The Error Function

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1 Introduction

This report

2 Analytical solution

The error function erf(x) is defined as

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \, dt. \tag{1}$$

3 Numerical solution

The error function can also be found by numerically solving the following differential equation:

$$u'(x) = \frac{2}{\sqrt{\pi}} e^{-x^2} \tag{2}$$

with the initial condition

$$u(0) = 0. \tag{3}$$

4 Plot visualization

Both the analytical and numerical solutions are shown in figure 1. The analytical solution (gsl erf(x)) is GSL's implementation of equation 1, while the numerical solution (myerf(x)) is computed by integration equation 2 and 3.

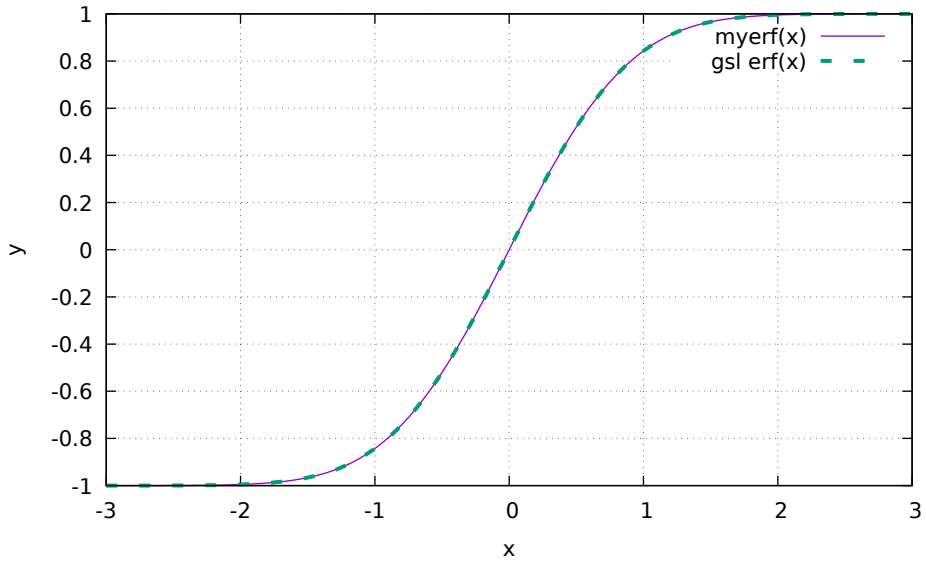

Figure 1: Numerical and analytical representations of the error function.