THE PHYSICS OF WIND ENERGY

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DEPARTMENT OF PHYSICS AND ASTRONOMY

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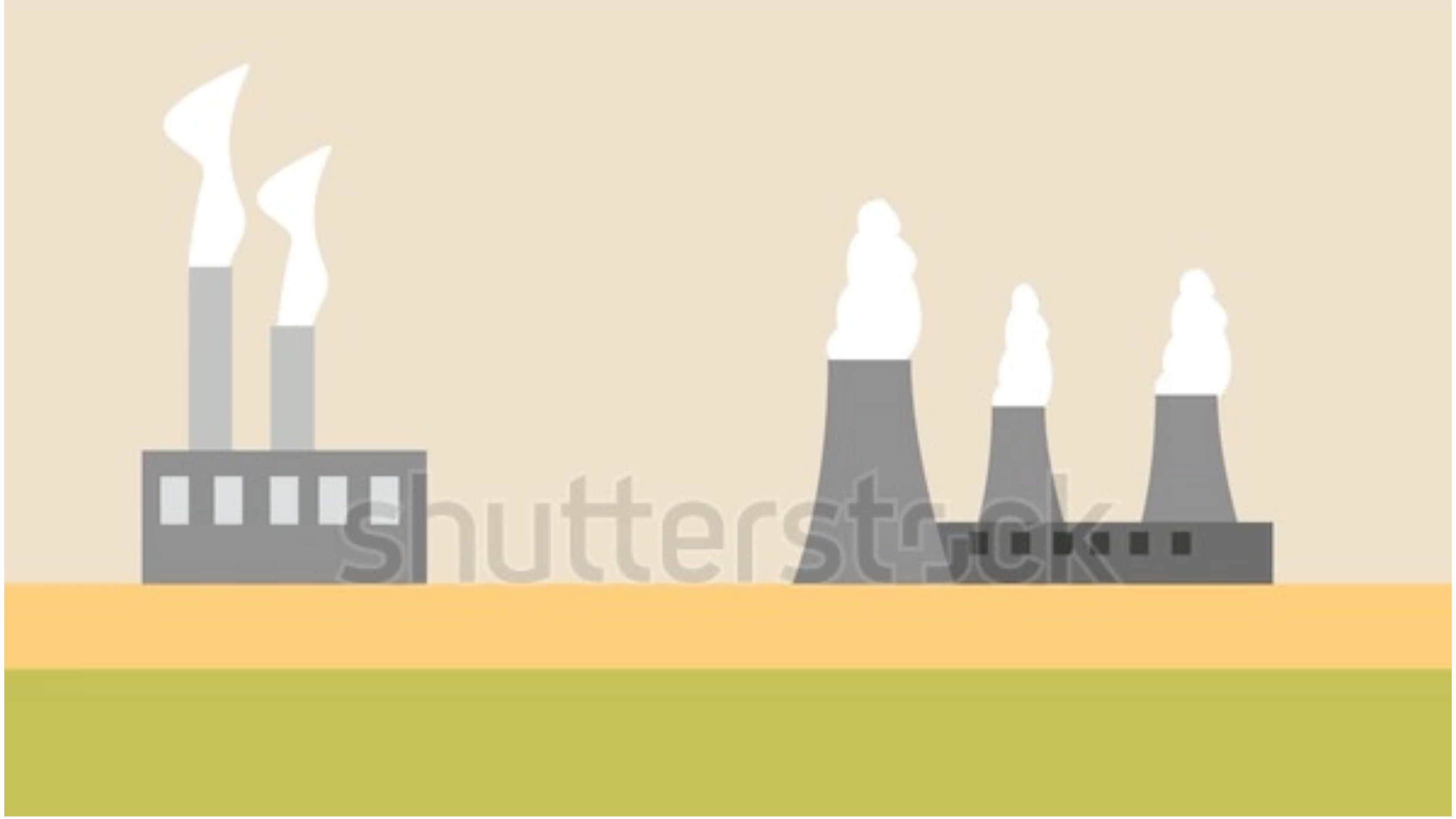
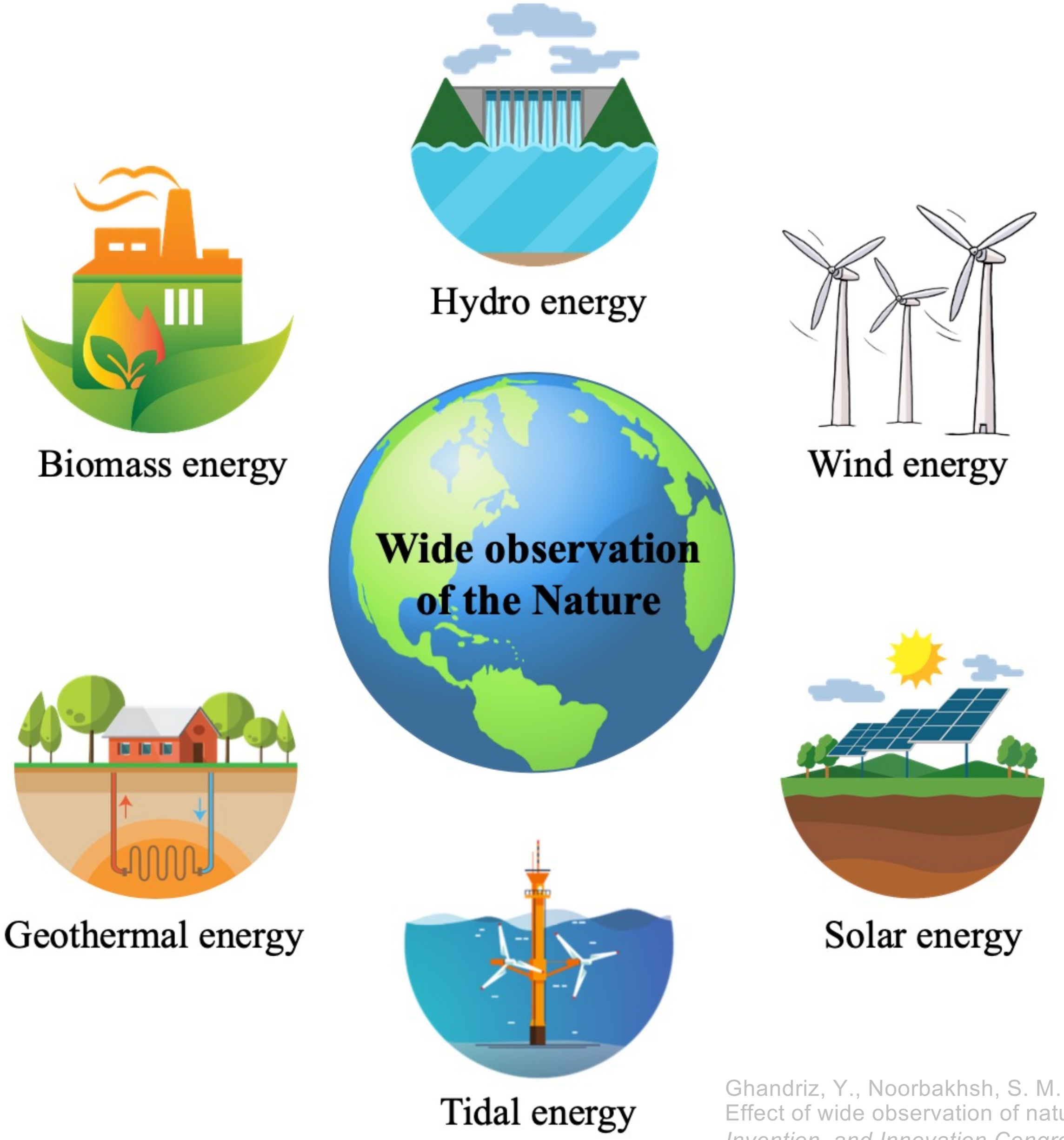
13 MARCH 2023

IDA DIXEN SKAARUP WÖLM
STUDENT

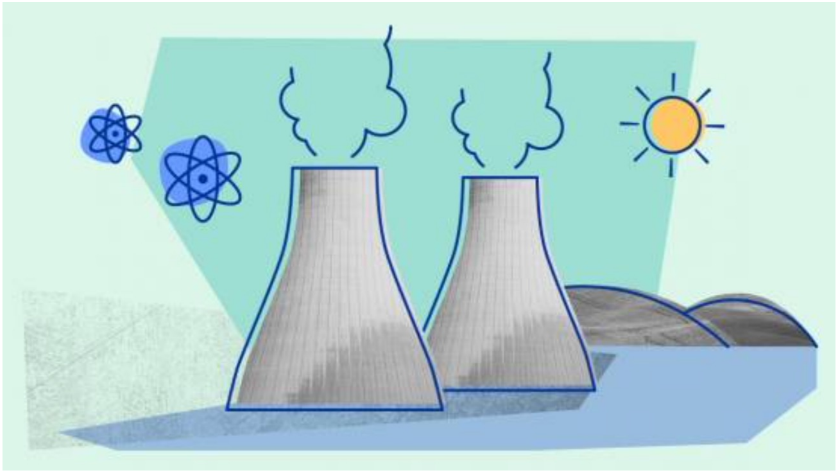
REPLACE FOSSILS WITH RENEWABLES



RENEABLE ENERGY SOURCES



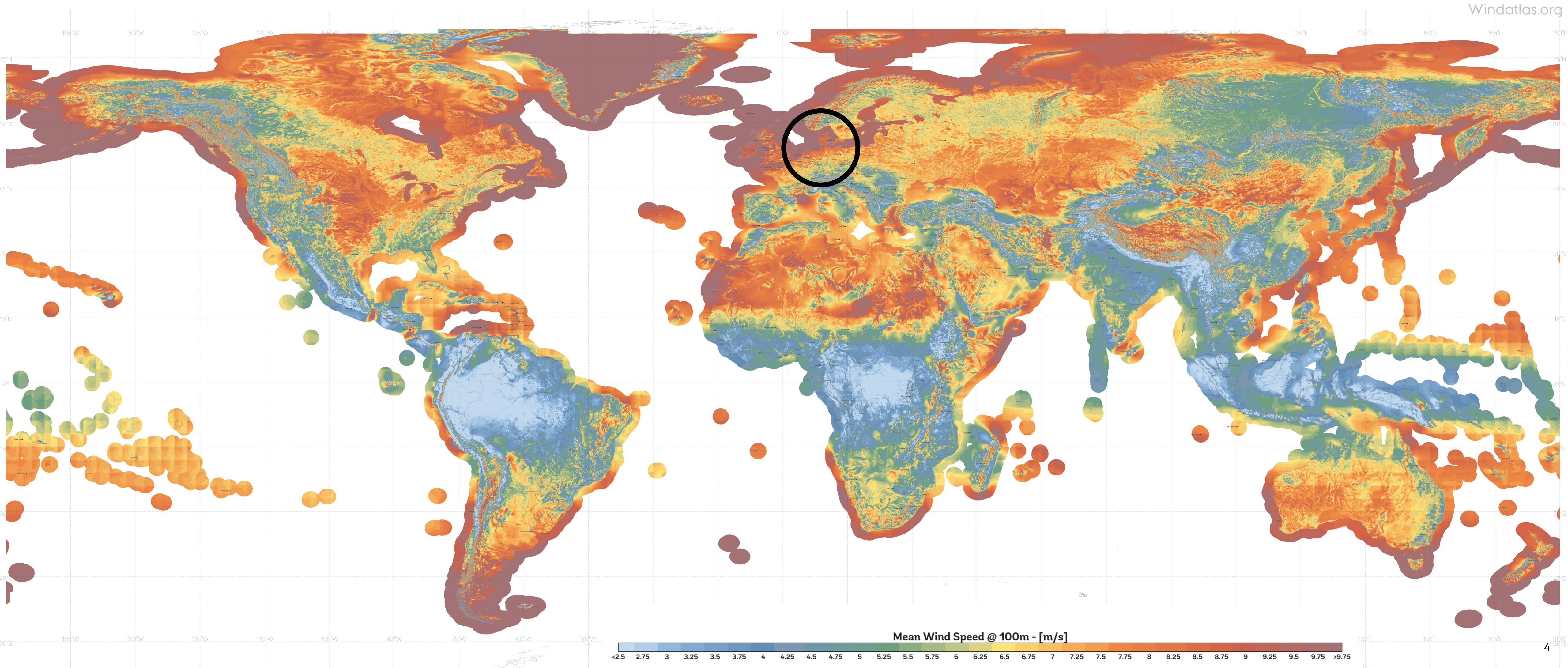
Ghandriz, Y., Noorbakhsh, S. M. Z., Gavagsaz-Ghoachani, R., & Phattanasak, M. (2021, September). Effect of wide observation of nature in renewable energy engineering education. In *2021 Research, Invention, and Innovation Congress: Innovation Electricals and Electronics (RI2C)* (pp. 193-198). IEEE.



https://www.iaea.org/newscenter/news/what-is-nuclear-energy-the-science-of-nuclear-power

DENMARK IS WINDY

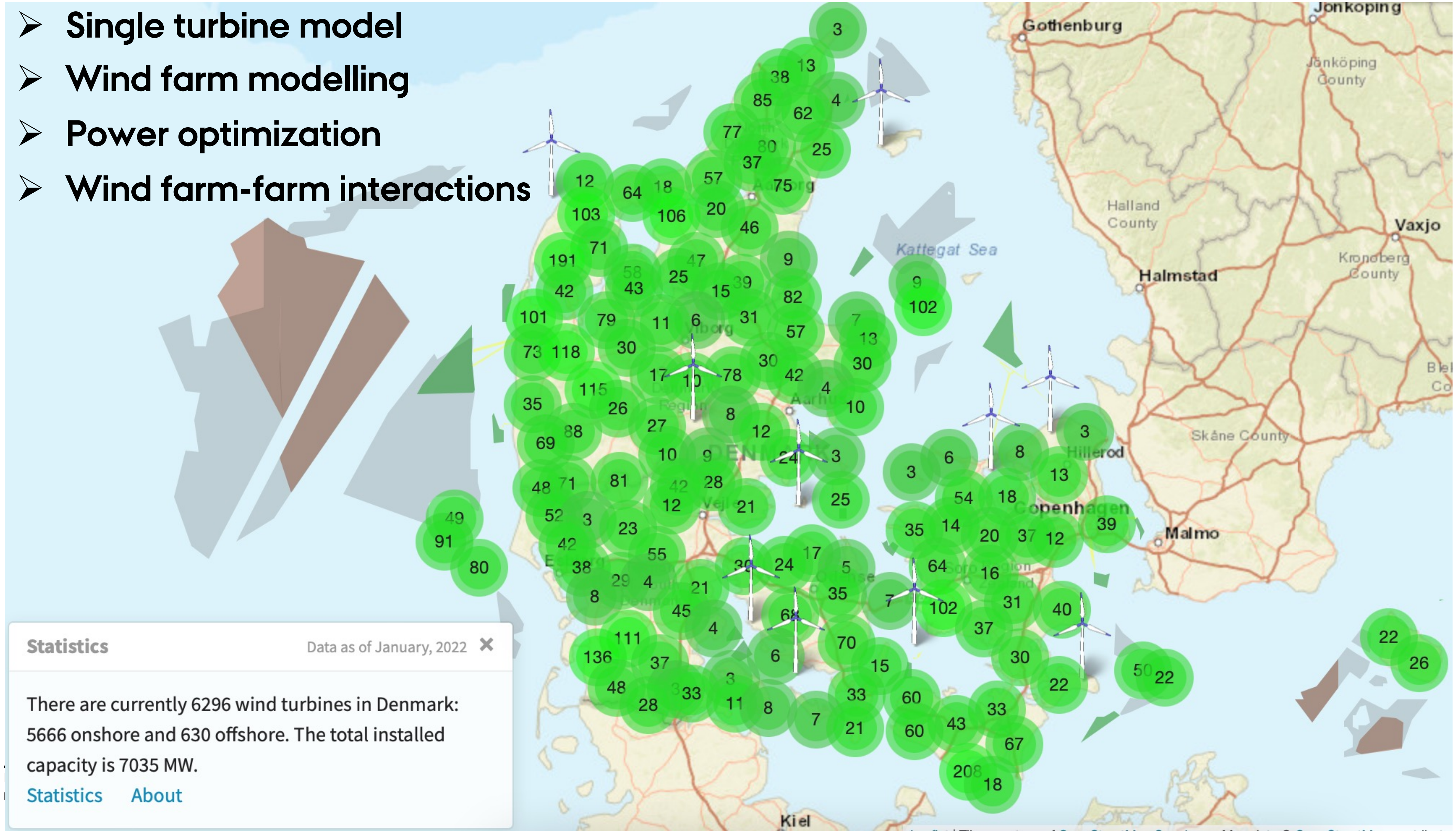
MEAN WIND SPEED



INSTALLED TURBINES

https://turbines.dk

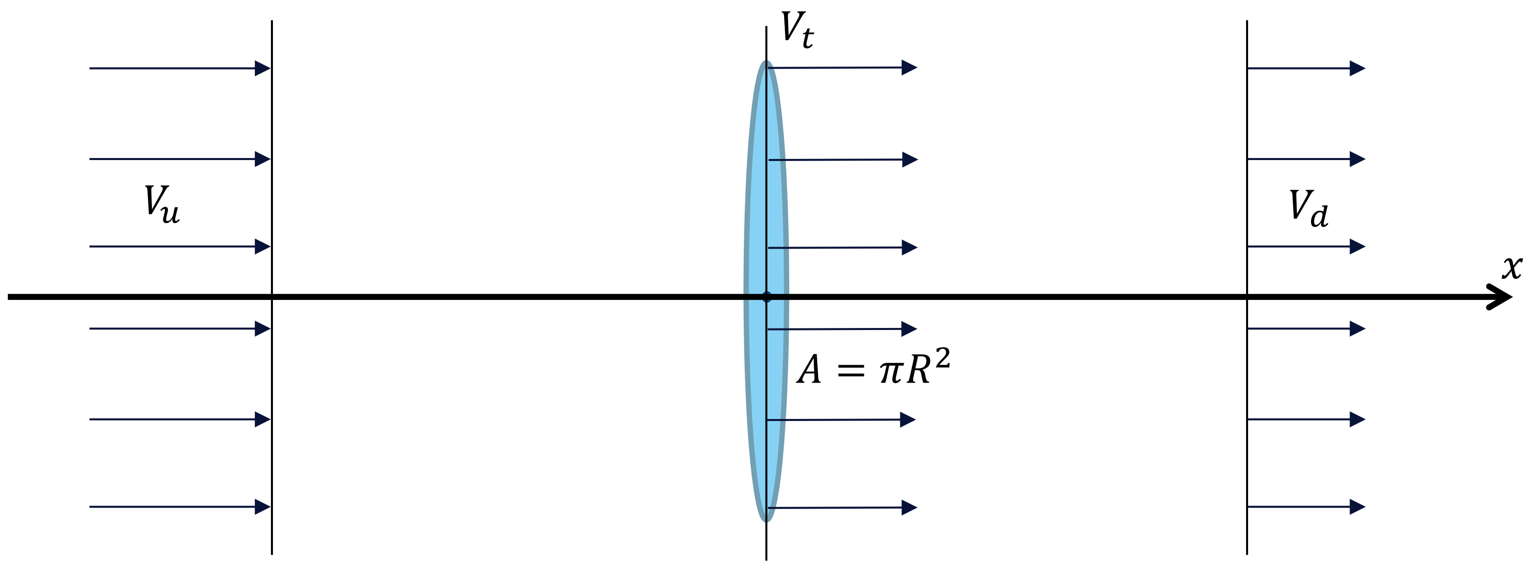
- Single turbine model
- Wind farm modelling
- Power optimization
- Wind farm-farm interactions

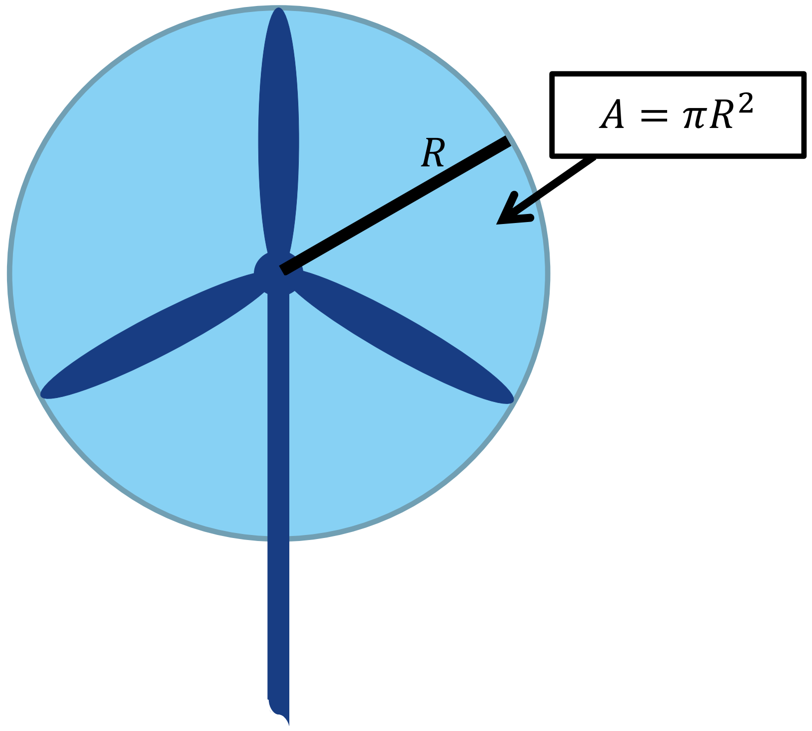


SINGLE TURBINE MODEL

- Classical Betz theory (1929)

- Ignore blade structure
- V_u is uniform

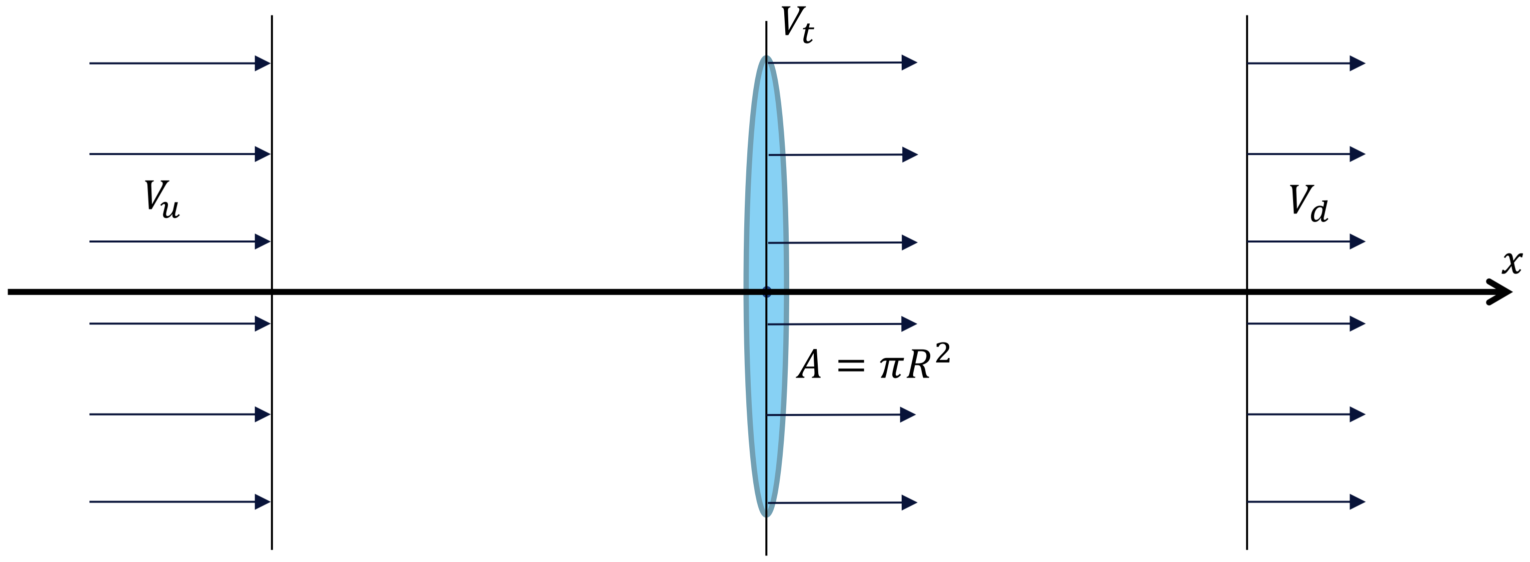




BETZ THEORY

Energy extracted

$$\Delta E = \frac{1}{2}\,\Delta m\left(V_u^2 - V_d^2\right)$$



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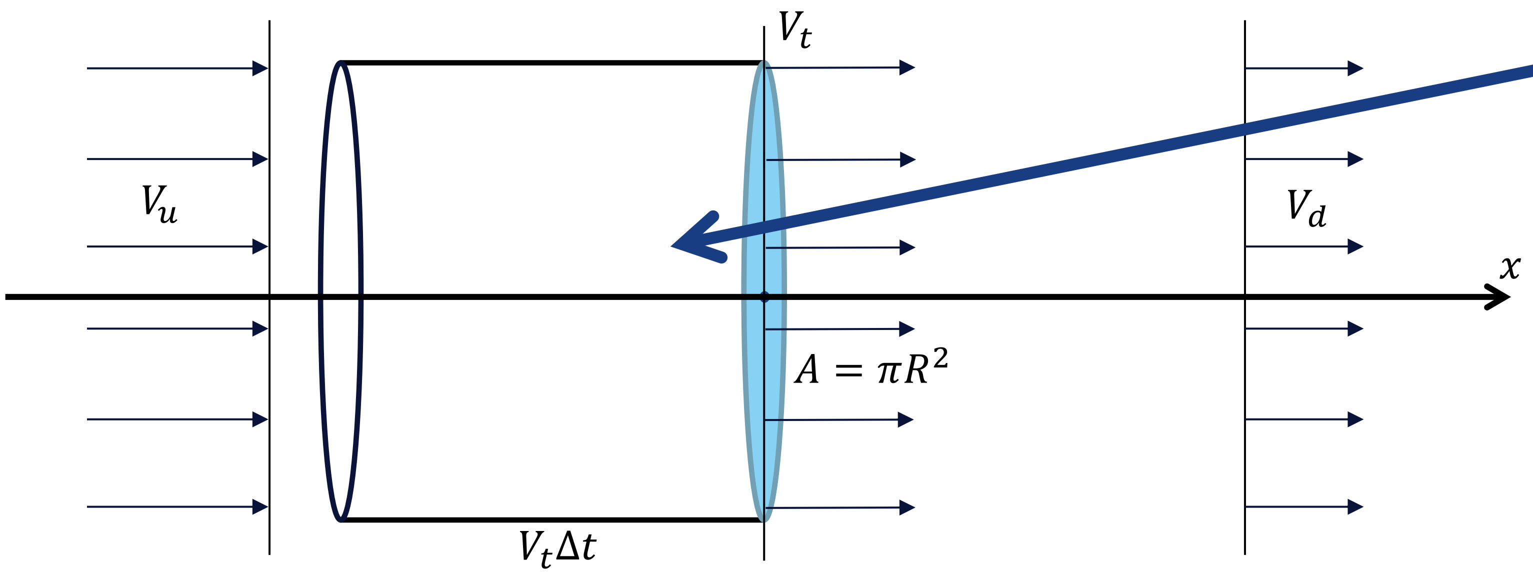


