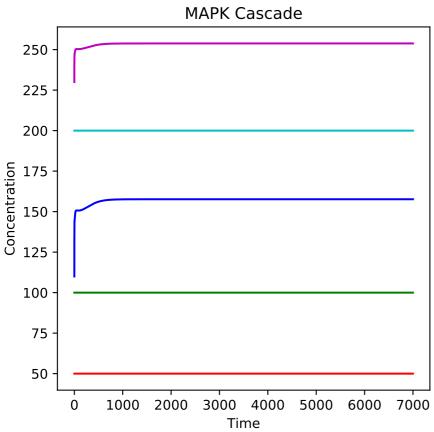




In [13]:

```
#Initial condition
initialCondition = [10, 20, 50, 20, 10, 15, 15, 10, 10, 60]
plotGraphsForTask7(initialCondition)
```

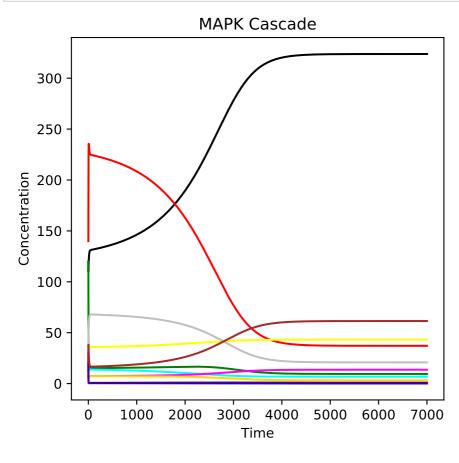


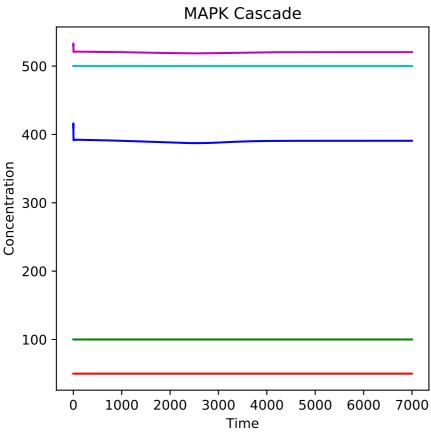


Task 8

In [14]:

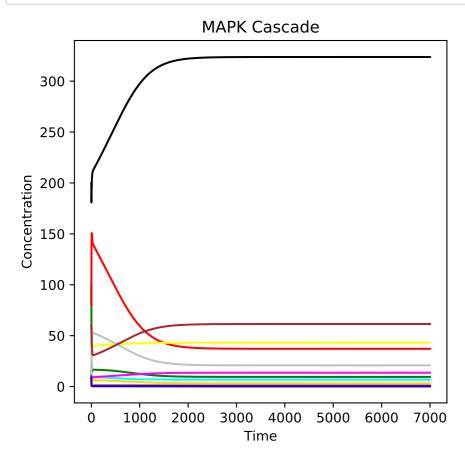
#Initial condition initialCondition = [120, 120, 140, 10, 20, 20, 20, 10, 10, 30, 30] plotGraphsForTask8(initialCondition)

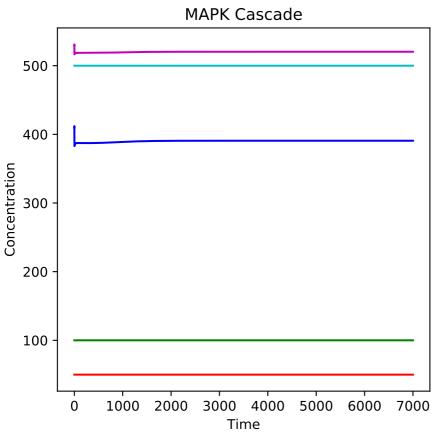




In [15]:

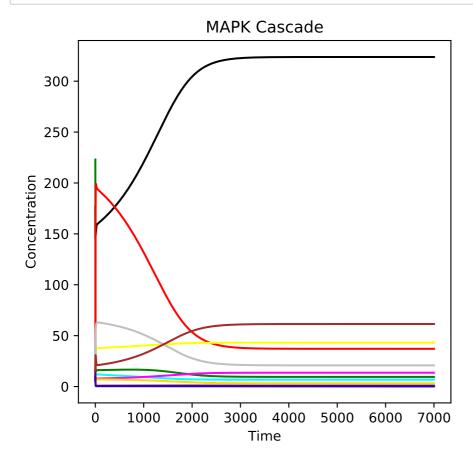
```
#Initial condition
initialCondition = [200, 100, 80, 20, 10, 15, 15, 10, 10, 60]
plotGraphsForTask8(initialCondition)
```

