MODULE II
Rotational Motion

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Basic Types of Motion

- Everything in the universe is moving and evolving. There are basically four different types of motion namely translational motion, rotational motion, oscillatory motion, and random motion.

a) Translational Motion

- Translational motion is a motion in which the location of the object changes with time.

Translational motion can again be classified into two types – rectilinear motion and curvilinear motion.

  - In rectilinear motion, the object moves along a straight line whereas, in curvilinear motion, the object moves along a curved path.
  - Circular motion is a special type of curvilinear motion in which an object moves along a circular path.
  - In translational motion, all objects are considered as point masses.

- Example: The motion of elevators in buildings (rectilinear motion)

  The motion of a basketball into the basket (curvilinear motion)

  The motion of satellites around the earth (circular motion).

b)Rotational motion

- Rotational motion is a motion in which the objects spin around an axis and the location of the object do not change with time. Rotational motion is always associated with rigid extended bodies.
- During rotational motion, each particle constituting the rigid body undergoes circular motion. Hence circular motion and rotational motion are closely related.

  Example: Motion of blades of the ceiling fan

c) Oscillatory motion

- Oscillatory motion is the to and fro motion of an object about a fixed point. It is a special

type of periodic motion – a motion that repeats itself in a regular interval of time.

Example: Oscillations of a pendulum

d) Random motion

- Random motion is a motion in which the particle moves in a zig-zag manner and the direction of motion changes continuously. This kind of motion is unpredictable in practice.

Example: Motion of honey bee

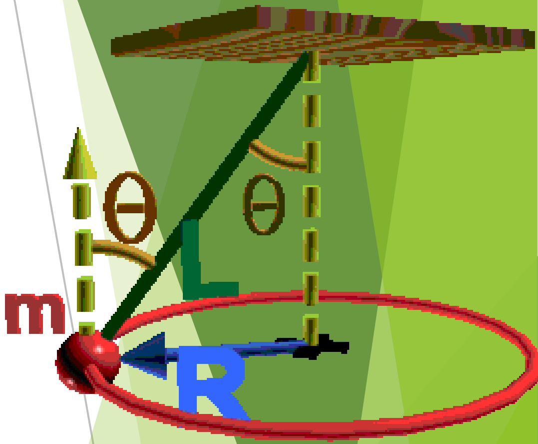
Circular Motion

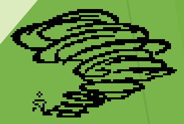
circular motion is said to be uniform when a particle moves along a circular path with a constant speed.

The point or line that is the center of the circle is the *axis of rotation*.

If the axis of rotation is *inside* the object, the object is *rotating (spinning)*.

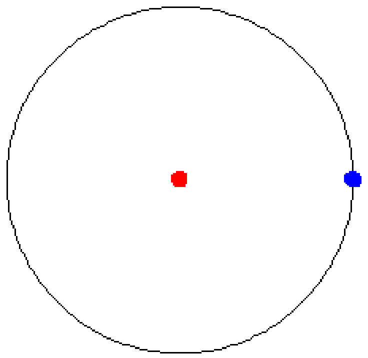
If the axis of rotation is *outside* the object, the object is *revolving*.

