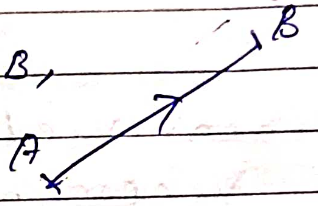
Vector Algebra

Two types of quantities one having only magnitude is called a scalar quantity.
example: mass, volume, time, temperature etc. have only magnitude.
The other having both magnitude and direction is called a vector quantity.
example - force, velocity, magnetic field etc have both magnitude and direction.
A vector is represented by a directed line segment whose end points are initial and terminal points. The length of the segment gives the magnitude of the vector.



$\overrightarrow{AB}$ read as vector AB, where A as the initial point and B as the terminal point. We associate two measure the distance between A and B and the direction from A to B.
vectors are also represented by small letters $\bar{a}, \bar{b}, \bar{c}$. The non negative number which is the measure of the magnitude of a vector is called modulus of the vector.

Different types of vectors.

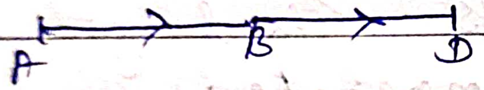
i. Zero vector: A vector whose modulus is zero is called a zero vector. The initial and terminal points of a zero vector ~~coin~~ coincides.
eg: $\overrightarrow{AA}$, $\overrightarrow{BB}$ → represent zero vector.

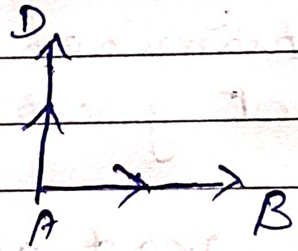
2- unit vector: A vector whose modulus is unity is called a unit vector. A unit vector in the direction of $\vec{a}$ is given as $\hat{a}$ read as a cap. unit vector in the direction of $\vec{a}$ is given by $\frac{\vec{a}}{|\vec{a}|}$

3 Co-initial vectors

Vectors having same initial point are called co-initial vectors.

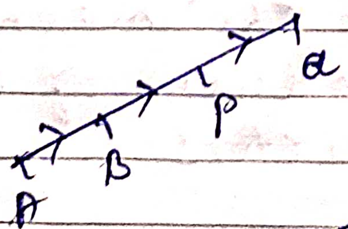
Example $\vec{AB}$ and $\vec{AD}$ are co-initial vectors.



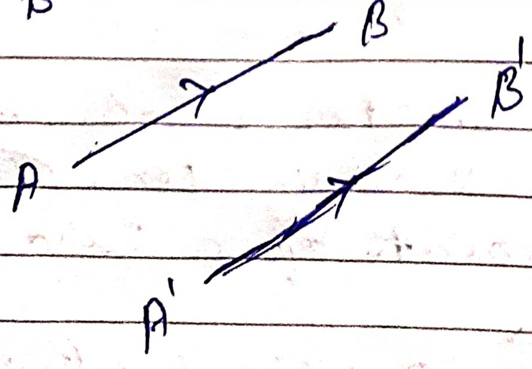


4. Collinear vectors.

Vectors having the same line of action or having the line of action parallel to one another irrespective of their magnitude and direction are called collinear vectors or parallel vectors.



$\vec{AB}$ and $\vec{PQ}$ are collinear vectors

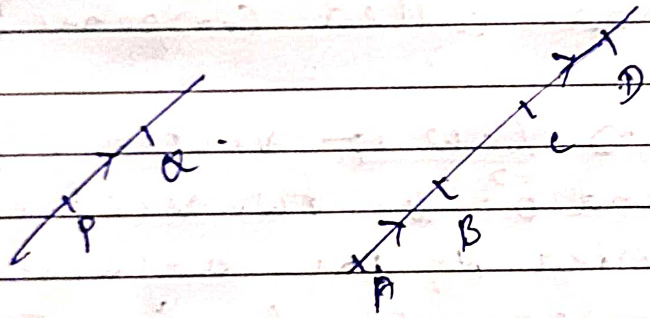


5. Like and unlike vectors.

Vectors having the same direction are called like vectors and vectors having opposite direction are called unlike vectors.

6. Equal vectors.

Two vectors are said to be equal if they have the same magnitude and the same direction irrespective of the position of their initial points.