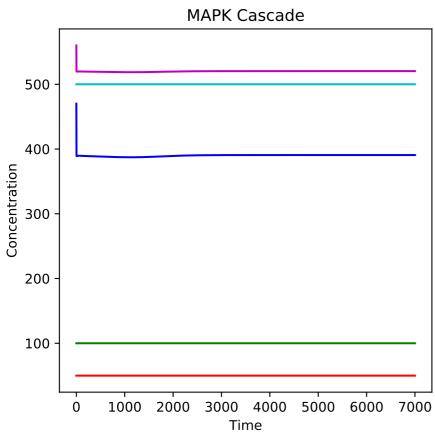




In [16]:

```
#Initial condition
initialCondition = [177, 223, 10, 40, 20, 5, 5, 25, 25, 15, 15]
plotGraphsForTask8(initialCondition)
```





Bertini

In [2]:

```
def parserBertiniOutput(file):
    lines = (line.strip() for line in file)
    nvariables = next(lines)
    dimensionSolution = int(nvariables[21:])
    variables = next(lines)
    rank = next(lines)
    skip1 = next(lines)
    dimensionTitle = next(lines)
    skip2 = next(lines)
    nonSingularTitle = next(lines)
    curline = next(lines)
    results = []
    while curline == '----':
        pathNumber = next(lines)
        componentNumber = next(lines)
        estimatedConditionNumber = next(lines)
        components = []
        for i in range(dimensionSolution):
            component = next(lines)
            real, imaginary = component.split(' ')
            component = float(real) + 1; * float(imaginary)
            components.append(component)
        results.append(components)
        multiplicity = next(lines)
        deflations = next(lines)
        curline = next(lines)
    return results
```

In [3]:

```
def modifiedResults(results, atol=1e-08):
    reducedResults = []
    for result in results:
        arrayForm = np.array(result)
        real = np.real(arrayForm)
        imaginary = np.imag(arrayForm)
        if np.allclose(np.zeros(shape=imaginary.shape), imaginary, atol=atol) and n
            reducedResults.append(real)
    return np.array(reducedResults)
```

In [4]:

```
#INPUT file for Bertini
bertiniInputFile = """
CONFIG
TRACKTYPE: 1;
INPUT
function y1, y2, y3, y4, y5, y6, y7, y8, y9, y10, y11, y12, y13, y14;
variable_group x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11;
constant k1, k2, k3, k4, k5, k6, k7, k8, k9, k10, k11, k12, k13, k14, k15, k16, k17
k1 = 0.02:
k2 = 1;
k3 = 0.01;
k4 = 0.032;
k5 = 1;
k6 = 15;
k7 = 0.045;
k8 = 1;
k9 = 0.092:
k10 = 1;
k11 = 0.01;
k12 = 0.01;
k13 = 1;
k14 = 0.5;
k15 = 0.086;
k16 = 0.0011;
k17 = 100;
k18 = 50;
k19 = {};
y1 = k2 * x6 + k15 * x11 - k1 * x1 * x4 - k16 * x1 * x5;
y2 = k3 * x6 + k5 * x7 + k10 * x9 + k13 * x10 - x2 * x5 * (k11 + k12) - k4 * x2 * x
y3 = k6 * x7 + k8 * x8 - k7 * x3 * x5;
y4 = x6 * (k2 + k3) + x7 * (k5 + k6) - k1 * x1 * x4 - k4 * x2 * x4;
y5 = k8 * x8 + k10 * x9 + k13 * x10 + k15 * x11 - x2 * x5 * (k11 + k12) - k7 * x3 *
y6 = k1 * x1 * x4 - x6 * (k2 + k3);
y7 = k4 * x2 * x4 - x7 * (k5 + k6);
y8 = k7 * x3 * x5 - x8 * (k8 + k9);
y9 = k9 * x8 - k10 * x9 + k11 * x2 * x5;
y10 = k12 * x2 * x5 - x10 * (k13 + k14);
y11 = k14 * x10 - k15 * x11 + k16 * x1 * x5;
y12 = x5 - k17 + x8 + x9 + x10 + x11;
y13 = x4 - k18 + x6 + x7;
y14 = x1 - k19 + x2 + x3 + x6 + x7 + x8 + x9 + x10 + x11;
END;
11 11 11
```

In [5]:

```
def computation(value, pathForData, subfolderPath):
    subfolder = os.path.join(pathForData, subfolderPath.format(0))
    if not os.path.exists(subfolder):
        os.mkdir(subfolder)
    os.chdir(subfolder)
    with open(os.path.join(subfolder, 'input'), 'w') as file:
        writeData = bertiniInputFile.format(value)
        file.write(writeData)
    subprocess.call(['/home/icxa/Documents/Software/BertiniLinux64_v1.5.1/./bertini
    with open(os.path.join(subfolder, 'main data')) as file:
        data = parserBertiniOutput(file)
        solNum = len(modifiedResults(data))
    return solNum
```

In [6]:

```
pathForData = os.path.abspath('/home/icxa/Documents/LabTasks5-16March/Data1')
if not os.path.exists(pathForData):
    os.mkdir(pathForData)
subfolderPath = 'input{:04}/'
```

In [7]:

```
k19PossibleValues = [200, 500]
for value in k19PossibleValues:
    solNum = computation(value, pathForData, subfolderPath)
    print("For k19 as ", value, "number of positive real solutions are:", solNum)
```

```
For k19 as 200 number of positive real solutions are: 1
For k19 as 500 number of positive real solutions are: 3
```

