Dynamics

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Dynamics

- Dynamics is a branch of mechanics that deals with the study of forces and its effect on the motion of bodies.
- In dynamics, Newton's laws of motion are three laws that describe the relationship between the motion of an object and the forces acting on it.

Newton's first law of motion

Newton's first law of motion states that everybody continues in its state of rest or of uniform motion along a straight line unless an unbalanced force acting on it.

Newton's first law of motion is also known as the law of inertia.

Inertia is an inherent property of all bodies.

Nobody can change its state of motion by itself and only an external force can change its state of motion.

Inertia is the resistance of a body to any change in its state of rest or uniform motion along a straight line. In the absence of a net external force, a body at rest continues to remain at rest and a body in motion continues to move with constant velocity.

- The inertia of an object is measured by its mass.
- The tendency of a body to remain in its existing state of rest is called inertia of rest.
- The tendency of a body to remain in its existing state of motion with constant velocity is called inertia of motion.
- Examples:
- A) A person standing in a stationary bus falls backward when the bus starts suddenly. This is because the lower part of his body moves forward with the bus, but the upper part of his body remains at rest due to inertia of rest, which results in the backward fall.
- b) A person trying to get down from a moving bus falls forward. The lower part suddenly comes to rest on touching the ground, but the upper part of his body remains in motion due to inertia of motion and the person falls forward.
- c) Fruits from a tree fall due to inertia of rest when the tree is shaken. Both the fruits and branches are at rest, but when shaken branches start moving whereas fruits remain in its state of rest and are separated from the branches.

Momentum(p)

- Momentum is the quantity of motion of a body.
- The momentum of a body is defined as the product of mass and velocity. It is a vector quantity and its unit is kg m/s.
- If a body of mass ‘m’ moving with a velocity ‘v’, then its momentum is given by $p = m\mathrm{v}$
- The momentum of a body at rest is zero.
- Force is related to momentum or more specifically change in momentum. Newton’s second law gives the relation between force and momentum.

Newton’s second law of motion

- *Newton’s second law of motion states that the rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction of the force.*

$$\text{force} \propto rate\ of\ change\ of\ momentum$$

force is proportional to the product of mass and acceleration

$$F \propto ma$$

$F = k\ ma$ where k is the constant of proportionality.

- By suitably defining the SI unit of force, we can take k as 1. Thus, $F = ma$
- One unit of force is defined as that which causes an acceleration of $1 m/s^2$ in a body of mass 1kg. This unit is known as newton (N). $1\ N = 1\ kg\ m/s^2$
- If ‘p’ is the momentum of the body, then Newton’s second law can be expressed in differential form as

$$F = dp\ /dt$$

- The second law implies that if Force F = 0, then acceleration, a = 0, which means the body is either at rest or in uniform motion.
- Thus, Newton’s first law can be derived from the second law. Newton’s first law gives a qualitative idea of force and the second law gives a mathematical expression for force.

Newton’s Third law of motion

<u>Newton’s third law of motion</u>

Newton’s third law of motion states that to every action, there is always an equal and opposite reaction.

- If a body B exerts a force, F_{AB} on a body A, then the body A exerts an equal and opposite force, F_{BA} on body B.

