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Problems.

Q. The x-intercept of a line is three times its y-intercept. The line passes through (−6, 3). Find its equation

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x-intercept of a line is three times its y-intercept.

$\therefore a = 3b.$

$\therefore \frac{x}{a} + \frac{y}{b} = 1$ is the intercept form —(1)

$\frac{x}{3b} + \frac{y}{b} = 1.$

This line passes through (−6, 3)

$(x, y) = (-6, 3)$

$\frac{-6}{3b} + \frac{3}{b} = 1.$

$\frac{-6b + 9b}{3b^2} = 1$

$\frac{3b}{3b^2} = 1 \Rightarrow 3b = 3b^2.$

$3b = 3$

$b = 3/3 = 1$

$\therefore a = 3b = 3 \times 1 = 3$

$\therefore (1) \Rightarrow \frac{x}{3} + \frac{y}{1} = 1$

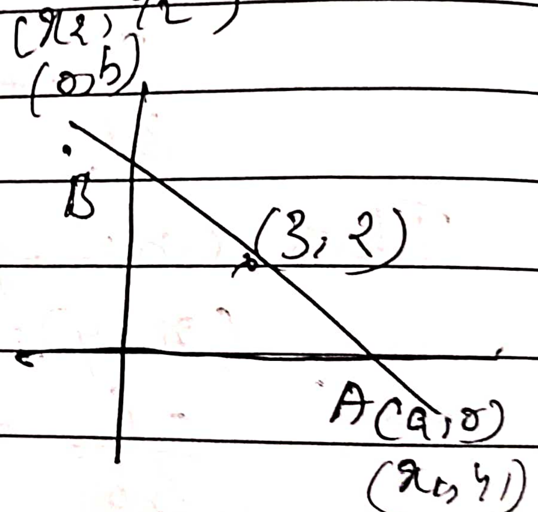
$$\frac{x+3y}{3}=1$$

$$x+3y=3$$

$$x+3y-3=0$$

Q. If a straight line cuts the co-ordinates axis at A and B and if (3, 2) is the midpoint of $\overline{AB}$, find the equation of $\overline{AB}$.

Let a be the x-intercept and b be the y-intercept. ∴ The co-ordinate of A are (a, 0) and B are (0, b). If (3, 2) is the midpoint of $\overline{AB}$. So using section formula.



$$(x, y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$(3, 2) = \left(\frac{a+0}{2}, \frac{0+b}{2}\right)$$

$$(3, 2) = (a/2, b/2)$$

$$a/2 = 3, \quad b/2 = 2$$

$\therefore a = 6,\ b = 4.$

$\therefore$ Intercept form $\frac{x}{a} + \frac{y}{b} = 1$

$$\Rightarrow \frac{x}{6} + \frac{y}{4} = 1.$$

$$\frac{4x+6y}{24} = 1$$

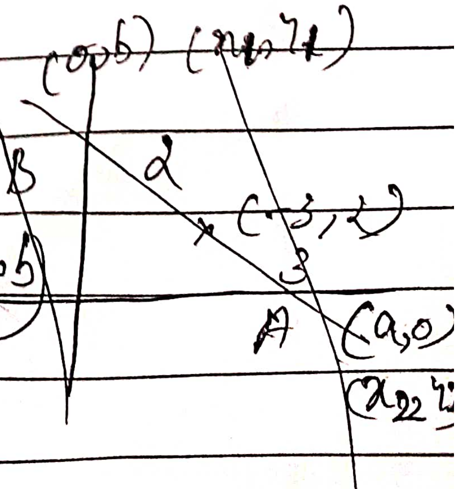
$$4x+6y = 24.$$

$$4x+6y-24=0.$$

$$2x+3y-12=0$$

Q. Find the equation of the straight line which passes through the point $(-3, 2)$ and is such that the portion between the x-axis and y-axis is divided internally at that point in the ratio $2:3$.

