Rotational dynamics continues…

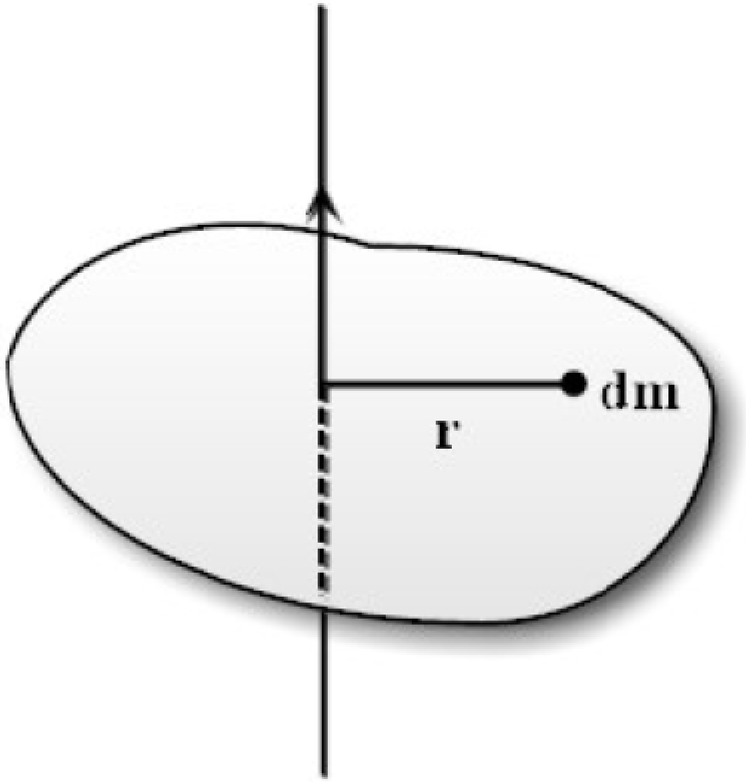
Moment of inertia of continuous mass distributions

If the body is continuous, its moment of inertia about a given axis can be obtained using the technique of integration. Consider a small element of the body of mass dm at perpendicular distance r from the axis of rotation. The moment of inertia of the element about the given axis is

$$dI = r^2 dm$$

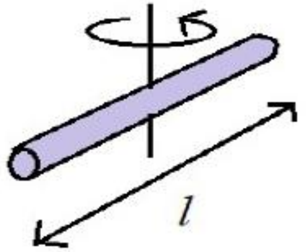
The moment of inertia of the rigid body about the given axis is obtained by integrating the above equation over appropriate limits to cover the whole body. Thus,

$$I = \int r^2 \, dm$$

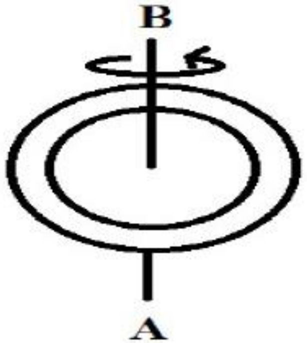


Moment of inertia of a continuous body

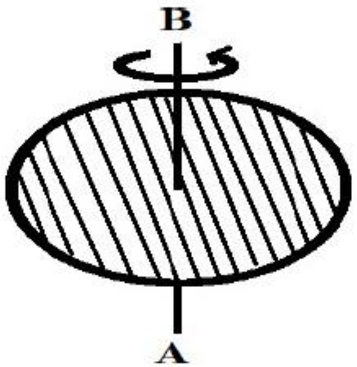
Moment of inertia of (a) a thin road (b) a ring (c) a circular disc (d) solid sphere and (e) hollow sphere about the given axis



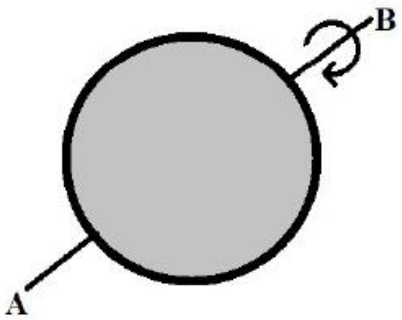
(a)



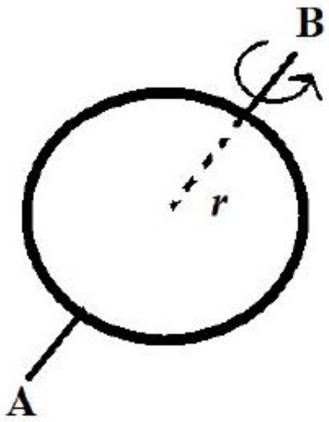
(b)



(c)



(d)



(e)

a) Moment of inertia of a thin uniform rod, about an axis through its centre and perpendicular to its length.

