Surface Tension

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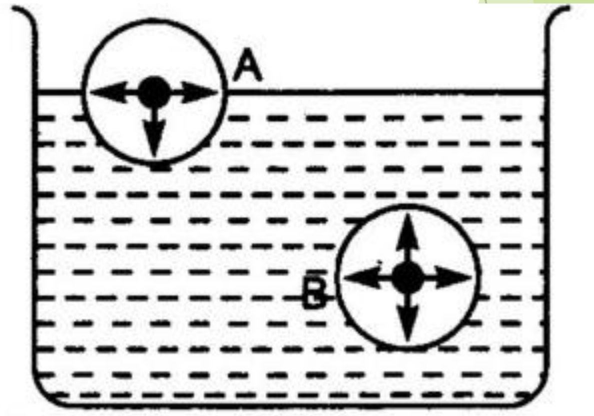
Cohesive force and Adhesive force

- Molecules in the liquid state experience strong intermolecular attractive forces. When those forces are between like molecules, they are referred to as cohesive forces.
  - For example, the molecules of a water droplet are held together by cohesive forces, and the especially strong cohesive forces at the surface constitute surface tension.
- When the attractive forces are between unlike molecules, they are said to be adhesive forces.
  - The adhesive forces between water molecules and the walls of a glass tube are stronger than the cohesive forces lead to an upward turning meniscus at the walls of the vessel and contribute to capillary action.

Surface tension

- Surface tension is the attractive force found in liquids which is responsible for pulling surface molecules in the rest of the liquid Liquids tend to acquire the least surface area possible.
- **The property of the surface of a liquid that allows it to resist an external force, due to the cohesive nature of its molecules.**
- The cohesive forces between liquid molecules are responsible for the phenomenon known as surface tension.

Surface tension is the tension of the surface film of a liquid caused due to the attraction of the particles in the surface layer by the bulk of the liquid, which tends to minimise surface area.



surface tension is given by the formula, $S = F/L$

Hence surface tension can be defined as the tangential force acting per unit length to the surface of a liquid.

SI unit is N/m

Surface energy= surface tension x Area of liquid surface

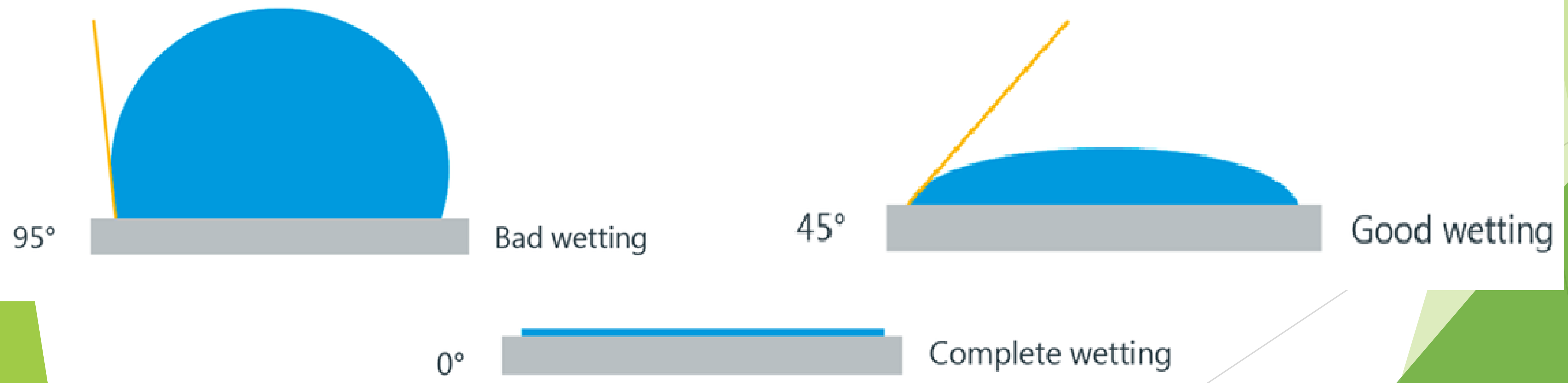
$U = S \times A$

Or $S = U/A$

Hence surface tension can be defined as the surface energy per unit area

Angle of Contact

- The angle subtended between the tangents drawn at liquid surface and at the solid surface inside the liquid at the point of contact is called angle of contact (θ)
- Angle of contact depends upon the nature of the liquid and solid in contact and the medium which exists above the free surface of the liquid.
- If θ is acute angle, i.e. θ<9o˚, then liquid meniscus will be concave upwards, then liquid will wet the solid surface.
- If θ is 90˚, then liquid meniscus will be plane.
- If θ is obtuse, i.e. θ>9o˚, then liquid meniscus will be convex upwards. θ>9o˚, then liquid will not wet the solid surface.

