Product formulae and Sum & difference formulae.

(A)

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ — (1)

$\sin(A-B) = \sin A \cos B - \cos A \sin B$ — (2)

(1) + (2) $\Longrightarrow$

$\sin(A+B) + \sin(A-B) = 2 \sin A \cos B$.

$A+B = C$ $\qquad$ $A+B = C$

$A-B = D$ $\qquad$ $-A+B = -D$

$2A = C+D$ $\qquad$ $2B = C-D$

$A = \frac{C+D}{2}$. $\qquad$ $B = \frac{C-D}{2}$.

(1) $\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$.

(1) − (2) $\Longrightarrow$

$\sin(A+B) - \sin(A-B) = 2 \cos A \sin B$

(2) $\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$.

(B)

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ — (3)

$\cos(A-B) = \cos A \cos B + \sin A \sin B$ — (4)

(3) + (4) $\Longrightarrow$

$\cos(A+B) + \cos(A-B) = 2 \cos A \cos B$

(3) $\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$.

(3) − (4) ⟹

$$\cos(A+B) - \cos(A-B) = -2\sin A \sin B.$$

(4) $$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}.$$

$$S + S = 2SC$$

$$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}.$$

$$S - S = 2CS$$

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}.$$

$$C + C = 2CC$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}.$$

$$C - C = -2SS$$

$$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}.$$

1. $\sin 4\theta + \sin 2\theta.$

$$= 2\sin\frac{4\theta+2\theta}{2}\cos\frac{4\theta-2\theta}{2}.$$

$$= 2\sin\frac{6\theta}{2}\cos\frac{2\theta}{2}.$$

$$= 2\sin 3\theta\cos\theta.$$

2. $\cos 65 + \cos 15 = 2\cos\frac{65+15}{2}\cos\frac{65-15}{2}.$

$$= 2\cos\frac{80}{2}\cos\frac{50}{2}.$$

$$= 2\cos 40\cos 25$$

3. $\sin 5\theta - \sin 3\theta = 2\cos\frac{5\theta+3\theta}{2} \cdot \sin\frac{5\theta-3\theta}{2}$

$= 2\cos\frac{8\theta}{2}\sin\frac{2\theta}{2}$

$= 2\cos 4\theta \sin\theta$

4. $\cos 8\theta - \cos 4\theta = -2\sin\frac{8\theta+4\theta}{2} \cdot \sin\frac{8\theta-4\theta}{2}$

$= -2\sin\frac{12\theta}{2}\sin\frac{4\theta}{2}$

$= -2\sin 6\theta \sin 2\theta$

5. Show that $\cos 5 - \sin 25 = \sin 35$

$\cos 5 = \sin 35 + \sin 25$

R.H.S $= \sin 35 + \sin 25$

$= 2\sin\frac{35+25}{2}\cos\frac{35-25}{2}$

$= 2\sin\frac{60}{2}\cos\frac{10}{2}$

$= 2\sin 30 \cdot \cos 5$

$= \cancel{2} \times \frac{1}{\cancel{2}} \cdot \cos 5$

$= \cos 5 = $ L.H.S

6. Prove that $\dfrac{\sin 4A + \sin 2A}{\cos 4A + \cos 2A} = \tan 3A$

L.H.S = $\dfrac{\sin 4A + \sin 2A}{\cos 4A + \cos 2A}$

$$= \frac{2 \sin \frac{4A+2A}{2} \cos \frac{4A-2A}{2}}{2 \cos \frac{4A+2A}{2} \cos \frac{4A-2A}{2}}$$

$$= \frac{\cancel{2} \sin \frac{6A}{2} \cos \frac{2A}{2}}{\cancel{2} \cos \frac{6A}{2} \cos \frac{2A}{2}}$$

$$= \frac{\sin 3A \cancel{\cos A}}{\cos 3A \cancel{\cos A}} = \frac{\sin 3A}{\cos 3A}$$

$$= \tan 3A$$

Q. Prove that $\dfrac{\sin 2\alpha + \sin 5\alpha - \sin \alpha}{\cos 2\alpha + \cos 5\alpha + \cos \alpha} = \tan 2\alpha$.

L.H.S = $\dfrac{\sin 2\alpha + \sin 5\alpha - \sin \alpha}{\cos 2\alpha + \cos 5\alpha + \cos \alpha}$.

$$= \frac{\sin 2\alpha + 2\cos\frac{5\alpha+\alpha}{2}\sin\frac{5\alpha-\alpha}{2}}{\cos 2\alpha + 2\cos\frac{5\alpha+\alpha}{2}\cos\frac{5\alpha-\alpha}{2}}$$

$$= \frac{\sin 2\alpha + 2\cos 3\alpha \sin 2\alpha}{\cos 2\alpha + 2\cos 3\alpha \cos 2\alpha}$$

$$= \frac{\sin 2\alpha\,(1 + 2\cos 3\alpha)}{\cos 2\alpha\,(1 + 2\cos 3\alpha)}$$

$$= \tan 2\alpha$$

Q. Prove that $\sin 50 - \sin 70 + \cos 80 = 0$

$$\sin 50 - \sin 70 + \cos 80 = 0$$

$$2\cos\frac{50+70}{2}\sin\frac{50-70}{2} + \cos 80 = 0$$

$$2\cos\frac{120}{2}\sin\frac{-20}{2} + \cos 80 = 0$$

$$2\cos 60\,\sin(-10) + \cos 80 = 0$$

$$\cancel{2} \times \tfrac{1}{\cancel{2}}\sin(-10) + \cos 80 = 0$$

$$-\sin 10 + \cos(90-10) = 0$$

$$-\sin 10 + \sin 70 = 0$$

$$0 = 0$$

L.H.S = R.H.S

Converse of product formulae

$\sin(A+B) + \sin(A-B) = 2\sin A\cos B$

(1) $\sin A\cos B = \frac{1}{2}[\sin(A+B) + \sin(A$

(2) $\sin(A+B) - \sin(A-B) = 2\cos A\sin B$.

$\cos A\sin B = \frac{1}{2}[\sin(A+B) - \sin(A$

3. $\cos(A+B) + \cos(A-B) = 2\cos A\cos B$.

$\cos A\cos B = \frac{1}{2}[\cos(A+B) + \cos(A$

4. $\cos(A+B) - \cos(A-B) = -2\sin A\sin B$.

$\sin A\sin B = -\frac{1}{2}[\cos(A+B) - \cos($

Q.1 $\sin 5x\cos 3x$ as a sum or differ

$\sin 5x\cos 3x = \frac{1}{2}[\sin(5x+3x) + \sin($

$= \frac{1}{2}(\sin 8x + \sin 2x)$

Q.2. $\cos 3\theta\cos\theta$ as a sum or differe

$\cos 3\theta\cos\theta = \frac{1}{2}[\cos(3\theta+\theta) + \cos$

$= \frac{1}{2}(\cos 4\theta + \cos 2\theta)$

3. Show that $\sin 10 \sin 50 \sin 70 = 1/8$

$\sin 10 [-1/2 (\cos 120 - \cos -20)]$

$\sin 10 [-1/2 (\cos (90+30) - \cos 20)]$

$\sin 10 [-1/2 (-\sin 30 - \cos 20)]$

$\sin 10 [-1/2 (-1/2 - \cos 20)]$

$\sin 10 [1/4 + 1/2 \cos 20]$

$\frac{1}{4} \sin 10 + \frac{1}{2} \sin 10 \cos 20$

$\frac{1}{4} \sin 10 + 1/2 [1/2 (\sin 30 + \sin -10)]$

$\frac{1}{4} \sin 10 + 1/2 [1/2 (1/2 - \sin 10)]$

$\frac{1}{4} \sin 10 + 1/2 [1/4 - 1/2 \sin 10]$

$\frac{1}{4} \sin 10 + 1/8 - 1/4 \sin 10$

$= 1/8$ //

Multiple Angles.

$2\theta, 3\theta, 4\theta$ etc are multiple angles.

① $\sin 2\theta = \sin(\theta+\theta)$

$= \sin\theta\cos\theta + \cos\theta\sin\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$.

② $\cos 2\theta = \cos(\theta+\theta)$

$= \cos\theta\cos\theta - \sin\theta\sin\theta$.

$\cos 2\theta = \cos^2\theta - \sin^2\theta \rightarrow (1)$

③ $\tan 2\theta = \tan(\theta+\theta)$

$= \dfrac{\tan\theta + \tan\theta}{1 - \tan\theta\tan\theta}$.

$\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$.

$\sin 2\theta$ and $\cos 2\theta$ in terms of $\tan\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$.

$= \dfrac{2\sin\theta\cos\theta}{1}$

$\sin 2\theta = \dfrac{2\sin\theta\cos\theta}{\sin^2\theta + \cos^2\theta}$.

