Equations of Motion

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Mechanics

- **Mechanics** : a tool to describe the behavior of an object in physics using various elements such as displacement, velocity, and acceleration.
- **Statics mechanics** deals with when the object is not in motion,
- **dynamic mechanics** describes in detail when the object is in motion.
- **Kinematics** is the study of motion without regard for the cause. On the other hand, **dynamics** is the study of the causes of motion.
- In kinematics, motion is mathematically described in terms of distance, displacement, speed, velocity, acceleration, and time.

Kinematics + dynamics = mechanics

Motion

- One dimensional motion: The motion of an object along a straight-line path is called motion in one dimension.
  - E.g: Motion of car along a straight road, a ball thrown vertically upwards, a freely falling body.
- Two Dimensional Motion: The motion of an object in plane is called motion in two dimension
  - E.g: **An ant moving on the top surface of a desk**
- Three Dimensional Motion: The motion of an object in space is called motion in three dimension.
  - E.g: Flying of bird
- If the velocity of the body remains a constant in one-dimensional motion, then it is called uniform motion. In uniform motion, the magnitude and direction of velocity remain constant and hence its acceleration is zero.

Equations of motion

- Consider the motion of a particle with initial velocity ‘u’ and uniform acceleration ‘a’.
- Let the displacement and velocity of the particle after a time ‘t’ second is ‘s’ and ‘v’ respectively.
- The motion of the particle along a straight-line path with uniform acceleration can be analyzed using the three equations of motion.

$$v = u + at$$

$$S = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2aS$$

Where *u* is the initial velocity, *v* final velocity, *S* the displacement and t time taken for this displacement

Derivation Equation 1

- From the definition for acceleration,

$$a = \frac{v-u}{t}$$

Cross multiplying, we get

$$at = v - u$$

$$v = u + at$$

Derivation Equation 2

Displacement = average velocity x time

Substitute, $v = u + at$

$$S = \left(\frac{u+v}{2}\right)t = \left(\frac{u+u+at}{2}\right)t$$

$$= \left(\frac{2u+at}{2}\right)t$$

$$= ut + \frac{1}{2}at^2$$

Derivation Equation 3

- From first equation

$$t = \frac{v-u}{a}$$

- Substituting for t in equation 2

$$S = u\frac{(v-u)}{a} + \frac{1}{2}a\left(\frac{(v-u)}{a}\right)^2$$

$$S = \frac{uv-u^2}{a} + \frac{1}{2}a(\frac{v^2-2uv+u^2}{a^2}) = \frac{uv-u^2}{a} + \frac{v^2-2uv+u^2}{2a}$$

$$= \frac{2uv-2u^2+v^2-2uv+u^2}{2a} = \frac{v^2-u^2}{2a}$$

- Cross multiplying, we get

$$2as = v^2 - u^2$$

$$v^2 = u^2 + 2aS$$

Equation 4

- This equation gives the distance travelled in t^th^ second = distance travelled in t seconds – distance travelled in (t-1)seconds.

- Distance travelled in t seconds S = $ut + \frac{1}{2}at^2$
-
  Distance travelled in (t-1) seconds S(t-1) = $u(t-1) + \frac{1}{2}a(t-1)^2$
- Solving the equation we get

$$S_t = \left[ut + \frac{1}{2}at^2\right] - \left[u(t-1) + \frac{1}{2}a(t-1)^2\right]$$

$$S_t = u + a(t - \frac{1}{2})$$

Errors in Measurements

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Errors in Measurements

- The measured value of the physical quantity is usually different from its true value.
- The result of every measurement by any measuring instrument is an approximate number, which contains some uncertainty.
- This uncertainty is called error.
- The difference between the true value and the measured value of a quantity is known as the error of measurement.
- Every calculated quantity, which is based on measured values, also has an error.
- Errors in measurements can be classified into two categories-- systematic errors and random errors.

1.Systamatic Errors

- Arises due to instrumental errors, incorrect experimental techniques, and personal errors.
- Instrumental errors: arise from the imperfect design or calibration of instruments, zero error of instruments, etc.
  - Examples: Zero error in vernier calipers or screw gauge and error due to measurement of length using a scale broken at one end
- Error due to incorrect experimental technique: occur due to inaccurate experimental procedures as well as external factors like pressure, temperature, humidity, wind, etc.
  - Eg.- measurement of body temperature by placing a thermometer under the armpit results in a lower temperature value than the actual value.

- Personal errors: arise due to personal bias, lack of proper setting of the apparatus, or individual’s carelessness in taking observations. These types of errors are also known as observational errors.

- Eg- when an observer holds his head towards the right (by habit) while reading the position of a needle on the scale, he introduces an error due to parallax.

- This type of error can be minimized by using better instruments, improving experimental techniques, and avoiding personal bias

2.Random Errors

- Random errors come from unpredictable changes in experimental conditions. It makes to give different results for same measurements taken repeatedly.
- Random errors are present in all experiments and are unpredictable.The random errors can be reduced by taking a greater number of measurements.
- These errors are also called statistical errors and can be removed by statistical methods like averaging.
- For example, unpredictable temperature changes can affect the electrical properties of instruments in an experiment involving electrical instruments.

3. Least Count Error

- The least count error is the error associated with the resolution of the instrument.
- The smallest value that can be measured by a measuring instrument is called its least count.
- Least count may not be sufficiently small. The maximum possible error is equal to the least count. All readings or values are good only up to this value.
    - For example, a vernier caliper has the least count of 0.01 cm and a screw gauge has a least count of 0.001 cm.
  - Using instruments of higher precision, improving experimental techniques, etc., we can reduce the least count error.
  - Repeating the observations several times and taking the arithmetic mean of all the observations, the mean value would be very close to the true value of the measured quantity.

4. Absolute Error

- If $a_1, a_2, a_3, \ldots\ldots\ldots, a_n$ be the values obtained for a physical quantity 'a' in an experiment repeated '*n*' times, then the arithmetic mean of the values is taken as the true value given by

$$a_0 = a_{mean} = \frac{a_1 + a_2 + \ldots\ldots + a_n}{n}$$

- The absolute error of a measurement is the difference between the individual measurement and the true value of that quantity denoted as $|\Delta a|$. The absolute errors in measurement values are

$$|\Delta a_1| = |a_0 - a_1|,\ |\Delta a_2| = |a_0 - a_2|,\ |\Delta a_3| = |a_0 - a_3|, \ldots\ldots, |\Delta a_n| = |a_0 - a_n|$$

Absolute error $|\Delta a|$ is always positive.

