7Q. Prove that $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$

L.H.S $= \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$.

$= \frac{\sin\theta \cdot \sin\theta + (1+\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(\sin\theta)}$.

$= \frac{\sin^2\theta + 1 + \cos\theta + \cos\theta + \cos^2\theta}{(1+\cos\theta)(\sin\theta)}$.

$= \frac{\sin^2\theta + \cos^2\theta + 1 + 2\cos\theta}{(1+\cos\theta)\sin\theta}$.

$= \frac{1 + 1 + 2\cos\theta}{(1+\cos\theta)\sin\theta} = \frac{2+2\cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta}$

$= \frac{2}{\sin\theta} = 2 \cdot \frac{1}{\sin\theta} = 2\operatorname{cosec}\theta$

$=$ R.H.S.

8 $\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2 \sec^2 A$

L.H.S $= \frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1}$

$= \frac{\operatorname{cosec} A (\operatorname{cosec} A + 1) + \operatorname{cosec} A (\operatorname{cosec} A + 1)}{(\operatorname{cosec} A - 1)(\operatorname{cosec} A + 1)}$

$= \frac{\operatorname{cosec}^2 A + \operatorname{cosec} A + \operatorname{cosec}^2 A - \operatorname{cosec} A}{\operatorname{cosec}^2 A - 1}$

$= \frac{2 \operatorname{cosec}^2 A}{\cot^2 A} = 2 \times \frac{1}{\sin^2 A} \times \tan^2 A$

$= 2 \times \frac{1}{\sin^2 A} \times \frac{\sin^2 A}{\cos^2 A}$

$= 2 \times \sec^2 A = R.H.S.$

9. Prove that $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$

L.H.S $= \sqrt{\frac{1 + \sin A}{1 - \sin A}}$

$= \frac{\sqrt{1 + \sin A}}{\sqrt{1 - \sin A}} \frac{\sqrt{1 + \sin A}}{\sqrt{1 + \sin A}}.$

$$= \frac{(1+\sin A)^2}{(1-\sin A)(1+\sin A)}$$

$$= \frac{(1+\sin A)^{1/2}}{1-\sin^2 A} = \frac{1+\sin A}{\cos^2 A}$$

$$= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A$$

Problems

1. If $\sin\alpha = 2/5$ and α is an acute angle, find $\cos\alpha$ and $\tan\alpha$.
2. If $\tan\alpha = 3$ and α is an acute angle, find $\sec\alpha$ and $\operatorname{cosec}\alpha$.
3. Prove the following identities
   (a) $\sin A \cot A = \cos A$
   (b) $\sin^2 A - \cos^2 A = 1 - 2\cos^2 A$
   (c) $(\sin A + \cos A)^2 = 1 + 2\sin A \cos A$
4. Prove that $\frac{1+\sin\alpha}{\cos\alpha} = \frac{\cos\alpha}{1-\sin\alpha}$.
5. Show that $\frac{1+\cos\alpha}{\sin\alpha} + \frac{\sin\alpha}{1+\cos\alpha} = 2\operatorname{cosec}\alpha$.
6. Prove that $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \sin A + \cos A$

Trigonometric functions of Standard angles:

θ	0°	$30^\circ = \pi/6$	$45^\circ = \pi/4$	$60^\circ = \pi/3$	$90^\circ = \pi/2$
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	not defined
$\cot\theta$	not defined	$\sqrt{3}$	1	$1/\sqrt{3}$	0
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	not defined
$\operatorname{cosec}\theta$	not defined	2	$\sqrt{2}$	$2/\sqrt{3}$	1

Evaluate the following.

(i) $3\sin 30^\circ - 4\cos^3 60^\circ$

$= 3 \times 1/2 - 4 \times (1/2)^3$

$= \frac{3}{2} - 4 \times \frac{1}{8} = \frac{3}{2} - \frac{1}{2}$

$= \frac{2}{2} = 1$ //.