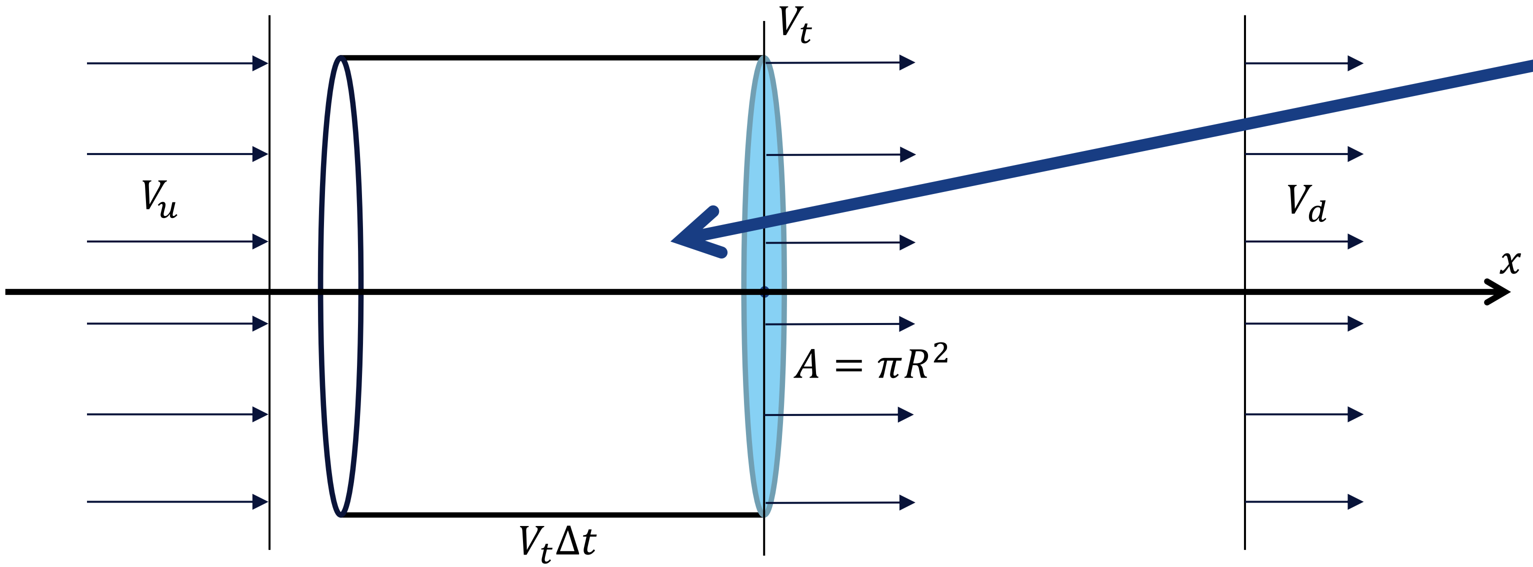
$$\mathcal{V} = \pi R^2 V_t \Delta t = \frac{\Delta m}{\rho}$$

BETZ THEORY

Energy extracted

$$\Delta E = \frac{1}{2}\,\Delta m\left(V_u^2 - V_d^2\right) = \frac{1}{2}\,\rho\pi R^2 V_t \Delta t\left(V_u^2 - V_d^2\right)$$

$$\mathcal{V} = \pi R^2 V_t \Delta t = \frac{\Delta m}{\rho}$$



BETZ THEORY

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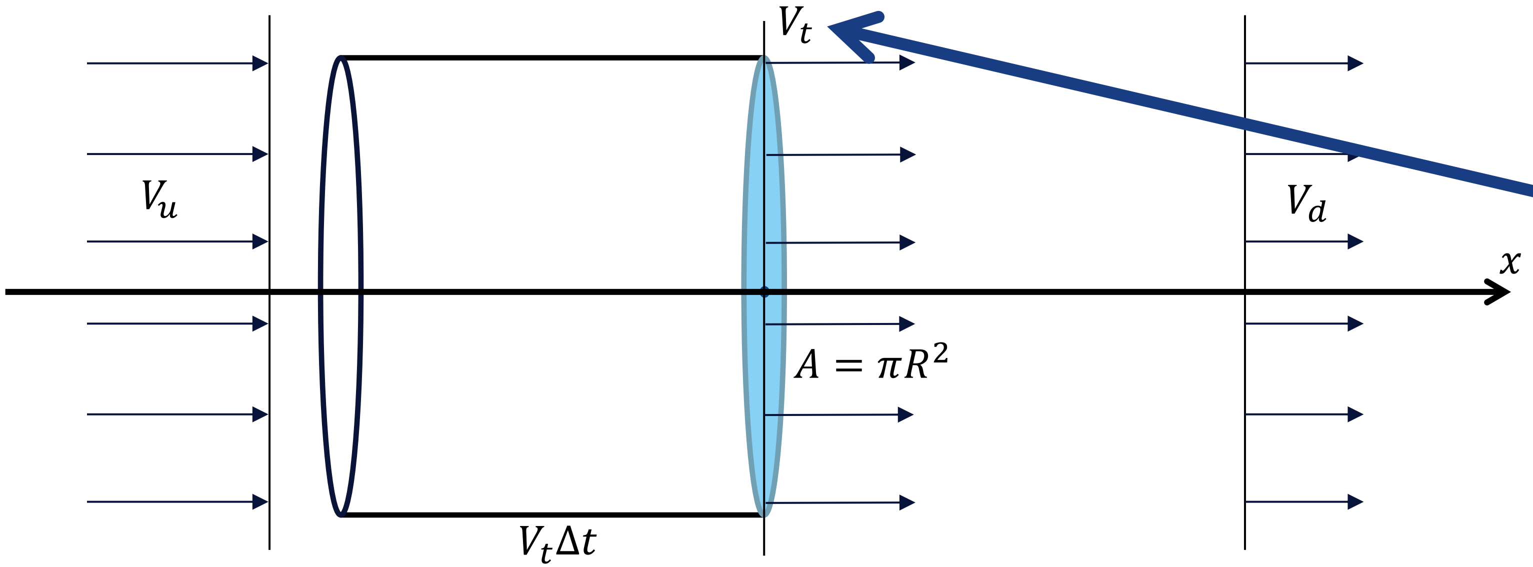


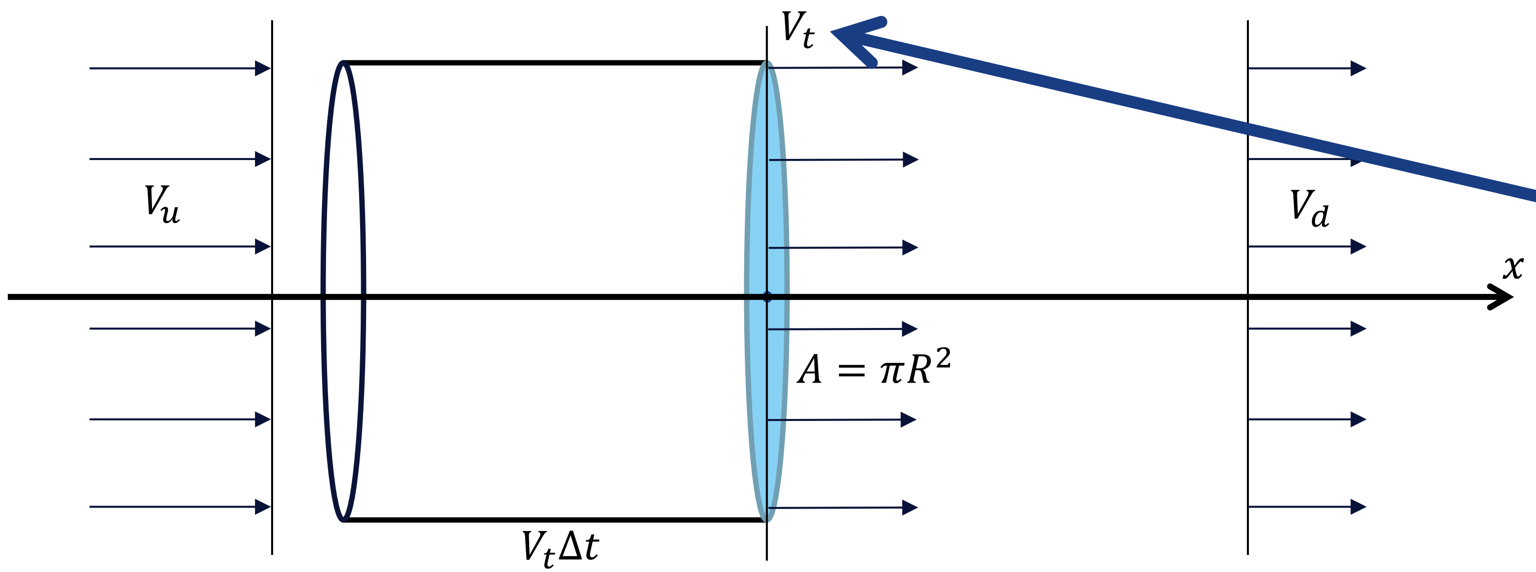
$$V_u < V_t < V_d$$

$$V_t = \frac{V_u + V_d}{2}$$

BETZ THEORY

Energy extracted

$$\Delta E = \frac{1}{2}\,\Delta m\left(V_u^2 - V_d^2\right) = \frac{1}{2}\,\rho\pi R^2 V_t \Delta t\left(V_u^2 - V_d^2\right) = \frac{\rho\pi R^2}{2}\left(\frac{V_u + V_d}{2}\right)\left(V_u^2 - V_d^2\right)\Delta \mathrm{t}$$



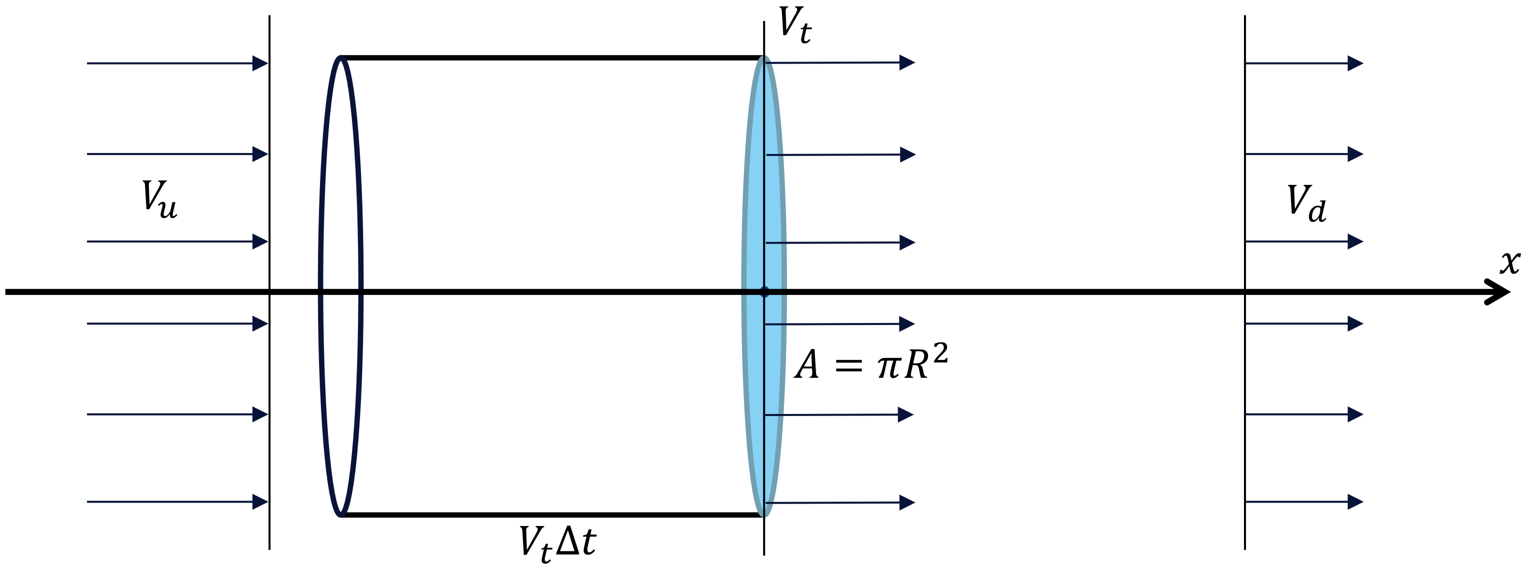
$$V_u < V_t < V_d$$

$$V_t = \frac{V_u + V_d}{2}$$

BETZ THEORY

Power output

$$P = \frac{\Delta E}{\Delta t} = \frac{\rho \pi R^2}{2}\left[\frac{1+\frac{V_d}{V_u}}{2}\left(1-\left(\frac{V_u}{V_d}\right)^2\right)\right] V_u^3$$

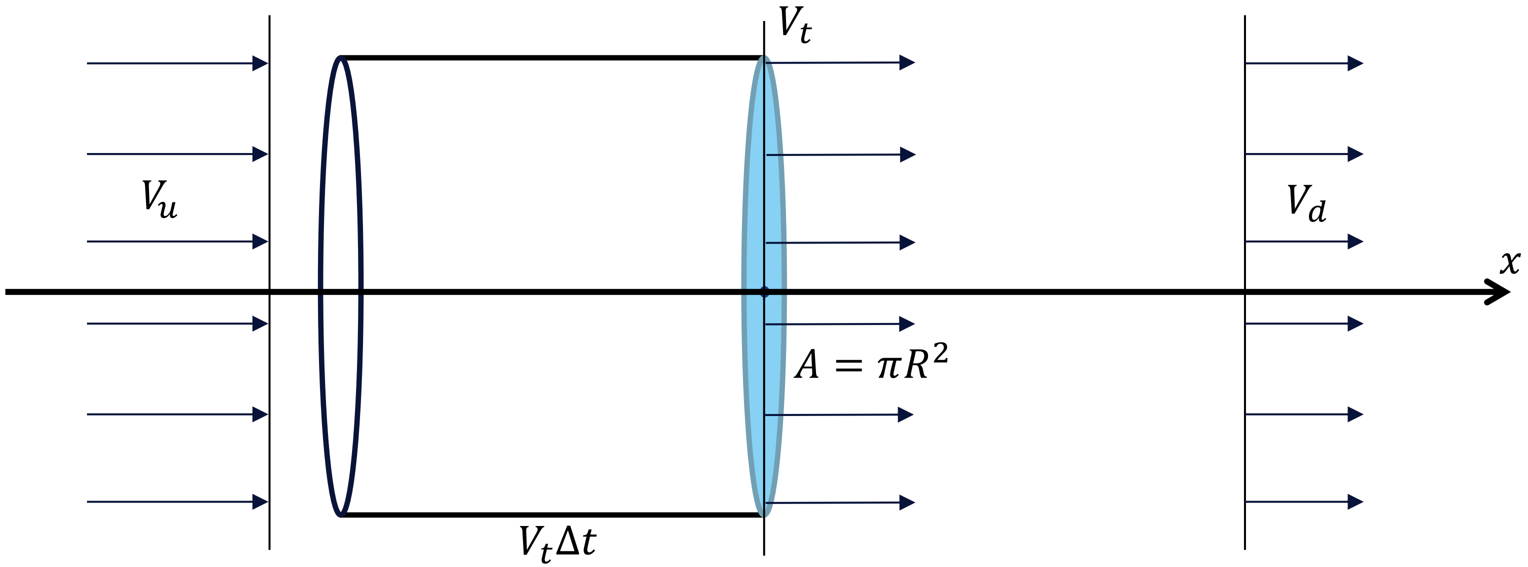


BETZ THEORY

Power output

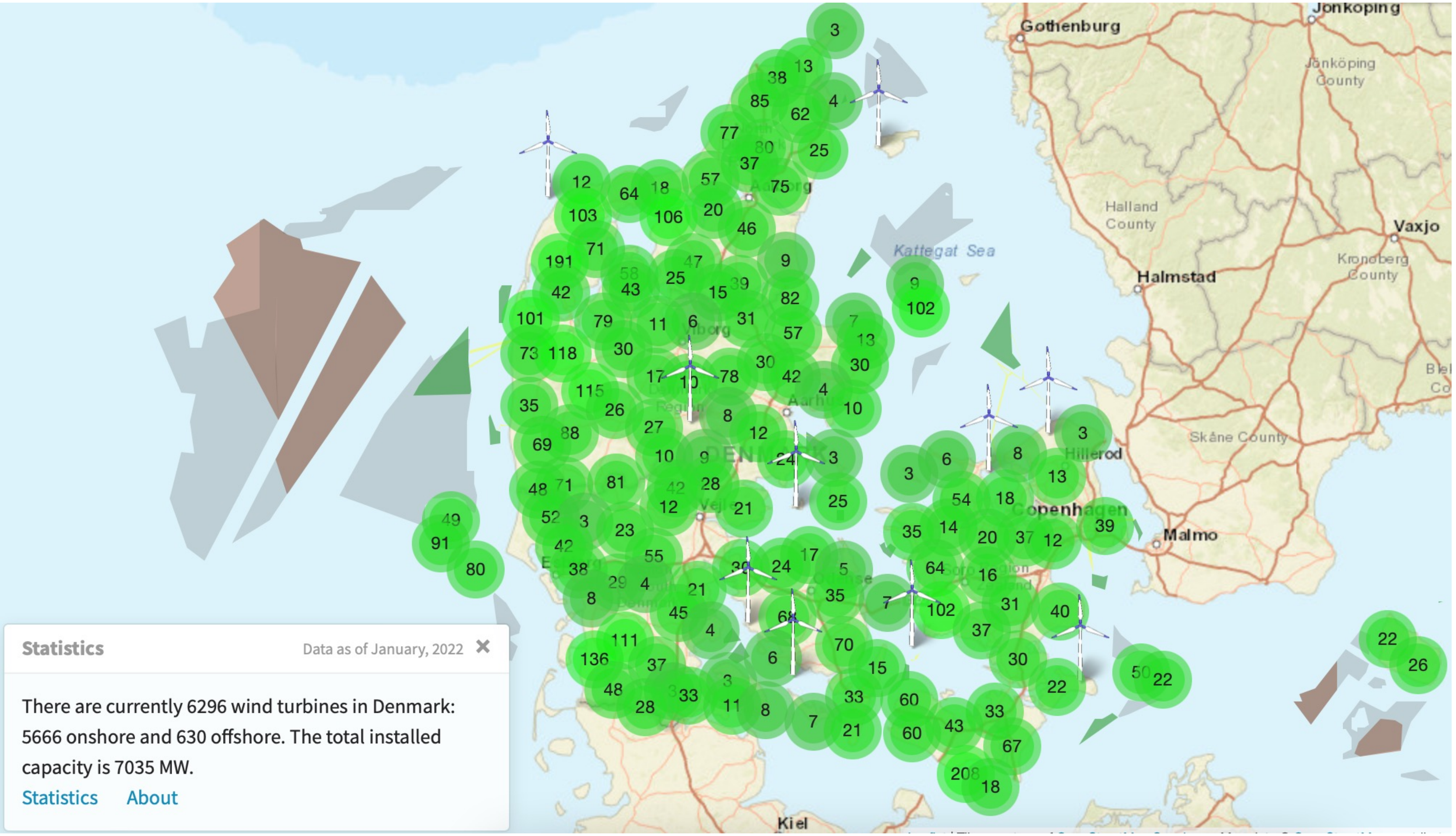
$$C_P = \frac{1+q}{2}(1-q^2), \qquad q = \frac{V_d}{V_u}$$

$$P = \frac{\Delta E}{\Delta t} = \frac{\rho \pi R^2}{2}\boxed{\left[\frac{1+\frac{V_d}{V_u}}{2}\left(1-\left(\frac{V_u}{V_d}\right)^2\right)\right]}V_u^3$$

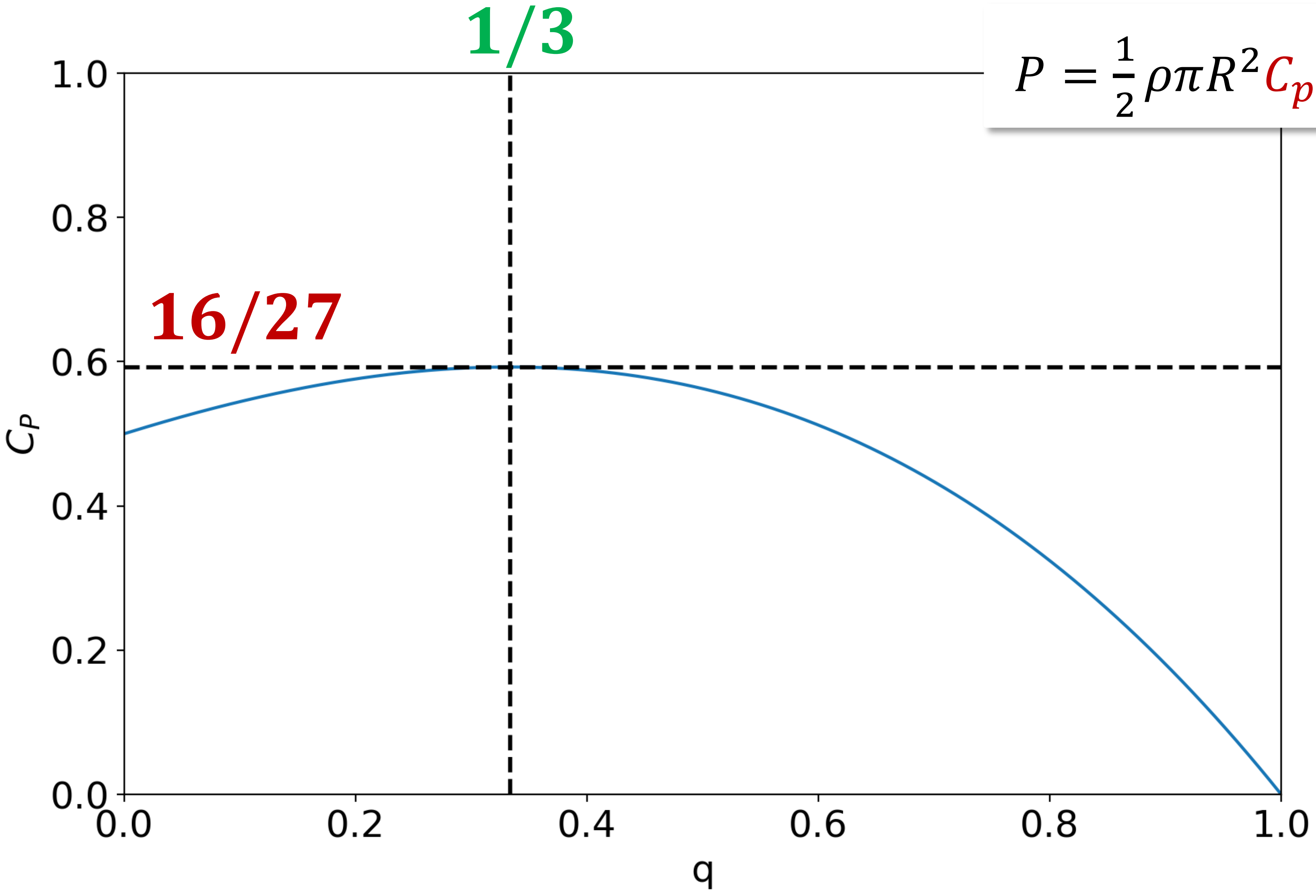


HOW TO MAXIMIZE POWER OUTPUT?