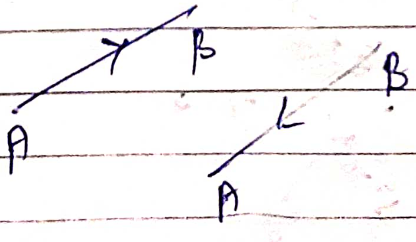


If $\vec{AB}$, $\vec{CD}$ and $\vec{PQ}$ are parallel segments and $AB = CD = PQ$, the vectors $\vec{AB}$, $\vec{CD}$ and $\vec{PQ}$ are equal.

7. Negative vectors.

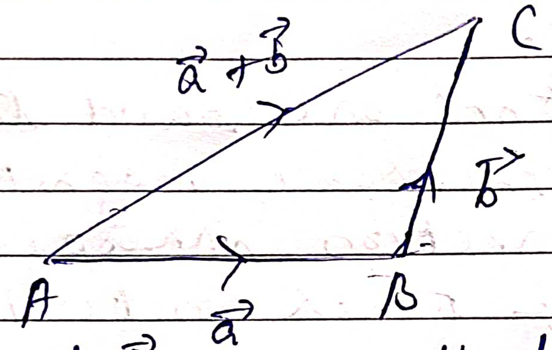
Vectors which have the same magnitude but direction opposite to each other are negative vectors.

If $\vec{AB} = a$,
$\vec{BA} = -a$.



Addition of vectors -

The vector $\vec{AB}$, the displacement of a body from a point A to another point B. If we have a situation to move it from B to C, the net displacement from A to C is $\vec{AC}$, $\vec{AC} = \vec{AB} + \vec{BC}$. This is known as triangle law of vector addition.



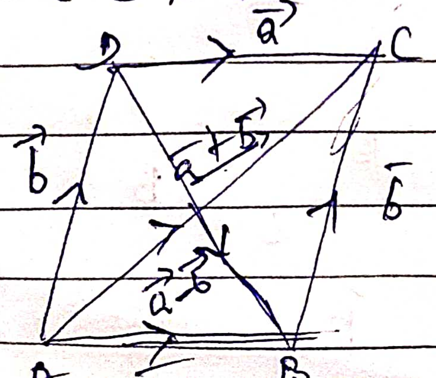
$\vec{a}$ and $\vec{b}$ are called component vectors.

Parallelogram law of vector addition

Let $\vec{a}$ and $\vec{b}$ be two vectors which are represented by the adjacent sides of a parallelogram, the sum of $\vec{a}$ and $\vec{b}$ is found out by completing the parallelogram. The principal diagonal gives the sum and off diagonal gives the difference of the vectors.

$$\vec{AC} = \vec{AB} + \vec{BC} = \vec{a} + \vec{b}$$

and $\vec{DB} = \vec{a} - \vec{b}$



Properties of vector addition.

Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors.

1. $\vec{a}+\vec{b} = \vec{b}+\vec{a}$
2. $\vec{a}+(\vec{b}+\vec{c}) = (\vec{a}+\vec{b})+\vec{c}$
3. $\vec{a}+\vec{0} = \vec{0}+\vec{a} = \vec{a}$
4. $\vec{a} + -\vec{a} = -\vec{a} + \vec{a} = 0$.

Properties of Scalar multiplication.

1. $m(n\vec{a}) = (mn)\vec{a}$
2. $m(\vec{a}+\vec{b}) = m\vec{a} + m\vec{b}$

Note: If two non zero vectors $\vec{a}$ and $\vec{b}$ are collinear then $\vec{a} = m\vec{b}$ where m is a scalar.

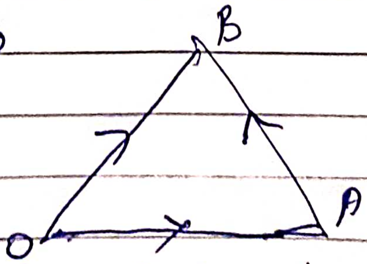
Position vector

The position vector of any point P with reference to the origin O is the vector $\vec{OP}$

Expression of a vector of the position vectors of its endpoints.

Let $\vec{AB}$ be a given vector. choose any point O as the origin. $\vec{OA}$ and $\vec{OB}$ are position vectors of A and B related to O.

By triangle law of addition.



$OA + AB = OB$

$AB = OB - OA$

= position vector of B − position vector of A