$$\frac{1}{6} \lim_{\theta \to 0} \left(4 \frac{\sin 4\theta}{4\theta} + 2 \cdot \frac{\sin 2\theta}{2\theta} \right).$$

$$= \frac{1}{6} \left[4 \lim_{4\theta \to 0} \frac{\sin 4\theta}{4\theta} + 2 \cdot \lim_{2\theta \to 0} \frac{\sin 2\theta}{2\theta} \right]$$

$$= \frac{1}{6} \left(4 + 2 \right] = 6/6 = 1$$

. Evaluate $\lim_{\theta \to \pi/2} \frac{\cos \theta}{(\pi/2 - \theta)}$.

$$\cos\theta = \sin(90 - \theta)$$
$$= \sin(\pi/2 - \theta).$$

$$\lim_{\theta \to \pi/2} \frac{\sin(\pi/2 - \theta)}{(\pi/2 - \theta)} = \lim_{\pi/2 - \theta \to 0} \frac{\sin(\pi/2 - \theta)}{(\pi/2 - \theta)}$$
$$= 1.$$

. Evaluate $\lim_{x \to 0} \frac{1 - \cos 2x}{x^2}$.

$$\sin^2 x = \frac{1 - \cos 2x}{2}.$$

$$\therefore 2\sin^2 x = 1 - \cos 2x.$$

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$$\lim_{x \to 0} 2 \frac{\sin^2 x}{x^2} = 2 \cdot \lim_{x \to 0} \frac{\sin^2 x}{x^2}$$

$$= 2 \cdot \lim_{x \to 0} \left(\frac{\sin x}{x} \cdot \frac{\sin x}{x} \right)$$

$$= 2 \left[\lim_{x \to 0} \frac{\sin x}{x} \cdot \lim_{x \to 0} \frac{\sin x}{x} \right]$$

$$= 2 [1 \times 1] = \underline{\underline{2}}.$$

Q. Evaluate $\lim_{\theta \to 0} \frac{\sin 3\theta \cos \theta}{\theta}$.

$$\lim_{\theta \to 0} \frac{\sin 3\theta}{\theta} \cdot \cos \theta.$$

$$\lim_{\theta \to 0} \frac{\sin 3\theta}{\theta} \cdot \lim_{\theta \to 0} \cos \theta.$$

Nr and Dr

xlied by 3

$$\lim_{3\theta \to 0} 3 \cdot \frac{\sin 3\theta}{3\theta} \cdot \lim_{\theta \to 0} \cos \theta.$$

$$3 \cdot \lim_{3\theta \to 0} \frac{\sin 3\theta}{3\theta} \cdot \lim_{\theta \to 0} \cos \theta.$$

$$= 3 \times 1 \times 1$$

$$= \underline{\underline{3}}$$

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Questions:

1. $\lim_{x \to 2} \frac{x^2 - 2x}{x - 2}$ 2. $\lim_{x \to 2} \frac{x^3 - 8}{x^2 - 4}$

3. $\lim_{x \to a} \frac{x^m - a^m}{x^n - a^n}$ 4. $\lim_{x \to \infty} \frac{x^2 - 2x + 3}{3x^2 - 2x}$.

5. $\lim_{\theta \to 0} \frac{\sin 5\theta}{\theta}$ 6. $\lim_{\theta \to 0} \frac{\sin 2\theta \cos 5\theta}{\theta}$.

7. $\lim_{\theta \to 0} \frac{\tan 3\theta}{\theta}$. 8. $\lim_{x \to 3} \frac{x^2 - 3x}{x^2 - 9}$

9. $\lim_{\theta \to 0} \frac{\sin a\theta}{\tan b\theta}$ 10. $\lim_{x \to \infty} \frac{(3x+1)(2x+4)}{(x+1)(x-7)}$

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Differentiation :

The differential coefficient or derivative of y with respect to x is denoted by $\frac{dy}{dx}$.

The process of finding $\frac{dy}{dx}$ is called differentiation.

Rules of differentiation.

1. $\frac{d}{dx}(u \pm v) = \frac{d}{dx}(u) \pm \frac{d}{dx}(v)$.

2. $\frac{d}{dx}(ku) = k \cdot \frac{du}{dx}$.

Formulae.

1. $\frac{d}{dx}(x^n) = n \cdot x^{n-1}$

2. $\frac{d}{dx}(x) = 1$

3. $\frac{d}{dx}(k) = 0$.

4. $\frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{1/2}) = \frac{1}{2} x^{1/2 - 1} = \frac{1}{2} x^{-1/2}$
$= \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$

5. $\frac{d}{dx}(\sin x) = \cos x$

6. $\frac{d}{dx}(\cos x) = -\sin x$.

1. Find the derivatives of

(a) x (b) x^4 (c) $\frac{1}{x^3}$ (d) $\frac{1}{\sqrt{x}}$ (e) 5.

(a) $\frac{d}{dx}(x) = 1$.

