7a. Prove that $\frac{\sin\alpha}{1+\cos\alpha} + \frac{1+\cos\alpha}{\sin\alpha} = 2\operatorname{cosec}\alpha$

$$L.H.S = \frac{\sin\alpha}{1+\cos\alpha} + \frac{1+\cos\alpha}{\sin\alpha}.$$

$$= \frac{\sin\alpha\cdot\sin\alpha + (1+\cos\alpha)(1+\cos\alpha)}{(1+\cos\alpha)(\sin\alpha)}.$$

$$= \frac{\sin^2\alpha + 1 + \cos\alpha + \cos\alpha + \cos^2\alpha}{(1+\cos\alpha)(\sin\alpha)}.$$

$$= \frac{\sin^2\alpha + \cos^2\alpha + 1 + 2\cos\alpha}{(1+\cos\alpha)\sin\alpha}.$$

$$= \frac{1+1+2\cos\alpha}{(1+\cos\alpha)\sin\alpha} = \frac{2+2\cos\alpha}{(1+\cos\alpha)\sin\alpha}$$

$$= \frac{2(1+\cos\alpha)}{(1+\cos\alpha)\sin\alpha}$$

$$= \frac{2}{\sin\alpha} = 2\times\frac{1}{\sin\alpha} = 2\operatorname{cosec}\alpha$$

$= R.H.S.$

8 $\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2\sec^2 A$

$L.H.S = \frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1}$

$= \frac{\operatorname{cosec} A(\operatorname{cosec} A + 1) + \operatorname{cosec} A(\operatorname{cosec} A - 1)}{(\operatorname{cosec} A - 1)(\operatorname{cosec} A + 1)}$

$= \frac{\operatorname{cosec}^2 A + \operatorname{cosec} A + \operatorname{cosec}^2 A - \operatorname{cosec} A}{\operatorname{cosec}^2 A - 1}$

$= \frac{2\operatorname{cosec}^2 A}{\cot^2 A} = 2 \times \frac{1}{\sin^2 A} \times \tan^2 A$

$= 2 \times \frac{1}{\sin^2 A} \times \frac{\sin^2 A}{\cos^2 A}$

$= 2 \times \sec^2 A = R.H.S.$

9. Prove that $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

$L.H.S = \sqrt{\frac{1+\sin A}{1-\sin A}}$

$= \frac{\sqrt{1+\sin A}}{\sqrt{1-\sin A}} \frac{\sqrt{1+\sin A}}{\sqrt{1+\sin A}}.$

$$= \frac{(\sqrt{1+\sin A})^2}{\sqrt{(1-\sin A)(1+\sin A)}}$$

$$= \frac{((1+\sin A)^{1/2})^2}{\sqrt{1-\sin^2 A}} = \frac{1+\sin A}{\sqrt{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A$$

Problems

1. If $\sin\alpha = 2/5$ and 'α' is an acute angle, find $\cos\alpha$ and $\tan\alpha$.
2. If $\tan\alpha = 3$ and α is an acute angle, find $\sec\alpha$ and $\operatorname{cosec}\alpha$.
3. Prove the following identities
   (a) $\sin A \cot A = \cos A$
   (b) $\sin^2 A - \cos^2 A = 1 - 2\cos^2 A$
   (c) $(\sin A + \cos A)^2 = 1 + 2\sin A \cos A$
4. Prove that $\dfrac{1+\sin\alpha}{\cos\alpha} = \dfrac{\cos\alpha}{1-\sin\alpha}$.
5. Show that $\dfrac{1+\cos\alpha}{\sin\alpha} + \dfrac{\sin\alpha}{1+\cos\alpha} = 2\operatorname{cosec}\alpha$.
6. Prove that $\dfrac{\cos A}{1-\tan A} + \dfrac{\sin A}{1-\cot A} = \sin A + \cos A$

Trigonometric functions of standard angles :-

θ	0°	$30^\circ = \pi/6$	$45^\circ = \pi/4$	$60^\circ = \pi/3$	$90^\circ = \pi/2$
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$.	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$.	$1/\sqrt{2}$.	$1/2$.	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	not defined
$\cot\theta$	not defined	$\sqrt{3}$	1	$1/\sqrt{3}$	0
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$.	2	not defined
$\csc\theta$	not defined.	2	$\sqrt{2}$.	$2/\sqrt{3}$	1

Evaluate the following.

(i) $3\sin 30^\circ - 4\cos^3 60^\circ$

$$= 3 \times 1/2 - 4 \times (1/2)^3$$

$$= \frac{3}{2} - 4 \times \frac{1}{8} = \frac{3}{2} - \frac{1}{2}$$

$$= \frac{2}{2} = 1 //.$$

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(2) $4\tan^2 60° - 3\tan^2 30° + \tan^2 45°$

$= 4 \times (\sqrt{3})^2 - 3 \times \left(\frac{1}{\sqrt{3}}\right)^2 + 1$

$= 4 \times 3 - 3 \times \frac{1}{3} + 1$

$= 12 - 1 + 1 = \underline{\underline{12}}.$

(3) $\tan^2 45° - \frac{1}{2}\cos 60° - \frac{3}{4}\tan^2 30°$

$= 1 - \frac{1}{2} \times \frac{1}{2} - \frac{3}{4} \times \left(\frac{1}{\sqrt{3}}\right)^2.$

$= 1 - \frac{1}{4} - \frac{3}{4} \times \frac{1}{3}$

$= 1 - \frac{1}{4} - \frac{1}{4} = 1 - \frac{2}{4} = 1 - \frac{1}{2}.$

$= \frac{2-1}{2}$

$= \underline{\underline{1/2}}.$

(4). $4\sin^3 \pi/3 - 3\cos \pi/6.$

$= 4\left(\sqrt{3}/2\right)^3 - 3\left(\sqrt{3}/2\right)$

$= \cancel{4} \times \frac{3\sqrt{3}}{\cancel{8}\,2} - \frac{3\sqrt{3}}{2}.$

$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = \underline{\underline{0}}.$