Product formulae and Sum + difference formulae.

(A)

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ —(1)

$\sin(A-B) = \sin A \cos B - \cos A \sin B$ —(2)

(1) + (2) $\Longrightarrow$

$\sin(A+B) + \sin(A-B) = 2 \sin A \cos B$.

$$\begin{array}{ll} A+B = C & A+B = C \\ A-B = D & -A+B = -D \\ \hline 2A = C+D & 2B = C-D \\ A = \dfrac{C+D}{2} & B = \dfrac{C-D}{2} \end{array}$$

(1) $\sin C + \sin D = 2 \sin \dfrac{C+D}{2} \cos \dfrac{C-D}{2}$.

(1) − (2) $\Longrightarrow$

$\sin(A+B) - \sin(A-B) = 2 \cos A \sin B$

(2) $\sin C - \sin D = 2 \cos \dfrac{C+D}{2} \sin \dfrac{C-D}{2}$.

(B)

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ —(3)

$\cos(A-B) = \cos A \cos B + \sin A \sin B$ —(4)

(3) + (4) $\Longrightarrow$

$\cos(A+B) + \cos(A-B) = 2 \cos A \cos B$

(3) $\cos C + \cos D = 2 \cos \dfrac{C+D}{2} \cos \dfrac{C-D}{2}$.

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(3) − (4) $\Longrightarrow$

$$\cos(A+B) - \cos(A-B) = -2\sin A \sin B.$$

(4) $$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2},$$

$$S+S = 2SC$$

$$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}.$$

$$S-S = 2CS$$

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}.$$

$$C+C = 2CC$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}.$$

$$C-C = -2SS$$

$$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}.$$

1. $\sin 4\alpha + \sin 2\alpha.$

$$= 2\sin\frac{4\alpha+2\alpha}{2}\cos\frac{4\alpha-2\alpha}{2}.$$

$$= 2\sin\frac{6\alpha}{2}\cos\frac{2\alpha}{2}.$$

$$= 2\sin 3\alpha\cos\alpha.$$

2. $\cos 65 + \cos 15 = 2\cos\frac{65+15}{2}\cos\frac{65-15}{2}.$

$$= 2\cos\frac{80}{2}\cos\frac{50}{2}.$$

$$= 2\cos 40\cos 25$$

3. $\sin 5\theta - \sin 3\theta = 2\cos\frac{5\theta+3\theta}{2} \sin\frac{5\theta-3\theta}{2}$

$= 2\cos\frac{8\theta}{2}\sin\frac{2\theta}{2}$

$= 2\cos 4\theta \sin\theta$.

4. $\cos 8\theta - \cos 4\theta = -2\sin\frac{8\theta+4\theta}{2} \sin\frac{8\theta-4\theta}{2}$

$= -2\sin\frac{12\theta}{2}\sin\frac{4\theta}{2}$

$= -2\sin 6\theta \sin 2\theta$

5. Show that $\cos 5 - \sin 25 = \sin 35$

$\cos 5 = \sin 35 + \sin 25$.

$R.H.S = \sin 35 + \sin 25$

$= 2\sin\frac{35+25}{2}\cos\frac{35-25}{2}$.

$= 2\sin\frac{60}{2}\cos\frac{10}{2}$.

$= 2\sin 30 \cdot \cos 5$

$= 2 \times \frac{1}{2} \cdot \cos 5$

$= \cos 5 = L.H.S$

6. Prove that $\dfrac{\sin 4A + \sin 2A}{\cos 4A + \cos 2A} = \tan 3A$

$$L.H.S = \frac{\sin 4A + \sin 2A}{\cos 4A + \cos 2A}$$

$$= \frac{2\sin\frac{4A+2A}{2}\cos\frac{4A-2A}{2}}{2\cos\frac{4A+2A}{2}\cos\frac{4A-2A}{2}}$$

$$= \frac{2\sin\frac{6A}{2}\cos\frac{2A}{2}}{2\cos\frac{6A}{2}\cos\frac{2A}{2}}$$