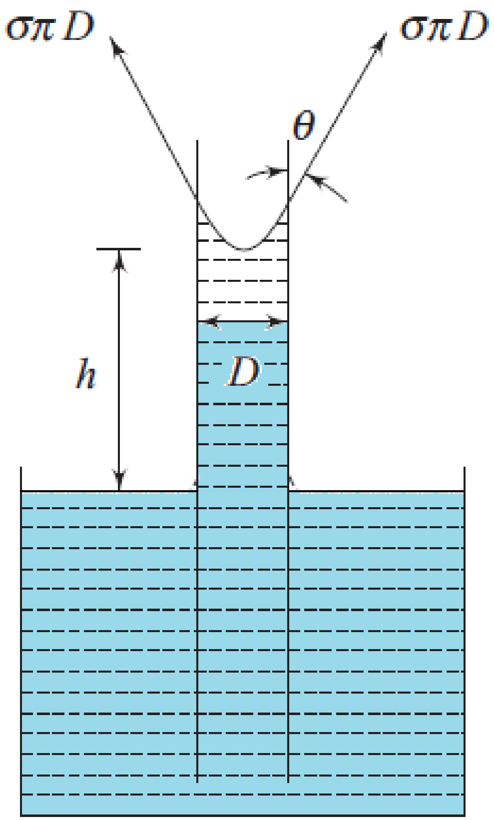


Examples: Use of Surface Tension in daily life

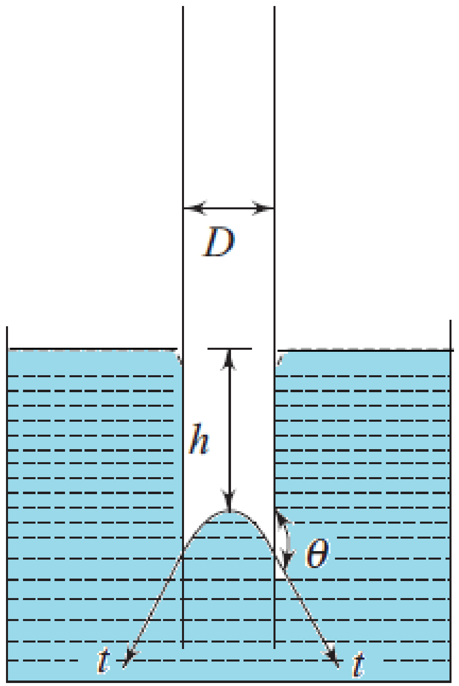
- Water striders can walk on water as their weight is considerably less to penetrate the water surface.
- Insects walking on water
- Floating a needle on the surface of the water.
- Rainproof tent materials where the surface tension of water will bridge the pores in the tent material
- Clinical test for jaundice
- Surface tension disinfectants (disinfectants are solutions of low surface tension).
- Cleaning of clothes by soaps and detergents which lowers the surface tension of the water
- Washing with cold water
- Round bubbles where the surface tension of water provides the wall tension for the formation of water bubbles.
- This phenomenon is also responsible for the shape of liquid droplets.

Capillarity

- One of the consequence of surface tension is capillarity or capillary action.
- Capillary action occurs when the adhesion to the surface material is stronger than the cohesive forces between the water molecules.
- The height to which capillary action will take water is limited by surface tension and gravity.



Capillary rise as Adhesive force is greater than cohesive force



Capillary fall as Cohesive force is more than Adhesive force

Ascent Formula

- The rising up of liquid through capillary tubes against gravity due to the surface tension at the boundary line of the liquid and solid tube surface. The elevation height of liquid in a capillary tube h is given by the ascent formula

$$h = \frac{2S\cos\theta}{r\rho g}$$

S --------is the surface tension

θ--------- is the angle of contact between the surface of the liquid and surface of the capillary tube at the point of contact

r ------is the inner radius of the capillary tube

ρ-------- is the density of the liquid rising through the capillary tube

g--------- is the acceleration due to gravity.

From the expression, it is clear that for a given substance, the elevation depends only on the radius of the capillary

Application of Surface Tension

- **Soaps and detergents**

Washing detergents are surfactants, **compounds that lower the surface tension between liquids and other substances**, making it easier for them to mix. When washing clothes, they help the water mix with and loosen dirt on the fabric.

- **Washing with cold water**

The major reason for using hot water for washing is that its surface tension is lower and it is a better wetting agent. But if the detergent lowers the surface tension, the heating may be un neccessary.

- **How does the ploughing of fields help in preservation of moisture in the soil?**

This is done **to break the tiny capillaries through which water can rise and finally evaporate**. The ploughing of field helps the solid to retain the moisture.

Pressure

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Pressure

Pressure (P) is defined as the perpendicular or normal force acting per unit area of a substance.

$$Pressure = \frac{Force}{Area}$$

$$P = \frac{F}{A}$$

- SI unit of pressure is N/m^2 or pascal (Pa).

One pascal is the pressure exerted by a force of 1 N on an area of 1 m^2.

- Pascal is a very small unit of pressure. Other commonly used units for pressure are
- $bar = 10^5 Pa$ and $millibar = 100\ Pa$.

Pressure inside a Fluid (Hydrostatic pressure)

- Hydrostatic pressure is the pressure that is exerted by a fluid at equilibrium at a given point within the fluid, due to the force of gravity or weight of the fluid.

$$hydrostatic\ pressure = \frac{Force\ inside\ the\ fluid}{area}$$

$$P = \frac{F_{fluid}}{A}$$

Due to the high mobility of the molecules of the fluid, the force acting at one position propagates immediately and isotropically with the same magnitude through the entire volume of the fluid.

Within a fluid at rest, the normal force exerted on a small surface has the same magnitude everywhere and is independent of the orientation of the surface. This fact is called Pascal's law.

This is the basic principle behind hydraulic machines like a crane, break, lift, etc.

Atmospheric Pressure (P_{atm})

- The pressure of the atmosphere at any point is equal to the weight of a column of air of unit cross-sectional area extending from that point to the top of the atmosphere.
- The atmospheric pressure decreases with an increase in altitude. The higher we climb up a mountain, the lesser will be the air above us and therefore, the lower will be the atmospheric pressure become. Mountaineers get bleed through their nose or ear due to this pressure difference.
- As you move deep into the water, it pushes you up. The pressure inside the water increases with depth.
- Atmospheric pressure can be measured with a mercury barometer in which pressure is measured in terms of the height of the mercury (Hg) column.

At sea level, atmospheric pressure corresponds to 76 cm of Hg which is equivalent to 1.013×10^5 Pa.

