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Complex Numbers.

Any number of the form $a+ib$, where a and b are real numbers is called a complex number.

Example

(1) $2+3i$ (2) $-1+2i$ (3) $-\sqrt{2}+i$

Real part and imaginary part of complex numbers

Let $z=a+ib$. Here a is called the real part of the complex number and b is called the imaginary part of the complex number.

Example

1. $z=3+5i$

3 is the real part

5 is the imaginary part.

2. $z=\sqrt{2}-3i$

$\sqrt{2}$ is the real part

-3 is the imaginary part.

Conjugate of a complex number

Let $z=a+ib$. The conjugate of the complex number z is given by

$\bar{z}=a-ib$

Find the conjugate

(1) $5+3i$
Conjugate is $5-3i$
2. $\sqrt{2}-2i$
Conjugate is $\sqrt{2}+2i$
3. $-\sqrt{3}-2i$
Conjugate is $-\sqrt{3}+2i$
4. $2-\sqrt{2}i$
Conjugate is $2+\sqrt{2}i$

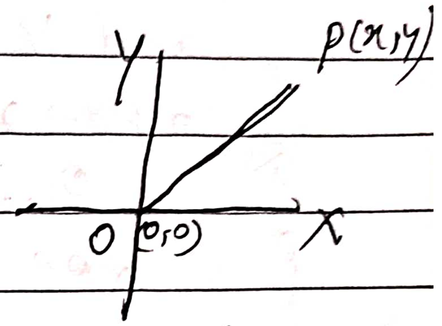
Cartesian representation of a complex number.

The plane having complex number assigned to each of its ~~pt~~ points is called Argand plane.
In the xy-plane, x-axis is called the real part of complex number. That is real axis. And the y-axis which represent the imaginary part of complex number is called imaginary axis.

modulus of a complex number.

The distance between the point P(x, y) from the origin O (0,0) is called modulus of a complex number and is denoted by $|z|$

$$|z| = \sqrt{(x-0)^2+(y-0)^2}$$
$$= \sqrt{x^2+y^2}.$$



Q. Find the modulus of $z = 3+2i$

$$|z| = \sqrt{(3)^2+(2)^2}$$
$$= \sqrt{9+4} = \sqrt{13}.$$

2. $z = -3-2i$

$$|z| = \sqrt{(-3)^2+(-2)^2}$$
$$= \sqrt{9+4} = \sqrt{13}.$$

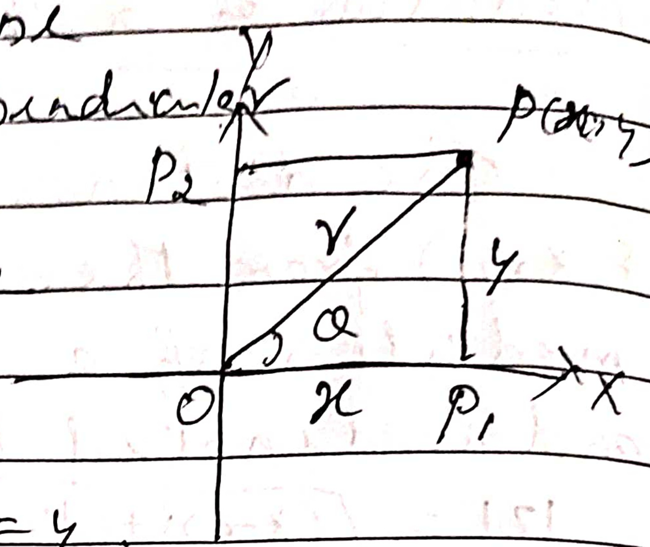
3. $z = 1+i$

$$|z| = \sqrt{1^2+1^2} = \sqrt{2}.$$

Polar representation of a complex number or modulus amplitude form number or modulus of a complex number

Consider a complex number $z = x+iy$. represent the point (x, y)

on the XY-plane
draw two perpendicular lines from the point P to the respective x and Y axis.



$OP_1 = x$ and $P_1P = y$.

Pythagorean relation.

$OP^2 = OP_1^2 + PP_1^2$

$r^2 = x^2 + y^2$

$r = \sqrt{x^2+y^2} = |z|$

is the modulus of complex number z

$\sin\alpha = y/r$

$y = r\sin\alpha$ —(1)

$\cos\alpha = x/r$

$x = r\cos\alpha$ —(2)

(1)/(2).

$y/x = \dfrac{r\sin\alpha}{r\cos\alpha}$

$\tan\alpha = y/x$

$z = x + iy$ becomes

$= r\cos\alpha + i\,r\sin\alpha$.

is called polar form of a complex number. where r is the modulus of the complex number and α is amplitude

∴ This form is also called modulus amplitude form of a complex number

Q. Represent the following complex numbers in polar form

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$z = \sqrt{3} + i$, $x = \sqrt{3}$, $y = 1$

polar form of a complex number

$z = r\cos\alpha + i\, r\sin\alpha$ comparing

with $r\cos\alpha = \sqrt{3}$. $z = \sqrt{3} + i$

$\therefore r\cos\alpha = \sqrt{3}$ — (1), $r\sin\alpha = 1$ — (2).