The arithmetic mean of all absolute errors of all the measurements is taken as the mean absolute error of the physical quantity '*a*'.

$$\Delta a_{mean} = \frac{\Delta a_1 + \Delta a_2 + \ldots\ldots + \Delta a_n}{n}$$

- The value of a physical quantity

$$a = a_{mean} \pm \Delta a_{mean}$$

5. Relative Error

The ratio of mean absolute error, Δa_{mean} to the mean value, a_{mean} of the physical quantity measured is called the relative error.

$$relative\ \ error = \frac{\Delta a_{mean}}{a_{mean}}$$

6. Percentage Error

The relative error of a physical quantity expressed in percentage is called percentage error.

$$percentage\ error = \frac{\Delta a_{mean}}{a_{mean}} \times 100$$

Problems

1. The measurement of length gives values of 2.54cm, 2.51cm, 2.48cm, 2.55cm, and 2.52cm. Find the absolute error, relative error, and percentage error.
2. The mean absolute error of a set of measurements is 0.85 and the mean value is 12.6. Find the relative error and percentage error.

Program : **Diploma in Engineering and Technology**	
Course Code : **1003**	Course Title: **Applied Physics-I**
Semester : **1**	Credits: **3**
Course Category: **Basic Science**	
Periods per week: **3 (L:3 T:0 P:0)**	Periods per semester: **45**

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Course Objectives:

- To provide students with a broad understanding of physical principles of the universe to help them develop critical thinking and quantitative reasoning skills.
- To help the diploma engineers in applying the basic concepts of physics to solve broad-based engineering problems

Module 1:

- Physical quantities - Fundamental and derived, Units and systems of units (CGS, MKS and SI units),
- Measurements - Errors in measurements- systematic and random errors (qualitative idea only), absolute error, relative error, percentage error, numerical problems
- Scalar and Vector quantities - Representation of vector, Collinear vectors, Coplanar vectors, equal vectors, unit vectors. Addition and Subtraction of Vectors, Triangle and Parallelogram law of addition, Resolution of a Vector.
- Equations of motion (elementary idea), Newton's laws of motion (no derivation), Force, Momentum, Statement and derivation of conservation of linear momentum, its applications - recoil of gun and rocket propulsion, Impulse and its examples (numerical problems related to force and momentum).

Questions?

- What is Physics?
  - Physics deals with the study of the basic laws of nature and their manifestation in different phenomena
- Scope?
  - Wide covering a tremendous range of magnitude of physical quantities
- Relation between Physics, technology and society
  - Technology gives rise to new physics: Physics generates new technology:- direct impact on society

Fundamental forces in nature

- Gravitational Force: force of attraction between objects (weakest force)
- Electromagnetic Force: force of attraction between charged particles
- Strong Nuclear Force: strongest force binds protons and neutrons
- Weak nuclear force: responsible for particle decay

Physical Quantities

- Any quantity which can be measured directly or indirectly in terms of which any laws of physics can be expressed is called physical quantity.
  - The property of an object that can be quantified.
    - Examples :
    - the length of a rod
    - the mass of a body.
    - The time taken to travel a distance
    - The current flowing through a conductor
    - The speed of a vehicle

- UNITS: Measurement of any physical quantity involves its comparison with a certain basic, reference standard called unit.
- Measurement: is the act of comparing a physical quantity with its unit.
- Measurement result is the value of a physical quantity obtained by means of measurement.

How are physical quantities classified?

- Scientists know many physical quantities, which are classified into **basic and derived.**
- The basic quantities are length, time, mass, electric current, temperature, luminous intensity and the amount of substance.
- Fundamental or base quantities are quantities which cannot be expressed in terms of any other physical quantities.

The measurements of physical quantities are expressed in terms of units, which are standardized values.

The distance of a race, which is a physical quantity, can be expressed in meters (for sprinters) or kilometers (for long distance runners).

Without standardized units, it would be extremely difficult for scientists to express and compare measured values in a meaningful way

Derived quantities

- They can be expressed in terms of fundamental quantities. The units of derived quantities are expressed in terms of fundamental units and they are called derived quantities.

Eg: Velocity, Force etc

International system of units (SI)

- This system of units was introduced in 1971 by the general conference on weights and measures and was internationally accepted.
- It has seven fundamental units along with two supplementary units.
- All physical quantities in the International System of Units (SI) are expressed in terms of combinations of seven fundamental physical units, which are units for: length, mass, time, electric current, temperature, amount of a substance, and luminous intensity

SI Base Units

Quantity	Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Temperature	kelvin	K
Amount of substance	mole	mol
Electric current	ampere	A
Luminous intensity	candela	cd
Plane angle	radian	rad
Solid angle	steradian	sr

Advantages of SI System of Units

- It is a coherent system of units.

  i.e., a system based on certain set of fundamental units.

- It is a rational system of units.

  i.e., it assigns only one unit to a particular physical quantity.

- SI is a metric system.

  i.e., multiples and submultiples of the system can be expressed as power of 10

System of Units

SYSTEM OF UNITS

	SI	MKS	CGS	BRITISH
LENGTH	METER	METER	CENTIMETER	FOOT
MASS	KILOGRAM	KILOGRAM	GRAM	POUND
TIME	SECOND	SECOND	SECOND	SECOND

Length – meter (m)

- The SI unit for length is the meter (m).
- The definition of the meter has changed over time to become more accurate and precise.
- It was first defined in 1791 as 1/10,000,000 of the distance from the equator to the North Pole.
- In 1889 it is redefined as the meter to be the distance between two engraved lines on a platinum-iridium bar. (The bar is now housed at the International Bureau of Weights and Measures, near Paris).

Length –meter (m)

- By 1960, some distances could be measured more precisely by comparing them to wavelengths of light. The meter was redefined as 1,650,763.73 wavelengths of orange light emitted by krypton atoms.
- In 1983, the meter was given its present definition as the distance

light travels in a vacuum in 1/ 299,792,458 of a second.

Mass- kilogram (kg)

- The SI unit for mass is the kilogram (kg).
- It is defined to be the mass of a platinum-iridium cylinder, housed at the International Bureau of Weights and Measures near Paris.
- Exact replicas of the standard kilogram cylinder are kept in numerous locations throughout the world, such as the National Institute of Standards and Technology in Gaithersburg, Maryland.
- The determination of all other masses can be done by comparing them with one of these standard kilograms.

Time : second (s)

- The SI unit for time, second (s).
- For many years it was defined as 1/86,400 of an average solar day. However, the average solar day is actually very gradually getting longer due to gradual slowing of Earth’s rotation.
- Accuracy in the fundamental units is essential, since all other measurements are derived from them. Therefore, a new standard was adopted to define the second in terms of a non-varying, or constant, physical phenomenon.
- One constant phenomenon is the very steady vibration of Cesium atoms, which can be observed and counted. This vibration forms the basis of the cesium atomic clock.
- In 1967, the second was redefined as the time required for 9,192,631,770 Cesium atom vibrations

- An atomic clock such as this one uses the vibrations of cesium atoms to keep time to a precision of one microsecond per year. The fundamental unit of time, the second, is based on such clocks. This image is looking down from the top of an atomic clock

Electric Current- ampere (A)

- Electric current is measured in the ampere (A), named after Andre Ampere.
- Understanding an ampere requires a basic understanding of electricity and magnetism.
- Basically, two parallel wires with an electric current running through them will produce an attractive force on each other.
- One ampere is defined as the amount of electric current that will produce an attractive force of 2.7 × 10–7 newton per meter of separation between the two wires (the newton is the derived unit of force).

