

# Module IV

# Properties of

# Matter

Fluid Dynamics

# What is a Fluid?

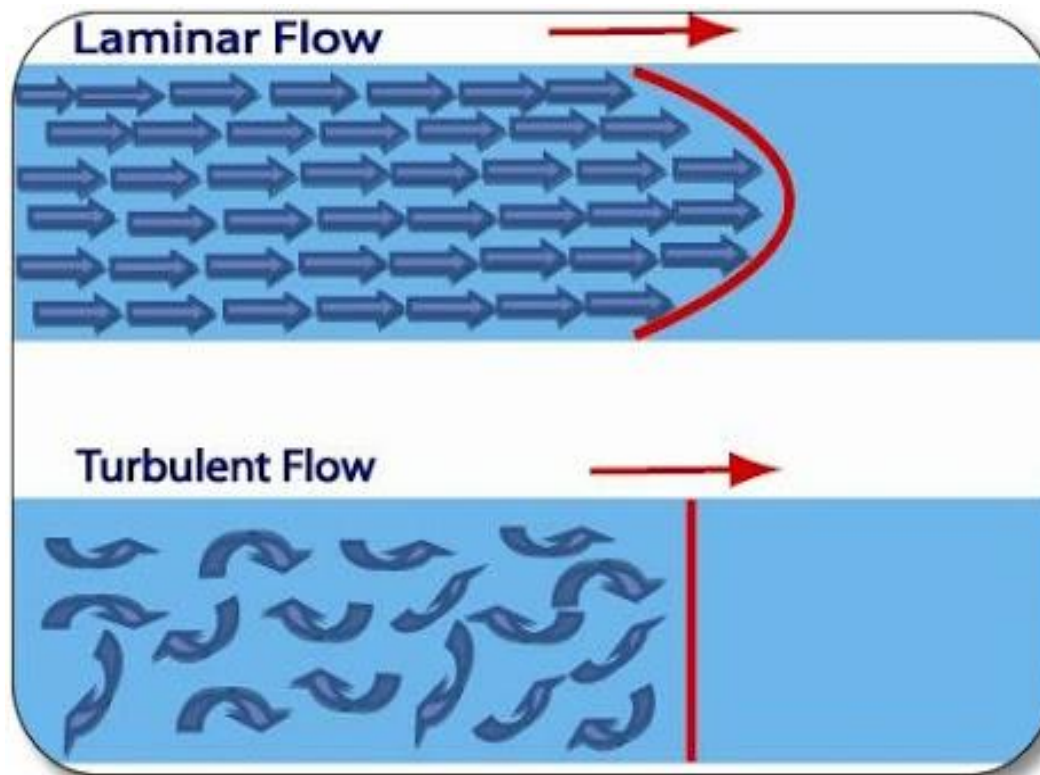
- A substance exists in three primary phases: solid, liquid, and gas. A substance in the liquid or gas phase is referred to as a **fluid**.
- Distinction between a solid and a fluid is made on the basis of the substance's ability to resist an applied shear (or tangential) stress that tends to change its shape.
- A solid can resist an applied shear stress by deforming, whereas a fluid deforms continuously under the influence of shear stress, no matter how small.
- In solids stress is proportional to *strain*, but in fluids stress is proportional to *strain rate*.

► **What is Streamline Flow?**

The flow of liquid is said to be steady or stream line if each particle of fluid follows a smooth path so that the velocity of the fluid at any point remains unchanged. The paths of the particles do not cross each other.

► **What is turbulent flow?**

The flow in which the fluid particles move randomly in any direction is called as Turbulent Flow.



# Comparison between streamline flow and turbulent flow

Streamline flow	Turbulent flow
It is the steady, smooth, regular, one-by-one flow of fluid particles.	The flow of particles is irregular and in a zigzag manner.
The speed of fluid motion is lower than the critical velocity. Critical velocity of fluids is the velocity above which the flow becomes irregular or turbulent.	The fluid motion speed is greater than the critical velocity.
The fluid layers do not cross each other	The fluid layer may cross each other
The particles reaching a given point in the path of flow have the same velocity with time. That means a single velocity of particles is possible at a given point in the flow of fluids.	The particles in the fluid can have different velocities at a given point in the fluid flow. That means multiple velocities at a given point in the flow can be possible.

