Problem 17 - Arctangent function by numerical root finding

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1 Introduction

This report covers a numerical implementation of the arctangent function

$$a = arctan(x), \tag{1}$$

by finding roots of the equation

$$tan(a) - x = 0. \tag{2}$$

That is, given x find a via eq. (2).

2 Implementation

The numerical solution is implemented via a root-finding algorithm in GSL, in this case the *gsl_multiroot_fsolver_hybrids* solver was used, which relies on finite differences for derivatives, which for a small problem like this was seen as adequate. However, the correct option would be to employ an algorithm using analytical derivatives, since the derivative of $tan(a)$ is known as $1 + tan(a)^2$.

The solver relies on a starting point (henceforth called a_0), which if not chosen carefully can lead to undesirable results. All root-finding algorithms in the GSL multiroot library uses some variation of the newton iteration

$$x_1 = x_0 - J(x_0)^{-1} f(x_0), \tag{3}$$

with x_0 and $f(x_0)$ being vector quantities and $J(x_0)$ being the Jacobian matrix of the system. Looking at figure ?? we see why we must choose a_0 with care. For instance, setting $a_0 = 0$ is fine for small values of x, but will result in a large error for the final value of a as x exceeds $\pm\pi$ (for $a_0 = 0$), since the *arctan* function only is defined for the initial repetition of the tangent function

$$x = tan(a) \quad -\frac{\pi}{2} < a < \frac{\pi}{2}. \tag{4}$$

Too alleviate this problem, the following definition for a_0 is made

$$a_0 = \tag{5}$$

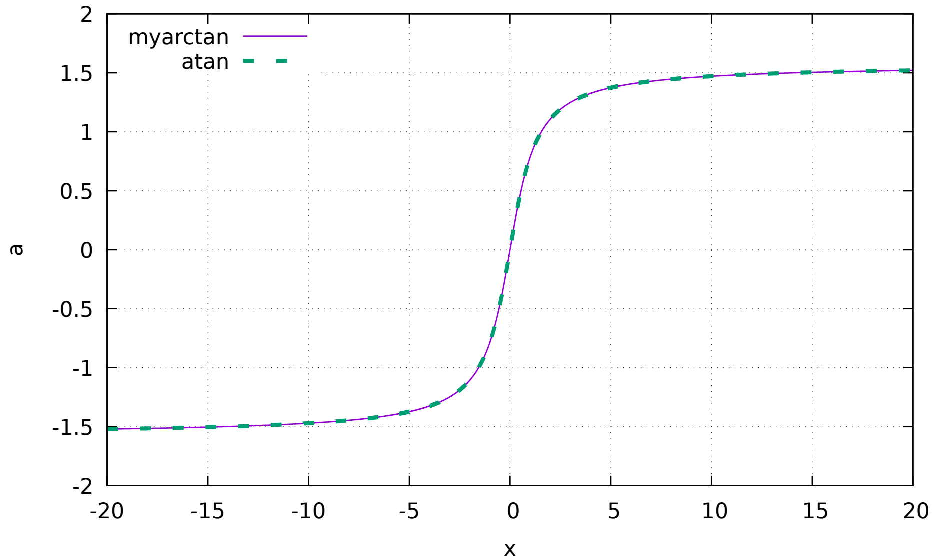

Figure 1: Visualization of the arctan function, showing both own numerical implementation (myarctan) and the atan definition from the math.h librabry

3 Plot visualization