Module IV
Properties of Matter

ELASTICITY

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Introduction

- A rigid body generally means a hard solid object having a definite shape and size.
- But in reality, bodies can be stretched, compressed and bent.
- Even the appreciably rigid steel bar can be deformed when a sufficiently large external force is applied on it.
- This means that solid bodies are not perfectly rigid.
- A solid has definite shape and size.
- In order to change (or deform) the shape or size of a body, a force is required.

Deforming Force and Restoring Force

- A force which produces a change in configuration (size or shape) of the object on applying it, is called a deforming force. ie, the force we applied to deform the material is called deforming force.
- The **restoring force** is a **force** which acts to bring a body back to its equilibrium position.
  - The internal reaction force developed inside an elastic material to resisit any change in its size or shape is called restoring force.

- **Elasticity**:- The property of a body, by virtue of which body regains its original size and shape when the applied force is removed, is known as **elasticity** and the body is known as **Elastic body**.

  Eg. Spring, rubber, skin, etc.

- **Plasticity**:- The property of a body, by virtue of which body does not regain its original size and shape when the applied force is removed, is known as **plasticity** and the body is known as **Plastic body**.

  Eg. Plastic paper, clay, putty, etc.

- **Rigidity:-** The property of a body, by virtue of which body does not change its original size and shape when the force is applied is known as **Rigidity.**

  Eg. Wall, Black board, duster, etc.

Stress:- The restoring force per unit area is known as stress.

If **F** is the force applied and **A** is the area of cross section of the body.

Stress = F/A

The SI unit of stress is N/m^2.

Strain:- It is defined as change in dimensions per unit original dimensions.

Strain= change in dimensions/original dimensions

Strain has no unit.

There are 3 types of stress:-

1) Tensile or Longitudinal Stress:-

If the applied force produces change in length of a body, the stress associated is called as Tensile Stress.

Longitudinal stress = $F/A = Mg/\pi r^2$

2) Volume stress :-

If the applied force produces change in volume of a body, the stress associated is called as Volume Stress.

Volume Stress = dP

3) Shear stress :-

If the applied force produces change in shape of a body, the stress associated is called as Shear Stress.

Shear Stress = Tangential force/ Area

There are 3 types of strain:-

1) Tensile or Longitudinal Strain:-

The change in the length per unit original length of the body is known as longitudinal strain.

Longitudinal strain = l/L

2) Volume strain:--

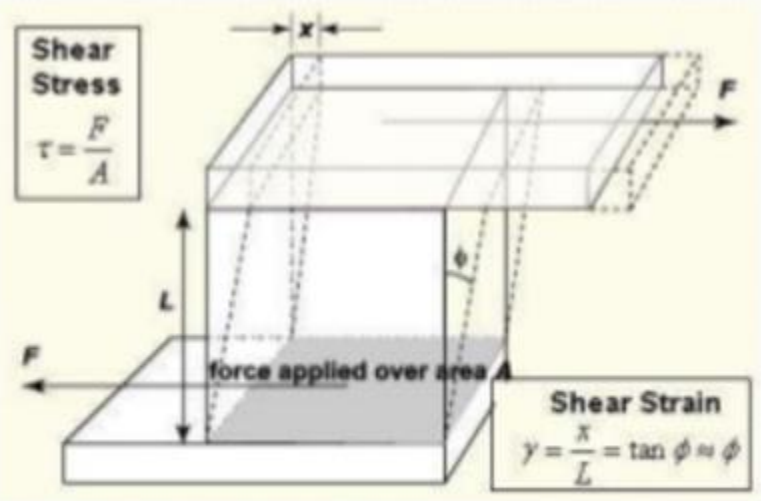
The change in the volume per unit original volume of the body is known as volume strain.

Volume Strain = dV/V

3) Shear strain:--

The ratio of relative displacement of any layer to its perpendicular distance from fixed surface is known as shear strain.

Shear Strain = X/L



Elastic Limit

- Elastic limit of a substance is defined as the maximum stress that can be applied to the substance before it becomes permanently deformed and does not return to its original state.
  - Material is elastic when the stress is below its elastic limit
  - Material is plastic if stress is above the elastic limit.

Elastic Fatigue

The loss of **elastic** properties because of the action of repeated alternating deforming force is called **elastic fatigue**.

Due to **elastic fatigue**: (i) Bridges are declared unsafe after a long time of their use.
(ii) Spring balances show wrong readings after they have been used for a long time.

HOOKE'S LAW:-

Statement:- *"Within elastic limit, stress is directly proportional to strain."*

Thus,

stress $\propto$ strain

stress = M × strain

where M = proportionality constant called as **modulus of elasticity.**

Therefore, M= Stress/strain

There are 3 types of elastic constants:-

1) Young's Modulus (Y)
2) Bulk Modulus (K)
3) Modulus of Rigidity(η)

The constant of proportionality depends on the material being deformed and on the nature of the deformation and is called modulus of elasticity or elastic modulus.

