Equations of Motion

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Mechanics

- **Mechanics** : a tool to describe the behavior of an object in physics using various elements such as displacement, velocity, and acceleration.
- **Statics mechanics** deals with when the object is not in motion,
- **dynamic mechanics** describes in detail when the object is in motion.
- **Kinematics** is the study of motion without regard for the cause. On the other hand, **dynamics** is the study of the causes of motion.
- In kinematics, motion is mathematically described in terms of distance, displacement, speed, velocity, acceleration, and time.

Kinematics + dynamics = mechanics

Motion

- One dimensional motion: The motion of an object along a straight-line path is called motion in one dimension.
  - E.g: Motion of car along a straight road, a ball thrown vertically upwards, a freely falling body.
- Two Dimensional Motion: The motion of an object in plane is called motion in two dimension
  - E.g: **An ant moving on the top surface of a desk**
- Three Dimensional Motion: The motion of an object in space is called motion in three dimension.
  - E.g: Flying of bird
- If the velocity of the body remains a constant in one-dimensional motion, then it is called uniform motion. In uniform motion, the magnitude and direction of velocity remain constant and hence its acceleration is zero.

Equations of motion

- Consider the motion of a particle with initial velocity ‘u’ and uniform acceleration ‘a’.
- Let the displacement and velocity of the particle after a time ‘t’ second is ‘s’ and ‘v’ respectively.
- The motion of the particle along a straight-line path with uniform acceleration can be analyzed using the three equations of motion.

$$v = u + at$$

$$S = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2aS$$

Where *u* is the initial velocity, *v* final velocity, *S* the displacement and t time taken for this displacement

Derivation Equation 1

- From the definition for acceleration,

$$a = \frac{v - u}{t}$$

Cross multiplying, we get

$$at = v - u$$

$$v = u + at$$

Derivation Equation 2

Displacement = average velocity x time

Substitute, v = u +at

$$S = \left(\frac{u+v}{2}\right)t = \left(\frac{u+u+at}{2}\right)t$$

$$= \left(\frac{2u+at}{2}\right)t$$

$$= ut + \frac{1}{2}at^2$$

Derivation Equation 3

- From first equation

$$t = \frac{v-u}{a}$$

- Substituting for t in equation 2

$$S = u\frac{(v-u)}{a} + \frac{1}{2}a\left(\frac{(v-u)}{a}\right)^2$$

$$S = \frac{uv-u^2}{a} + \frac{1}{2}a\left(\frac{v^2-2uv+u^2}{a^2}\right) = \frac{uv-u^2}{a} + \frac{v^2-2uv+u^2}{2a}$$

$$= \frac{2uv-2u^2+v^2-2uv+u^2}{2a} = \frac{v^2-u^2}{2a}$$

- Cross multiplying, we get

$$2as = v^2 - u^2$$

$$v^2 = u^2 + 2aS$$

Equation 4

- This equation gives the distance travelled in t^{th} second = distance travelled in t seconds – distance travelled in (t-1)seconds.

- Distance travelled in t seconds S = $ut + \frac{1}{2}at^2$
- Distance travelled in (t-1) seconds S(t-1) = $u(t-1) + \frac{1}{2}a(t-1)^2$

- Solving the equation we get

$$S_t = \left[ut + \frac{1}{2}at^2\right] - \left[u(t-1) + \frac{1}{2}a(t-1)^2\right]$$

$$S_t = u + a(t - \frac{1}{2})$$