Varying K19 values upto one digit of accuracy

In [29]:

```
lowerBound = 200
upperBound = 501
interval = 1
possibleValues = np.arange(lowerBound, upperBound, interval)
```

```
In [30]:
```

```
print(len(possibleValues))
```

301

In [31]:

```
print(possibleValues)
[200 201 202 203 204 205 206 207 208 209 210 211 212 213 214 215 216 2
218 219 220 221 222 223 224 225 226 227 228 229 230 231 232 233 234 2
35
 236 237 238 239 240 241 242 243 244 245 246 247 248 249 250 251 252 2
53
254 255 256 257 258 259 260 261 262 263 264 265 266 267 268 269 270 2
272 273 274 275 276 277 278 279 280 281 282 283 284 285 286 287 288 2
 290 291 292 293 294 295 296 297 298 299 300 301 302 303 304 305 306 3
07
 308 309 310 311 312 313 314 315 316 317 318 319 320 321 322 323 324 3
 326 327 328 329 330 331 332 333 334 335 336 337 338 339 340 341 342 3
344 345 346 347 348 349 350 351 352 353 354 355 356 357 358 359 360 3
 362 363 364 365 366 367 368 369 370 371 372 373 374 375 376 377 378 3
79
 380 381 382 383 384 385 386 387 388 389 390 391 392 393 394 395 396 3
97
 398 399 400 401 402 403 404 405 406 407 408 409 410 411 412 413 414 4
416 417 418 419 420 421 422 423 424 425 426 427 428 429 430 431 432 4
33
 434 435 436 437 438 439 440 441 442 443 444 445 446 447 448 449 450 4
51
452 453 454 455 456 457 458 459 460 461 462 463 464 465 466 467 468 4
470 471 472 473 474 475 476 477 478 479 480 481 482 483 484 485 486 4
488 489 490 491 492 493 494 495 496 497 498 499 500]
```

In [32]:

```
pathForData = os.path.abspath('/home/icxa/Documents/LabTasks5-16March/Data2')
if not os.path.exists(pathForData):
    os.mkdir(pathForData)
subfolderPath = 'input{:04}/'
```

In [33]:

```
results = []
for value in possibleValues:
    solNum = computation(value, pathForData, subfolderPath)
    results.append((value, solNum))
```

In [34]:

print(results)

```
[(200, 1), (201, 1), (202, 1), (203, 0), (204, 1),
                                                    (205, 1),
                                                              (206, 1),
(207, 1), (208, 1), (209, 1), (210, 1), (211, 1), (212, 1), (213, 1),
(214, 1), (215,
                1), (216, 1), (217, 1), (218, 0), (219, 1), (220, 1),
(221,
     1), (222,
                1), (223, 1), (224, 1), (225,
                                              1), (226,
                                                         1), (227,
                                                                   1),
(228, 1), (229, 1), (230, 1), (231, 0), (232, 1), (233, 1), (234, 1),
(235, 1), (236, 1), (237, 1), (238, 1), (239, 1), (240, 1), (241, 1),
                1), (244, 1), (245, 1), (246, 1), (247, 1),
                                                             (248, 1),
(242,
     1), (243,
          (250, 1),
                    (251, 1), (252, 1), (253, 1),
(249, 1),
                                                   (254,
                                                             (255, 1),
                                                         1),
(256, 1), (257, 1), (258, 1), (259, 1), (260, 1), (261, 1),
                                                            (262, 1),
(263, 1), (264, 1), (265, 1), (266, 1), (267, 1), (268, 1), (269, 1),
(270, 1), (271, 0), (272, 1), (273, 1), (274, 1), (275, 1)
                                                         1),
                                                             (276,
                                                                   1),
                1), (279, 1), (280, 1), (281,
                                              1),
(277, 1), (278,
                                                   (282,
                                                         1),
                                                             (283,
                                                            (290, 1),
(284, 1), (285, 1), (286, 1), (287, 1), (288, 1), (289, 1),
(291, 1), (292, 1), (293, 1), (294, 1), (295, 1), (296, 1),
                                                             (297, 1),
(298, 1), (299, 1), (300, 1), (301, 1), (302, 1), (303,
                                                         1),
                                                             (304, 1),
(305, 1), (306, 1), (307, 0), (308, 1), (309, 1), (310, 1), (311, 1),
(312, 1), (313, 1), (314, 1), (315, 1), (316, 1), (317, 1), (318, 1),
(319, 1), (320, 1), (321, 1), (322, 1), (323, 1), (324, 1), (325, 1),
(326, 1), (327,
                1), (328, 1), (329, 1), (330,
                                              1),
                                                   (331,
                                                         1),
                                                             (332,
                                                  (338, 1),
(333, 1), (334, 1), (335, 1), (336, 1), (337, 1),
                                                             (339, 1),
(340, 1), (341, 1), (342, 1), (343, 1), (344, 1), (345, 1),
                                                             (346, 1),
(347, 1), (348,
                1), (349, 1), (350, 1), (351, 1),
                                                   (352,
                                                         1),
                                                             (353, 1),
(354, 1),
                    (356, 1), (357, 1),
                                        (358, 1),
          (355, 1),
                                                   (359,
                                                         1),
                                                             (360, 1),
(361, 1), (362, 1), (363, 1), (364, 1), (365, 1),
                                                  (366, 1),
                                                            (367, 1),
(368, 1), (369, 1), (370, 1), (371, 0), (372, 1), (373, 1), (374, 1),
(375, 1), (376,
                1), (377, 1), (378, 1), (379, 1),
                                                   (380,
                                                         1),
                                                             (381,
                                                                   1),
(382, 1), (383, 1), (384, 1), (385, 1), (386, 1),
                                                   (387,
                                                             (388,
                                                         1),
(389, 1), (390, 1), (391, 1), (392, 1), (393, 1), (394, 1),
                                                            (395, 1),
(396, 1), (397, 1), (398, 1), (399, 1), (400, 1),
                                                   (401, 1),
                                                             (402, 1),
                1), (405, 1), (406, 1), (407,
(403,
          (404,
                                              1),
                                                   (408, 1),
                                                             (409, 1),
     1),
                                                   (415, 3),
(410, 3), (411, 3), (412, 3), (413, 3), (414, 3),
                                                             (416, 3),
(417, 3), (418, 3), (419, 3), (420, 3), (421, 2),
                                                  (422, 3),
                                                             (423, 3),
(424, 3), (425, 3), (426, 1), (427, 3), (428, 3), (429, 3),
                                                             (430,
     3),
          (432, 3), (433, 3), (434, 3), (435,
                                              3),
                                                   (436,
                                                         3),
                                                             (437,
     3), (439, 2), (440, 3), (441, 3), (442,
                                              3),
                                                         3),
(438,
                                                  (443,
                                                             (444,
                                                             (451, 3),
(445, 3),
          (446, 3), (447, 3), (448, 3), (449, 3),
                                                   (450, 3),
                    (454, 3), (455, 3),
     3),
          (453, 3),
                                         (456, 3),
                                                   (457,
                                                         3),
                                                                   3),
(452,
                                                             (458,
                    (461, 3), (462, 3),
                                         (463, 1),
                                                         3),
          (460, 3),
                                                             (465,
                                                   (464,
(466, 3), (467, 3), (468, 3), (469, 3), (470, 3),
                                                  (471, 3),
                                                            (472,
                                                                   3),
(473, 3), (474, 3), (475, 3), (476, 3), (477, 3), (478, 3), (479, 3),
          (481, 3), (482, 3), (483, 3), (484, 3), (485,
                                                         3),
(480,
     2),
                                                             (486,
(487, 3), (488, 3), (489, 1), (490, 3), (491, 3), (492, 3),
                                                            (493,
```

(494, 3), (495, 3), (496, 3), (497, 3), (498, 3), (499, 3), (500, 3)]

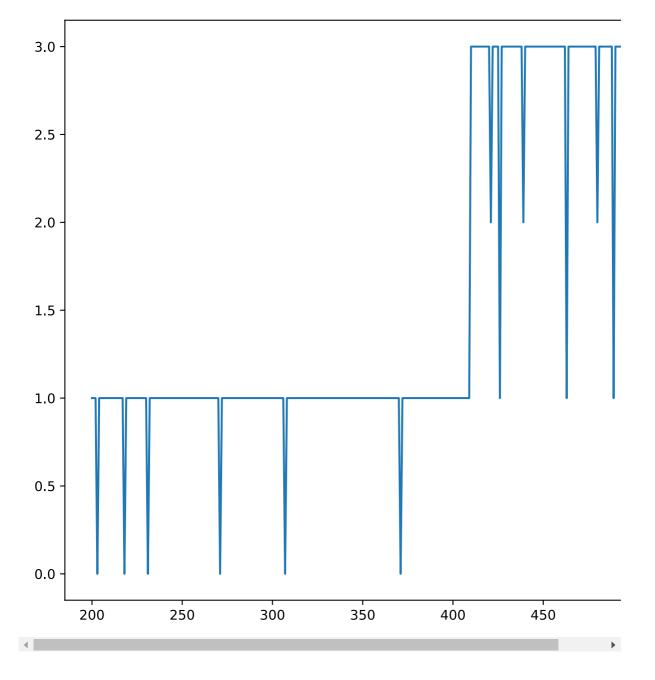
In [35]:

```
for i in range(len(results)):
    value1, solNum1 = results[i]
    for j in range(i, len(results)):
        value2, solNum2 = results[j]
        if solNum1 == 1 and solNum2 == 3 and value2-value1 == 1:
            print(value1, solNum1, value2, solNum2)
            break
```

```
409 1 410 3
426 1 427 3
463 1 464 3
489 1 490 3
```

In [37]:

```
x, y = zip(*results)
fig, axes = plt.subplots(1,1, figsize=(8, 8))
plt.plot(x, y)
plt.show()
```



