Statistical Machine Learning, Fall 2018 - Problem set 4

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Kalman Filter Implementation

Consider a particle moving in a line under random forces and damping. More specifically, the 2-dimensional state x of a particle at a given time step is:

$$x = \begin{pmatrix} p \\ \dot{p} \end{pmatrix}$$

where p represents the particle's location in the line and $\dot{p}$ is its velocity. The state evolves as $p_t = p_{t-1} + \dot{p}_{t-1}$ and the velocity evolves as $\dot{p}_t = 0.98 \cdot \dot{p}_{t-1} + v_t$. The observations y are (very) noisy measurements of the particle position. In detail, we have the following linear dynamical system:

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 0.98 \end{pmatrix} \quad , \quad C = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad , \quad R = 100$$

For initial condition assume that $x_1 \sim N(0, I)$. In this problem we implement the Kalman filter to estimate the particle state from the noisy measurements.

1. Simulate 100 samples of the particle state and the measurements. Let $x_1, ..., x_{100}$ be the true location and $y_1, ..., y_{100}$ the measurements respectively, from data generated from the model.

2. Plot the evolution of the particle true position. The next plot should be plotted on top of this one.

3. Implement the Kalman filter and find $x_{t|t}$ for each t. Plot the resulting estimate of the particle positions across time.

4. Compute for $T = 100$:

   $$\frac{1}{T}\sum_t (y_t - x_t)^2 \quad , \quad \frac{1}{T}\sum_t (x_{t|t} - x_t)^2$$

   Comment on the relative qualities of using the measurements y_t directly as estimates of the particle position and the Kalman filter estimates.