Introduction to Vectors and Scalars

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Physical quantities

- Vectors : Both magnitude and direction
  - Eg: displacement , velocity, force, acceleration
- Scalars: Quantities that have only magnitude
  - Eg: distance, speed, work, energy, power
- A vector quantity is denoted either using bold letters (**A, B**) or putting a small arrow → on the top of the symbol ($\vec{a}$) used for the representation of the quantity. The magnitude of a vector quantity, say F, is denoted by |**F**| or F

Multiplication of two scalars will always be a scalar

Multiplication of a scalar with a vector is a scalar

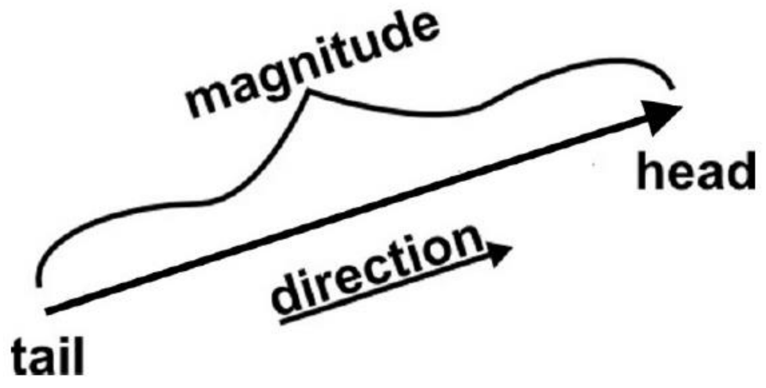
Examples

- **Distance** is the length of the path taken by an object
- **Displacement** is the **distance** between where the object started and where it ended up(depends on direction of motion).
- Displacement is defined as the change of position of a body in a direction
- For example, suppose you are travelling in a bus, 5 km east and then 3 km west. Then came back to your home through the same way. Then the total distance travelled is 16km while the total displacement is 0.

- **Speed (s)** is a scalar quantity,
  - It is the rate at which an object covers distance.
  - The average **speed** is the distance (a scalar quantity) per time ratio.
  - **Speed** is ignorant of direction.
  - It is the distance travelled by a body per unit time
  - **velocity** (v) is a vector quantity; it is direction dependent.
- It is defined as the displacement of a body per unit time
- Unit of both speed and velocity are the same

Geometric representation of a vector

- A vector quantity is represented graphically by a straight line with an Arrowhead
- The length of the straight line represents the magnitude of the vector and the arrowhead gives the direction of the vector.

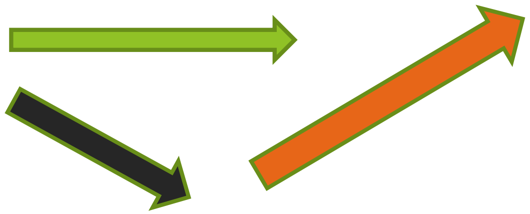


- Arrow mark is called the head and the other end is called the tail of the vector
- A vector can be displaced parallel to itself. Moving a vector parallel to itself does not change the magnitude and direction of the vector.

Type of Vectors

- Collinear vectors

Two or more vectors lying on the same line are called collinear vectors. They can have the same or different magnitude and the direction can be either the same or opposite.



- Equal vectors

Two vectors of the same magnitude and direction are called equal vectors.

- Negative of a vector

The negative of a vector is defined as another vector having the same magnitude but opposite in direction to the given vector.

Unit Vector

- A unit vector is a vector of unit magnitude and points in a particular direction.
- It is used just to specify a direction and hence it is also called a direction vector
- The unit $\hat{a}$ of a vector $\vec{A}$ is defined as $\hat{a} = \frac{\vec{A}}{|\vec{A}|}$
- The commonly used unit vectors are $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ which indicates X, Y, and Z directions respectively.

Addition of Vectors

- Addition of vectors results in getting the combined effect of the vectors known as resultant.
- **It is that single vector that would have the same effect as all the original vectors taken together**
- Vectors are added geometrically since directions are to be taken into account
- Methods of additions
  - Tail to head method
  - Triangle method
  - Parallellogram method

  - When we multiply two or more vectors, it is important to determine whether we want a product that has a scalar quantity or vector quantity.