One atmosphere (atm) is a standard unit of pressure equal to the mean atmospheric pressure at sea level.

$$1 atm = 1.013 \times 10^5\ Pa = 76\ cm\ of\ Hg$$

Gauge Pressure and Absolute Pressure

- To find the pressure at a depth,

consider a fluid at rest in a container. Point 1 is at height h above point 2. The pressures at points 1 and 2 are P1 and P2 respectively. Consider a cylindrical element of fluid having base area A and height h. As the fluid is at rest the resultant horizontal forces should be zero and the resultant vertical forces should balance the weight of the element.

The forces (Remember, F=PA, since pressure P=F/A)

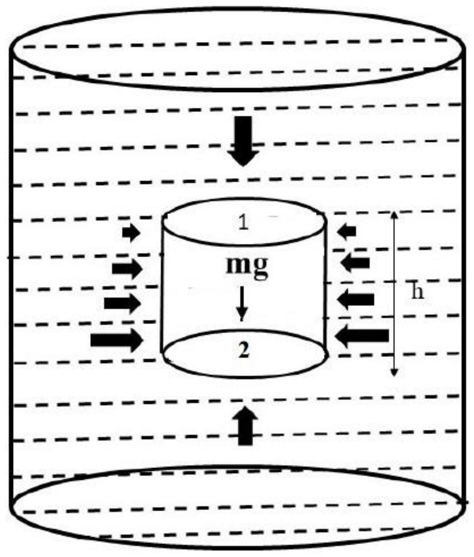
acting in the vertical direction are due to the fluid pressure

at the top (P_1A) acting downward, at the bottom (P_2A)

acting upward. If mg is the weight of the fluid in the cylinder, then

$(P_2 - P_1)A = mg$

Now, if ρ is the mass density of the fluid



Pressure inside a fluid at rest in a container

$$Density\ (\rho) = \frac{Mass}{Volume} = \frac{m}{V}$$

Therefore, the mass of fluid,

$$m = \rho V = \rho h A$$

So, the above equation can rewrite as,

$$(P_2 - P1)A = \rho h A g$$

$$(P_2 - P1) = \rho g h$$

Pressure difference (P_2 -P_1) depends on the vertical distance h between the points (1 and 2), the density of the fluid ρ, and acceleration due to gravity g. If point 1 is shifted to the top of the fluid which is open to the atmosphere, P_1 may be replaced by atmospheric pressure (P_{atm}) and we replace P_2 by P.
Then the above equation turns to

$$(P - P_{\text{atm}}) = \rho g h$$
$$P = P_{\text{atm}} + \rho g h$$

Thus, the pressure P, at depth below the surface of a liquid open to the atmosphere is greater than atmospheric pressure by an amount $\rho g h$. This P is called absolute pressure at that point. Absolute pressure at a point is pressure measured with respect to zero pressure or absolute vacuum.

The excess of pressure,($P - P_{\text{atm}}$) at depth, h is called a gauge pressure at that point.
Gauge pressure at a point is the pressure measured relative to the atmospheric pressure.

Module IV Properties of Matter

Fluid Dynamics

What is a Fluid?

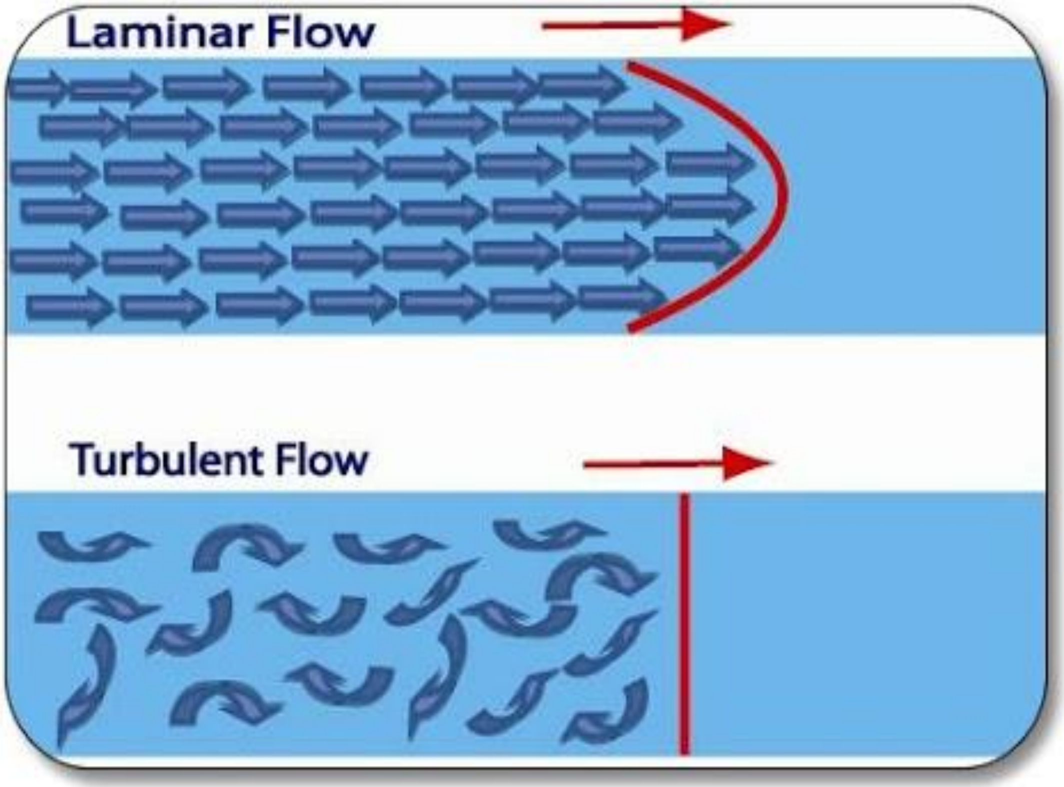
- A substance exists in three primary phases: solid, liquid, and gas. A substance in the liquid or gas phase is referred to as a **fluid.**
- Distinction between a solid and a fluid is made on the basis of the substance's ability to resist an applied shear (or tangential) stress that tends to change its shape.
- A solid can resist an applied shear stress by deforming, whereas a fluid deforms continuously under the influence of shear stress, no matter how small.
- In solids stress is proportional to *strain, but in fluids stress is proportional* to *strain rate*.

