Complex Numbers.

Any number of the form $a+ib$, where a and b are real numbers is called a complex number.

Example

(1) $2+3i$ (2) $-1+2i$ (3) $-\sqrt{2}+i$

Real part and imaginary part of complex numbers

Let $z=a+ib$. Here a is called the real part of the complex number and b is called the imaginary part of the complex number.

Example

1. $z=3+5i$

3 is the real part

5 is the imaginary part.

2. $z=\sqrt{2}-3i$

$\sqrt{2}$ is the real part

-3 is the imaginary part.

Conjugate of a complex number

Let $z=a+ib$. The conjugate of the complex number z is given by

$\bar{z}=a-ib$

Find the conjugate

(1) $5+3i$

Conjugate is $5-3i$

2. $\sqrt{2}-2i$

Conjugate is $\sqrt{2}+2i$

3. $-\sqrt{3}-2i$

Conjugate is $-\sqrt{3}+2i$

4. $2-\sqrt{2}i$

Conjugate is $2+\sqrt{2}i$

Cartesian representation of a complex number.

The plane having complex number assigned to each of its ~~pt~~ points is called Argand plane.

In the xy-plane, x-axis is called the real part of complex number. That is real axis. And the y-axis which represent the imaginary part of complex number is called imaginary axis.

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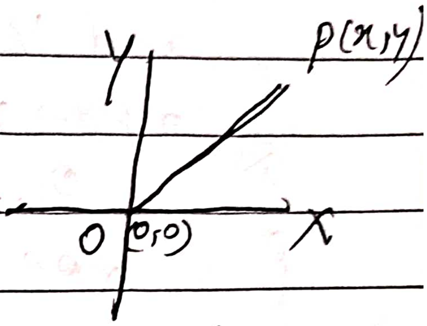
modulus of a complex number.

The distance between the point P(x,y) from the origin O (0,0) is called modulus of a complex number and is denoted by |z|



$$|z| = \sqrt{(x-0)^2+(y-0)^2}$$
$$= \sqrt{x^2+y^2}.$$

Q. Find the modulus of $z = 3+2i$

$$|z| = \sqrt{(3)^2+(2)^2}$$
$$= \sqrt{9+4} = \sqrt{13}.$$

2. $z = -3-2i$

$$|z| = \sqrt{(-3)^2+(-2)^2}$$
$$= \sqrt{9+4} = \sqrt{13}.$$

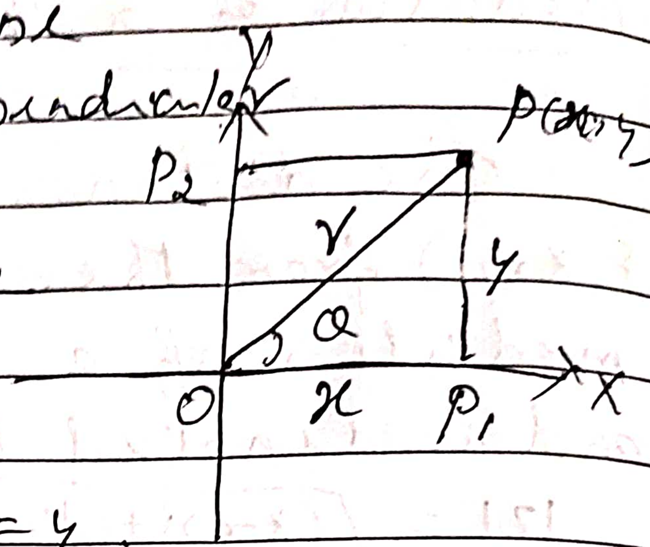
3. $z = 1+i$

$$|z| = \sqrt{1^2+1^2} = \sqrt{2}.$$

Polar representation of a complex number or modulus amplitude form

Consider a complex number $z = x+iy$. represent the point (x,y)

on the XY-plane
draw two perpendicular lines from the point P to the respective x and Y axis.



$OP_1 = x$ and $P_1P = y$.

Pythagorean relation.

$$OP^2 = OP_1^2 + PP_1^2$$

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2+y^2} = |z|$$

is the modulus of complex number z

$$\sin\alpha = y/r$$

$$y = r\sin\alpha \quad -(1)$$

$$\cos\alpha = x/r$$

$$x = r\cos\alpha \quad -(2)$$

(1)/(2). $\frac{y}{x} = \frac{r\sin\alpha}{r\cos\alpha}$

$\tan\alpha = y/x$

$z = x + iy$ becomes

$= r\cos\alpha + i\,r\sin\alpha.$

is called polar form of a complex number. where r is the modulus of the complex number and α is amplitude

∴ This form is also called modulus amplitude form of a complex number

Q. Represent the following complex numbers in polar form

$z = \sqrt{3} + i$, $x = \sqrt{3}$, $y = 1$

polar form of a complex number

$z = r\cos\alpha + i\, r\sin\alpha$ comparing with $r\cos\alpha = \sqrt{3}$. $z = \sqrt{3} + i$

$\therefore r\cos\alpha = \sqrt{3}$, $r\sin\alpha = 1$

— (1) — (2).

$(2)/(1) \Rightarrow \dfrac{r\sin\alpha}{r\cos\alpha} = \dfrac{1}{\sqrt{3}}$.

$\tan\alpha = 1/\sqrt{3}$.

$\alpha = 30^\circ$.

$= 30 \times \pi/180 = \pi/6^c$

or

$\tan\alpha = y/x$

$= 1/\sqrt{3}$

$\alpha = 30^\circ$

$(1)^2 + (2)^2 \Rightarrow r^2\cos^2\alpha + r^2\sin^2\alpha = 3 + 1$

$r^2(\sin^2\alpha + \cos^2\alpha) = 4$

$r^2 = 4$

$r = \pm 2$ // only take

$r = 2$ //.

or

$r = \sqrt{x^2 + y^2}$

$= \sqrt{3+1} = \sqrt{4} = 2$

$\therefore z = 2\cos\pi/6 + i\cdot 2\sin\pi/6$

$= 2(\cos\pi/6 + i\sin\pi/6)$

(2) $z = 1+i$, $x = 1, y = 1$

Comparing with polar form of a complex number, we get

$r\cos\theta = 1$, — (1) $\quad r\sin\theta = 1$ — (2)

$(2)/(1) \Rightarrow \dfrac{r\sin\theta}{r\cos\theta} = 1$

or

$\tan\theta = y/x = 1$ $\quad \tan\theta = 1$

$\theta = 45^\circ$ $\quad \theta = 45^\circ = \pi/4^c$

$(1)^2 + (2)^2 \Rightarrow r^2\cos^2\theta + r^2\sin^2\theta = 2$

$r^2(\sin^2\theta + \cos^2\theta) = 2$

or $\quad r^2 = 2$

$r = \sqrt{x^2+y^2} = \sqrt{2}$ $\quad r = \pm\sqrt{2} = \sqrt{2}$

$\therefore z = r\cos\theta + i r\sin\theta$

$= r(\cos\theta + i\sin\theta)$

$= \sqrt{2}(\cos \pi/4^c + i\sin \pi/4^c)$

Equality of two complex number:

Two complex numbers are equal if only if their real parts and imaginary parts are equal.

$z_1 = a+ib, \quad z_2 = c+id$.
$z_1 = z_2 \Longrightarrow a = c$ and
$b = d$.

Find the value of x and y if two complex numbers are equal.

1. $z_1 = x+3i$ and $z_2 = 3+3i$
$z_1 = z_2$.
$x+3i = 3+3i$
$x = 3$ //
if $z_1 = z_2$.

2. $z_1 = 5-2i$, $z_2 = 5-yi$
$z_1 = z_2$.
$5-2i = 5-yi$
$y = 2$ //

Addition of complex numbers.

$z_1 = a+ib, \quad z_2 = c+id$.
$z_1 + z_2 = a+ib + c+id$
$= \underline{\underline{a+c}} + i\underline{\underline{(b+d)}}$.

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Add the following :

1. $2+3i$, $1-4i$ and $-2+i$

$$(2+3i)+(1-4i)+(-2+i)$$
$$(2+1-2)+i(3-4+1)$$
$$1+0i = \underline{\underline{1}}$$

2. $2-3i$, $-3+5i$, $4+6i$

$$(2-3i)+(-3+5i)+(4+6i)$$
$$(2-3+4)+i(-3+5+6)$$
$$\underline{\underline{3+8i}}$$

Substraction of complex numbers

$z_1 = a+bi$ and $z_2 = c+id$.

$$z_1 - z_2 = (a+bi)-(c+id)$$
$$= (a-c)+i(b-d)$$

Substract the following.