# Reynold's Number

- ▶ Osborne Reynolds (1842-1912) defined a dimensionless number that has a vital role in predicting whether a liquid has stream line flow or turbulent flow.
- ▶ This number is called the Reynolds number (Re).
- ▶ Reynolds number is defined as the ratio of inertial forces to viscous forces within flowing fluid.

$$R_e = \frac{\text{inertial force}}{\text{viscous force}}$$

$$R_e = \frac{\rho v d}{\eta}$$

*Incompressible liquid = liquids with constant density*

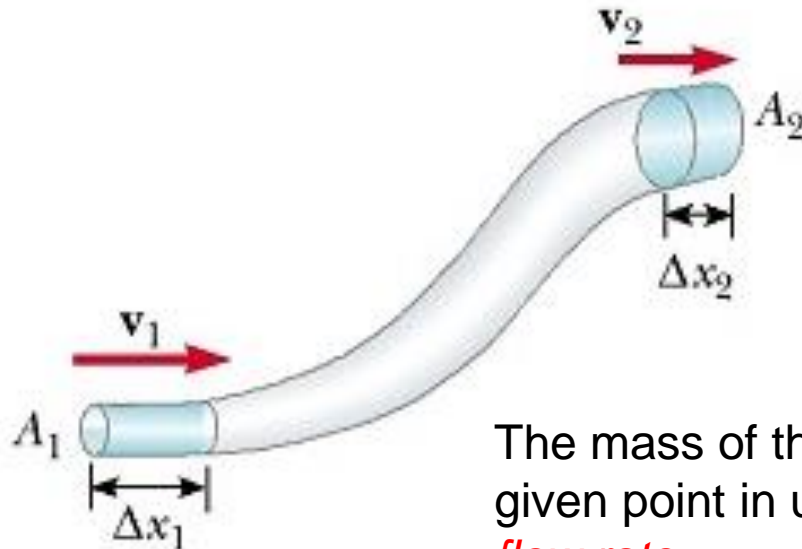
**Compressible fluids:** are the **fluids** with variable density

where  $\rho$  is the density of the fluid flowing with a speed  $v$ ,  $d$  is the diameter of the pipe, and  $\eta$  is the coefficient of viscosity of the fluid.

It is found that flow is streamline or laminar for Re less than 1000. The flow becomes unsteady for Re between 1000 and 2000. The flow is turbulent for Re greater than 2000.

# Equation of Continuity

- ▶ Equation of continuity in fluid dynamics states that when an incompressible and non-viscous fluid moving with steady flow through a pipe of varying cross-sectional area, the product of the area of cross-section and velocity of flow remains a constant at every point in the pipe.
- ▶ Consider a pipe of varying cross sectional area . Let  $v_1$  represents the fluids velocity as it passes the cross sectional area  $A_1$  and  $v_2$  that through the area  $A_2$ ,



The mass of the liquid that possesses a given point in unit time is called the **mass flow rate**.

- ▶ Let  $\Delta x_1$  is the distance moves in a time  $\Delta t$  at section 1. Let the volume of the liquid passed is  $A_1 \Delta x_1$ .
- ▶ Mass of the liquid passed along section 1 is  $\rho_1 A_1 \cdot \Delta x_1$  where  $\rho_1$  is the density of the fluid.
- ▶ Mass of the liquid passed in unit time or flow rate  $= \rho_1 \cdot A_1 \cdot (\Delta x_1 / \Delta t) = \rho_1 \cdot A_1 \cdot v_1$
- ▶ Similarly, at the point 2, the flow rate is  $\rho_2 \cdot A_2 \cdot v_2$ . Since no fluid flows in or out through the sides, the flow rate section 1 should be same as the flow rate at section 2.

Thus  $\rho_1 \cdot A_1 \cdot v_1 = \rho_2 \cdot A_2 \cdot v_2$ . This is called **the equations of continuity**.

If the fluid is incompressible,  $\rho_1 = \rho_2$ ,

*then the equation becomes  $A_1 v_1 = A_2 v_2$*

$$Av = \text{constant}$$

where,  $A$  is the cross-sectional area of the fluid flow, and  $v$  is the velocity of the fluid at that point.

$Av$  is defined as the **volume flux** or **the flow rate of the fluid**, which remains constant for steady flow.

When the area of the cross-section is greater, the velocity of the liquid is lesser and vice versa.

# ENERGY OF A LIQUID

- ▶ A liquid in motion possess kinetic energy. Due to its position, it has gravitational PE, The liquid also possess pressure energy due to its pressure

## A) Kinetic Energy:

- ▶ KE for a body of mass  $m$  moving with velocity  $v = \frac{1}{2} mv^2$
- ▶ KE for a liquid/unit volume =  $\frac{1}{2} \rho v^2$
- ▶ KE/unit mass =  $\frac{1}{2} v^2$

## B) Potential energy (with a reference to a reference level)

A body of mass  $m$  positioned at a height  $h$  possesses a potential energy equal to  $mgh$ .