$$= \frac{\sin 3A\,\cancel{\cos A}}{\cos 3A\,\cancel{\cos A}} = \frac{\sin 3A}{\cos 3A}$$

$$= \underline{\underline{\tan 3A}}.$$

Q. Prove that $\dfrac{\sin 2\alpha + \sin 5\alpha - \sin\alpha}{\cos 2\alpha + \cos 5\alpha + \cos\alpha} = \tan 2\alpha.$

$$L.H.S = \frac{\sin 2\alpha + \sin 5\alpha - \sin\alpha.}{\cos 2\alpha + \cos 5\alpha + \cos\alpha.}$$

$$= \frac{\sin 2\alpha + 2\cos\frac{5\alpha+\alpha}{2}\sin\frac{5\alpha-\alpha}{2}}{\cos 2\alpha + 2\cos\frac{5\alpha+\alpha}{2}\cos\frac{5\alpha-\alpha}{2}}$$

$$= \frac{\sin 2\alpha + 2\cos 3\alpha \sin 2\alpha}{\cos 2\alpha + 2\cos 3\alpha \cos 2\alpha}$$

$$= \frac{\sin 2\alpha \cancel{(1 + 2\cos 3\alpha)}}{\cos 2\alpha \cancel{(1 + 2\cos 3\alpha)}}$$

$$= \tan 2\alpha$$

Q.. Prove that $\sin 50 - \sin 70 + \cos 80 = 0$

$\sin 50 - \sin 70 + \cos 80 = 0$

$2\cos\frac{50+70}{2}\sin\frac{50-70}{2} + \cos 80 = 0$

$2\cos\frac{120}{2}\sin\frac{-20}{2} + \cos 80 = 0$

$2\cos 60 \sin(-10) + \cos 80 = 0$

$\cancel{2} \times \frac{1}{\cancel{2}} \sin(-10) + \cos 80 = 0$

$-\sin 10 + \cos(90-10) = 0$

$-\sin 10 + \sin 10 = 0$

$0 = 0$

$L \cdot H \cdot S = R \cdot H \cdot S$

Converse of product formulae

$\sin(A+B) + \sin(A-B) = 2\sin A \cos B$

① $\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A$

② $\sin(A+B) - \sin(A-B) = 2\cos A \sin B.$

$\cos A \sin B = \frac{1}{2}[\sin(A+B) - \sin(A$

3. $\cos(A+B) + \cos(A-B) = 2\cos A \cos B.$

$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A$

4. $\cos(A+B) - \cos(A-B) = -2\sin A \sin B.$

$\sin A \sin B = -\frac{1}{2}[\cos(A+B) - \cos($

Q.1 $\sin 5x \cos 3x$ as a sum or differ

$\sin 5x \cos 3x = \frac{1}{2}[\sin(5x+3x) + \sin($

$= \frac{1}{2}(\sin 8x + \sin 2x)$

Q.2. $\cos 3\alpha \cos\alpha$ as a sum or differe

$\cos 3\alpha \cos\alpha = \frac{1}{2}[\cos(3\alpha+\alpha) + \cos$

$= \frac{1}{2}(\cos 4\alpha + \cos 2\alpha)$

3. Show that $\sin 10 \sin 50 \sin 70 = 1/8$

$\sin 10[-1/2(\cos 120 - \cos 20)]$

$\sin 10[-1/2(\cos(90+30) - \cos 20)]$

$\sin 10[-1/2(-\sin 30 - \cos 20)]$

$\sin 10[-1/2(-1/2 - \cos 20)]$

$\sin 10[1/4 + 1/2 \cos 20]$

$\frac{1}{4}\sin 10 + \frac{1}{2}\sin 10 \cos 20$.

$\frac{1}{4}\sin 10 + 1/2[1/2(\sin 30 + \sin -10)]$

$\frac{1}{4}\sin 10 + 1/2[1/2(1/2 - \sin 10)]$

$\frac{1}{4}\sin 10 + 1/2[1/4 - 1/2 \sin 10]$

$\frac{1}{4}\sin 10 + 1/8 - 1/4 \sin 10$

$= 1/8$ //.