Dividing numerator and denominator by $\cos^2\theta$

$$\sin 2\theta = \frac{\frac{2\sin\theta\cos\theta}{\cos^2\theta}}{\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta}}.$$

$$= \frac{2\tan\theta}{\frac{\sin^2\theta}{\cos^2\theta}+\frac{\cos^2\theta}{\cos^2\theta}} = \frac{2\tan\theta}{\tan^2\theta+1}$$

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}.$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta.$$

$$= \frac{\cos^2\theta - \sin^2\theta}{1}$$

$$\cos 2\theta = \frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta+\cos^2\theta}.$$

Dividing numerator and denominator by $\cos^2\theta$.

$$\cos 2\theta = \frac{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta}}.$$

$$= \frac{\frac{\cos^2\theta}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}+\frac{\cos^2\theta}{\cos^2\theta}}.$$

(5) $\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta} \rightarrow (2)$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = 1-\sin^2\theta - \sin^2\theta$

(5A) $\cos 2\theta = 1-2\sin^2\theta \rightarrow (3)$

or

$1-2\sin^2\theta = \cos 2\theta$

$-2\sin^2\theta = \cos 2\theta - 1$

$2\sin^2\theta = 1-\cos 2\theta$

(6) $\therefore \sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \cos^2\theta - (1-\cos^2\theta)$

$= \cos^2\theta - 1 + \cos^2\theta$

(6A) $\cos 2\theta = 2\cos^2\theta - 1 \rightarrow (4)$

or $2\cos^2\theta - 1 = \cos 2\theta$

$2\cos^2\theta = \cos 2\theta + 1$

(7) $\therefore \cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

$(6) \div (7)$

$\dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{(1-\cos 2\theta)/2}{(1+\cos 2\theta)/2}$

(8) $\tan^2\theta = \dfrac{1-\cos 2\theta}{1+\cos 2\theta}$

1. If $\tan\alpha = 2$ find $\sin 2\alpha$, $\cos 2\alpha$ and $\tan 2\alpha$.

(1) $\sin 2\alpha = \dfrac{2\tan\alpha}{1+\tan^2\alpha}$, $\tan\alpha = 2$.

$= \dfrac{2\times 2}{1+4} = \dfrac{4}{5}$.

(2) $\cos 2\alpha = \dfrac{1-\tan^2\alpha}{1+\tan^2\alpha}$.

$= \dfrac{1-4}{1+4} = -3/5$.

(3) $\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha} = \dfrac{2\times 2}{1-4}$

$= \dfrac{4}{-3} = -4/3$.

or

$\tan 2\alpha = \dfrac{\sin 2\alpha}{\cos 2\alpha} = \dfrac{4/5}{-3/5}$

$= \dfrac{4}{5} \times \dfrac{-5}{3} = -4/3$

Date
Page

2. If $\tan A = 3/4$ (A is acute) and $\sin B = 5/13$ (B lies in 2nd quadrant). Find $\sin 2A$ and $\sin 2B$.

(1) $\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$

Given $\tan A = 3/4$.

$\sin 2A = \dfrac{2 \times 3/4}{1 + 9/16}$

$= \dfrac{3/2}{(16+9)/16}$.

$= \left(\dfrac{3}{2}\right) / (25/16)$

$= 3/2 \times \dfrac{16}{25} = \dfrac{24}{25}$

(2) $\sin 2B = 2 \sin B \cos B$

Given $\sin B = 5/13$.

B lies in 2nd quadrant

$\cos^2 B = 1 - \sin^2 B$

$= 1 - 25/169$

$= \dfrac{169-25}{169} = \dfrac{144}{169}$.

$\cos B = \mp 12/13$

B lies on 2nd quadrant

$\cos B = -12/13$.

$\therefore \sin 2B = 2 \sin B \cos B$

$= 2 \times \frac{5}{13} \times \frac{-12}{13}$.

$= \frac{-120}{169}$.

3. Prove that $\frac{1+\cos 2A}{\sin 2A} = \cot A$ and deduce the value of $\cot 15^\circ$.

$L.H.S = \frac{1+\cos 2A}{\sin 2A}$.

$\cos^2 A = \frac{1+\cos 2A}{2}$

$\therefore 1+\cos 2A = 2\cos^2 A$.

$\sin 2A = 2 \sin A \cos A$.

$L.H.S = \frac{2\cos^2 A}{2 \sin A \cos A} = \cot A$

$= R.H.S$.

$\cot A = \frac{1+\cos 2A}{\sin 2A}$

$A = 15^\circ$.

$$\cot 15^\circ = \frac{1+\cos 2\times 15^\circ}{\sin 2\times 15^\circ}$$

$$= \frac{1+\cos 30^\circ}{\sin 30^\circ}$$

$$= \frac{1+\sqrt{3}/2}{1/2}$$

$$= \frac{(2+\sqrt{3})/2}{1/2} = \frac{2+\sqrt{3}}{2}\times\frac{2}{1}$$

$$= \underline{\underline{2+\sqrt{3}}}.$$

4. Prove that $\cos 4\theta = 1 - 8\sin^2\theta\cos^2\theta$.

$$\cos 2\theta = 1 - 2\sin^2\theta.$$

$$\theta = 2\theta.$$

$$\cos 4\theta = 1 - 2\sin^2 2\theta.$$

$$= 1 - 2(2\sin 2\theta)^2$$

$$= 1 - 2(2\sin\theta\cos\theta)^2 \qquad \sin^2\theta = (\sin\theta)^2.$$

$$= 1 - 2(4\sin^2\theta\cos^2\theta)$$

$$= 1 - 8\sin^2\theta\cos^2\theta$$

Expressions of $\sin 3A$, $\cos 3A$

$\sin 3A = \sin(2A + A)$

$= \sin 2A \cos A + \cos 2A \sin A$

$= 2\sin A \cos A \cdot \cos A + (1 - 2\sin^2 A)\sin A$

$= 2\sin A \cos^2 A + \sin A - 2\sin^3 A$

$= 2\sin A(1 - \sin^2 A) + \sin A - 2\sin^3 A$

$= 2\sin A - 2\sin^3 A + \sin A - 2\sin^3 A$

$\sin 3A = 3\sin A - 4\sin^3 A$

$\cos 3A = \cos(2A + A)$

$= \cos 2A \cos A - \sin 2A \sin A$

$= (2\cos^2 A - 1)\cos A - 2\sin A \cos A \cdot \sin A$

$= 2\cos^3 A - \cos A - 2\sin^2 A \cos A$

$= 2\cos^3 A - \cos A - 2(1 - \cos^2 A)\cos A$

$= 2\cos^3 A - \cos A - 2(\cos A - \cos^3 A)$

$= 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A$

$\cos 3A = 4\cos^3 A - 3\cos A$

5. Prove that $\dfrac{\cos 3A + \cos A}{\sin 3A - \sin A} = \cot A$

L.H.S $= \dfrac{\cos 3A + \cos A}{\sin 3A - \sin A}$

$$= \frac{4\cos^3 A - 3\cos A + \cos A}{3\sin A - 4\sin^3 A - \sin A}$$

$$= \frac{4\cos^3 A - 2\cos A}{2\sin A - 4\sin^3 A}$$

$$= \frac{2\cos A(2\cos^2 A - 1)}{2\sin A(1 - 2\sin^2 A)}$$

$$= \frac{\cos A \cdot \cos 2A}{\sin A \cos 2A} = \frac{\cos A}{\sin A}$$

$$= \cot A = R.H.S$$

6. Prove that $\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x} = 2.$

$$L.H.S = \frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x}.$$

$$= \frac{3\sin x - 4\sin^3 x}{\sin x} - \frac{4\cos^3 x - 3\cos x}{\cos x}$$

$$= \frac{\sin x(3 - 4\sin^2 x)}{\sin x} - \frac{\cos x(4\cos^2 x - 3)}{\cos x}$$

$$= 3 - 4\sin^2 x - (4\cos^2 x - 3)$$

$$= 3 - 4\sin^2 x - 4\cos^2 x + 3$$

$$= 6 - 4(\sin^2 x + \cos^2 x)$$

$$= 6 - 4 \times 1 = 6 - 4$$

$$= 2$$

Date
Page

Workout problems.

1. If $\cos\theta = 1/2$, θ is acute, find $\sin 2\theta$, $\cos 2\theta$, $\sin 3\theta$ and $\cos 3\theta$.
2. If $\tan B = 1$, B lies in 3rd quadrant, find $\sin 2B$, $\cos 2B$ and $\tan 2B$.
3. If $\tan\theta = 2$, find $\cos 2\theta$.
4. Prove that $\dfrac{\sin 2A}{1+\cos 2A} = \tan A$ and deduce the value of $\tan 15^\circ$.
5. If $\cos 2\theta = -3/5$ find $\tan\theta$.
6. Prove that $\dfrac{\sin 3A}{\sin A} + \dfrac{\cos 3A}{\cos A} + 4\cos 2A$.
7. Prove that $\cos^4 A - \sin^4 A = \cos 2A$

classmate

Date

Page

To convert degree measure

1. $\left(\frac{7\pi}{6}\right)^c = \frac{7\pi}{6} \times 1^c = \frac{7\pi}{6} \times \frac{180}{\pi}$ $\quad 1^c = \frac{180^\circ}{\pi}$

$= 210^\circ$

2. $\left(\frac{11}{16}\right)^c = \frac{11}{16} \times 1^c = \frac{11}{16} \times \frac{180}{\pi}$

$= \frac{11 \times 45 \times 7}{4 \times 22}$

$= \frac{315}{8}$

$= \left(39\frac{3}{8}\right)^\circ = 39^\circ \frac{3}{8}^\circ$

$= 39^\circ (3/8 \times 60)'$ $\quad 1^\circ = 60'$

$= 39^\circ \frac{180}{8}'$

$= 39^\circ (22\ 1/2)'$

$= 39^\circ 22' (1/2 \times 60)''$

$= 39^\circ 22' 30''$

3. $\left(\frac{\pi}{10}\right)^c = \frac{\pi}{10} \times 1^c = \frac{\pi}{10} \times \frac{180}{\pi}$

$= 18^\circ$.

4. $\left(\frac{\pi}{8}\right)^c = \frac{\pi}{8} \times 1^c = \frac{\pi}{8} \times \frac{180^\circ}{\pi} = \frac{180}{8}$

$= (22\ 1/2)^\circ = 22^\circ (1/2 \times 60)$

$= 22^\circ 30'$

To convert radian measure.