- What is Streamline Flow?

The flow of liquid is said to be steady or stream line if each particle of fluid follows a smooth path so that the velocity of the fluid at any point remains unchanged. The paths of the particles do not cross each other.

- What is turbulent flow?

The flow in which the fluid particles move randomly in any direction is called as Turbulent Flow.



Comparison between streamline flow and turbulent flow

Streamline flow	Turbulent flow
It is the steady, smooth, regular, one-by-one flow of fluid particles.	The flow of particles is irregular and in a zigzag manner.
The speed of fluid motion is lower than the critical velocity. Critical velocity of fluids is the velocity above which the flow becomes irregular or turbulent.	The fluid motion speed is greater than the critical velocity.
The fluid layers do not cross each other	The fluid layer may cross each other
The particles reaching a given point in the path of flow have the same velocity with time. That means a single velocity of particles is possible at a given point in the flow of fluids.	The particles in the fluid can have different velocities at a given point in the fluid flow. That means multiple velocities at a given point in the flow can be possible.

Reynold's Number

- Osborne Reynolds (1842-1912) defined a dimensionless number that has a vital role in predicting whether a liquid has stream line flow or turbulent flow.
- This number is called the Reynolds number (Re).
- Reynolds number is defined as the ratio of inertial forces to viscous forces within flowing fluid.

$$R_e = \frac{inertial\ force}{viscous\ force}$$

$$R_e = \frac{\rho v d}{\eta}$$

Incompressible liquid = liquids with constant density

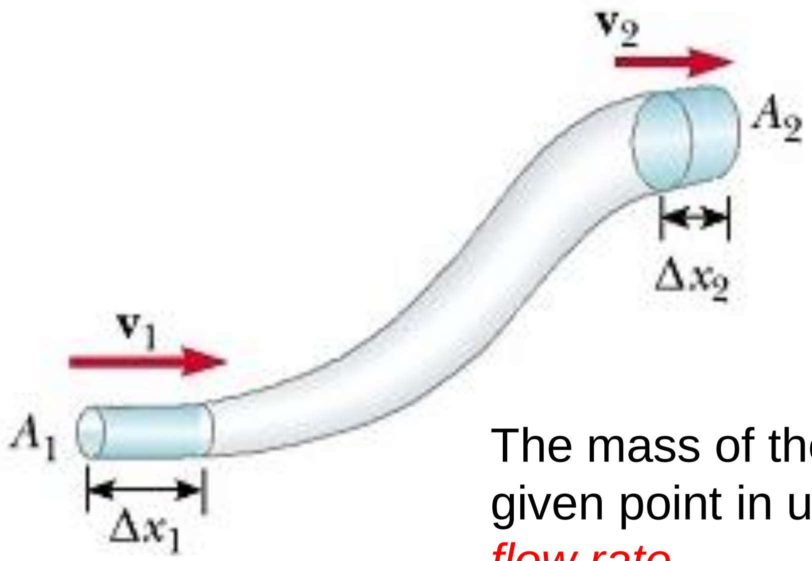
Compressible fluids: are the **fluids** with variable density

where ρ is the density of the fluid flowing with a speed v, d is the diameter of the pipe, and η is the coefficient of viscosity of the fluid.

It is found that flow is streamline or laminar for Re less than 1000.The flow becomes unsteady for Re between 1000 and 2000.The flow is turbulent for Re greater than 2000.

Equation of Continuity

- Equation of continuity in fluid dynamics states that when an incompressible and non-viscous fluid moving with steady flow through a pipe of varying cross-sectional area, the product of the area of cross-section and velocity of flow remains a constant at every point in the pipe.
- Consider a pipe of varying cross sectional area . Let v_1 represents the fluids velocity as it passes the cross sectional area A_1 and v_2 that through the area A_2



The mass of the liquid that possesses a given point in unit time is called the *mass flow rate*.

- Let Δx_1 is the distance moves in a time Δt at section 1. Let the volume of the liquid passed is $A_1 \Delta x_1$.
- Mass of the liquid passed along section 1 is $\rho_1 A_1 . \Delta x_1$ where ρ_1 is the density of the fluid.
- Mass of the liquid passed in unit time or flow rate$=\rho_1 . A_1 .(\Delta x_1 / \Delta t)= \rho_1 . A_1 . v_1$
- Similarly , at the point 2, the flow rate is $\rho_2 . A_2 . v_2$. Since no fluid flows in or out through the sides, the flow rate section 1 should be same as the flow rate at section 2.

Thus $\rho_1 . A_1 . v_1 = \rho_2 . A_2 . v_2$. This is called the equations of continuity.

If the fluid is incompressible, $\rho_1 = \rho_2$,

then the equation becomes $A_1 v_1 = A_2 v_2$

$$Av = constant$$

where, A is the cross-sectional area of the fluid flow, and v is the velocity of the fluid at that point.

Av is defined as the volume flux or the flow rate of the fluid, which remains constant for steady flow.

When the area of the cross-section is greater, the velocity of the liquid is lesser and vice versa.

ENERGY OF A LIQUID

- A liquid in motion possess kinetic energy. Due to its position, it has gravitational PE, The liquid also possess pressure energy due to its pressure

A) Kinetic Energy:

- KE for a body of mass m moving with velocity v = $\frac{1}{2} mv^2$
- KE for a liquid/unit volume= $\frac{1}{2} \rho v^2$
- KE/unit mass= $\frac{1}{2} v^2$

B)Potential energy (with a reference to a reference level)

A body of mass m positioned at a height h possesses a potential energy equal to mgh.

The potential energy/unit volume of a liquid =ρgh where ρ is the density of the liquid.