Varying K19 values upto one decimal digit of accuracy

```
In [10]:
```

```
lowerBound = 200.0
upperBound = 500.1
interval = 0.1
possibleValues = np.arange(lowerBound, upperBound, interval)
```

In [11]:

```
print(len(possibleValues))
```

3001

In [12]:

```
print(possibleValues)
```

```
[ 200.
        200.1 200.2 .... 499.8 499.9
                                       500. 1
```

```
In [13]:
```

```
pathForData = os.path.abspath('/home/icxa/Documents/LabTasks5-16March/Data4')
if not os.path.exists(pathForData):
    os.mkdir(pathForData)
subfolderPath = 'input{:04}/'
```

In [14]:

```
results = []
for value in possibleValues:
    solNum = computation(value, pathForData, subfolderPath)
    results.append((value, solNum))
```

In [15]:

print(results)

[(200.0, 1), (200.099999999999, 1), (200.199999999999, 1), (200.29999999999, 1), (200.39999999999, 1), (200.49999999999, 1), (200.599999999997, 1), (200.69999999996, 1), (200.79999999 99995, 1), (200.899999999995, 1), (200.99999999994, 1), (201.09 9999999994, 1), (201.199999999993, 1), (201.299999999993, 1), (201.399999999992, 1), (201.499999999991, 1), (201.59999999999 1, 0), (201.69999999999, 1), (201.79999999999, 1), (201.8999999 999989, 1), (201.99999999999989, 1), (202.099999999988, 1), (202.1 99999999997, 1), (202.299999999987, 1), (202.399999999986, 1), (202.499999999986, 1), (202.599999999985, 1), (202.69999999998 5, 1), (202.799999999994, 1), (202.8999999999984, 1), (202.999999 99999983, 1), (203.0999999999982, 1), (203.199999999982, 1), (20 3.299999999981, 1), (203.399999999981, 1), (203.49999999998, 1), (203.59999999999, 1), (203.69999999999, 1), (203.799999999 9978, 1), (203.8999999999978, 1), (203.999999999977, 1), (204.099 9999999977, 1), (204.1999999999976, 1), (204.299999999976, 1), (204.399999999975, 1), (204.499999999974, 1), (204.59999999997 4, 1), (204.6999999999973, 1), (204.799999999973, 1), (204.899999 99999972, 1), (204.999999999972, 1), (205.0999999999971, 1), (20 /205 20000000000007 **1302 3000000000000000** 1 \ 1 \

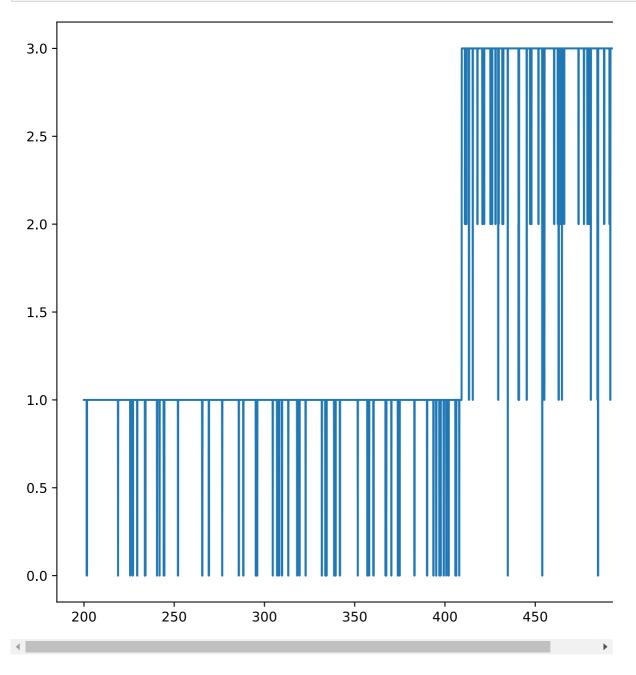
In [19]:

```
for i in range(len(results)):
    value1, solNum1 = results[i]
    for j in range(i, len(results)):
        value2, solNum2 = results[j]
        if solNum1 == 1 and solNum2 == 3 and value2-value1 < 0.2:</pre>
            print(value1, solNum1, value2, solNum2)
            break
```

```
409.2 1 409.3 3
413.3 1 413.4 3
415.4 1 415.5 3
429.5 1 429.6 3
441.0 1 441.1 3
445.3 1 445.4 3
455.0 1 455.2 3
455.1 1 455.2 3
463.0 1 463.1 3
464.8 1 464.9 3
480.8 1 480.9 3
484.5 1 484.6 3
491.6 1 491.7 3
495.1 1 495.2 3
```

In [20]:

```
x, y = zip(*results)
fig, axes = plt.subplots(1,1, figsize=(8, 8))
plt.plot(x, y)
plt.show()
```



Varying K17 in 2 orders of magnitude

In [38]:

```
lowerBound = 0
upperBound = 1001
interval = 1
possibleValuesK17 = list(np.arange(lowerBound, upperBound, interval))
```