Multiplication of Vectors

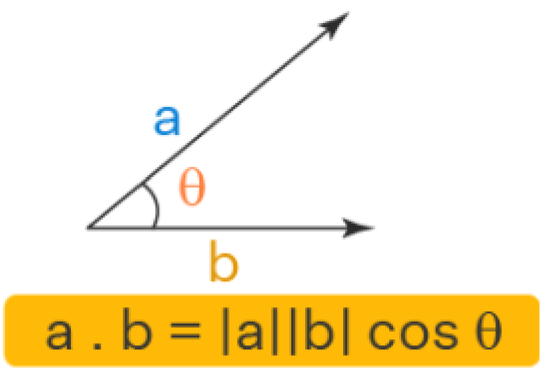
- There are actually three possible products in vector multiplication:
  - vector multiplied by a scalar factor giving a vector, (kA = $\boldsymbol{B}$)
  - vector multiplied by a vector giving a scalar quantity (dot product)

  $A.B = C$
- vector multiplied by a vector giving a vector quantity (cross product)

  $A \times B = C$
- Cross Product between two vectors A and B is defined as

$$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = |\vec{a}|\,|\vec{b}|\,\sin\theta\,\hat{n}$$

- Dot Product between two vectors A and B is defined as



- Where θ is the angle between the two vectors and is a unit vector perpendicular to both A and $\boldsymbol{B}$
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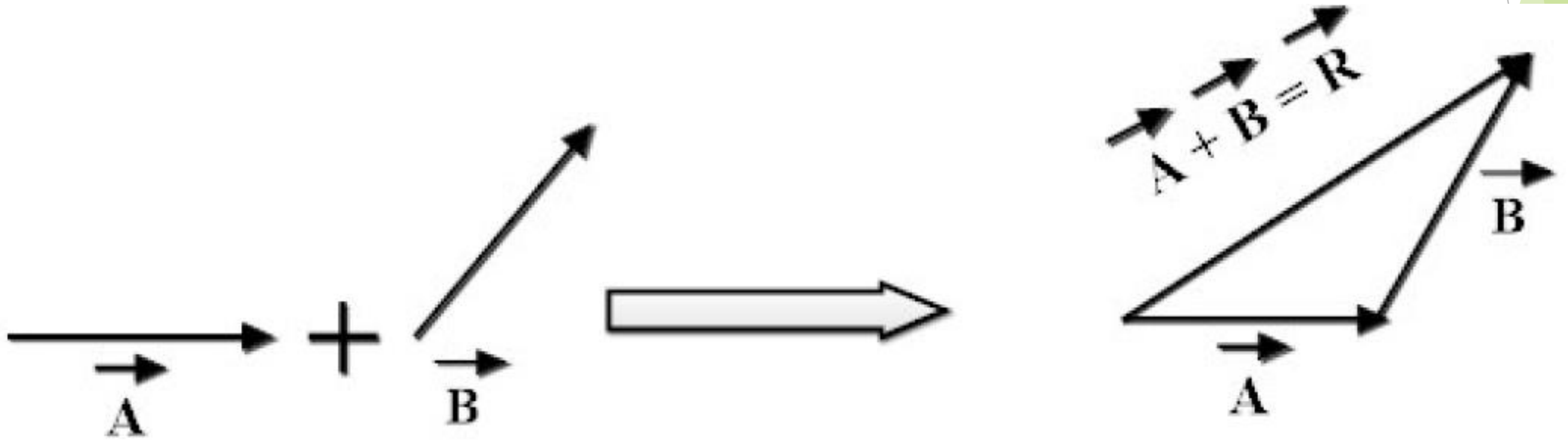
Triangle method

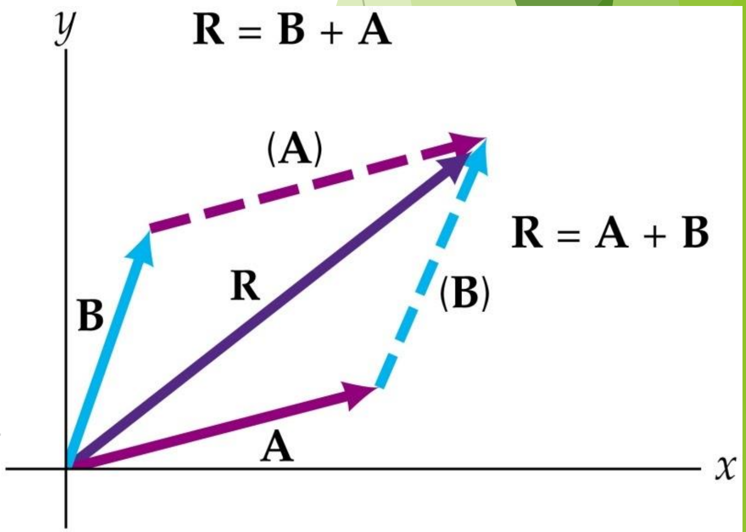
Let $\vec{A}$ and $\vec{B}$ are two non-parallel vectors. To find the vector sum using the triangle method, place the vectors such that the tail of one vector coincides with the head of the other vector. Complete the triangle by drawing the third side. The third side gives the resultant vector $\vec{R}$.

The triangular law of vector addition states that if two vectors are represented by the adjacent sides of a triangle taken in order, then the resultant vector is represented both in magnitude and direction by the third side of the triangle taken in the reverse order.



