(3) If α is an acute angle and $\sin\alpha = 0.4$, find the value of $\sec\alpha + \tan\alpha$.

$\sin\alpha = 0.4 = 2/5$

$\sin^2\alpha + \cos^2\alpha = 1$

$\cos^2\alpha = 1 - \sin^2\alpha$

$= 1 - (\sin\alpha)^2$

$= 1 - (2/5)^2 = 1 - \frac{4}{25}$

$= \frac{25-4}{25} = \frac{21}{25}$

$\cos\alpha = \sqrt{21/25} = \frac{\sqrt{21}}{5}$

$\sec\alpha = \frac{1}{\cos\alpha} = \frac{5}{\sqrt{21}}$ //.

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{2/5}{\sqrt{21}/5} = \frac{2}{5} \times \frac{5}{\sqrt{21}}$

$= \frac{2}{\sqrt{21}}$ //.

$\sec\alpha + \tan\alpha = \frac{5}{\sqrt{21}} + \frac{2}{\sqrt{21}} = \frac{7}{\sqrt{21}}$

(4) If α is an acute angle and $\cos\alpha = 1/2$, find the value of $3\sin\alpha - 4\tan\alpha$.

$\sin^2\alpha + \cos^2\alpha = 1$

$\sin^2\alpha = 1 - \cos^2\alpha = 1 - (\cos\alpha)^2$

$\sin^2\theta = 1 - \frac{1}{4} = 3/4$

$\sin\theta = \sqrt{3/4} = \sqrt{3}/2$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{3}/2}{1/2} = \frac{\sqrt{3}}{2} \times \frac{2}{1}$

$= \sqrt{3}$ //

$\therefore 3\sin\theta - 4\tan\theta = 3 \times \frac{\sqrt{3}}{2} - 4 \times \sqrt{3}$

$= \frac{3\sqrt{3} - 8\sqrt{3}}{2} = \frac{5\sqrt{3}}{2}$

Prove the following identities.

(1) $\cos A \tan A = \sin A$

$L.H.S = \cos A \tan A$

$= \cos A \frac{\sin A}{\cos A}$

$= \sin A = R.H.S.$

$L.H.S = R.H.S.$

(2) $\sin A \tan A \operatorname{cosec}^2 A = \sec A$

$L.H.S = \sin A \tan A \operatorname{cosec}^2 A$

$= \sin A \frac{\sin A}{\cos A} \frac{1}{\sin^2 A} = \frac{1}{\cos A}$

Date ______
Page ______

$= \sec A$

$= R \cdot H \cdot S$

$L \cdot H \cdot S = R \cdot H \cdot S$

(3) $\cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha$.

$L \cdot H \cdot S = \cos^2\alpha - \sin^2\alpha$

$= (1 - \sin^2\alpha) - \sin^2\alpha$.

$= 1 - \sin^2\alpha - \sin^2\alpha$.

$= 1 - 2\sin^2\alpha$

(4) $\tan x + \cot x = \sec x \operatorname{cosec} x$.

$L \cdot H \cdot S = \tan x + \cot x$

$= \dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}$

$= \dfrac{\sin^2 x + \cos^2 x}{\cos x \sin x} = \dfrac{1}{\cos x \sin x}$

$= \dfrac{1}{\cos x} \cdot \dfrac{1}{\sin x} = \sec x \cdot \operatorname{cosec} x$

$= R \cdot H \cdot S$

(5) $\sec^2\alpha + \operatorname{cosec}^2\alpha = \sec^2\alpha \operatorname{cosec}^2\alpha$.

$L \cdot H \cdot S = \sec^2\alpha + \operatorname{cosec}^2\alpha$.

$= \dfrac{1}{\cos^2\alpha} + \dfrac{1}{\sin^2\alpha}$.

$$= \frac{\sin^2\alpha + \cos^2\alpha}{\sin^2\alpha \cos^2\alpha}$$

$$= \frac{1}{\sin^2\alpha \cos^2\alpha} = \frac{1}{\cos^2\alpha} \cdot \frac{1}{\sin^2\alpha}$$

$$= \sec^2\alpha \operatorname{cosec}^2\alpha.$$

Q. Prove that $\frac{1+\cos A}{\sin A} = \frac{\sin A}{1-\cos A}$.

$$L.H.S = \frac{1+\cos A}{\sin A}$$

Numerator and denominator multiplied by $1-\cos A$.

$$= \frac{(1+\cos A)(1-\cos A)}{\sin A(1-\cos A)}$$

$$= \frac{1-\cos^2 A}{\sin A(1-\cos A)}.$$

$$= \frac{\sin^2 A}{\sin A(1-\cos A)} \quad (1-\cos^2 A = \sin^2 A)$$

$$= \frac{\sin A}{1-\cos A}$$

$$= R.H.S$$