https://turbines.dk



OPTIMAL PITCH ANLGE

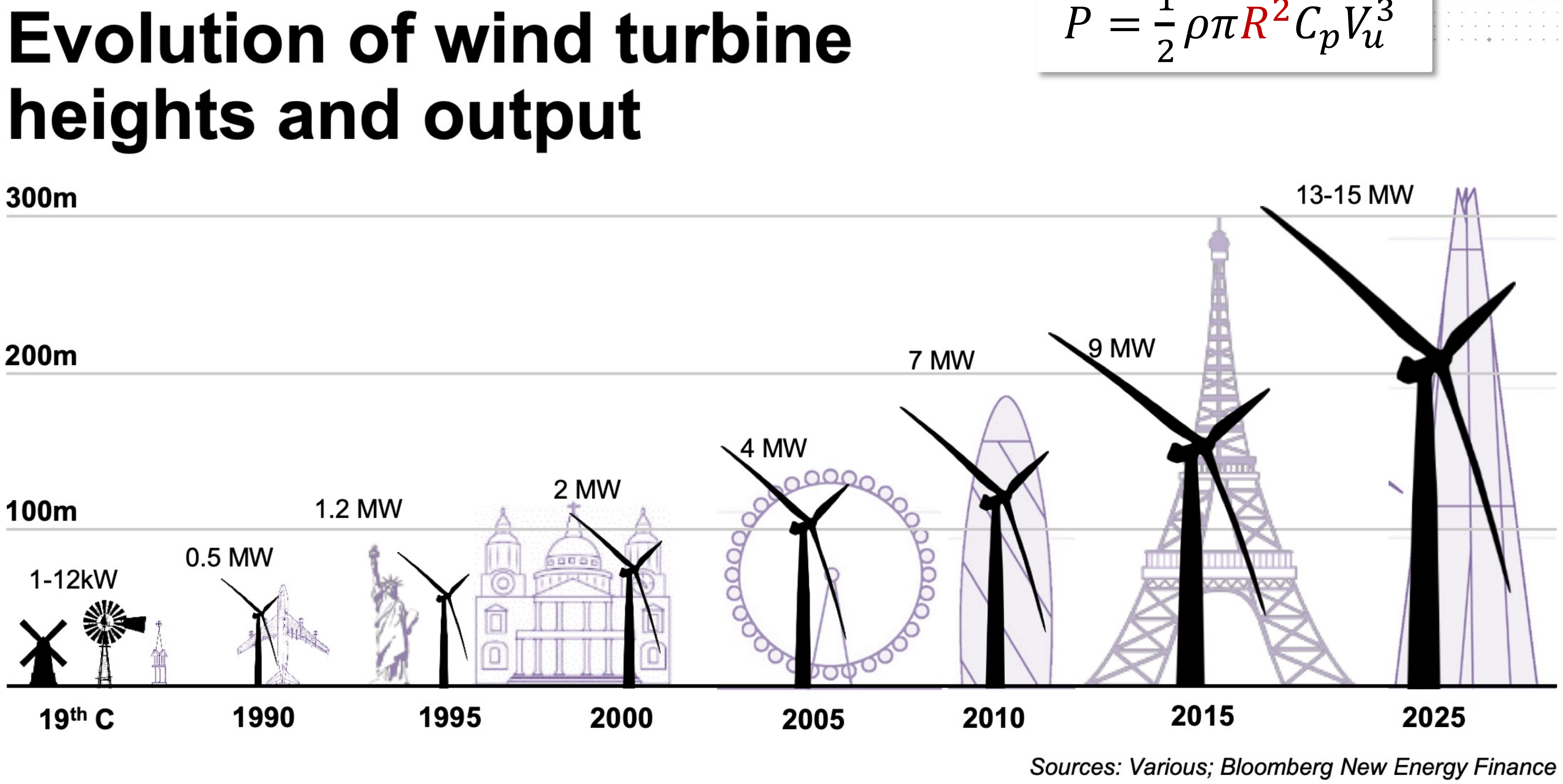


$$P = \frac{1}{2}\rho\pi R^2 C_p V_u^3$$

$$q = \frac{V_d}{V_u}$$

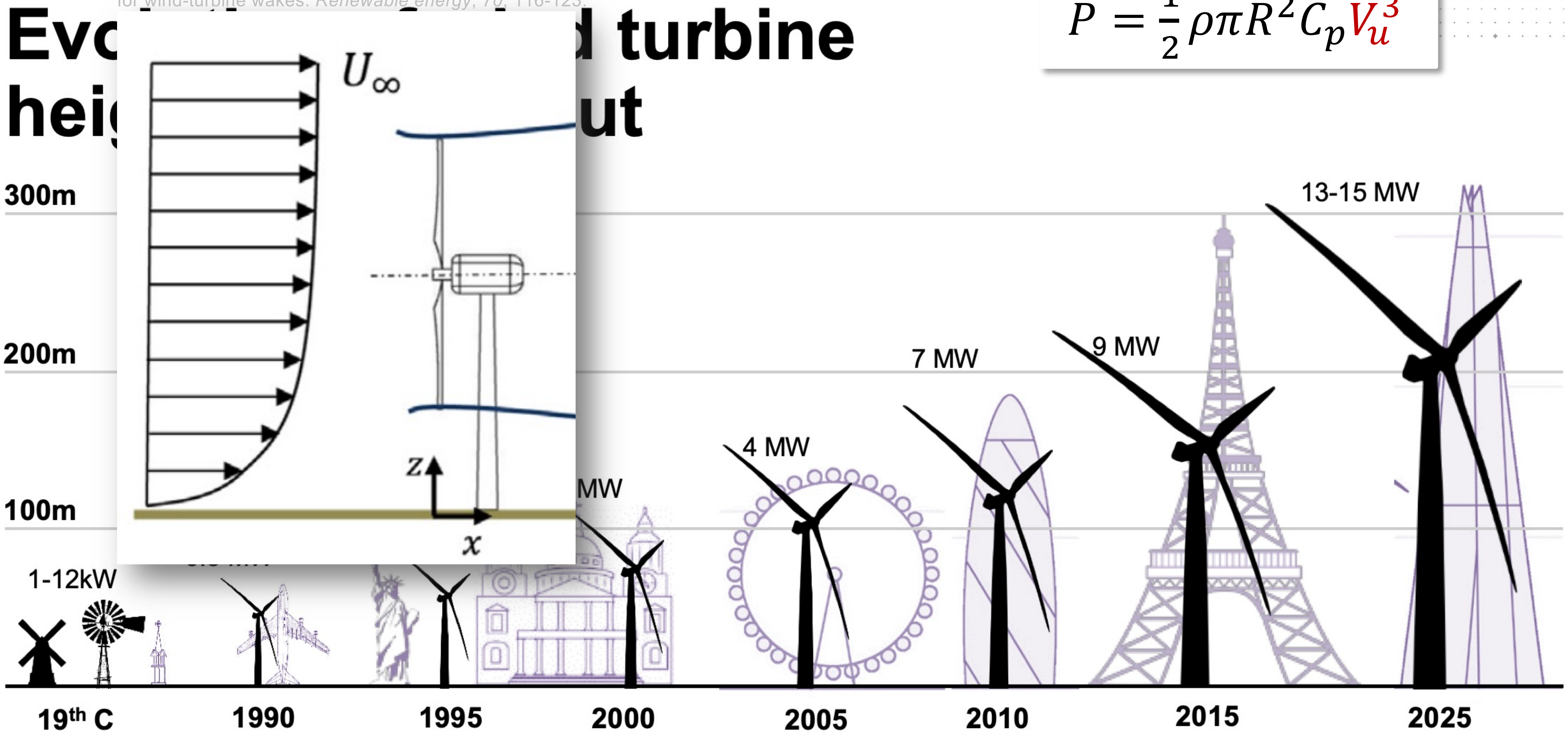
INCREASE R

$$P = \frac{1}{2}\rho\pi R^2 C_p V_u^3$$



INCREASE HUB HEIGHT

Bastankhah, M., & Porté-Agel, F. (2014). A new analytical model for wind-turbine wakes. *Renewable energy*, *70*, 116-123.

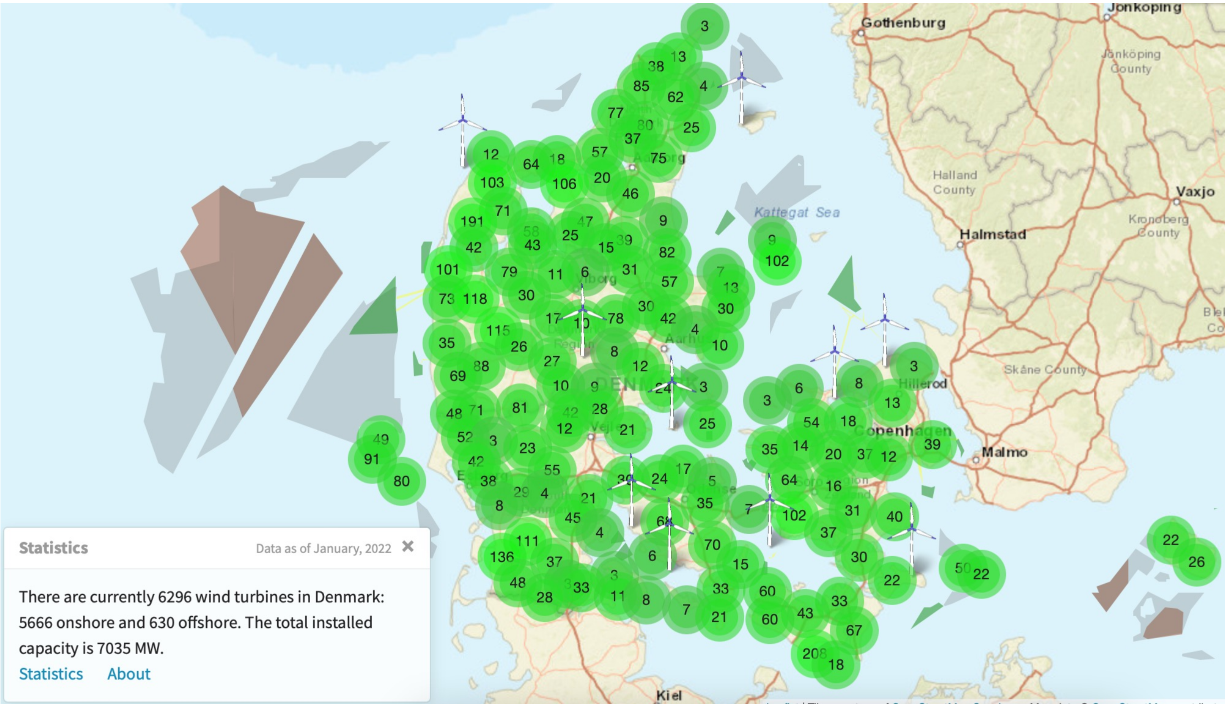


$$P = \frac{1}{2}\rho\pi R^2 C_p V_u^3$$

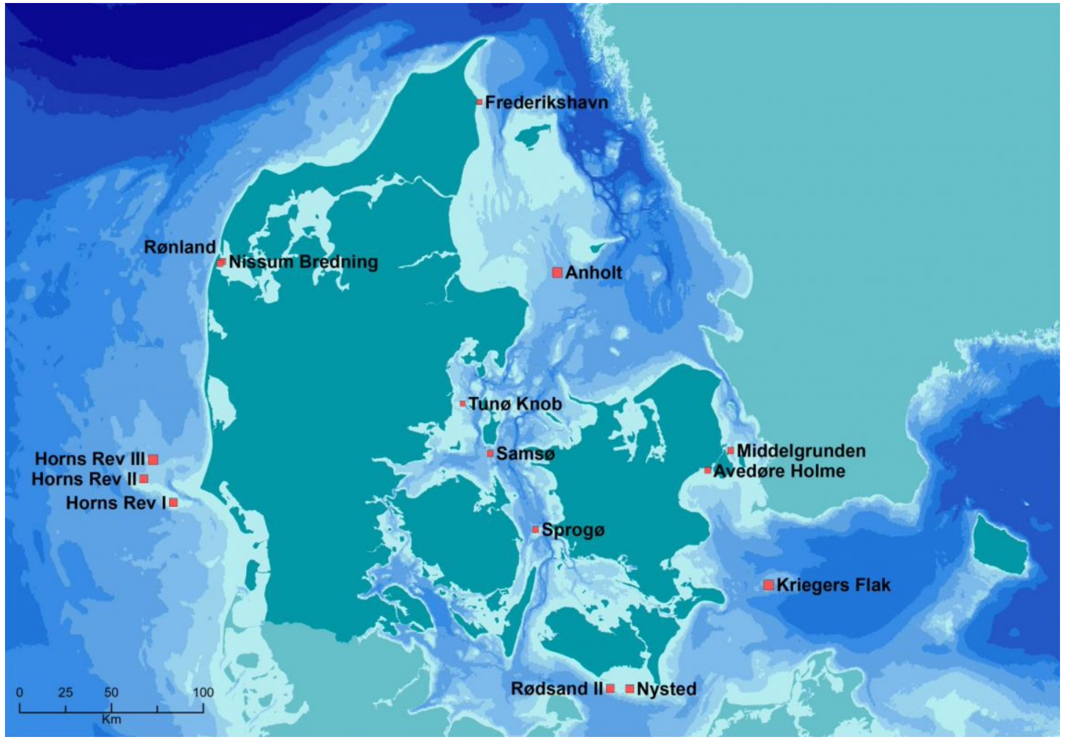
Sources: Various; Bloomberg New Energy Finance

BETTER WIND CONDITIONS OFFSHORE

https://turbines.dk

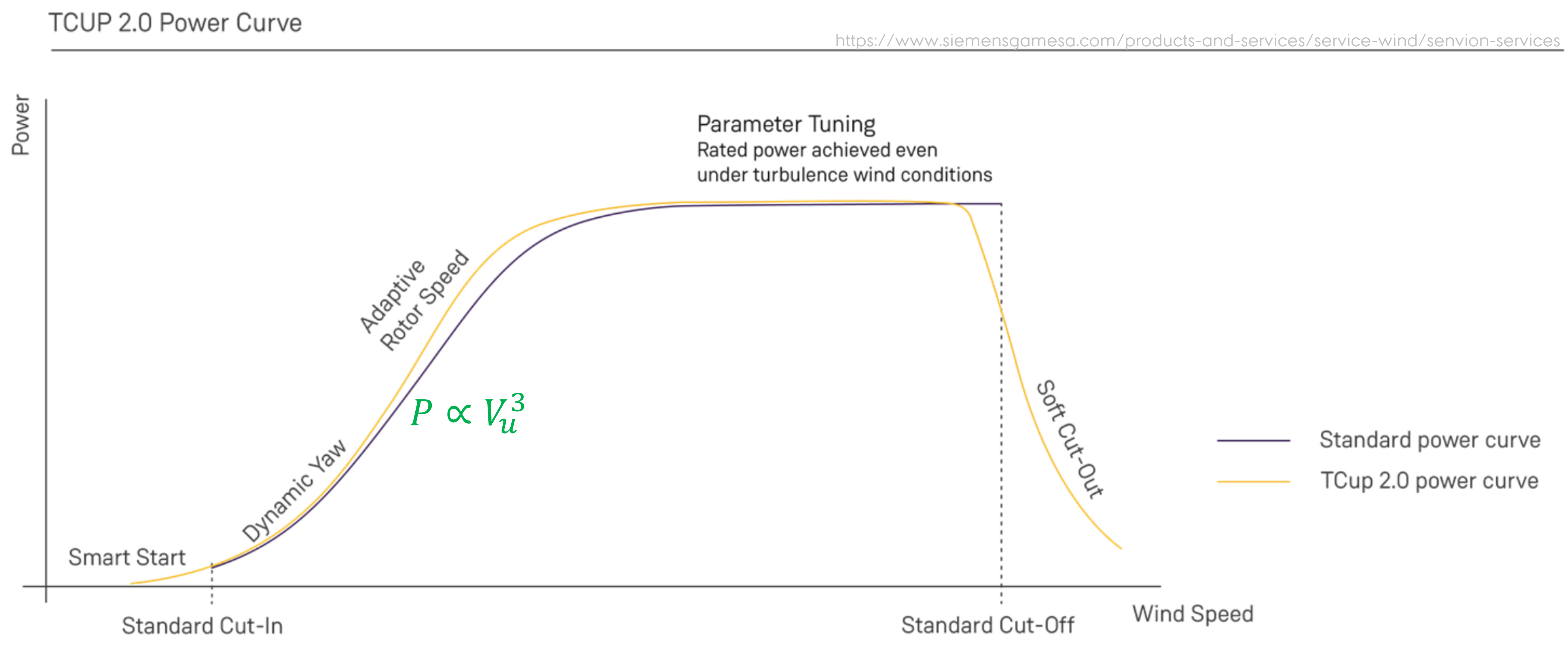


$$P = \frac{1}{2}\rho\pi R^2 C_p V_u^3$$



https://ens.dk/ansvarsomraader/vindmoeller-paa-hav/etablerede-havvindmoelleparker

POWER CURVE

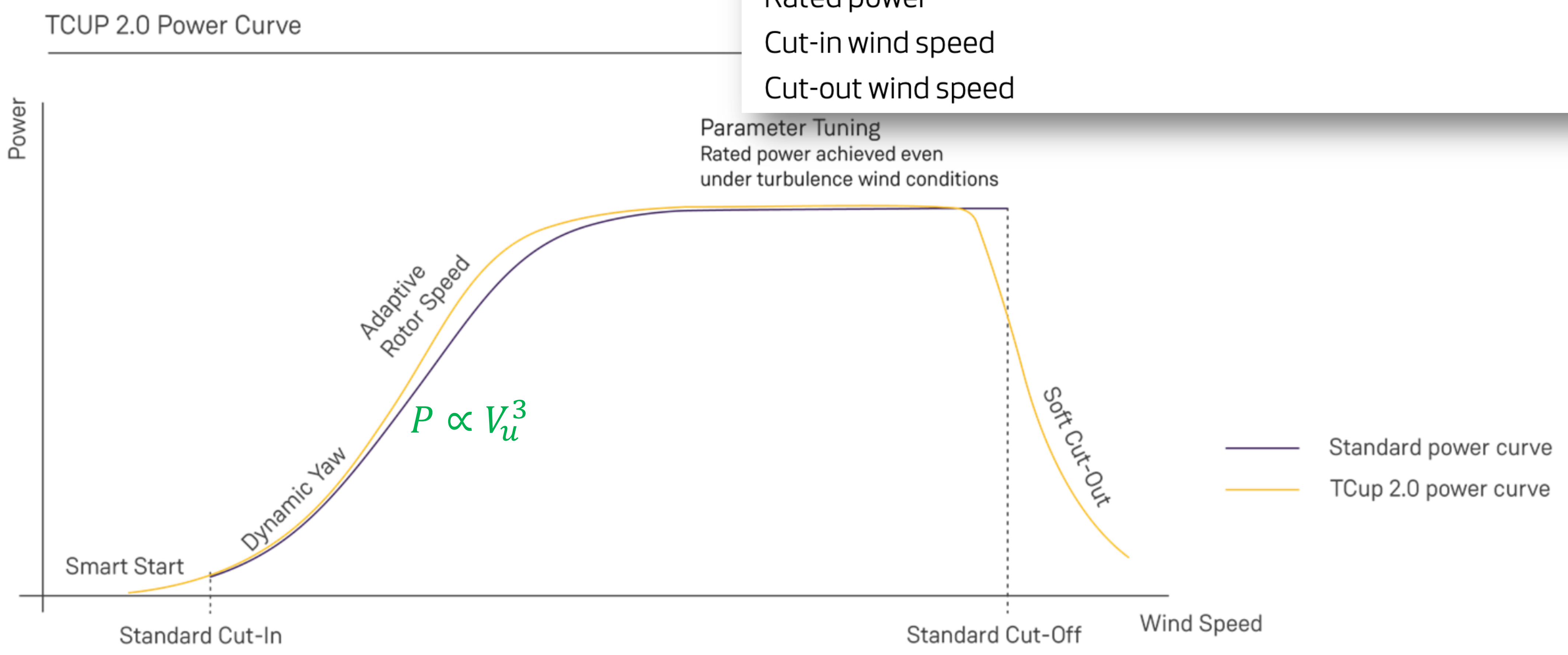


TCUP 2.0 Power Curve
https://www.siemensgamesa.com/products-and-services/service-wind/senvion-services
Power
Parameter Tuning
Rated power achieved even
under turbulence wind conditions
Adaptive
Rotor Speed
$P \propto V_u^3$
Dynamic Yaw
Smart Start
Soft Cut-Out
Standard power curve
TCup 2.0 power curve
Standard Cut-In
Standard Cut-Off
Wind Speed

POWER CURVE

Vestas 4 MW platform brochure: https://www.vestas.com/en/products/4-mw-platform

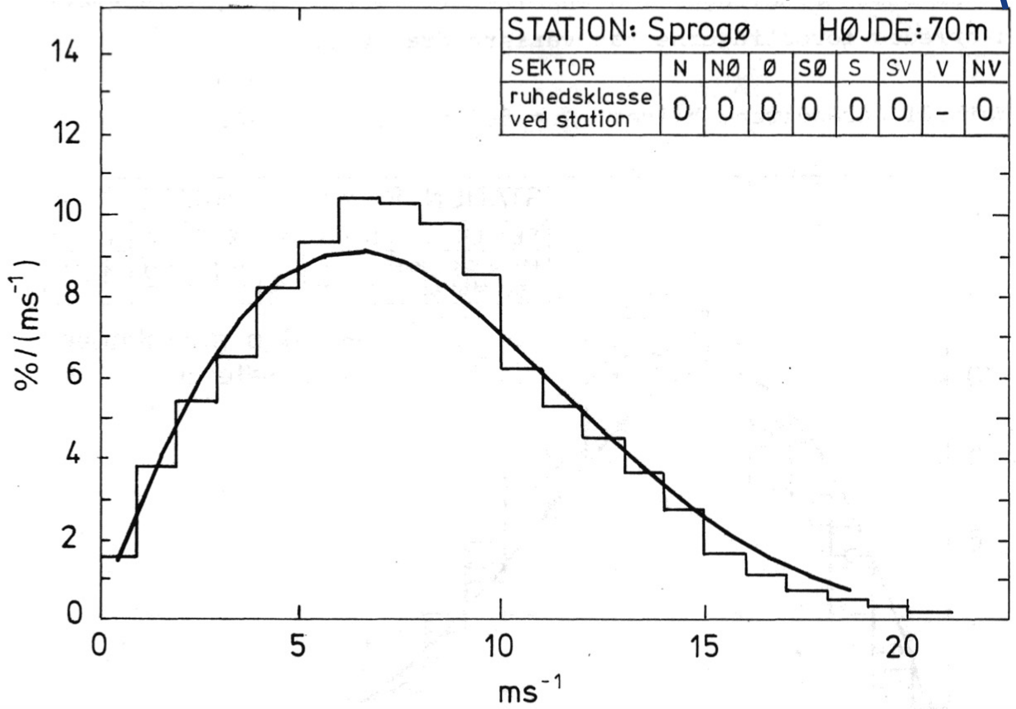
Power regulation	Pitch regulated with variable speed
Operating data	
Rated power	4,500kW
Cut-in wind speed	3m/s
Cut-out wind speed	24m/s

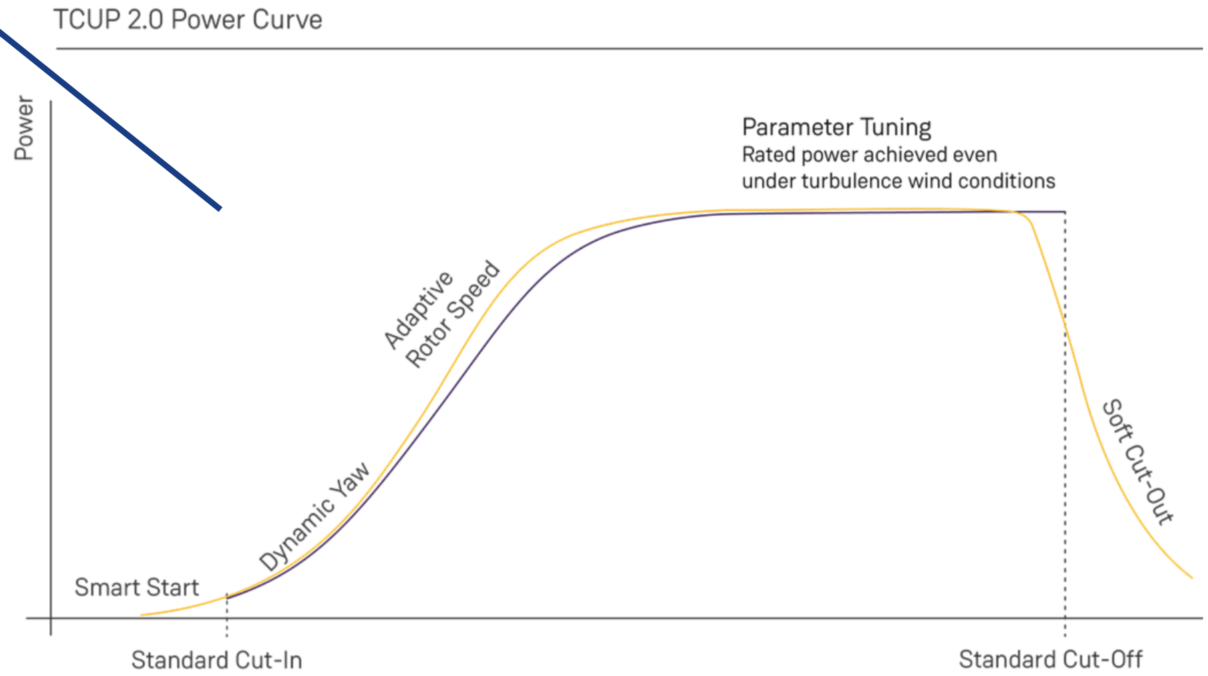


ANNUAL ENERGY PRODUCTION

$$\text{AEP} = 8760 \int_0^{\infty} p(u)P(u)\,\text{d}u$$

Petersen, E. L., Troen, I., Prandsen, S., & Hedegaard, K. (1981). Windatlas for Denmark RISO. *Roskilde, Denmark-RISO National Laboratory*.