By the standard formula

$$\frac{d}{dx}(x^n) = n \cdot x^{n-1}$$

$$\frac{d}{dx}(x^1) = 1 \cdot x^{1-1} = 1 \cdot x^0 = 1 \cdot 1$$

$$= 1$$

(b) $\frac{d}{dx}(x^4)$

$= 4x^3$

$$\frac{d}{dx}(x^4) = 4 \cdot x^{4-1}$$

$$= 4 \cdot x^3$$

(c) $\frac{d}{dx}\left(\frac{1}{x^3}\right) = \frac{d}{dx}(x^{-3})$

$= -3 \cdot x^{-3-1}$

$= -3x^{-4}$

$= \frac{-3}{x^4}$

(d) $\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right) = \frac{d}{dx}\left(\frac{1}{x^{1/2}}\right) = \frac{d}{dx}(x^{-1/2})$

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$= \frac{-1}{2} x^{-1/2 - 1}$

$= \frac{-1}{2} x^{-3/2}$

$= \frac{-1}{2 \times x^{3/2}}$

(d) $\frac{d}{dx}(5) = 0 \implies \frac{d}{dx}(K) = 0$

Find the derivatives of

(a) $\frac{d}{dx}\; x^2 + 2x + 1$ (b) $3x^6 + \frac{1}{x} - 5$

(c) $\frac{4}{x^3} + 5x^{10} - \frac{1}{\sqrt{x}}$ (d) $4x^{10} - 3x^5 - 2.$

(a) $\frac{d}{dx}(x^2 + 2x + 1) = \frac{d}{dx}(x^2) + \frac{d}{dx}(2x) + \frac{d}{dx}(1)$

$= 2 \cdot x^{2-1} + 2 \cdot \frac{d}{dx}(x) + 0$

$= 2x + 2 \times 1 \cdot x^{1-1} + 0$

$= 2x + 2x^0 = 2x + 2$

(b) $\frac{d}{dx}\left(3x^6 + \frac{1}{x} - 5\right) = 3 \cdot \frac{d}{dx}(x^6) + \frac{d}{dx}(1/x)$

$= 3 \cdot 6 \cdot x^{6-1} + \frac{d}{dx}(x^{-1})$

$= 18x^5 + -1 \cdot x^{-1-1} = 18x^5 - x^{-2}$

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$= 18x^5 - \frac{1}{x^2}$

c) $\frac{d}{dx}\left(\frac{4}{x^3} + 5x^{10} - \frac{1}{\sqrt{x}}\right)$.

$4 \cdot \frac{d}{dx}\left(\frac{1}{x^3}\right) + 5 \cdot \frac{d}{dx}(x^{10}) - \frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)$

$= 4 \frac{d}{dx}(x^{-3}) + 5 \cdot 10 \cdot x^{10-1} - \frac{d}{dx}\left(\frac{1}{x^{1/2}}\right)$

$= 4\left(-3 \cdot x^{-3-1}\right) + 5 \cdot 10x^9 - \frac{d}{dx}(x^{-1/2})$

$= 4(-3 \cdot x^{-4}) + 50x^9 - \left(-\frac{1}{2} x^{-1/2-1}\right)$

$= 12x^{-4} + 50x^9 + \frac{1}{2}x^{-3/2}$

$= \frac{12}{x^4} + 50x^9 + \frac{1}{2 \cdot x^{3/2}}$

d) $\frac{d}{dx}(4x^{10} - 3x^5 - 2)$.

$4 \cdot \frac{d}{dx}(x^{10}) - 3\frac{d}{dx}(x^5) - \frac{d}{dx}(2)$.

$= 4 \cdot 10x^{10-1} - 3(5 \cdot x^{5-1}) - 0$

$= 4 \times 10 x^9 - 3(5x^4)$

$= 40x^9 - 15x^4$

Q. Find the derivative of $x^2+\sin x$.

$$\frac{d}{dx}(x^2+\sin x)$$

$$=\frac{d}{dx}(x^2)+\frac{d}{dx}(\sin x)$$

$$=2\cdot x^{2-1}+\cos x$$

$$=2x+\cos x$$

Q. Find the (a) $\frac{d}{dx}(2+\cos x)$

(b) $\frac{d}{dx}(x^2+2\cos x+5)$

(c) $\frac{d}{dx}(\sqrt{x}+2\sin x+5)$

(a) $\frac{d}{dx}(2+\cos x)=\frac{d}{dx}(2)+\frac{d}{dx}(\cos x)$

$$=0-\overset{\sin x}{\cancel{\cos x}}=-\sin x$$

$$\cancel{=-\cos x}$$

(b) $\frac{d}{dx}(x^2+2\cos x+5)$

$$=\frac{d}{dx}(x^2)+2\cdot\frac{d}{dx}(\cos x)+\frac{d}{dx}(5)$$

$$=2x^{2-1}+2\cdot(-\sin x)+0$$

$$=2x-2\sin x$$

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(c) $\frac{d}{dx}(\sqrt{x}) + 2\cdot\frac{d}{dx}(\sin x) + \frac{d}{dx}(5)$

$\frac{d}{dx}(x^{1/2}) + 2\cdot(\cos x) + 0.$

$\frac{1}{2}x^{1/2-1} + 2\cos x.$

$\frac{1}{2}x^{-1/2} + 2\cos x$

$= \frac{1}{2x^{1/2}} + 2\cos x$

$= \frac{1}{2\sqrt{x}} + 2\cos x.$

Product rule.

