SMFD Assignment 2.1

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Question 1: Two-State Loop

(a) **Transition Matrix Q:**

$$Q = \begin{bmatrix} 0.5 & 0.5 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 \\ 0 & 0 & 0.25 & 0.75 \\ 0 & 0 & 0.75 & 0.25 \end{bmatrix}$$

(b) **Recurrent and Transient States:**

States 1 and 2 form a closed communicating class, so they are **recurrent**.

States 3 and 4 form another closed communicating class, so they are also **recurrent**.

There are no transient states.

(c) **Two Different Stationary Distributions:**

First stationary distribution (supported on states 1 and 2):

$$\pi^{(1)} = \left[\frac{1}{3}, \frac{2}{3}, 0, 0\right]$$

Second stationary distribution (supported on states 3 and 4):

$$\pi^{(2)} = \left[0, 0, \frac{3}{5}, \frac{2}{5}\right]$$

Question 4: The Wandering King

The king moves randomly on an 8×8 chessboard, choosing one of the legal moves from each square with equal probability. The number of legal moves depends on the square type:

- **Corner squares (4 total):** 3 legal moves

- **Edge squares (non-corner, 24 total):** 5 legal moves
- **Interior squares (36 total):** 8 legal moves

Let the stationary probability be proportional to the number of legal moves. Compute the total weight:

$$4 \cdot 3 + 24 \cdot 5 + 36 \cdot 8 = 12 + 120 + 288 = 420$$

So the stationary probabilities for each square type are:

- **Corner square:** $\frac{3}{420} = \frac{1}{140}$
- **Edge (non-corner):** $\frac{5}{420} = \frac{1}{84}$
- **Interior square:** $\frac{8}{420} = \frac{2}{105}$