Problems of the above section.

1. For what values of k shall the three lines $5x+2y-4=0$, $2x+ky+11=0$ and $3x-4y-18=0$ are concurrent.

Consider $5x+2y=4$ and $3x-4y=18$

$$\Delta = \begin{vmatrix} 5 & 2 \\ 3 & -4 \end{vmatrix} = -20-6=-26.$$

$$\Delta_1 = \begin{vmatrix} 4 & 2 \\ 18 & -4 \end{vmatrix} = -16-36 = -52$$

$$\Delta_2 = \begin{vmatrix} 5 & 4 \\ 3 & 18 \end{vmatrix} = 90-12 = \underline{\underline{78}}$$

$$\therefore x = \frac{\Delta_1}{\Delta} = \frac{-52}{-26} = 2 //$$

$$y = \frac{\Delta_2}{\Delta} = 78/_{-26} = -3 //$$

$(2,-3)$ is the point of intersection of two lines. Substitute these points in the equation ~~$2x$~~ $2x+ky+11=0$.

$$(x,y) = (2,-3)$$

$$2\times2 + k\times-3+11=0.$$

$$4-3k+11=0.$$

$15-3k=0$, $+3k=+15 \implies k=15/3=5 //$

2. Find the value of p if $(2p+1)x-(5-p)y=8$ and $(5p-1)x-(p+1)y=3$ are parallel.

Two lines are parallel, if

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}.$$

$a_1 = 2p+1$, $b_1 = -(5-p)$

$a_2 = 5p-1$, $b_2 = -(p+1)$.

$$\frac{2p+1}{5p-1} = \frac{+(5-p)}{+(p+1)}$$

$$(2p+1)(p+1) = (5-p)(5p-1).$$

$$2p^2+2p+p+1 = 25p-5-5p^2+p$$

$$2p^2+3p+1 = -5-5p^2+26p.$$

$$2p^2+\cancel{3p}+1+5+5p^2-26p=0.$$

$$7p^2-23p+6=0.$$

$$p=\frac{-b\pm\sqrt{b^2-4ac}}{2a},\quad p=\frac{23\pm\sqrt{529-168}}{14}$$

$$p=\frac{23\pm\sqrt{361}}{14}=\frac{23\pm19}{14}$$

$$=\frac{23+19}{14},\ \frac{23-19}{14}$$

$$=\frac{42}{14},\ \frac{4}{14}$$

$$=3,\ 2/7$$

$$\begin{array}{r}24\\\times 7\\\hline 168\end{array}\qquad\begin{array}{r}23\\\times 23\\\hline 69\\46\\\hline 529\end{array}$$

Q. Find the value of q for which the straight lines $8qx + (2-3q)y + 1 = 0$ and $qx + 8y + 7 = 0$ are perpendicular

Two straight lines are perpendicular, the product of two slope is equal to -1

or.

$\therefore a_1 a_2 + b_1 b_2 = 0.$

$a_1 = 8q \quad a_2 = q, \quad b_1 = (2-3q), \quad b_2 = 8.$

$8q \times q + (2-3q) \cdot 8 = 0.$

$8q^2 + 16 - 24q = 0.$

$8q^2 - 24q + 16 = 0.$

$q^2 - 3q + 2 = 0 \qquad \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}.$

$\dfrac{3 \pm \sqrt{9-8}}{2} = \dfrac{3+1}{2}, \dfrac{3-1}{2}.$

$= 2, 1$

Q. Find the equation of the lines passing through the point of intersection of the lines $x - y + 1 = 0$ and $2x + 3y + 2 = 0$. and parallel to $x + y - 6 = 0$

The point of intersection of

$x-y+1=0$ and $2x+3y+2=0$ is

$x-y=-1$, $2x+3y=-2$.

$$\Delta = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3+2 = 5$$

$$\Delta_1 = \begin{vmatrix} -1 & -1 \\ -2 & 3 \end{vmatrix} = -3-2 = -5.$$

$$\Delta_2 = \begin{vmatrix} 1 & -1 \\ 2 & -2 \end{vmatrix} = -2+2 = 0$$

$$x = \frac{\Delta_1}{\Delta}, \quad y = \frac{\Delta_2}{\Delta}$$

$x = -5/5$, $y = 0/5$

$x = -1$, $y = 0$

$(-1, 0)$ is the point of intersection

Any line parallel to $x+y-6=0$ is

$x+y+k=0$ ...(1). Since it passes through

$(-1,0) \Longrightarrow x+y+k=0$.