If u and v are two functions of x, then

$$\frac{d}{dx}(uv) = u\cdot\frac{d}{dx}(v) + v\cdot\frac{d}{dx}(u)$$

(1^{st} function x derivative of 2^{nd} + 2^{nd} function x derivative of 1^{st})

Q. $\frac{d}{dx}(x^2 \sin x) = x^2\cdot\frac{d}{dx}(\sin x) + \sin x\cdot\frac{d}{dx}(x^2)$

$= x^2\cdot(\cos x) + \sin x\cdot 2\cdot x^{2-1}$

$= x^2\cos x + \sin x\cdot 2\cdot x$

$= x^2\cos x + 2x\sin x$

$$\frac{d}{dx}(\sin x \cdot \cos x).$$

$$= \sin x \cdot \frac{d}{dx}(\cos x) + \cos x \cdot \frac{d}{dx}(\sin x)$$

$$= \sin x \cdot (-\sin x) + \cos x \cdot (\cos x)$$

$$= -(\sin x)^2 + (\cos x)^2$$

$$= -\sin^2 x + \cos^2 x$$

$$\frac{d}{dx}(x^2 + 2x\sin x + \cos x).$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(2x\sin x) + \frac{d}{dx}\cos x$$

$$2x + 2 \cdot \frac{d}{dx}(x\sin x) + (-\sin x)$$

$$= 2x + 2\left[x \cdot \frac{d}{dx}(\sin x) + \sin x \cdot \frac{d}{dx}(x)\right] - \sin x$$

$$= 2x + 2\left[x \cdot \cos x + \sin x\right] - \sin x$$

$$= 2x + 2x\cos x + 2\sin x - \sin x$$

$$= 2x + 2x\cos x + \sin x$$

Algebraical limit

$$\lim_{n\to\infty} \frac{x^n - a^n}{x - a} = n.a^{n-1}$$ for all rational values of n.

1. Evaluate. $\lim_{x\to 2} \frac{x^4 - 16}{x - 2}$

$$\lim_{x\to 2} \frac{x^4 - 2^4}{x - 2} = 4.2^{4-1} \quad n = 4, a = 2$$

$$= 4 \times 2^3$$

$$= 4 \times 8 = \underline{\underline{32}}$$

2. Evaluate $\lim_{x\to 3} \frac{x^3 - 27}{x^2 - 9}$

$$\lim_{x\to 3} \frac{x^3 - 3^3}{x^2 - 3^2}$$

Divide the numerator and denominator by $(x-3)$

$$\lim_{x\to 3} \frac{(x^3 - 3^3)/x-3}{(x^2 - 3^2)/x-3}$$

$$\frac{\lim_{x\to 3} (x^3 - 3^3)/x-3}{\lim_{x\to 3} (x^2 - 3^2)/x-3} = \frac{3.3^{3-1}}{2.3^{2-1}} = \frac{\cancel{3}\times 9}{2\times \cancel{3}} = \underline{\underline{\frac{9}{2}}}$$

3. Evaluate. $\lim_{x \to 4} \frac{x^3-64}{x^2-4^2}$

$$\lim_{x \to 4} \frac{x^3-4^3}{x^2-4^2}.$$

Devide the numerator and denominator by $(x-4)$

$$\lim_{x \to 4} \frac{(x^3-4^3)/x-4}{(x^2-4^2)/x-4}$$

$$\frac{\lim_{x \to 4} (x^3-4^3)/x-4}{\lim_{x \to 4} (x^2-4^2)/x-4} = \frac{3 \cdot 4^{3-1}}{2 \cdot 4^{2-1}}$$

$$= \frac{3 \cdot \cancel{4}^{\cancel{2}\,2}}{2 \times 4} = \underline{\underline{6}}.$$

4. Evaluate $\lim_{x \to 2} \frac{x\sqrt{x}-2\sqrt{2}}{x-2}.$

$$\lim_{x \to 2} \frac{x^1 \cdot x^{1/2} - 2^1 \times 2^{1/2}}{x-2}.$$

$x^m \cdot x^n = x^{m+n}$ $\cancel{x^m \cdot x^n = x^{m+}}$

$$\lim_{x \to 2.} \frac{x^{3/2} - 2^{3/2}}{x-2} = 3/2\, 2^{3/2-1}$$

$$= \frac{3}{2} \cdot 2^{1/2}$$

$$= \frac{3\sqrt{2}}{2} = \frac{3\times\sqrt{2}\times\sqrt{2}}{2\sqrt{2}}$$

$$= \frac{3\times 2}{2\sqrt{2}} = 3/\sqrt{2}$$

Trigonometrical limit

$$\lim_{\theta\to 0}\left(\frac{\sin\theta}{\theta}\right) = 1$$

Q. Evaluate $\lim_{\theta\to 0} \frac{\sin m\theta}{\sin n\theta}$.

$$\lim_{\theta\to 0} \frac{\sin m\theta/\theta}{\sin n\theta/\theta}.$$

$\lim_{\theta\to 0} \frac{\sin m\theta}{\theta} \longrightarrow$ multiply Nr and Dr by m

