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Neural Fuzzy Models of Feed Forward Networks for Function Approximation using Sigmoidal Signals

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Abstract

Approximation by Neuro-Fuzzy models is studied by many researchers in the recent years. In this paper, we introduced Feed forward Neural Network with Additive Takagi-Sugeno type with right sigmoidal signals and left sigmoidal signal as an activation function in the hidden layer. This new model is the Universal Approximator and it approximate the set of all continuous function defined on R^n. Its performance is better than the four layer network.

Keywords: Universal Approximation, Artificial Neural Network, Neuro-Fuzzy modeling, Fuzzy Inference System, Left Sigmoidal Signal, Right Sigmoidal Signal.

1. Introduction

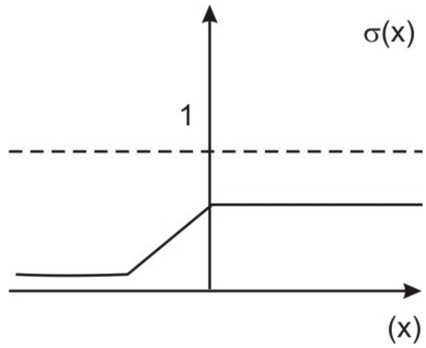
Since 1994 the research on approximating ability of regular Fuzzy Neural Network has been attracting many scholars. Our main objective of this paper solving the approximation problem related to Regular Fuzzy Neural Network with two hidden layers using left sigmoidal and right sigmoidal signals as an activation function which gives better approximation. Approximation using Neural Network Model Chen et al. (1995). Based on Chen Ramakrishnan et al (2006) introduced new sigmoidal signals called left sigmoidal signals and right sigmoidal signal used for better approximation. Regular Neuro Fuzzy Models we use left sigmoidal signals and right sigmoidal signals for approximation which gives better approximation than the existing models. Extended neuro-fuzzy models of multilayer perceptrons are studied by Dong Zhang et al(2004).

2. Preliminaries

2.1. Left Sigmoidal Signals

The function $\sigma : R \rightarrow R$ is said to be left sigmodial if $\lim\limits_{x \to -\alpha} \sigma(x) = 0$

The following is an example of left generalized sigmoidal signals.



$$\sigma(x) = \begin{cases} e^{\alpha x}, \text{ for } x < 0, \ \alpha > 0 \\ \beta, \text{ for } x \geq 0, \ \beta > 0 \end{cases}$$

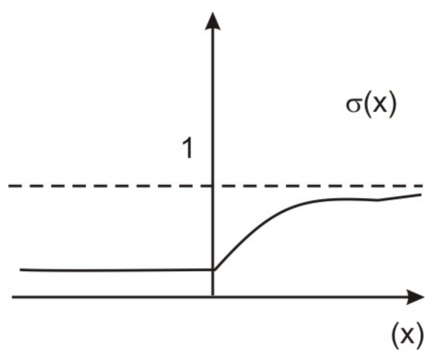
2.2. Right Sigmoidal Signals

The function $\sigma : R \to R$ is said to be right sigmoidal

if $\lim\limits_{x \to +\alpha} \sigma(x) = 1$

The following is an example of right generalized sigmoidal signals.

$$\sigma(x) = \begin{cases} 1 + e^{-\alpha x}, \text{ for } x > 0, \ \alpha > 0 \\ \beta, \text{ for } x < 0, \ \beta > 0 \end{cases}$$



3. Three Layer Feed Forward Network

Three layer feed forward Neural Network is a single hidden layer consisting of n nodes and using sigmoidal signals as an activation function. The sigmoidal signals as an activation function. The sigmoidal sigal to here we used left sigmoidal signal and right sigmoidal sigal. The output layer consist only one node. The three layer network is written in the form

$$\text{TNN} = \sum_{i=1}^{N} W_i \sigma \left(\sum_{i=1}^{n} C_i \ x_i + v_i \right) \tag{1.1}$$

where x_i is the input variables f(n) is the output of the given input, W_i, C_i is the weight value and is the right and left sigmoidal signal. The right sigmoidal and left sigmoidal signals are introduced by Ramakrishnan et al (2006).

θ is the bias term.

The output of the three layer network is

$$y = \sum_{i=1}^{N} W_i \sigma \left(\sum_{i=1}^{n} C_i \ x_i \ + v_i \right) \tag{1.2}$$

The three layer adaptive type network.