1. 340°

$340^\circ = 340 \times 1^\circ$ $\qquad 1^\circ = \frac{\pi}{180}^c$

$= \overset{17}{\cancel{340}} \times \frac{\pi}{\cancel{180}\,9}$

$= \left(\frac{17\pi}{9}\right)^c$

$= \frac{17\pi}{9}$ radian.

2. $75^\circ = \overset{155}{\cancel{75}} \times \frac{\pi}{\cancel{180}\,\cancel{36}12} = \left(\frac{5\pi}{12}\right)^c$

$= \frac{5\pi}{12}$ radian

3. $-37^\circ 30' = -\left(+37 + \frac{\cancel{30}}{\cancel{60}\,2}\right)^\circ$

$= -\left(+37 + \frac{1}{2}\right)^\circ$

$= -\left(\frac{+74+1}{2}\right)^\circ = -\frac{75}{2}^\circ$

$= -\frac{\overset{\cancel{25}\,5}{\cancel{75}}}{2} \times \frac{\pi}{\cancel{180}\,\cancel{60}12}^c = \left(\frac{5\pi}{24}\right)^c$

$= \frac{5\pi}{24}$ radian

1. $\sin\theta = 2/5$ $\cos\theta$, $\tan\theta$?

$\cos^2\theta = 1 - \sin^2\theta = 1 - (\sin\theta)^2$

$= 1 - (2/5)^2 = 1 - 4/25$

$= \frac{25-4}{25} = \frac{21}{25}$

$\therefore \cos\theta = \sqrt{21/25} = \frac{\sqrt{21}}{\sqrt{25}} = \frac{\sqrt{21}}{5}$ //.

$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2/5}{\sqrt{21}/5}$

$= \frac{2}{5} \times \frac{5}{\sqrt{21}} = 2/\sqrt{21}$.

$\tan\theta = 3$.

$\sec^2\theta = 1 + \tan^2\theta = 1 + (\tan\theta)^2$

$= 1 + 3^2 = 1 + 9 = 10$

$\therefore \sec\theta = \sqrt{10}$ //.

$\therefore \cos\theta = \frac{1}{\sec\theta} = \frac{1}{\sqrt{10}}$ //.

$\sin^2\theta = 1 - \cos^2\theta = 1 - (\cos\theta)^2$

$= 1 - \left(\frac{1}{\sqrt{10}}\right)^2 = 1 - \frac{1}{10} = \frac{10-1}{10}$

$= 9/10$.

$\sin\theta = \sqrt{9/10} = 3/\sqrt{10}$.

$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{3/\sqrt{10}} = \sqrt{10}/3$

(a) $\sin A \cot A = L.H.S$

$L.H.S = \sin A \times \frac{\cos A}{\sin A} = \cos A = R.H.S$

$L.H.S = R.H.S$

(b) $\sin^2 A - \cos^2 A = L.H.S$

$L.H.S = \sin^2 A - \cos^2 A$

$= (1 - \cos^2 A) - \cos^2 A$

$= 1 - \cos^2 A - \cos^2 A$

$= 1 - 2\cos^2 A = R.H.S$

$L.H.S = R.H.S$

(c) $(\sin A + \cos A)^2 = L.H.S$

$L.H.S = (\sin A + \cos A)^2$

$= (\sin A)^2 + 2\sin A \cos A + (\cos A)^2$

$= \sin^2 A + 2\sin A \cos A + \cos^2 A$

$= \sin^2 A + \cos^2 A + 2\sin A \cos A$

$= 1 + 2\sin A \cos A = R.H.S$

$L.H.S = R.H.S$

$\frac{1+\sin\theta}{\cos\theta} = L.H.S$

$L.H.S = \frac{1+\sin\theta}{\cos\theta}$

Numerator and denominator multiplied by $1-\sin\theta$.

$$L.H.S = \frac{(1+\sin\theta)(1-\sin\theta)}{\cos\theta(1-\sin\theta)}$$

$$= \frac{1-(\sin\theta)^2}{\cos\theta(1-\sin\theta)} = \frac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)}$$

$$= \frac{\cos^2\theta}{\cos\theta(1-\sin\theta)} = \frac{(\cos\theta)^2}{\cos\theta(1-\sin\theta)}$$

$$= \frac{\cos\theta \cdot \cos\theta}{\cos\theta(1-\sin\theta)} = \frac{\cos\theta}{1-\sin\theta} = R.H.S.$$

5 $$\frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta}.$$

$$= \frac{(1+\cos\theta)(1+\cos\theta) + \sin\theta \cdot \sin\theta}{\sin\theta(1+\cos\theta)}.$$

$$= \frac{1+\cos\theta+\cos\theta+\cos^2\theta+\sin^2\theta}{\sin\theta(1+\cos\theta)}.$$

$$= \frac{1+2\cos\theta+1}{\sin\theta(1+\cos\theta)} = \frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)} = \frac{2}{\sin\theta}.$$

$$= 2 \cdot \operatorname{cosec}\theta.$$

$$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

6. $\dfrac{\cos A}{1-\tan A} + \dfrac{\sin A}{1-\cot A}$

$$= \frac{\cos A}{1-\frac{\sin A}{\cos A}} + \frac{\sin A}{1-\frac{\cos A}{\sin A}}$$

$$= \frac{\cos A}{\frac{\cos A - \sin A}{\cos A}} + \frac{\sin A}{\frac{\sin A - \cos A}{\sin A}}$$

$$= \frac{\cos A \cdot \cos A}{\cos A - \sin A} + \frac{\sin A \cdot \sin A}{\sin A - \cos A}$$

$$= \frac{\cos A \cos A}{\cos A - \sin A} - \frac{\sin A \sin A}{\cos A - \sin A}$$

$$= \cancel{\cos^2 A - \sin^2 A}$$

$$= \frac{(\cos A - \sin A)\cos^2 A - (\cos A - \sin A)\sin^2 A}{(\cos A - \sin A)(\cos A - \sin A)}$$

$$= \frac{\cancel{(\cos A - \sin A)}(\cos^2 A - \sin^2 A)}{\cancel{(\cos A - \sin A)}(\cos A - \sin A)}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$$

$$= \frac{(\cos A - \sin A)(\cos A + \sin A)}{\cos A - \sin A}$$

$$= \underline{\underline{\cos A + \sin A}}$$

7a. Prove that $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$

L.H.S $= \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$.

$= \frac{\sin\theta \cdot \sin\theta + (1+\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(\sin\theta)}$.

$= \frac{\sin^2\theta + 1 + \cos\theta + \cos\theta + \cos^2\theta}{(1+\cos\theta)(\sin\theta)}$.

$= \frac{\sin^2\theta + \cos^2\theta + 1 + 2\cos\theta}{(1+\cos\theta)\sin\theta}$.

$= \frac{1 + 1 + 2\cos\theta}{(1+\cos\theta)\sin\theta} = \frac{2+2\cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta}$

$= \frac{2}{\sin\theta} = 2 \times \frac{1}{\sin\theta} = \underline{\underline{2\operatorname{cosec}\theta}}$

$=$ R.H.S.

8 $\dfrac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \dfrac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2\sec^2 A$

$$L.H.S = \frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1}$$

$$= \frac{\operatorname{cosec} A(\operatorname{cosec} A + 1) + \operatorname{cosec} A(\operatorname{cosec} A - 1)}{(\operatorname{cosec} A - 1)(\operatorname{cosec} A + 1)}$$

$$= \frac{\operatorname{cosec}^2 A + \operatorname{cosec} A + \operatorname{cosec}^2 A - \operatorname{cosec} A}{\operatorname{cosec}^2 A - 1}$$

$$= \frac{2\operatorname{cosec}^2 A}{\cot^2 A} = 2 \times \frac{1}{\sin^2 A} \times \tan^2 A$$

$$= 2 \times \frac{1}{\sin^2 A} \times \frac{\sin^2 A}{\cos^2 A}$$

$$= 2 \times \sec^2 A = R.H.S.$$

9. Prove that $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

$$L.H.S = \sqrt{\frac{1+\sin A}{1-\sin A}}$$

$$= \frac{\sqrt{1+\sin A}}{\sqrt{1-\sin A}} \frac{\sqrt{1+\sin A}}{\sqrt{1+\sin A}}.$$

$$= \frac{(\sqrt{1+\sin A})^2}{\sqrt{(1-\sin A)(1+\sin A)}}$$

$$= \frac{((1+\sin A)^{1/2})^2}{\sqrt{1-\sin^2 A}} = \frac{1+\sin A}{\sqrt{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A$$

Problems

1. If $\sin\theta = 2/5$ and 'θ' is an acute angle, find $\cos\theta$ and $\tan\theta$.
2. If $\tan\theta = 3$ and θ is an acute angle, find $\sec\theta$ and $\operatorname{cosec}\theta$.
3. Prove the following identities
   (a) $\sin A \cot A = \cos A$
   (b) $\sin^2 A - \cos^2 A = 1 - 2\cos^2 A$
   (c) $(\sin A + \cos A)^2 = 1 + 2\sin A \cos A$
4. Prove that $\frac{1+\sin\theta}{\cos\theta} = \frac{\cos\theta}{1-\sin\theta}$.
5. Show that $\frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta} = 2\operatorname{cosec}\theta$.
6. Prove that $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \sin A + \cos A$

Trigonometric functions of standard angles:

θ	0°	$30^\circ = \pi/6$	$45^\circ = \pi/4$	$60^\circ = \pi/3$	$90^\circ = \pi/2$
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	not defined
$\cot\theta$	not defined	$\sqrt{3}$	1	$1/\sqrt{3}$	0
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	not defined
$\operatorname{cosec}\theta$	not defined	2	$\sqrt{2}$	$2/\sqrt{3}$	1

Evaluate the following.