$\lim_{\theta\to 0} \frac{\sin n\theta}{\theta} \longrightarrow$ multiply Nr and Dr by n.

$$\lim_{\theta\to 0} \frac{m \sin m\theta}{m\theta} = m \cdot \lim_{m\theta\to 0} \frac{\sin m\theta}{m\theta}.$$

$$\lim_{\theta\to 0} \frac{n \cdot \sin n\theta}{n\theta}. \quad n \cdot \lim_{n\theta\to 0} \frac{\sin n\theta}{n\theta}$$

$= \frac{m \times 1}{n \times 1} = m/n$

Q. Evaluate $\lim_{x \to 0} \frac{\tan x}{x}$

$\tan x = \frac{\sin x}{\cos x}$

$\lim_{x \to 0} \frac{\sin x/\cos x}{x}$ $\quad || \frac{a/b}{c} = \frac{a}{bc}$

$= \lim_{x \to 0} \frac{\sin x}{x \cos x}$

$= \lim_{x \to 0} \frac{\sin x}{x} \times \lim_{x \to 0} \frac{1}{\cos x}$

$= 1 \times 1 = 1$

Q. Evaluate $\lim_{\alpha \to 0} \frac{\sin 4\alpha + \sin 2\alpha}{6\alpha}$

$\lim_{\alpha \to 0} \left(\frac{\sin 4\alpha}{6\alpha} + \frac{\sin 2\alpha}{6\alpha} \right)$

$\frac{1}{6} \lim_{\alpha \to 0} \left(\frac{\sin 4\alpha}{\alpha} + \frac{\sin 2\alpha}{\alpha} \right)$

Nr + Dr mlt by 4 $\qquad$ Nr + Dr mlt by 2.

$$\frac{1}{6} \lim_{\theta \to 0} \left(4 \frac{\sin 4\theta}{4\theta} + 2 \cdot \frac{\sin 2\theta}{2\theta} \right).$$

$$= \frac{1}{6} \left[4 \lim_{4\theta \to 0} \frac{\sin 4\theta}{4\theta} + 2 \cdot \lim_{2\theta \to 0} \frac{\sin 2\theta}{2\theta} \right]$$

$$= \frac{1}{6} \left[4 + 2 \right] = 6/6 = 1$$

51, 56, 52, 53, 54, 55, 57, 61, 59

Limits

Variable: A variable is a varying quantity, whose value changes during any mathematical investigation.

eg: 1. the atmospheric temperature.

2. The angle between the hour hand and minute hand of a working clock.

Dependent and Independent variables

A variable whose value is chosen arbitrarily is called independent variable.

A dependent variable depends on independent variable.

eg: $y = f(x)$.

$y = x^2$

Constant is a fixed quantity whose value remains unchanged throughout a mathematical investigation

eg: $\pi = 3.14$

Function: If a variable y ~~depends~~ depends on a variable x in such a way that each value of x determines exactly one value of y, then we say that y is a function of x, it can be written as $y = f(x)$.

(1) $y = f(x)$ are called explicit functions

eg: $y = e^x$, $y = x^2 + x + 1$

(2) If a function in x and y is given in such a way that x and y cannot be seperated, then the function is called implicit function.

eg: $x^2 + y^2 = 25$, $x^2 + xy + y^2 = 0$

Parametric function

~~If t~~

If two variables x and y are expressed in terms of a third variable (θ, t), then the variable is called the parameter and the function containing the parameter is known as parametric function.

eg: (1) $x = at^2$, $y = bt^2$

parametric function with parameter t.

(2) $x = a\cos\theta$, $y = b\sin\theta$.

with parameter θ.

Limits

Consider a variable x ~~ta~~ taking an infinite number of values according to a certain rule, as x approaches 'a' after some time, we say that x tends to a or limit of x equal to a

$\text{Lt}\, x = a$ or $\lim x = a$

Limits of a function

The sequence of numbers $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ As x increases, the value of $1/x$ becomes smaller and smaller or the value approaches to "zero",

$$\lim_{x \to \infty} \frac{1}{x} = 0$$

The numbers $1, \frac{1}{0.1}, \frac{1}{0.01}, \ldots$ As x decreases, the value of $1/x$ become larger and larger or the value approaches to infinity.

$$\lim_{x \to 0} (1/x) = \infty$$

Properties

1. $\lim (u+v) = \lim u + \lim v$
2. $\lim (u-v) = \lim u - \lim v$
3. $\lim (uv) = \lim u \cdot \lim v$
4. $\lim (u/v) = \dfrac{\lim u}{\lim v}, \quad v \neq 0$