The potential energy per unit mass of the liquid is *gh*

C)Pressure energy

- It is the **energy** in/of a fluid due to the applied **pressure** (force per area).
- So if you have a static fluid in an enclosed container, the **energy** of the system is only due to the **pressure**;
- If the fluid is moving along a flow, then the **energy** of the system is the kinetic **energy** as well as the **pressure**.
- Pressure energy = $P.V = Pm/\rho$

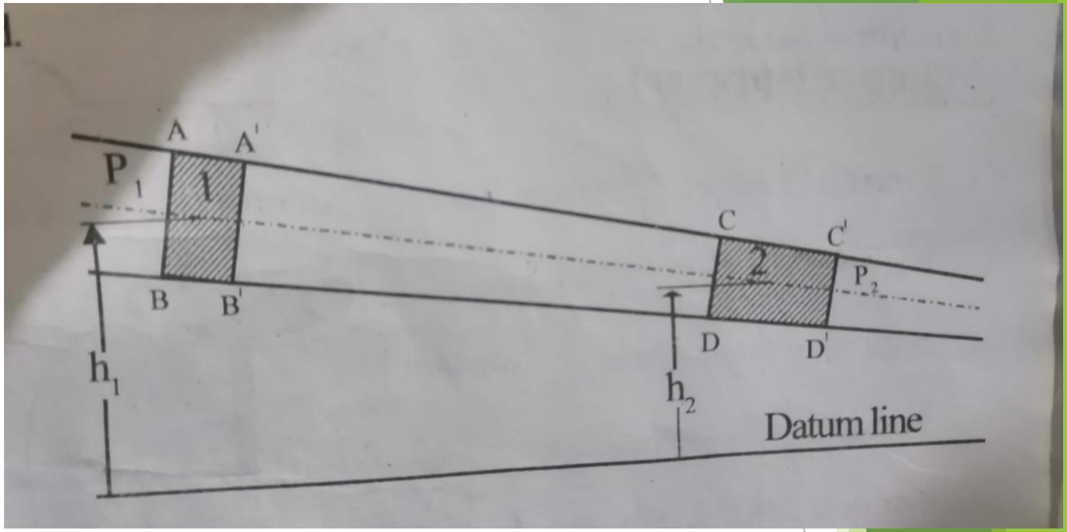
Bernoulli's Theorem

- The total energy of a small amount of liquid flowing from one point to another along a streamline without friction remains the same through out the motion.
- Potential energy + pressure energy +kinetic energy= a constant
- $mgh+PV+ \frac{1}{2} mv^2 = a$ constant
- $mgh+mP/\rho+ \frac{1}{2} mv^2 = a$ constant

Bernoulli's theorem states that the sum of kinetic energy, potential energy, and pressure energy of an ideal fluid in a streamline flow remains constant throughout the flow.

Bernoulli's theorem Proof

Consider an incompressible fluid of density ρ flowing as a continuous stream through a pipe as shown in figure. Consider two sections AB and CD. 1 and 2 are the midpoints of the two sections.



Let the liquid column *ABDC* displaces to the right and occupies the new position A'B'D'C'

As the liquid flows down , there is loss in both PE and pressure energy. This loss should appear as a gain in KE of the liquid.

Loss in pressure energy = $mP_1/\rho - mP_2/\rho$

Loss in PE= $mgh_1 - mgh_2$

Gain in KE= $\frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2$

Where v_1 and v_2 are the velocities of the liquids at points 1and 2 respectively

According to the law of conservation of energy, the total loss in energy = total gain in energy

i.e, $(mP_1/\rho - mP_2/\rho)+(mgh_1-mgh_2)=(\frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2)$

$mP_1/\rho + mgh_1 + \frac{1}{2} mv_1^2 = mP_2/\rho + \frac{1}{2} mv^2{}_2 + mgh_2$

This becomes the general form of Bernaulli's equation. Dividing throughout by m, we get

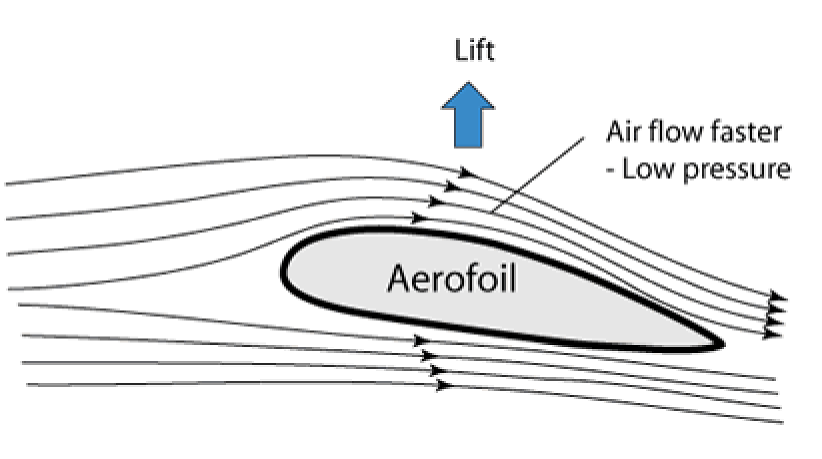
$\mathbf{P_1/\rho + gh_1 + \frac{1}{2} v_1^2 = P_2/\rho + \frac{1}{2} v^2{}_2 + gh_2}$