ANNUAL ENERGY PRODUCTION

$$\text{AEP} = 8760 \int_0^\infty p(u)P(u)\,\mathrm{d}u = 8760 \cdot \text{CF} \cdot P_{rated}$$

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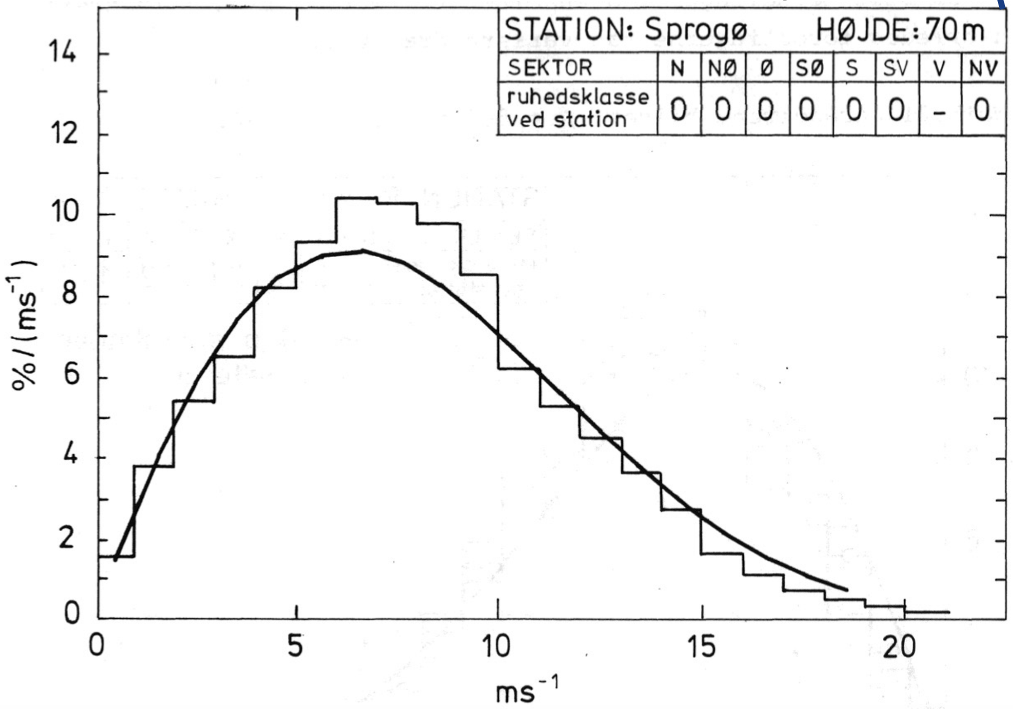

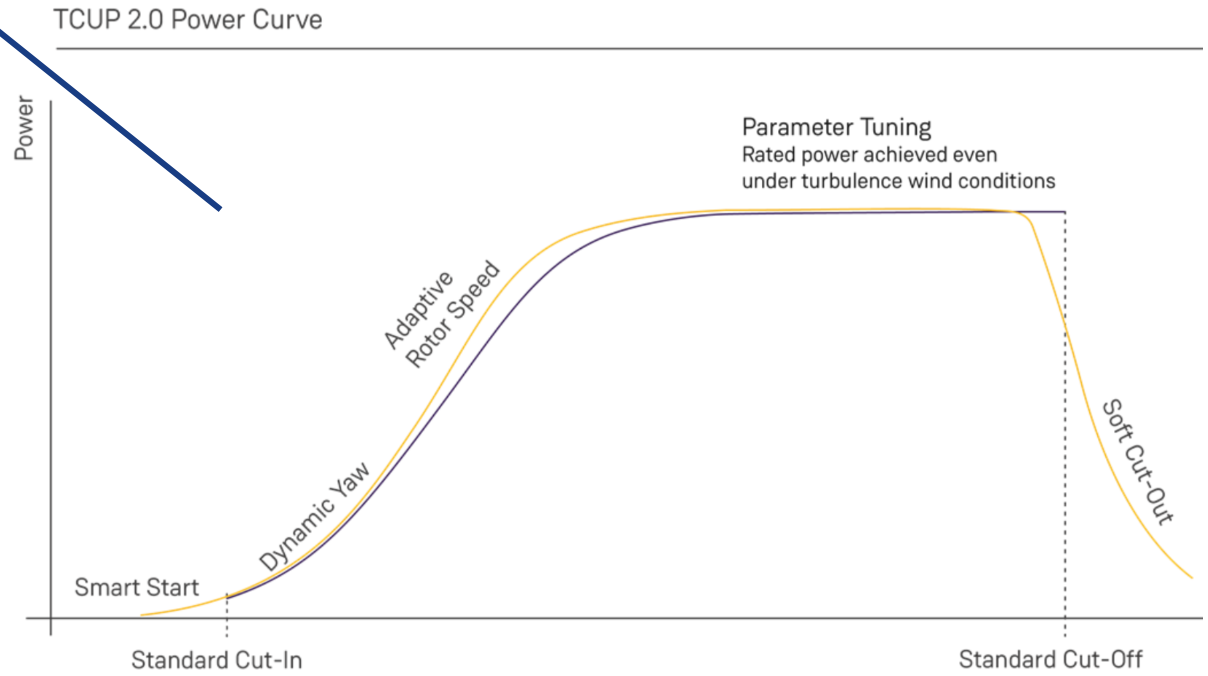

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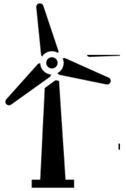
Typical turbine

- $P_{rated} = 4500\ \mathrm{kW}$
- $\mathrm{CF} \sim 0.3$
- $\mathrm{AEP} = 11.8\ \mathrm{GWh/yr}$



My household: $\sim 2600\ \mathrm{kWh}$

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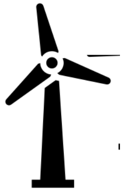

$\times\ 4500$

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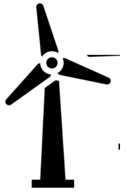

$\times\ 4500$

My household: $\sim 2600\ \mathrm{kWh}$

Typical family: $\sim 4000\ \mathrm{kWh}$

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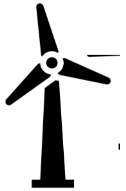


$\times\ 4500$

My household: $\sim 2600\ \mathrm{kWh}$

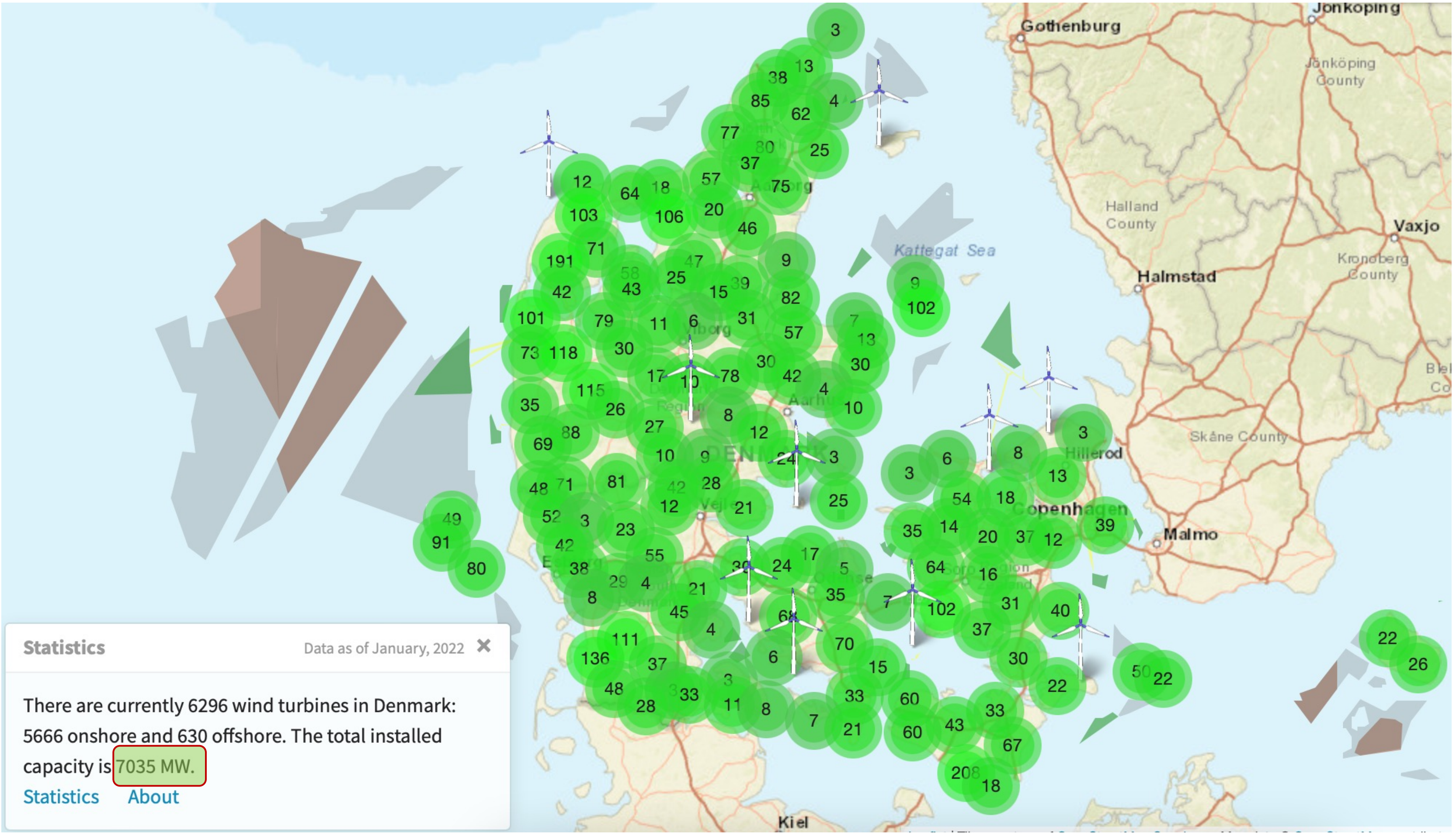
$\times\ 4000$

Typical family: $\sim 4000\ \mathrm{kWh}$

ANNUAL ENERGY PRODUCTION

$$\text{AEP} = 18.5 \text{ TWh/yr}$$

https://turbines.dk



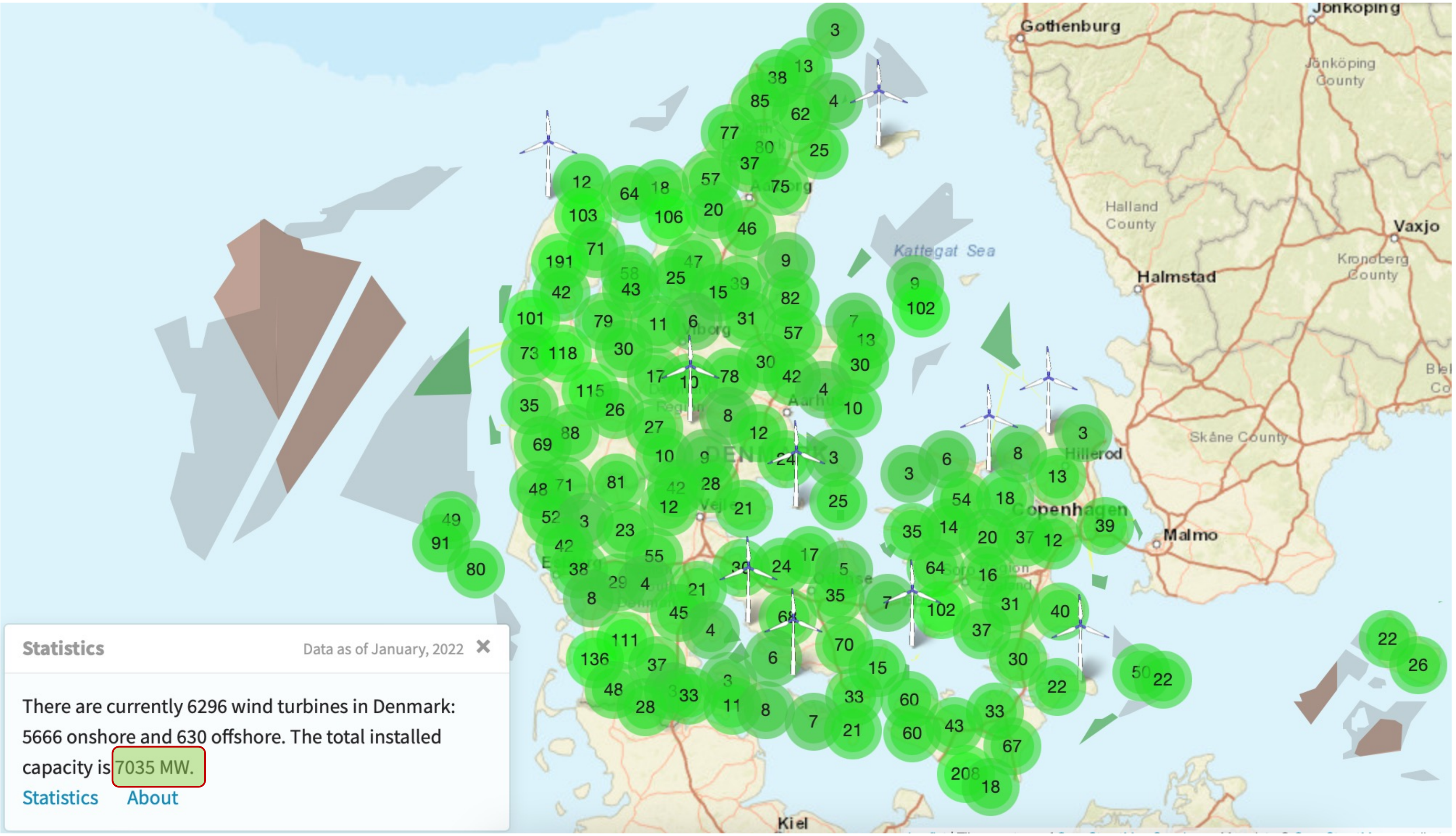
ANNUAL ENERGY PRODUCTION

$$\text{AEP} = 18.5\ \text{TWh/yr}$$

Electricity consumption in 2022:

35 TWh

https://turbines.dk



ANNUAL ENERGY PRODUCTION

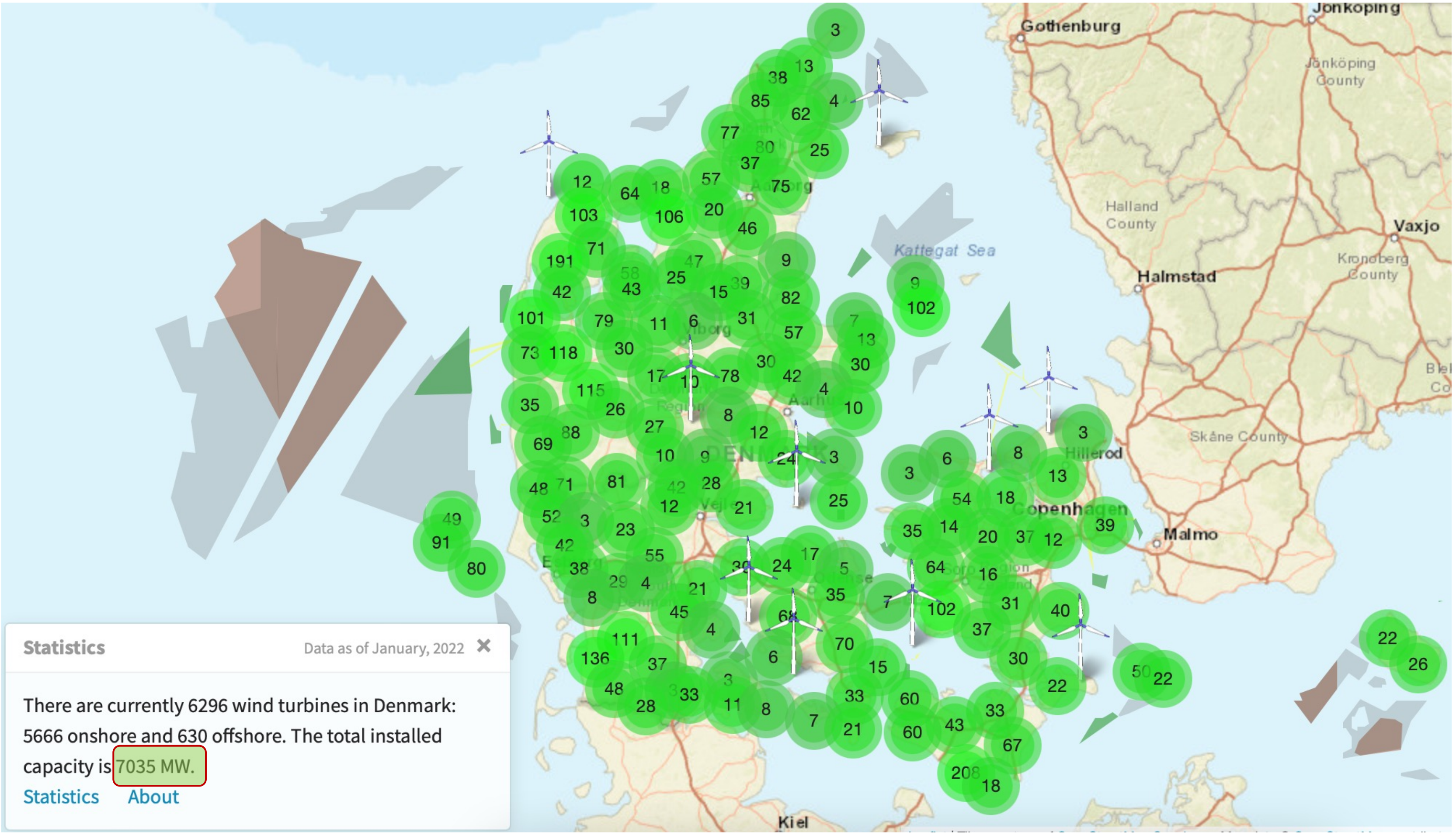
$$\text{AEP} = 18.5\ \text{TWh/yr}$$

Electricity consumption in 2022:

$$35\ \text{TWh}$$

⇒ 53% covered by wind

https://turbines.dk



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Electricity consumption in 2022:

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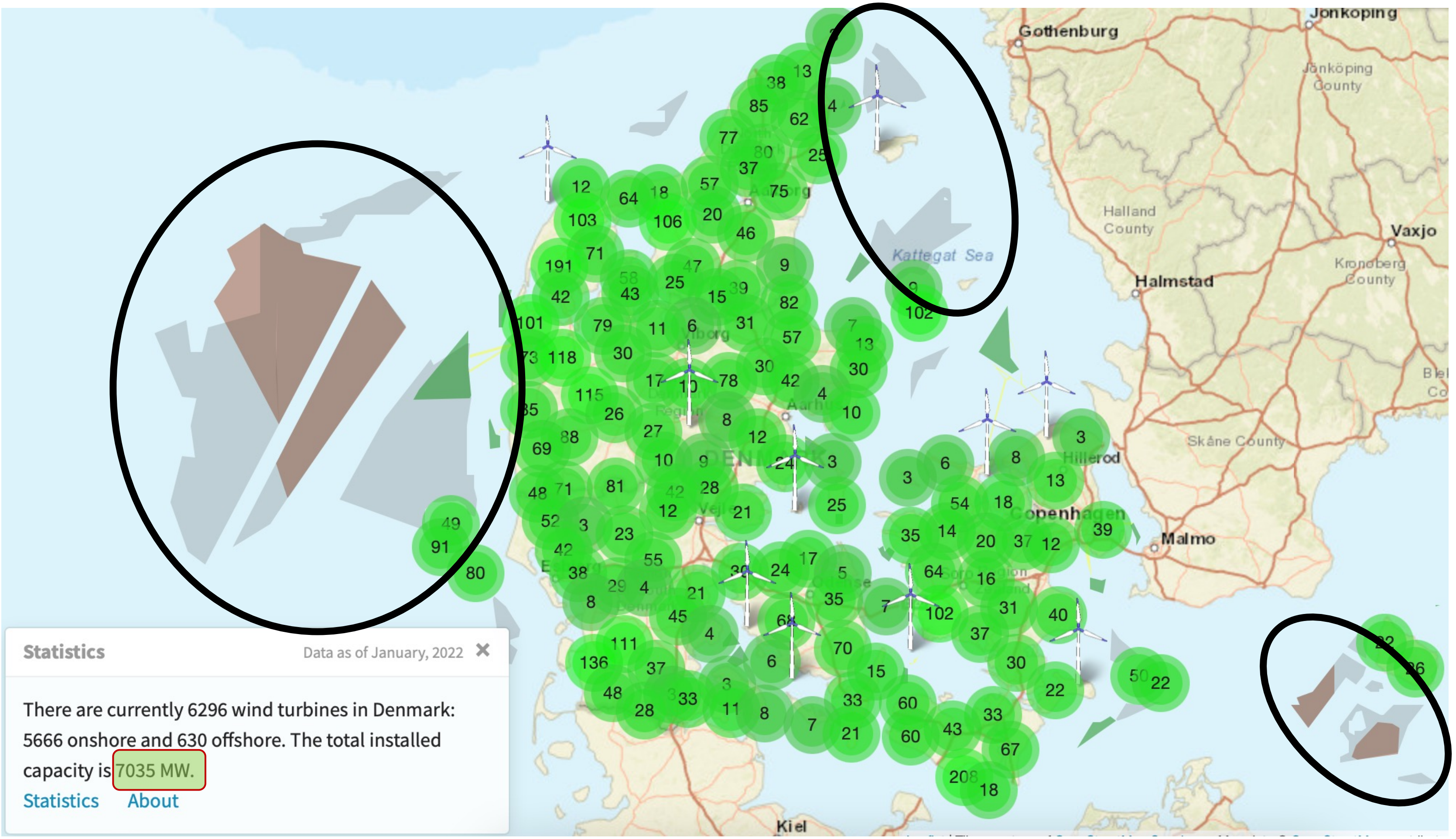
$\Rightarrow$ 53% covered by wind

https://turbines.dk



SINGLE WAKE MODEL

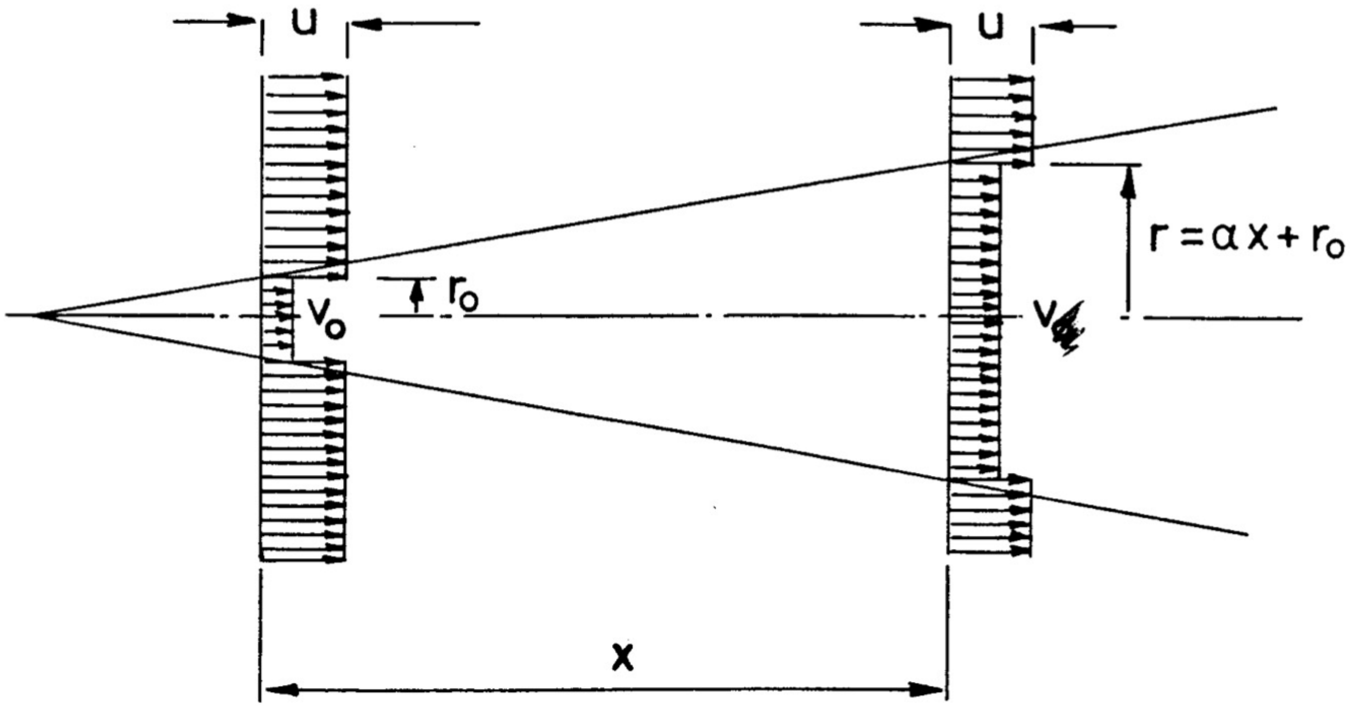
- N. O. Jensen (1983)

Hasager, C. B., Rasmussen, L., Peña, A., Jensen, L. E., & Réthoré, P. E. (2013). Wind farm wake: The Horns Rev photo case. *Energies*, *6*(2), 696-716.

JENSEN WAKE MODEL

$V_u = u$

- Axial symmetry
- Wake increases linearly



Jensen, N. O. (1983). *A note on wind generator interaction* (Vol. 2411). Roskilde, Denmark: Risø National Laboratory.