(1) $3\sin 30^\circ - 4\cos^3 60^\circ$

$$= 3 \times 1/2 - 4 \times (1/2)^3$$

$$= \frac{3}{2} - 4 \times \frac{1}{8} = \frac{3}{2} - \frac{1}{2}$$

$$= \frac{2}{2} = 1$$

(2) $4\tan^2 60^\circ - 3\tan^2 30^\circ + \tan^2 45^\circ$

$= 4 \times (\sqrt{3})^2 - 3 \times \left(\frac{1}{\sqrt{3}}\right)^2 + 1$

$= 4 \times 3 - 3 \times \frac{1}{3} + 1$

$= 12 - 1 + 1 = \underline{\underline{12}}.$

(3) $\tan^2 45^\circ - \frac{1}{2}\cos 60^\circ - \frac{3}{4}\tan^2 30^\circ$

$= 1 - \frac{1}{2} \times \frac{1}{2} - \frac{3}{4} \times \left(\frac{1}{\sqrt{3}}\right)^2.$

$= 1 - \frac{1}{4} - \frac{3}{4} \times \frac{1}{3}$

$= 1 - \frac{1}{4} - \frac{1}{4} = 1 - \frac{2}{4} = 1 - \frac{1}{2}.$

$= \frac{2-1}{2}$

$= \underline{\underline{1/2}}.$

(4). $4\sin^3 \pi/3 - 3\cos \pi/6.$

$= 4(\sqrt{3}/2)^3 - 3(\sqrt{3}/2)$

$= \cancel{4} \times \frac{3\sqrt{3}}{\cancel{8}\,2} - \frac{3\sqrt{3}}{2}.$

$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = \underline{\underline{0}}.$

(5) Prove that $\dfrac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} = 2 - \sqrt{3}$

$$\frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} = \frac{1 - 1/\sqrt{3}}{1 + 1/\sqrt{3}}$$

$$= \frac{(\sqrt{3} - 1)/\sqrt{3}}{(\sqrt{3} + 1)/\sqrt{3}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

~~Dr~~ ~~or~~

Numerator and denominator multiplied by $\sqrt{3} - 1$

$$= \frac{(\sqrt{3} - 1)(\sqrt{3} - 1)}{(\sqrt{3} + 1)(\sqrt{3} - 1)} = \frac{3 - 2\sqrt{3} + 1}{3 - 1}$$

$$= \frac{4 - 2\sqrt{3}}{2} = \frac{2(2 - \sqrt{3})}{2}$$

$$= 2 - \sqrt{3}.$$

(6). Prove that $\dfrac{\tan 60 - \tan 45}{1 + \tan 60 \tan 45} = 2 - \sqrt{3}$

$$\frac{\tan 60 - \tan 45}{1 + \tan 60 \tan 45} = \frac{\sqrt{3} - 1}{1 + \sqrt{3}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

Numerator and denominator multiplied by $\sqrt{3}-1$

$$= \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$= \frac{3-2\sqrt{3}+1}{3-1} = \frac{4-2\sqrt{3}}{2}.$$

$$= \frac{2(2-\sqrt{3})}{2} = \underline{\underline{2-\sqrt{3}}}.$$

Q. Evaluate $\sin^3 \pi/3 \cos^2 \pi/4 \tan \pi/6$.

$\sin^3 \pi/3 \cos^2 \pi/4 \tan \pi/6$

$$= \sin^3 60^\circ \cos^2 45^\circ \tan 30^\circ$$

$$= (\sqrt{3}/2)^3 (1/\sqrt{2})^2 \cdot 1/\sqrt{3}$$

$$= \frac{3\sqrt{3}}{8} \times \frac{1}{2} \times \frac{1}{\sqrt{3}}$$

$$= \underline{\underline{3/16}}.$$

Workout problems.

1. If $\tan\theta = 3$, θ is acute find $\sin\theta$ and $\cos\theta$.
2. Show that $\frac{\sin\theta}{1-\cos\theta} + \frac{1-\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$.
3. Prove that $\frac{\sec\theta}{\sec\theta+1} + \frac{\sec\theta}{\sec\theta-1} = 2\operatorname{cosec}^2\theta$.
4. Evaluate. $\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$
5. Evaluate $4\sin^3 \pi/6 - 3\cos \pi/6$.
6. If $\theta = 30^\circ$, verify that $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$.
7. Evaluate $\sin \pi/2 \times \cos \pi/4 \times \tan \pi/3$
8. Evaluate $2\operatorname{cosec}^2 \pi/2 - 4\sec^2 \pi/4$.
9. Evaluate $3\cot^2 \pi/3 - 2\tan^2 \pi/6$.

when n is an odd integer, $n = 1, 3, 5 \ldots$

$$\sin(n \cdot 90 \pm \theta) = \pm \cos\theta.$$

$$\cos(n \cdot 90 \pm \theta) = \pm \sin\theta$$

$$\tan(n \cdot 90 \pm \theta) = \pm \cot\theta.$$

when n is an even integer, $n = 2, 4, 6 \ldots$

$$\sin(n \cdot 90 \pm \theta) = \pm \sin\theta$$

$$\cos(n \cdot 90 \pm \theta) = \pm \cos\theta$$

$$\tan(n \cdot 90 \pm \theta) = \pm \tan\theta.$$

positive or negative sign depends on the quadrant in which the angle lies.

Evaluate the following.

1. $\sin 120^\circ = \sin(90 + 30)$

$= \cos 30 = \sqrt{3}/2.$

$\boxed{\text{or}}$

$\sin(120)^\circ = \sin(180 - 60)$

$= \sin 60^\circ$

$= \sqrt{3}/2$

2. $\cos 135^\circ = \cos(90 + 45)$

$= -\sin 45 = -1/\sqrt{2}$

$\boxed{\text{or}}$

$\cos 135^\circ = \cos(180 - 45)$

$= -\cos 45 = -1/\sqrt{2}$

3. $\sin 300^\circ = \sin(360-60)$

$= -\sin 60 = -\sqrt{3}/2$.

$\boxed{\text{OR}}$

$\sin 300^\circ = \sin(270+30^\circ)$.

$= -\cos 30^\circ = -\sqrt{3}/2$

4. $\cos 330^\circ = \cos(360-30)$.

$= \cos 30 = \sqrt{3}/2$.

$\boxed{\text{OR}}$.

$\cos 330^\circ = \cos(270+60)$.

$= \sin 60 = \sqrt{3}/2$

5. $\tan 150^\circ = \tan(90+60)$

$= -\cot 60 = -1/\sqrt{3}$

$\boxed{\text{OR}}$

$\tan 150^\circ = \tan(180-30)$

$= -\tan 30 = -1/\sqrt{3}$.

6. $\operatorname{cosec} 120^\circ = \cancel{\frac{1}{\cos}} = \frac{1}{\sin 120^\circ}$

$= \frac{1}{\sin(180-60)} = \frac{1}{\sin 60}$

$= \frac{1}{\sqrt{3}/2} = 2/\sqrt{3}$.

Date
Page

7. $\sin 450^\circ = \sin(360+90)$
$= \sin 90 = 1$ //.

8. $\cos 510^\circ = \cos(360+150)$
$= \cos 150$
$= \cos(90+60)$
$= -\sin 60$
$= -\sqrt{3}/2$ //.

9. $\sin 510^\circ = \sin(360+150)$
$= \sin 150$
$= \sin(90+60)$
$= \cos 60 = 1/2$ //

10. $\sec 750 = \dfrac{1}{\cos 750}$

$= \dfrac{1}{\cos(360+360+30)}$

$= \dfrac{1}{\cos(360+30)}$

$= \dfrac{1}{\cos 30} = \dfrac{1}{\sqrt{3}/2} = 2/\sqrt{3}$

Note

1. $\sin(\theta-90) = \sin-(90-\theta)$
$= -\sin(90-\theta)$
$= -\cos\theta$

2. $\cos(\theta-90) = \cos-(90-\theta)$

$= \cos(90-\theta)$

$= \underline{\underline{\sin\theta}}$.

3. $\tan(\theta - 270) = \tan-(270-\theta)$

$= -\tan(270-\theta)$.

$= -\cot\theta$

1. Evaluate $\sin 120^\circ \cos 330^\circ + \cos 240^\circ \sin 330^\circ$

$\sin 120^\circ = \sin(90+30^\circ)$

$= \cos 30^\circ = \underline{\underline{\sqrt{3}/2}}$.

$\cos 330^\circ = \cos(360-30)$

$= \cos 30 = \sqrt{3}/2$.

$\cos 240^\circ = \cos(180+60)$

$= -\cos 60 = -1/2$

$\sin 330^\circ = \sin(360-30)$

$= -\sin 30 = -1/2$.

$\therefore \sin 120^\circ \cos 330^\circ + \cos 240^\circ \sin 330^\circ$

$= \sqrt{3}/2 \times \sqrt{3}/2 + \frac{-1}{2} \times -\frac{1}{2}$.

$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$ //.

$= \cos(90-\theta)$

$= \underline{\underline{\sin\theta}}$.

3. $\tan(\theta - 270) = \tan-(270-\theta)$

$= -\tan(270-\theta)$.

$= -\cot\theta$

1. Evaluate $\sin 120^\circ \cos 330^\circ + \cos 240^\circ \sin 330^\circ$

$\sin 120^\circ = \sin(90+30^\circ)$

$= \cos 30^\circ = \underline{\underline{\sqrt{3}/2}}$.

$\cos 330^\circ = \cos(360-30)$

$= \cos 30 = \sqrt{3}/2$.

