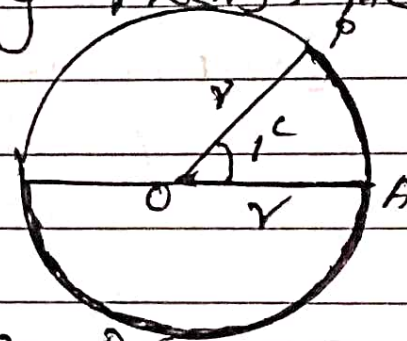
Relation between Degree and Radian

Consider a circle with centre O and radius r. Let A be a point on the circle. Now cut off an arc AP whose length is equal to the radius r of the circle. Thus $\angle AOP = 1^c$. To produce AO to meet the circle at B. Thus arc AB is half of the circumference and hence its length is πr. Also $\angle AOB$ = a straight line $= 180^\circ$

We know that angles at the centre of a circle are proportional to the arcs subtending them. Hence



$$\frac{\angle AOP}{\angle AOB} = \frac{\text{arc } AP}{\text{arc } AB}.$$

$$\frac{1^c}{180^\circ} = \frac{r}{\pi r}$$

$$1^c = \frac{180^\circ}{\pi} = \left(\frac{180}{\pi}\right)^\circ \quad \left(\pi = \frac{22}{7}\right)$$

or $1^\circ = \left(\frac{\pi}{180}\right)^c$ ═══.

The relation between degree measures and radian measures of some standard angles

Degree	0	30°	45°	60°	90°	180°	270°	360°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π

$\rightarrow 1^\circ = (\pi/180)^c$

$30^\circ = \frac{30 \times \pi}{180} = (\pi/6)^c$

$180^\circ = \frac{180 \times \pi}{180} = \pi^c$

$270^\circ = \frac{270 \times \pi}{180} = 3\pi/2^c$

$30^\circ = 30 \times 1^\circ$

$= 30 \times \frac{\pi}{180}$

Problems

① Express the following angles in degree measure.

(a) $\left(\frac{2\pi}{15}\right)^c$

$\frac{2\pi}{15} \times 1^c = \frac{2\pi}{15} \times \frac{180^\circ}{\pi} = \frac{360}{15} = 24^\circ$

(b) $\left(\frac{1}{4}\right)^c$

$\frac{1}{4} \times 1^c = \frac{1}{4} \times \frac{180}{\pi} = \frac{45}{\pi} = \frac{45}{22/7} = \frac{315}{22}$

$= \left(\frac{315}{22}\right)^\circ = \left(14.\frac{7}{22}\right)^\circ$

$= 14^\circ \frac{7}{22} \times 1^\circ$

$22 \overline{)315}$ (14, 22, 88, 7)

$= 14^\circ \frac{7}{22} \times 60'$

$= 14^\circ \frac{210}{11}'$

$= 14^\circ (19\frac{1}{11})'$

$= 14^\circ 19' \frac{1}{11} \times 1'' \quad = 14^\circ 19' (\frac{1}{11} \times 60)''$

$= 14^\circ 19' 5''$ approximately

(c) 6^c

$6^c = 6 \times 1^c$

$= 6 \times \frac{180}{11} = 6 \times \frac{180}{22/7}$

$= \frac{3\,6 \times 7 \times 180}{22\,11} = \frac{3780^\circ}{11}$

$= (343\frac{7}{11})^\circ$

$= 343^\circ \frac{7}{11} \times 60'$

$= 343^\circ \frac{420}{11}'$

$= 343^\circ (38\frac{2}{11})'$

$= 343^\circ 38' \left(\frac{3}{11} \times 60\right)''$

$= 343^\circ 38' 41''$

$11 \overline{)120}$ (11, 10) $1\frac{10}{11}$

II Express the following in radian measure

(a) 55°

$55^\circ = 55 \times 1^\circ$

$= 55 \times \left(\frac{\pi}{180}\right)^c$

$= \left(\frac{11\pi}{36}\right)^c$ //

(b) 240°

$240^\circ = 240 \times 1^\circ$

$= 240 \times \left(\frac{\pi}{180}\right)^c$

$= \frac{240 \times \pi}{180} = \left(\frac{4\pi}{3}\right)^c$

(c) $40^\circ 20' = \left(40^\circ + \frac{20}{60}\right)^\circ = \left(40 + \frac{1}{3}\right)^\circ$ $\left(1' = \frac{1}{60}^\circ\right)$

$= \left(\frac{121}{3}\right)^\circ = \frac{121}{3} \times \frac{\pi}{180}^c$

$= \left(\frac{121\pi}{540}\right)^c$

(d) $75^\circ 30' = \left(75 + \frac{30}{60}\right)^\circ$

$= \left(75 + \frac{1}{2}\right)^\circ = \left(\frac{151}{2}\right)^\circ$

$= \frac{151}{2} \times \frac{\pi}{180} = \left(\frac{151\pi}{360}\right)^c$

(e) $12^\circ 12' 20'' = \left(12 + \frac{12}{60} + \frac{20}{60\times 60}\right)^\circ$

$= \left(12 + \frac{1}{5} + \frac{1}{180}\right)^\circ$

$= \left(\frac{61}{5} + \frac{1}{180}\right)^\circ \quad \left(1'' = \frac{1}{60\times 60}^\circ\right)$

$= \left(\frac{2197}{180}\right)^\circ$

$= \left(\frac{2197}{180} \times \frac{\pi}{180}\right)^c$

$= \left(\frac{2197\pi}{32400}\right)^c$

$5 \overline{)180,5}$
$36,1$

36
61
36
216
2196