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1. Calculate $\lim_{x\to\infty} \frac{x^2+2x+1}{x^2+x-3}$

$$\lim_{x\to\infty} \frac{\cancel{x^2}(1+2/x+1/x^2)}{\cancel{x^2}(1+1/x-3/x^2)}$$

$$\lim_{x\to\infty} \frac{(1+2/x+1/x^2)}{(1-1/x-3/x^2)}$$

$= 1/1 = \underline{1}$

$$= \frac{x^2+2x+1}{x^2}$$

$$= \frac{x^2}{x^2}+\frac{2x}{x^2}+\frac{1}{x^2}$$

$$= 1+\frac{2}{x}+\frac{1}{x^2}$$

$$= \frac{1+0+0}{1+0-0} = \frac{1}{1}$$

$\lim_{x\to\infty} \frac{1}{x} = 0$

Similarly

$\lim_{x\to\infty} 1/x^2 = 0$

2. Evaluate $\lim_{x\to\infty} \frac{x^2-2x+8}{4x^3-3}$

$$\lim_{x\to\infty} \frac{x^2(1-2/x+8/x^2)}{x^3(4-3/x^3)}$$

$$\lim_{x\to\infty} \frac{1}{x}\left(\frac{1-2/x+8/x^2}{4-3/x^3}\right)$$

$$\frac{1}{4}\lim_{x\to\infty} 1/x = \frac{1}{4}\times 0 = \underline{\underline{0}}$$

$$\frac{x^2-2x+8}{x^2}$$

$$= \frac{x^2/x^2 - \frac{2x}{x^2}+\frac{8}{x^2}}{\frac{4x^3}{x^3}-\frac{3}{x^3}}$$

$$= \frac{1-2/x+8/x^2}{4-3/x^3}$$

3. Evaluate $\lim_{x\to\infty} \frac{3x^2+5}{x^2-2}$

$$\lim_{x\to\infty} \frac{\cancel{x^2}(3+5/x^2)}{\cancel{x^2}(1-2/x^2)}$$

$$\lim_{x\to\infty}\left(\frac{3+5/x^2}{1-2/x^2}\right) = \frac{3+0}{1-0} = 3/1 = \underline{\underline{3}}$$

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4. Evaluate $\lim_{x\to\infty} \frac{x^2-2x+3}{3x^2-2x}$.

$$\lim_{x\to\infty} \frac{x^2(1-2/x+3/x^2)}{x^2(3-2/x)}$$

$$\lim_{x\to\infty} \frac{1-2/x+3/x^2}{3-2/x} \qquad \lim_{x\to\infty} \frac{1}{x}=0$$

$$= \frac{1-0+0}{3-0} = 1/3$$

5. $\lim_{x\to\infty} \frac{x^3-2x+3}{2x^3-4x+6}$.

$$\lim_{x\to\infty} \frac{\cancel{x^3}(1-2/x^2+3/x^3)}{\cancel{x^3}(2-4/x^2+6/x^3)}$$

$$\frac{x^3-2x+3}{x^3} = x^3/x^3 - \frac{2x}{x^{3}{}_2} + \frac{3}{x^3} = 1-\frac{2}{x^2}+\frac{3}{x^3}$$

$$\lim_{x\to\infty} \frac{1-2/x^2+3/x^3}{2-4/x^2+6/x^3}$$

$$= \frac{1-0+0}{2-0+0} = 1/2$$

6. Calculate $\lim_{x\to 0} \frac{ax+b}{cx+d}$.

$$\lim_{x\to 0} \frac{ax+b}{cx+d} = \frac{a\times 0+b}{c\times 0+d} = \frac{0+b}{0+d}$$

$$= b/d$$

2. Calculate $\lim_{x \to 1} \frac{2x+3}{4x-1}$

$$\lim_{x \to 1} \frac{2x+3}{4x-1} = \frac{2 \times 1 + 3}{4 \times 1 - 1}$$

$$= \frac{2+3}{4-1} = \underline{\underline{\frac{5}{3}}}$$

3. $\lim_{x \to 2} (2x+3)$

$$\lim_{x \to 2} (2x+3) = 2 \times 2 + 3$$

$$= 4 + 3 = \underline{\underline{7}}$$

4. $\lim_{x \to 2} \frac{x^2-4}{x-2}$

$$\lim_{x \to 2} \frac{x^2-4}{x-2} = \frac{4-4}{2-2} = \frac{0}{0} \neq 0$$

$$\lim_{x \to 2} \frac{x^2-2^2}{x-2}$$

~~$a^2-b^2=(a-b)($~~

$a^2-b^2=(a-b)(a+b)$

$$\lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$$

$$\lim_{x \to 2} x+2 = 2+2 = \underline{\underline{4}}$$

5. $\lim_{x \to 1} \frac{x^2+4x-5}{x^2+x-2}$

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$$\lim_{x \to 1} \frac{x^2+4x-5}{x^2+x-2} = \frac{1^2+4\times1-5}{1^2+1-2} = \frac{1+4-5}{1+1-2}$$

$$= \frac{5-5}{2-2} = 0/0 \neq 0$$

numbers -1 and 5 pdt $= -5$

$$\lim_{x \to 1} \frac{(x-1)(x+5)}{(x-1)(x+2)}$$

sum $= 4$

numbers 1 and 5

pdt $= 1\times5 = 5$

sum $= 1+5 = 6$

But pdt $= 1\times-5 = -5$

sum $= 1-5 = -4$

pdt $= -1\times5 = -5$

sum $= -1+5 = 4$

pdt $= -2$

sum $= 1$

numbers 1 and -2

pdt $= 1\times-2 = -2$.

sum $= 1-2 = -1$

~~pdt~~ numbers -1 and 2

pdt $= -1\times2 = -2$.