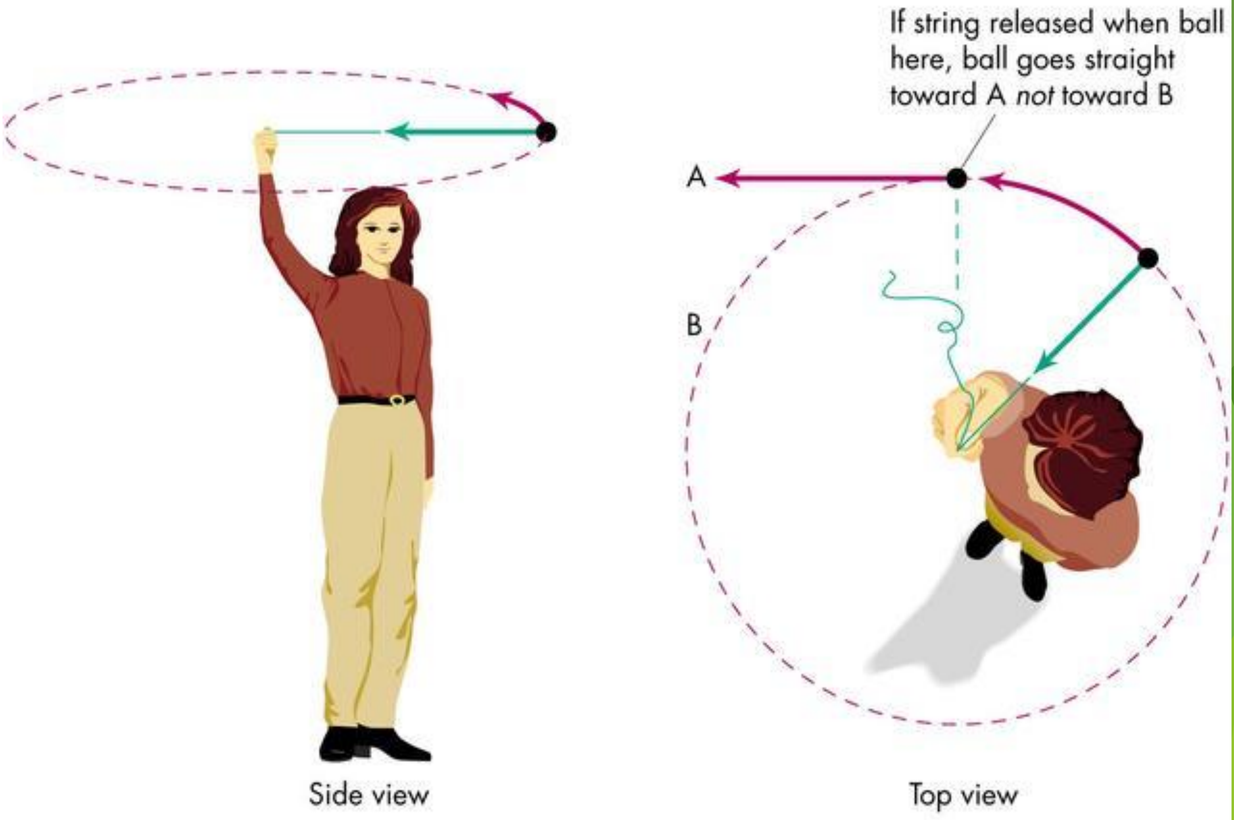




Uniform circular motion

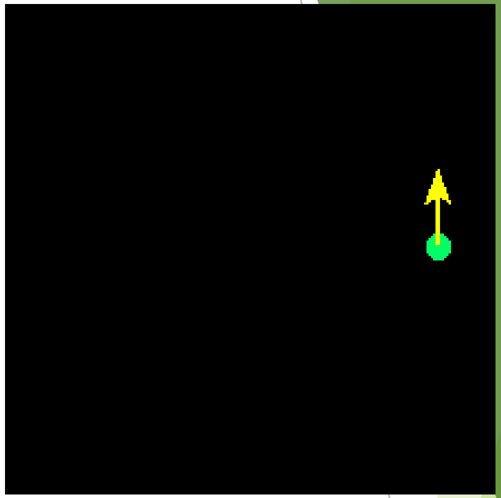
- An object which moves in a circle at constant speed is said to be executing uniform circular motion.
- Magnitude of the velocity remains constant, the direction of the velocity continuously changes.