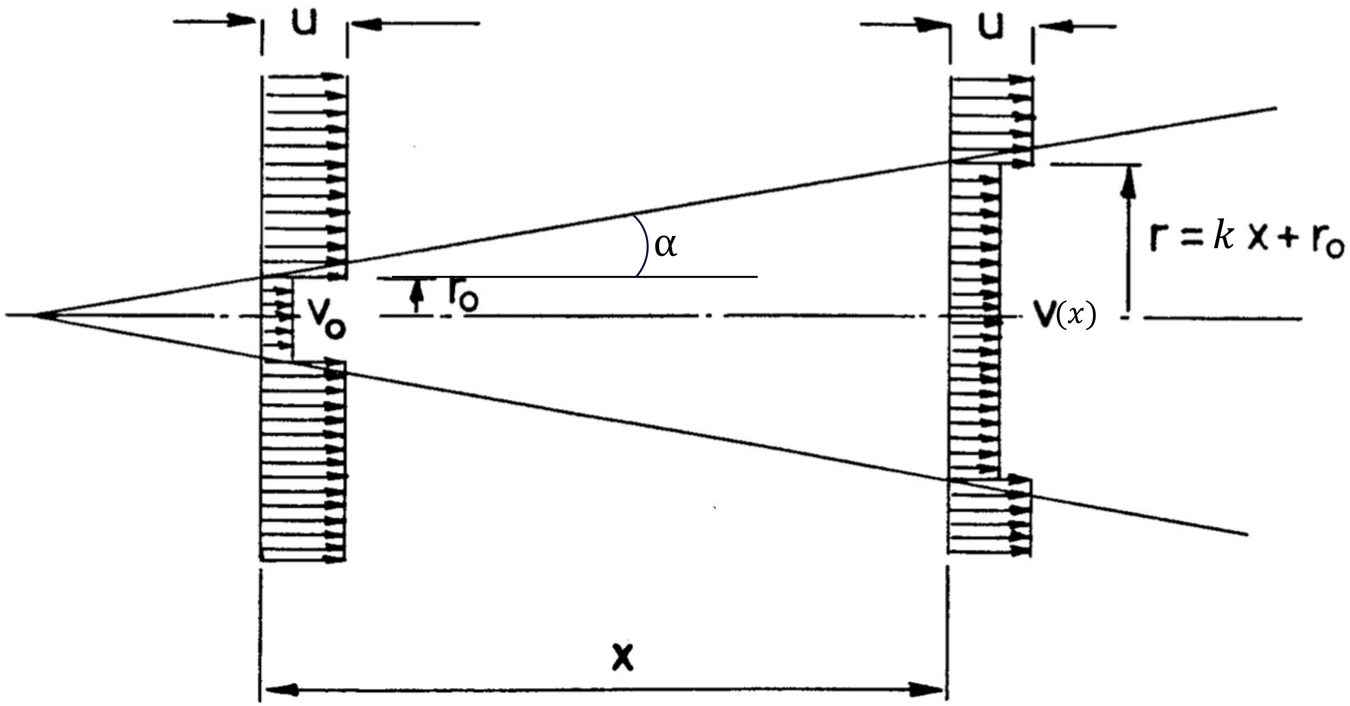
JENSEN WAKE MODEL

- Axial symmetry
- Wake increases linearly

$$r(x) = r_0 + kx$$

where

$$k = \tan(\alpha)$$



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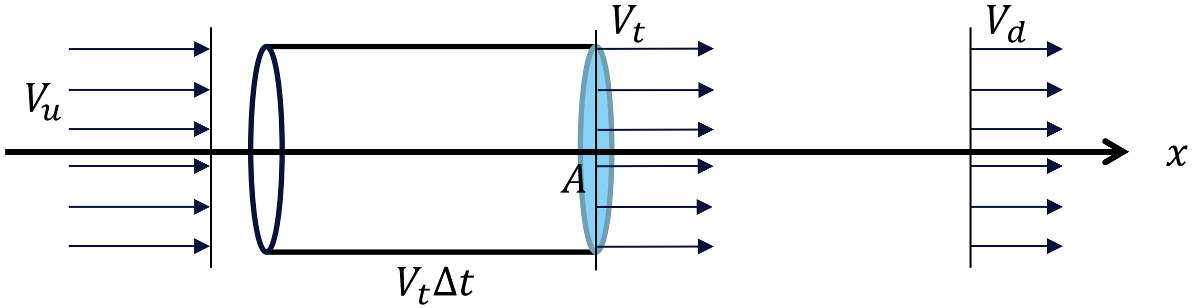
JENSEN WAKE MODEL



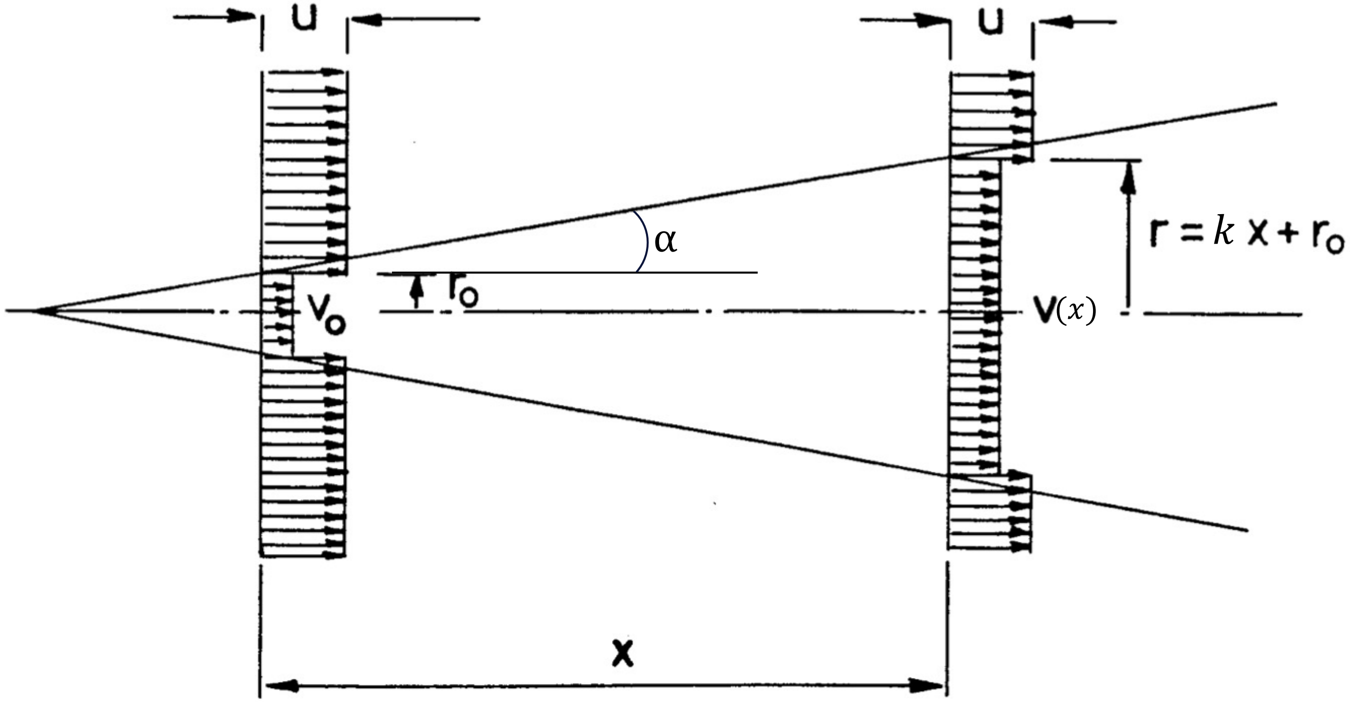
Behind turbine

$$V_0 = qu$$

Downstream wind

$$V(x,q) = u\left[1 - \frac{1-q}{\left(1+\frac{kx}{R}\right)^2}\right]$$

$$r_0 = R$$



Jensen, N. O. (1983). *A note on wind generator interaction* (Vol. 2411). Roskilde, Denmark: Risø National Laboratory.

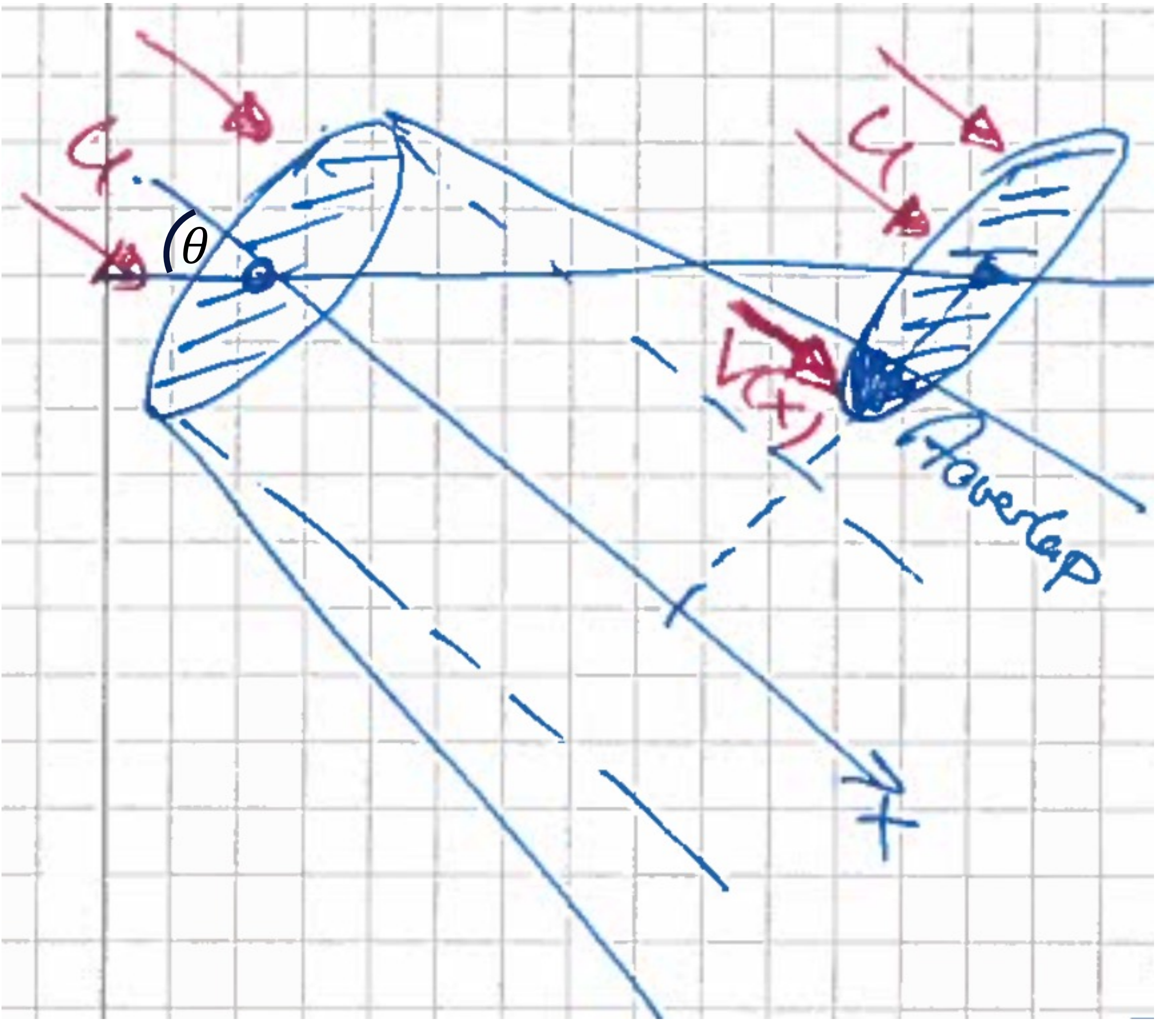
PARTIAL WAKE

Upstream turbine

$$P_1 = \frac{1}{2}\rho A C_p u^3$$

Downwind turbine

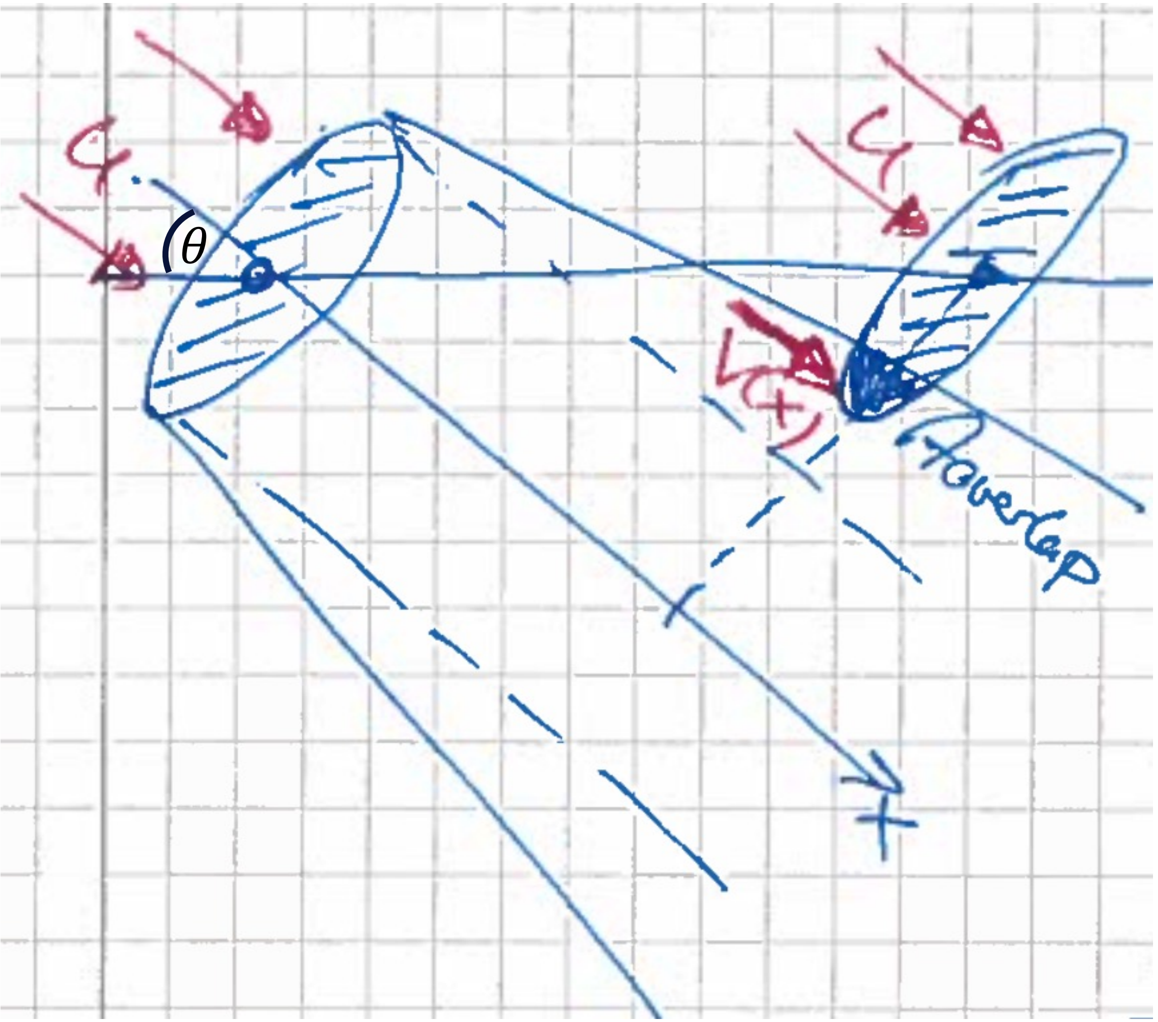
$$P_2 = \frac{1}{2}\rho A C_p \bar{V}^3(\theta, q)$$



PARTIAL WAKE

Effective velocity at downwind turbine

$$\bar{V}(\theta, q) = V(x, q)\frac{A_{overlap}(\theta)}{A} + u\frac{A - A_{overlap}(\theta)}{A}$$

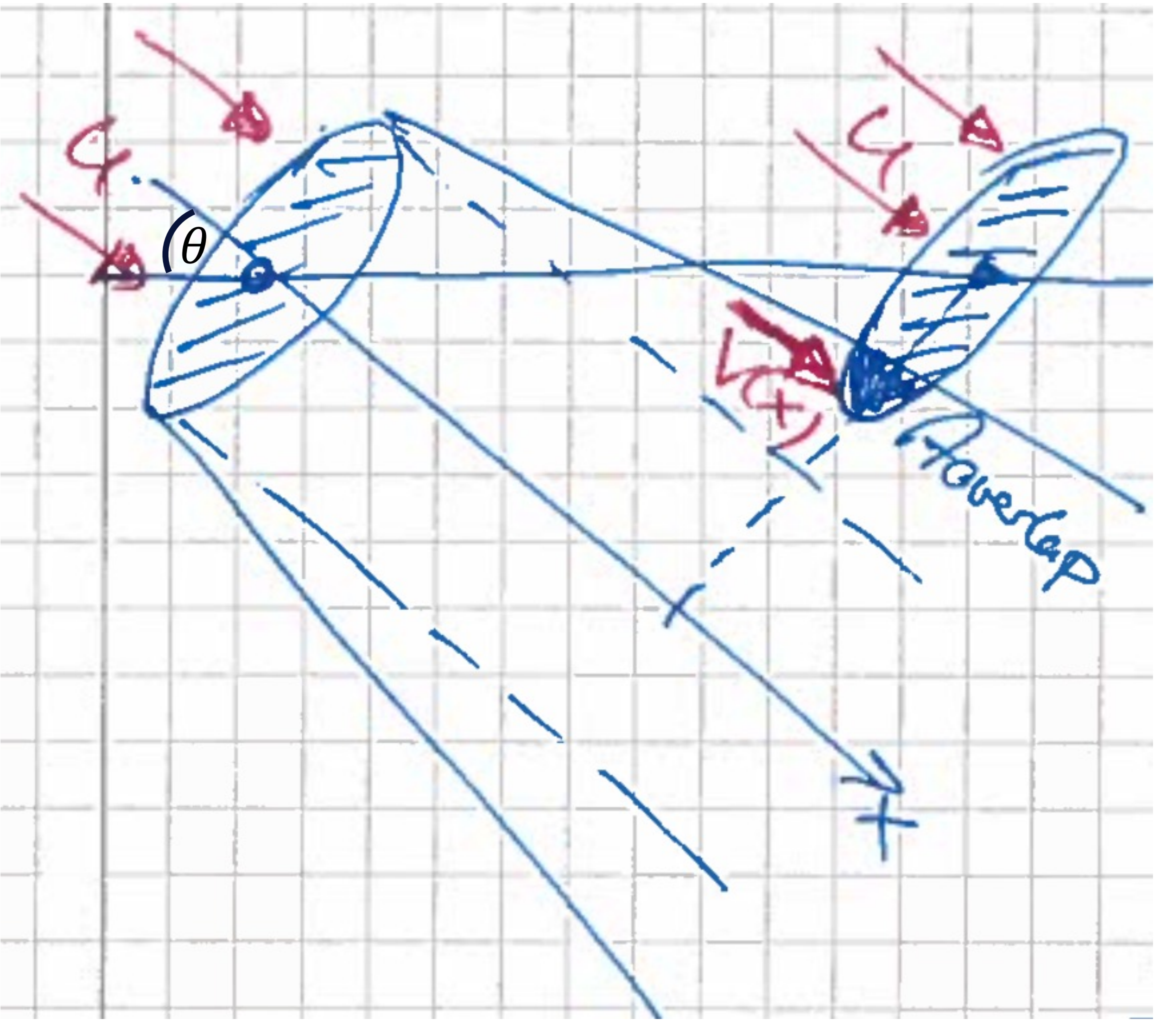


PARTIAL WAKE

Effective velocity at downwind turbine

$$\bar{V}(\theta, q) = V(x, q)\frac{A_{overlap}(\theta)}{A} + u\frac{A - A_{overlap}(\theta)}{A}$$

$$= u\left[1 - \frac{1-q}{\left(1 + \frac{k\,x(\theta)}{R}\right)^2}\frac{A_{overlap}(\theta)}{A}\right]$$



PARTIAL WAKE

Effective velocity at downwind turbine

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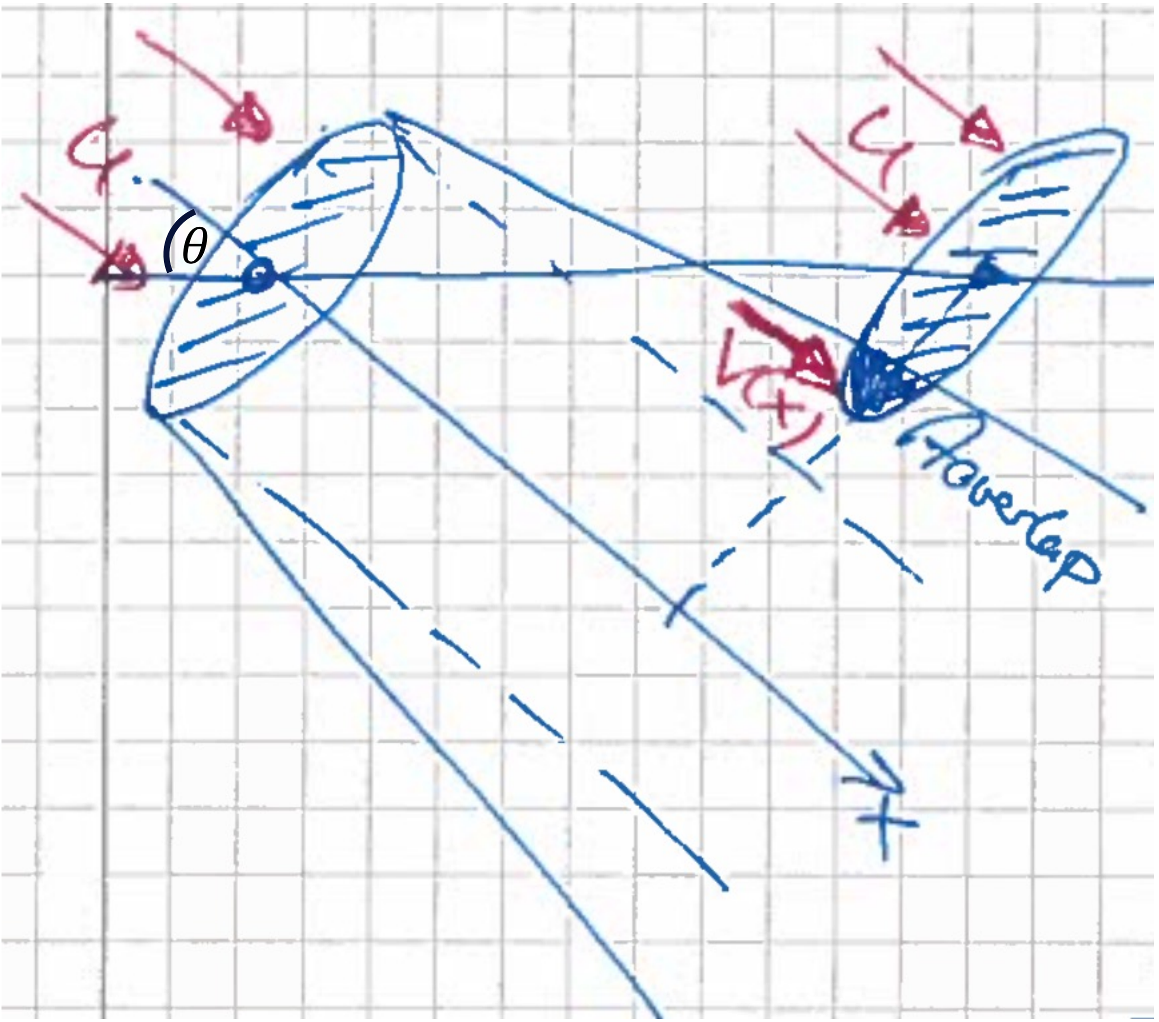
$$= u\left[1 - \boxed{\frac{1-q}{\left(1+\frac{k\,x(\theta)}{R}\right)^2}\,\frac{A_{overlap}(\theta)}{A}}\right]$$

Wind velocity deficit $\delta(\theta,q)$



WAKE INTERACTIONS

- Katic emperical rule (1986)

Hasager, C. B., Rasmussen, L., Peña, A., Jensen, L. E., & Réthoré, P. E. (2013). Wind farm wake: The Horns Rev photo case. *Energies*, *6*(2), 696-716.