Consider a thin uniform rod of length l and mass M as shown in Fig. (a).Its moment of inertia about an axis through its centre and perpendicular to its length is given by

$$I = \frac{1}{12} Ml^2$$

b) Moment of inertia of a ring, about an axis through the centre and perpendicular to its plane.

Consider a circular ring of radius R and mass M as shown in Fig.(b). Its moment of inertia about an axis through the centre and perpendicular to its plane is given by

$$I = MR^2$$

c) MI of a uniform circular disc

c) Consider a uniform circular disc of radius R and mass M as shown in Fig. (c).Its moment of inertia about an axis through the centre and perpendicular to its plane is given by

$$I = \frac{1}{2}MR^2$$

d) Moment of inertia of a solid sphere, about any diameter.

Consider a solid sphere of radius R and mass M as shown in Fig. (d). Its moment of inertia about any diameter is given by

$$I = \frac{2}{5}MR^2$$

e) Moment of inertia of a hollow sphere, about any diameter.

Consider a hollow sphere of radius R and mass M. Its moment of inertia about any diameter is given by

$$I = \frac{2}{3}MR^2$$

Moment of Inertia(MI) of a circular Ring

A) about an axis passing through the center and perpendicular to its plane

Let M be the mass and R be the radius of the ring. Consider an infinitesimally small element of mass dm of the ring.

M.I of the element = R^2dm

Then MI of the ring about an axis perpendicular to the plane of the ring

and passing through the center of mass , I = $\int R^2 dm$

I = MR^2

B) About a diameter

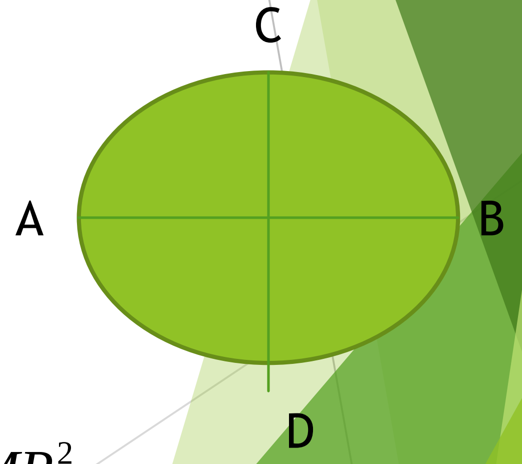
Let $$I_x = I_y = I_d$$

Using perpendicular axis theorem,

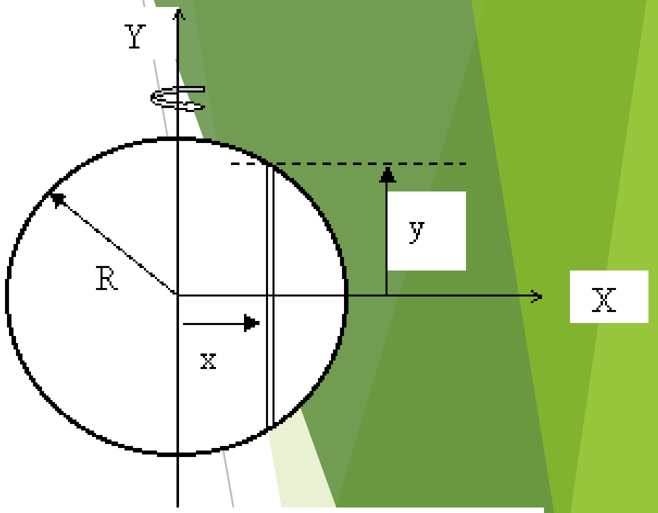
Or,

$$I_x + I_y = MR^2$$

$$2I_d = MR^2 \ \ so \ \ I_d = \frac{MR^2}{2}$$



MI of a uniform circular disc



A disc rotating about its diameter

Figure 5

- Consider a circular disc of mass M and radius R.
- The area of the disc is πR^2.
- Mass per unit area of
- the disc = $M/\pi R^2$
- Now, consider a small ring of breadth dx at a distance x from the centre.
- The area of the small ring= $2\pi x dx$
- The mass of the small ring, dm =

$$\frac{2\pi x dx M}{\pi R^2} = \frac{2M\,xdx}{R^2}$$

- Moment of inertia of this ring $=x^2dm$
- Assuming that the whole disc is made up of a number of small rings whose radii *x* varies from 0 to *R*, the MI of the whole disc can be obtained by integrating the expression within the limits *x*=0 to *x*=*R*.
- *i.e*

$$I=\int_0^R\left[\frac{2M}{R^2}\right]x^3dx$$

$$=\frac{2M}{R^2}\left[\frac{x^4}{4}\right]_0^R=\frac{2M}{R^2}\frac{R^4}{4}$$

- *Or*

$$I=\frac{MR^2}{2}$$