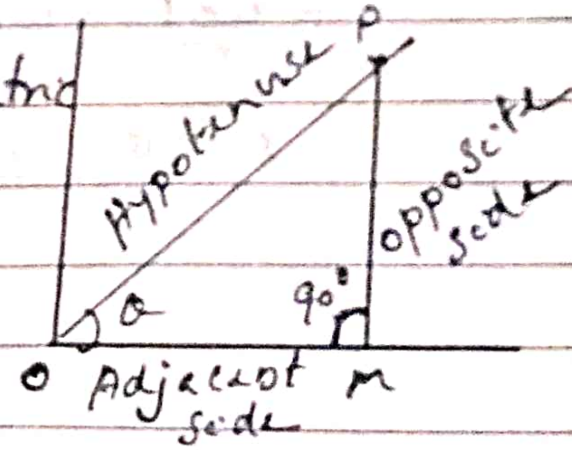
Trigonometric functions of an acute angle.

Consider an acute angle of measure θ in the standard position. Let P be any point on the terminal side. From P draw a perpendicular to meet the x-axis at M. Thus POM is a right triangle, right angled at M. and $\angle MOP = \theta$. With respect to the angle θ, OM is the adjacent side, MP is the opposite side and OP is the hypotenuse.

There are six trigonometric functions, namely $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$, $\sec\theta$ and $\operatorname{cosec}\theta$.



$$\sin\theta = \frac{MP}{OP} = \frac{\text{Opposite Side}}{\text{Hypotenuse}} \longrightarrow A_1$$

$$\cos\theta = \frac{OM}{OP} = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} \longrightarrow B_1$$

$$\tan\theta = \frac{MP}{OM} = \frac{\text{Opposite side}}{\text{Adjacent Side}} \longrightarrow C_1$$

$$\cot\theta = \frac{OM}{MP} = \frac{\text{Adjacent Side}}{\text{opposite Side}} \longrightarrow C_2$$

$$\sec\theta = \frac{OP}{OM} = \frac{\text{Hypotenuse}}{\text{Adjacent Side}} \longrightarrow B_2$$

$$\operatorname{cosec}\theta = \frac{OP}{MP} = \frac{\text{Hypotenuse}}{\text{opposite side}} \longrightarrow A_2$$

The above formulae, we get A_1 and A_2, B_1 and B_2, C_1 and C_2 are reciprocal.

$$\sin\theta = \frac{1}{\operatorname{cosec}\theta}, \quad \cos\theta = \frac{1}{\sec\theta}.$$

$$\tan\theta = \frac{1}{\cot\theta}.$$

Quotient Relations.

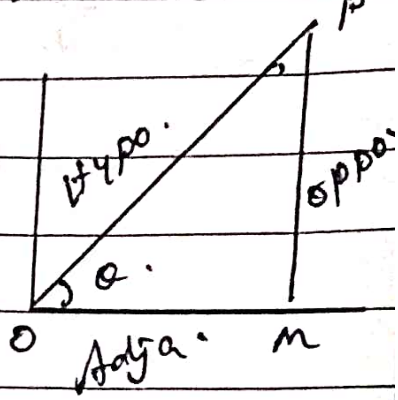
$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \quad \cot\theta = \frac{\cos\theta}{\sin\theta}.$$

Classmate

Date ______
Page ______

Pythagorean Relations.

Using Pythagorus theorem, from the right triangle POM. We have



$$MP^2 + OM^2 = OP^2 \longrightarrow (1)$$

(A) Dividing (1) by OP^2, we get.

$$\frac{MP^2}{OP^2} + \frac{OM^2}{OP^2} = \frac{OP^2}{OP^2}$$

$$\left(\frac{MP}{OP}\right)^2 + \left(\frac{OM}{OP}\right)^2 = 1$$

$$(\sin\theta)^2 + (\cos\theta)^2 = 1$$

$$\sin^2\theta + \cos^2\theta = 1$$

Thus $\sin^2\theta = 1 - \cos^2\theta$

and $\cos^2\theta = 1 - \sin^2\theta$.

