$$\frac{1}{6} \lim_{\theta \to 0} \left(4 \frac{\sin 4\theta}{4\theta} + 2 \cdot \frac{\sin 2\theta}{2\theta} \right)$$

$$= \frac{1}{6} \left[4 \lim_{4\theta \to 0} \frac{\sin 4\theta}{4\theta} + 2 \cdot \lim_{2\theta \to 0} \frac{\sin 2\theta}{2\theta} \right]$$

$$= \frac{1}{6} [4 + 2] = 6/6 = 1$$

. Evaluate $\lim_{\theta \to \pi/2} \frac{\cos \theta}{(\pi/2 - \theta)}$.

$$\cos \theta = \sin(90 - \theta)$$
$$= \sin(\pi/2 - \theta).$$

$$\lim_{\theta \to \pi/2} \frac{\sin(\pi/2 - \theta)}{(\pi/2 - \theta)} = \lim_{\pi/2 - \theta \to 0} \frac{\sin(\pi/2 - \theta)}{(\pi/2 - \theta)}$$
$$= 1.$$

. Evaluate $\lim_{x \to 0} \frac{1 - \cos 2x}{x^2}$.

$$\sin^2 x = \frac{1 - \cos 2x}{2}.$$

$$\therefore 2 \sin^2 x = 1 - \cos 2x.$$

$$\lim_{x\to 0} 2\frac{\sin^2 x}{x^2} = 2\cdot\lim_{x\to 0}\frac{\sin^2 x}{x^2}$$

$$= 2\cdot\lim_{x\to 0}\left(\frac{\sin x}{x}\cdot\frac{\sin x}{x}\right)$$

$$= 2\left[\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{\sin x}{x}\right]$$

$$= 2[1\times 1] = \underline{\underline{2}}.$$

Q. Evaluate $\lim_{\theta\to 0}\frac{\sin 3\theta\cos\theta}{\theta}$.

$$\lim_{\theta\to 0}\frac{\sin 3\theta}{\theta}\cdot\cos\theta.$$

$$\lim_{\theta\to 0}\frac{\sin 3\theta}{\theta}\cdot\lim_{\theta\to 0}\cos\theta.$$

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$$\lim_{3\theta\to 0} 3\cdot\frac{\sin 3\theta}{3\theta}\cdot\lim_{\theta\to 0}\cos\theta.$$

$$3\cdot\lim_{3\theta\to 0}\frac{\sin 3\theta}{3\theta}\cdot\lim_{\theta\to 0}\cos\theta.$$

$$= 3\times 1\times 1$$

$$= \underline{\underline{3}}$$

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Questions:

1. $\lim_{x \to 2} \frac{x^2 - 2x}{x - 2}$

2. $\lim_{x \to 2} \frac{x^3 - 8}{x^2 - 4}$

3. $\lim_{x \to a} \frac{x^m - a^m}{x^n - a^n}$

4. $\lim_{x \to \infty} \frac{x^2 - 2x + 3}{3x^2 - 2x}$.

5. $\lim_{\theta \to 0} \frac{\sin 5\theta}{\theta}$

6. $\lim_{\theta \to 0} \frac{\sin 2\theta \cos 5\theta}{\theta}$.

7. $\lim_{\theta \to 0} \frac{\tan 3\theta}{\theta}$.

8. $\lim_{x \to 3} \frac{x^2 - 3x}{x^2 - 9}$

9. $\lim_{\theta \to 0} \frac{\sin a\theta}{\tan b\theta}$

10. $\lim_{x \to \infty} \frac{(3x+1)(2x+4)}{(x+1)(x-7)}$

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Differentiation :

The differential coefficient or derivative of y with respect to x is denoted by $\frac{dy}{dx}$.

The process of finding $\frac{dy}{dx}$ is called differentiation.

Rules of differentiation.

1. $\frac{d}{dx}(u \pm v) = \frac{d}{dx}(u) \pm \frac{d}{dx}(v)$.

2. $\frac{d}{dx}(ku) = k \cdot \frac{du}{dx}$.

Formulae.