Temperature- kelvin (K)

- The SI unit of temperature is the kelvin (or kelvins, but not degrees kelvin).
- This scale is named after physicist William Thomson, Lord Kelvin, who was the first to call for an absolute temperature scale.
- The Kelvin scale is based on absolute zero. This is the point at which all thermal energy has been removed from all atoms or molecules in a system.
- This temperature, 0 K, is equal to −273.15 °C and −459.67 °F. Conveniently, the Kelvin scale actually changes in the same way as the Celsius scale.
- For example, the freezing point (0 °C) and boiling points of water (100°C) are 100 degrees apart on the Celsius scale. These two temperatures are also 100 kelvins apart (freezing point = 273.15 K; boiling point = 373.15 K).

Luminous intensity -candela(cd)

- The candela is the SI's base unit for photometry.
- Defined as the luminous intensity in a given direction of a source that emits monochromatic radiation of frequency 540 × 10^12 hertz and has a radiant intensity in that same direction of 1/683 watt per steradian (unit solid angle).
- The candela is used to measure the visual intensity of light sources, like light bulbs or the bulbs in torches. It is the only SI base unit based on human perception.
- The candela converts the power of optical radiation to perceived luminance, originally defined as one candlepower (with the candle made of sperm whale wax)

- The ‘new candle’, or ‘candela’, was introduced in 1948 and later modified in 1979.
- The human eye has different sensitivities to different frequencies of light.
- The peak sensitivity is at approximately 540THz, which is in the greeny-yellow region of the spectrum. So we see this light more intensely than other colours of the same physical power.
- The ‘lumen’ is derived from the candela and measures total light in all directions from a source.

Amount of Substance- mole(mol)

- One mole (mol) contains exactly $6.02214076 \times 10^{23}$ elementary entities. This number is the fixed numerical value of the Avogadro constant, N_A, when expressed in the unit mol^{-1} and is called the Avogadro number.
- The amount of substance, symbol n, of a system is a measure of the number of specified elementary entities. ▫ An elementary entity may be an atom, a molecule, an ion, an electron, any other particle or specified group of particles.

- When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles. The SI unit of concentration (of amount of substance) is the mole per cubic meter (mol/m3).
- The SI unit of concentration (of amount of substance) is the mole per cubic meter (mol/m3)
- One mole (mol) contains exactly 6.02214076 × 10^23 elementary entities. This number is the fixed numerical value of the Avogadro constant, NA, when expressed in the unit mol−1 and is called the Avogadro number.
- The amount of substance, symbol n, of a system is a measure of the number of specified elementary entities. An elementary entity may be an atom, a molecule, an ion, an electron, any other particle or specified group of particles.

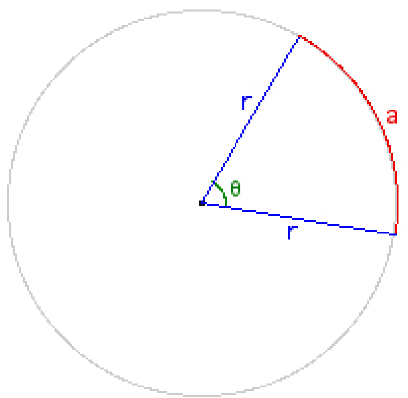
Plane angle -radian(rad)

- SI unit of plane angle is radian
- Radian describes the plane angle subtended by a circular arc, as the length of the arc divided by the radius of the arc.
- One radian is the angle subtended at the center of a circle by an arc that is equal in length to the radius of the circle.

One radian is $\frac{180°}{\pi}$ [approximately 57.3°].

One degree is $\frac{\pi}{180}$ rad [approximately 0.0175 rad].

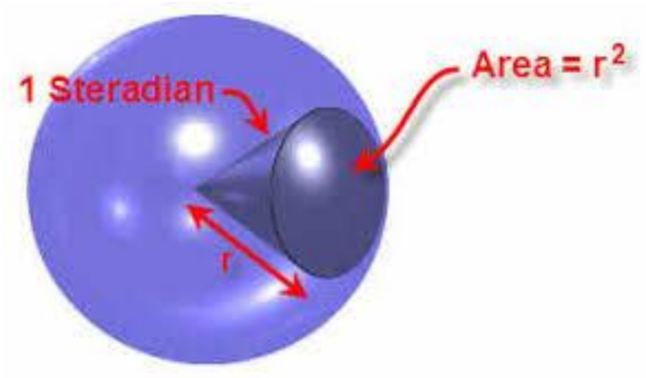
In the illustration below, θ is an angle described in radian measure.



$$\theta = \frac{a}{r}$$

Solid angle- steradian(sr)

- Steradian, unit of solid-angle
- measure in the International System of Units (SI), defined as the solid angle of a sphere subtended by a portion of the surface whose area is equal to the square of the sphere's radius.
- A steradian is (180/π)2 square degrees or about 3282.8 square degrees.



- Steradian, unit of solid-angle measure in the International System of Units(SI),
- Since the complete surface area of a sphere is 4π times the square of its radius, the total solid angle about a point is equal to 4π steradians.
- Derived from the Greek for solid and the English word radian, a steradian is, in effect, a solid radian;
- The radian is an SI unit of plane-angle measurement defined as the angle of a circle subtended by an arc equal in length to the circle's radius.

Introduction to Vectors and Scalars

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Physical quantities

- Vectors : Both magnitude and direction
  - Eg: displacement , velocity, force, acceleration
- Scalars: Quantities that have only magnitude
  - Eg: distance, speed, work, energy, power
- A vector quantity is denoted either using bold letters (**A, B**) or putting a small arrow → on the top of the symbol ($\vec{a}$) used for the representation of the quantity. The magnitude of a vector quantity, say F, is denoted by |**F**| or F

  Multiplication of two scalars will always be a scalar

  Multiplication of a scalar with a vector is a scalar

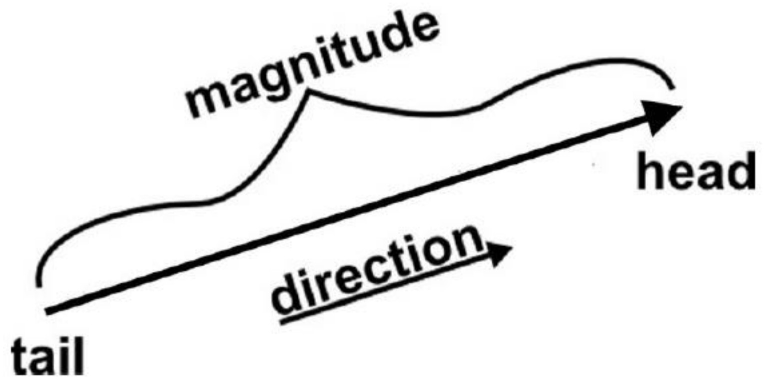
Examples

- **Distance** is the length of the path taken by an object
- **Displacement** is the **distance** between where the object started and where it ended up(depends on direction of motion).
- Displacement is defined as the change of position of a body in a direction
- For example, suppose you are travelling in a bus, 5 km east and then 3 km west. Then came back to your home through the same way. Then the total distance travelled is 16km while the total displacement is 0.

