

### **Additional Exercises for Lab 3: Do either one of the two questions below, or do Ex2 & Ex3 of the original Lab 3.**

#### **PCA with Image Reconstruction**

One simple application of PCA is to perform image reconstruction in applications like compression, denoising, etc. In this exercise, you are expected to work on a gray image of your choice, represented as a matrix of size  $m \times n$  (features  $\times$  num\_samples).

You are required first to modify the myPCA function used in Exercise 1 as follows:

$$[A\_mean, A\_normalized, D, V] = \text{myPCA}(A)$$

Where the myPCA function returns the mean of the matrix A along the feature dimension, the normalized matrix A, the diagonal matrix D and the matrix V containing corresponding eigenvalues and eigenvectors.

Create the following function then:

$$[Ar] = \text{reconstruct}(A\_mean, A\_normalized, D, V, \text{keep\_ratio})$$

Where it takes the mean, the normalized gray image matrix A, eigenvalues and eigenvectors and reconstruct the gray image by the keep\_ratio which controls the number of components to keep. The reconstructed image is returned as Ar.

For image compression problem, apply the above functions on a random gray image of your choice first. Try to retain from 95% to 5% of the components and then reconstruct the image using those reduced variances. See what happens in the image quality? Provide an image quality measure using Euclidean distance.

For image denoising problem, firstly corrupt the image with random gaussian noises using “imnoise” function from MATLAB, starting with small gaussian noises. Then reconstruct the image using your implemented functions. Experiment with different keep\_ratio and see the reconstruction quality against the uncorrupted image.

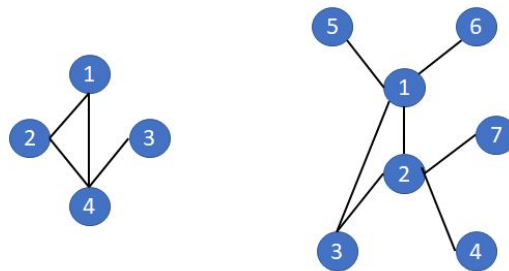
#### **Degree Centrality vs. Eigenvector Centrality**

We introduce the problem of **Centrality Measures** during social network analysis. Centrality Measures address the problem where we care about the most important node, also known as the central node, in a network. A vast number of methods for centrality measures have been introduced over the years. In this exercise, we look at the practice **Eigenvector Centrality**. We start with some toy network graphs before going into more complex network settings.

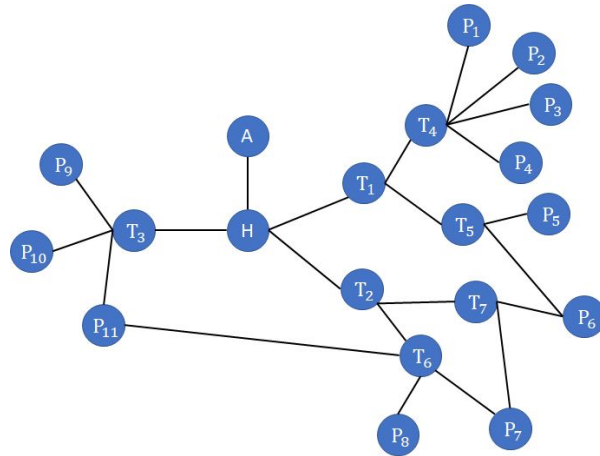
More mathematical details in Centrality problem can be referred in P. 169 of the book here [\[http://math.sjtu.edu.cn/faculty/xiaodong/course/Networks%20An%20introduction.pdf\]](http://math.sjtu.edu.cn/faculty/xiaodong/course/Networks%20An%20introduction.pdf).

**Figure 1 provides some simple toy networks. Figure 2** provides a network illustrating a website structure. And Figure 3 provides the map of Edmonton LRT Transit System. Choose 2 out of 3 figures and perform network analysis with the following steps:

1. Start by constructing the adjacency matrix  $A$  for the given network.
2. Calculate the degree centrality for each node  $v$ . For an undirected graph  $G = (V, E)$ , the degree centrality of a given node  $v$  is defined as:  $CD(v) = \text{degree}(v)$ .
3. Calculate the eigenvector centrality for each node  $v$ . The calculation is done by Formula 7.6 of the Network book (P. 170).
4. Given the analysis of both degree centrality and eigenvector centrality, which node in the network graph is more important and why? Compare and contrast degree and eigenvector centrality? What does each of the centrality measure indicate individually for your network? Which centrality measure is good for the network?



**Fig 1.** Two toy examples.



**Fig 2.** Network structure mined from a website.