$\cos 240^\circ = \cos(180+60)$

$= -\cos 60 = -1/2$

$\sin 330^\circ = \sin(360-30)$

$= -\sin 30 = -1/2$.

$\therefore \sin 120^\circ \cos 330^\circ + \cos 240^\circ \sin 330^\circ$

$= \sqrt{3}/2 \times \sqrt{3}/2 + \frac{-1}{2} \times \frac{-1}{2}$.

$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$ //.

2- Evaluate $\cos 570^\circ \sin 510^\circ - \sin 330 \cos 390^\circ$

$\cos 570^\circ = \cos(360 + 210)$

$= \cos 210$

$= \cos(270 - 60)$

$= -\sin 60 = -\sqrt{3}/2.$

$\sin 510 = \sin(360 + 150)$

$= \sin 150$

$= \sin(90 + 60)$

$= \cos 60 = 1/2$

$\sin 330 = \sin(360 - 30)$

$= -\sin 30$

$= -1/2.$

$\cos 390 = \cos(360 + 30)$

$= \cos 30$

$= \sqrt{3}/2.$

$\cos 570^\circ \sin 510^\circ - \sin 330 \cos 390$

$= -\sqrt{3}/2 \times 1/2 - -\frac{1}{2} \times \sqrt{3}/2.$

$= -\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = 0$

3. Simplify $\dfrac{\sin(180+A)\cos(90-A)\tan(270+A)}{\cos(90+A)\cos(360+A)}$.

$\sin(180+A) = -\sin A$

$\cos(90-A) = \sin A$

$\tan(270+A) = -\cot A$

$\cos(90+A) = -\sin A$

$\cos(360+A) = \cos A$.

$$\frac{\sin(180+A)\cos(90-A)\tan(270+A)}{\cos(90+A)\cos(360+A)}$$

$$= \frac{\cancel{-}\sin A \times \sin A \times -\cot A}{\cancel{-}\sin A \times \cos A}$$

$$= \frac{-\sin A \times \cot A}{\cos A}$$

$$= \frac{-\sin A \times \frac{\cos A}{\sin A}}{\cos A}$$

$$= \frac{-\sin A \times \cos A}{\sin A \times \cos A}$$

$$= -1.$$

$$\frac{a/b}{c} = \frac{a}{bc}$$

$$\frac{a}{(b/c)} = \frac{ac}{b}.$$

Compound Angles

Definition An angle which is made up of sum or difference of two or more angles is called a compound angle.

example:

(1) $A+B$, $A-B$, $A+B+C$ etc. are compound angles

(2) $(45+30)$, $(60-45)$, $(30+20+25)$ etc. are compound angles

Addition formula.

1. $\sin(A+B) = \sin A \cos B + \cos A \sin B$
2. $\cos(A+B) = \cos A \cos B - \sin A \sin B$
3. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A . \tan B}$

Subtraction formula.

1. $\sin(A-B) = \sin A \cos B - \cos A \sin B$
2. $\cos(A-B) = \cos A \cos B + \sin A \sin B$
3. $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

1. If $\cos A = \frac{3}{5}$, $\tan B = \frac{5}{12}$, A and B are acute angles, find the values of $\sin(A+B)$ and $\cos(A-B)$.

Given

$\cos A = 3/5$.

$\sin^2 A = 1 - \cos^2 A$

$= 1 - (3/5)^2$

$= 1 - \frac{9}{25} = \frac{25-9}{25}$

$\sin^2 A = \frac{16}{25}$

$\therefore \sin A = \sqrt{16/25} = \pm 4/5$

A is acute

$\therefore \sin A = 4/5$.

Given

$\tan B = \frac{5}{12}$.

$\sec^2 B = 1 + \tan^2 B$

$= 1 + (5/12)^2 = 1 + \frac{25}{144}$

$= \frac{144+25}{144} = \frac{169}{144}$

$\sec B = \sqrt{\frac{169}{144}} = \pm \frac{13}{12}$

B is acute, $\sec B = \frac{13}{12}$.

$\cos B = 1/\sec B = \frac{1}{13/12} = \underline{\underline{12/13}}.$

$\sin^2 B = 1 - \cos^2 B$

$= 1 - (12/13)^2 = 1 - \frac{144}{169}.$

$= \frac{169-144}{169} = \frac{25}{169}.$

$\sin B = \sqrt{\frac{25}{169}} = \pm\frac{5}{13}$, B is acute.

$\sin B = \underline{\underline{5/13}}.$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$= \frac{4}{5} \times \frac{12}{13} + \frac{3}{5} \times \frac{5}{13}$

$= \frac{48}{65} + \frac{15}{65}$

$= \frac{65\times 48 + 65 \times 15}{65 \times 65}.$

$= \frac{\cancel{65}(48+15)}{\cancel{65} \times 65} = \frac{63}{65}$ //.

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$= \frac{3}{5} \times \frac{12}{13} + \frac{4}{5} \times \frac{5}{13}.$

$= \frac{36}{65} + \frac{20}{65} = \frac{65\times 36 + 20 \times 65}{65 \times 65}$

$$= \frac{65(36+20)}{65 \times 65} = \frac{56}{65}$$

2. If $\tan A = 3/4$, $\tan B = 5/12$, A and B are acute angles, find $\tan(A-B)$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{3/4 - 5/12}{1 + 3/4 \times 5/12}$$

$$= \frac{\frac{36-20}{48}}{1 + \frac{15}{48}} = \frac{16/48}{\frac{48+15}{48}}$$

$$= \frac{16/48}{63/48} = 16/63$$

3. If $\tan A = 2$, $\tan B = 1$, A and B are acute angles. Find the value of $\tan(A-B)$ and $\cos(A-B)$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{2-1}{1+2\times 1} = \frac{1}{1+2} = 1/3$$

Date
Page

$1 + \tan^2 A = \sec^2 A$.

$A = A - B$.

$\sec^2(A-B) = 1 + \tan^2(A-B)$.

$= 1 + (1/3)^2 = 1 + 1/9$.

$= \frac{9+1}{9} = \cancel{10/3}\ \underline{\underline{10/9}}$.

$\sec(A-B) = \sqrt{10}/3$.

$\therefore \sec A = 1/\cos A$, $A = A - B$.

$\sec(A-B) = 1/\cos(A-B)$

$\therefore \cos(A-B) = 1/\sec(A-B)$

$= \underline{\underline{3/\sqrt{10}}}$.

4. If A and B are acute angles where $\tan A = 1/2$, $\tan B = 1/3$, show that $A + B = \pi/4^c$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$= \frac{1/2 + 1/3}{1 - 1/2 \times 1/3} = \frac{\frac{3+2}{6}}{1 - 1/6}$

Date
Page

$$= \frac{5/6}{6-1/6} = \frac{5/6}{5/6} = 1$$

$$\tan(A+B) = 1$$

$$A+B = 45^\circ = \pi/4^c$$

Q. Find the value of tan 75 and show that tan 75 + cot 75 = 4

$$\tan 75 = \tan(45+30)$$

$$= \frac{\tan 45 + \tan 30}{1 - \tan 45 \cdot \tan 30} = \frac{1 + 1/\sqrt{3}}{1 - 1/\sqrt{3}} = \frac{\sqrt{3}+1/\sqrt{3}}{\sqrt{3}-1/\sqrt{3}}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

Numerator and denominator multiplied by $\sqrt{3}+1$

$$= \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{3+\sqrt{3}+\sqrt{3}+1}{3-1} = \frac{4+2\sqrt{3}}{2}$$

$$= \frac{2(2+\sqrt{3})}{2} = 2+\sqrt{3}$$

$$\cot 75 = \frac{1}{\tan 75} = \frac{1}{2+\sqrt{3}}.$$

Numerator and denominator multiplied by $2-\sqrt{3}$.

$$\cot 75 = \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})} = \frac{2-\sqrt{3}}{4-3} = \underline{\underline{2-\sqrt{3}}}$$

$$\therefore \tan 75 + \cot 75 = 2+\sqrt{3}+2-\sqrt{3}$$
$$= \underline{\underline{4}}.$$

$L.H.S = R.H.S$

Converse of product formulae

$\sin(A+B) + \sin(A-B) = 2\sin A\cos B$

① $\sin A\cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

② $\sin(A+B) - \sin(A-B) = 2\cos A\sin B$.

$\cos A\sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$

3. $\cos(A+B) + \cos(A-B) = 2\cos A\cos B$.

$\cos A\cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

4. $\cos(A+B) - \cos(A-B) = -2\sin A\sin B$.

