Question 1: Functional Dependencies I . . . . . [15 points]

Consider the following legal instance of a relational schema S with attributes XYZ:

S	X	Y	Z
	m	20	T
	m	10	F
	o	30	T
	n	30	T
	o	20	T

Table 1: Legal instance of schema S for question 2.1

(a) Which of the following dependencies are *violated* by the instances of S in Table 1?

i. **[2 points]** ☑ Yes ☐ No : $X \rightarrow Y$ is violated.

ii. **[2 points]** ☑ Yes ☐ No : $Z \rightarrow X$ is violated.

iii. **[2 points]** ☐ Yes ☑ No : $Y \rightarrow Z$ is violated.

iv. **[2 points]** ☐ Yes ☑ No : $XY \rightarrow Z$ is violated.

v. **[2 points]** ☑ Yes ☐ No : $YZ \rightarrow X$ is violated.

vi. **[2 points]** ☑ Yes ☐ No : $XZ \rightarrow Y$ is violated.

(b) **[3 points]** By only observing the instance of S in Table 1, can you identify the functional dependencies that hold on schema S? Why?

☐ Yes ☑ No

Because Table 1 is just a legal instance of schema S.

There may be counterexamples in other instances of schema S.

Question 2: Functional Dependencies II ..... [32 points]

For the next set of questions consider the relational schema $\mathcal{R} = \{P, Q, R, S, T, U, V, W\}$ and the set of functional dependencies FD:

$$
\begin{aligned}
Q &\rightarrow U && (1)\\
U &\rightarrow V && (2)\\
PQ &\rightarrow WST && (3)\\
SU &\rightarrow TR && (4)\\
VT &\rightarrow RW && (5)\\
R &\rightarrow W && (6)
\end{aligned}
$$

(a) **[8 points]** Which of the following is a minimum cover of the FD? Mark all that qualify; if none, mark accordingly, and give your *own*. answer.

i. The given FDs (Eq 1-6), is a minimum cover already.

ii. $\{Q \rightarrow U, U \rightarrow V, PQ \rightarrow S, SU \rightarrow T, SU \rightarrow R, VT \rightarrow R, VT \rightarrow W, R \rightarrow W\}$

iii. $\{Q \rightarrow U, U \rightarrow V, PQ \rightarrow S, SU \rightarrow T, PQ \rightarrow W, VT \rightarrow R, PQ \rightarrow T, R \rightarrow W\}$

iv. $\{Q \rightarrow U, U \rightarrow V, PQ \rightarrow S, SU \rightarrow T, VT \rightarrow R, R \rightarrow W\}$

v. $\{Q \rightarrow U, U \rightarrow V, PQ \rightarrow S, SU \rightarrow T, SU \rightarrow R, VT \rightarrow R, PQ \rightarrow T, R \rightarrow W\}$

vi. none of the above - the cover is _____

(b) Yes/No: Which of the following functional dependencies can be deduced, from the above set of functional dependencies (Eq. (1)-(6))?

i. **[3 points]** ☑ Yes ☐ No : $Q \rightarrow V$

ii. **[3 points]** ☐ Yes ☑ No : $QU \rightarrow R$

iii. **[3 points]** ☑ Yes ☐ No : $SQ \rightarrow T$

iv. **[3 points]** ☑ Yes ☐ No : $SQ \rightarrow W$

v. **[3 points]** ☑ Yes ☐ No : $PQ \rightarrow R$

vi. **[3 points]** ☐ Yes ☑ No : $VT \rightarrow Q$

(c) **[3 points]** True or False: The attribute closure $\{Q\}^+$ is $\{Q, U, V\}$.

☑ True ☐ False

(d) **[3 points]** True or False: The attribute closure $\{PQ\}^+$ is $\{P, Q, W, S, T\}$.

☐ True ☑ False

Question 3: Decompositions.....[20 points]

For this set of questions, consider the relation with attributes, $\mathcal{X} = \{A, B, C, D, E, F\}$, Let the following functional dependencies FD be defined over the relation $\mathcal{X}$:

$$A \rightarrow B$$
$$B \rightarrow CD$$
$$E \rightarrow F$$

(a) **[2 points]** Provide the attribute closure of $\{AB\}$. $\{AB\}^+ = \{ABCD\}$

(b) Consider the decomposition AB, BCD, EF. Mark 'True' or 'False':

i. **[3 points]** ☐ True ☑ False : It is lossless

ii. **[3 points]** ☑ True ☐ False : It is dependency-preserving

(c) Consider the decomposition AB, $BCDF$, EF. Mark 'True' or 'False':

i. **[3 points]** ☐ True ☑ False : It is lossless

ii. **[3 points]** ☑ True ☐ False : It is dependency-preserving

(d) Consider the decomposition $ABCEF$, EBD. Mark 'True' or 'False':

i. **[3 points]** ☑ True ☐ False : It is lossless

ii. **[3 points]** ☑ True ☐ False : It is dependency-preserving

Question 4: Normal Forms.....[33 points]

Consider the relation with attributes, $\mathcal{E} = \{P, Q, R, S\}$. Suppose that the following functional dependencies hold:

$$PQ \rightarrow R \quad (7)$$
$$PQ \rightarrow S \quad (8)$$
$$R \rightarrow P \quad (9)$$
$$S \rightarrow Q \quad (10)$$

(a) **[6 points]** List *all* the candidate key(s) for $\mathcal{E}$. $\{PQ\}, \{QR\}, \{RS\}, \{PS\}$

(b) **[2 points]** Is the relation $\mathcal{E}$ in BCNF? ☐ Yes ☑ No

(c) From the list below, select all applicable choices to justify whether $\mathcal{E}$ is (or is not) in BCNF.

Note: when we refer to the *main requirement* for BCNF, we mean: *every determinant is a super key*.

i. **[1 point]** ☐ True ☑ False : All FD's satisfy the main requirement.
ii. **[1 point]** ☐ True ☑ False : FD (7) violates the main requirement.
iii. **[1 point]** ☐ True ☑ False : FD (8) violates the main requirement.
iv. **[1 point]** ☑ True ☐ False : FD (9) violates the main requirement.
v. **[1 point]** ☑ True ☐ False : FD (10) violates the main requirement.

(d) **[2 points]** Is the relation $\mathcal{E}$ in 3NF? ☑ Yes ☐ No

(e) From the list below, select all applicable choices to justify whether $\mathcal{E}$ is (or is not) in 3NF.

Note: when we refer to the *secondary requirement* for 3NF, we mean: *for every FD $X \rightarrow A$, A is part of a candidate key*.

i. **[1 point]** ☑ True ☐ False : All FD's satisfy the secondary requirement.
ii. **[1 point]** ☐ True ☑ False : FD (7) violates the secondary requirement.
iii. **[1 point]** ☐ True ☑ False : FD (8) violates the secondary requirement.
iv. **[1 point]** ☐ True ☑ False : FD (9) violates the secondary requirement.
v. **[1 point]** ☐ True ☑ False : FD (10) violates the secondary requirement.

(f) **[5 points]** Give a 3NF decomposition of $\mathcal{E}$ that is lossless, dependency preserving, and has as few tables as possible. $R_1 = \{P, Q, R, S\}$

(g) **[8 points]** Give a BCNF decomposition of $\mathcal{E}$ that is lossless, and has as few tables as possible.

$R_1 = \{P, R\}$

$R_2 = \{Q, S\}$

$R_3 = \{R, S\}$