- **Speed (s)** is a scalar quantity,
  - It is the rate at which an object covers distance.
  - The average **speed** is the distance (a scalar quantity) per time ratio.
  - **Speed** is ignorant of direction.
  - It is the distance travelled by a body per unit time
  - **velocity** (v) is a vector quantity; it is direction dependent.
- It is defined as the displacement of a body per unit time
- Unit of both speed and velocity are the same

Geometric representation of a vector

- A vector quantity is represented graphically by a straight line with an Arrowhead
- The length of the straight line represents the magnitude of the vector and the arrowhead gives the direction of the vector.

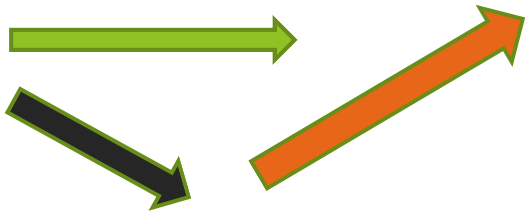


- Arrow mark is called the head and the other end is called the tail of the vector
- A vector can be displaced parallel to itself. Moving a vector parallel to itself does not change the magnitude and direction of the vector.

Type of Vectors

- **Collinear vectors**

Two or more vectors lying on the same line are called collinear vectors. They can have the same or different magnitude and the direction can be either the same or opposite.



- **Equal vectors**

Two vectors of the same magnitude and direction are called equal vectors.

- **Negative of a vector**

The negative of a vector is defined as another vector having the same magnitude but opposite in direction to the given vector.

Unit Vector

- A unit vector is a vector of unit magnitude and points in a particular direction.
- It is used just to specify a direction and hence it is also called a direction vector
- The unit $\hat{a}$ of a vector $\vec{A}$ is defined as $\hat{a} = \frac{\vec{A}}{|\vec{A}|}$
- The commonly used unit vectors are $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ which indicates X, Y, and Z directions respectively.

Addition of Vectors

- Addition of vectors results in getting the combined effect of the vectors known as resultant.
- **It is that single vector that would have the same effect as all the original vectors taken together**
- Vectors are added geometrically since directions are to be taken into account
- Methods of additions
  - Tail to head method
  - Triangle method
  - Parallellogram method

  - When we multiply two or more vectors, it is important to determine whether we want a product that has a scalar quantity or vector quantity.

Multiplication of Vectors

- There are actually three possible products in vector multiplication:
  - vector multiplied by a scalar factor giving a vector, (kA = B)
  - vector multiplied by a vector giving a scalar quantity (dot product)

  $A.B = C$

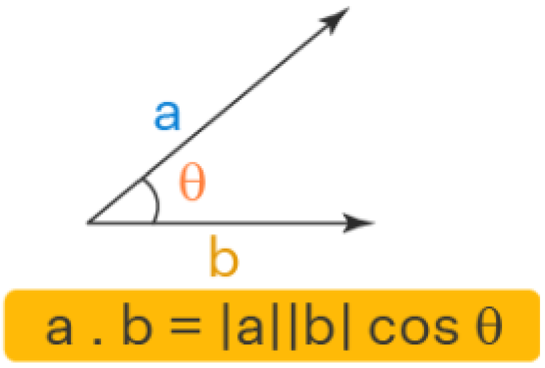
- vector multiplied by a vector giving a vector quantity (cross product)

  $A \times B = C$

- Cross Product between two vectors A and B is defined as

$$\vec{a} \times \vec{b} = |\vec{a}|\,|\vec{b}|\,\sin\theta\,\hat{n}$$

- Dot Product between two vectors A and B is defined as

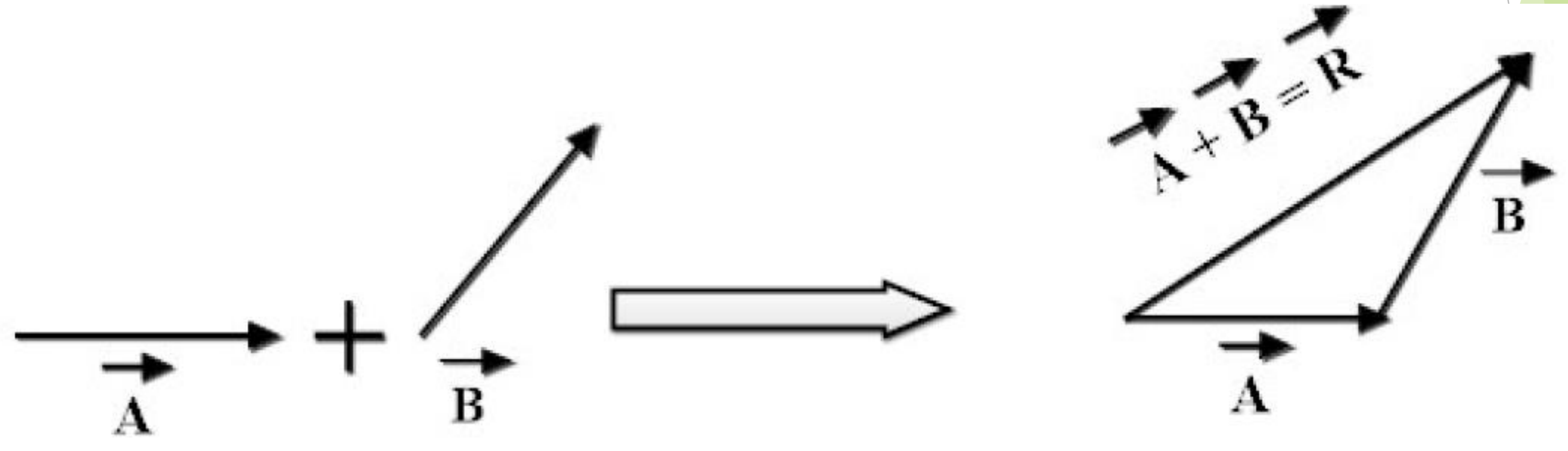


- Where θ is the angle between the two vectors and is a unit vector perpendicular to both A and B