Or **$P/\rho + gh + \frac{1}{2} v^2$ = a constant**

which is the equation for unit mass of the liquid

AIR FOIL

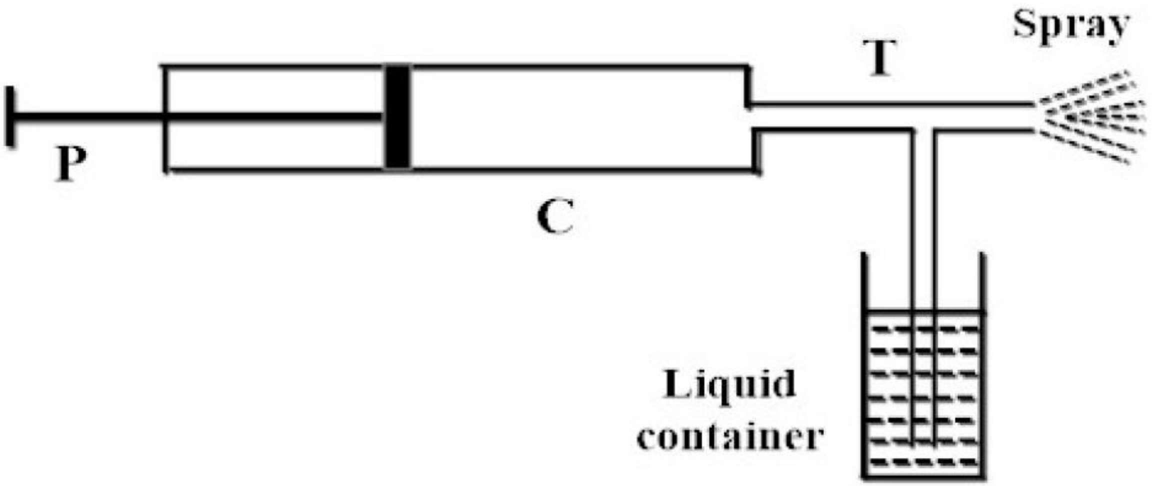
- An **airfoil** is the cross-sectional shape of a wing, blade (of a propeller, rotor, or turbine), or sail . An **airfoil**-shaped body moving through a fluid produces an aerodynamic force. It works based on Bernauli's principle. Due to the specifically designed shape of the wings, air over the top surface moves fast.



This is faster than it flows past under surface. This causes a reduction in pressure above the wing which gives rise to about two third of the lift. The wings are also designed in such a way that the air striking at the bottom gives additional lift.

Atomiser

- An atomizer is a device to spray a liquid or make a liquid into a jet of tiny droplets. The principle behind the working of atomizers is Bernoulli's theorem. According to Bernoulli's theorem, the velocity of a fluid increases in a region, the pressure decreases in that region, and vice versa in order to maintain the total energy as constant.



Schematic diagram of an atomizer

- The schematic diagram of an atomizer is shown in the figure.
- It works on Bernoulli's **principle**.
- When high speed horizontal air passes over a vertical tube, it creates a low pressure and draws the air and liquid inside the vertical tube upward.
- **Atomizer** has a nozzle at the end of the horizontal tube which causes the liquid to break up into small drops and mixes it with the air.
- The air inside the **perfume** bottle is moving relatively slowly; therefore, according to **Bernoulli's principle**, its pressure is relatively high, and it exerts a strong downward force on the **perfume** itself. In an **atomizer** there is a narrow tube running from near the bottom of the bottle to the top.

Viscosity

The internal friction of real fluid is called Viscosity

Frictional force between different layers of fluids as they move fast one another.

In liquids, it is due to the cohesive forces between molecules.

In gases it is due to collision between molecules

Eg of Viscous fluids -Honey, Grease, engine oil

Coefficient of Viscosity

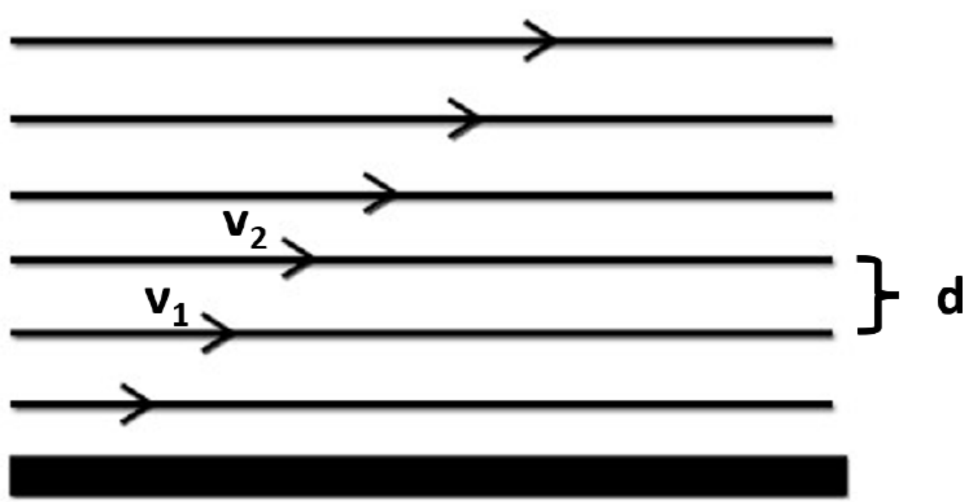
- The flow of a liquid through a pipe can be considered to be in the form of concentric layers of the same axis
- The inner most layer will be moving with maximum velocity whereas the outermost layer (the layer in contact with the pipe) will be moving with minimum velocity.
- From layer to layer the velocity gradually changes
- **Viscosity** is a measure of a fluid's resistance to flow
- Consider two two parallel layers of liquid, one sliding over the other with different velocities.
- A tangential viscous force appears in between the layers trying to minimise the relative velocity between the layers.
- The viscous force depends on the velocity gradient, the area of the layers in contact and the nature of the liquid

- Let the layers moves with velocity v_1 and v_2 respectively separated by a distance d .The rate of change of velocity with dstance is called the *velocity gradient*
- Hence the velocity gradient = $(v_1-v_2)/d$
- The viscous force is directly proportional to the velocity gradient and the area of each layer
- If A is the area of each layer in contact, then

$$F \alpha A,$$

$$F \alpha A \frac{(v_1 - v_2)}{d}$$

$$F = \eta A \frac{(v_1 - v_2)}{d}$$



Where η is a constant which depends upon the nature of the liquid

- co-efficient of viscosity η can be defined as that much resistive force developed between the liquid layers of unit area when they are moving with a unit velocity gradient.
- The SI unit of coefficient of viscosity η is $kgm^{-1}s^{-1}$ or Nsm^{-2}.