$\sin A\sin B = -\frac{1}{2}[\cos(A+B) - \cos(A-B)]$

Q.1 $\sin 5x\cos 3x$ as a sum or difference

$\sin 5x\cos 3x = \frac{1}{2}[\sin(5x+3x) + \sin(5x-3x)]$

$= \frac{1}{2}(\sin 8x + \sin 2x)$

Q.2. $\cos 3\theta\cos\theta$ as a sum or difference

$\cos 3\theta\cos\theta = \frac{1}{2}[\cos(3\theta+\theta) + \cos(3\theta-\theta)]$

$= \frac{1}{2}(\cos 4\theta + \cos 2\theta)$

3. Show that $\sin 10 \sin 50 \sin 70 = 1/8$

$\sin 10 [-1/2 (\cos 120 - \cos 20)]$

$\sin 10 [-1/2 (\cos (90+30) - \cos 20)]$

$\sin 10 [-1/2 (-\sin 30 - \cos 20)]$

$\sin 10 [-1/2 (-1/2 - \cos 20)]$

$\sin 10 [1/4 + 1/2 \cos 20]$

$\frac{1}{4} \sin 10 + \frac{1}{2} \sin 10 \cos 20$

$\frac{1}{4} \sin 10 + 1/2 [1/2 (\sin 30 + \sin -10)]$

$\frac{1}{4} \sin 10 + 1/2 [1/2 (1/2 - \sin 10)]$

$\frac{1}{4} \sin 10 + 1/2 [1/4 - 1/2 \sin 10]$

$\frac{1}{4} \sin 10 + 1/8 - 1/4 \sin 10$

$= 1/8$ //

4. Show that $\cos 55 + \cos 65 + \cos 175 = 0$

$= \cos 55 + 2 \cos \frac{65+175}{2} \cos \frac{65-175}{2}$

$= \cos 55 + 2 \cos \frac{240}{2} \cos -\frac{110}{2}$

$= \cos 55 + 2 \cos 120 \cos (-55)$

$= \cos 55 + 2 \cos 120 \cos 55$

$= \cos 55 (1 + 2 \cos 120)$

$= \cos 55 (1 + 2 \cos (90+30))$

$= \cos 55 (1 + -2 \sin 30)$

$= \cos 55 (1 - 2 \times 1/2)$

$= \cos 55 (1 - 1)$

$= 0$

Q. Prove that $\sin 20 \sin 40 \sin 80 = \sqrt{3}/8$.

$\sin 20 [-1/2 (\cos 120 - \cos -40)]$

$\sin 20 [-1/2 (\cos 120 - \cos 40)]$

$\sin 20 [-1/2 (-\sin 30 - \cos 40)]$

$= \sin 20 [\frac{1}{2} \sin 30^\circ + 1/2 \cos 40^\circ)$

$= \sin 20 [1/2 \times 1/2 + \frac{1}{2} \cos 40^\circ]$

$= \frac{1}{4} \sin 20 + \frac{1}{2} \sin 20 \cos 40^\circ$.

$= \frac{1}{4} \sin 20 + \frac{1}{2} (1/2 [\sin 60 + \sin(-20)])$

$= \frac{1}{4} \sin 20 + \frac{1}{4} \times \sqrt{3}/2 + \frac{-1}{4} \sin 20$

$= \sqrt{3}/8$

classmate

Date
Page

Workout problems

1. Prove that $\dfrac{\cos 3A - \cos A}{\sin A - \sin 3A} = \tan 2A$

2. Prove that $\dfrac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A} = \tan 3A$

3. Prove that $\sin 50 - \sin 70 + \sin 10 = 0$.

4. Prove that $\cos 80 + \cos 40 - \cos 20 = 0$

5. Prove that $\cos 20 \cos 40 \cos 80 = 1/8$

7a. Prove that $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$

L.H.S $= \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$.

$= \frac{\sin\theta \cdot \sin\theta + (1+\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(\sin\theta)}$.

$= \frac{\sin^2\theta + 1 + \cos\theta + \cos\theta + \cos^2\theta}{(1+\cos\theta)(\sin\theta)}$.

$= \frac{\sin^2\theta + \cos^2\theta + 1 + 2\cos\theta}{(1+\cos\theta)\sin\theta}$.

$= \frac{1 + 1 + 2\cos\theta}{(1+\cos\theta)\sin\theta} = \frac{2+2\cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta}$

$= \frac{2}{\sin\theta} = 2 \times \frac{1}{\sin\theta} = \underline{\underline{2\operatorname{cosec}\theta}}$

$=$ R.H.S.

8 $\dfrac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \dfrac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2\sec^2 A$

$$L.H.S = \frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1}$$

$$= \frac{\operatorname{cosec} A(\operatorname{cosec} A + 1) + \operatorname{cosec} A(\operatorname{cosec} A - 1)}{(\operatorname{cosec} A - 1)(\operatorname{cosec} A + 1)}$$

$$= \frac{\operatorname{cosec}^2 A + \operatorname{cosec} A + \operatorname{cosec}^2 A - \operatorname{cosec} A}{\operatorname{cosec}^2 A - 1}$$

$$= \frac{2\operatorname{cosec}^2 A}{\cot^2 A} = 2 \times \frac{1}{\sin^2 A} \times \tan^2 A$$

$$= 2 \times \frac{1}{\sin^2 A} \times \frac{\sin^2 A}{\cos^2 A}$$

$$= 2 \times \sec^2 A = R.H.S.$$

9. Prove that $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

$$L.H.S = \sqrt{\frac{1+\sin A}{1-\sin A}}$$

$$= \frac{\sqrt{1+\sin A}}{\sqrt{1-\sin A}} \frac{\sqrt{1+\sin A}}{\sqrt{1+\sin A}}.$$

$$= \frac{(\sqrt{1+\sin A})^2}{\sqrt{(1-\sin A)(1+\sin A)}}$$

$$= \frac{((1+\sin A)^{1/2})^2}{\sqrt{1-\sin^2 A}} = \frac{1+\sin A}{\sqrt{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A$$

Problems

1. If $\sin\theta = 2/5$ and 'θ' is an acute angle, find $\cos\theta$ and $\tan\theta$.
2. If $\tan\theta = 3$ and θ is an acute angle, find $\sec\theta$ and $\operatorname{cosec}\theta$.
3. Prove the following identities
   (a) $\sin A \cot A = \cos A$
   (b) $\sin^2 A - \cos^2 A = 1 - 2\cos^2 A$
   (c) $(\sin A + \cos A)^2 = 1 + 2\sin A \cos A$
4. Prove that $\frac{1+\sin\theta}{\cos\theta} = \frac{\cos\theta}{1-\sin\theta}$.
5. Show that $\frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta} = 2\operatorname{cosec}\theta$.
6. Prove that $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \sin A + \cos A$

Trigonometric functions of standard angles:

θ	0°	$30^\circ = \pi/6$	$45^\circ = \pi/4$	$60^\circ = \pi/3$	$90^\circ = \pi/2$
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	not defined
$\cot\theta$	not defined	$\sqrt{3}$	1	$1/\sqrt{3}$	0
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	not defined
$\operatorname{cosec}\theta$	not defined	2	$\sqrt{2}$	$2/\sqrt{3}$	1

Evaluate the following.

(1) $3\sin 30^\circ - 4\cos^3 60^\circ$

$$= 3 \times 1/2 - 4 \times (1/2)^3$$

$$= \frac{3}{2} - 4 \times \frac{1}{8} = \frac{3}{2} - \frac{1}{2}$$

$$= \frac{2}{2} = 1$$

(2) $4\tan^2 60^\circ - 3\tan^2 30^\circ + \tan^2 45^\circ$

$= 4 \times (\sqrt{3})^2 - 3 \times \left(\frac{1}{\sqrt{3}}\right)^2 + 1$

$= 4 \times 3 - 3 \times \frac{1}{3} + 1$

$= 12 - 1 + 1 = \underline{\underline{12}}.$

(3) $\tan^2 45^\circ - \frac{1}{2}\cos 60^\circ - \frac{3}{4}\tan^2 30^\circ$

$= 1 - \frac{1}{2} \times \frac{1}{2} - \frac{3}{4} \times \left(\frac{1}{\sqrt{3}}\right)^2.$

$= 1 - \frac{1}{4} - \frac{3}{4} \times \frac{1}{3}$

$= 1 - \frac{1}{4} - \frac{1}{4} = 1 - \frac{2}{4} = 1 - \frac{1}{2}.$

$= \frac{2-1}{2}$

$= \underline{\underline{1/2}}.$

(4). $4\sin^3 \pi/3 - 3\cos \pi/6.$

$= 4(\sqrt{3}/2)^3 - 3(\sqrt{3}/2)$

$= \cancel{4} \times \frac{3\sqrt{3}}{\cancel{8}\,2} - \frac{3\sqrt{3}}{2}.$

$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = \underline{\underline{0}}.$

Date
Page

Quadrantal angles:

If the terminal ray of an angle coincide with one of the co-ordinate axes, the angle is said to be a quadrantal angle.

$0°, 90°, 180°, 270°, 360°$ etc are quadrantal angles.

Table of result

T-fun \ θ	0°	90°	180°	270°	360°
Sin	0	1	0	−1	0
Cos	1	0	−1	0	1
tan	0	∞	0	∞	0
Cot	∞	0	∞	0	∞
Sec	1	∞	−1	∞	1
Cosec	∞	1	∞	−1	∞.

Trigonometric functions of angles in quadrants.

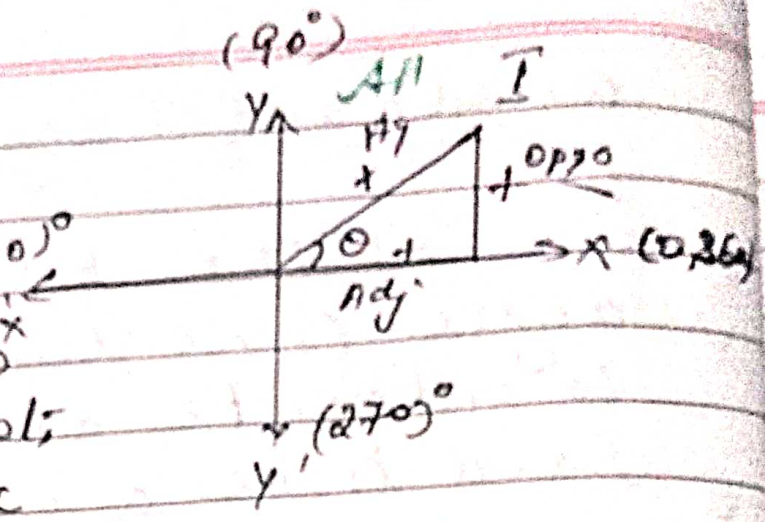
(1) Suppose that θ lies in 1st quadrant

$\sin\theta = +ve$

$\cos\theta = +ve$.