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Young's Modulus: Elasticity of Length

- Consider a long bar of cross-sectional area A and initial length L that is clamped at one end. When an external force is applied perpendicular to the cross-section, internal forces in the bar resist distortion ("stretching"), but the bar reaches an equilibrium situation in which its final length L_f, which is greater than L and in which the external force is exactly balanced by internal forces.
- longitudinal stress as the ratio of the magnitude of the external force F to the cross-sectional area A.

$$Longitudinal\ stress = \frac{F}{A}$$

The longitudinal strain is defined as the ratio of the change in length ΔL to the original length L. Strain can be tensile (increase in length) or compressive (decrease in length).

$$Longitudinal\ strain = \frac{\Delta L}{L}$$

Young's modulus (Y) is a measure of the length elasticity of a material. Young's Modulus is defined as the ratio of longitudinal stress to the longitudinal strain under relatively small deforming force.

$$Young's\ Modulus = \frac{Longitudinal\ \ stress}{Longitudinal\ \ strain}$$

$$Y = \frac{\left(\frac{F}{A}\right)}{\left(\frac{\Delta L}{L}\right)}$$

$$Y = \frac{FL}{A\,\Delta L}$$

Rigidity Modulus: Elasticity of Shape

- Rigidity modulus or shear modulus (η) is defined as the ratio of shear stress to shear strain.

$$shear\ stress = \frac{tangential\ force}{area} = \frac{F}{A}$$

$$shear\ strain = \frac{\Delta x}{h}$$

If the deformation is very small, the shear strain can be recognized by the angular deformation (θ).

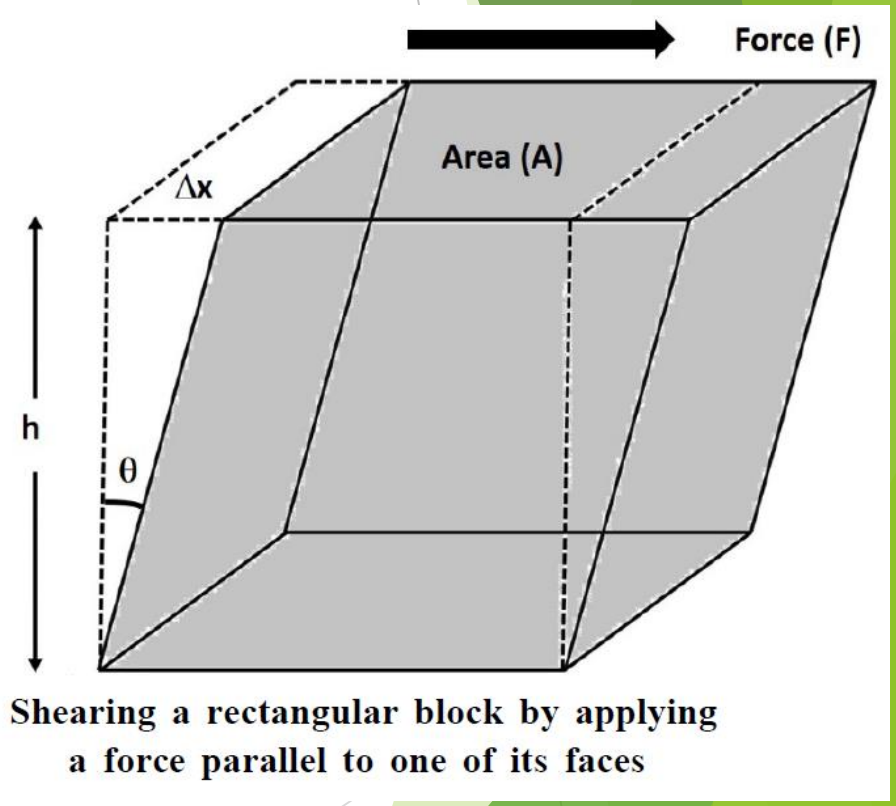
$$Rigidity\ Modulus = \frac{shear\ stress}{shear\ strain}$$

$$\theta = \frac{arc\ length}{radius} = \frac{\Delta x}{h}$$

$$\therefore\ shear\ strain = \theta$$

$$\eta = \frac{\left(\frac{F}{A}\right)}{\theta}$$

$$\eta = \frac{F}{A\theta}$$



Shearing a rectangular block by applying a force parallel to one of its faces

Bulk Modulus: Volume Elasticity

- volume stress as the ratio of the magnitude of the total force (F) exerted on a surface to the area (A) of the surface. The perpendicular force per unit area is called pressure (P).

$$Volume\ stress = \frac{F}{A} = P$$

- Under pressure, the object experiences a volume change ΔV. The volume strain or bulk strain is defined as the ratio of the change in volume to the original volume.

$$volume\ strain = \frac{\Delta V}{V}$$

Bulk modulus (B) is defined as the ratio of volume stress to volume strain.

$$Bulk\ Modulus = \frac{Volume\ stress}{Volume\ stain}$$

$$B = -\frac{P}{\left(\frac{\Delta V}{V}\right)}$$

$$B = -\frac{PV}{\Delta V}$$

The reciprocal of the bulk modulus is called the compressibility and is denoted by k

$$k = \frac{1}{B} = -\frac{\Delta V}{PV}$$

The SI unit of compressibility is that of reciprocal of pressure, Pa^{-1}. Materials with small bulk modulus and large compressibility are easier to compress.