In [39]:

print(possibleValuesK17)

```
2, 3, 4, 5, 6,
                      7, 8, 9, 10, 11, 12, 13, 14,
                                                            17,
                                                   15,
                                                       16,
                                                                18,
                                                                    19,
20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36,
                                                                     3
7, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54,
55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71,
2, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89,
90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105,
106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119,
120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133,
134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147,
148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161,
162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175,
176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189,
190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203,
204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217,
218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231,
232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245,
246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259,
260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273,
274, 275,
         276, 277, 278, 279, 280, 281, 282, 283, 284,
                                                       285,
                                                            286, 287,
288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301,
302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315,
316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329,
330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343,
344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357,
358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371,
372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383,
                                                            384, 385,
386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399,
400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413,
414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427,
428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441,
442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455,
456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469,
470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483,
         486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497,
484. 485.
498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511,
512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525,
526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539,
540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553,
554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567,
568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581,
582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593,
                                                            594, 595,
596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609,
610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623,
624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637,
638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651,
652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665,
666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679,
680, 681, 682, 683, 684, 685,
                              686, 687, 688, 689, 690,
                                                       691,
                                                            692, 693,
694, 695, 696, 697, 698, 699,
                              700, 701,
                                        702, 703, 704, 705,
                                                            706, 707,
708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721,
722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735,
736, 737,
         738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749,
750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763,
764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777,
778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791,
792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805,
806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819,
```

```
820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000]
```

In [40]:

```
possibleValuesK19 = [200,500]
```

In [41]:

possibleVariations = [(k17, k19) for k17 in possibleValuesK17 for k19 in possibleVa

In [42]:

```
print(possibleVariations)
[(0, 200), (0, 500), (1, 200), (1, 500), (2, 200), (2, 500), (3, 20)]
0), (3, 500), (4, 200), (4, 500), (5, 200), (5, 500), (6, 200), (6,
500), (7, 200), (7, 500), (8, 200), (8, 500), (9, 200), (9, 500), (1
0, 200), (10, 500), (11, 200), (11, 500), (12, 200), (12, 500), (13,
200), (13, 500), (14, 200), (14, 500), (15, 200), (15, 500), (16, 20
0), (16, 500), (17, 200), (17, 500), (18, 200), (18, 500), (19, 20
0), (19, 500), (20, 200), (20, 500), (21, 200), (21, 500), (22, 20
0), (22, 500), (23, 200), (23, 500), (24, 200), (24, 500), (25, 20
0), (25, 500), (26, 200), (26, 500), (27, 200), (27, 500), (28, 20
0), (28, 500), (29, 200), (29, 500), (30, 200), (30, 500), (31, 20
0), (31, 500), (32, 200), (32, 500), (33, 200), (33, 500), (34, 20
0), (34, 500), (35, 200), (35, 500), (36, 200), (36, 500), (37, 20
0), (37, 500), (38, 200), (38, 500), (39, 200), (39, 500), (40, 20
0), (40, 500), (41, 200), (41, 500), (42, 200), (42, 500), (43, 20
0), (43, 500), (44, 200), (44, 500), (45, 200), (45, 500), (46, 20
0), (46, 500), (47, 200), (47, 500), (48, 200), (48, 500), (49, 20
0), (49, 500), (50, 200), (50, 500), (51, 200), (51, 500), (52, 20
0), (52, 500), (53, 200), (53, 500), (54, 200), (54, 500), (55, 20
0), (55, 500), (56, 200), (56, 500), (57, 200), (57, 500), (58,
               /50
                    2001
                          / 50
                               5001
                                     160
                                          2007
                                                160
```

In [43]:

```
#INPUT file for Bertini
bertiniInputFileT10 = """
CONFIG
TRACKTYPE: 1;
INPUT
function y1, y2, y3, y4, y5, y6, y7, y8, y9, y10, y11, y12, y13, y14;
variable_group x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11;
constant k1, k2, k3, k4, k5, k6, k7, k8, k9, k10, k11, k12, k13, k14, k15, k16, k17
k1 = 0.02:
k2 = 1;
k3 = 0.01;
k4 = 0.032;
k5 = 1;
k6 = 15;
k7 = 0.045;
k8 = 1;
k9 = 0.092:
k10 = 1;
k11 = 0.01;
k12 = 0.01;
k13 = 1;
k14 = 0.5;
k15 = 0.086;
k16 = 0.0011;
k17 = \{\};
k18 = 50;
k19 = {};
y1 = k2 * x6 + k15 * x11 - k1 * x1 * x4 - k16 * x1 * x5;
y2 = k3 * x6 + k5 * x7 + k10 * x9 + k13 * x10 - x2 * x5 * (k11 + k12) - k4 * x2 * x
y3 = k6 * x7 + k8 * x8 - k7 * x3 * x5;
y4 = x6 * (k2 + k3) + x7 * (k5 + k6) - k1 * x1 * x4 - k4 * x2 * x4;
y5 = k8 * x8 + k10 * x9 + k13 * x10 + k15 * x11 - x2 * x5 * (k11 + k12) - k7 * x3 *
y6 = k1 * x1 * x4 - x6 * (k2 + k3);
y7 = k4 * x2 * x4 - x7 * (k5 + k6);
y8 = k7 * x3 * x5 - x8 * (k8 + k9);
y9 = k9 * x8 - k10 * x9 + k11 * x2 * x5;
y10 = k12 * x2 * x5 - x10 * (k13 + k14);
y11 = k14 * x10 - k15 * x11 + k16 * x1 * x5;
y12 = x5 - k17 + x8 + x9 + x10 + x11;
y13 = x4 - k18 + x6 + x7;
y14 = x1 - k19 + x2 + x3 + x6 + x7 + x8 + x9 + x10 + x11;
END;
11 11 11
```