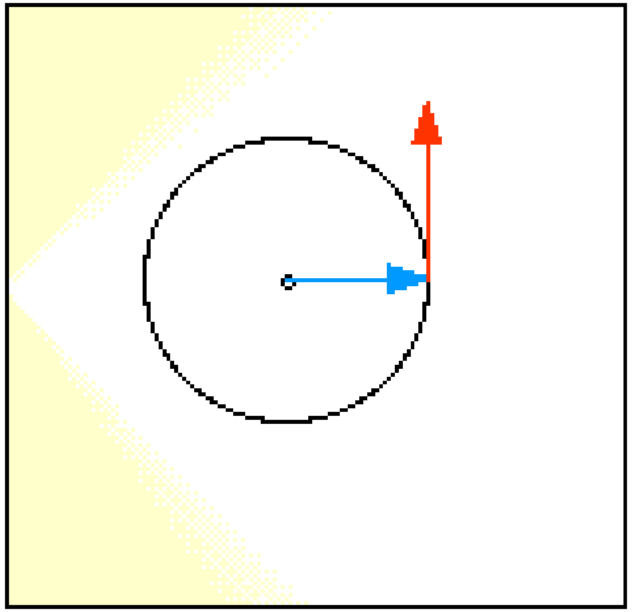


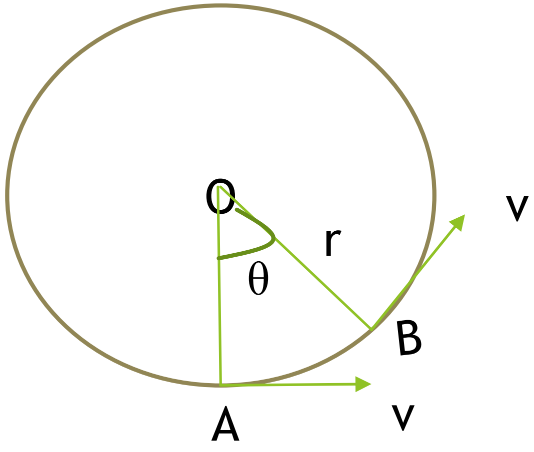


The direction of v changes continually!

- The instantaneous velocity is in a direction tangential to the circular path and hence called tangential velocity.
- Consider a body moving in a circle of radius r with a uniform speed v. The velocity at any instant is at right angles to the radius of the circular path.







The velocity is always tangent to the path

- During circular motion, the radius vector traces an angle at the centre. This angle is **angular displacement.**

  Let the body displaces from A to B in a time t.

  The angle traced is θ. Here angular displacement is θ

  Unit is rad.

  (1 radian is the angle subtended by an arc whose length is equal to the radius of the circle. 2л rad = 360

- Angular velocity (ω) : It is defined as the angular displacement in unit time.
- If θ is the angular displacement in a time t, $\omega = \theta/t$ rad/s

- Angular velocity of a rotating mechanism is expressed in ***revolution per minute (r.p.m)***.

  The angular velocity of a second hand in a clock is 1 r.p.m

  1 r.p.m = 1 revolution/1 minute = 2 л rad /60 second

  = л/30 rad/s

- ***Relation between linear velocity and angular velocity***

  - Let the body travels along a circle of radius r with uniform speed v. The body travels from A to B in a time t, the angular displacement being θ.

$$\omega = \frac{\theta}{t}$$

Since the body is travelling along AB in a time t, AB = v x t---------------(1)

Applying the general formula

Angle = $\frac{arc}{radius}$, for the sector AOB,

$\frac{AB}{r}$ = θ or AB = rθ--------------------(2)

Equating the RHS of both equations, we get

$V\,t = r\theta$

Or $v = r(\frac{\theta}{t})$

ie, $v = r\omega$

Linear velocity = radius x angular velocity

Angular acceleration (α)

It is defined as the change in angular velocity in unit time.

If ω_1 and ω_2 are the initial and final angular velocities during time t,

$$\text{Angular acceleration } (\alpha) = \frac{\omega_2 - \omega_1}{t}$$

Relation between linear acceleration and angular acceleration

- Rate of change of linear velocity is linear acceleration.

  Rate of change of angular velocity is angular acceleration.

  Ie, $a = \frac{v_2 - v_1}{t}$ $\alpha = \frac{\omega_2 - \omega_1}{t}$

Consider a body moving along a circle of radius r with an angular velocity ω_1, the corresponding linear velocity being v_1

Then $v_1 = r\omega_1$, $v_2 = r\omega_2$

We have $\alpha = (\omega_2 - \omega_1)/t = \{\frac{v_2}{r} - \frac{v_1}{r}\}/t$

$\frac{\frac{1}{r}(v_2 - v_1)}{t} = \frac{1}{r}\,a$ or $a = r\alpha$

- Period (T) : Period is the time taken for a single cycle.

Unit is second

Frequency (f): The number of cycles completed in one second is called frequency.

Unit is Hertz

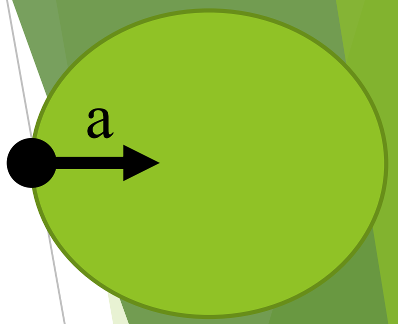
$f = 1/T$ and $T = 1/f$

Relation between T, ω and f

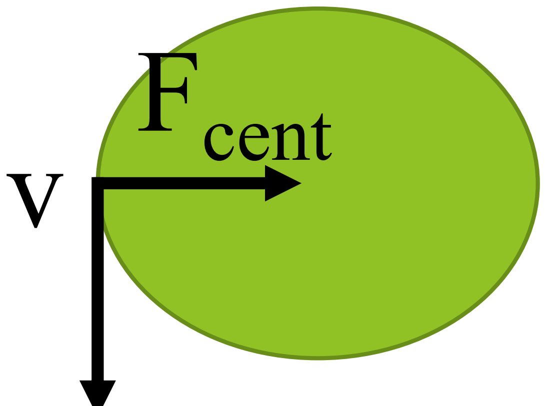
$$\omega = \frac{\theta}{t} \quad , \theta = 2\pi \text{ rad}$$

$$\omega = 2\pi/T \quad \text{or } \omega = 2\pi f$$

Centripetal Acceleration & Centripetal Force



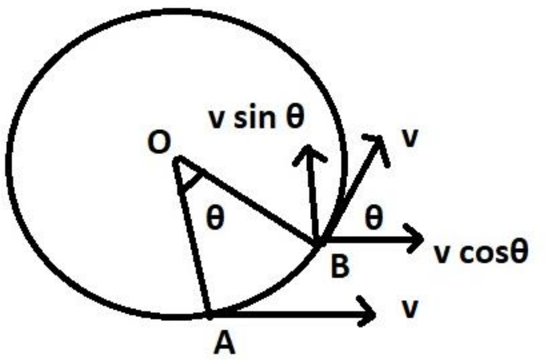
- During circular motion the magnitude of velocity remains constant, but its acceleration changes continuously. This change leads to an acceleration which is directed towards the centre. This acceleration is known as centripetal acceleration.
- A force is necessary to change the direction of a body. Therefore an external force must be acting inwards to keep the body in a circular path. This inward force acting on the body is called the centripetal force.



Magnitude of centripetal acceleration

- Imagine a body of mass m moving along a circle of radius r with a velocity v. Initially the body is at A, the velocity at this point of time does not have a component towards the centre, because v cos90 =0.
- After a small time interval t, let the body changes its position to B. θ is the angular displacement .

Angular velocity, $\omega = \theta / t$



At the position B, the tangential velocity can be resolved into two components. v cosθ along the horizontal direction and v sinθ along the vertical direction.

If the time interval is very small, the point B will not be far away from point A

- The angle θ also will be very small.

At point B the vertical component can be considered to be pointing towards the centre. This means that a change in velocity has occurred during transition from A to B

Change in velocity towards the centre = $v \sin\theta - 0 = v \sin\theta$

For small values $\sin\theta$ can be approximated to θ.

Therefore change in velocity = $v\theta$

Centripetal acceleration = change in velocity /time

$= v\theta/t \quad = v\omega$

Since $v = r\omega$ it can be also written as $r\omega^2$ or v^2/r

But $F = ma$ Hence the expression for centripetal force becomes

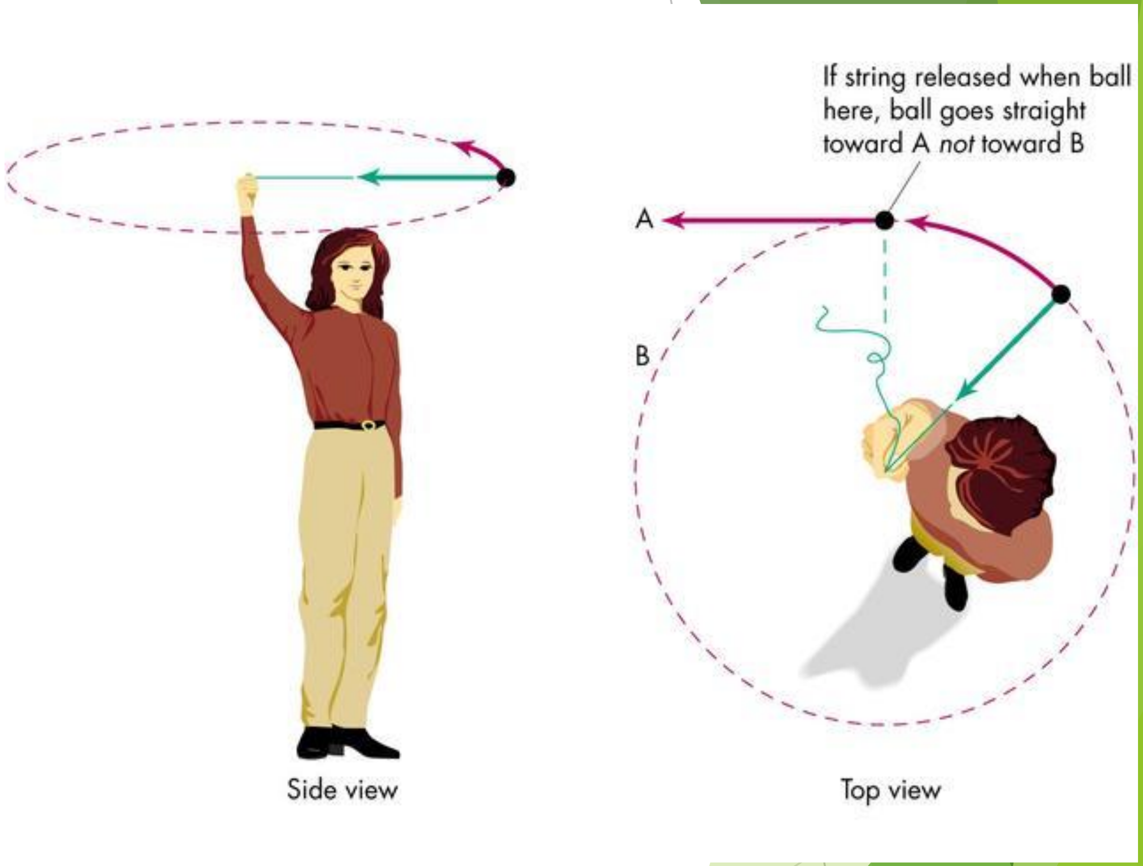
$F = mv\omega$ and $F = mv^2/r$ and $F = mr\omega^2$

The centripetal acceleration depends on:

The speed of the object.

The radius of the circle

- For a body execute uniform circular motion an agency should supply the necessary centripetal force.
- Example shown in figure
- In this case the centripetal force is supplied by the tension in the string.



Tie a stone to a string and whirl it around

Banking of Roads and Rails

- **Banking of roads** is defined as the phenomenon in which the edges are raised for the curved **roads** above the inner edge to provide the necessary centripetal force to the vehicles so that they take a safe turn. ... The angle at which the vehicle is inclined is defined as the **bank** angle.
- The angle of banking is the angle made by the elevated path with the horizontal. Let AB and AC represent the horizontal and banked paths respectively as shown in figure. Let θ be the angle of banking. Consider a vehicle of mass m takes a curved path of radius r with a speed v. The weight of the vehicle mg acts vertically downwards. The normal reaction N of the road on the vehicle will be perpendicular to the AC. The normal reaction can be resolved into vertical and horizontal components

$$\text{Tan}\theta = \frac{v^2}{Rg} \quad \textit{where R is the radius of the curve}$$

The vertical componentis equal to the weight of the body.

$$N\cos\theta = mg$$

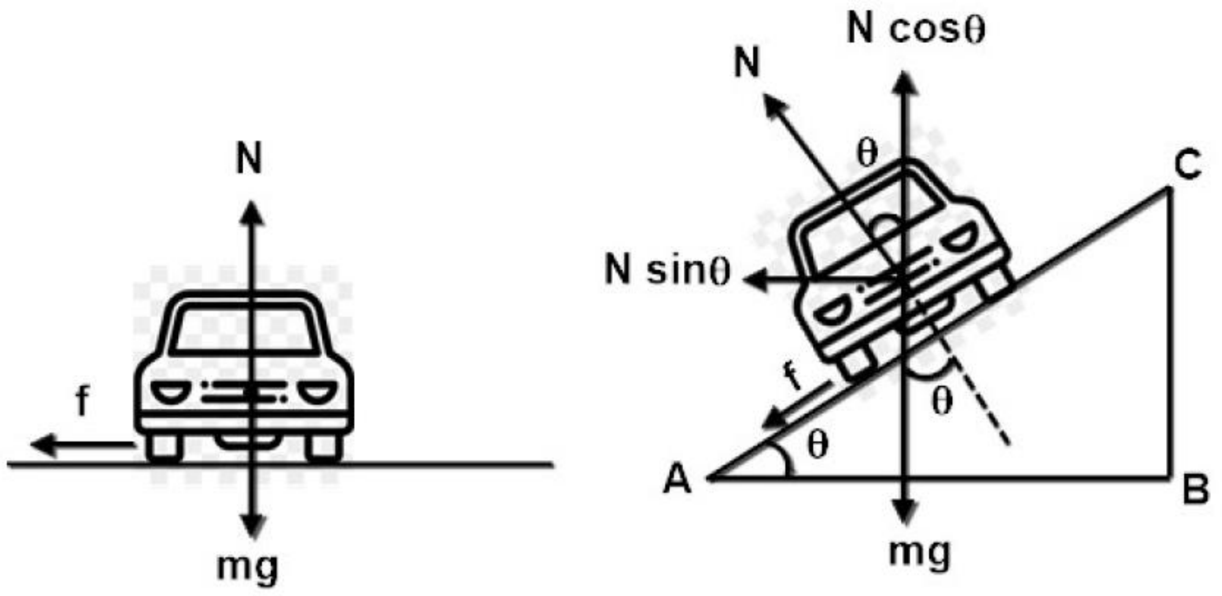
The horizontal component provides the centripetal force

$$N\sin\theta = \frac{mv^2}{r}$$

Dividing the second equation by the first gives

$$\tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left\{\frac{v^2}{rg}\right\}$$



Horizontal curve

Banked curve

The angle of banking depends on the radius of the curve of the road and the speed of the vehicle.

- Roads are most often banked for the average speed of vehicles passing over them. Nevertheless, if the speed of a vehicle is lesser or more than this, the self-adjusting state friction will operate between tyre and road and vehicle will not skid.

Banking of Railway Tracks

- In the case of a train moving through a curved track, centripetal force is required towards the centre of the circular track.
- This force is provided by the thrust exerted by the side of the outer rail against the flange of the outer wheel.
- When a fast-moving train takes a curved path, it tends to move away tangentially off the track.
- To avoid this, the outer rail is raised above the level of the inner rail. This is known as the banking of railway tracks.
- The banking of railway tracks avoids skidding and reduces the wear and tear of the wheels. In the case of a curved railway track, the level of the outer rail is higher than that of the inner one.
- The height of the outer rail above the inner rail in the banked rail track is called super elevation (S). If d is the distance between the rails and θ be the angle of super elevation.

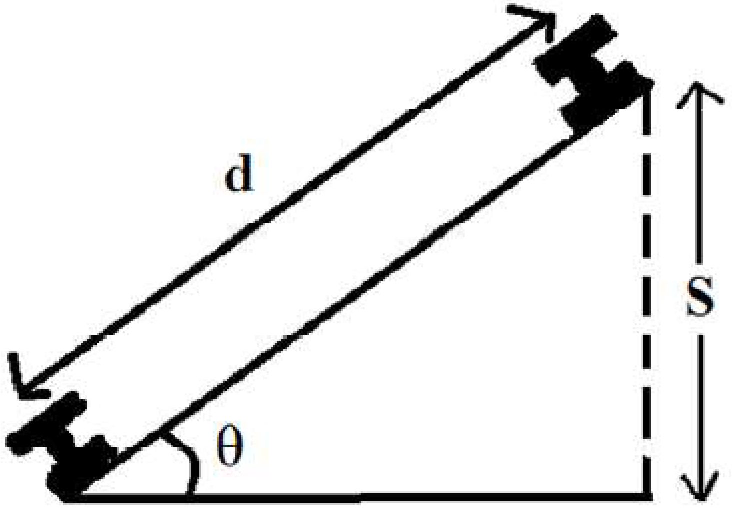
$$\sin\theta = \frac{S}{d}$$

or

$$S = d\sin\theta$$

Since θ is usually small for banked rail tracks, $\sin\theta$ approximately equal to $\tan\theta$

$$\tan\theta = \frac{S}{d}$$



But the equation for the angle of banking is given by

$$\tan\theta = \frac{v^2}{rg}$$

$$\frac{S}{d} = \frac{v^2}{rg}$$

$$S = \frac{v^2 d}{rg}$$