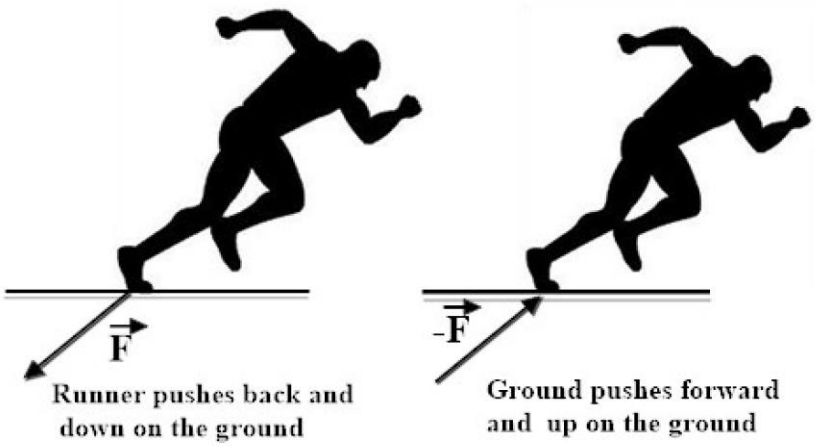
$$F_{AB} = -F_{BA}$$

The main properties of action and reaction
forces are:
a) Action and reaction are the simultaneously occurring pair of forces acting between two objects.
b) Forces always occur in pairs and a single force doesn’t exist in the universe. This is an important property of forces.
c) Action and reaction are always equal in magnitude and opposite in direction.
d) There is no cause-effect relation implied in the third law. Both action and reaction occur at the same time. So, any of the two forces can be called action and the other reaction.
e) The action and reaction forces, though equal and opposite, never adds up to get zero. Action and reaction do not cancel each other since they act on different objects.

Examples

1. When a man jumps off a boat to the shore, he exerts a force on the boat. The boat exerts an equal and opposite force on the man which makes the jump possible. The boat moves backward due to the force exerted by the man.

2. A runner exerts a force on the ground and the reaction force of the ground on the runner pushes him forward.



Law of conservation of momentum

- Newton's second law leads to Law of conservation of momentum
- Statement: Total momentum before collision between two or more bodies is equal to total momentum after collision

Or

When a group of bodies are exerting force to each other, their total momentum remains conserved in the absence of external forces.

$$force = \frac{Change\ in\ momentum}{time}$$

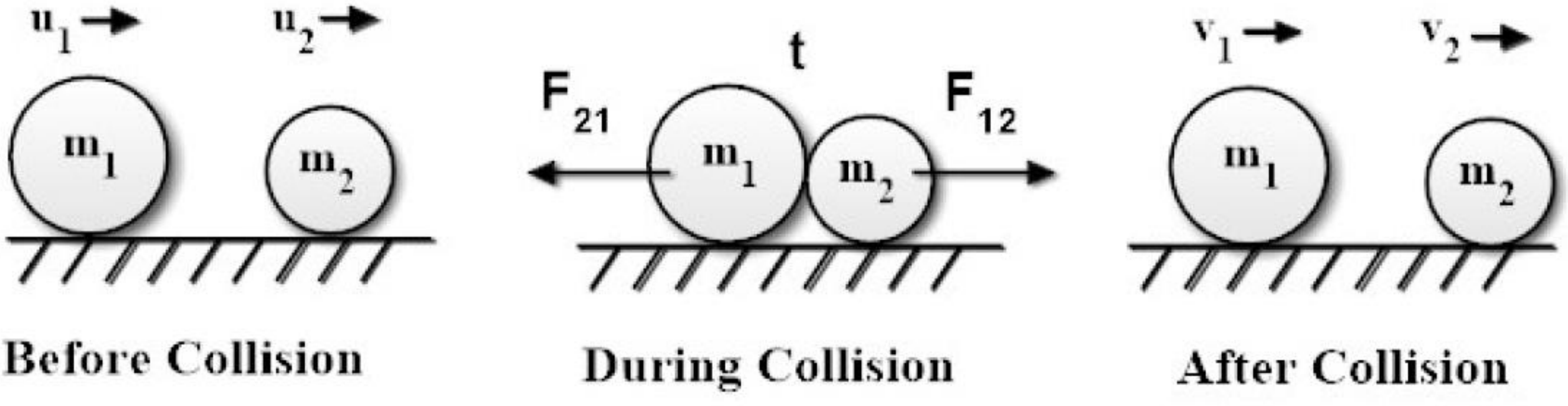
If the net force acting on the system is zero, then the change in momentum also becomes zero. Ie,

$$final\ momentum = initial\ momentum$$

- Thus, the law of conservation of momentum states that if the net external force acting on a system is zero, its linear momentum remains constant.

Proof of law of conservation of momentum

- Consider two bodies of masses m_1 and m_2 moving along a straight line with velocities u_1 and u_2 respectively. Let the bodies collide for a time t seconds. After the collision, the velocities become v_1 and v_2 respectively for masses m_1 and m_2 along the same direction as shown in Figure.