KATIC WAKE SUPERPOSITION

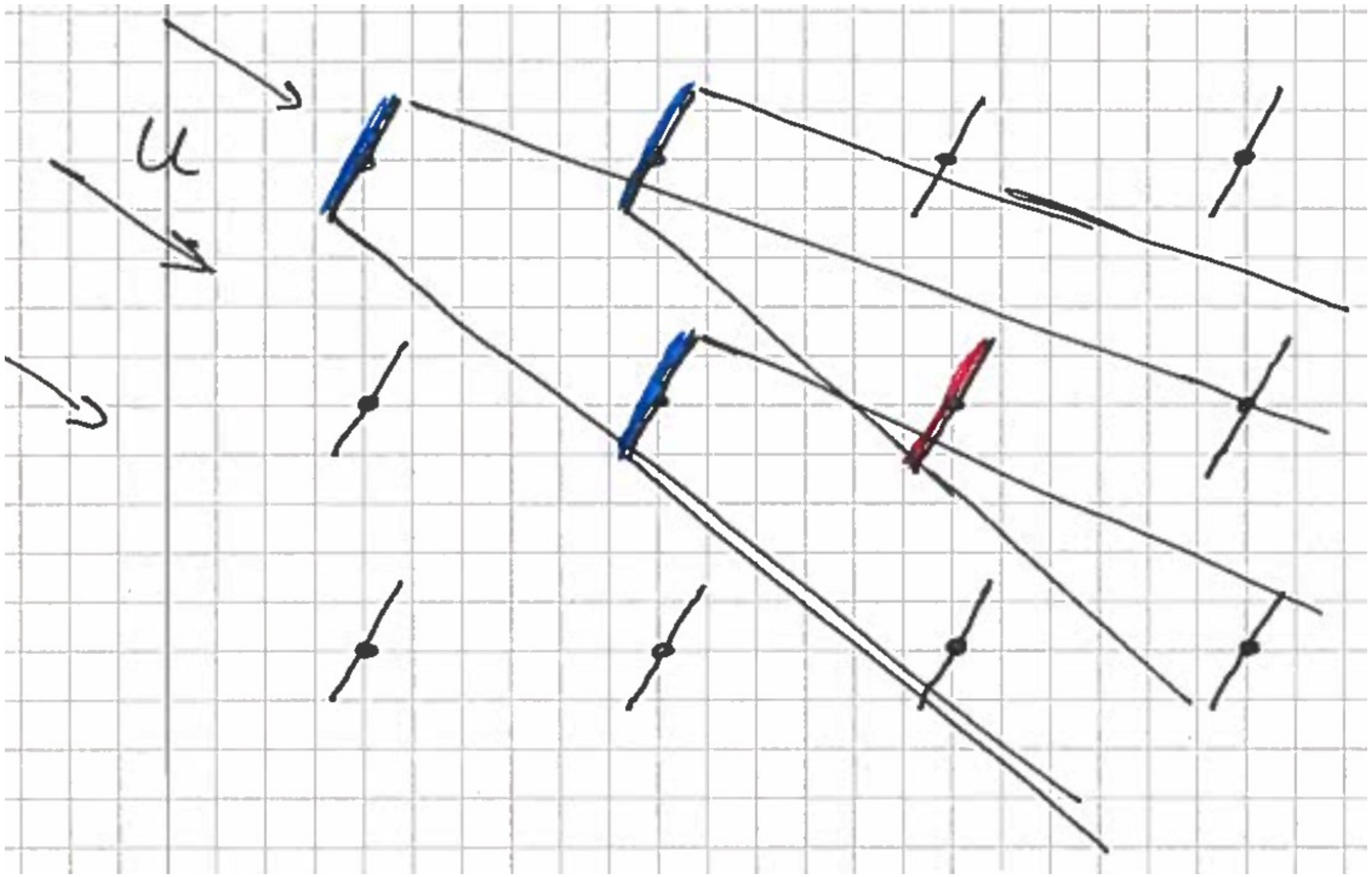
- N identical turbines

Effective deficit at turbine i

$$\delta_i^2(\theta, \boldsymbol{q}) = \sum_{i \neq j} \delta_{ij}^2(\theta, q_j)$$

where

$$\boldsymbol{q} = (q_1, \dots, q_N)$$

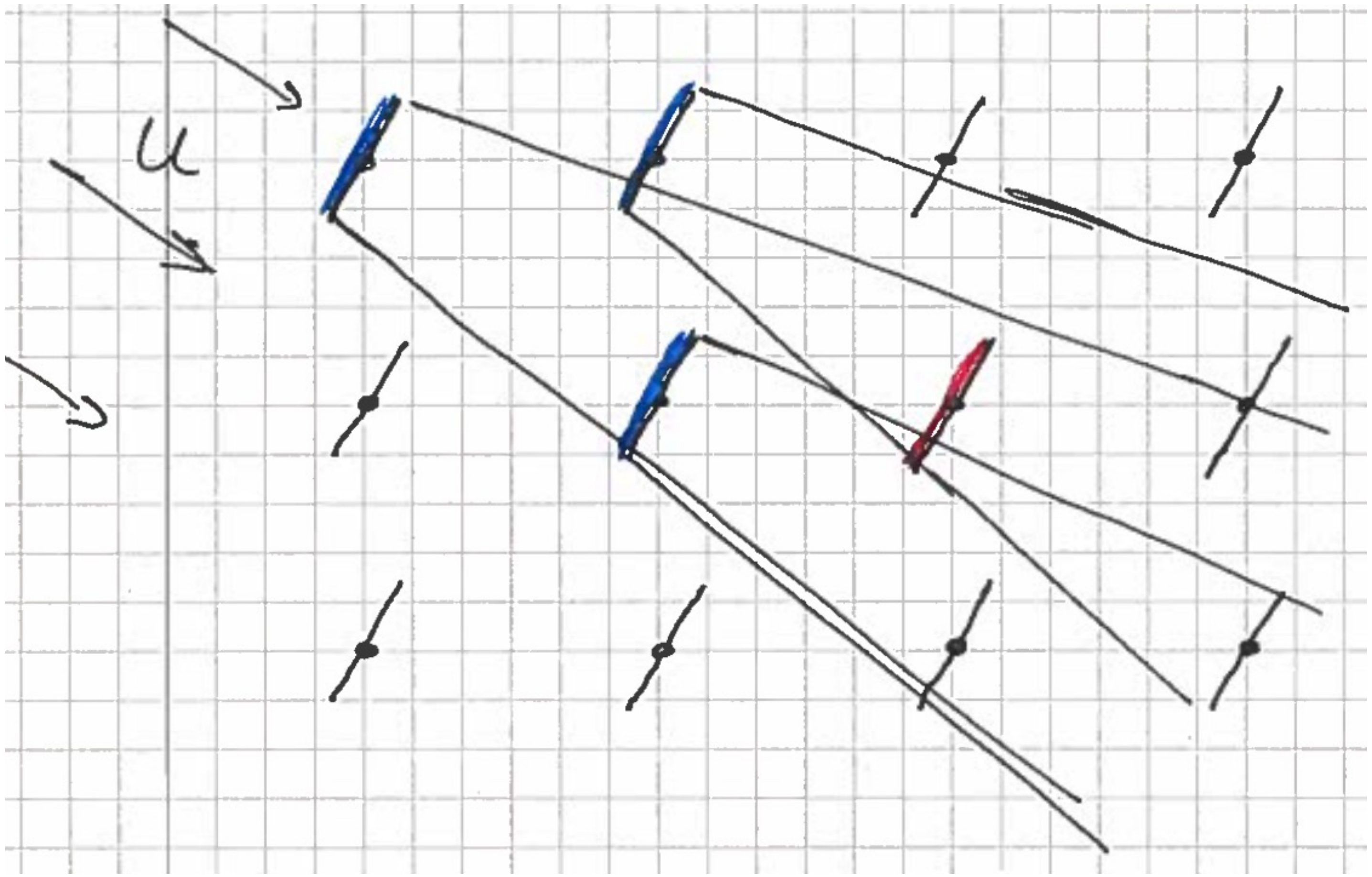


KATIC WAKE SUPERPOSITION

- N identical turbines

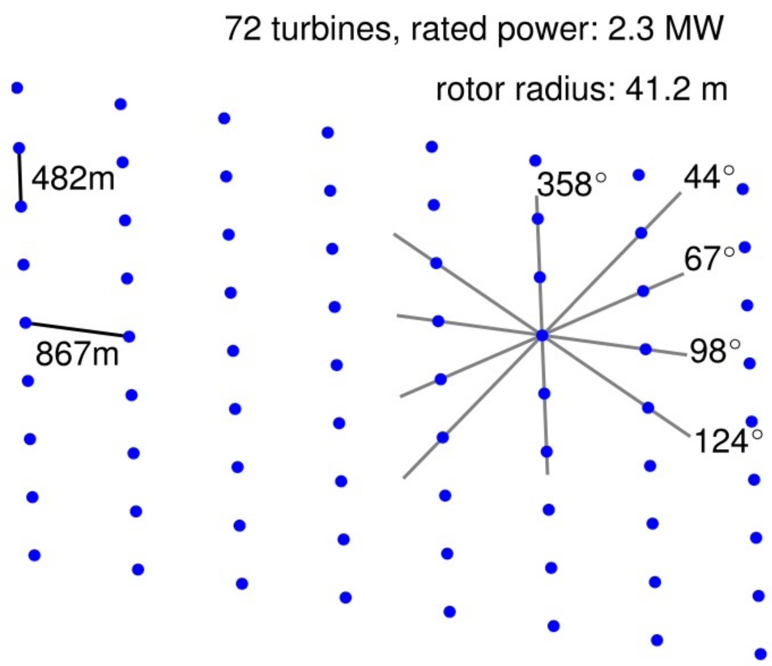
Total power output

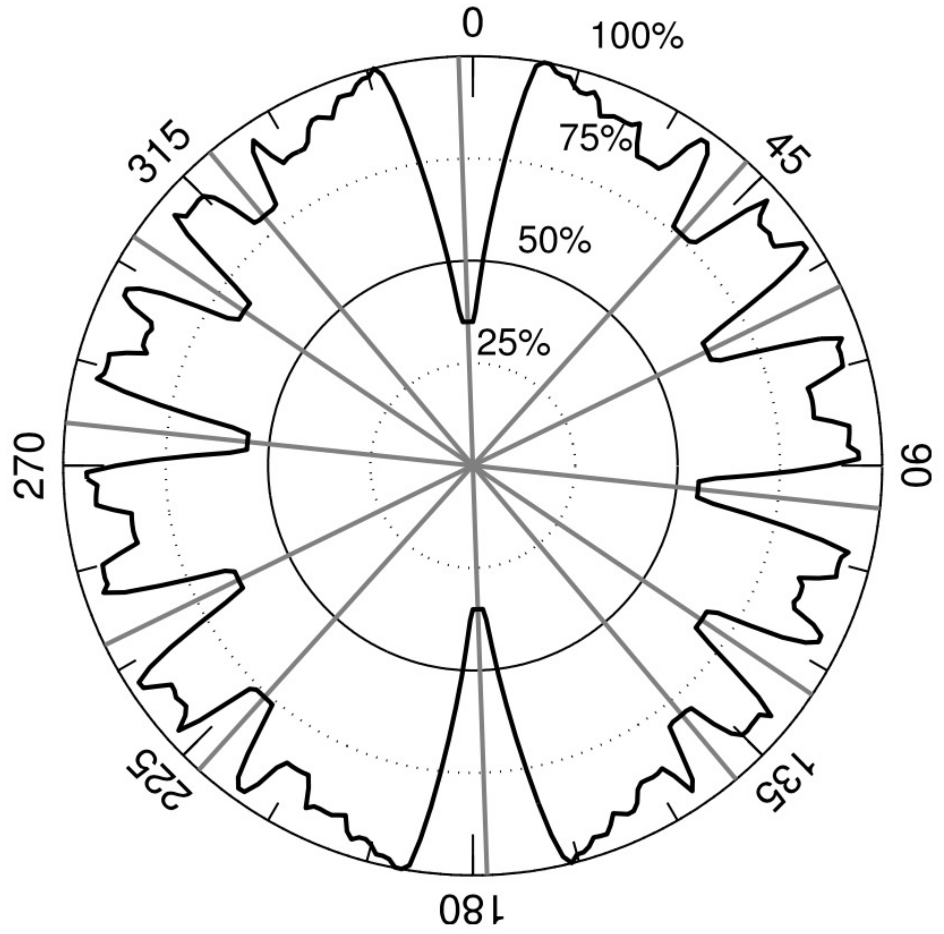
$$P_{tot}(u,\theta,\boldsymbol{q}) = \sum_{i=1}^{N} P_i(u_i(u,\boldsymbol{q}),\theta,q_i)$$



NYSTED OFFSHORE WIND FARM

$$\eta(\theta) = \frac{P_{farm}^{Betz}(u,\theta)}{N\, P^{Betz}(u)}$$





Herp, J., Poulsen, U. V., & Greiner, M. (2015). Wind farm power optimization including flow variability. *Renewable Energy*, *81*, 173-181.

WIND FARM POWER OPTIMIZATION

- Operational
- Layout

Hasager, C. B., Rasmussen, L., Peña, A., Jensen, L. E., & Réthoré, P. E. (2013). Wind farm wake: The Horns Rev photo case. *Energies*, *6*(2), 696-716.

PITCH ANGLE OPTIMIZATION

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Wind farm power optimization including flow variability

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[d] Department of Mathematics, Aarhus University, Ny Munkegade 118, DK-8000 Aarhus C, Denmark

Algorithm 1 Sequential Optimization

Require: wind speed u and direction θ, and initial guess for $\mathbf{q}$
Ensure: P_{farm} maximal
  while P_{farm} increases **do**
    for i = most upwind **to** most downwind **do**
      update $q_i \leftarrow \arg\max P_{\text{farm}}(q_i)$
    end for
  end while

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    for i = most upwind to most downwind do
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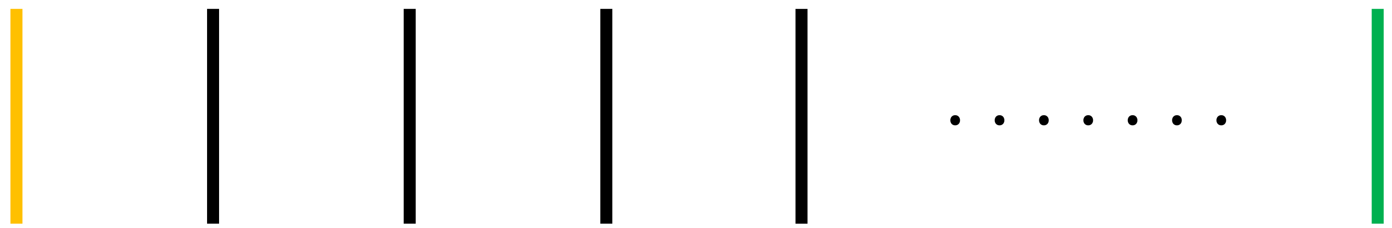
Herp, J., Poulsen, U. V., & Greiner, M. (2015). Wind farm power optimization including flow variability. *Renewable Energy*, *81*, 173-181.

$$q = \frac{V_d}{V_u}$$

q_1 q_2 q_3 q_4 q_5 $q_N = \frac{1}{3}$



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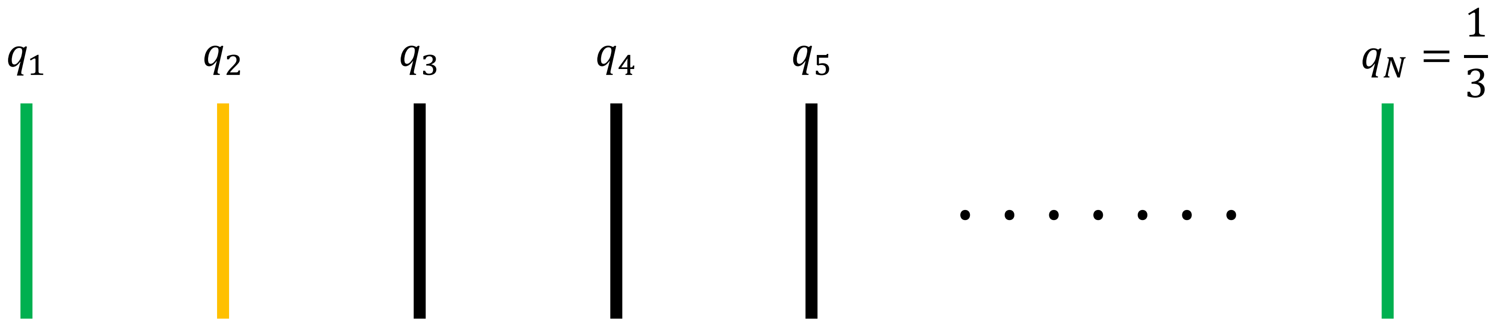

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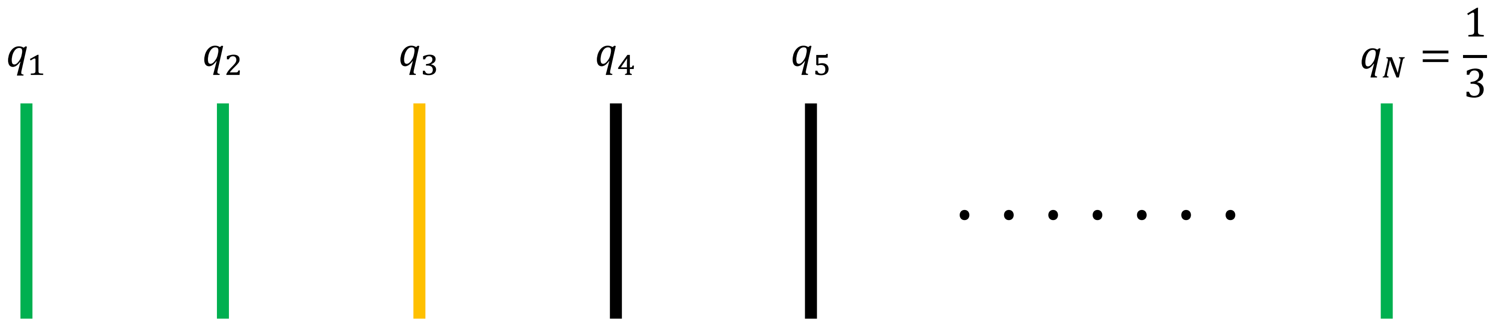

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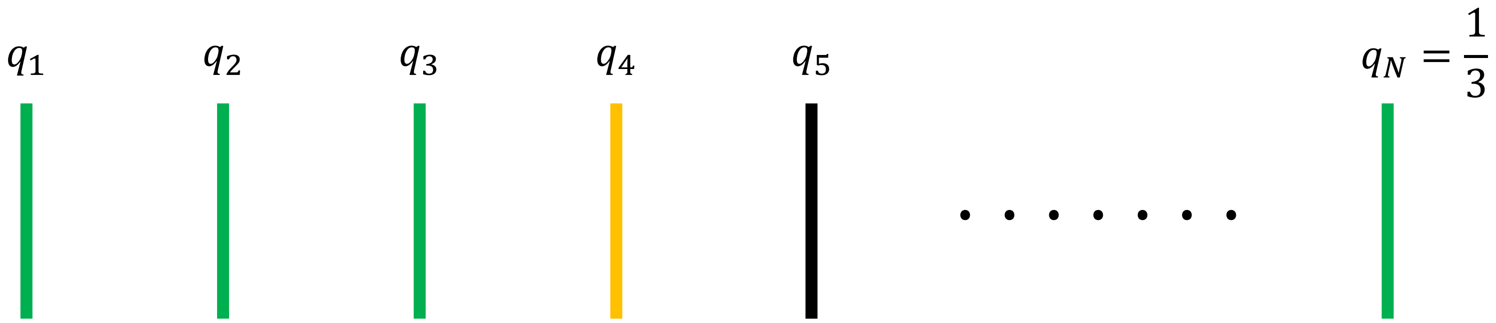

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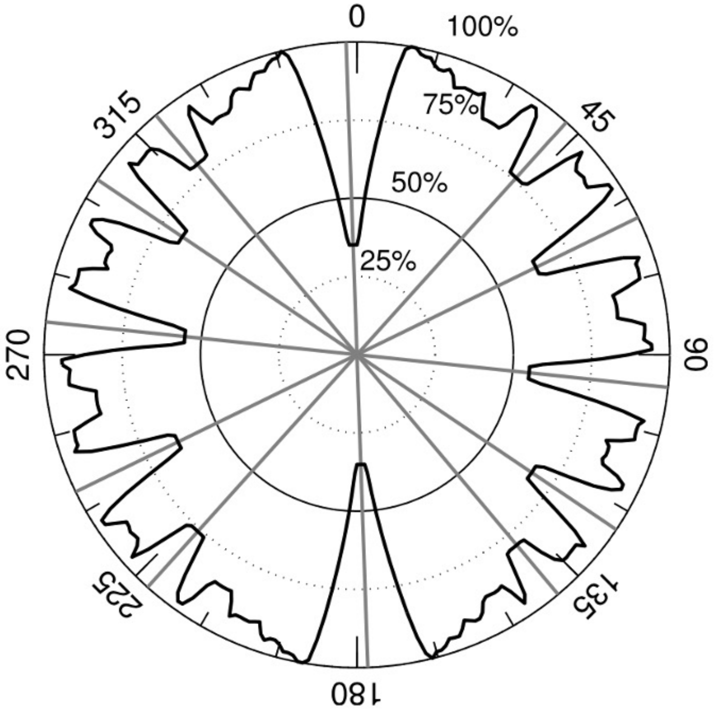
PITCH ANGLE OPTIMIZATION

Optimization gain

$$P_{farm}^{opt} = (1 + \xi) P_{farm}^{Betz}$$



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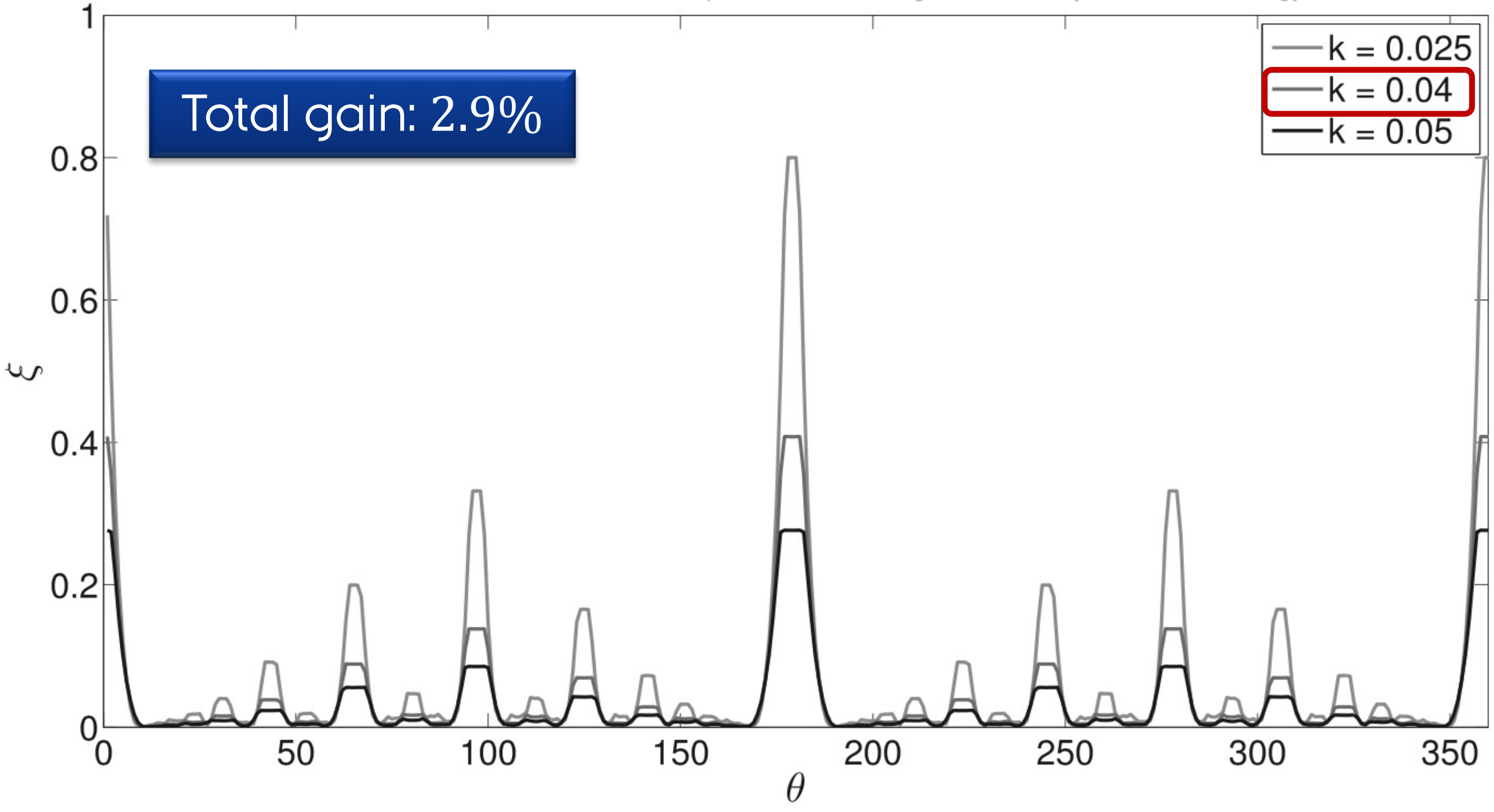
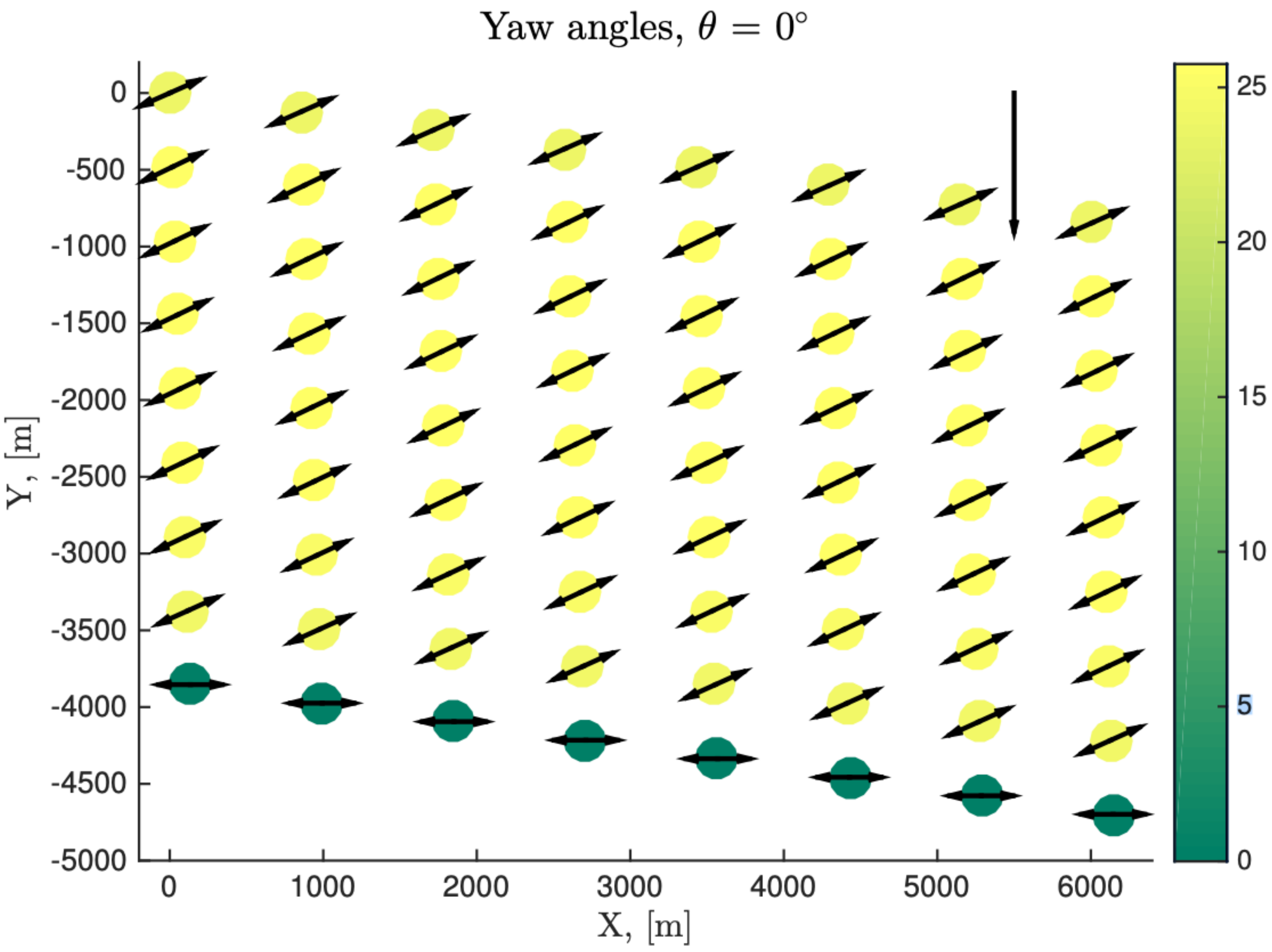


YAW ANGLE OPTIMIZATION

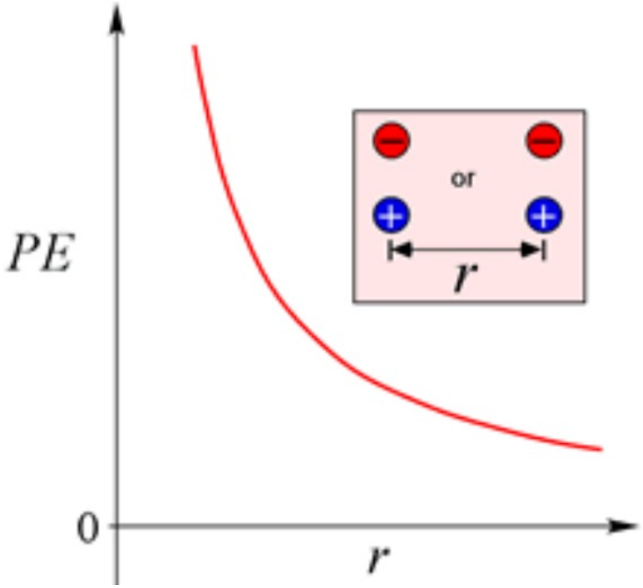
Total gain: 6.7%



Thøgersen, E., Tranberg, B., Herp, J., & Greiner, M. (2017, May). Statistical meandering wake model and its application to yaw-angle optimisation of wind farms. In *Journal of Physics: Conference Series* (Vol. 854, No. 1, p. 012017). IOP Publishing.