Using section formula



$$(-3,2) = \left(\frac{2\times a+3\times 0}{2+3}, \frac{2\times 0+3\times b}{2+3}\right)$$

$$(-3,2) = \left(\frac{2a}{\;}, \frac{3b}{\;}\right)$$

$\therefore a = 6, b = 4.$

$\therefore$ Intercept form $\frac{x}{a} + \frac{y}{b} = 1$

$$\Rightarrow \frac{x}{6} + \frac{y}{4} = 1.$$

$$\frac{4x+6y}{24} = 1$$

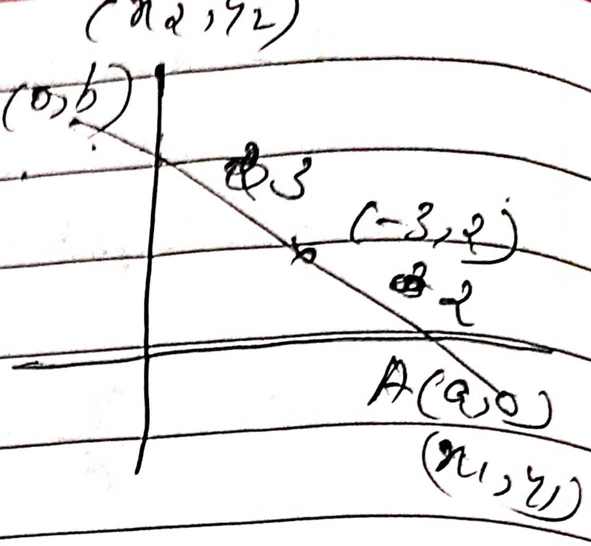
$$4x + 6y = 24.$$

$$4x + 6y - 24 = 0.$$

$$2x + 3y - 12 = 0$$

Q. Find the equation of the straight line which passes through the point $(-3, 2)$ and is such that the portion between the x-axis and y-axis is divided internally at that point in the ratio $2:3$.

using section formula.



$$(-3,2) = \left(\frac{2\times 0 + 3\times a}{2+3}, \frac{2\times b + 3\times 0}{2+3}\right)$$

$$(-3,2) = \left(3a/5, 2b/5\right)$$

$3a/5 = -3$ $\quad$ $2b/5 = 2$.

$3a = -15$ $\quad$ $2b = 10$

$\underline{\underline{a = -5}}$ $\quad$ $\underline{\underline{b = 5}}$

∴ The equation of AB is

$$\frac{x}{-5} + \frac{y}{5} = 1$$

$$\frac{5x - 5y}{-25} = 1$$

$$5x - 5y = -25$$

$$5x - 5y + 25 = 0.$$

$$\underline{\underline{x - y + 5 = 0}}$$

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Q. Find the slope and intercept of the eqn (1) $2x-3y+5=0$

$-3y=-2x-5$

$y=\frac{-2}{-3}x-\frac{5}{-3}$

$y=\frac{2}{3}x+5/3$ to compare the slope-intercept form $y=mx+c$

$m=2/3$ //.

$2x-3y+5=0$.

$2x-3y=-5$.

$\frac{2x}{-5}-\frac{3y}{-5}=1$

$\frac{x}{(-5/2)}+\frac{y}{(5/3)}=1$ to compare the intercept form $x/a+y/b=1$.

x-intercept $=-5/2$.

y-intercept $=+5/3$

(2) $7x-3y+42=0$.

$-3y=-7x-42$.

$y=-7/-3\,x-42/-3$

$y=7/3\,x-14$

$m=7/3$

$7x - 3y = -42.$

$$\frac{7x}{-42} - \frac{3y}{-42} = 1$$

$$\frac{x}{-42/7} + \frac{y}{42/3} = 1.$$

x-intercept $a = -42/7$

y-intercept $b = 42/3$

2. Find the equation of the line passing through $(1,-2)$ and $(-2,1)$ and also find intercepts on the axis.

using two point form

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Given $(x_1, y_1) = (1,-2)$

$(x_2, y_2) = (-2,1)$

$$\therefore\ y + 2 = \frac{1+2}{-2-1}(x-1)$$

$$y + 2 = \frac{\cancel{3}}{-\cancel{3}}(x-1)$$

$$y + 2 = -1(x-1).$$

$y + 2 = -x + 1 \Rightarrow y + 2 + x - 1 = 0$

$\therefore x + y + 1 = 0$ is the required equation:

To find intercept on the axis.

The we get the equation

$x+y+1=0$

$x+y=-1$

$\frac{x}{-1}+\frac{y}{-1}=1$

$\therefore$ x-intercept $a=-1$

y-intercept $b=-1$

2. The straight line through (4,3) makes intercepts of 4a and 3a on the x-axis and y-axis respectively. Find a.

x-intercept 4a

y-intercept 3a.

The intercept form is $\frac{x}{a}+\frac{y}{b}=1$

$\frac{x}{4a}+\frac{y}{3a}=1$.

If the line passes through

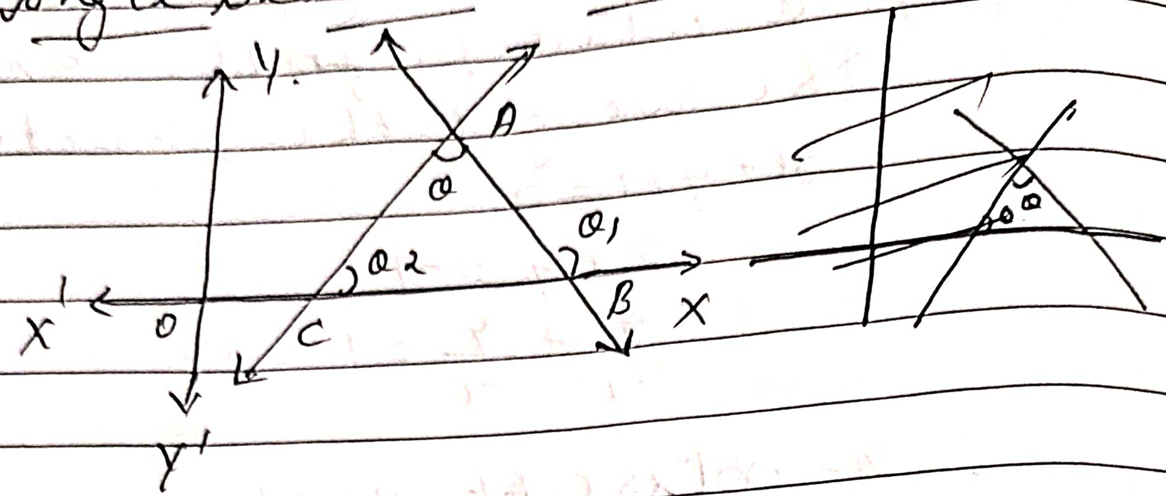
(4,3) $\Rightarrow \frac{4}{4a}+\frac{3}{3a}=1$.

$\cancel{\frac{12a+12a}{12a^2}} \Rightarrow \frac{1}{a}+\frac{1}{a}=1$

$\frac{a+a}{a^2}=1 \Rightarrow \frac{2a}{a^2}=1$

$2a=a^2 \Rightarrow a=2$

Angle between two lines.



The straight line $\overleftrightarrow{AB}$ makes an angle of inclination θ_1 with the x-axis and straight line $\overleftrightarrow{AC}$ makes an angle θ_2 with the x-axis.

Slope of $\overleftrightarrow{AB}$

$m_1 = \tan\theta_1$

Slope of $\overleftrightarrow{AC}$

$m_2 = \tan\theta_2$

From the figure. $\theta + \theta_2 = \theta_1$

$\theta = \theta_1 - \theta_2$.

Taking tangent function on both sides, we get $\tan\theta = \tan(\theta_1 - \theta_2)$

$$\tan\theta = \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1 \tan\theta_2}.$$

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}.$$

$$\therefore \theta = \tan^{-1}\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)$$

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Q. Find the angle between two lines with slopes $\sqrt{3}$ and $1/\sqrt{3}$.

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}.$$

Given $m_1 = \sqrt{3}$, $m_2 = 1/\sqrt{3}$.

$$\tan\theta = \frac{\sqrt{3} - 1/\sqrt{3}}{1 + \sqrt{3} \times 1/\sqrt{3}} = \frac{\frac{3-1}{\sqrt{3}}}{1+1}$$

$$\tan\theta = \frac{2/\sqrt{3}}{2} = \frac{2}{\sqrt{3} \times 2} = 1/\sqrt{3}$$

$$\theta = 30^\circ$$

The condition for parallelism and perpendicularity of two straight lines.

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} \quad -(1)$$

1. If two lines are parallel, angle between them, $\theta = 0$. Then $\tan\theta = \tan 0 = 0$.

$$(1) \Longrightarrow \frac{m_1 - m_2}{1 + m_1 m_2} = 0.$$

$$m_1 - m_2 = 0(1 + m_1 m_2) = 0.$$

$$m_1 = m_2$$

Two lines are parallel, the slopes are equa

2. If two lines lines are perpendicular

$\theta = 90^\circ$.

$\tan\theta = \tan 90^\circ = \infty = \frac{1}{0}$.

$(1) \Rightarrow \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{1}{0}$

$(m_1 - m_2) \times 0 = 1 + m_1 m_2$

$0 = 1 + m_1 m_2$

or $1 + m_1 m_2 = 0$

$m_1 m_2 = -1$

Two lines are perpendicular, the product of the slopes is -1

Note (1) Two lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ are parallel.

By the statement $m_1 = m_2$ — (1)

The slope of $a_1 x + b_1 y + c_1 = 0$

$b_1 y = -a_1 x - c_1$

$y = -a_1/b_1 \, x - c_1/b_1$

$\therefore m_1 = -a_1/b_1$

The slope of $a_2 x + b_2 y + c_2 = 0$

$\Rightarrow m_2 = -a_2/b_2$

$(1) \Rightarrow \frac{+a_1}{b_1} = +a_2/b_2$

or $a_1/a_2 = b_1/b_2$

Two lines are perpendicular. By the statement $m_1 m_2 = -1$

$$\therefore -\frac{a_1}{b_1} \times -\frac{a_2}{b_2} = -1$$

$$\frac{a_1 a_2}{b_1 b_2} = -1$$

$$\therefore a_1 a_2 = -b_1 b_2$$

$$\Rightarrow a_1 a_2 + b_1 b_2 = 0$$

Note (1) Equation of a line parallel to $ax+by+c=0$ is $ax+by+k=0$, where k is constant.

(2) Equation of a line ~~perpendicular~~ perpendicular to $ax+by+c=0$ is $\rightarrow bx-ay+k=0$ where k is constant.

1. Find the equation to the straight line passing through $(4,5)$ which is parallel to $2x+3y=4$

Any line parallel to $2x+3y-4=0$ has the form $2x+3y+k=0$ — (1). This line passes through $(4,5)$.

$$(x,y) = (4,5)$$

$$\therefore 2\times4 + 3\times5 + k = 0$$

$$8+15+K=0 \Longrightarrow K+23=0$$
$$\therefore K=-23$$

$$(1) \Longrightarrow 2x+3y-23=0$$

Q. Find the equation to a straight line passing through (4,5) and perpendicular to $2x+3y=4$

Any line perpendicular to $2x+3y-4=0$ has the form $3x-2y+K=0$ — (1). This line passes through (4,5)

$$(x,y)=(4,5).$$

$$(1) \Longrightarrow 3x-2y+K=0$$
$$3\times4-2\times5+K=0$$
$$12-10+K=0$$
$$2+K=0 \Longrightarrow K=-2$$

$(1) \Longrightarrow 3x-2y-2=0$ is required equation.

Point of intersection of two lines:

Q. Find the point of intersection of two straight lines $3x-y+5=0$

and $x+3y-2=0$

$$3x-y=-5, \quad x+3y=2.$$

$$\Delta = \begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix} = 9+1 = 10$$

$$\Delta_1 = \begin{vmatrix} -5 & -1 \\ 2 & 3 \end{vmatrix} = -15+2 = -13$$

$$\Delta_2 = \begin{vmatrix} 3 & -5 \\ 1 & 2 \end{vmatrix} = 6+5 = 11$$

$$x = \frac{\Delta_1}{\Delta} = -13/10 \qquad y = \frac{\Delta_2}{\Delta} = \frac{4}{10}.$$

$(-13/10, 11/10)$ is the point of intersection of ~~two~~ the above two lines.

Concurrent lines.

Consider the straight lines $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$ and $a_3x+b_3y+c_3=0$. ~~intersection of~~

If the ~~two~~ point of intersection of any two lines lies on the third line, we can say that the lines are concurrent

Q. Prove the lines $2x-3y-7=0$, $3x-4y-10=0$ and $8x+11y-5=0$ are concurrent.

Solve any two lines.

$2x-3y=7$, $3x-4y=10$.

$$\Delta = \begin{vmatrix} 2 & -3 \\ 3 & -4 \end{vmatrix} = -8+9 = \underline{\underline{1}}$$

$$\Delta_1 = \begin{vmatrix} 7 & -3 \\ 10 & -4 \end{vmatrix} = -28+30 = 2$$

$$\Delta_2 = \begin{vmatrix} 2 & 7 \\ 3 & 10 \end{vmatrix} = 20-21 = -1$$

$$x = \frac{\Delta_1}{\Delta} = \frac{2}{1} = 2, \quad y = \frac{\Delta_2}{\Delta} = \frac{-1}{1} = \underline{\underline{-1}}$$

$(2,-1)$ is the point of intersection of lines. Substitute these point in the third line, we get.

$$8x+11y-5=0.$$

$$(x,y) = (2,-1)$$

$$8\times 2 + 11\times -1 - 5$$

$$= 16-11-5$$

$$= 16-16$$

$$= 0$$