

$$\begin{aligned}
 &= \sqrt{x} \cos x + \sin x \left( \frac{1}{2} x^{-1/2} \right) \\
 &= \sqrt{x} \cos x + \frac{1}{2\sqrt{x}} \sin x \\
 &= \sqrt{x} \cos x + \frac{1}{2\sqrt{x}} \sin x
 \end{aligned}$$

Quotient rule.

If  $u$  and  $v$  any two functions of  $x$   
then  $\frac{d}{dx} \left( \frac{u}{v} \right) = \frac{v \cdot \frac{d}{dx}(u) - u \cdot \frac{d}{dx}(v)}{v^2}$

OR

$$\frac{d}{dx} \left( \frac{uv}{v} \right) = \frac{v \cdot \frac{d}{dx}(uv) - uv \cdot \frac{d}{dx}(v)}{(v)^2}$$

$$\begin{aligned}
 \text{Q. } \frac{d}{dx} \left( \frac{x^2}{\sin x} \right) &= \frac{\sin x \cdot \frac{d}{dx}(x^2) - x^2 \cdot \frac{d}{dx}(\sin x)}{(\sin x)^2} \\
 &= \frac{\sin x \cdot (2x) - x^2 \cdot (\cos x)}{\sin^2 x} \\
 &= \frac{2x \sin x - x^2 \cos x}{\sin^2 x}
 \end{aligned}$$

$$\text{Q. } \frac{d}{dx} \left( \frac{x-1}{x+1} \right) = \frac{(x+1) \frac{d}{dx}(x-1) - (x-1) \frac{d}{dx}(x+1)}{(x+1)^2}$$

$$= \frac{(x+1) \left( \frac{d}{dx}(x) - \frac{d}{dx}(1) \right) - (x-1) \left( \frac{d}{dx}(x) \frac{d}{dx}(1) \right)}{(x+1)^2}$$

$$= \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2}$$

$$= \frac{(x+1) - (x-1)}{(x+1)^2} = \frac{x+1-x+1}{(x+1)^2} = \frac{2}{(x+1)^2}$$

$$2. \frac{d}{dx} \left( \frac{\cos x}{x} \right) = \frac{x \cdot \frac{d}{dx}(\cos x) - \cos x \frac{d}{dx}(x)}{(x)^2}$$

$$= \frac{x \cdot (-\sin x) - \cos x \cdot \frac{d}{dx}(x^{1/2})}{x^2}$$

$$= \frac{-x \sin x - \cos x \left( \frac{1}{2} x^{1/2-1} \right)}{x^2}$$

$$= \frac{-x \sin x - \cos x \cdot \frac{1}{2} x^{-1/2}}{x^2}$$

$$= \frac{-x \sin x - \cos x \cdot \frac{1}{2\sqrt{x}}}{x^2}$$

$$= \frac{-x \sin x - \frac{1}{2\sqrt{x}} \cos x}{x^2}$$



# Differential coefficient of sec x, sec x, tan x and cot x

$$1. \frac{d}{dx}(\operatorname{cosec} x) = \frac{d}{dx}\left(\frac{1}{\sin x}\right)$$

$$= \sin x \cdot \frac{d}{dx}(1) - 1 \cdot \frac{d}{dx}(\sin x)$$

$$= \sin x \times 0 - \cos x = -\cos x$$

$$= -\frac{\cos x}{\sin x} \times \frac{1}{\sin x}$$

$$= -\cot x \cdot \operatorname{cosec} x$$

$$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$

$$2. \frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$$

$$= \cos x \cdot \frac{d}{dx}(1) - 1 \cdot \frac{d}{dx}(\cos x)$$

$$= \cos x \times 0 - 1 \cdot (-\sin x)$$

$$= \frac{\sin x}{(\cos x)^2} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$= \tan x \sec x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$Q. \frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right).$$

$$= \cos x \cdot \frac{d}{dx}(\sin x) - \sin x \cdot \frac{d}{dx}(\cos x)$$

$$\frac{(\cos x)^2}{(\cos x)^2} =$$

$$= \cos x \cdot \cos x - \sin x \cdot (-\sin x)$$

$$\frac{(\cos x)^2}{(\cos x)^2} =$$

$$= \frac{\cos^2 x + \sin^2 x}{(\cos x)^2} = \frac{1}{\cos^2 x}.$$

$$= \sec^2 x //$$

$$\frac{d}{dx}(\tan x) = \underline{\underline{\sec^2 x}}$$

$$Q. \frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right).$$

$$= \sin x \cdot \frac{d}{dx}(\cos x) - \cos x \cdot \frac{d}{dx}(\sin x)$$

$$\frac{(\sin x)^2}{(\sin x)^2} =$$

$$= \sin x (-\sin x) - \cos x \cdot \cos x$$

$$\frac{(\sin x)^2}{(\sin x)^2} =$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\frac{(\sin^2 x + \cos^2 x)}{\sin^2 x}.$$