$\tan\theta = +ve$.

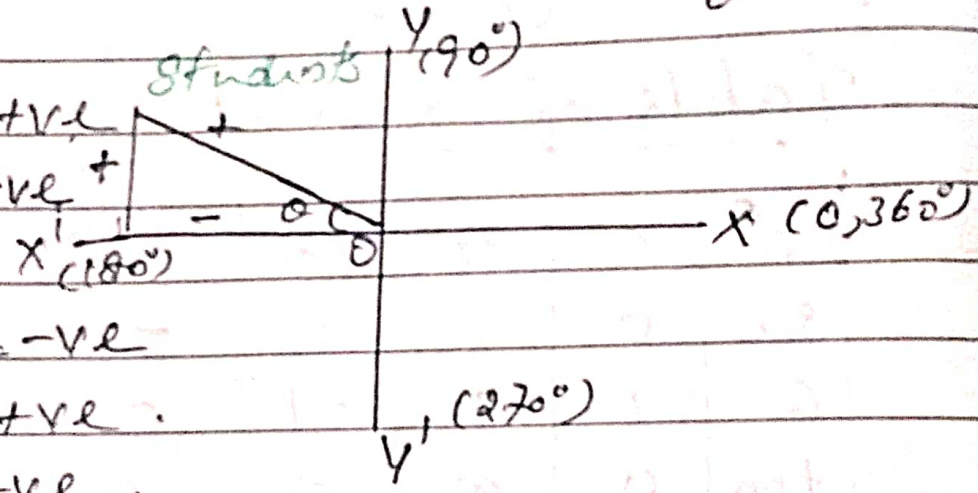
when θ lies on the 1st quadrant; all trigonometric functions are positive.



(2). Suppose that θ lies in 2nd quadrant

$\sin\theta = \frac{+}{+} = +ve$

$\cos\theta = \frac{-}{+} = -ve$



$\tan\theta = \frac{+}{-} = -ve$

$\operatorname{cosec}\theta = +ve$.

$\sec\theta = -ve$.

when θ lies on the 2nd quadrant $\sin\theta$ and $\operatorname{cosec}\theta$ are positive and other t-functions are negative.

(3) Suppose that θ lies on 3rd quadrant

$\sin\theta = \frac{-}{+} = -ve$

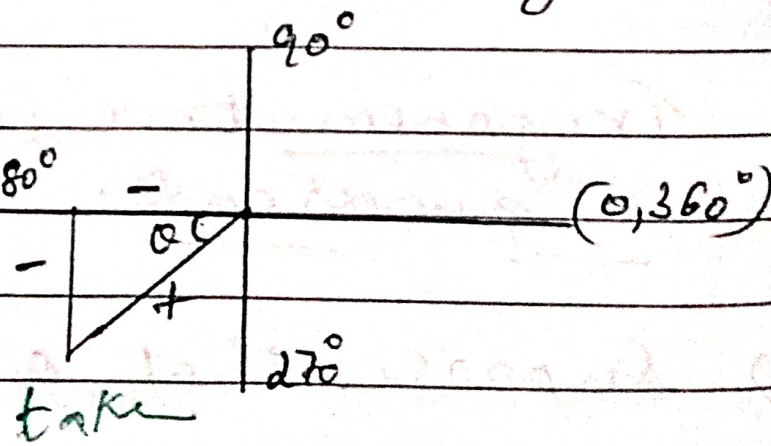
$\cos\theta = \frac{-}{+} = -ve$

$\tan\theta = \frac{-}{-} = +ve$

$\cot\theta = +ve$.

$\sec\theta = -ve$

$\operatorname{cosec}\theta = -ve$



when θ lies in 3rd quadrant $\tan\theta$ and $\cot\theta$ are positive and other t-functions are negative.

(4). Suppose that θ lies in 4th quadrant

$\sin\theta = \frac{-}{+} = -ve$

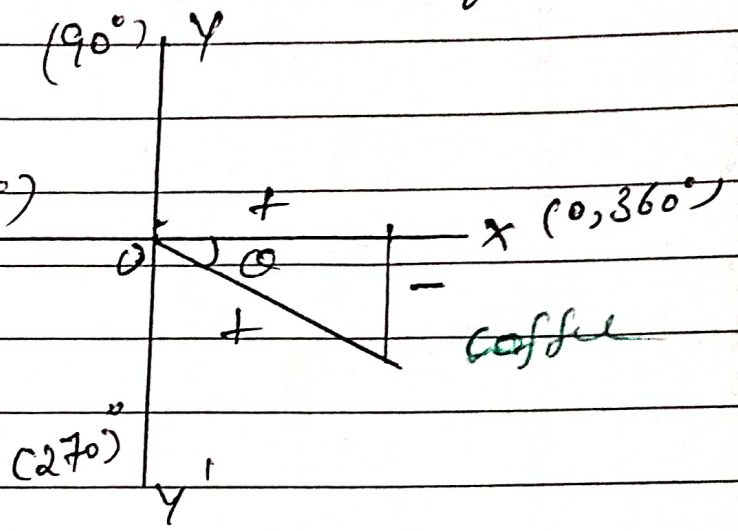
$\cos\theta = \frac{+}{+} = +ve$

$\tan\theta = \frac{-}{+} = -ve$

$\cot\theta = \frac{+}{-} = -ve$.

$\sec\theta = +ve$

$\operatorname{cosec}\theta = -ve$.



when θ lies in 4th quadrant $\cos\theta$ and $\sec\theta$ are positive and all other t-functions are negative.

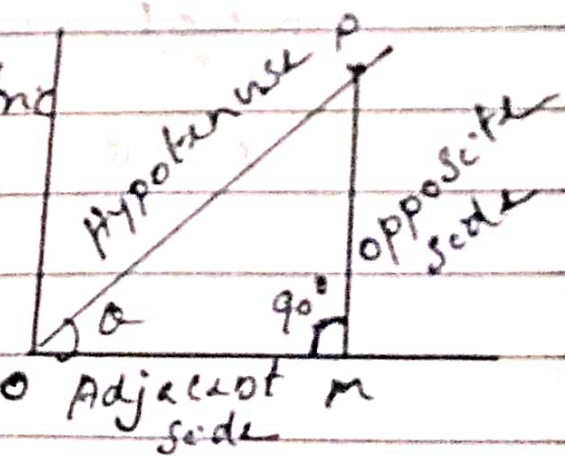
<u>Note</u>

All Students take Coffee.

Trigonometric functions of an acute angle.

Consider an acute angle of measure θ in the standard position. Let P be any point on the terminal side. From P draw a perpendicular to meet the x-axis at M. Thus POM is a right triangle, right angled at M. and $\angle MOP = \theta$. With respect to the angle θ, OM is the adjacent side, MP is the opposite side and OP is the hypotenuse.

There are six trigonometric functions, namely $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$, $\sec\theta$ and $\operatorname{cosec}\theta$.



$\sin\theta = \frac{MP}{OP} = \frac{\text{Opposite Side}}{\text{Hypotenuse}} \longrightarrow A_1$

$\cos\theta = \frac{OM}{OP} = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} \longrightarrow B_1$

$\tan\theta = \frac{MP}{OM} = \frac{\text{Opposite side}}{\text{Adjacent side}} \longrightarrow C_1$

$\cot\theta = \frac{OM}{MP} = \frac{\text{Adjacent side}}{\text{opposite side}} \longrightarrow C_2$

$\sec\theta = \frac{OP}{OM} = \frac{\text{Hypotenuse}}{\text{Adjacent side}} \longrightarrow B_2$

$\operatorname{cosec}\theta = \frac{OP}{MP} = \frac{\text{Hypotenuse}}{\text{opposite side}} \longrightarrow A_2$

The above formulae, we get A_1 and A_2, B_1 and B_2, C_1 and C_2 are reciprocal.

$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$, $\cos\theta = \frac{1}{\sec\theta}$.

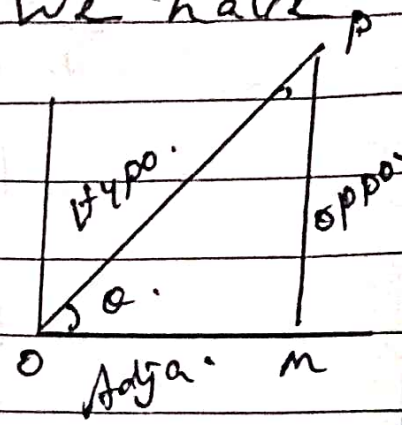
$\tan\theta = \frac{1}{\cot\theta}$.

Quotient Relations.

$\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\cot\theta = \frac{\cos\theta}{\sin\theta}$.

Pythagorean Relations.

Using Pythagoras theorem, from the right triangle POM. We have



$$MP^2 + OM^2 = OP^2 \longrightarrow (1)$$

(A) Dividing (1) by OP^2, we get.

$$\frac{MP^2}{OP^2} + \frac{OM^2}{OP^2} = \frac{OP^2}{OP^2}$$

$$\left(\frac{MP}{OP}\right)^2 + \left(\frac{OM}{OP}\right)^2 = 1$$

$$(\sin\theta)^2 + (\cos\theta)^2 = 1$$

$$\sin^2\theta + \cos^2\theta = 1$$

Thus $\sin^2\theta = 1 - \cos^2\theta$

and $\cos^2\theta = 1 - \sin^2\theta$.