(2) / (1) $\Rightarrow$ $\dfrac{r\sin\alpha}{r\cos\alpha} = \dfrac{1}{\sqrt{3}}$.

$\tan\alpha = 1/\sqrt{3}$.

$\alpha = 30^\circ$.

$= 30 \times \pi/180 = \pi/6^c$.

or

$\tan\alpha = y/x$

$= 1/\sqrt{3}$

$\alpha = 30^\circ$

$(1)^2 + (2)^2 \Rightarrow r^2\cos^2\alpha + r^2\sin^2\alpha = 3 + 1$

$r^2(\sin^2\alpha + \cos^2\alpha) = 4$

$r^2 = 4$

$r = \pm 2$ only take

$r = 2$.

or

$r = \sqrt{x^2 + y^2}$

$= \sqrt{3+1} = \sqrt{4} = 2$

$\therefore z = 2\cos\pi/6 + i\, 2\sin\pi/6$

$= 2(\cos\pi/6 + i\sin\pi/6)$

(2) $z = 1+i$, $x = 1, y = 1$

Comparing with polar form of a complex number, we get

$r\cos\theta = 1$, — (1) $r\sin\theta = 1$ — (2)

$(2)/(1) \Rightarrow \dfrac{r\sin\theta}{r\cos\theta} = 1$

$\tan\theta = 1$

$\theta = 45^\circ = \pi/4^c$

or

$\tan\theta = y/x$
$= 1$
$\theta = \underline{45^\circ}$

$(1)^2 + (2)^2 \Rightarrow r^2\cos^2\theta + r^2\sin^2\theta = 2$

$r^2(\sin^2\theta + \cos^2\theta) = 2$

$r^2 = 2$

$r = \pm\sqrt{2} = \sqrt{2}$

or

$r = \sqrt{x^2+y^2} = \underline{\sqrt{2}}$

$\therefore z = r\cos\theta + ir\sin\theta$

$= r(\cos\theta + i\sin\theta)$

$= \sqrt{2}(\cos \pi/4^c + i\sin \pi/4^c)$

Equality of two complex number:

Two complex numbers are equal if only if their real parts and imaginary parts are equal.

$z_1 = a+ib, \quad z_2 = c+id$.

$z_1 = z_2 \implies a = c$ and $b = d$.

Find the value of x and y if two complex numbers are equal.

1. $z_1 = x+3i$ and $z_2 = 3+3i$

$z_1 = z_2$.

$x+3i = 3+3i$

$x = 3$

if $z_1 = z_2$.

2. $z_1 = 5-2i$, $z_2 = 5-yi$

$z_1 = z_2$.

$5-2i = 5-yi$

$y = 2$

Addition of complex numbers.

$z_1 = a+ib, \quad z_2 = c+id$.

$z_1 + z_2 = a+ib+c+id$

$= a+c+i(b+d)$.

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Add the following:

1. $2+3i$, $1-4i$ and $-2+i$

$$(2+3i)+(1-4i)+(-2+i)$$
$$(2+1-2)+i(3-4+1)$$
$$1+0i = 1$$

2. $2-3i$, $-3+5i$, $4+6i$

$$(2-3i)+(-3+5i)+(4+6i)$$
$$(2-3+4)+i(-3+5+6)$$
$$3+8i$$

Substraction of complex numbers

$z_1 = a+bi$ and $z_2 = c+id$.

$$z_1 - z_2 = (a+bi)-(c+id)$$
$$=(a-c)+i(b-d)$$

Substract the following.

1. $(5-6i)$ from $3-5i$

$$(3-5i)-(5-6i)$$
$$=(3-5)+i(-5+6)$$
$$= -2+i$$

2. $8-2i$ from $7-3i$

$$(7-3i)-(8-2i)$$
$$= 7-3i-8+2i$$
$$= \underline{\underline{-1-i}}.$$

multiplication of two complex numbers. or product of complex nos

$z_1 = a+ib$, $z_2 = c+id$.

$$z_1 z_2 = (a+ib)(c+id)$$
$$= ac+iad+ibc+i^2bd$$
$$= ac+iad+ibc-bd \qquad \underline{\underline{i^2=-1}}$$
$$= \underline{\underline{(ac-bd)+i(ad+bc)}}.$$

1. $(3+3i)$ and $(5+2i)$.

$$(3+3i)(5+2i)$$
$$= 15+6i+15i+6i^2$$
$$= 15+6i+15i-6$$
$$= \underline{\underline{9+21i}}.$$

2. $(5+2i)$ and $(1+2i)$.

$$(5+2i)(1+2i)$$
$$= 5+10i+2i+4i^2$$
$$= 5+10i+2i-4 = \underline{\underline{1+12i}}.$$

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workout problems.

1. Find the modulus and amplitude of the following and express them in polar form.

   a. $5+12i$ b. $-3-3i$

2. Find the conjugate of the following complex numbers

   a. $2+4i$ b. $-i$

3. If $a+7i = 5+bi$, find a and b.

4. Add the following.

   a. $2+3i, 1+i$ b. $1+i, 1-i$

5. Substract

   a. $8+i$ from $10+2i$

   b. i from $2-i$

6. multiply

   a. $5+6i, 3+2i$

   b. $i-1, i+1$

   c. $i, -i$