- Since there is no external force acting on the system of two colliding bodies, the bodies apply internal forces on each other during the collision. Let the force acting on the mass $m1$ (applied by m_2) be F_{12} and the force acting on the mass m_2 (applied by m_1) be F_{21}. From Newton's second law of motion,

$$\text{Force} = \frac{change\ in\ momentum}{time}$$

$$F_{12} = m_1 \frac{(u_1 - v_1)}{t} \qquad F_{21} = m_2 \frac{(u_2 - v_2)}{t}$$

- According to Newtons third law $F_{12} = -F_{21}$

$$m_2 \frac{(u_2 - v_2)}{t} = -m_1 \frac{(u_1 - v_1)}{t}$$

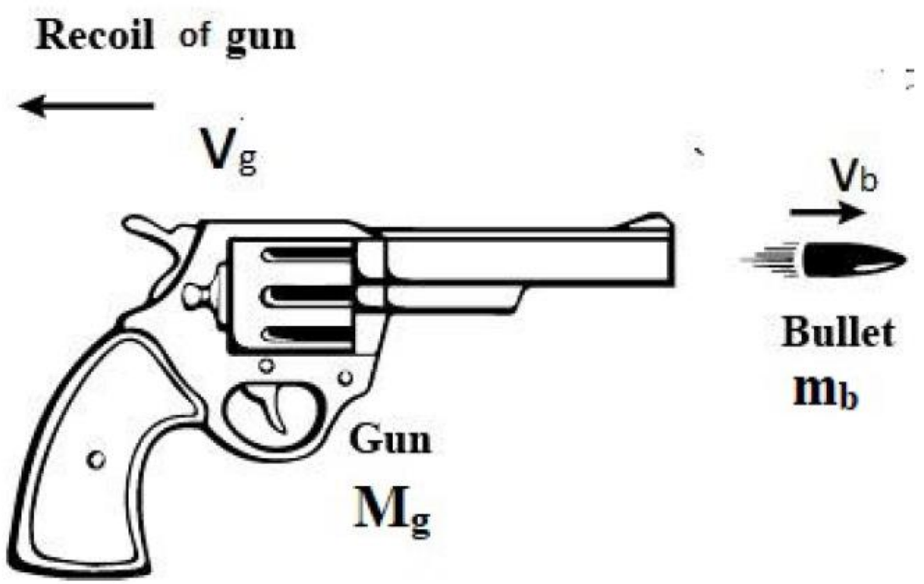
$$m_2 (u_2 - v_2) = -m_1 (u_1 - v_1)$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

Ie., Total momentum before collision = Total momentum after collision

Recoil of Gun

- The backward motion of a gun when a bullet is fired from it is called the recoil of the gun. It can be explained using the principle of conservation of linear momentum.
- The total momentum of the gun and bullet before firing is zero. Since no external force acts on the gun and the bullet, its momentum should be conserved.
- After firing the bullet moves with a velocity producing momentum in the forward direction.
- To balance the momentum change, the gun moves backward with a velocity, such that the total momentum is zero



Let M_g and m_b are masses of the gun and bullet respectively. Suppose, a bullet is fired from the gun with a velocity v_b and the gun recoils with a velocity V_g.

$$Total\ momenta\ before\ firing = 0$$

$$Total\ momenta\ after\ firing = M_g V_g + m_b v_b$$

By the law of conservation of momentum, the total momenta after firing must be equal to the total momenta before firing.

$$M_g V_g + m_b v_b = 0$$

$$M_g V_g = -m_b v_b$$

$$V_g = -\frac{m_b v_b}{M_g}$$

The negative sign shows that the **direction of recoil velocity of** the gun is opposite to the direction of the velocity of the bullet.

Rocket Propulsion

- Rockets are used to launch artificial satellites and space shuttles, deliver explosive warheads to their targets, and also for human space flight and scientific exploration of outer space.
- The principle behind rocket propulsion is the law of conservation of momentum (external force on rocket is zero and effect of gravity is neglected).

  - Rocket can be considered as a system of particles in which mass is varying during its motion
  - In a rocket fuel is burned and the exhaust gas is expelled out from the rear end of the rocket
  - The force exerted by the exhaust gas on the rocket is equal and opposite to the force exerted by the rocket to expel it.
  - This force exerted by the exhaust gas on the rocket propels the rocket forwards
  - When more gas is ejected from the rocket ,the mass of the rocket decreases.