~~substituting~~ $(-1)+0+k=0$

$k=1$

$(1) \Longrightarrow x+y+1=0$ is the required equation.

2. Find the equation of the line through the intersection of the lines $2x+3y=1$, $3x+4y=6$ and perpendicular to $5x-2y=7$.

The point of intersection of $2x+3y=1$ and $3x+4y=6$ is

$$2x+3y=1$$
$$3x+4y=6.$$

$\Delta = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = 8-9 = -1$, $\quad x = \frac{\Delta_1}{\Delta} = \frac{-14}{-1} = \underline{\underline{14}}$

$\Delta_1 = \begin{vmatrix} 1 & 3 \\ 6 & 4 \end{vmatrix} = 4-18 = -14$ $\quad y = \frac{\Delta_2}{\Delta} = \frac{9}{-1} = \underline{\underline{-9}}$

$\Delta_2 = \begin{vmatrix} 2 & 1 \\ 3 & 6 \end{vmatrix} = 12-3 = 9$

$(14,-9)$ is the point of intersection. Any line perpendicular to $5x-2y=7$ is $-2x-5y+k=0$. Since it passes through $(14,-9)$.

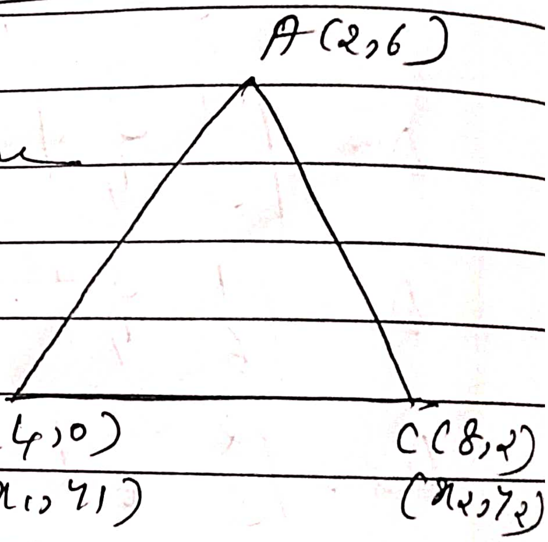
$$-2\times14-5\times-9+k=0.$$
$$-28+45+k=0.$$
$$17+k=0$$
$$k=-17$$
$$-2x-5y-17=0.$$
$$\underline{\underline{2x+5y+17=0}}$$

Q. A(2,6), B(4,0), C(8,2) are the vertices of a triangle. $\overline{AD}$ is drawn perpendicular to $\overline{BC}$.

(1) Find the slope of $\overline{BC}$

(2) Hence write down the equation to $\overline{BC}$

3. Using the equation $\overline{BC}$, find the equation of $\overline{AD}$.



1. Two pair of points are given, then

Slope $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{2-0}{8-4}$

$= 2/4 = 1/2$

2. ~~Slope and (4,0) are given, the Slope-point form~~

2. Two-point form:

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 0 = \frac{1}{2}(x - 4)$

$2(y - 0) = (x - 4)$

$2y = x - 4$

$\therefore x - 2y - 4 = 0$

3. Any line perpendicular $-2x - y + k = 0$
This line passes through (2,6),

$$\Rightarrow (x, y) = (2,6)$$

$$-2 \times 2 - 6 + k = 0$$

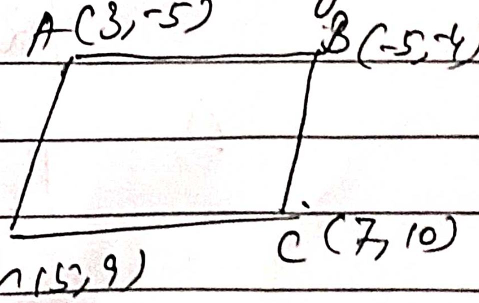
$$-4 - 6 + k = 0 \Rightarrow k = 10$$

$$\therefore -2x - y + 10 = 0$$

$$\therefore 2x + y - 10 = 0$$

Q. Prove that the points (3, -5), (-5, -4), (7, 10) and (15, 9) taking in order are the vertices of a parallelogram.

ABCD is a parallelogram of the opposite sides are parallel. In this case to prove that the slope of $\overline{AB}$ and slope of $\overline{CD}$ are equal and slope of $\overline{AD}$ and slope of $\overline{BC}$ are equal.