COULOMB OPTIMIZATION

- Master thesis by Jonathan Muff



http://guweb2.gonzaga.edu/faculty/cronk/CHEM101pub/energy.html

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COULOMB OPTIMIZATION

Electric potential energy

$$U_E = q_i V_E$$

Coulomb potential

$$V_E = \frac{1}{4\pi\epsilon_0}\frac{q_j}{r}$$

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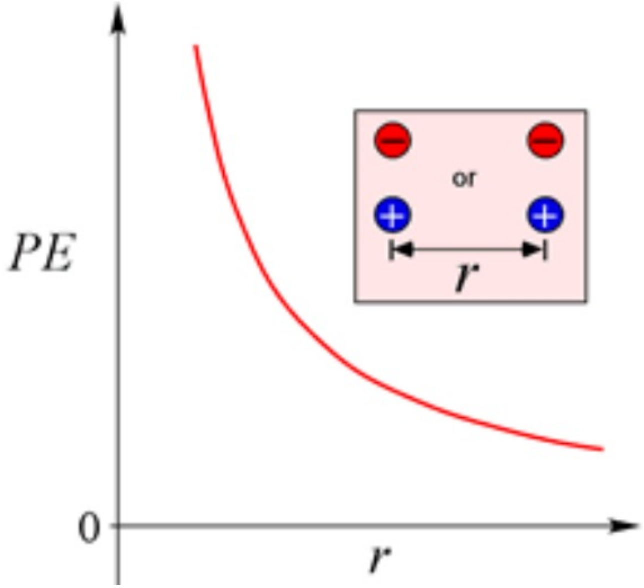
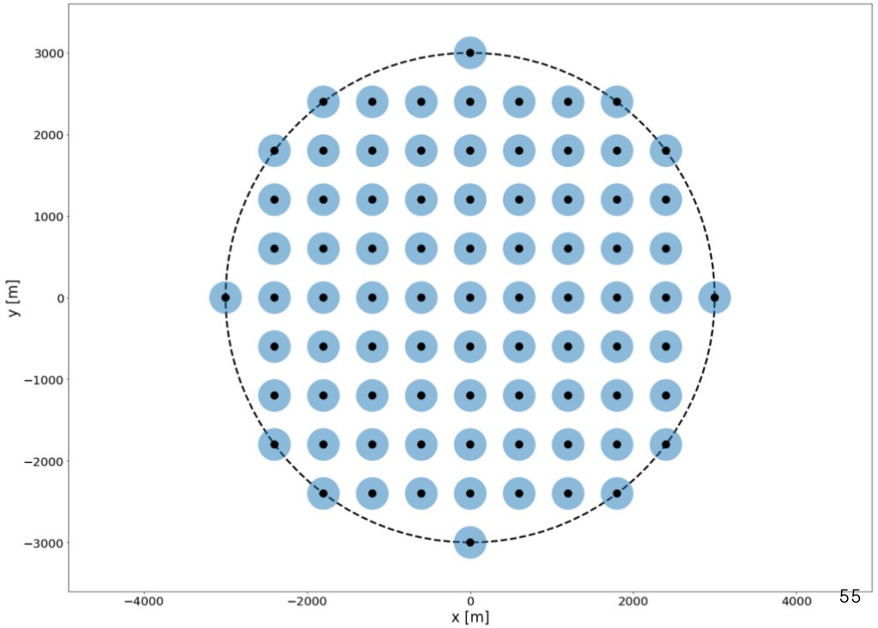


Jonathan Muff, Coulomb Layouts of Wind Farms (master thesis, 2023)



COULOMB OPTIMIZATION

Electric potential energy

$$U_E = q_i V_E$$

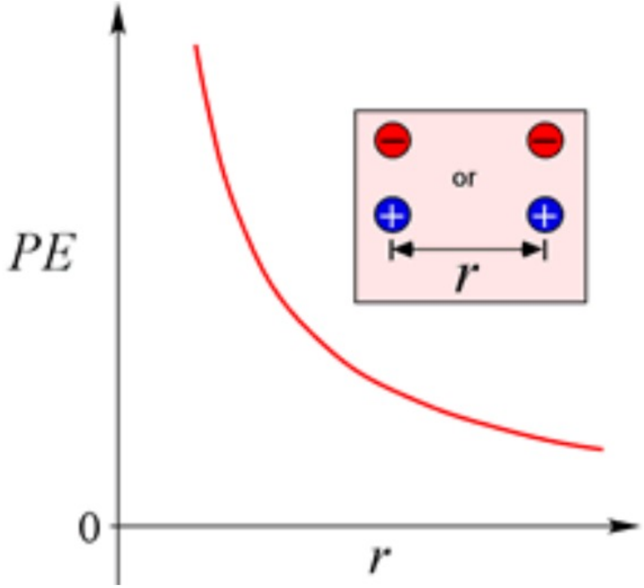
Coulomb potential

$$V_E = \frac{1}{4\pi\epsilon_0}\frac{q_j}{r}$$

For N particles

$$\Rightarrow U_E = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\sum_{j>i}^{N}\frac{q_i q_j}{r_{ij}}$$

http://guweb2.gonzaga.edu/faculty/cronk/CHEM101pub/energy.html



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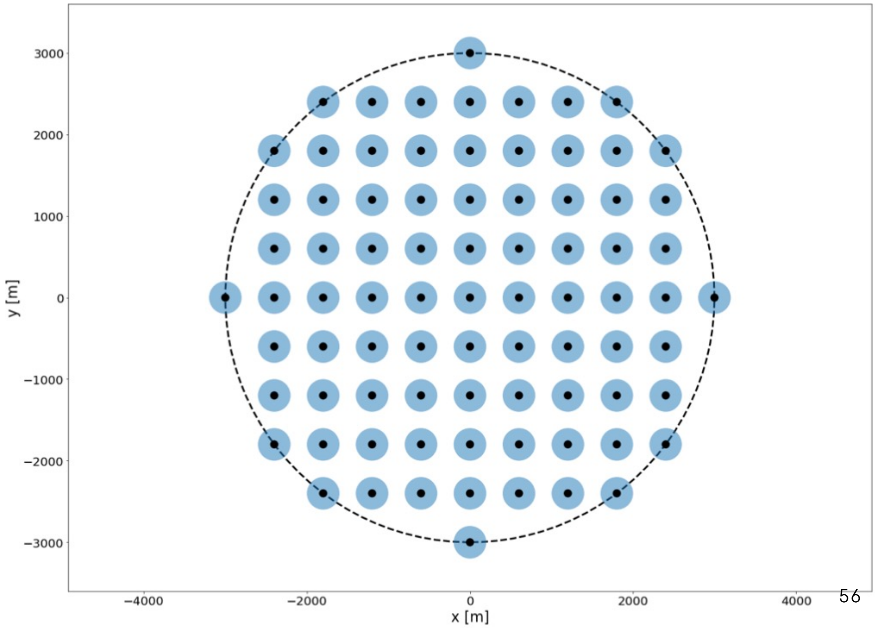

COULOMB OPTIMIZATION

Optimize layout of N turbines

$$U_{E,min} = \min_{r} \sum_{i=1}^{N} \sum_{j>i}^{N} \frac{1}{r_{ij}}$$

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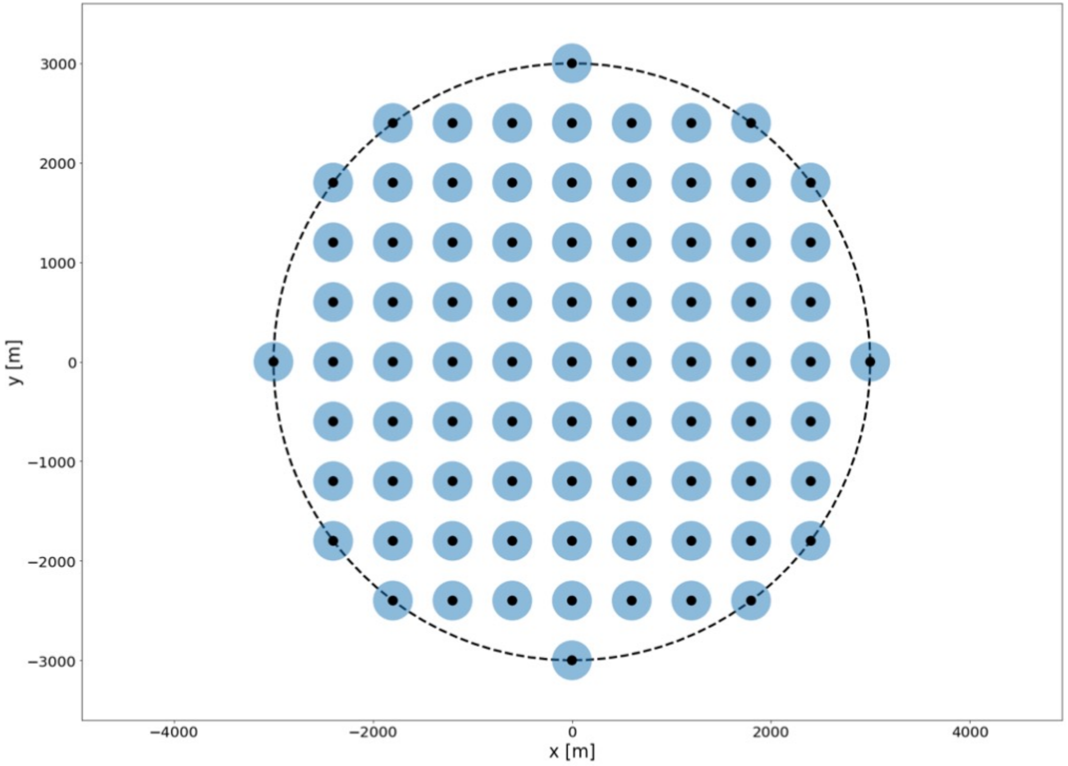

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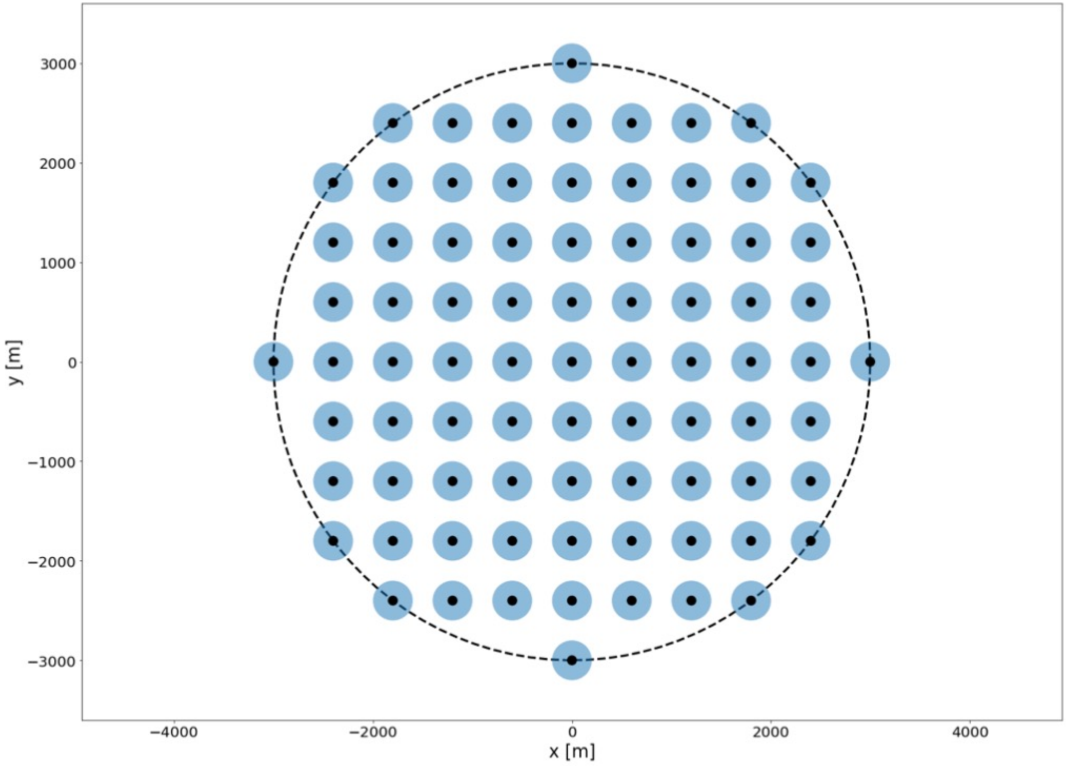

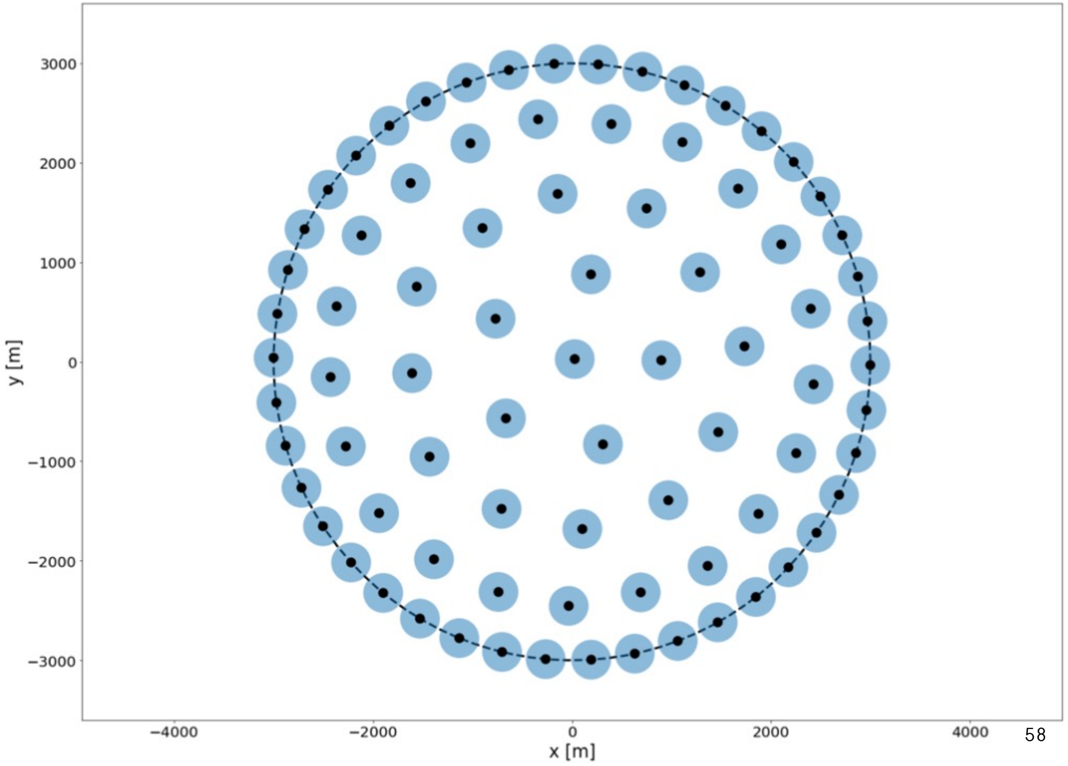

COULOMB LAYOUTS

v/etablerede-havvindmoelleparker

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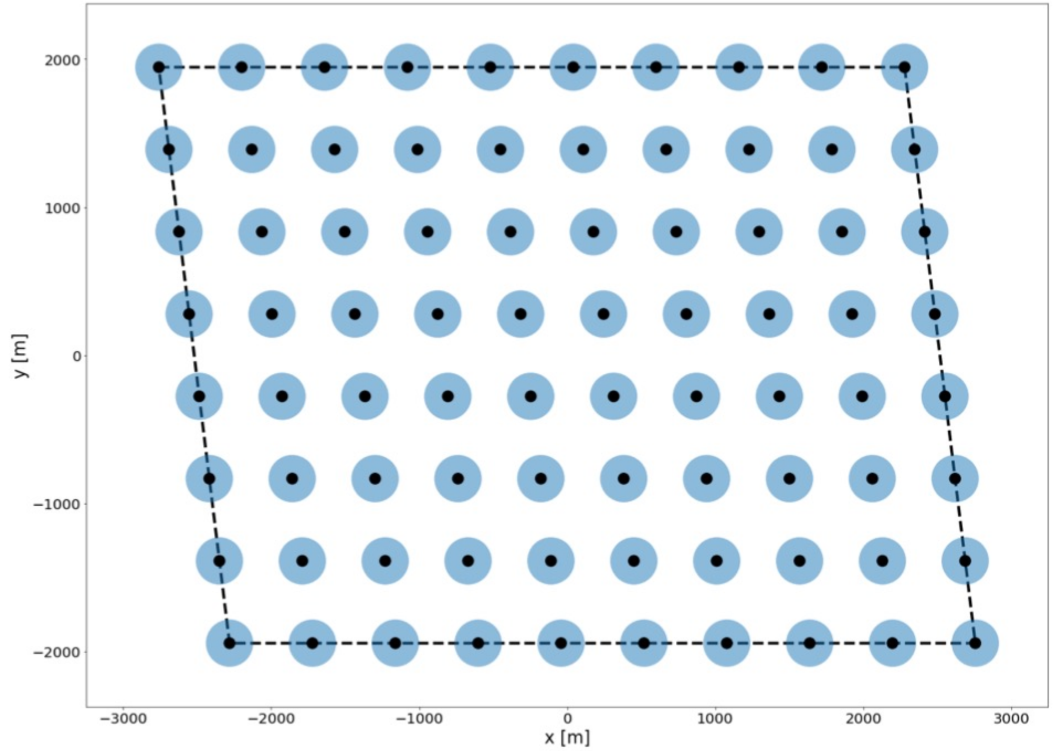

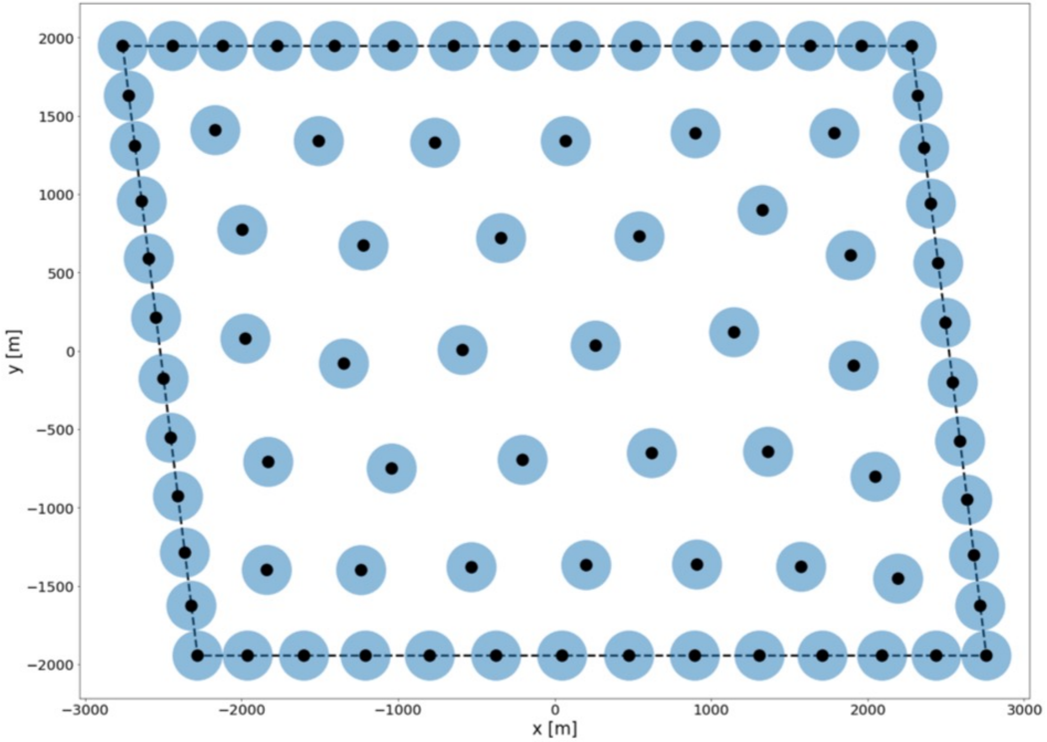

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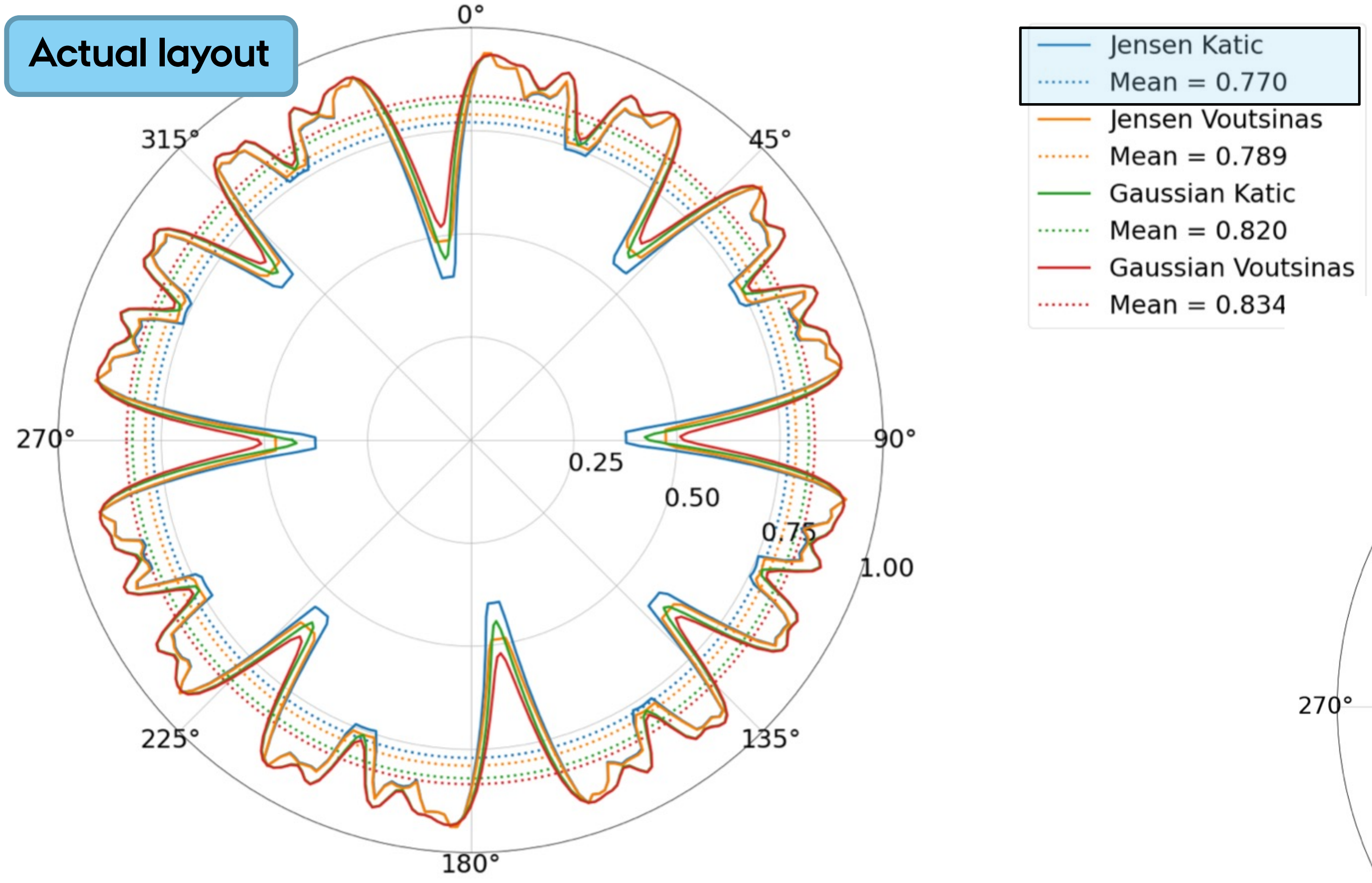