In [44]:

```
def computationT10(value, pathForData, subfolderPath):
    subfolder = os.path.join(pathForData, subfolderPath.format(0))
    if not os.path.exists(subfolder):
        os.mkdir(subfolder)
    os.chdir(subfolder)
    with open(os.path.join(subfolder, 'input'), 'w') as file:
        writeData = bertiniInputFileT10.format(*value)
        file.write(writeData)
    subprocess.call(['/home/icxa/Documents/Software/BertiniLinux64_v1.5.1/./bertini
    with open(os.path.join(subfolder, 'main data')) as file:
        data = parserBertiniOutput(file)
        solNum = len(modifiedResults(data))
    return solNum
```

In [45]:

```
pathForData = os.path.abspath('/home/icxa/Documents/LabTasks5-16March/Data3')
if not os.path.exists(pathForData):
    os.mkdir(pathForData)
subfolderPath = 'inputFolder/'
```

In [46]:

```
results = []
for value in possible Variations:
    solNum = computationT10(value, pathForData, subfolderPath)
    results.append((value[0], value[1], solNum))
```

In [47]:

print(results)

```
[(0, 200, 0), (0, 500, 0), (1, 200, 1), (1, 500, 1), (2, 200, 1),
(2, 500, 1), (3, 200, 1), (3, 500, 1), (4, 200, 1), (4, 500, 1), (5,
200, 1), (5, 500, 1), (6, 200, 1), (6, 500, 1), (7, 200, 1), (7, 50
0, 1), (8, 200, 1), (8, 500, 1), (9, 200, 1), (9, 500, 1), (10, 200,
1), (10, 500, 1), (11, 200, 1), (11, 500, 1), (12, 200, 1), (12, 50
0, 1), (13, 200, 1), (13, 500, 1), (14, 200, 1), (14, 500, 1), (15,
200, 1), (15, 500, 1), (16, 200, 1), (16, 500, 1), (17, 200, 1), (1
7, 500, 1), (18, 200, 1), (18, 500, 1), (19, 200, 1), (19, 500, 1),
(20, 200, 1), (20, 500, 1), (21, 200, 1), (21, 500, 1), (22, 200,
1), (22, 500, 1), (23, 200, 1), (23, 500, 1), (24, 200, 1), (24, 50
0, 1), (25, 200, 1), (25, 500, 1), (26, 200, 1), (26, 500, 1), (27,
200, 1), (27, 500, 1), (28, 200, 1), (28, 500, 1), (29, 200, 1), (2
9, 500, 1), (30, 200, 1), (30, 500, 1), (31, 200, 1), (31, 500, 1),
(32, 200, 1), (32, 500, 1), (33, 200, 1), (33, 500, 1), (34, 200,
1), (34, 500, 1), (35, 200, 1), (35, 500, 1), (36, 200, 1), (36, 50
0, 1), (37, 200, 1), (37, 500, 0), (38, 200, 1), (38, 500, 1), (39,
200, 1), (39, 500, 1), (40, 200, 1), (40, 500, 1), (41, 200, 1), (4
1, 500, 1), (42, 200, 1), (42, 500, 1), (43, 200, 1), (43, 500, 1),
(44, 200, 1), (44, 500, 1), (45, 200, 1), (45, 500, 1), (46, 200, 1), (46, 500, 1), (47, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (48, 500, 1), (4
```