1. $(5-6i)$ from $3-5i$

$$(3-5i)-(5-6i)$$
$$= (3-5)+i(-5+6)$$
$$= \underline{\underline{-2+i}}$$

2. $8-2i$ from $7-3i$

$(7-3i)-(8-2i)$

$= 7-3i-8+2i$

$= \underline{\underline{-1-i}}$.

multiplication of two complex numbers: or product of complex nos

$z_1 = a+ib$, $z_2 = c+id$.

$z_1 z_2 = (a+ib)(c+id)$

$= ac+iad+ibc+i^2bd$

$= ac+iad+ibc-bd$ $\quad \underline{\underline{i^2=-1}}$

$= \underline{\underline{(ac-bd)+i(ad+bc)}}$.

1. $(3+3i)$ and $(5+2i)$.

$(3+3i)(5+2i)$

$= 15+6i+15i+6i^2$

$= 15+6i+15i-6$

$= \underline{\underline{9+21i}}$.

2. $(5+2i)$ and $(1+2i)$.

$(5+2i)(1+2i)$

$= 5+10i+2i+4i^2$

$= 5+10i+2i-4 = \underline{\underline{1+12i}}$.

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Workout problems.

1. Find the modulus and amplitude of the following and express them in polar form.
   a. $5+12i$ b. $-3-3i$
2. Find the conjugate of the following complex numbers
   a. $2+4i$ b. $-i$
3. If $a+7i = 5+bi$, find a and b.
4. Add the following.
   a. $2+3i, 1+i$ b. $1+i, 1-i$
5. Substract
   a. $8+i$ from $10+2i$
   b. i from $2-i$
6. multiply
   a. $5+6i, 3+2i$
   b. $i-1, i+1$
   c. $i, -i$

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Problems.

Q. The x-intercept of a line is three times its y-intercept. The line passes through (-6,3). Find its equation

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x-intercept of a line is three times its y-intercept.

$a = 3b$.

$\therefore \frac{x}{a} + \frac{y}{b} = 1$ is the intercept form —(1)

$\frac{x}{3b} + \frac{y}{b} = 1.$

This line passes through (-6,3)

$(x,y) = (-6,3)$

$\frac{-6}{3b} + \frac{3}{b} = 1.$

$\frac{-6b+9b}{3b^2} = 1$

$\frac{3b}{3b^2} = 1 \Rightarrow 3b = 3b^2.$

$3b = 3$

$b = 3/3 = 1$

$\therefore a = 3b = 3 \times 1 = 3$

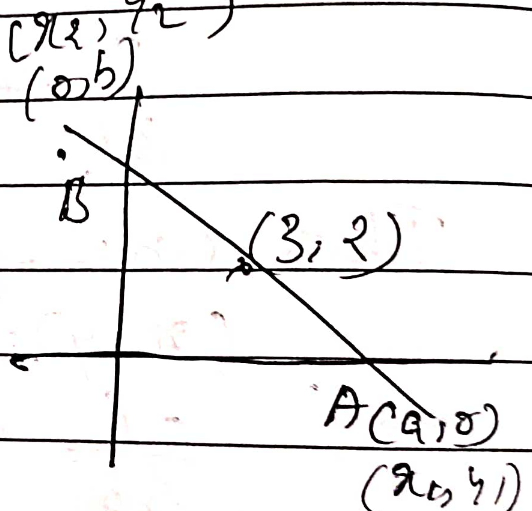
$\therefore (1) \Rightarrow \frac{x}{3} + \frac{y}{1} = 1$

$$\frac{x+3y}{3}=1$$

$$x+3y=3$$

$$x+3y-3=0$$

Q. If a straight line cuts the co-ordinates axis at A and B and if (3,2) is the midpoint of $\overline{AB}$, find the equation of $\overline{AB}$.



Let a be the x-intercept and b be the y-intercept
∴ The co-ordinate of A are (a,0) and B are (0,b). If (3,2) is the midpoint of $\overline{AB}$. So using section formula.

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$(3,2)=\left(\frac{a+0}{2},\frac{0+b}{2}\right)$$

$$(3,2)=\left(a/2,\ b/2\right)$$

$$a/2=3,\quad b/2=2$$

$\therefore a = 6, b = 4.$

$\therefore$ Intercept form $\frac{x}{a} + \frac{y}{b} = 1$

$$\Rightarrow \frac{x}{6} + \frac{y}{4} = 1.$$

$$\frac{4x + 6y}{24} = 1$$

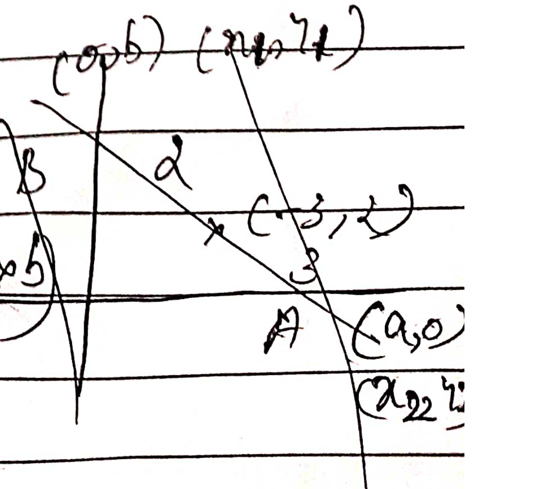
$$4x + 6y = 24.$$

$$4x + 6y - 24 = 0.$$

$$2x + 3y - 12 = 0$$

2. Find the equation of the straight line which passes through the point $(-3, 2)$ and is such that the portion between the x-axis and y-axis is divided internally at that point in the ratio $2:3$.

Using section formula



$$(-3, 2) = \left(\frac{2 \times a + 3 \times 0}{2+3}, \frac{2 \times 0 + 3 \times b}{2+3}\right)$$

$$(-3, 2) = \left(\frac{2a}{5}, \frac{3b}{5}\right)$$

$\therefore a = 6, \ b = 4.$

$\therefore$ Intercept form $\frac{x}{a} + \frac{y}{b} = 1$

$$\Rightarrow \frac{x}{6} + \frac{y}{4} = 1.$$

$$\frac{4x + 6y}{24} = 1$$

$$4x + 6y = 24.$$

$$4x + 6y - 24 = 0.$$

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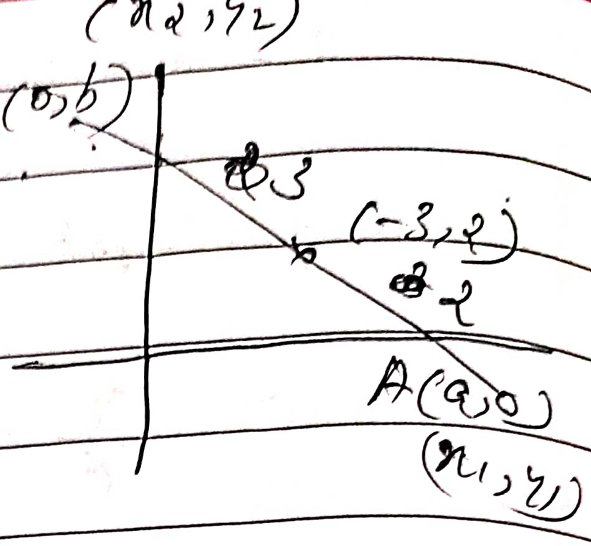

using Section formula.

$$(-3,2) = \left(\frac{2\times 0 + 3\times a}{2+3}, \frac{2\times b + 3\times 0}{2+3} \right)$$

$$(-3,2) = \left(3a/5, 2b/5 \right)$$

$3a/5 = -3$ $\quad$ $2b/5 = 2$.