The potential energy/unit volume of a liquid =  $\rho gh$  where  $\rho$  is the density of the liquid.

The potential energy per unit mass of the liquid is  $gh$



## C) Pressure energy

- ▶ It is the **energy** in/of a fluid due to the applied **pressure** (force per area).
- ▶ So if you have a static fluid in an enclosed container, the **energy** of the system is only due to the **pressure**;
- ▶ If the fluid is moving along a flow, then the **energy** of the system is the kinetic **energy** as well as the **pressure**.
- ▶ Pressure energy =  $P.V = Pm/\rho$

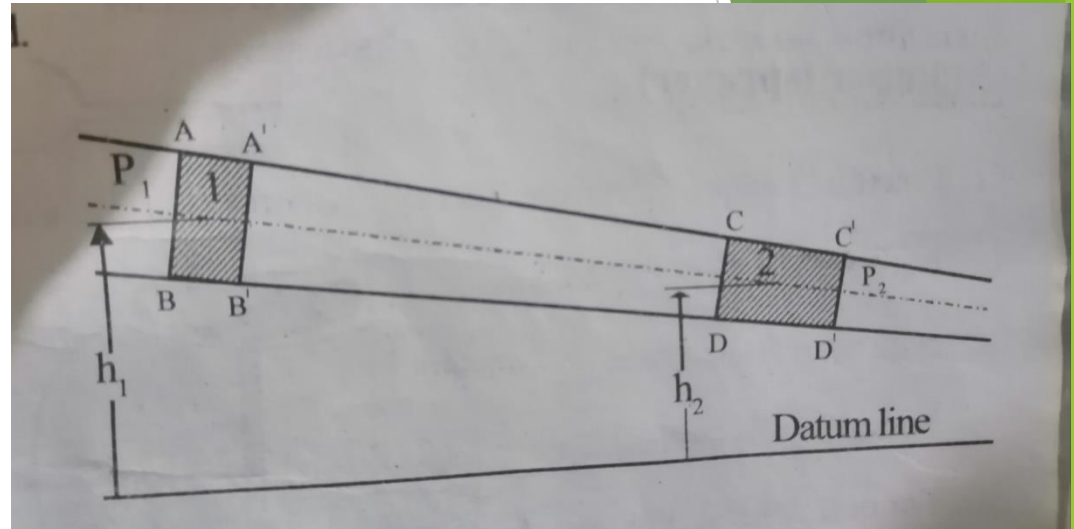
# Bernoulli's Theorem

- ▶ The total energy of a small amount of liquid flowing from one point to another along a streamline without friction remains the same through out the motion.
- ▶ Potential energy + pressure energy +kinetic energy= a constant
- ▶  $mgh + PV + \frac{1}{2} mv^2 = \text{a constant}$
- ▶  $mgh + mP/\rho + \frac{1}{2} mv^2 = \text{a constant}$

Bernoulli's theorem states that the sum of kinetic energy, potential energy, and pressure energy of an ideal fluid in a streamline flow remains constant throughout the flow.

# Bernoulli's theorem Proof

Consider an incompressible fluid of density  $\rho$  flowing as a continuous stream through a pipe as shown in figure. Consider two sections AB and CD. 1 and 2 are the midpoints of the two sections.



Let the liquid column  $ABDC$  displace to the right and occupies the new position  $A'B'D'C'$

As the liquid flows down, there is loss in both PE and pressure energy. This loss should appear as a gain in KE of the liquid.

$$\text{Loss in pressure energy} = mP_1/\rho - mP_2/\rho$$

$$\text{Gain in KE} = \frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2$$

$$\text{Loss in PE} = mgh_1 - mgh_2$$

Where  $v_1$  and  $v_2$  are the velocities of the liquids at points 1 and 2 respectively

According to the law of conservation of energy, the total loss in energy = total gain in energy

i.e,  $(mP_1/\rho - mP_2/\rho) + (mgh_1 - mgh_2) = (\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2)$

$$mP_1/\rho + mgh_1 + \frac{1}{2}mv_1^2 = mP_2/\rho + \frac{1}{2}mv_2^2 + mgh_2$$

This becomes the general form of Bernaulli's equation. Dividing throughout by m, we get