sum $= -1+2 = 1$//

numbers -1 and 2

$$\lim_{x \to 1} \frac{x+5}{x+2} = \frac{1+5}{1+2} = 6/3 = 2//$$

Q. Evaluate $\lim_{x \to 2} \frac{x^2-5x+6}{x^2+x-6}$

$$\lim_{x \to 2} \frac{x^2-5x+6}{x^2+x-6} = \frac{2^2-5\times2+6}{2^2+2-6}$$

$$= \frac{4-10+6}{4+2-6} = \frac{0}{0} \neq 0.$$

$$\lim_{x \to 2} \frac{\cancel{(x-2)}(x-3)}{(x+3)\cancel{(x-2)}}$$

pdt = 6
Sum = 5
numbers
−2 and −3
pdt = −2×−3 = 6
Sum = −2−3 = −5

$$\lim_{x \to 2} \frac{x-3}{x+3} = \frac{2-3}{2+3}$$

$$= \frac{-1}{5}$$

Q. $\lim_{x \to 1} \frac{x^2+x-2}{x^2+2x-3}$

$$\lim_{x \to 1} \frac{x^2+x-2}{x^2+2x-3} = \frac{1^2+1-2}{1^2+2\times1-3} = \frac{1+1-2}{1+2-3}$$

$$= \frac{2-2}{3-3} = 0/0 \neq 0.$$

$$\lim_{x \to 1} \frac{(x+2)\cancel{(x-1)}}{\cancel{(x-1)}(x+3)} = \lim_{x \to 1} \frac{x+2}{x+3}$$

$$= \frac{1+2}{1+3} = 3/4$$

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Differential coefficient of inverse t-functions.

(1) Find $\frac{dy}{dx}$, if $y = \sin^{-1}x$.

Let $y = \sin^{-1}x$.

$\Rightarrow x = \sin y$.

Both side differentiating with respect to y.

$$\frac{d}{dy}(x) = \frac{d}{dy}(\sin y).$$

$$\frac{dx}{dy} = \cos y.$$

$$= \sqrt{\cos^2 y}$$ $\because \cos^2 y + \sin^2 y = 1$

$$= \sqrt{1 - \sin^2 y}$$

$$\frac{dx}{dy} = \sqrt{1 - x^2}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}.$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{1}{\sqrt{1 - x^2}}.$$

$$\Rightarrow \frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1 - x^2}}.$$

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Result

1. $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}.$

2. $\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$

3. $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}.$

4. $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}.$

Q. Find the derivative of $e^x \sin^{-1}x$.

$$\frac{d}{dx}(e^x \sin^{-1}x)$$

$$= e^x \cdot \frac{d}{dx}(\sin^{-1}x) + \sin^{-1}x \cdot \frac{d(e^x)}{dx}$$

$$= e^x \cdot \frac{1}{\sqrt{1-x^2}} + \sin^{-1}x \cdot e^x.$$

$$= \frac{e^x}{\sqrt{1-x^2}} + e^x \cdot \sin^{-1}x$$

Q. Find the derivative of $\frac{\sin^{-1}x}{x}$

$$\frac{d}{dx}\left(\frac{\sin^{-1}x}{x}\right) = \frac{x \cdot \frac{d}{dx}(\sin^{-1}x) - \sin^{-1}x \cdot \frac{d(x)}{dx}}{(x)^2}.$$

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$$= \frac{x \cdot \frac{1}{\sqrt{1-x^2}} - \sin^{-1}x \cdot 1}{x^2}$$

$$= \frac{x/\sqrt{1-x^2} - \sin^{-1}x}{x^2}$$

Q. Find the derivative of $(x^2+3)\tan^{-1}x$.

$$\frac{d}{dx}[(x^2+3)\tan^{-1}(x)]$$

$$= (x^2+3)\cdot\frac{d}{dx}(\tan^{-1}(x)) + \tan^{-1}x \cdot \frac{d}{dx}(x^2+3)$$

$$= (x^2+3)\frac{1}{1+x^2} + \tan^{-1}x\left(\frac{d}{dx}(x^2) + \frac{d}{dx}(3)\right)$$

$$= \frac{x^2+3}{1+x^2} + \tan^{-1}x\,(2x + 0)$$

$$= \frac{x^2+3}{1+x^2} + 2x\tan^{-1}x$$

Q. Find the derivative of $\frac{x\sin^{-1}x}{(1+x^2)}$

$$\frac{d}{dx}\left(\frac{x\sin^{-1}x}{1+x^2}\right)$$

$$= \frac{(1+x^2)\frac{d}{dx}(x\sin^{-1}x) - x\sin^{-1}x\frac{d}{dx}(1+x^2)}{(1+x^2)^2}$$

$$= \frac{(1+x^2)\left(x \cdot \frac{d}{dx}(\sin^{-1}x) + \sin^{-1}x \frac{d}{dx}(x)\right) - x\sin^{-1}x\left(\frac{d}{dx}(1) + \frac{d}{dx}(x^2)\right)}{(1+x^2)^2}$$

$$= \frac{(1+x^2)\left(x \frac{1}{\sqrt{1-x^2}} + \sin^{-1}x \cdot 1\right) - x\sin^{-1}x(2x)}{(1+x^2)^2}$$

$$= \frac{(1+x^2)\left(\frac{x}{\sqrt{1-x^2}} + \sin^{-1}x\right) - 2x^2\sin^{-1}x}{(1+x^2)^2}$$

