Module IV Area.

To find the equation of a curve if the slope of the curve is known. The area bounded by a curve with the x or y can be obtained by integration.

1. Area enclosed by the curve $y = f(x)$, the x-axis and the ordinates at $x = a$ and $x = b$ is $\int_a^b y\,dx$ or $\int_a^b f(x)\,dx$.

2. Area enclosed by the curve $x = f(y)$ the y-axis and the abscissae at

$y=a$, $y=b$ is $\int_a^b x\,dy$ or

$\int_a^b f(y)\,dy$.

Q. Find the area bounded by the curve $x=y^2-2y$, the y-axis and the abscissae at $y=1$ and $y=2$.

$$A = \int x\,dy$$

$$= \int_1^2 y^2-2y\,dy$$

$$= \int y^2 dy - 2\int y\,dy$$

$$= y^3/3 - 2\cdot y^2/2 \Big|_1^2$$

$$= \left| \frac{y^3}{3} - y^2 \right|_1^2$$

$$= \left(\frac{8}{3}-4\right) - \left(1/3 - 1\right)$$

$$= \left(\frac{8-12}{3}\right) - \left(\frac{1-3}{3}\right)$$

$$= \frac{-4}{3} - \frac{-2}{3} = \frac{-4}{3} + \frac{2}{3}$$

$$= \frac{-2}{3}$$

classmate
Date
Page

A = 2/3 Sq.units.

Find the area bounded by the curve $y = x^2 + x$ and the x-axis

The equation to the x-axis

$y = 0$.

$y = x^2 + x = 0$.

$x(x+1) = 0$

$x = 0, x + 1 = 0$.

$x = 0, x = -1$

Area = $\int_{-1}^{0} y\,dx$

$= \int_{-1}^{0} (x^2 + x)\,dx$

$= \cancel{\int} \int x^2 dx + \int x\,dx$

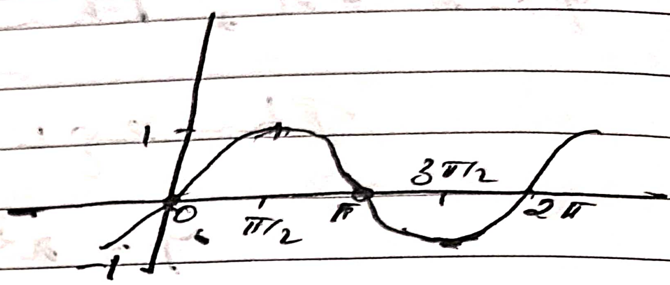
$= \left(x^3/3 + x^2/2\right)_{-1}^{0}$

$= (0 + 0) - (-1/3 + 1/2)$

$= -\left(\frac{-2+3}{6}\right) = -1/6$

A = 1/6 Sq.units

Q. Find the area included between one arch of the curve $y = \sin x$ and the x-axis.



$$\text{Area} = \int y \, dx$$

$$= \int_0^{\pi} \sin x \, dx$$

$$= (-\cos x)_0^{\pi} = -(\cos \pi - \cos 0)$$

$$= -(-1-1) = 2 \text{ Sq. units.}$$

Q. Find the area bounded by the curve $x = 4 - y^2$ about the y-axis.

The Equation of y-axis is $x = 0$.

$$\therefore x = 4 - y^2$$

$$\therefore 4 - y^2 = 0$$

$$4 = y^2 \quad \therefore y = \pm 2$$

$$\text{Area} = \int_{-2}^{2} (4 - y^2) \, dy$$

$$= \left(4y - y^3/3\right)_{-2}^{2}$$

$= (4\times 2 - 8/3) - (4\times -2 - {}^{-}8/3)$

$= (8 - 8/3) - (-8 + 8/3)$

$= \left(\frac{24-8}{3}\right) - \left(\frac{-24+8}{3}\right)$

$= \frac{48+16}{3} = 32/3$ sq. units

Q. Find the area enclosed by the curve $y = x + \sin x$ with the x-axis and the ordinates at $x=0$ and $x=\pi/2$

Area $= \int y \, dx$

$= \int_0^{\pi/2} (x + \sin x) \, dx$

$= \left(x^2/2 - \cos x\right)_0^{\pi/2}$

$= \left(\frac{\pi^2}{8} - \cos \pi/2\right) - (0 - \cos 0)$

$= \left(\pi^2/8 - 0\right) - (0 - 1)$

$= \frac{\pi^2}{8} + 1$

Q. Find the area bounded by the curve $y = 2x^2+1$ with the y-axis and between the lines $y=1$ and $y=9$.

$$y=1 \implies 2x^2 = 0$$
$$\implies x = 0$$
$$y=9 \implies 2x^2 = 8$$
$$x^2 = 4$$

Area $= \int x \, dy$.

$$y = 2x^2+1 \implies 2x^2 = y-1$$
$$x^2 = \frac{y-1}{2}$$
$$x = \sqrt{\frac{y-1}{2}}$$

Area $= \frac{1}{\sqrt{2}} \int_1^9 \sqrt{y-1} \, dy$.

$y-1 = u$

$dy = du$

$$= \frac{1}{\sqrt{2}} \int_1^9 \sqrt{u} \, du = \frac{1}{\sqrt{2}} \int_1^9 u^{1/2} du$$

$$= \frac{1}{\sqrt{2}} \left(\frac{u^{3/2}}{3/2} \right)$$

$$= \frac{2}{3\sqrt{2}} \left((y-1)^{3/2} \right)$$

$$= \frac{2}{3\sqrt{2}} \left((8)^{3/2} \right)$$

$$= \frac{2}{3\sqrt{2}} \times 8^{3/2}$$