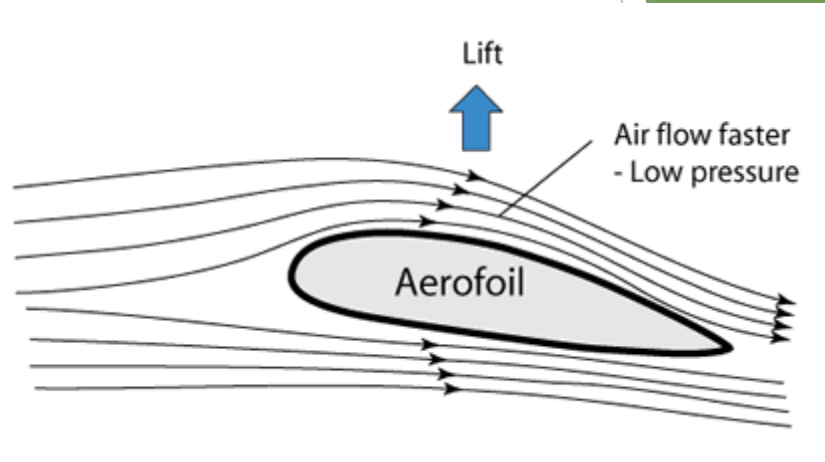
$$P_1/\rho + gh_1 + \frac{1}{2}v_1^2 = P_2/\rho + \frac{1}{2}v_2^2 + gh_2$$

Or  $P/\rho + gh + \frac{1}{2}v^2 = \text{a constant}$

which is the equation for unit mass of the liquid

# AIR FOIL

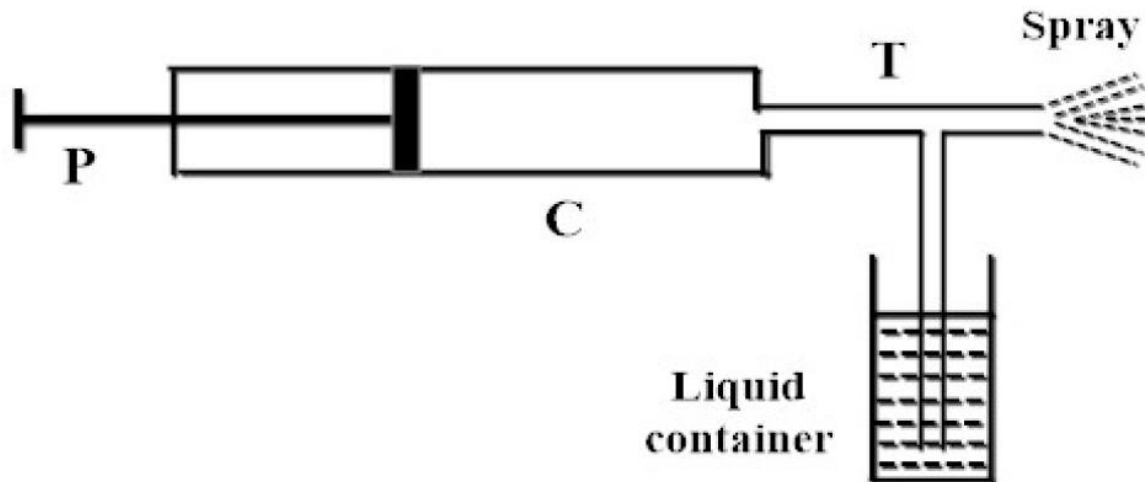
- An **airfoil** is the cross-sectional shape of a wing, blade (of a propeller, rotor, or turbine), or sail. An **airfoil**-shaped body moving through a fluid produces an aerodynamic force. It works based on Bernauli's principle. Due to the specifically designed shape of the wings, air over the top surface moves fast.



This is faster than it flows past under surface. This causes a reduction in pressure above the wing which gives rise to about two third of the lift. The wings are also designed in such a way that the air striking at the bottom gives additional lift.

# Atomiser

- An atomizer is a device to spray a liquid or make a liquid into a jet of tiny droplets. The principle behind the working of atomizers is Bernoulli's theorem. According to Bernoulli's theorem, the velocity of a fluid increases in a region, the pressure decreases in that region, and vice versa in order to maintain the total energy as constant.



Schematic diagram of an atomizer

- ▶ The schematic diagram of an atomizer is shown in the figure.
- ▶ It works on Bernoulli's **principle**.
- ▶ When high speed horizontal air passes over a vertical tube, it creates a low pressure and draws the air and liquid inside the vertical tube upward.
- ▶ **Atomizer** has a nozzle at the end of the horizontal tube which causes the liquid to break up into small drops and mixes it with the air.
- ▶ The air inside the **perfume** bottle is moving relatively slowly; therefore, according to **Bernoulli's principle**, its pressure is relatively high, and it exerts a strong downward force on the **perfume** itself. In an **atomizer** there is a narrow tube running from near the bottom of the bottle to the top.

# Viscosity

The internal friction of real fluid is called Viscosity

Frictional force between different layers of fluids as they move past one another.