$3a = -15$ $\quad$ $2b = 10$

$\underline{\underline{a = -5}}$ $\quad$ $\underline{\underline{b = 5}}$

∴ The equation of AB is

$$\frac{x}{-5} + \frac{y}{5} = 1$$

$$\frac{5x - 5y}{-25} = 1$$

$$5x - 5y = -25$$

$$5x - 5y + 25 = 0.$$

$$\underline{\underline{x - y + 5 = 0}}$$

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Q. Find the slope and intercept of the eqn (1) $2x - 3y + 5 = 0$

$$-3y = -2x - 5$$

$$y = -\frac{2}{-3}x - \frac{5}{-3}$$

$y = \frac{2}{3}x + \frac{5}{3}$ to compare the slope-intercept form $y = mx + c$

$m = 2/3$ //.

$$2x - 3y + 5 = 0$$

$$2x - 3y = -5$$

$$\frac{2x}{-5} - \frac{3y}{-5} = 1$$

$\frac{x}{(-5/2)} + \frac{y}{(5/3)} = 1$ to compare the intercept form $x/a + y/b = 1$.

x-intercept $= -5/2$.

y-intercept $= +5/3$

(2) $7x - 3y + 42 = 0$

$$-3y = -7x - 42$$

$$y = -\frac{7}{-3}x - \frac{42}{-3}$$

$$y = \frac{7}{3}x - 14$$

$$m = 7/3$$

$$7x - 3y = -42$$

$$\frac{7x}{-42} - \frac{3y}{-42} = 1$$

$$\frac{x}{-42/7} + \frac{y}{42/3} = 1$$

x-intercept $a = -42/7$

y-intercept $b = 42/3$

2. Find the equation of the line passing through $(1,-2)$ and $(-2,1)$ and also find intercepts on the axis.

using two point form

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Given $(x_1, y_1) = (1, -2)$

$(x_2, y_2) = (-2, 1)$

$$\therefore \quad y + 2 = \frac{1+2}{-2-1}(x-1)$$

$$y + 2 = \frac{\cancel{3}}{-\cancel{3}}(x-1)$$

$$y + 2 = -1(x-1).$$

$$y + 2 = -x + 1 \implies y + 2 + x - 1 = 0$$

$x + y + 1 = 0$ is the required

equation :

To find intercept on the axis.

Then we get the equation

$x+y+1=0$

$x+y=-1$

$\frac{x}{-1}+\frac{y}{-1}=1$

$\therefore$ x-intercept $a=-1$

y-intercept $b=-1$

Q. The straight line through $(4,3)$ makes intercepts of $4a$ and $3a$ on the x-axis and y-axis respectively. Find a.

x-intercept $4a$

y-intercept $3a$.

The intercept form is $\frac{x}{a}+\frac{y}{b}=1$

$\frac{x}{4a}+\frac{y}{3a}=1$.

If the line passes through

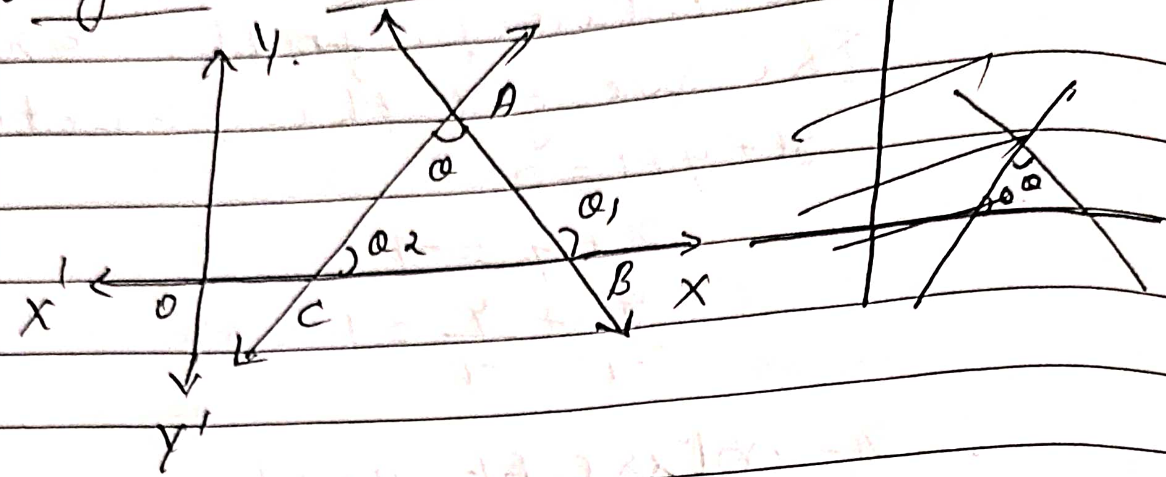
$(4,3) \Rightarrow \frac{\cancel{4}}{4a}+\frac{\cancel{3}}{3a}=1$.

$\cancel{\frac{12a+12a}{12a^2}} \Rightarrow \frac{1}{a}+\frac{1}{a}=1$

$\frac{a+a}{a^2}=1 \Rightarrow \frac{2a}{a^2}=1$

$2a=a^2 \Rightarrow a=2$

Angle between two lines.



The straight line $\overleftrightarrow{AB}$ makes an angle of inclination θ_1 with the x-axis and straight line $\overleftrightarrow{AC}$ makes an angle θ_2 with the x-axis.

Slope of $\overleftrightarrow{AB}$

$m_1 = \tan\theta_1$

Slope of $\overleftrightarrow{AC}$

$m_2 = \tan\theta_2$

From the figure. $\theta + \theta_2 = \theta_1$

$\theta = \theta_1 - \theta_2$.

Taking tangent function on both sides, we get $\tan\theta = \tan(\theta_1 - \theta_2)$

$$\tan\theta = \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1 \tan\theta_2}.$$

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}.$$

$$\therefore \theta = \tan^{-1}\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)$$

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Q. Find the angle between two lines with slopes $\sqrt{3}$ and $1/\sqrt{3}$.

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}.$$

Given $m_1 = \sqrt{3}$, $m_2 = 1/\sqrt{3}$.

$$\tan\theta = \frac{\sqrt{3} - 1/\sqrt{3}}{1 + \sqrt{3} \times 1/\sqrt{3}} = \frac{\frac{3-1}{\sqrt{3}}}{1+1}$$

$$\tan\theta = \frac{2/\sqrt{3}}{2} = \frac{2}{\sqrt{3} \times 2} = 1/\sqrt{3}$$

$$\theta = 30^{\circ}$$

The condition for parallelism and perpendicularity of two straight lines.

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} \quad \text{--- (1)}$$

1. If two lines are parallel, angle between them, $\theta = 0$. Then $\tan\theta = \tan 0 = 0$.

$$(1) \Longrightarrow \frac{m_1 - m_2}{1 + m_1 m_2} = 0.$$

$$m_1 - m_2 = 0(1 + m_1 m_2) = 0.$$

$$m_1 = m_2$$

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Two lines are parallel, the slopes are equal

2. If two lines are perpendicular

$\theta = 90°$

$\tan\theta = \tan 90° = \infty = \frac{1}{0}$.

$(1) \Rightarrow \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{1}{0}$

$(m_1 - m_2) \times 0 = 1 + m_1 m_2$

$0 = 1 + m_1 m_2$

or $1 + m_1 m_2 = 0$

$m_1 m_2 = -1$

Two lines are perpendicular, the product of the slopes is -1

Note (1) Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel.

By the statement $m_1 = m_2$ — (1)

The slope of $a_1x + b_1y + c_1 = 0$

$b_1 y = -a_1 x - c_1$

$y = -a_1/b_1\, x - c_1/b_1$

$\therefore m_1 = -a_1/b_1$

The slope of $a_2x + b_2y + c_2 = 0$

$\Rightarrow$ ~~by~~ $m_2 = -a_2/b_2$

$(1) \Rightarrow \frac{+a_1}{b_1} = +a_2/b_2$

or $a_1/a_2 = b_1/b_2$

Two lines are perpendicular. By the statement $m_1 m_2 = -1$

$$\therefore -\frac{a_1}{b_1} \times -\frac{a_2}{b_2} = -1$$

$$\frac{a_1 a_2}{b_1 b_2} = -1$$

$$\therefore a_1 a_2 = -b_1 b_2$$

$$\Rightarrow a_1 a_2 + b_1 b_2 = 0$$

Note (1) Equation of a line parallel to $ax+by+c=0$ is $ax+by+k=0$. where k is constant.

(2) Equation of a line ~~perpend~~ perpendicular to $ax+by+c=0$ is $\rightarrow bx-ay+k=0$ where k is constant.

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1. Find the equation to the straight line passing through $(4,5)$ which is parallel to $2x+3y=4$

Any line parallel to $2x+3y-4=0$ has the form $2x+3y+k=0$ —(1). This line passes through $(4,5)$.

$(x,y)=(4,5)$

$\therefore 2\times 4 + 3\times 5 + k = 0.$

$8+15+K=0 \Rightarrow K+23=0$

$\therefore K=-23$

$(1) \Rightarrow 2x+3y-23=0$

Q. Find the equation to a straight line passing through (4,5) and perpendicular to $2x+3y=4$

Any line perpendicular to $2x+3y-4=0$ has the form $3x-2y+K=0$ —(1). This line passes through (4,5)

$(x,y)=(4,5)$.