Triangle method

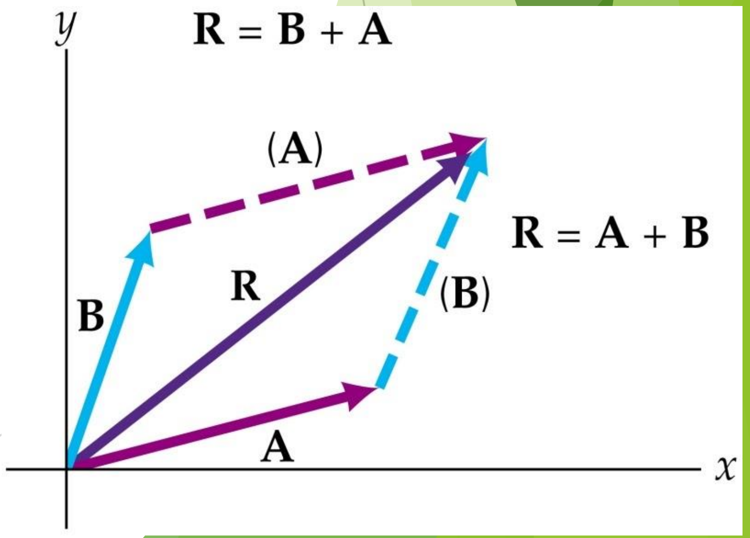
Let $\vec{A}$and $\vec{B}$ are two non-parallel vectors. To find the vector sum using the triangle method, place the vectors such that the tail of one vector coincides with the head of the other vector . Complete the triangle by drawing the third side. The third side gives the resultant vector$\vec{R}$.

The triangular law of vector addition states that if two vectors are represented by the adjacent sides of a triangle taken in order, then the resultant vector is represented both in magnitude and direction by the third side of the triangle taken in the reverse order.



Law of Parallelogram Vectors

- This method is based on the parallelogram law of vector addition.
- The parallelogram law of vector addition states that if two vectors are represented both in magnitude and direction by the two sides of a parallelogram drawn from a point, then the resultant vector is represented both in magnitude and direction by the diagonal of the parallelogram passing through the point.
- According to parallelogram law of vectors , their resultant vector will be represented by the diagonal of the parallelogram .



Parallelogram law of vector addition

- Consider two vectors $\vec{P}$ and $\vec{Q}$. Let θ be the angle between the two vectors. The resultant, $\vec{R}$, of the two vectors can be obtained by the parallelogram method as shown.

From right angled triangle *OCD*,

$$OC^2 = OD^2 + CD^2$$

$$= (OA + AD)^2 + CD^2$$

$$= OA^2 + AD^2 + 2.OA.AD + CD^2 \quad \ldots(1)$$

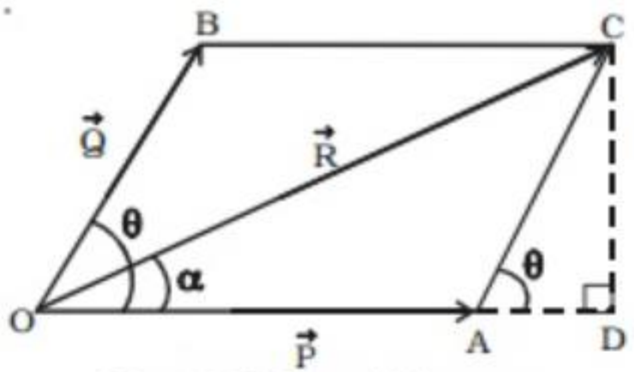


Fig 2.15 Parallelogram law of vectors

In Fig. 2.15 $\angle BOA = \theta = \angle CAD$

From right angled Δ CAD,

$$AC^2 = AD^2 + CD^2 \quad \ldots(2)$$

Substituting (2) in (1)

$$OC^2 = OA^2 + AC^2 + 2OA.AD \quad \ldots(3)$$

From ΔACD,

$$CD = AC \sin\theta \quad \ldots(4)$$

$$AD = AC \cos\theta \quad \ldots(5)$$

Substituting (5) in (3) $OC^2 = OA^2 + AC^2 + 2\,OA.AC\cos\theta$

Substituting $OC = R$, $OA = P$,

$OB = AC = Q$ in the above equation

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

(or) $$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \qquad \text{...(6)}$$

Equation (6) gives the magnitude of the resultant. From $\Delta\, OCD$,

$$\tan\alpha = \frac{CD}{OD} = \frac{CD}{OA + AD}$$

Substituting (4) and (5) in the above equation,

$$\tan\alpha = \frac{AC\sin\theta}{OA + AC\cos\theta} = \frac{Q\sin\theta}{P + Q\cos\theta}$$

The direction of the resultant vector is specified by the angle α with respect to the vector $\vec{P}$. The angle α is given by the expression

(or) $$\alpha = \tan^{-1}\left[\frac{Q\sin\theta}{P + Q\cos\theta}\right] \qquad \text{...(7)}$$

Equation (7) gives the direction of the resultant.

Special Cases

a) If two vectors are in the same direction, then $\theta = 0$ and hence, $cos\theta = 1$

The magnitude of the resultant, $R = \sqrt{A^2 + B^2 + 2ABcos\theta}$

$$R = \sqrt{A^2 + B^2 + 2AB}$$

$$R = \sqrt{(A + B)^2}$$

$$R = A + B$$

The magnitude of the resultant is the sum of the magnitudes of the two vectors.

b) If two vectors are in opposite direction, then $\theta = 180^{\circ}$ and hence, $cos\theta = -1$

The magnitude of the resultant, $R = \sqrt{A^2 + B^2 + 2ABcos\theta}$

$$R = \sqrt{A^2 + B^2 - 2AB}$$

$$R = \sqrt{(A - B)^2}$$

$$R = A - B$$

The magnitude of the resultant is the difference of the magnitudes of the two vectors.