In liquids, it is due to the cohesive forces between molecules.

In gases it is due to collision between molecules

Eg of Viscous fluids -Honey, Grease, engine oil



# Coefficient of Viscosity

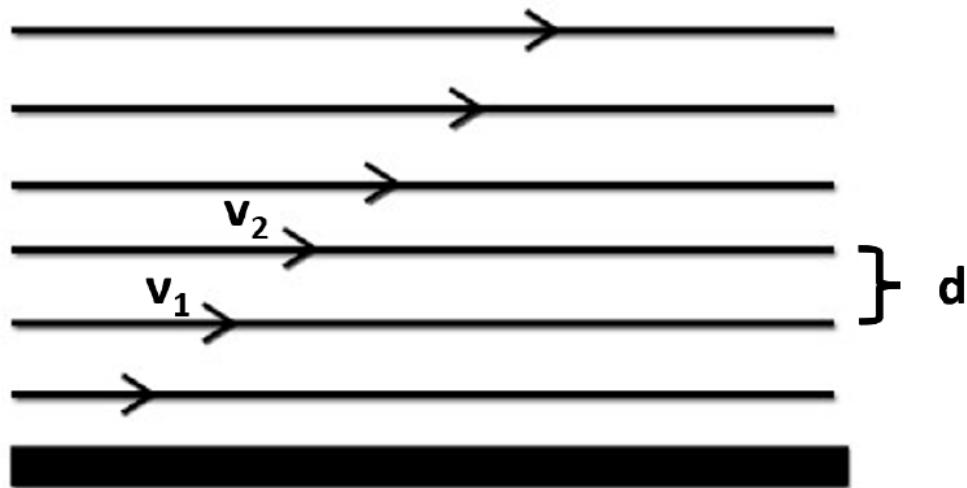
- ▶ The flow of a liquid through a pipe can be considered to be in the form of concentric layers of the same axis
- ▶ The inner most layer will be moving with maximum velocity whereas the outermost layer (the layer in contact with the pipe) will be moving with minimum velocity.
- ▶ From layer to layer the velocity gradually changes
- ▶ **Viscosity** is a measure of a fluid's resistance to flow
- ▶ Consider two two parallel layers of liquid, one sliding over the other with different velocities.
- ▶ A tangential viscous force appears in between the layers trying to minimise the relative velocity between the layers.
- ▶ The viscous force depends on the velocity gradient, the area of the layers in contact and the nature of the liquid

- ▶ Let the layers moves with velocity  $v_1$  and  $v_2$  respectively separated by a distance  $d$  .The rate of change of velocity with dstance is called the *velocity gradient*
- ▶ Hence the velocity gradient =  $(v_1 - v_2)/d$
- ▶ The viscous force is directly proportional to the velocity gradient and the area of each layer
- ▶ If A is the area of each layer in contact, then

$$F \propto A,$$

$$F \propto A \frac{(v_1 - v_2)}{d}$$

$$F = \eta A \frac{(v_1 - v_2)}{d}$$



Where  $\eta$  is a constant which depends upon the nature of the liquid

- ▶ co-efficient of viscosity  $\eta$  can be defined as that much resistive force developed between the liquid layers of unit area when they are moving with a unit velocity gradient.
- ▶ The SI unit of coefficient of viscosity  $\eta$  is  $\text{kgm}^{-1}\text{s}^{-1}$  or  $\text{Nsm}^{-2}$ .

## Terminal Velocity

Terminal velocity is defined as the highest velocity attained by an object falling through a fluid. It is observed when the sum of drag force and buoyancy is equal to the downward gravity force acting on the object. The acceleration of the object is zero as the net force acting on the object is zero.

# STOKE'S FORMULA

- ▶ Stoke's Law is a mathematical equation that expresses the settling velocities of the small spherical particles in a fluid medium.
- ▶ The law is derived considering the forces acting on a particular particle as it sinks through the liquid column under the influence of gravity.
- ▶ When a small object falls through a liquid, it gradually picks up speed. As its velocity increases, the viscous force acting upwards also increases until it just balances the effective weight of the object. At this point , net force acting on the object becomes zero. Hence the acceleration becomes zero.

- ▶ The other quantities on which the force  $F$  depends on viscosity  $\eta$  of the fluid and radius  $a$  of the sphere.
- ▶ Sir George G. Stokes (1819-1903), an English scientist introduced an expression for the viscous drag force  $F$  as given below.

$$F = 6\pi\eta r v$$

Where  $\eta$  is the co-efficient of viscosity of the medium,  $r$  is the radius of the spherical body and  $v$  is the uniform velocity after equilibrium or terminal velocity.