$(1) \Rightarrow 3x-2y+K=0$

$3\times4-2\times5+K=0$

$12-10+K=0$

$2+K=0 \Rightarrow K=-2$

$(1) \Rightarrow 3x-2y-2=0$ is required equation.

Point of intersection of two lines:

Q. Find the point of intersection of two straight lines $3x-y+5=0$

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and $x+3y-2=0$

$3x-y=-5, \quad x+3y=2.$

$$\Delta = \begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix} = 9+ = 10$$

$$\Delta_1 = \begin{vmatrix} -5 & -1 \\ 2 & 3 \end{vmatrix} = -15+2 = -13$$

$$\Delta_2 = \begin{vmatrix} 3 & -5 \\ 1 & 2 \end{vmatrix} = 6+5 = 11$$

$$x = \frac{\Delta_1}{\Delta} = -13/10 \qquad y = \frac{\Delta_2}{\Delta} = \frac{4}{10}.$$

$(-13/10, 11/10)$ is the point of intersection of ~~two~~ the above two lines.

Concurrent lines.

Consider the straight lines $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$ and $a_3x+b_3y+c_3=0$. ~~intersection of~~

If the ~~two~~ point of intersection of any two lines lies on the third line, we can say that the lines are concurrent

Q. Prove the lines $2x-3y-7=0$, $3x-4y-10=0$ and $8x+11y-5=0$ are concurrent.

Solve any two lines.

$2x-3y=7$, $3x-4y=10$.

$$\Delta = \begin{vmatrix} 2 & -3 \\ 3 & -4 \end{vmatrix} = -8+9 = 1$$

$$\Delta_1 = \begin{vmatrix} 7 & -3 \\ 10 & -4 \end{vmatrix} = -28+30 = 2$$

$$\Delta_2 = \begin{vmatrix} 2 & 7 \\ 3 & 10 \end{vmatrix} = 20-21 = -1$$

$$x = \frac{\Delta_1}{\Delta} = \frac{2}{1} = 2, \quad y = \frac{\Delta_2}{\Delta} = \frac{-1}{1} = -1$$

$(2,-1)$ is the point of intersection of lines. Substitute these point in the third line, we get.

$$8x+11y-5=0.$$

$$(x,y) = (2,-1)$$

$$8\times 2 + 11\times -1 - 5$$

$$= 16-11-5$$

$$= 16-16$$

$$= 0$$

Problems of the above section.

1. For what values of k shall the three lines $5x+2y-4=0$, $2x+ky+11=0$ and $3x-4y-18=0$ are concurrent.

Consider $5x+2y=4$ and $3x-4y=18$

$$\Delta = \begin{vmatrix} 5 & 2 \\ 3 & -4 \end{vmatrix} = -20-6 = -26.$$

$$\Delta_1 = \begin{vmatrix} 4 & 2 \\ 18 & -4 \end{vmatrix} = -16-36$$
$$= -52$$

$$\Delta_2 = \begin{vmatrix} 5 & 4 \\ 3 & 18 \end{vmatrix} = 90-12 = \underline{\underline{78}}$$

$\therefore$ $x = \dfrac{\Delta_1}{\Delta} = \dfrac{-52}{-26} = 2$ //

$y = \dfrac{\Delta_2}{\Delta} = 78/_{-26} = -3$ //

$(2,-3)$ is the point of intersection of two lines. Substitute these points in the equation $2x+ky+11=0$.

$(x, y) = (2, -3)$

$2\times2 + k\times-3 + 11 = 0.$

$4 - 3k + 11 = 0.$

$15 - 3k = 0$, $+3k = +15 \implies k = 15/3 = 5$ //

2. Find the value of p if $(2p+1)x - (5-p)y = 8$ and $(5p-1)x - (p+1)y = 3$ are parallel ______.

Two lines are parallel, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}.$$

$a_1 = 2p+1$, $b_1 = -(5-p)$

$a_2 = 5p-1$, $b_2 = -(p+1)$.

$$\begin{array}{r} 24 \\ \times 7 \\ \hline 168 \end{array} \qquad \begin{array}{r} 23 \\ \times 23 \\ \hline 69 \\ 46 \\ \hline 529 \end{array}$$

$$\frac{2p+1}{5p-1} = \frac{+(5-p)}{+(p+1)}$$

$$(2p+1)(p+1) = (5-p)(5p-1).$$

$$2p^2+2p+p+1 = 25p-5-5p^2+p$$

$$2p^2+3p+1 = -5-5p^2+26p.$$

$$2p^2+3p+1+5+5p^2-26p = 0.$$

$$7p^2 - 23p + 6 = 0.$$

$$p = \frac{-b \pm \sqrt{b^2-4ac}}{2a}, \quad p = \frac{23 \pm \sqrt{529-168}}{14}$$

$$p = \frac{23 \pm \sqrt{361}}{14} = \frac{23 \pm 19}{14}$$

$$= \frac{23+19}{14}, \frac{23-19}{14}$$

$$= \frac{42}{14}, \frac{4}{14}$$

$$= 3, 2/7$$

Q. Find the value of q for which the straight lines $8qx+(2-3q)y+1=0$ and $qx+8y+7=0$ are perpendicular

Two straight lines are perpendicular, the product of two slope is equal to -1

or.

$\therefore a_1a_2+b_1b_2=0.$

$a_1=8q$ $a_2=q$, $b_1=(2-3q)$, $b_2=8.$

$8q\times q+(2-3q)\cdot 8=0.$

$8q^2+16-24q=0.$

$8q^2-24q+16=0.$

$q^2-3q+2=0$. $\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$

$\frac{3\pm\sqrt{9-8}}{2}=\frac{3+1}{2},\frac{3-1}{2}.$

$=2,1$

Q. Find the equation of the lines passing through the point of intersection of the lines $x-y+1=0$ and $2x+3y+2=0.$ and parallel to $x+y-6=0$

The point of intersection of $x-y+1=0$ and $2x+3y+2=0$ is

$x-y=-1, \quad 2x+3y=-2.$

$$\Delta = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3+2 = 5$$

$$\Delta_1 = \begin{vmatrix} -1 & -1 \\ -2 & 3 \end{vmatrix} = -3-2 = -5.$$

$$\Delta_2 = \begin{vmatrix} 1 & -1 \\ 2 & -2 \end{vmatrix} = -2+2 = 0$$

$x = \frac{\Delta_1}{\Delta}, \quad y = \frac{\Delta_2}{\Delta}$

$x = -5/5, \quad y = 0/5$

$x = -1, \quad y = 0$

$(-1, 0)$ is the point of intersection

Any line parallel to $x+y-6=0$ is $x+y+k=0$; ——(1). Since it passes through $(-1, 0) \Longrightarrow x+y+k=0$.

~~substituting~~ $(-1)+0+k=0$

$k=1$

$(1) \Longrightarrow x+y+1=0$ is the required equation.

2. Find the equation of the line through the intersection of the lines $2x+3y=1$, $3x+4y=6$ and perpendicular to $5x-2y=7$.

The point of intersection of $2x+3y=1$ and $3x+4y=6$ is

$$2x+3y=1$$
$$3x+4y=6.$$

$\Delta = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = 8-9 = -1,$ $\quad x = \frac{\Delta_1}{\Delta} = \frac{-14}{-1} = 14$

$\Delta_1 = \begin{vmatrix} 1 & 3 \\ 6 & 4 \end{vmatrix} = 4-18 = -14$ $\quad y = \frac{\Delta_2}{\Delta} = \frac{9}{-1} = -9$

$\Delta_2 = \begin{vmatrix} 2 & 1 \\ 3 & 6 \end{vmatrix} = 12-3 = 9$

$(14,-9)$ is the point of intersection.
Any line perpendicular to $5x-2y=7$ is $-2x-5y+k=0$. Since it passes through $(14,-9)$.