Q. $\frac{d}{dx}(xe^x \sin^{-1}x) = xe^x \cdot \frac{d}{dx}(\sin^{-1}x) + \sin^{-1}x \frac{d}{dx}(xe^x)$.

$$= xe^x \cdot \frac{1}{\sqrt{1-x^2}} + \sin^{-1}x\left(x\frac{d}{dx}(e^x) + e^x \cdot \frac{d}{dx}(x)\right)$$

$$= \frac{xe^x}{\sqrt{1-x^2}} + \sin^{-1}x(xe^x + e^x \cdot 1)$$

$$= \frac{xe^x}{\sqrt{1-x^2}} + (xe^x + e^x)\sin^{-1}x$$

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$= \sqrt{x}\cos x + \sin x\left(\frac{1}{2}x^{-1/2}\right)$.

$= \sqrt{x}\cos x + \frac{1}{2x^{1/2}}\sin x$.

$= \sqrt{x}\cos x + \frac{1}{2\sqrt{x}}\sin x$

Quotient rule.

If u and v any two functions of x then $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \frac{d}{dx}(u) - u\frac{d}{dx}(v)}{v^2}$

or

$$\frac{d}{dx}\left(\frac{Nr}{Dr}\right) = \frac{Dr \cdot \frac{d}{dx}(Nr) - Nr \cdot \frac{d}{dx}(Dr)}{(Dr)^2}.$$

Q. $\frac{d}{dx}\left(\frac{x^2}{\sin x}\right) = \frac{\sin x \cdot \frac{d}{dx}(x^2) - x^2 \cdot \frac{d}{dx}(\sin x)}{(\sin x)^2}$.

$$= \frac{\sin x \cdot (2x) - x^2 \cdot (\cos x)}{\sin^2 x}$$

$$= \frac{2x\sin x - x^2\cos x}{\sin^2 x}$$

Q. $\frac{d}{dx}\left(\frac{x-1}{x+1}\right) = \frac{(x+1)\frac{d}{dx}(x-1) - (x-1)\frac{d}{dx}(x+1)}{(x+1)^2}$

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$$= \frac{(x+1)\left(\frac{d}{dx}(x) - \frac{d}{dx}(1)\right) - (x-1)\left(\frac{d}{dx}(x) + \frac{d}{dx}(1)\right)}{(x+1)^2}.$$

$$= \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2}.$$

$$= \frac{(x+1) - (x-1)}{(x+1)^2} = \frac{x+1-x+1}{(x+1)^2} = \frac{2}{(x+1)^2}$$

Q. $\frac{d}{dx}\left(\frac{\cos x}{\sqrt{x}}\right) = \frac{\sqrt{x}\cdot\frac{d}{dx}(\cos x) - \cos x\frac{d}{dx}(\sqrt{x})}{(\sqrt{x})^2}.$

$$= \frac{\sqrt{x}\cdot(-\sin x) - \cos x\cdot\frac{d}{dx}(x^{1/2})}{x}.$$

$$= \frac{-\sqrt{x}\sin x - \cos x\left(\frac{1}{2}x^{1/2-1}\right)}{x}.$$

$$= \frac{-\sqrt{x}\sin x - \cos x\cdot\frac{1}{2}\times x^{-1/2}}{x}$$

$$= \frac{-\sqrt{x}\sin x - \cos x\cdot\frac{1}{2x^{1/2}}}{x}$$

$$= \frac{-\sqrt{x}\sin x - \frac{1}{2\sqrt{x}}\cdot\cos x.}{x}$$

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Differential coefficient of cosec x, sec x, tan x and cot x

1. $\frac{d}{dx}(\operatorname{cosec} x) = \frac{d}{dx}\left(\frac{1}{\sin x}\right)$

$= \frac{\sin x \cdot \frac{d}{dx}(1) - 1 \cdot \frac{d}{dx}(\sin x)}{(\sin x)^2}$

$= \frac{\sin x \times 0 - \cos x}{(\sin x)^2} = \frac{-\cos x}{(\sin x)^2}$

$= \frac{-\cos x}{\sin x} \times \frac{1}{\sin x}$

$= -\cot x \cdot \operatorname{cosec} x$

$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

2. $\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$

$= \frac{\cos x \cdot \frac{d}{dx}(1) - 1 \cdot \frac{d}{dx}(\cos x)}{(\cos x)^2}$

$= \frac{\cos x \times 0 - 1 \cdot (-\sin x)}{(\cos x)^2}$

$= \frac{\sin x}{(\cos x)^2} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$

$= \tan x \sec x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

Q. $\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$.

$= \frac{\cos x \cdot \frac{d}{dx}(\sin x) - \sin x \cdot \frac{d}{dx}(\cos x)}{(\cos x)^2}$

$= \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{(\cos x)^2}$.