1. $\frac{d}{dx}(x^n) = n \cdot x^{n-1}$

2. $\frac{d}{dx}(x) = 1$

3. $\frac{d}{dx}(k) = 0$.

4. $\frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{1/2}) = \frac{1}{2} x^{1/2 - 1} = \frac{1}{2} x^{-1/2}$

$= \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$

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5. $\frac{d}{dx}(\sin x) = \cos x$

6. $\frac{d}{dx}(\cos x) = -\sin x$.

1. Find the derivatives of
(a) x (b) x^4 (c) $\frac{1}{x^3}$ (d) $\frac{1}{\sqrt{x}}$ (e) 5.

(a) $\frac{d}{dx}(x) = \underline{\underline{1}}$.

By the standard formula

$\frac{d}{dx}(x^n) = n \cdot x^{n-1}$

$\frac{d}{dx}(x^1) = 1 \cdot x^{1-1} = 1 \cdot x^0 = 1 \cdot 1$

$= \underline{\underline{1}}$

(b) $\frac{d}{dx}(x^4)$

$= 4x^3$ $\quad \frac{d}{dx}(x^4) = 4 \cdot x^{4-1}$

$= 4 \cdot x^3$

(c) $\frac{d}{dx}\left(\frac{1}{x^3}\right) = \frac{d}{dx}(x^{-3})$

$= -3 \cdot x^{-3-1}$

$= -3x^{-4}$

$= \underline{\underline{\frac{-3}{x^4}}}$.

(d) $\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right) = \frac{d}{dx}\left(\frac{1}{x^{1/2}}\right) = \frac{d}{dx}\left(x^{-1/2}\right)$

$= \frac{-1}{2} x^{-1/2 - 1}$

$= \frac{-1}{2} x^{-3/2}$

$= \frac{-1}{2 \times x^{3/2}}$

(d) $\frac{d}{dx}(5) = 0.$ $\Rightarrow \frac{d}{dx}(K) = 0$

Find the derivatives of

(a) $\frac{d}{dx}$ $x^2 + 2x + 1$ (b) $3x^6 + \frac{1}{x} - 5$

(c) $\frac{4}{x^3} + 5x^{10} - \frac{1}{\sqrt{x}}$ (d) $4x^{10} - 3x^5 - 2.$

(a) $\frac{d}{dx}(x^2 + 2x + 1) = \frac{d}{dx}(x^2) + \frac{d}{dx}(2x) + \frac{d}{dx}(1)$

$= 2 \cdot x^{2-1} + 2 \cdot \frac{d}{dx}(x) + 0$

$= 2x + 2 \times 1 \cdot x^{1-1} + 0$

$= 2x + 2x^0 = \underline{\underline{2x + 2}}$

(b) $\frac{d}{dx}\left(3x^6 + \frac{1}{x} - 5\right) = 3 \cdot \frac{d}{dx}(x^6) + \frac{d}{dx}(1/x)$

$= 3 \cdot 6 \cdot x^{6-1} + \frac{d}{dx}(x^{-1})$

$= 18x^5 + -1 \cdot x^{-1-1} = 18x^5 - x^{-2}$

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$$= 18x^5 - \frac{1}{x^2}$$

(c) $\frac{d}{dx}\left(\frac{4}{x^3} + 5x^{10} - \frac{1}{\sqrt{x}}\right)$.

$$4 \cdot \frac{d}{dx}\left(\frac{1}{x^3}\right) + 5 \cdot \frac{d}{dx}(x^{10}) - \frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)$$

$$= 4 \frac{d}{dx}(x^{-3}) + 5 \cdot 10 \cdot x^{10-1} - \frac{d}{dx}\left(\frac{1}{x^{1/2}}\right)$$

$$= 4\left(-3 \cdot x^{-3-1}\right) + 5 \cdot 10x^9 - \frac{d}{dx}(x^{-1/2})$$

$$= 4(-3 \cdot x^{-4}) + 50x^9 - \left(-\frac{1}{2} x^{-1/2-1}\right)$$

$$= 12x^{-4} + 50x^9 + \frac{1}{2} x^{-3/2}$$

$$= \frac{12}{x^4} + 50x^9 + \frac{1}{2 \cdot x^{3/2}}$$

(d) $\frac{d}{dx}(4x^{10} - 3x^5 - 2)$.