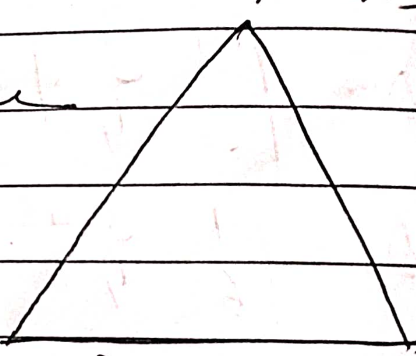
$$-2\times14-5\times-9+k=0.$$
$$-28+45+k=0.$$
$$17+k=0$$
$$k=-17$$
$$-2x-5y-17=0.$$
$$2x+5y+17=0$$

Q. A(2,6), B(4,0), C(8,2) are the vertices of a triangle. $\overline{AD}$ is drawn perpendicular to $\overline{BC}$.

(1) Find the slope of $\overline{BC}$

(2) Hence write down the equation to $\overline{BC}$

3. using the equation $\overline{BC}$, find the equation of $\overline{AD}$



1 - Two pair of points are given, then

Slope $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{2-0}{8-4}$

$= 2/4 = 1/2$//

2 - ~~Slope and (4,0) are given, the Slope - point form~~

2. Two - point form:

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$.

$y - 0 = 1/2(x - 4)$

$2(y - 0) = (x - 4)$

$2y = x - 4$

$\therefore x - 2y - 4 = 0$.

3. Any line perpendicular $-2x - y + k = 0$
This line passes through $(2,6)$,

$$\Rightarrow (x, y) = (2,6)$$

$$-2\times 2 - 6 + k = 0$$

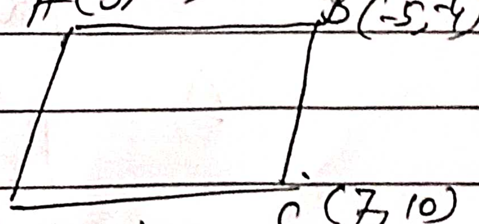
$$-4 - 6 + k = 0 \Rightarrow k = 10.$$

$$\therefore -2x - y + 10 = 0$$

$$\therefore 2x + y - 10 = 0$$

Q. Prove that the points $(3,-5)$, $(-5,-4)$, $(7,10)$ and $(15,9)$ taking in order are the vertices of a parallelogram.

ABCD is a parallelogram if the opposite sides are parallel. In this case to prove that the slope of $\overline{AB}$ and slope of $\overline{CD}$ are equal and slope of $\overline{AD}$ and slope of $\overline{BC}$ are equal.



Slope of $\overline{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

$$\cancel{= \frac{-4+5}{-4+5} = \frac{-8}{1}}$$

$$= \frac{-4+5}{-5-3} = -\frac{1}{8}.$$

Slope of $\overline{DC} = \frac{10-9}{7-15} = -\frac{1}{8}$

$\overline{AB}$ and $\overline{DC}$ are equal.

Slope of $AD = \frac{9+5}{15-3} = \frac{14}{12} = 7/6$.

Slope of $BC = \frac{10-^{-}4}{7+5} = \frac{14}{12} = 7/6$.

$\overline{AD}$ and $\overline{BC}$ are equal.

Therefore ABCD is a parallelogram

Q Show that the three lines $3x+4y=13$, $2x-7y+1=0$ and $5x-y=14$ are concurrent.

Find the point of intersection of the lines $3x+4y=13$ and $2x-7y=-1$

$$\Delta = \begin{vmatrix} 3 & 4 \\ 2 & -7 \end{vmatrix} = -21-8 = -29.$$

$\frac{13}{7} \atop 91$

$$\Delta_1 = \begin{vmatrix} 13 & 4 \\ -1 & -7 \end{vmatrix} = -91+4 = -87$$

$$\Delta_2 = \begin{vmatrix} 3 & 13 \\ 2 & -1 \end{vmatrix} = -3-26 = -29$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-87}{-29} = +3, \quad y = \frac{\Delta_2}{\Delta} = \frac{-29}{-29} = 1$$

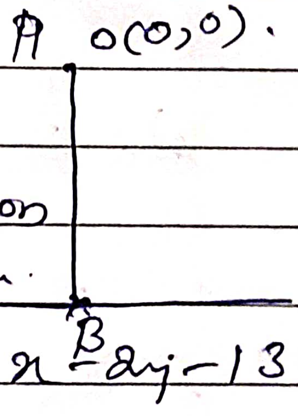
(3,1) is the point of intersection.
Substitute (3,1) in the equation
$5x-y-14=0$ — (1)

$\therefore (x,y)=(3,1)$ in (1).

$5\times3-1-14 = 15-1-14$
$= 14-14$
$= 0$.

$\therefore$ these lines are Concurrent.

Q- Find the foot of the perpendicular from the origin to the line $3x-2y-13=0$.



To find the point B.
B is the point of intersection
Given line is $3x-2y-13=0$.
First we have to find the equation of $\overline{AB}$.
$\overline{AB}$ is perpendicular to the $3x-2y-13=0$. Any line perpendicular to $3x-2y-13=0$ is $-2x-3y+k=0$.
Since it passes through (0,0). — (1)
$(x,y)=(0,0)$.
Substitute (x,y) in (1).
$-2\times0-3\times0+k=0 \Rightarrow \therefore k=0$

∴ The equation is $-2x-3y=0$.

∴ $2x+3y=0$.

Next to find the foot of the point of intersection of these lines.

$3x-2y=13$ and $2x+3y=0$.

$$\Delta = \begin{vmatrix} 3 & -2 \\ 2 & 3 \end{vmatrix} = 9+4 = 13 //$$

$$\Delta_1 = \begin{vmatrix} 13 & -2 \\ 0 & 3 \end{vmatrix} = 39 \cancel{+0} = \underline{\underline{39}}$$

$$\Delta_2 = \begin{vmatrix} 3 & 13 \\ 2 & 0 \end{vmatrix} = 3\times 0 - 26$$
$$= -26 //$$

$$\therefore x = \frac{\Delta_1}{\Delta} = \frac{39}{13} = 3 //$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-26}{13} = -2 //$$

∴ The foot of the perpendicular is $(3, -2)$ or the point of intersection.

or

Slope is ∞ for the y-axis and any line parallel to y-axis.

Q. $(-3,2), (4,2)$.

$(x_1,y_1) = (-3,2)$

$(x_2,y_2) = (4,2)$.

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{2-2}{4-{-3}} = \frac{0}{7} = 0.$$

$\tan\theta = 0.$

The line parallel to x-axis

or

Slope is 0 for the y-axis and any line perpendicular to y-axis.

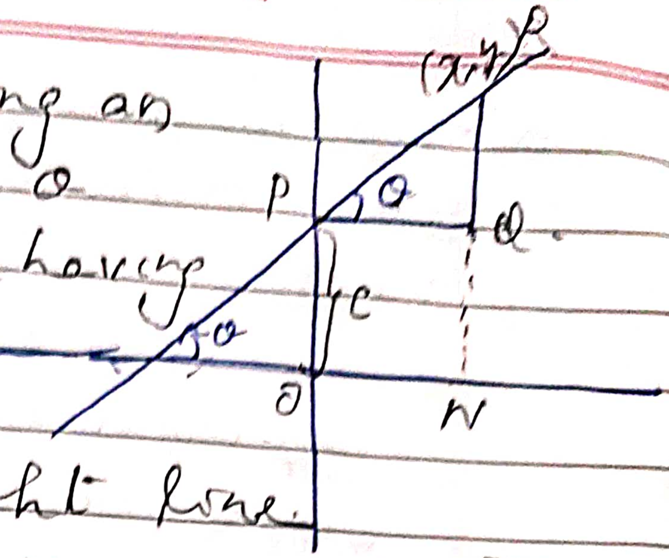
The Slop is also called gradient.

Equations of straight lines.

Case (i) Slope-Intercept form

Given Slope and y-intercept

Straight line having an angle of inclination θ with the x-axis and having y intercept c.



Let (x,y) be any point on the straight line. Draw RN ⊥r to the x-axis and $\overline{PQ}$ ⊥r to RN. $OP = C$.

From the right angle PQR, $\angle P = \theta$.

$$\tan\theta = \frac{RQ}{PQ} = \frac{RN - NQ}{ON} = \frac{RN - OP}{ON}$$

$$= \frac{y - c}{x}$$

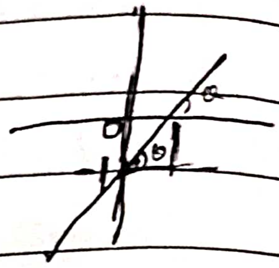
$\tan\theta = m$.

$$m = \frac{y - c}{x}.$$

$$mx = y - c \Rightarrow \underline{\underline{y = mx + c}}$$

Remark. y-intercept is the distance between the intersecting point of straight line and y-axis, from the origin.

Q: Write down the equation of the line with slope $\frac{1}{2}$ and y-intercept -1.