C) When P and Q are in perpendicular directions, θ= 90

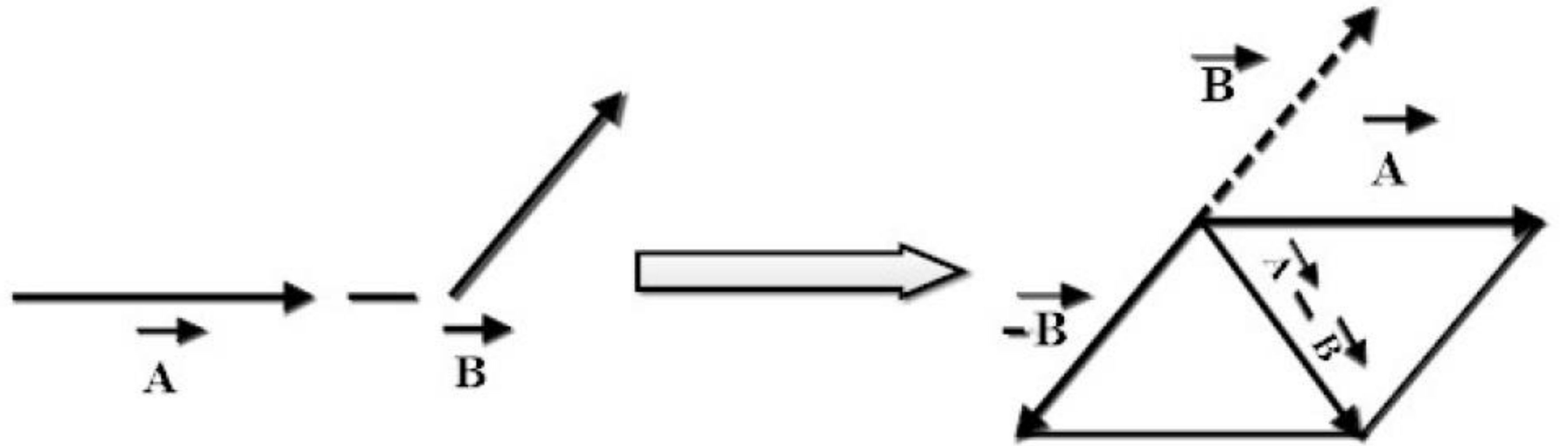
Resultant $R = \sqrt{P^2 + Q^2}$

we get minimum effect

Subtraction of two vectors

Subtraction of two vectors also involves addition. To subtract $\vec{B}$ from $\vec{A}$ first, take the negative of $\vec{B}$ and then add it to $\vec{A}$. Hence, subtraction of two vectors is the same as the addition of a vector with the negative of the second vector.

$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$



Resolution of a vector

- Two or more vectors can be combined to form a single vector through addition.
- Similarly, a given vector can be represented as the sum of two or more vectors acting along different directions.
- The process of splitting a given vector into two or more vectors along different directions is called the resolution of a vector.
- The vectors obtained by the resolution of the given vector are called component vectors.

- A vector lying in a plane is usually resolved along two mutually perpendicular directions. The resolution of a vector along mutually perpendicular directions is called rectangular resolution.
- The two perpendicular components are called rectangular components. The rectangular components are taken along the X-axis and Y-axis.
- Consider a vector $\vec{A}$ making an angle θ with the X-axis.

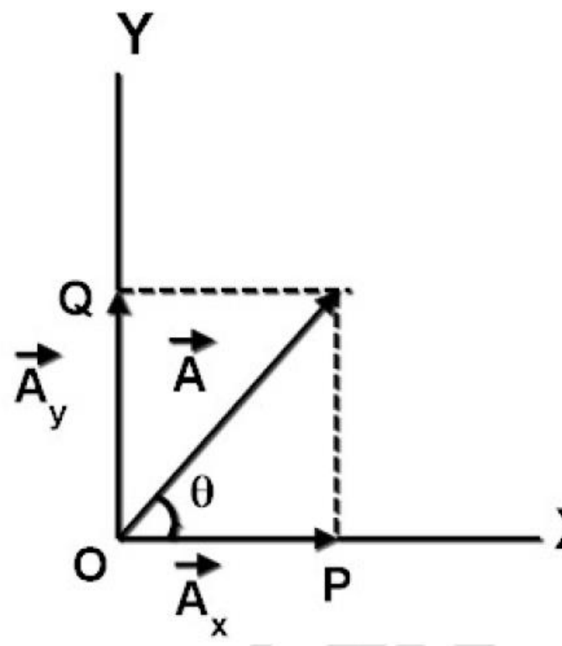
Draw perpendiculars from the head of the vector $\vec{A}$

to X-axis and Y axis to meet at the points P and Q respectively.

Then, if OP and OQ are taken as two vectors $\vec{A_x}$ and $\vec{A_y}$

respectively, then by parallelogram law of vector addition,

$\vec{A}$ is the resultant vector.



- $\vec{A} = \vec{A_x} + \vec{A_y}$
- Thus and $\vec{A_x}$ and $\vec{A_y}$ are vector components of $\vec{A}$.
- Magnitudes of $\vec{A_x}$ and $\vec{A_y}$ are called scalar components.
- A_x and A_y are called x-component and y-component respectively.
- Using simple trigonometric relations, x-component and y-component of vector $\vec{A}$ is given by

$$A_x = A\,cos\theta$$

$$A_y = A\,sin\theta$$

Dynamics

Dr.Jinchu.I

Dynamics

- Dynamics is a branch of mechanics that deals with the study of forces and its effect on the motion of bodies.
- In dynamics, Newton's laws of motion are three laws that describe the relationship between the motion of an object and the forces acting on it.

Newton's first law of motion

Newton's first law of motion states that everybody continues in its state of rest or of uniform motion along a straight line unless an unbalanced force acting on it.

Newton's first law of motion is also known as the law of inertia.

Inertia is an inherent property of all bodies.

Nobody can change its state of motion by itself and only an external force can change its state of motion.

Inertia is the resistance of a body to any change in its state of rest or uniform motion along a straight line. In the absence of a net external force, a body at rest continues to remain at rest and a body in motion continues to move with constant velocity.

- The inertia of an object is measured by its mass.
- The tendency of a body to remain in its existing state of rest is called inertia of rest.
- The tendency of a body to remain in its existing state of motion with constant velocity is called inertia of motion.
- Examples:
- A) A person standing in a stationary bus falls backward when the bus starts suddenly. This is because the lower part of his body moves forward with the bus, but the upper part of his body remains at rest due to inertia of rest, which results in the backward fall.
- b) A person trying to get down from a moving bus falls forward. The lower part suddenly comes to rest on touching the ground, but the upper part of his body remains in motion due to inertia of motion and the person falls forward.
- c) Fruits from a tree fall due to inertia of rest when the tree is shaken. Both the fruits and branches are at rest, but when shaken branches start moving whereas fruits remain in its state of rest and are separated from the branches.

Momentum(p)

- Momentum is the quantity of motion of a body.
- The momentum of a body is defined as the product of mass and velocity. It is a vector quantity and its unit is kg m/s.
- If a body of mass ‘m’ moving with a velocity ‘v’, then its momentum is given by $p = m\mathrm{v}$
- The momentum of a body at rest is zero.
- Force is related to momentum or more specifically change in momentum. Newton’s second law gives the relation between force and momentum.

Newton's second law of motion

- *Newton's second law of motion states that the rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction of the force.*

force $\propto$ $rate\ of\ change\ of\ momentum$

force is proportional to the product of mass and acceleration

$$F \propto ma$$

$F = k\ ma$ where k is the constant of proportionality.

- By suitably defining the SI unit of force, we can take k as 1. Thus, $F = ma$
- One unit of force is defined as that which causes an acceleration of $1m/s^2$ in a body of mass 1kg. This unit is known as newton (N). $1\ N = 1\ kg\ m/s^2$
- If 'p' is the momentum of the body, then Newton's second law can be expressed in differential form as

$$F = dp\ /dt$$

- The second law implies that if Force F = 0, then acceleration, a = 0, which means the body is either at rest or in uniform motion.
- Thus, Newton's first law can be derived from the second law. Newton's first law gives a qualitative idea of force and the second law gives a mathematical expression for force.