Terminal Velocity

Terminal velocity is defined as the highest velocity attained by an object falling through a fluid. It is observed when the sum of drag force and buoyancy is equal to the downward gravity force acting on the object. The acceleration of the object is zero as the net force acting on the object is zero.

STOKE'S FORMULA

- Stoke's Law is a mathematical equation that expresses the settling velocities of the small spherical particles in a fluid medium.
- The law is derived considering the forces acting on a particular particle as it sinks through the liquid column under the influence of gravity.
- When a small object falls through a liquid, it gradually picks up speed. As its velocity increases, the viscous force acting upwards also increases until it just balances the effective weight of the object. At this point , net force acting on the object becomes zero. Hence the acceleration becomes zero.

- The other quantities on which the force F depends on viscosity η of the fluid and radius a of the sphere.
- Sir George G. Stokes (1819-1903), an English scientist introduced an expression for the viscous drag force F as given below.

$$F = 6\pi\eta r v$$

Where η is the co-efficient of viscosity of the medium, r is the radius of the spherical body and v is the uniform velocity after equilibrium or terminal velocity.

Module IV
Properties of Matter

ELASTICITY

Dr.Jinchu.I

Introduction

- A rigid body generally means a hard solid object having a definite shape and size.
- But in reality, bodies can be stretched, compressed and bent.
- Even the appreciably rigid steel bar can be deformed when a sufficiently large external force is applied on it.
- This means that solid bodies are not perfectly rigid.
- A solid has definite shape and size.
- In order to change (or deform) the shape or size of a body, a force is required.

Deforming Force and Restoring Force

- A force which produces a change in configuration (size or shape) of the object on applying it, is called a deforming force. ie, the force we applied to deform the material is called deforming force.
- The **restoring force** is a **force** which acts to bring a body back to its equilibrium position.
  - The internal reaction force developed inside an elastic material to resisit any change in its size or shape is called restoring force.

- **Elasticity**:- The property of a body, by virtue of which body regains its original size and shape when the applied force is removed, is known as **elasticity** and the body is known as **Elastic body**.

  Eg. Spring, rubber, skin, etc.
- **Plasticity**:- The property of a body, by virtue of which body does not regain its original size and shape when the applied force is removed, is known as **plasticity** and the body is known as **Plastic body**.

  Eg. Plastic paper, clay, putty, etc.
- **Rigidity:-** The property of a body, by virtue of which body does not change its original size and shape when the force is applied is known as **Rigidity.**

  Eg. Wall, Black board, duster, etc.

Stress:- The restoring force per unit area is known as stress.
If **F** is the force applied and **A** is the area of cross section of the body.

Stress = F/A

The SI unit of stress is N/m^2.

Strain:- It is defined as change in dimensions per unit original dimensions.

Strain= change in dimensions/original dimensions

Strain has no unit.

There are 3 types of stress:-

1) Tensile or Longitudinal Stress:-

If the applied force produces change in length of a body, the stress associated is called as Tensile Stress.

Longitudinal stress = $F/A = Mg/\prod r^2$

2) Volume stress :-

If the applied force produces change in volume of a body, the stress associated is called as Volume Stress.

Volume Stress = dP

3) Shear stress :-

If the applied force produces change in shape of a body, the stress associated is called as Shear Stress.

Shear Stress = Tangential force/ Area

There are 3 types of strain:-

1) Tensile or Longitudinal Strain:-
The change in the length per unit original length of the body is known as longitudinal strain.

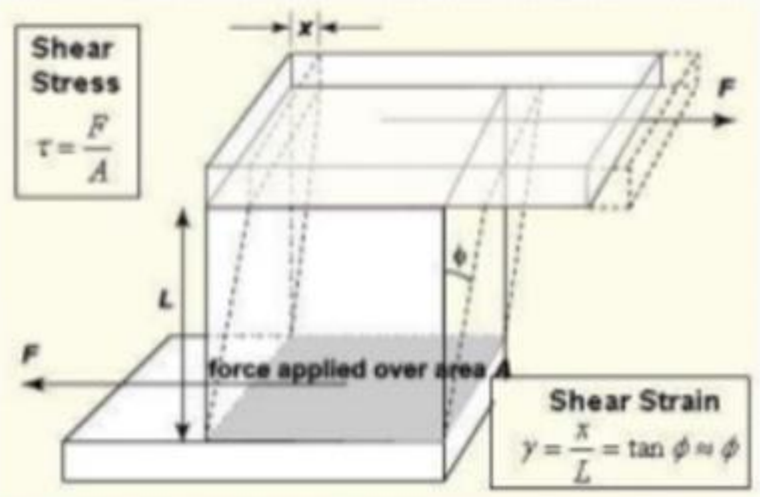
Longitudinal strain = l/L

2) Volume strain:--
The change in the volume per unit original volume of the body is known as volume strain.

Volume Strain = dV/V

3) Shear strain:--
The ratio of relative displacement of any layer to its perpendicular distance from fixed surface is known as shear strain.

Shear Strain = X/L



Elastic Limit

- Elastic limit of a substance is defined as the maximum stress that can be applied to the substance before it becomes permanently deformed and does not return to its original state.
  - Material is elastic when the stress is below its elastic limit
  - Material is plastic if stress is above the elastic limit.

Elastic Fatigue

The loss of **elastic** properties because of the action of repeated alternating deforming force is called **elastic fatigue**.