Slope $= m = \frac{1}{2}$

y-intercept, $c = -1$.

Slope-intercept form

$$y = mx + c$$

$$= \tfrac{1}{2}x - 1 = \frac{x}{2} - 1$$

$$y = \frac{x-2}{2}$$

$$2y = x - 2.$$

$$\therefore \cancel{2y+}\ x - 2y - 2 = 0$$

Q: Find the equation of a line with angle of inclination 45° with the x-axis and y-intercept -1.

$\theta = 45^\circ$, $c = -1$

$m = \tan\theta = \tan 45^\circ = 1$

slope-intercept form

$$y = mx + c$$

$$y = 1 \times x - 1$$

$$y = x - 1$$

$$x - y - 1 = 0$$

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Q. Write down the equation of a line which makes an angle 150° with the x-axis and cutting the y-axis at ~~the point (0, 2)~~.

$\theta = 150°$

$m = \tan\theta$

$= \tan 150°$

$= \tan(90 + 60)$

$= -\cot 60° = -1/\sqrt{3}$

y-intercept $c = -2$.

Slope - intercept form

$y = mx + c$

$y = \frac{-1}{\sqrt{3}} x - 2$.

$y = \frac{-x - 2\sqrt{3}}{\sqrt{3}}$, $-x - 2\sqrt{3} = \sqrt{3}y$

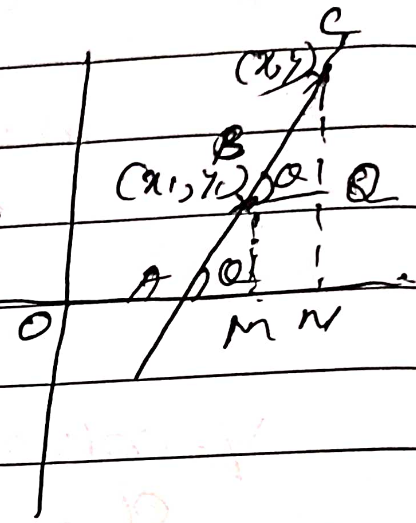
$-x - 2\sqrt{3} - y\sqrt{3} = 0$

$x + \sqrt{3}y + 2\sqrt{3} = 0$

Case II

Slope - point form

Given slope and any one point (x_1, y_1) on the line.



It having an inclination θ with the x-axis and passing through a given point (x_1, y_1). Let (x, y) be any other point on the line.

Draw $\overline{BM}$, $\overline{CN}$ $\perp$r to x-axis and $\overline{BQ}$ $\perp$r to CN.

ΔBQC, $\angle B = \theta$

$$\tan\theta = \frac{CQ}{BQ} = \frac{CN - QN}{MN}$$

$$\tan\theta = \frac{y - BM}{ON - OM} = \frac{y - y_1}{x - x_1}$$

$$\tan\theta = m = \frac{y - y_1}{x - x_1}$$

$$y - y_1 = m(x - x_1)$$

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Q. Write down the equation of the line determined by the slope -2 and passing through $(-2,3)$.

$$m = -2, \quad (x_1, y_1) = (-2, 3)$$

$$m = \frac{y - y_1}{x - x_1}$$

$$-2 = \frac{y-3}{x+2}$$

$$-2(x+2) = y-3$$

$$-2x - 4 = y - 3$$

$$-2x - 4 - y + 3 = 0$$

$$-2x - y - 1 = 0$$

$$2x + y + 1 = 0$$

Q. Write down the equation of the line determined by the inclination to x-axis is 45° and passing through $(2,3)$.

$$\theta = 45^\circ$$

$$m = \tan\theta = \tan 45^\circ = 1$$

$$(x_1, y_1) = (2, 3)$$

$$m = \frac{y - y_1}{x - x_1} \Rightarrow 1 = \frac{y-3}{x-2}$$

$$x - 2 = y - 3 \Rightarrow x - y - 2 + 3 = 0$$

$x - y + 1 = 0$

Q. A straight line is inclined at 135° with the x-axis and it passes through (3, -4). Find the equation.

$\theta = 135°$

$m = \tan\theta = \tan 135°$

$= \tan(90 + 45)$

$= -\cot 45$

$m = -1$

$(x_1, y_1) = (3, -4)$

$m = \dfrac{y - y_1}{x - x_1}$

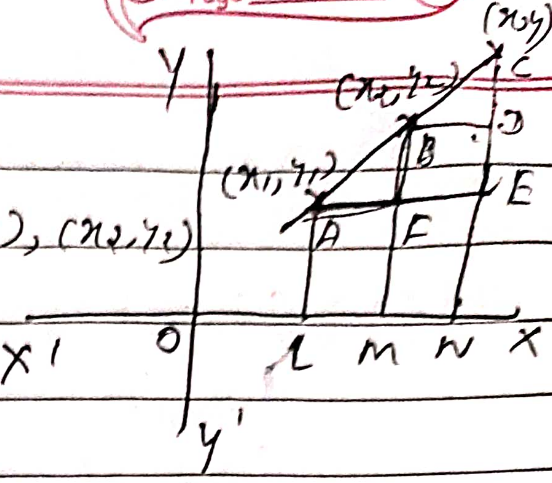
$-1 = \dfrac{y + 4}{x - 3}$

$-(x - 3) = y + 4$

$-x - y + 3 - 4 = 0$

$\therefore -x - y - 1 = 0 \Rightarrow x + y + 1 = 0$

Case III



Two point form

given, Two points are $(x_1,y_1), (x_2,y_2)$

$$\Rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Q. Write down the equation to the lines joining the pairs of points

① $(3,8), (6,12)$.

$(x_1,y_1) \longrightarrow (3,8)$

$(x_2,y_2) \longrightarrow (6,12)$

Two-point form is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

$$y - 8 = \frac{12 - 8}{6 - 3}(x - 3)$$

$$\frac{y-8}{1} = \frac{4}{3}(x - 3).$$

Cross multiplying, we get

$$3(y-8) = 4(x-3)\times 1 = 4(x-3)$$

$$3y - 24 = 4x - 12$$

$$3y - 24 - 4x + 12 = 0.$$

$$-4x + 3y - 12 = 0.$$

$4x - 3y + 12 = 0$ is the required

required equation

(2) $(2,-1), (-6,3)$.

$(x_1, y_1) \longrightarrow (2,-1)$

$(x_2, y_2) \longrightarrow (-6,3)$

The two point form is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y + 1 = \frac{3+1}{-6-2}(x-2)$$

$$y + 1 = \frac{4}{-8}(x-2).$$

$$-8(y+1) = 4(x-2)$$

$$-8y - 8 = 4x - 8.$$

$$-8y - \cancel{8} - 4x + \cancel{8} = 0.$$

$$-4x - 8y = 0.$$

$$4x + 8y = 0$$

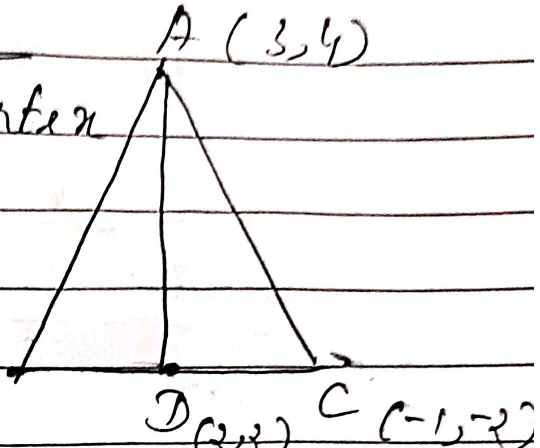
$$4(x + 2y) = 0.$$

$x + 2y = 0.$ is the

required equation.

(3) The vertices of a triangle are $A(3,4)$, $B(5,6)$ and $C(-1,-2)$. Find the equation to the median through A.

medians → the line segment from any vertex of a triangle to the mid-point of the opposite side.



$\vec{AD}$ is the median.

D is the mid-point of $\vec{BC}$. The points of D are $= \left(\frac{-1+5}{2}, \frac{-2+6}{2}\right)$.