$$4 \cdot \frac{d}{dx}(x^{10}) - 3\frac{d}{dx}(x^5) - \frac{d}{dx}(2).$$

$$= 4 \cdot 10x^{10-1} - 3(5 \cdot x^{5-1}) - 0$$

$$= 4 \times 10x^9 - 3(5x^4)$$

$$= 40x^9 - 15x^4$$

Q. Find the derivative of $x^2 + \sin x$.

$$\frac{d}{dx}(x^2 + \sin x)$$

$$= \frac{d}{dx}(x^2) + \frac{d}{dx}(\sin x)$$

$$= 2 \cdot x^{2-1} + \cos x.$$

$$= 2x + \cos x.$$

Q. Find the (a) $\frac{d}{dx}(2 + \cos x)$.

(b) $\frac{d}{dx}(x^2 + 2\cos x + 5)$

(c) $\frac{d}{dx}(\sqrt{x} + 2\sin x + 5)$

(a) $\frac{d}{dx}(2 + \cos x) = \frac{d}{dx}(2) + \frac{d}{dx}(\cos x)$

$$= 0 - \cancel{\cos x}\ \sin x = -\sin x$$

$$= \cancel{-\cos x}.$$

(b). $\frac{d}{dx}(x^2 + 2\cos x + 5)$

$$= \frac{d}{dx}(x^2) + 2 \cdot \frac{d}{dx}(\cos x) + \frac{d}{dx}(5)$$

$$= 2x^{2-1} + 2 \cdot (-\sin x) + 0$$

$$= 2x - 2\sin x.$$

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(e) $\frac{d}{dx}(\sqrt{x}) + 2 \cdot \frac{d}{dx}(\sin x) + \frac{d}{dx}(5)$

$\frac{d}{dx}(x^{1/2}) + 2 \cdot (\cos x) + 0.$

$\frac{1}{2} x^{1/2 - 1} + 2\cos x.$

$\frac{1}{2} x^{-1/2} + 2\cos x$

$= \frac{1}{2x^{1/2}} + 2\cos x$

$= \frac{1}{2\sqrt{x}} + 2\cos x.$

Product rule.

If u and v are two functions of x, then

$$\frac{d}{dx}(uv) = u \cdot \frac{d}{dx}(v) + v \cdot \frac{d}{dx}(u)$$

(1^{st} function $\times$ derivative of 2^{nd} + 2^{nd} function $\times$ derivative of 1^{st})

Q. $\frac{d}{dx}(x^2 \sin x) = x^2 \cdot \frac{d}{dx}(\sin x) + \sin x \cdot \frac{d}{dx}(x^2)$

$= x^2 \cdot (\cos x) + \sin x \cdot 2 \cdot x^{2-1}$

$= x^2 \cos x + \sin x \cdot 2 \cdot x$

$= x^2 \cos x + 2x \sin x$

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$$\frac{d}{dx}(\sin x \cdot \cos x).$$

$$= \sin x \cdot \frac{d}{dx}(\cos x) + \cos x \cdot \frac{d}{dx}(\sin x)$$

$$= \sin x \cdot (-\sin x) + \cos x \cdot (\cos x)$$

$$= -(\sin x)^2 + (\cos x)^2$$

$$= -\sin^2 x + \cos^2 x$$

$$\frac{d}{dx}(x^2 + 2x\sin x + \cos x).$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(2x\sin x) + \frac{d}{dx}\cos x$$

$$2x + 2\cdot\frac{d}{dx}(x\sin x) + (-\sin x)$$

$$= 2x + 2\left[x \cdot \frac{d}{dx}(\sin x) + \sin x \cdot \frac{d}{dx}(x)\right] - \sin x$$

$$= 2x + 2[x \cdot \cos x + \sin x] - \sin x$$

$$= 2x + 2x\cos x + 2\sin x - \sin x$$

$$= 2x + 2x\cos x + \sin x$$