- Let *T* be the reactional force(Thrust)
- *Mg*, the **weight** of the rocket
- Then the force that drives the rocket =*T-Mg*
- Then according to Newton's law *Ma= T-Mg*
- Then the acceleration of the rocket

$$a = \frac{T - Mg}{M}$$

- The mass of the rocket decreases due to consumption of fuel . Also as the rocket goes up, g decreases. But the thrust remains the same. Hence the acceleration of the rocket increases as time advance

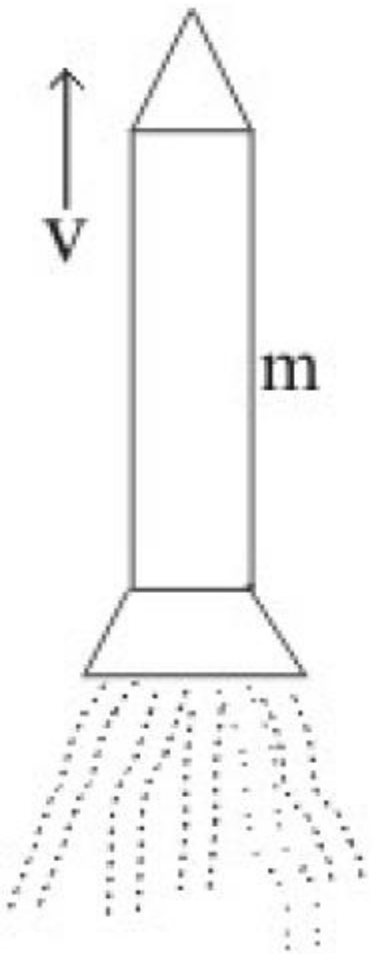


Figure 2a. Rocket at time t after takeoff with mass m and velocity v in upwards direction

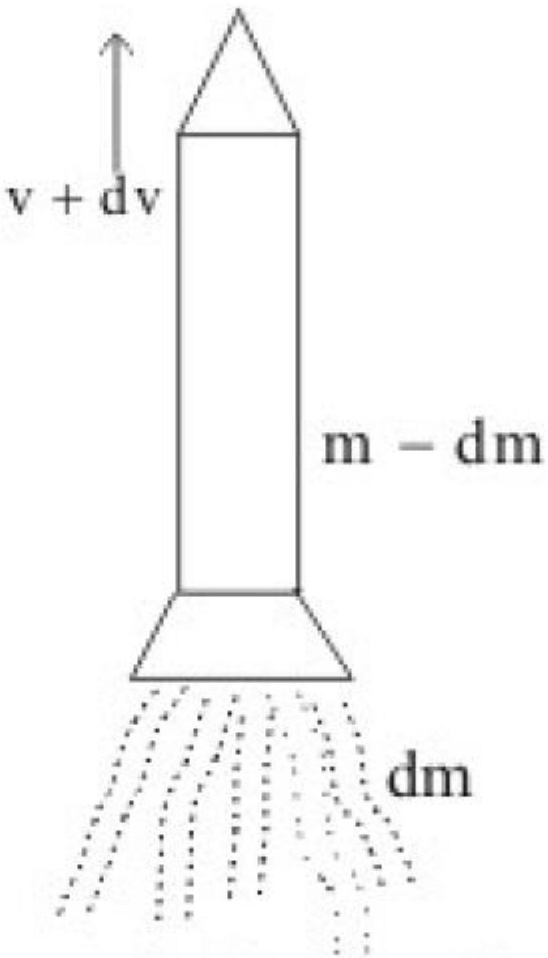


Figure 2b. Rocket at time t+Δt after takeoff with mass m – dm velocity v+dv in upwards direction

Impulse

- A large force acting for a short interval of time is called an impulsive force.

Impulse = Force x time

Unit= Newton second (Ns)

$$I = Ft = ma \times t = m\frac{(v-u)}{t}t = m(v-u) = mv - mu$$

Impulse = change in momentum

- Examples of impulsive forces are
- Kicking a football: A footballer exerts a large force on the ball, but only for a very short interval of time.
- Striking a nail with a hammer: For fixing a nail, a hammer is used to exert a large force for a small time.
- Striking a ball with a bat: The momentary force exerted by the bat changes the direction of the ball in a small interval.