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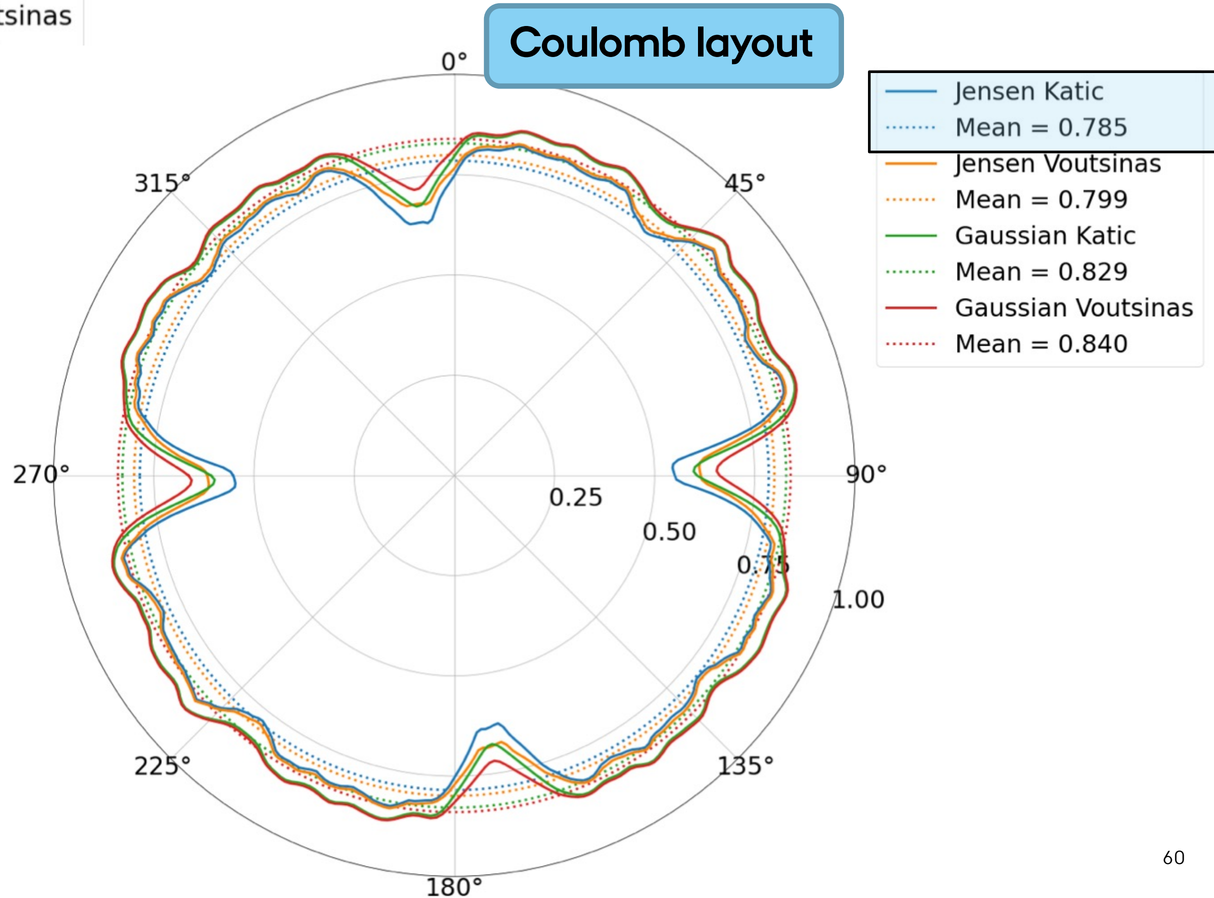

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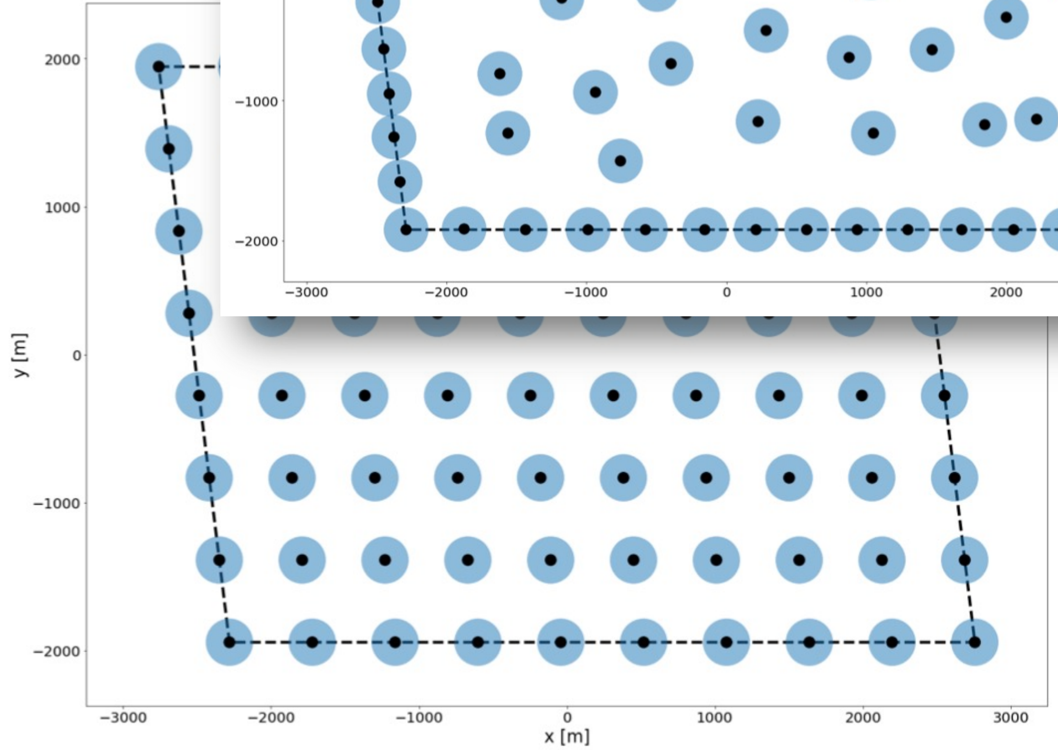

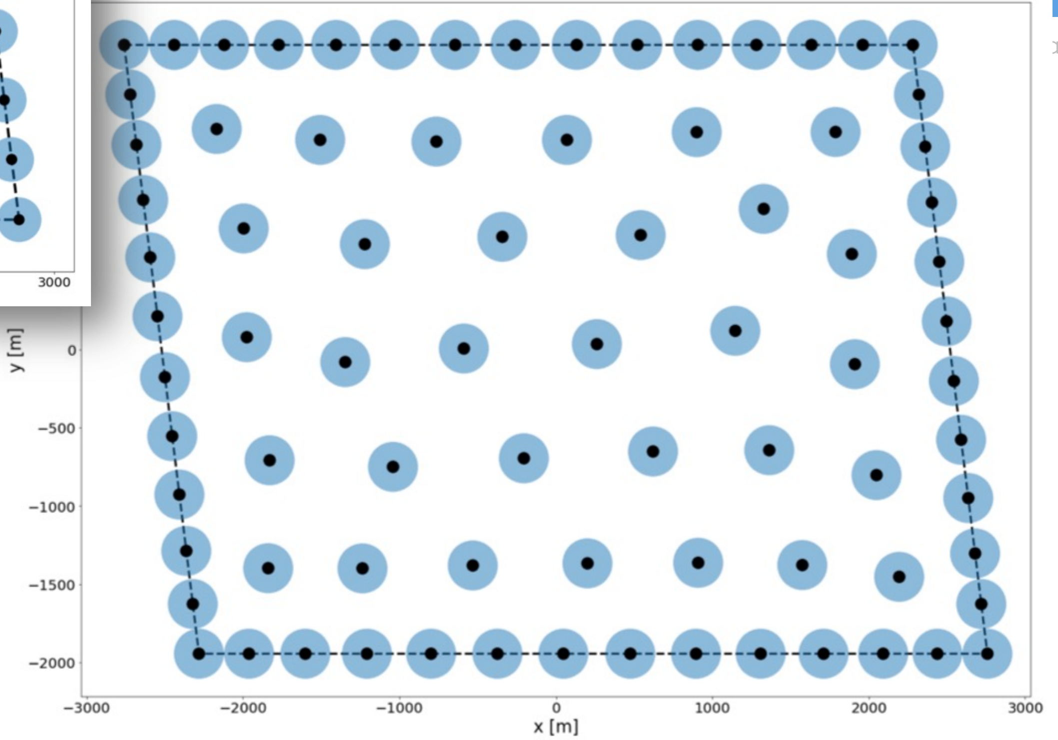

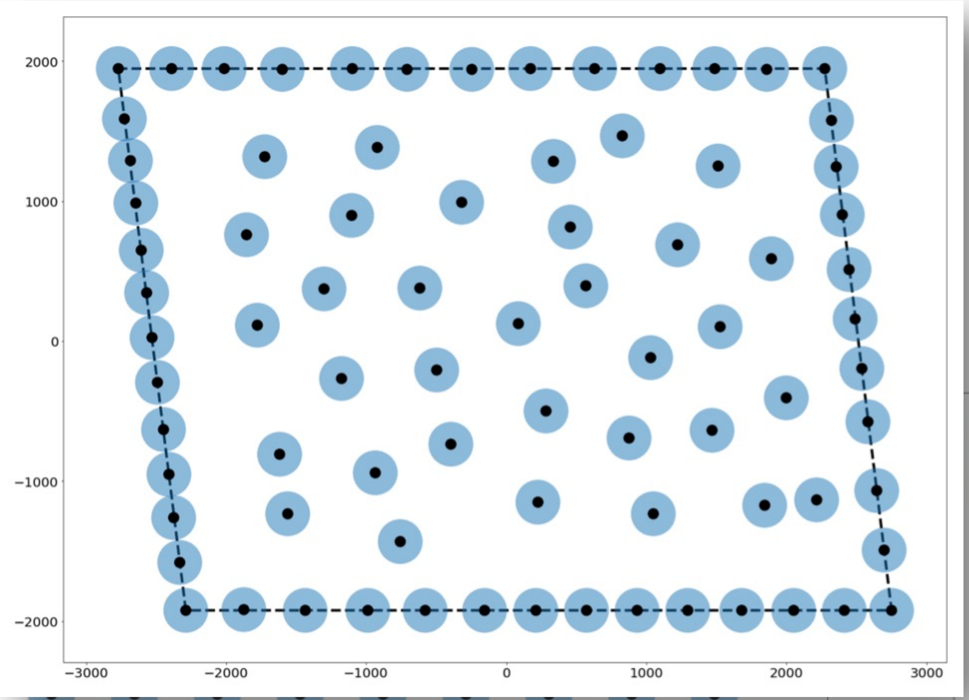

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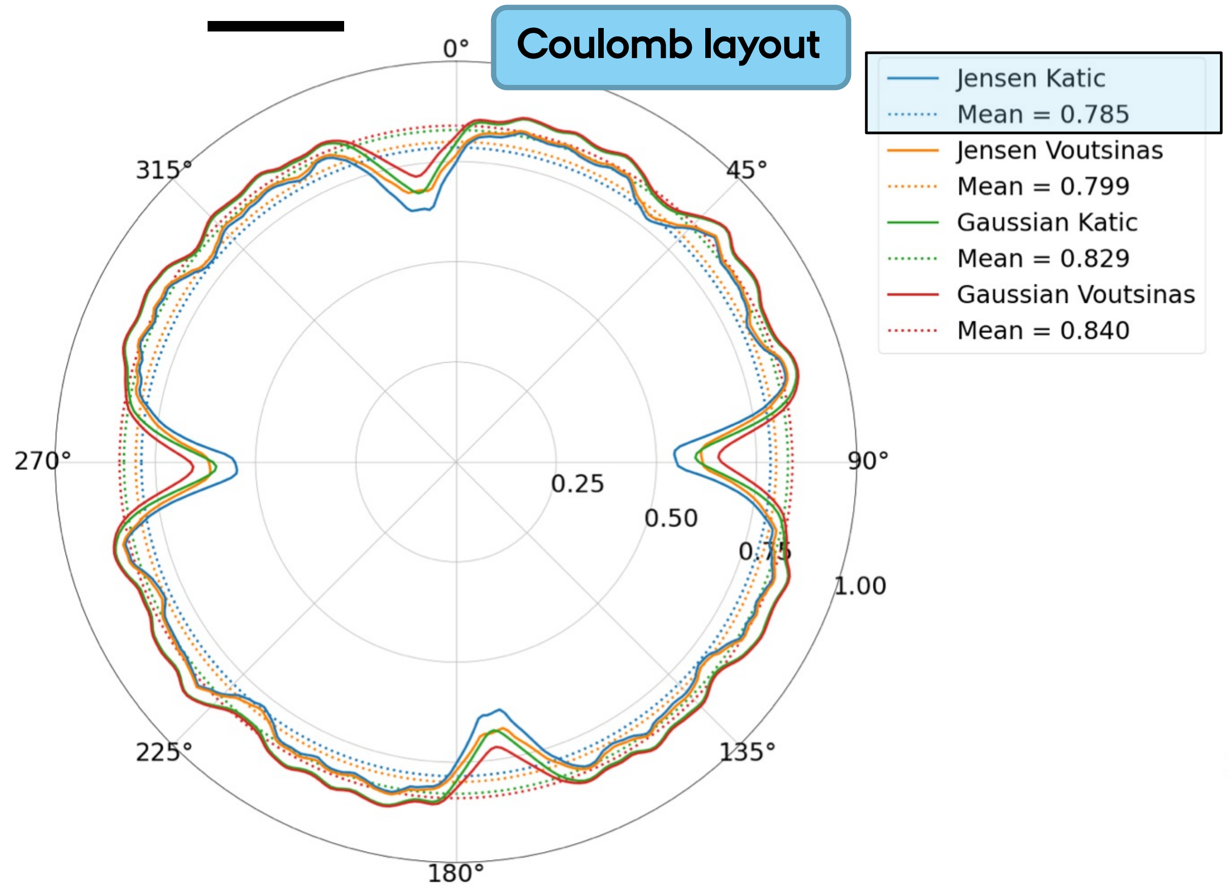

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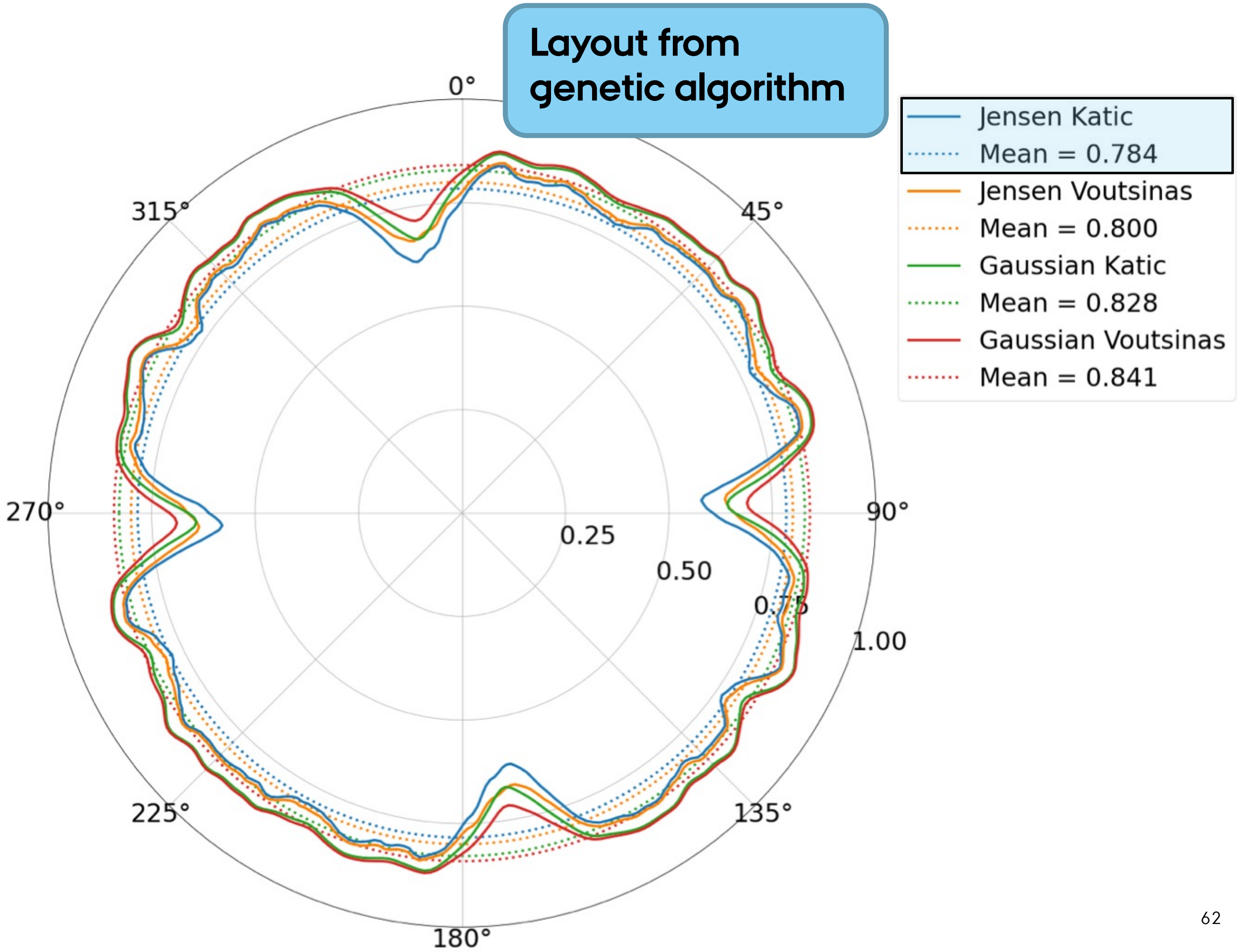
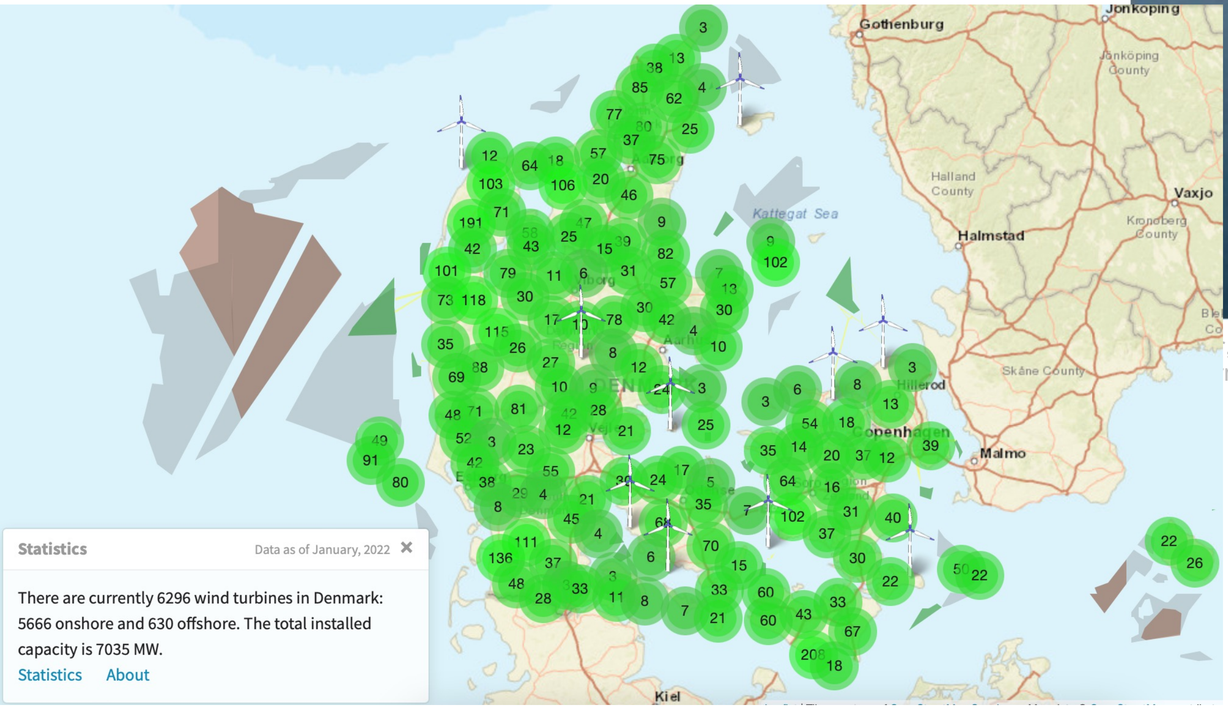


OUTLOOK

https://turbines.dk



, C. B., Rasmussen, L., Peña, A., Jensen, L. E., & Réthoré, P. E. (2013).
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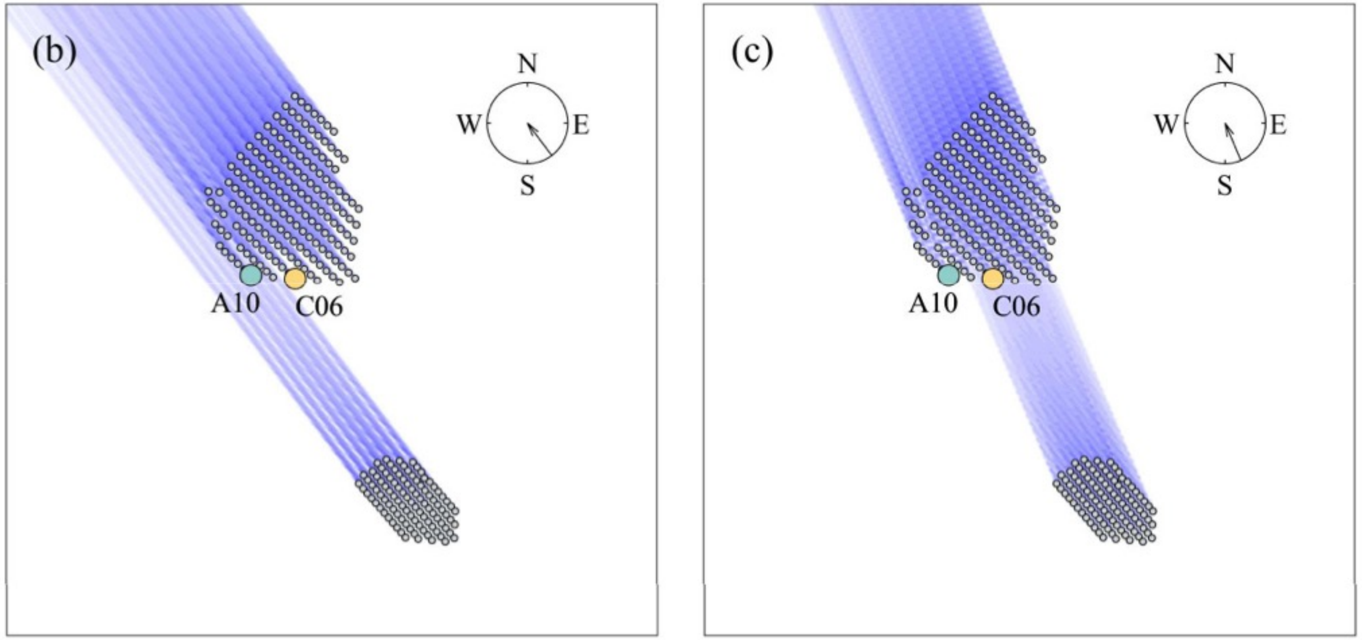
OUTLOOK

- Wind farm-farm wake interactions



Pedersen, J. G., Svensson, E., Poulsen, L., & Nygaard, N. G. (2022, May). Turbulence Optimized Park model with Gaussian wake profile. In *Journal of Physics: Conference Series* (Vol. 2265, No. 2, p. 022063). IOP Publishing.

The pessimist complains about the wind;
the optimist expects it to change;
the realist adjusts the sails

- WILLIAM ARTHUR WARD

https://www.nrel.gov/research/re-wind.html

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