Due to **elastic fatigue**: (i) Bridges are declared unsafe after a long time of their use.
(ii) Spring balances show wrong readings after they have been used for a long time.

HOOKE'S LAW:-

Statement:- "*Within elastic limit, stress is directly proportional to strain.*"

Thus,

stress ∝ strain

stress = M × strain

where M = proportionality constant called as **modulus of elasticity.**

Therefore, M= Stress/strain

There are 3 types of elastic constants:-

1) Young's Modulus (Y)
2) Bulk Modulus (K)
3) Modulus of Rigidity(η)

The constant of proportionality depends on the material being deformed and on the nature of the deformation and is called modulus of elasticity or elastic modulus.

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Due to **elastic fatigue**: (i) Bridges are declared unsafe after a long time of their use. (ii) Spring balances show wrong readings after they have been used for a long time.

Young's Modulus: Elasticity of Length

- Consider a long bar of cross-sectional area A and initial length L that is clamped at one end. When an external force is applied perpendicular to the cross-section, internal forces in the bar resist distortion ("stretching"), but the bar reaches an equilibrium situation in which its final length L_f, which is greater than L and in which the external force is exactly balanced by internal forces.
- longitudinal stress as the ratio of the magnitude of the external force F to the cross-sectional area A.

$$Longitudinal\ stress = \frac{F}{A}$$

The longitudinal strain is defined as the ratio of the change in length ΔL to the original length L. Strain can be tensile (increase in length) or compressive (decrease in length).

$$Longitudinal\ strain = \frac{\Delta L}{L}$$

Young’s modulus (Y) is a measure of the length elasticity of a material. Young’s Modulus is defined as the ratio of longitudinal stress to the longitudinal strain under relatively small deforming force.

$$Young's\ Modulus = \frac{Longitudinal\ \ stress}{Longitudinal\ \ strain}$$

$$Y = \frac{\left(\frac{F}{A}\right)}{\left(\frac{\Delta L}{L}\right)}$$

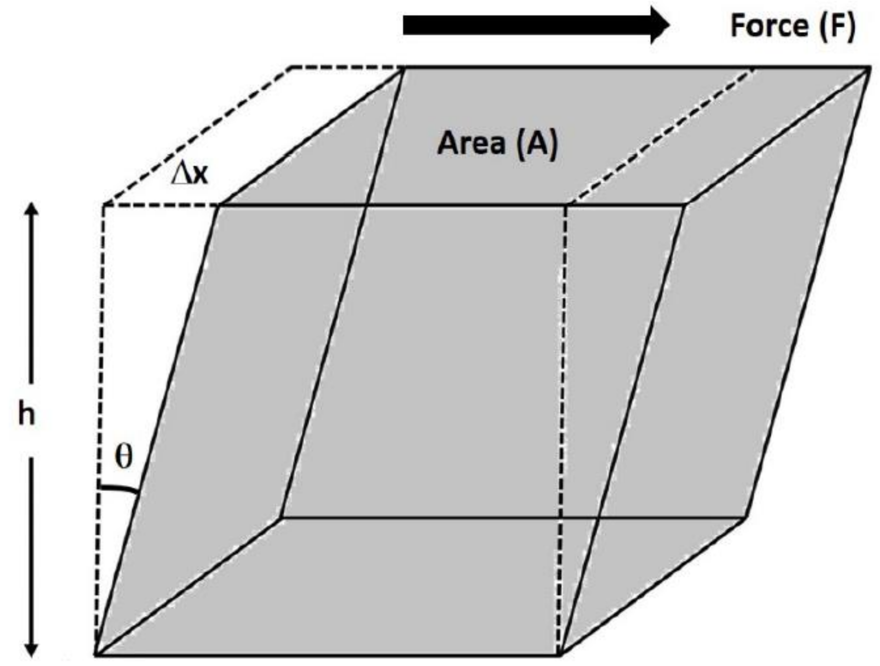
$$Y = \frac{F\,L}{A\ \Delta L}$$

Rigidity Modulus: Elasticity of Shape

- Rigidity modulus or shear modulus (η) is defined as the ratio of shear stress to shear strain.

$$shear\ stress = \frac{tangential\ force}{area} = \frac{F}{A}$$

$$shear\ strain = \frac{\Delta x}{h}$$



Shearing a rectangular block by applying a force parallel to one of its faces

If the deformation is very small, the shear strain can be recognized by the angular deformation (θ).

$$\theta = \frac{arc\ length}{radius} = \frac{\Delta x}{h}$$

$$\therefore\ shear\ strain = \theta$$

$$Rigidity\ Modulus = \frac{shear\ stress}{shear\ strain}$$

$$\eta = \frac{\left(\frac{F}{A}\right)}{\theta}$$

$$\eta = \frac{F}{A\theta}$$

Bulk Modulus: Volume Elasticity

- volume stress as the ratio of the magnitude of the total force (F) exerted on a surface to the area (A) of the surface. The perpendicular force per unit area is called pressure (P).

$$Volume\ stress = \frac{F}{A} = P$$

- Under pressure, the object experiences a volume change ΔV. The volume strain or bulk strain is defined as the ratio of the change in volume to the original volume.

$$volume\ strain = \frac{\Delta V}{V}$$

Bulk modulus (B) is defined as the ratio of volume stress to volume strain.

$$Bulk\ Modulus = \frac{Volume\ stress}{Volume\ stain}$$

$$B = -\frac{P}{\left(\frac{\Delta V}{V}\right)}$$

$$B = -\frac{PV}{\Delta V}$$

The reciprocal of the bulk modulus is called the compressibility and is denoted by k

$$k = \frac{1}{B} = -\frac{\Delta V}{PV}$$

The SI unit of compressibility is that of reciprocal of pressure,Pa^{-1}. Materials with small bulk modulus and large compressibility are easier to compress.