Newton’s Third law of motion

<u>Newton’s third law of motion</u>

Newton’s third law of motion states that to every action, there is always an equal and opposite reaction.

- If a body B exerts a force, F_{AB} on a body A, then the body A exerts an equal and opposite force, F_{BA} on body B.

$$F_{AB} = -F_{BA}$$

The main properties of action and reaction forces are:

a) Action and reaction are the simultaneously occurring pair of forces acting between two objects.

b) Forces always occur in pairs and a single force doesn’t exist in the universe. This is an important property of forces.

c) Action and reaction are always equal in magnitude and opposite in direction.

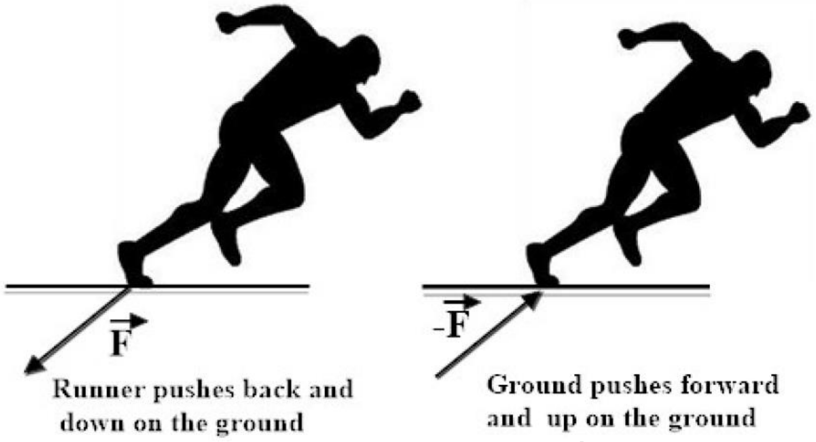
d) There is no cause-effect relation implied in the third law. Both action and reaction occur at the same time. So, any of the two forces can be called action and the other reaction.

e) The action and reaction forces, though equal and opposite, never adds up to get zero. Action and reaction do not cancel each other since they act on different objects.

Examples

1. When a man jumps off a boat to the shore, he exerts a force on the boat. The boat exerts an equal and opposite force on the man which makes the jump possible. The boat moves backward due to the force exerted by the man.

2. A runner exerts a force on the ground and the reaction force of the ground on the runner pushes him forward.



Law of conservation of momentum

- Newton's second law leads to Law of conservation of momentum
- Statement: Total momentum before collision between two or more bodies is equal to total momentum after collision

Or

When a group of bodies are exerting force to each other, their total momentum remains conserved in the absence of external forces.

$$force = \frac{Change\ in\ momentum}{time}$$

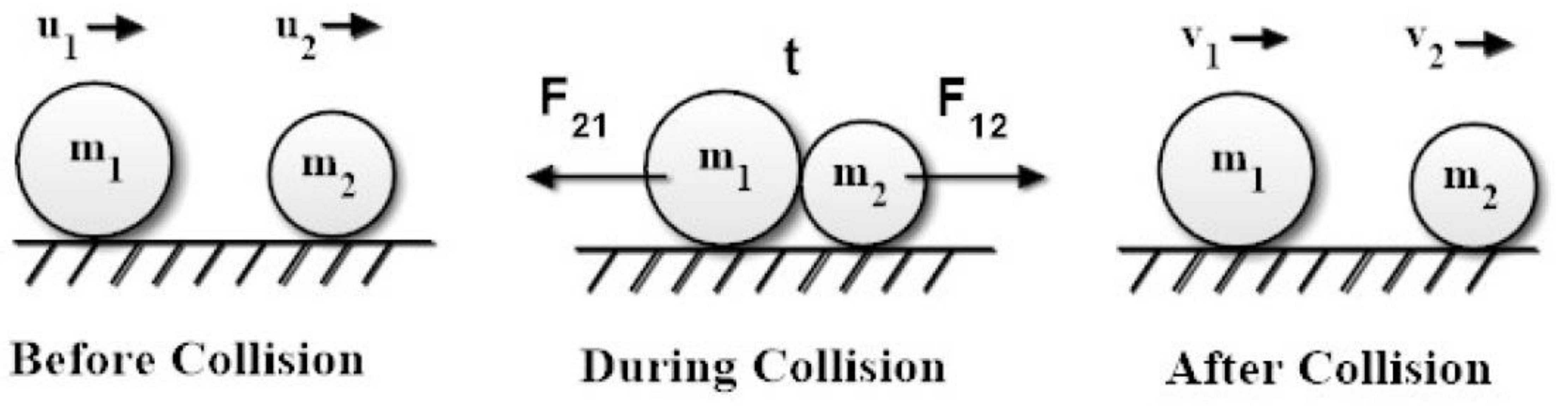
If the net force acting on the system is zero, then the change in momentum also becomes zero. Ie,

$$final\ momentum = initial\ momentum$$

- Thus, the law of conservation of momentum states that if the net external force acting on a system is zero, its linear momentum remains constant.

Proof of law of conservation of momentum

- Consider two bodies of masses $m_1$and m_2 moving along a straight line with velocities u_1 and u_2 respectively. Let the bodies collide for a time t seconds. After the collision, the velocities become v_1 and v_2 respectively for masses $m_1$and m_2 along the same direction as shown in Figure.



- Since there is no external force acting on the system of two colliding bodies, the bodies apply internal forces on each other during the collision. Let the force acting on the mass $m1$ (applied by m_2) be F_{12} and the force acting on the mass m_2 (applied by m_1) be F_{21}. From Newton's second law of motion,

$$\text{Force} = \frac{change\ in\ momentum}{time}$$

$$F_{12} = m_1 \frac{(u_1 - v_1)}{t} \qquad F_{21} = m_2 \frac{(u_2 - v_2)}{t}$$

- According to Newtons third law F_{12} =$-F_{21}$

$$m_2 \frac{(u_2 - v_2)}{t} = -m_1 \frac{(u_1 - v_1)}{t}$$

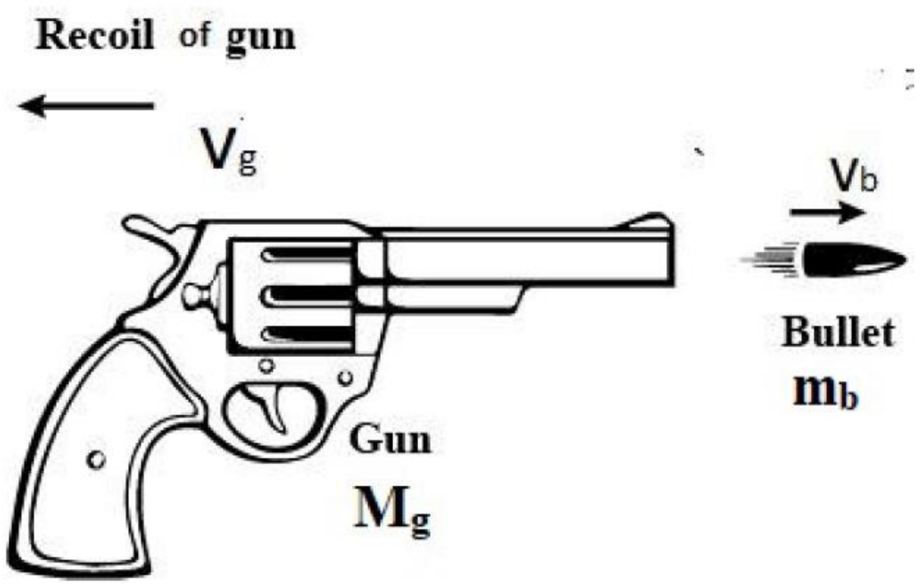
$$m_2 (u_2 - v_2) = -m_1 (u_1 - v_1)$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