$= \frac{\cos^2 x + \sin^2 x}{(\cos x)^2} = \frac{1}{\cos^2 x}$.

$= \sec^2 x$ //

$\frac{d}{dx}(\tan x) = \sec^2 x$

Q. $\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$.

$= \frac{\sin x \cdot \frac{d}{dx}(\cos x) - \cos x \cdot \frac{d}{dx}(\sin x)}{(\sin x)^2}$.

$= \frac{\sin x(-\sin x) - \cos x \cdot \cos x}{(\sin x)^2}$.

$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$.

$= \frac{-1}{\sin^2 x} = -\operatorname{cosec}^2 x$

$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

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Result

1. $\frac{d}{dx}(x^n) = n \cdot x^{n-1}$ 2. $\frac{d}{dx}(k) = 0$.

3. $\frac{d}{dx}(\sin x) = \cos x$ 4. $\frac{d}{dx}(\cos x) = -\sin x$.

5. $\frac{d}{dx}(\tan x) = \sec^2 x$ 6. $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

7. $\frac{d}{dx}(\sec x) = \sec x \tan x$. 8. $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x$.

8. $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

9. $\frac{d}{dx}(e^x) = e^x$ 10. $\frac{d}{dx}(\log x) = \frac{1}{x}$.

Q. $\frac{d}{dx}(x + \sin x + 5\tan x)$.

$= \frac{d}{dx}(x) + \frac{d}{dx}(\sin x) + \frac{d}{dx}(5\tan x)$

$= 1 + \cos x + 5 \cdot \frac{d}{dx}(\tan x)$

$= 1 + \cos x + 5 \cdot \sec^2 x$

Q. $\frac{d}{dx}(4e^x - 3\operatorname{cosec} x)$.

$= \frac{d}{dx}(4e^x) - \frac{d}{dx}(3\operatorname{cosec} x)$

$= 4 \cdot \frac{d}{dx}(e^x) - 3 \cdot \frac{d}{dx}(\operatorname{cosec} x)$

$= 4 \cdot e^x - 3 \cdot (-\operatorname{cosec} x \cot x)$

$= 4e^x + 3 \operatorname{cosec} x \cot x$

Q. $\frac{d}{dx}\left(\frac{\tan x}{x}\right)$

$= \frac{x \cdot \frac{d}{dx}(\tan x) - \tan x \cdot \frac{d}{dx}(x)}{x^2}$

$= \frac{x \cdot (\sec^2 x) - \tan x \cdot 1}{x^2}$

$= \frac{x \sec^2 x - \tan x}{x^2}$

Q. $\frac{d}{dx}(x^n \log x)$

$= x^n \cdot \frac{d}{dx}(\log x) + \log x \cdot \frac{d}{dx}(x^n)$

$= x^n \cdot \frac{1}{x} + \log x \cdot n \cdot x^{n-1}$

$= x^n \cdot x^{-1} + n \cdot x^{n-1} \log x$

$= x^{n-1} + n x^{n-1} \log x$

$= x^{n-1}(1 + n \log x)$

Q. $\frac{d}{dx}(x^3 e^x)$

$= x^3 \cdot \frac{d}{dx}(e^x) + e^x \cdot \frac{d}{dx}(x^3)$

$= x^3 \cdot e^x + e^x \cdot 3 \cdot x^{3-1}$

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$$= x^3 e^x + e^x \cdot 3x^2.$$
$$= e^x (x^3 + 3x^2)$$

Q. $\frac{d}{dx}\left(\frac{\cos x}{x+\sin x}\right)$

$$= \frac{(x+\sin x)\cdot\frac{d}{dx}(\cos x) - \cos x\cdot\frac{d}{dx}(x+\sin x)}{(x+\sin x)^2}.$$

$$= \frac{(x+\sin x)(-\sin x) - \cos x(1+\cos x)}{(x+\sin x)^2}.$$

$$= \frac{-x\sin x - (\sin x)^2 - (\cos x + (\cos x)^2)}{(x+\sin x)^2}.$$

$$= \frac{-x\sin x - \sin^2 x - \cos x - \cos^2 x}{(x+\sin x)^2}.$$

$$= \frac{-x\sin x - \cos x - (\sin^2 x + \cos^2 x)}{(x+\sin x)^2}.$$

$$= \frac{-x\sin x - \cos x - 1}{(x+\sin x)^2}$$

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Q. $\frac{d}{dx}\left(\frac{x\sec x}{3x+2}\right)$

$$= \frac{(3x+2)\frac{d}{dx}(x\sec x) - x\sec x \cdot \frac{d}{dx}(3x+2)}{(3x+2)^2}.$$

$$= \frac{(3x+2)\left[x\frac{d}{dx}\sec x + \sec x \cdot \frac{d}{dx}(x)\right] - x\sec x\left(3\cdot\frac{d}{dx}(x) + \frac{d}{dx}(2)\right)}{(3x+2)^2}.$$

$$= \frac{(3x+2)(x\cdot\sec x\tan x + \sec x) - x\sec x(3+0)}{(3x+2)^2}.$$

$$= \frac{(3x+2)(x\sec x\tan x + \sec x) - 3x\sec x}{(3x+2)^2}.$$