(B) Dividing (1) by OM^2, we get

$$\frac{MP^2}{OM^2} + \frac{OM^2}{OM^2} = \frac{OP^2}{OM^2}$$

$$\left(\frac{MP}{OM}\right)^2 + 1 = \left(\frac{OP}{OM}\right)^2$$

$$(\tan\theta)^2 + 1 = (\sec\theta)^2$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$\tan^2\theta = \sec^2\theta - 1$

and $\sec^2\theta - \tan^2\theta = 1$

Date
Page

(c) Dividing (1) by MP^2

$$\frac{MP^2}{MP^2} + \frac{OM^2}{MP^2} = \frac{OP^2}{MP^2}$$

$$1 + \left(\frac{OM}{MP}\right)^2 = \left(\frac{OP}{MP}\right)^2$$

$$1 + (\cot\alpha)^2 = (\operatorname{cosec}\alpha)^2.$$

$$1 + \cot^2\alpha = \operatorname{cosec}^2\alpha.$$

Thus $\cot^2\alpha = \operatorname{cosec}^2\alpha - 1$

and $\operatorname{cosec}^2\alpha - \cot^2\alpha = 1$

using these formulae or right triangle figure to calculate the following problems.

(1). If $\cos\alpha = 4/5$ and α is an acute angle, find $\sin\alpha$ and $\tan\alpha$.

using formulae.

$$\sin^2\alpha + \cos^2\alpha = 1.$$

$$\sin^2\alpha = 1 - \cos^2\alpha = 1 - (\cos\alpha)^2$$

$$= 1 - (4/5)^2$$

$$= 1 - \frac{16}{25} = \frac{25-16}{25} = \frac{9}{25}$$

$$\sin\alpha = \sqrt{9/25}$$

$$\therefore \sin\alpha = \pm\frac{3}{5}$$

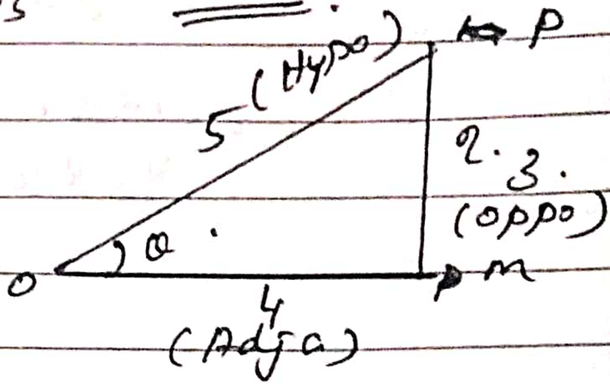
Take +ve value.

$\sin\alpha = 3/5$

$\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{3/5}{4/5} = 3/4$

$\boxed{\text{or}}$

using figure.



$MP^2 + OM^2 = OP^2$

$MP^2 + 16 = 25$

$MP^2 = 25 - 16 = 9$

$MP = 3$

$\therefore \sin\alpha = \dfrac{\text{oppo}}{\text{Hypo}} = \dfrac{3}{5}$

$\tan\alpha = \dfrac{\text{oppo}}{\text{Adja}} = \dfrac{3}{4}$

(To calculate any method)

(2) If $\tan\alpha = 12/5$ and α is an acute angle, find $\sin\alpha$ and $\cos\alpha$.

$1 + \tan^2\alpha = \sec^2\alpha$

$\sec^2\alpha = 1 + \tan^2\alpha = 1 + (12/5)^2$

$= 1 + \dfrac{144}{25} = \dfrac{25+144}{25}$

$= \dfrac{169}{25}$

$$\sec\theta = \sqrt{169/25} = 13/5.$$

$$\therefore \cos\theta = \frac{1}{\sec\theta} = \frac{1}{13/5} = \underline{\underline{5/13}}.$$

$$\sin^2\theta + \cos^2\theta = 1$$

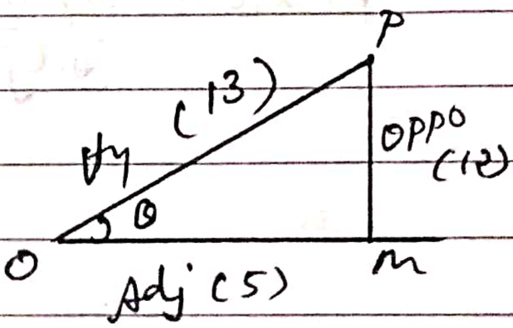
$$\sin^2\theta = 1 - \cos^2\theta = 1 - (5/13)^2$$

$$= 1 - \frac{25}{169} = \frac{169-25}{169}$$

$$= \frac{144}{169}.$$

$$\sin\theta = \sqrt{\frac{144}{169}} = 12/13 //.$$

$\boxed{\text{or}}$



$$mp^2 + om^2 = op^2$$

$$(12)^2 + (5)^2 = op^2$$

$$op^2 = 144 + 25$$

$$= 169.$$

$$op = \sqrt{169} = 13.$$

$$\therefore \sin\theta = \frac{\text{oppo}}{\text{Hypo}} = \frac{12}{13}$$

$$\cos\theta = \frac{\text{Adj}}{\text{Hypo}} = \frac{5}{13} //$$