In [49]:

```
x, y, z = zip(*results)
print(x,y,z)
(0, 0, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, 7, 7, 8, 8, 9, 9, 10, 10,
11, 11, 12, 12, 13, 13, 14, 14, 15, 15, 16, 16, 17, 17, 18, 18, 19,
19, 20, 20, 21, 21, 22, 22, 23, 23, 24, 24, 25, 25, 26, 26, 27, 27,
28, 28, 29, 29, 30, 30, 31, 31, 32, 32, 33, 33, 34, 34, 35, 35, 36,
36, 37, 37, 38, 38, 39, 39, 40, 40, 41, 41, 42, 42, 43, 43, 44, 44,
45, 45, 46, 46, 47, 47, 48, 48, 49, 49, 50, 50, 51, 51, 52, 52, 53,
53, 54, 54, 55, 55, 56, 56, 57, 57, 58, 58, 59, 59, 60, 60, 61, 61,
62, 62, 63, 63, 64, 64, 65, 65, 66, 66, 67, 67, 68, 68, 69, 69, 70,
70, 71, 71, 72, 72, 73, 73, 74, 74, 75, 75, 76, 76, 77, 77, 78, 78,
79, 79, 80, 80, 81, 81, 82, 82, 83, 83, 84, 84, 85, 85, 86, 86, 87,
87, 88, 88, 89, 89, 90, 90, 91, 91, 92, 92, 93, 93, 94, 94, 95, 95,
96, 96, 97, 97, 98, 98, 99, 99, 100, 100, 101, 101, 102, 102, 103, 1
03, 104, 104, 105, 105, 106, 106, 107, 107, 108, 108, 109, 109, 110,
110, 111, 111, 112, 112, 113, 113, 114, 114, 115, 115, 116, 116, 11
7, 117, 118, 118, 119, 119, 120, 120, 121, 121, 122, 122, 123, 123,
124, 124, 125, 125, 126, 126, 127, 127, 128, 128, 129, 129, 130, 13
0, 131, 131, 132, 132, 133, 133, 134, 134, 135, 135, 136, 136, 137,
137, 138, 138, 139, 139, 140, 140, 141, 141, 142, 142, 143, 143, 14
4, 144, 145, 145, 146, 146, 147, 147, 148, 148, 149, 149, 150, 150,
                   150
                        160 164 164 166
                                             1 5 5
                                                 156
In [52]:
storeV1k17 = []
```

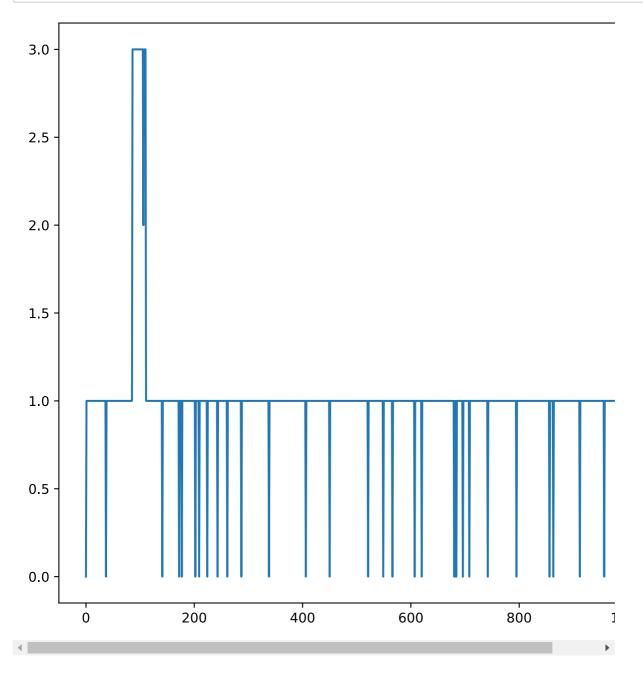
```
storeV1numSol = []
storeV2k17 = []
storeV2numSol = []
for i in range(len(results)):
    k17 = results[i][0]
    k19 = results[i][1]
    numSol = results[i][2]
    if k19 == 500:
        storeV1k17.append(k17)
        storeV1numSol.append(numSol)
    elif k19 == 200:
        storeV2k17.append(k17)
        storeV2numSol.append(numSol)
print(len(storeV1k17))
print(len(storeV2k17))
```

1001

1001

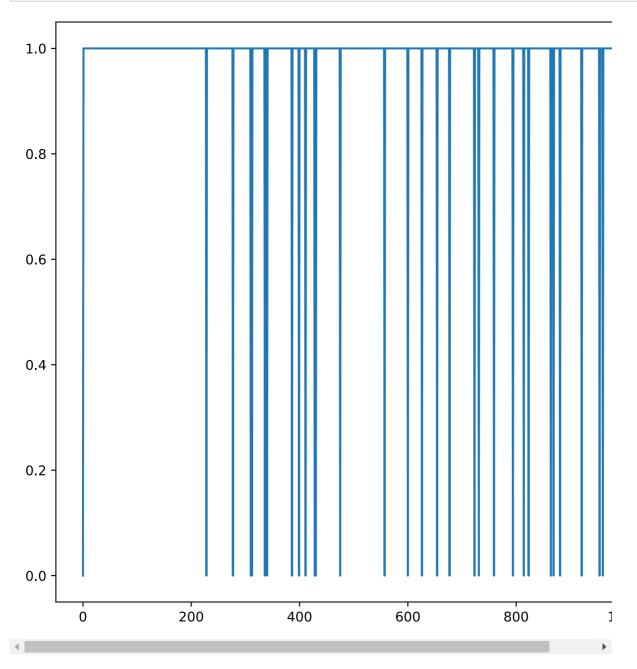
In [55]:

```
fig, axes = plt.subplots(1,1, figsize=(8, 8))
plt.plot(storeV1k17, storeV1numSol)
plt.show()
```



In [56]:

```
fig, axes = plt.subplots(1,1, figsize=(8, 8))
plt.plot(storeV2k17, storeV2numSol)
plt.show()
```



In []: