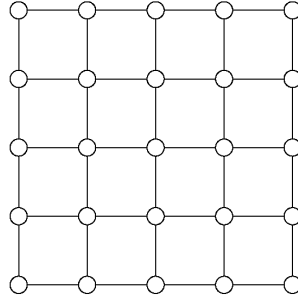


Statistical Machine Learning, Fall 2018 - Problem set 3

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You are given a 5×5 grid binary 0-1 MRF.

$$p(x) = \frac{1}{Z} \prod_{ij \in E} \phi_{ij}(x_i, x_j) = \frac{1}{Z} \exp \left(\sum_{ij \in E} \varphi_{ij}(x_i, x_j) \right)$$

$$p(x|y) = \frac{1}{Z} \prod_{ij \in E} \phi_{ij}(x_i, x_j) \prod_i \phi_i(x_i, y_i) = \frac{1}{Z} \exp \left(\sum_{ij \in E} \varphi_{ij}(x_i, x_j) + \sum_i \varphi_i(x_i, y_i) \right)$$

$$\varphi_{ij}(x_i, x_j) = 1_{\{x_i = x_j\}}, \quad \varphi_i(x_i, y_i) = -0.5(x_i - y_i)^2$$

- Simulate data (x, y) . First create x using Gibbs sampling, then create y , note the $y_i | x_i \sim N(x_i, 1)$.
- Compute the correct marginal $p(x_i | y)$.
- Apply Gibbs sampling to compute $\hat{p}(x_i | y)$.
- Plot $\sum_i (p(x_i | y) - \hat{p}(x_i | y))^2$ as a function of the numbers of samples.