3.1. TS (Takagi-Sugeno) Type Fuzzy Inference Systems

A TS-type fuzzy inference system comprises four parts in total: a fuzzification interface, a rule base, a fuzzy reasoning engine and a defuzzification interface. A typical multi-input single-output fuzzy rule in the rule base has the form

$$\text{Rj : If } \ x1 \text{ is } \ A_j^{(1)} \ \theta x_2 \ \text{ is } A_j^{(2)} \ \theta ... x_n \ \text{ is } A_j^{(1)} \ \text{ then } \ y = f_j(\overline{x}),$$

where σ is a Left sigmoidal signal is the fuzzy connective operator, is the fuzzy set defined on the universe of xj for the j^{th} rule, and y = f_i ($\overline{x}$) is a crisp function in the consequent of the j^{th} rule.

A TS-type fuzzy inference system forms a crisp nonlinear function from the input universe to the output universe. By use of the singleton fuzzification method and the fuzzy reasoning method proposed by Takagi et al. the function can be expressed as

$$y = \frac{\sum_{i=1}^{m} \mu_j(\overline{x}) \ f_j(\overline{x})}{\sum_{j=1}^{m} \mu_j \ (\overline{x})} \tag{1.3}$$

where

$$\mu_j(\overline{x}) = A_j^{(1)} \ (x_1) \ \theta \ A_j^{(2)} (x_2) \ \theta \ ... A_j^{(n)} \ (x_n) \tag{1.4}$$

Is the firing strength of the jth rule, θ is the fuzzy connective operator, and $A_j^{(i)}$ (x_i) is the membership function for the fuzzy set $A_j^{(i)}$.

In practice, sometimes the weighted average operator is replaced with the weighted sum operator to reduce the computational load. Hence, eq. (1.3) is transformed into

$$y = \sum_{i=1}^{m} \ \mu_j(\overline{x}) \ f_j(\overline{x}) \tag{1.5}$$

Fuzzy inference systems with the above functional form are called additive TS-type inference systems in this paper.

4. The Additive--TS-Type Multilayer Network

4.1. Network Structure

Three layer feed forward Neural Network with additive TS-type using Left-sigmoidal signals and Right-sigmoidal signals is denoted by FFNATSLS and FFNATSRS network is in the following form it contains a four layer network and is defined as follows a multi-input single-output FFNATSLS and FFNATSRS comprises four layer in total :

1) The first layer is the input layer consisting of n nodes.
2) The second layer is a hidden layer and it consisting of n hidden nodes with left and Right sigmoidal signal is an activation function and its output is in the form of

$$b_i = \sigma \left(\sum_{i=1}^{m} C_i \ x_i \right)$$

where σ is a Left sigmoidal signal is the left sigmoidal signal and right sigmoidal signal.

3) The third layer is the gain layer consisting of n hidden nodes and it is in the form of

$$g_i = \sum_{i=1}^{n} v_i \; b_i$$

where b_i is the output from the hidden layer, v_i weight vector associated with the b_i.

4) The last layer is the output layer consisting of only one node. The output is the form of

$$y = g^T\left(\bar{x}\right)$$

where g is the set of outputs from the third layer $\bar{x}$ is the transpose of the given input x_1, x_2, … x=.

The output of the above network is

$$y_i = \sum_{i=1}^{n} g_i \; (\bar{x}) \; x_i$$

$$= \sum_{i=1}^{n} \left(\sum_{i=1}^{n} V_i \; \sigma\left(\sum_{i=1}^{n} c_i \, x_i \right)\right) x_i$$

where I = 1, 2, … n (2.1)

and σ is the left sigmoidal signal and right sigmoidal signal is an activation function in the second layer.

4.2. Universal Approximation

Let $C(R^n)$ be the set of all real continuous function defined on R_n $F_1(\Omega)$ be the set of all multiple-input – single-output in three layer feed forward Neural Network with Additive TS-type with left sigmoidal signal and right sigmoidal signal as an activation function.

Theorem 4.2.1.

For arbitrary $\varepsilon > 0$ and $f(n) \in C(R^n)$, there exists $NN(n) \in F_1$ (x) such that

$f(n) – NN(x) | < \in$ for any $x \in C (R_n)$

Proof

FFNATSLS is a feed forward Neural Network with Additive TS type with left sigmoidal signals.

$$y = g(n) = \sum_{i=1}^{n} V_i \; \sigma\left(\sum_{i=1}^{n} c_i \, x_i \right) \tag{2.2}$$

FNATSLS is in the form of FFN.

Hence we get $F (C(R^n)) C F_1 (c (R^n))$

that is $| f(x) – NN(x) | < \varepsilon$

Theorem 4.2.2.

