Compound Angles

Definition An angle which is made up of sum or difference of two or more angles is called a compound angle.
example:

(1) $A+B$, $A-B$, $A+B+C$ etc. are compound angles

(2) $(45+30)$, $(60-45)$, $(30+20+25)$ etc. are compound angles

Addition formula.

1. $\sin(A+B) = \sin A \cos B + \cos A \sin B$
2. $\cos(A+B) = \cos A \cos B - \sin A \sin B$
(3) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

Subtraction formula.

1. $\sin(A-B) = \sin A \cos B - \cos A \sin B$
2. $\cos(A-B) = \cos A \cos B + \sin A \sin B$
3. $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

1. If $\cos A = \frac{3}{5}$, $\tan B = \frac{5}{12}$, A and B are acute angles, find the values of $\sin(A+B)$ and $\cos(A-B)$.

Given

$\cos A = 3/5$.

$\sin^2 A = 1 - \cos^2 A$

$= 1 - (3/5)^2$

$= 1 - \frac{9}{25} = \frac{25-9}{25}$

$\sin^2 A = \frac{16}{25}$

$\therefore \sin A = \sqrt{16/25} = \pm 4/5$

A is acute

$\therefore \sin A = 4/5$.

Given

$\tan B = \frac{5}{12}$.

$\sec^2 B = 1 + \tan^2 B$

$= 1 + (5/12)^2 = 1 + \frac{25}{144}$

$= \frac{144+25}{144} = \frac{169}{144}$

$\sec B = \sqrt{\frac{169}{144}} = \pm \frac{13}{12}$

B is acute, $\sec B = \frac{13}{12}$.

$$\cos B = \frac{1}{\sec B} = \frac{1}{13/12} = \underline{\underline{12/13}}.$$

$$\sin^2 B = 1 - \cos^2 B$$

$$= 1 - (12/13)^2 = 1 - \frac{144}{169}.$$

$$= \frac{169-144}{169} = \frac{25}{169}.$$

$$\sin B = \sqrt{\frac{25}{169}} = \pm\frac{5}{13}$$, B is acute.

$$\sin B = \underline{\underline{5/13}}.$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$= \frac{4}{5} \times \frac{12}{13} + \frac{3}{5} \times \frac{5}{13}$$

$$= \frac{48}{65} + \frac{15}{65}$$

$$= \frac{65 \times 48 + 65 \times 15}{65 \times 65}.$$

$$= \frac{\cancel{65}(48+15)}{\cancel{65} \times 65} = \frac{63}{65} \,//.$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$= \frac{3}{5} \times \frac{12}{13} + \frac{4}{5} \times \frac{5}{13}.$$

$$= \frac{36}{65} + \frac{20}{65} = \frac{65 \times 36 + 20 \times 65}{65 \times 65}$$

$$= \frac{\cancel{65}(36+20)}{\cancel{65} \times 65} = \frac{56}{65}$$

2\. If $\tan A = 3/4$, $\tan B = 5/12$, A and B are acute angles, find $\tan(A-B)$

$$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$$

$$= \frac{3/4 - 5/12}{1 + 3/4 \times 5/12}$$

$$= \frac{\frac{36-20}{48}}{1+\frac{15}{48}} = \frac{16/48}{\frac{48+15}{48}}$$

$$= \frac{16/48}{63/48} = 16/63$$

3\. If $\tan A = 2$, $\tan B = 1$, A and B are acute angles. Find the value of $\tan(A-B)$ and $\cos(A-B)$

$$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$$

$$= \frac{2-1}{1+2\times 1} = \frac{1}{1+2} = 1/3$$

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$1 + \tan^2 A = \sec^2 A$.

$A = A - B$.

$\sec^2(A-B) = 1 + \tan^2(A-B)$.

$= 1 + (1/3)^2 = 1 + 1/9$.

$= \frac{9+1}{9} = \cancel{10/9}\ 10/9$.

$\sec(A-B) = \sqrt{10}/3$.

$\therefore \sec A = 1/\cos A$, $A = A - B$.

$\sec(A-B) = 1/\cos(A-B)$

$\therefore \cos(A-B) = 1/\sec(A-B)$

$= 3/\sqrt{10}$

4. If A and B are acute angles where $\tan A = 1/2$, $\tan B = 1/3$, show that $A + B = \pi/4^c$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{1/2 + 1/3}{1 - 1/2 \times 1/3} = \frac{\frac{3+2}{6}}{1 - 1/6}$$

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$$= \frac{5/6}{6 \cdot 1/6} = \frac{5/6}{5/6} = 1$$

$$\tan(A+B) = 1$$

$$A+B = 45^\circ = \pi/4^c$$

Q. Find the value of tan 75 and show that $\tan 75 + \cot 75 = 4$.

$$\tan 75 = \tan(45+30)$$

$$= \frac{\tan 45 + \tan 30}{1 - \tan 45 \cdot \tan 30}$$

$$= \frac{1 + 1/\sqrt{3}}{1 - 1/\sqrt{3}} = \frac{\sqrt{3}+1/\sqrt{3}}{\sqrt{3}-1/\sqrt{3}}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

Numerator and denominator multiplied by $\sqrt{3}+1$

$$= \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{3+\sqrt{3}+\sqrt{3}+1}{3-1} = \frac{4+2\sqrt{3}}{2}$$

$$= \frac{2(2+\sqrt{3})}{2} = 2+\sqrt{3}$$

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$\cot 75 = \frac{1}{\tan 75} = \frac{1}{2+\sqrt{3}}$.

Numerator and denominator multiplied by $2-\sqrt{3}$.

$\cot 75 = \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})} = \frac{2-\sqrt{3}}{4-3} = \underline{\underline{2-\sqrt{3}}}$

$\therefore \tan 75 + \cot 75 = 2+\sqrt{3}+2-\sqrt{3}$

$= \underline{\underline{4}}$.