Law of Parallelogram Vectors

- This method is based on the parallelogram law of vector addition.
- The parallelogram law of vector addition states that if two vectors are represented both in magnitude and direction by the two sides of a parallelogram drawn from a point, then the resultant vector is represented both in magnitude and direction by the diagonal of the parallelogram passing through the point.
- According to parallelogram law of vectors , their resultant vector will be represented by the diagonal of the parallelogram .



Parallelogram law of vector addition

- Consider two vectors $\vec{P}$ and $\vec{Q}$. Let θ be the angle between the two vectors. The resultant, $\vec{R}$, of the two vectors can be obtained by the parallelogram method as shown.

From right angled triangle OCD,

$$OC^2 = OD^2 + CD^2$$

$$= (OA + AD)^2 + CD^2$$

$$= OA^2 + AD^2 + 2.OA.AD + CD^2 \quad ...(1)$$

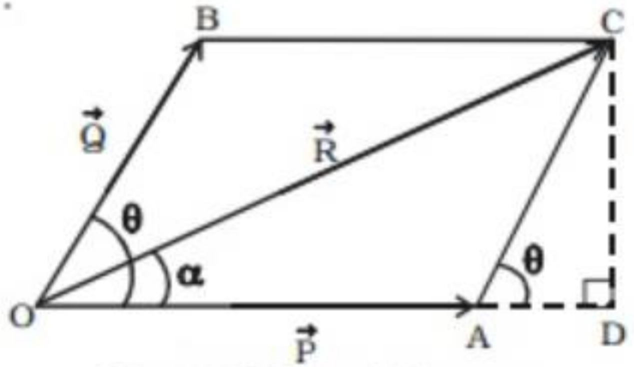


Fig 2.15 Parallelogram law of vectors

In Fig. 2.15 $\angle BOA = \theta = \angle CAD$

From right angled Δ CAD,

$$AC^2 = AD^2 + CD^2 \quad ...(2)$$

Substituting (2) in (1)

$$OC^2 = OA^2 + AC^2 + 2OA.AD \quad ...(3)$$

From ΔACD,

$$CD = AC \sin\theta \quad ...(4)$$

$$AD = AC \cos\theta \quad ...(5)$$

Substituting (5) in (3) $OC^2 = OA^2 + AC^2 + 2\ OA.AC\cos\theta$

Substituting $OC = R$, $OA = P$,

$OB = AC = Q$ in the above equation

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

(or) $$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \quad \ldots(6)$$

Equation (6) gives the magnitude of the resultant. From $\Delta\, OCD$,

$$\tan\alpha = \frac{CD}{OD} = \frac{CD}{OA + AD}$$

Substituting (4) and (5) in the above equation,

$$\tan\alpha = \frac{AC\sin\theta}{OA + AC\cos\theta} = \frac{Q\sin\theta}{P + Q\cos\theta}$$

The direction of the resultant vector is specified by the angle α with respect to the vector $\vec{P}$. The angle α is given by the expression

(or) $$\alpha = \tan^{-1}\left[\frac{Q\sin\theta}{P + Q\cos\theta}\right] \quad \ldots(7)$$

Equation (7) gives the direction of the resultant.

Special Cases

a) If two vectors are in the same direction, then $\theta = 0$ and hence, $cos\theta = 1$

The magnitude of the resultant, $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$$R = \sqrt{A^2 + B^2 + 2AB}$$

$$R = \sqrt{(A + B)^2}$$

$$R = A + B$$

The magnitude of the resultant is the sum of the magnitudes of the two vectors.

b) If two vectors are in opposite direction, then $\theta = 180^\circ$ and hence, $cos\theta = -1$

The magnitude of the resultant, $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$$R = \sqrt{A^2 + B^2 - 2AB}$$

$$R = \sqrt{(A - B)^2}$$

$$R = A - B$$

The magnitude of the resultant is the difference of the magnitudes of the two vectors.

