Statistical Machine Learning, Fall 2018 - Problem set 5

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Multiple Cause Model

Consider a multiple cause model with $k = 6$ binary hidden variables x_i and 2-dimensional real-valued observed vector y, such that:

$$p(x_i = 0) = p(x_i = 1) = 0.5 \qquad\qquad i = 1, .., k$$
$$y|x_1, ..., x_k \sim N(Ax, I_{2\times 2})$$

$$A = \begin{pmatrix} -0.3 & -1.0 & 1.3 & 0.5 & -0.2 & 0.5 \\ -1.0 & -2.5 & 2.3 & 0.8 & 1.5 & -0.5 \end{pmatrix}$$

- Given that $y = (2.64, 2.56)$, compute $\log p(y)$ and the exact posterior distributions $p(x_i|y) \qquad i = 1, ..., k$
- Validate that the mean-field iteration is:

  $$q_i(x_i) \propto \exp(-\frac{1}{2}\|\sum_{j\neq i} A_j q_j(1) + A_i x_i - y\|^2)$$

  such that A_i is the i-th column of A.
- Apply the mean-field approximation to obtain an approximation $q(x) = \prod_i q_i(x_i) \approx p(x|y)$. Start from the uniform density $q_i(x_i) = 0.5$. Plot the ELBO lower bound $L(q) = \sum_x q(x) \log f(x, y) + H(q)$ as a function of the iterations. Compare it to $\log p(y)$.
- After MF convergence compare between q_i and $p(x_i|y)$ for $i = 1, .., 6$.