$$= \frac{-1}{\sin^2 x} = -\underline{\underline{\csc^2 x}}$$

$$\frac{d}{dx}(\cot x) = \underline{\underline{-\csc^2 x}}$$



## Result

1.  $\frac{d}{dx}(x^n) = n \cdot x^{n-1}$

2.  $\frac{d}{dx}(c) = 0$

3.  $\frac{d}{dx}(\sin x) = \cos x$

4.  $\frac{d}{dx}(\cos x) = -\sin x$

5.  $\frac{d}{dx}(\tan x) = \sec^2 x$

6.  $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

7.  $\frac{d}{dx}(\sec x) = \sec x \tan x$

8.  $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

9.  $\frac{d}{dx}(e^x) = e^x$

10.  $\frac{d}{dx}(\log x) = \frac{1}{x}$

Q.  $\frac{d}{dx}(x + \sin x + 5 \tan x)$

$$= \frac{d}{dx}(x) + \frac{d}{dx}(\sin x) + \frac{d}{dx}(5 \tan x)$$

$$= 1 + \cos x + 5 \cdot \frac{d}{dx}(\tan x)$$

$$= 1 + \cos x + 5 \cdot \sec^2 x$$

Q.  $\frac{d}{dx}(4e^x - 3 \operatorname{cosec} x)$

$$= \frac{d}{dx}(4e^x) - \frac{d}{dx}(3 \operatorname{cosec} x)$$

$$= 4 \cdot \frac{d}{dx}(e^x) - 3 \cdot \frac{d}{dx}(\operatorname{cosec} x)$$

$$= 4 \cdot e^x - 3 \cdot (-\operatorname{cosec} x \cot x)$$

$$= 4e^x + 3 \sec x \cot x$$

$$Q. \frac{d}{dx} \left( \frac{\tan x}{x} \right)$$

$$= x \cdot \frac{d}{dx} (\tan x) - \tan x \cdot \frac{d}{dx} (x)$$

$$= x \cdot (\sec^2 x) - \tan x \cdot 1$$

$$= \frac{x \sec^2 x - \tan x}{x^2}$$

$$Q. \frac{d}{dx} (x^n \log x)$$

$$= x^n \cdot \frac{d}{dx} (\log x) + \log x \cdot \frac{d}{dx} (x^n)$$

$$= x^n \cdot \frac{1}{x} + \log x \cdot n \cdot x^{n-1}$$

$$= x^n \cdot x^{-1} + n \cdot x^{n-1} \log x$$

$$= x^{n-1} + n x^{n-1} \log x$$

$$= x^{n-1} (1 + n \log x)$$

$$Q. \frac{d}{dx} (x^3 e^x)$$

$$= x^3 \cdot \frac{d}{dx} (e^x) + e^x \cdot \frac{d}{dx} (x^3)$$

$$= x^3 \cdot e^x + e^x \cdot 3 \cdot x^{3-1}$$



$$= x^3 e^x + e^x \cdot 3x^2$$

$$= e^x (x^3 + 3x^2)$$

Q.  $\frac{d}{dx} \left( \frac{\cos x}{x + \sin x} \right)$

$$= \frac{(x + \sin x) \cdot \frac{d}{dx}(\cos x) - \cos x \cdot \frac{d}{dx}(x + \sin x)}{(x + \sin x)^2}$$

$$= \frac{(x + \sin x)(-\sin x) - \cos x(1 + \cos x)}{(x + \sin x)^2}$$

$$= \frac{-x \sin x - (\sin x)^2 - (\cos x + \cos^2 x)}{(x + \sin x)^2}$$

$$= \frac{-x \sin x - \sin^2 x - \cos x - \cos^2 x}{(x + \sin x)^2}$$

$$= \frac{-x \sin x - \cos x - (\sin^2 x + \cos^2 x)}{(x + \sin x)^2}$$

$$= \frac{-x \sin x - \cos x - 1}{(x + \sin x)^2}$$

$$Q. \frac{d}{dx} \left( \frac{x \sec x}{3x+2} \right)$$

$$= \frac{(3x+2) \frac{d}{dx} (x \sec x) - x \sec x \cdot \frac{d}{dx} (3x+2)}{(3x+2)^2}$$

$$= (3x+2) \left[ x \frac{d}{dx} \sec x + \sec x \cdot \frac{d}{dx} (x) \right] -$$

$$\frac{x \sec x \left( 3 \cdot \frac{d}{dx} (x) + \frac{d}{dx} (2) \right)}{(3x+2)^2}$$

$$= (3x+2) (x \cdot \sec x \tan x + \sec x) -$$

$$\frac{x \sec x (3+0)}{(3x+2)^2}$$

$$= (3x+2) (x \sec x \tan x + \sec x) - \frac{3x \sec x}{(3x+2)^2}$$