$= \left(4/2, 4/2\right) = (2,2)$

The pairs of A and D are $(3,4)$ and $(2,2)$. using two point form

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$(x_1, y_1) \rightarrow (3,4)$, $(x_2, y_2) \rightarrow (2,2)$

$$y - 4 = \frac{2-4}{2-3}(x-3)$$

$$y - 4 = \frac{+2}{+1}(x-3)$$

$y-4 = 2(x-3)$

$y-4 = 2x-6$

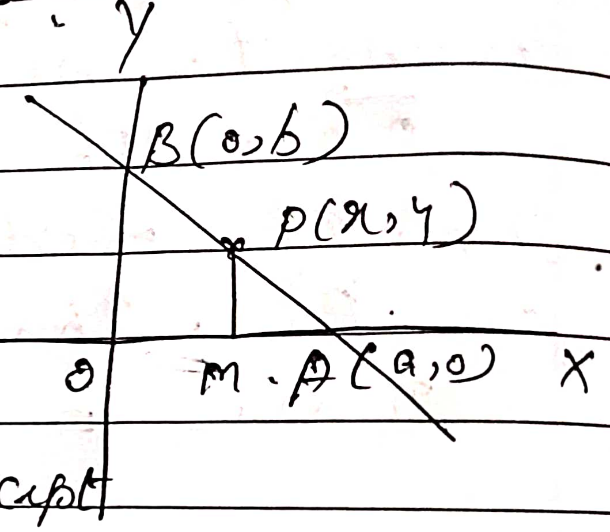
$y-4-2x+6=0$

$-2x+y+2=0$

$\therefore 2x-y-2=0$

Case IV

Intercept form



$\frac{x}{a}+\frac{y}{b}=1$

Here a is called x-intercept and b is called y-intercept

The x-intercept 'a' and y-intercept 'b' are given.

Q. Write down the equation of a line which has x-intercept 3 and y-intercept 5

x-intercept $\rightarrow$ 3, $a=3$

y-intercept $\rightarrow$ 5, $b=5$

Intercept form is

$$\frac{x}{a}+\frac{y}{b}=1.$$

$$\frac{x}{3}+\frac{y}{5}=1.$$

$$\frac{5x+3y}{15}=1$$

$$5x+3y=15.$$

$5x+3y-15=0$ is the required equation.

2. Write down the equation of a line having x-intercept 5 and passing through (3, -2).

Here x-intercept 5 and passing through $(x,y)=(3,-2)$ are given.
y-intercept = unknown.
Intercept form is

$$\frac{x}{a}+\frac{y}{b}=1.$$

$\frac{x}{5}+\frac{y}{b}=1$. But this line ~~to~~ passes through $(3,-2)$.

$\therefore (x,y) \rightarrow (3,-2)$

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$\frac{3}{5} + \frac{-2}{b} = 1$.

$\frac{3b-10}{5b} = 1$

$\Rightarrow 3b - 10 = 5b$.

$3b - 5b = 10$.

$-2b = 10 \Rightarrow b = \frac{10}{-2} = \underline{-5}$

$\therefore a = 5,\ b = -5$ $\therefore$ The required equation, using intercept form

$\frac{x}{a} + \frac{y}{b} = 1$, ~~to find the equation~~

$\Rightarrow \frac{x}{5} + \frac{y}{-5} = 1$

$\frac{-5x + 5y}{-25} = 1$

$\Rightarrow -5x + 5y = 25$

$-5x + 5y + 25 = 0$.

$5x - 5y - 25 = 0$.

$5(x - y - 5) = 0$.

$\Rightarrow (x - y - 5) = 0$

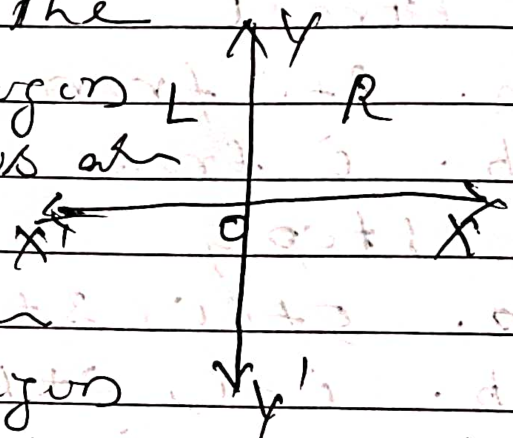
is the required equation.

Co-ordinate Geometry

Co-ordinate System

A rectangular co-ordinate system has two axis, XOX' (the x-axis) and YOY' (the y-axis). The axis are mutually ~~perap~~ perpendicular to each other. The intersecting point of the two axis is called the origin of the co-ordinate system. It is denoted by O.

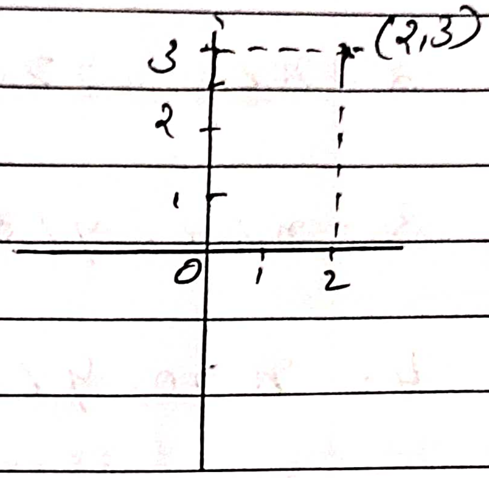
The values in the right of the origin along the x-axis are positive and the values on the left of the origin along the x-axis are negative.



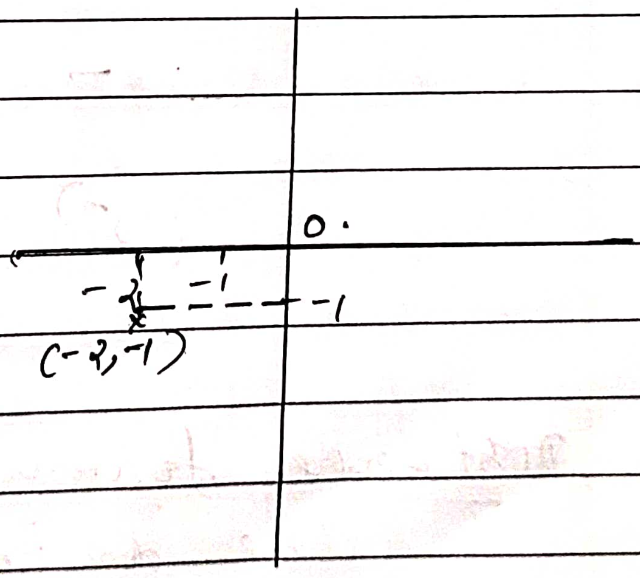
Similarly the values above the original along the y-axis are positive and values below the origin on the y-axis are negative. The x-value is also known as abscissa and y-value is ordinate.

The two value together are called Co-ordinate. At the x-axis y value is zero and the y-axis x-value is zero.

Example(1). Fix (2,3) in a Co-ordinate System.



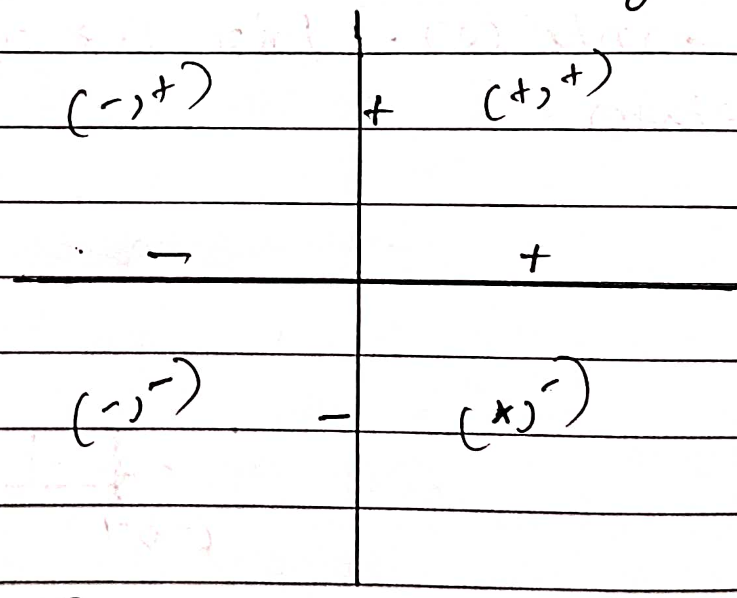
Example (2). Fix (-2,-1) on a Co-ordinate system.