Slope of $\overline{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{\cancel{-4+5}}{-4+5} = \frac{-8}{1}$$

$$= \frac{-4+5}{-5-3} = -\frac{1}{8}$$

Slope of $\overline{DC} = \frac{10-9}{7-15} = -\frac{1}{8}$

$\overline{AB}$ and $\overline{DC}$ are equal.

Slope of $AD = \frac{9+5}{15-3} = \frac{14}{12} = 7/6$.

Slope of $BC = \frac{10-(-4)}{7+5} = \frac{14}{12} = 7/6$.

$\overline{AD}$ and $\overline{BC}$ are equal.
Therefore $ABCD$ is a parallelogram.

Q Show that the three lines $3x+4y=13$, $2x-7y+1=0$ and $5x-y=14$ are concurrent.

Find the point of intersection of the lines ~~2x~~ $3x+4y=13$ and $2x-7y=-1$

$$\Delta = \begin{vmatrix} 3 & 4 \\ 2 & -7 \end{vmatrix} = -21-8 = -29.$$

$$\frac{13}{\underline{7}}\ 91$$

$$\Delta_1 = \begin{vmatrix} 13 & 4 \\ -1 & -7 \end{vmatrix} = -91+4 = -87$$

$$\Delta_2 = \begin{vmatrix} 3 & 13 \\ 2 & -1 \end{vmatrix} = -3-26 = -29$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-87}{-29} = +3, \quad y = \frac{\Delta_2}{\Delta} = \frac{-29}{-29} = 1$$

$(3,1)$ is the point of intersection.
Substitute $(3,1)$ in the equation
$5x-y-14=0 \quad -(1)$

$\therefore (x,y)=(3,1)$ in (1).

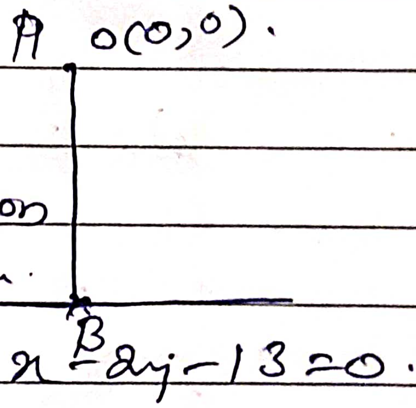
$5\times 3-1-14 = 15-1-14$

$= 14-14$

$= 0$.

$\therefore$ these lines are concurrent.

Q- Find the foot of the perpendicular from the origin to the line $3x-2y-13=0$.



To find the point B.
B is the point of intersection
Given line is $3x-2y-13=0$.
First we have to find the equation of $\overline{AB}$.
$\overline{AB}$ is perpendicular to the $3x-2y-13=0$. Any line perpendicular to $3x-2y-13=0$ is $-2x-3y+k=0$.
Since it passes through $(0,0)$. $-(1)$

$(x,y)=(0,0)$.

Substitute (x,y) in (1).

$-2\times 0-3\times 0+k=0 \Rightarrow \therefore k=0$

∴ The equation is $-2x-3y=0$.

∴ $2x+3y=0$.

Next to find the foot of the point of intersection of these lines.

$3x-2y=13$ and $2x+3y=0$.

$$\Delta = \begin{vmatrix} 3 & -2 \\ 2 & 3 \end{vmatrix} = 9+4 = 13$$

$$\Delta_1 = \begin{vmatrix} 13 & -2 \\ 0 & 3 \end{vmatrix} = 39 \cancel{+0} = \underline{\underline{39}}$$

$$\Delta_2 = \begin{vmatrix} 3 & 13 \\ 2 & 0 \end{vmatrix} = 3\times0 - 26$$
$$= -26$$

$$\therefore x = \frac{\Delta_1}{\Delta} = \frac{39}{13} = 3$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-26}{13} = -2$$

∴ The foot of the perpendicular is $(3,-2)$ or the point of intersection.