Ie., Total momentum before collision = Total momentum after collision

Recoil of Gun

- The backward motion of a gun when a bullet is fired from it is called the recoil of the gun. It can be explained using the principle of conservation of linear momentum.
- The total momentum of the gun and bullet before firing is zero. Since no external force acts on the gun and the bullet, its momentum should be conserved.
- After firing the bullet moves with a velocity producing momentum in the forward direction.
- To balance the momentum change, the gun moves backward with a velocity, such that the total momentum is zero



Let M_g and m_b are masses of the gun and bullet respectively. Suppose, a bullet is fired from the gun with a velocity v_b and the gun recoils with a velocity V_g.

$$Total\ momenta\ before\ firing = 0$$

$$Total\ momenta\ after\ firing = M_g V_g + m_b v_b$$

By the law of conservation of momentum, the total momenta after firing must be equal to the total momenta before firing.

$$M_g V_g + m_b v_b = 0$$

$$M_g V_g = -m_b v_b$$

$$V_g = -\frac{m_b v_b}{M_g}$$

The negative sign shows that the **direction of recoil velocity of** the gun is opposite to the direction of the velocity of the bullet.

Rocket Propulsion

- Rockets are used to launch artificial satellites and space shuttles, deliver explosive warheads to their targets, and also for human space flight and scientific exploration of outer space.
- The principle behind rocket propulsion is the law of conservation of momentum (external force on rocket is zero and effect of gravity is neglected).

  - Rocket can be considered as a system of particles in which mass is varying during its motion
  - In a rocket fuel is burned and the exhaust gas is expelled out from the rear end of the rocket
  - The force exerted by the exhaust gas on the rocket is equal and opposite to the force exerted by the rocket to expel it.
  - This force exerted by the exhaust gas on the rocket propels the rocket forwards
  - When more gas is ejected from the rocket ,the mass of the rocket decreases.

- Let T be the reactional force(Thrust)
- Mg, the **weight** of the rocket
- Then the force that drives the rocket $= T\text{-}Mg$
- Then according to Newton's law $Ma = T\text{-}Mg$
- Then the acceleration of the rocket

$$a = \frac{T - Mg}{M}$$

- The mass of the rocket decreases due to consumption of fuel . Also as the rocket goes up, g decreases. But the thrust remains the same. Hence the acceleration of the rocket increases as time advance

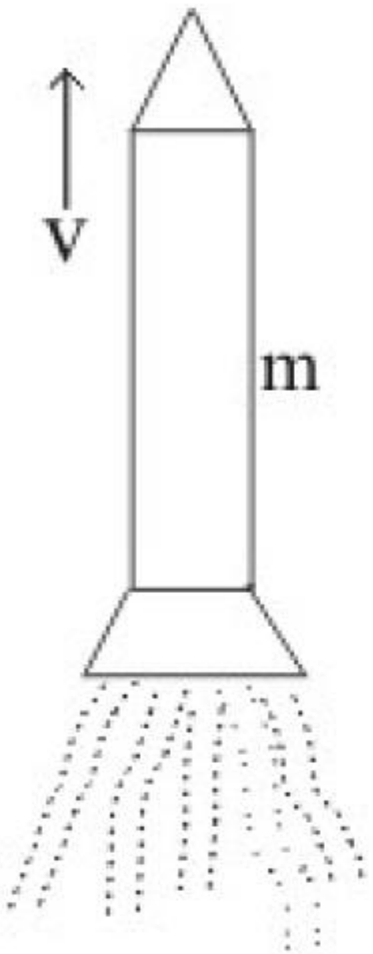


Figure 2a. Rocket at time t after takeoff with mass m and velocity v in upwards direction

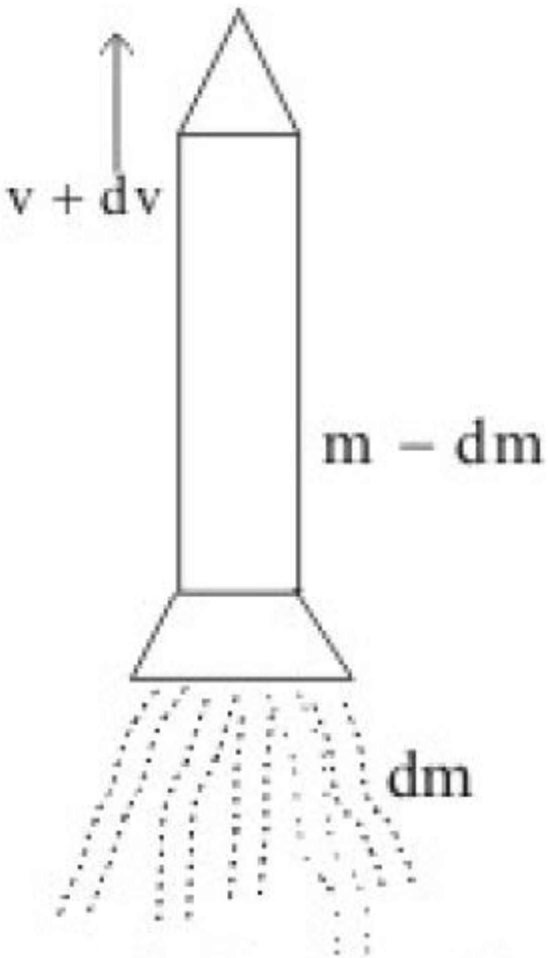


Figure 2b. Rocket at time t+Δt after takeoff with mass $m - dm$ velocity v+dv in upwards direction

Impulse

- A large force acting for a short interval of time is called an impulsive force.

Impulse = Force x time

Unit= Newton second (Ns)

$$I = Ft = ma \times t = m\frac{(v-u)}{t}t = m(v-u) = mv - mu$$

Impulse = change in momentum

- Examples of impulsive forces are
- Kicking a football: A footballer exerts a large force on the ball, but only for a very short interval of time.
- Striking a nail with a hammer: For fixing a nail, a hammer is used to exert a large force for a small time.
- Striking a ball with a bat: The momentary force exerted by the bat changes the direction of the ball in a small interval.