Conclusion

If a point (x, y) lies on a rectangular co-ordinate system, we can see that:

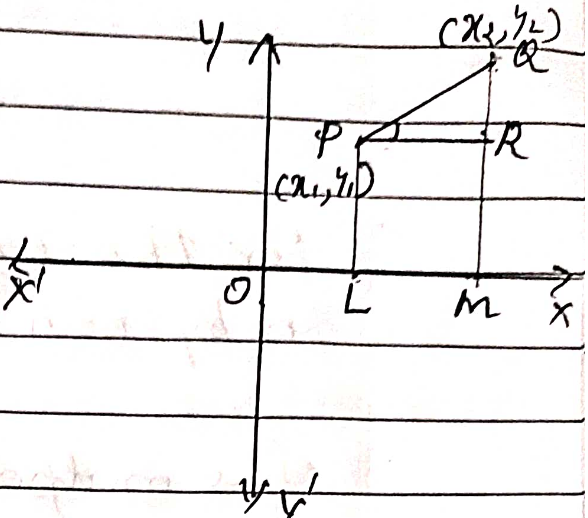
1. $x > 0, y > 0$ means (x, y) lies on 1^{st} quadrant
2. $x < 0, y > 0$ means (x, y) lies on 2^{nd} quadrant
3. $x < 0, y < 0$ means (x, y) lies on 3^{rd} quadrant
4. $x > 0, y < 0$ means (x, y) lies on 4^{th} quadrant



Distance formula

The distance between two points.

we have $OL = x_1$, $LP = y_1$
$OM = x_2$, $MQ = y_2$.
$PR = LM = OM - OL$
$= x_2 - x_1$.
$QR = QM - MR$
$= QM - LP$
$= y_2 - y_1$.



from ΔPQR

$$PQ^2 = PR^2 + QR^2 \quad \text{(pythagorus theorem)}$$
$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

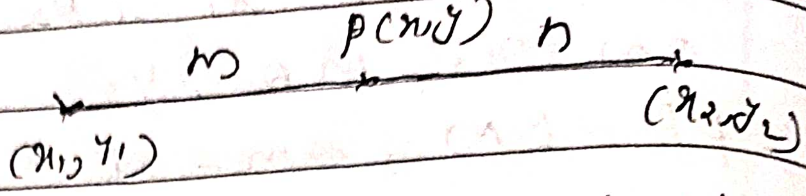
$\therefore$ Distance between two points (x_1, y_1) and (x_2, y_2) is
$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Note : The distance of a point (x, y) from the origin is $\sqrt{x^2 + y^2}$

Section formula

If a point $P(x, y)$ divides the segment joining (x_1, y_1) and (x_2, y_2) in the ratio $m:n$ internally the co-ordinates of P are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

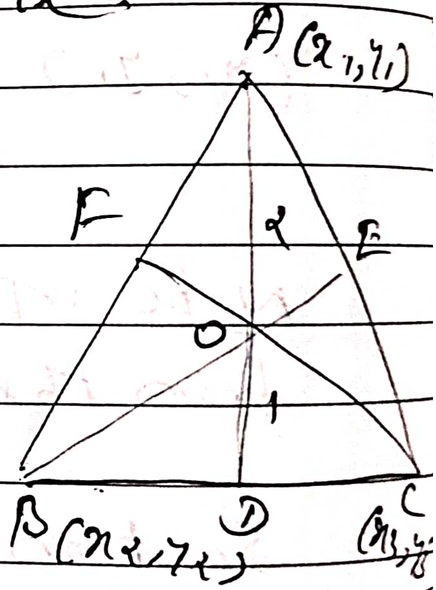


(For the external division, replace n by $(-n)$.

The midpoint of a segment joining (x_1, y_1) and (x_2, y_2) is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Centroid of a triangle.



The centroid of a triangle is the point of intersection of the three medians (median is the segment drawn from a vertex to the midpoint of the opposite side. The centroid divides each median in the ratio 2 : 1

Consider AD. The centroid ~~of~~ O divides AD in the ~~rat~~

ratio 2:1. The co-ordinates of D are $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

Since D is the mid point BC

O is the point which divides the segment $\overline{AD}$ in the ratio 2:1

$\therefore$ The co-ordinates of O

$$\left(\frac{2\cdot\frac{x_2+x_3}{2}+1\cdot x_1}{2+1}, \frac{2\cdot\frac{y_2+y_3}{2}+1\cdot y_1}{2+1}\right)$$

$$=\left(\frac{x_2+x_3+x_1}{3}, \frac{y_2+y_3+y_1}{3}\right)$$

$\therefore \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$ is the centroid of $\triangle ABC$.

Q. The points (-4,5), (2,-3) are at the ends of a diameter of a circle. Find its radius.

using distance formula.

$$S=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$=\sqrt{(2+4)^2+(-3-5)^2}$$

$$=\sqrt{36+64}=\sqrt{100}$$

$$=10$$

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diameter = 10
radius = diameter / 2.
= 10/2 = 5.

Q. Find the co-ordinates of the point which divides the segment joining (2,5) and (4,-2) internally in the ratio 3:4.

Co-ordinates pt
$= \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$

$(x_1, y_1) = (2,5)$, $(x_2, y_2) = (4,-2)$.
$m:n = 3:4$.
Co-ordinate pt
$= \left(\frac{3\times4 + 4\times2}{3+4}, \frac{3\times-2 + 4\times5}{3+4} \right)$
$= \left(\frac{12+8}{7}, \frac{-6+20}{7} \right)$
$= \left(\frac{20}{7}, \frac{14}{7} \right) = \left(\frac{20}{7}, 2 \right)$

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Q. Find the centroid of a triangle having vertices (2, 6), (4, 0) and (8, 2).

Centroid of a triangle

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$= \left(\frac{2+4+8}{3}, \frac{6+0+2}{3}\right)$$

$$= \left(14/3, 8/3\right)$$

Straight lines:

Two mutually ⊥r straight lines $\overleftrightarrow{XOX'}$ and $\overleftrightarrow{YOY'}$ on a coordinate system. The equation of $\overleftrightarrow{XOX'}$ is $y=0$ and the equation of $\overleftrightarrow{YOY'}$ is $x=0$. The equation of any straight line parallel to x-axis is $y=k$ and the equation of any straight line parallel to y-axis is $x=k$.

Slope of a Segment: If a straight line is inclined at an angle α with the x-axis, the slope of that straight

line is given by $\tan\alpha$. It is represented by
$m = \tan\alpha$.

∴ If two points in any straight line are given, the slope of that straight line is given by
$m = \frac{y_2 - y_1}{x_2 - x_1}$, if two points are given

The slope is also called gradient.

Q. Find the slope and the angle of inclination of the pair of points.

$\Rightarrow (x_2, y_2) = (6,5)$

1. $(5,-2), (6,5) \Rightarrow (x_1, y_1) = (5,-2)$

$x_1 = 5, y_1 = -2.$

$x_2 = 6, y_2 = 5$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{5-(-2)}{6-5} = \frac{5+2}{1} = 7$

Slope of tangent

$m = \tan\alpha$

$\tan\alpha = 7$

$\theta = \tan^{-1}(7)$

2. $(-5, 2), (9, 4) \Rightarrow (x_1, y_1) = (-5, 2), (x_2, y_2) = (9, 4)$

$x_1 = -5, x_2 = 9.$

$y_1 = 2, y_2 = 4.$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{9-(5)}$$

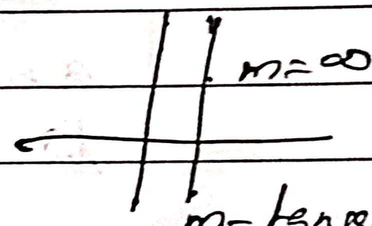
$$= \frac{2}{14} = 1/7.$$

$m = \tan\alpha.$

$\tan\alpha = m = 1/7.$

$\theta = \tan^{-1}(1/7)$

3. $(2, -3), (2, 4).$



$(x_1, y_1) = (2, -3)$

$(x_2, y_2) = (2, 4).$

$x_1 = 2, y_1 = -3, x_2 = 2, y_2 = 4.$

$m = \tan\alpha.$

$= \tan 90^\circ$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - -3}{2-2} = \frac{4+3}{0.}$$

$$= 7/0.$$

$m = \infty.$

$\tan\alpha = m = \infty.$

$\therefore$ The line is perpendicular to x-axis.

or

Slope is ∞ for the Y-axis and any line parallel to Y-axis.

2. $(-3, 2), (4, 2)$.

$(x_1, y_1) = (-3, 2)$

$(x_2, y_2) = (4, 2)$.

$m = 0$.

$\tan\theta = 0$

$m = \tan\theta$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-2}{4-(-3)} = \frac{0}{7} = 0.$$

$\tan\theta = 0$.

The line parallel to x-axis

or

Slope is 0 for the Y-axis and any line perpendicular to Y-axis.