For arbitrary $\varepsilon > 0$ and $f(n) \in C(R^n)$ there exist $NN(n) \in F_1(x)$

such that $| f(n) – NN(x) | < \varepsilon$

for any $x \in C(R^n)$

Proof

FFNATSRS is a Feed Forward Neural Network with Additive TS type with right sigmoidal signals.

$$y = g(n) = \sum_{i=1}^{n} V_i \; \sigma\left(\sum_{i=1}^{n} c_i \, x_i \right) \tag{2.3}$$

FFNATSRS is in the form of FFN.

Hence we get

$F(C (R^n)) \subset F_1 (C (R^n))$

that is $| f(x) – NN(x) | < \varepsilon$

Theorem 4.2.3.
FFNATSLS is a multi-input – single-output system, there exists a multiple-input and single-output additive TS-type fuzzy inference system, such that they are functionally equal.

Proof
Let a FFNATSLS is given

We transform Eq. (2.1) into the following form

$$y = \sum_{i=1}^{n} g_i(\bar{x})\, x_i$$

$$= \sum_{i=1}^{n} \left(\sum_{i=1}^{n} V_i\, \sigma \left(\sum_{i=1}^{n} c_i\, x_i \right) \right) x_i$$

where σ is the left sigmoidal signal

$$= \sum_{i=1}^{n} \sigma \left(\sum_{i=1}^{n} c_i\, x_i \right) \left(\sum_{i=1}^{n} V_i\, x_i \right) x_i \tag{2.4}$$

Define

$$f_i(\bar{x}) = \sum_{i=1}^{n} V_i\, x_i \tag{2.5}$$

Where $(\bar{x}) = [x_1, \ldots, x_n]T$, f_i is a first order polynomial.

Then we can rewrite Eq.(2.4) as

$$y = \sum_{i=1}^{n} \sigma \left(\sum_{i=1}^{n} c_i\, x_i \right) f_i(\bar{x}) \tag{2.6}$$

Based on the Eq. (2.6), we can give a fuzzy rule base as follows:

$$R_1 : \text{if } \sum_{i=1}^{n} W_i\, x_i \text{ is A then } y = f_i(\bar{x}) \tag{2.7}$$

Where A is a fuzzy set defined on R^n, whose membership function is the σ is the left sigmoidal function.

The additive fuzzy inference system is based on the left sigmoidal signal is

$$\sigma = \left(\sum_{i=1}^{n} c_i\, x_i \right) \text{ and the total output y is}$$

$$y_i = \sum_{i=1}^{n} \sigma \left(\sum_{i=1}^{n} c_i\, x_i \right) f_i(\bar{x}) \tag{2.8}$$

FFNATSLS equal to TS-type fuzzy inference system.

So far, the proof would have been already completed. However, some more comprenhensive fuzzy rules are always expected, so we find a decomposition of the premise of each rule resulting in the rule base :

$$R_1 : \text{If } x_1 \text{ is } A_i^{(1)}\, \theta.\, x_i, \text{ is } A_j^{(2)}\, \theta \ldots \theta\, x_n \text{ is } A_i^{(n)} \text{ then } y = f_i(\bar{x}) \tag{2.9}$$

Where θ is a logical operator and $A_i^{(i)}$ are fuzzy sets defined on the universes of x_i's.

By using the fuzzy operator denoted by *, we obtain the following equation :

$$\sigma \left(\sum_{i=1}^{n} c_i\, x_i \right) = \sigma(c_1 x_1) * \sigma(c_2 x_2) * \ldots \sigma(c_n x_n) \tag{2.10}$$

where σ is a Left sigmoidal signal.

$$R_i : \text{If } x_1 \text{ is } A_i^{(1)} * x_2 \text{ is } A_i^{(2)} * \ldots x_n \text{ is } A_i^{(n)} \text{ then } y = f_i(\bar{x}) \tag{2.11}$$

Hence FFNATSLS is functionally equivalent to additive TS fuzzy inference system.

5. Conclusion

In this work we used three layer networks with left and right sigmoidal signal as activation function. The approximation capabilities are better than four layer network.

References

[1] Approximation capability in C(Rn) by multilayer feedforward networks and related problems, IEEE Trans. Neural Networks 6 (1) (1995) 25-30.

[2] Puyin Liu , Universal approximations of continuous fuzzy-valued functions by multilayer regular fuzzy neural networks , Fuzzy Sets and Systems 119 (2001) 313-320.

[3] Ramakrishnan M., Velu C.M., Prabakaran N., Ekambavanan K., Thangavelu P. and Vivekanandan P (2006), ‘Function approximation using feedforward Neural Networks with sigmoidal signals’, International Journal of soft computing Vol. 1, No.1,pp.76-82.

[4] Dong Zhang, Xiao-Li Bai, Kai-Yuan Cai(2004) “Extended neuro-fuzzy models of multilayer perceptrons” Fuzzy Sets and Systems 142(2004) 221-242.