(2) $4\tan^2 60° - 3\tan^2 30° + \tan^2 45°$

$= 4 \times (\sqrt{3})^2 - 3 \times \left(\frac{1}{\sqrt{3}}\right)^2 + 1$

$= 4 \times 3 - 3 \times \frac{1}{3} + 1$

$= 12 - 1 + 1 = \underline{12}$.

(3) $\tan^2 45° - \frac{1}{2}\cos 60° - \frac{3}{4}\tan^2 30°$

$= 1 - \frac{1}{2} \times \frac{1}{2} - \frac{3}{4} \times \left(\frac{1}{\sqrt{3}}\right)^2$.

$= 1 - \frac{1}{4} - \frac{3}{4} \times \frac{1}{3}$

$= 1 - \frac{1}{4} - \frac{1}{4} = 1 - \frac{2}{4} = 1 - \frac{1}{2}$.

$= \frac{2-1}{2}$

$= \underline{1/2}$.

(4). $4\sin^3 \pi/3 - 3\cos \pi/6$.

$= 4(\sqrt{3}/2)^3 - 3(\sqrt{3}/2)$

$= 4 \times \frac{3\sqrt{3}}{8} - \frac{3\sqrt{3}}{2}$.

$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = \underline{0}$.

(5) Prove that $\frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} = 2 - \sqrt{3}$

$$\frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} = \frac{1 - 1/\sqrt{3}}{1 + 1/\sqrt{3}}$$

$$= \frac{(\sqrt{3} - 1)/\sqrt{3}}{(\sqrt{3} + 1)/\sqrt{3}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

Or

Numerator and denominator multiplied by $\sqrt{3} - 1$

$$= \frac{(\sqrt{3} - 1)(\sqrt{3} - 1)}{(\sqrt{3} + 1)(\sqrt{3} - 1)} = \frac{3 - 2\sqrt{3} + 1}{3 - 1}$$

$$= \frac{4 - 2\sqrt{3}}{2} = \frac{2(2 - \sqrt{3})}{2}$$

$$= 2 - \sqrt{3}.$$

(6) Prove that $\frac{\tan 60 - \tan 45}{1 + \tan 60 \tan 45} = 2 - \sqrt{3}$

$$\frac{\tan 60 - \tan 45}{1 + \tan 60 \tan 45} = \frac{\sqrt{3} - 1}{1 + \sqrt{3}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

Numerator and denominator multiplied by $\sqrt{3}-1$

$$\therefore = \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$= \frac{3-2\sqrt{3}+1}{3-1} = \frac{4-2\sqrt{3}}{2}.$$

$$= \frac{2(2-\sqrt{3})}{2} = \underline{\underline{2-\sqrt{3}}}.$$

Q. Evaluate $\sin^3 \pi/3 \cos^2 \pi/4 \tan \pi/6$.

$$\sin^3 \pi/3 \cos^2 \pi/4 \tan \pi/6$$

$$= \sin^3 60^\circ \cos^2 45^\circ \tan 30^\circ$$

$$= (\sqrt{3}/2)^3 (1/\sqrt{2})^2 \cdot 1/\sqrt{3}$$

$$= \frac{3\sqrt{3}}{8} \times \frac{1}{2} \times \frac{1}{\sqrt{3}}$$

$$= \underline{\underline{3/16}}.$$

Workout problems.

1. If $\tan\theta = 3$, θ is acute find $\sin\theta$ and $\cos\theta$.
2. Show that $\frac{\sin\theta}{1-\cos\theta} + \frac{1-\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$.
3. Prove that $\frac{\sec\theta}{\sec\theta+1} + \frac{\sec\theta}{\sec\theta-1} = 2\operatorname{cosec}^2\theta$.
4. Evaluate. $\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$
5. Evaluate $4\sin^3 \pi/6 - 3\cos \pi/6$.
6. If $\theta = 30^\circ$, Verify that $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$.
7. Evaluate $\sin \pi/2 \times \cos \pi/4 \times \tan \pi/3$
8. Evaluate $2\operatorname{cosec}^2 \pi/2 - 4\sec^2 \pi/4$.
9. Evaluate $3\cot^2 \pi/3 - 2\tan^2 \pi/6$.