C) When P and Q are in perpendicular directions, θ= 90

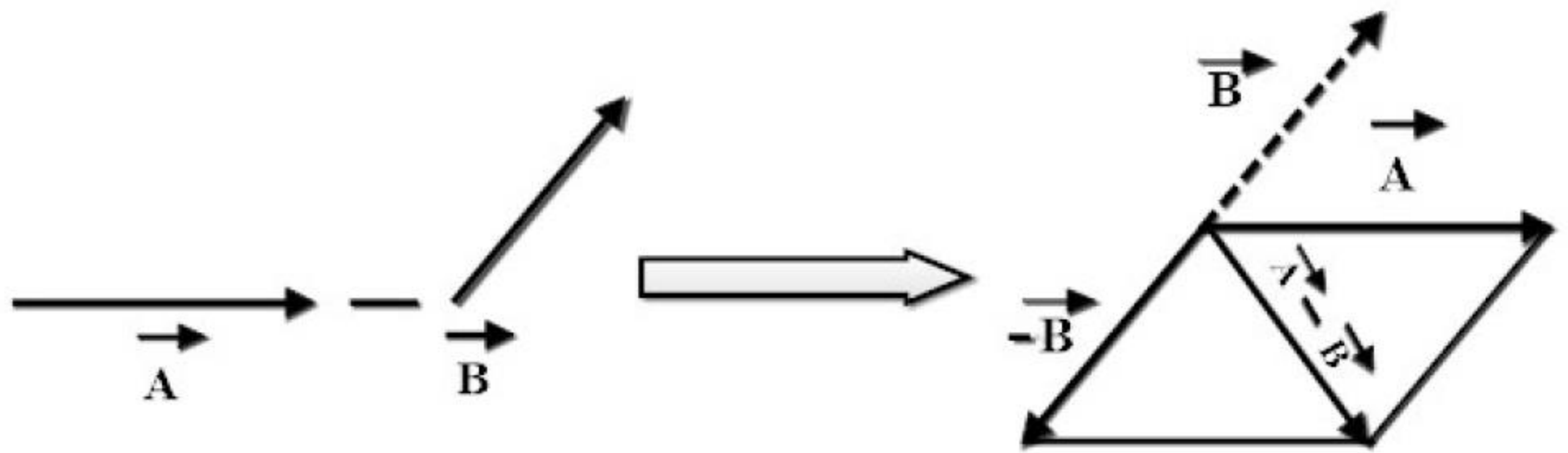
Resultant $R = \sqrt{P^2 + Q^2}$

we get minimum effect

Subtraction of two vectors

Subtraction of two vectors also involves addition. To subtract $\vec{B}$ from $\vec{A}$ first, take the negative of $\vec{B}$ and then add it to $\vec{A}$. Hence, subtraction of two vectors is the same as the addition of a vector with the negative of the second vector.

$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$



Resolution of a vector

- Two or more vectors can be combined to form a single vector through addition.
- Similarly, a given vector can be represented as the sum of two or more vectors acting along different directions.
- The process of splitting a given vector into two or more vectors along different directions is called the resolution of a vector.
- The vectors obtained by the resolution of the given vector are called component vectors.

- A vector lying in a plane is usually resolved along two mutually perpendicular directions. The resolution of a vector along mutually perpendicular directions is called rectangular resolution.
- The two perpendicular components are called rectangular components. The rectangular components are taken along the X-axis and Y-axis.
- Consider a vector $\vec{A}$ making an angle θ with the X-axis.

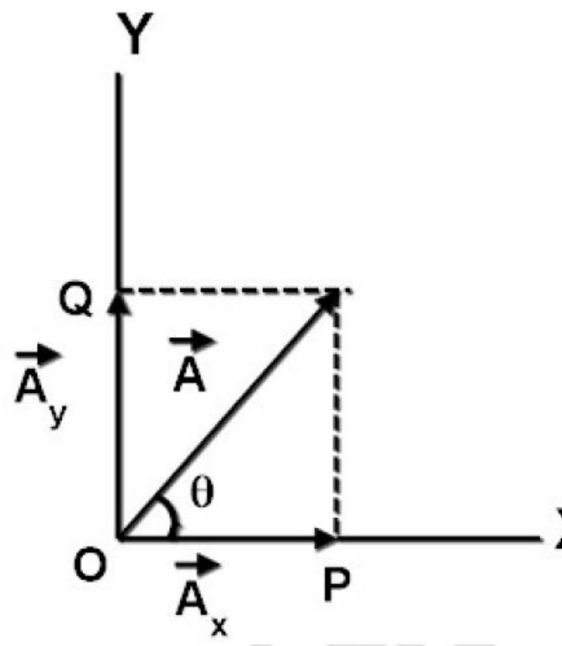
Draw perpendiculars from the head of the vector $\vec{A}$

to X-axis and Y axis to meet at the points P and Q respectively.

Then, if OP and OQ are taken as two vectors $\vec{A_x}$ and $\vec{A_y}$

respectively, then by parallelogram law of vector addition,

$\vec{A}$ is the resultant vector.



- $\vec{A} = \vec{A_x} + \vec{A_y}$
- Thus and $\vec{A_x}$ and $\vec{A_y}$ are vector components of $\vec{A}$.
- Magnitudes of $\vec{A_x}$ and $\vec{A_y}$ are called scalar components.
- A_x and A_y are called x-component and y-component respectively.
- Using simple trigonometric relations, x-component and y-component of vector $\vec{A}$ is given by

$$A_x = A\,cos\theta$$

$$A_y = A\,sin\theta$$