(B) Dividing (1) by OM^2, we get

$$\frac{MP^2}{OM^2} + \frac{OM^2}{OM^2} = \frac{OP^2}{OM^2}$$

$$\left(\frac{MP}{OM}\right)^2 + 1 = \left(\frac{OP}{OM}\right)^2$$

$$(\tan\theta)^2 + 1 = (\sec\theta)^2$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\tan^2\theta = \sec^2\theta - 1$$

and $\sec^2\theta - \tan^2\theta = 1$

(c) Dividing (1) by MP^2

$$\frac{MP^2}{MP^2} + \frac{OM^2}{MP^2} = \frac{OP^2}{MP^2}$$

$$1 + \left(\frac{OM}{MP}\right)^2 = \left(\frac{OP}{MP}\right)^2$$

$$1 + (\cot\theta)^2 = (\operatorname{cosec}\theta)^2.$$

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta.$$

Thus $\cot^2\theta = \operatorname{cosec}^2\theta - 1$

and $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

using These formulae or right triangle figure to calculate the following problems.

(1). If $\cos\theta = 4/5$ and θ is an acute angle, find $\sin\theta$ and $\tan\theta$.

using formulae.

$$\sin^2\theta + \cos^2\theta = 1.$$

$$\sin^2\theta = 1 - \cos^2\theta = 1 - (\cos\theta)^2$$

$$= 1 - (4/5)^2$$

$$= 1 - \frac{16}{25} = \frac{25-16}{25} = \frac{9}{25}$$

$$\sin\theta = \sqrt{9/25}$$

$$\therefore \sin\theta = \pm\frac{3}{5}$$

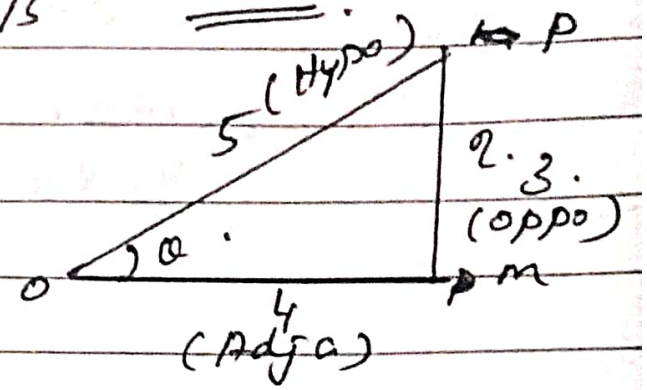
Take the value.

$\sin\alpha = 3/5$

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{3/5}{4/5} = 3/4$

$\boxed{\text{OR}}$

using figure.



$MP^2 + OM^2 = OP^2$.

$MP^2 + 16 = 25$

$MP^2 = 25 - 16 = 9$.

$MP = 3$.

$\therefore \sin\alpha = \frac{\text{oppo}}{\text{Hypo}} = \frac{3}{5}$.

$\tan\alpha = \frac{\text{oppo}}{\text{Adja}} = \frac{3}{4}$

(To calculate any method)

(2) If $\tan\alpha = 12/5$ and α is an acute angle, find $\sin\alpha$ and $\cos\alpha$.

$1 + \tan^2\alpha = \sec^2\alpha$.

$\sec^2\alpha = 1 + \tan^2\alpha = 1 + (12/5)^2$.

$= 1 + \frac{144}{25} = \frac{25 + 144}{25}$

$= \frac{169}{25}$

$\sec\theta = \sqrt{169/25} = 13/5$.

$\therefore \cos\theta = \frac{1}{\sec\theta} = \frac{1}{13/5} = \underline{5/13}$.

$\sec^2\theta + \cos^2\theta = 1$

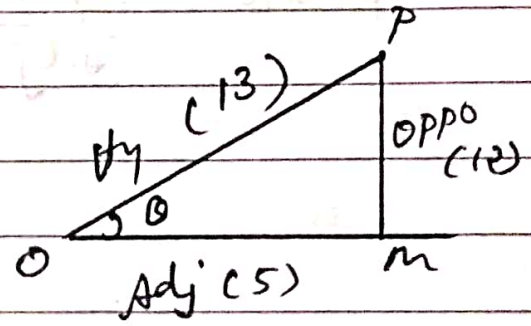
$\sin^2\theta = 1 - \cos^2\theta = 1 - (5/13)^2$

$= 1 - \frac{25}{169} = \frac{169-25}{169}$

$= \frac{144}{169}$.

$\sin\theta = \sqrt{\frac{144}{169}} = 12/13$ //.

$\boxed{\text{OR}}$



$MP^2 + OM^2 = OP^2$

$(12)^2 + (5)^2 = OP^2$

$OP^2 = 144 + 25$

$= 169$.

$OP = \sqrt{169} = 13$.

$\therefore \sin\theta = \frac{\text{oppo}}{\text{Hypo}} = \frac{12}{13}$

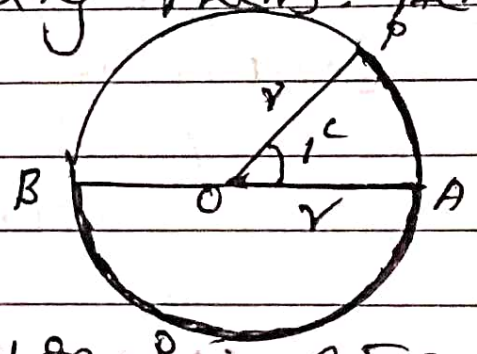
$\cos\theta = \frac{\text{Adj}}{\text{Hypo}} = \frac{5}{13}$ //

Relation between Degree and Radian

Consider a circle with centre O and radius r. Let A be a point on the circle. Now cut off an arc AP whose length is equal to the radius r of the circle. Thus $\angle AOP = 1^c$. To produce AO to meet the circle at B. Thus arc AB is half of the circumference and hence its length is πr. Also $\angle AOB$ = a straight line $= 180^\circ$

We know that angles at the centre of a circle are proportional to the arcs subtending them. Hence

$$\frac{\angle AOP}{\angle AOB} = \frac{\text{arc } AP}{\text{arc } AB}.$$



$$\frac{1^c}{180^\circ} = \frac{r}{\pi r}$$

$$1^c = \frac{180^\circ}{\pi} = \left(\frac{180}{\pi}\right)^\circ \quad \left(\pi = \frac{22}{7}\right)$$

or $1^\circ = \left(\frac{\pi}{180}\right)^c$

The relation between degree measures and radian measures of some standard angles

Degree	0	30°	45°	60°	90°	180°	270°	360°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π

$\Rightarrow 1^\circ = (\pi/180)^c$

$30^\circ = \frac{30 \times \pi}{180} = (\pi/6)^c$

$270^\circ = \frac{270 \times \pi}{180} = 3\pi/2^c$

$180^\circ = \frac{180 \times \pi}{180} = \pi^c$

$30^\circ = 30 \times 1^\circ = 30 \times \frac{\pi}{180}$

Problems

① Express the following angles in degree measure.

(a) $\left(\frac{2\pi}{15}\right)^c$

$\frac{2\pi}{15} \times 1^c = \frac{2\pi}{15} \times \frac{180^\circ}{\pi} = \frac{360}{15} = 24^\circ$

(b) $\left(\frac{1}{4}\right)^c$

$\frac{1}{4} \times 1^c = \frac{1}{4} \times \frac{180}{\pi} = \frac{45}{\pi} = \frac{45}{22/7} = \frac{315}{22}$

$= \left(\frac{315}{22}\right)^\circ = \left(14\frac{7}{22}\right)^\circ$

$= 14^\circ \frac{7}{22} \times 1^\circ$

$$\begin{array}{r} 14 \\ 22\overline{)315} \\ 22 \\ \hline 95 \\ 88 \\ \hline 7 \end{array}$$

$= 14^\circ \frac{7}{22} \times 60'$

$= 14^\circ \frac{210}{11}'$

$= 14^\circ (19 \frac{1}{11})'$

$= 14^\circ 19' \frac{1}{11} \times 1'' = 14^\circ 19' (\frac{1}{11} \times 60)''$

$= 14^\circ 19' 5''$ approximately

(c) 6^c

$6^c = 6 \times 1^c$

$= 6 \times \frac{180}{\pi} = 6 \times \frac{180}{22/7} =$

$= \frac{6 \times 7 \times 180}{22} = \frac{3780}{11}^\circ$

$= (343 \frac{7}{11})^\circ$

$= 343^\circ \frac{7}{11} \times 60'$

$= 343^\circ \frac{420}{11}'$

$= 343^\circ (38 \frac{2}{11})'$

$= 343^\circ\, 38' \left(\frac{3}{11} \times 60\right)''$

$= 343^\circ\, 38'\, 1''$

$11 \overline{)120}$
$\quad 11$
$\quad 10$

$1\frac{10}{11}$

Express the following in radian measure

(a) 55°

$55^\circ = 55 \times 1^\circ$

$= 55 \times \left(\frac{\pi}{180}\right)^c$

$= \left(\frac{11\pi}{36}\right)^c$

(b) 240°

$240^\circ = 240 \times 1^\circ$

$= 240 \times \left(\frac{\pi}{180}\right)^c$

$= \frac{240 \times \pi}{180} = \left(\frac{4\pi}{3}\right)^c$

(c) $40^\circ 20' = \left(40^\circ + \frac{20}{60}\right)^\circ = \left(40 + \frac{1}{3}\right)^\circ$ $\left(1' = \frac{1}{60}^\circ\right)$

$= \left(\frac{121}{3}\right)^\circ = \frac{121}{3} \times \frac{\pi}{180}^c$

$= \left(\frac{121\pi}{540}\right)^c$

(d) $75^\circ 30' = \left(75 + \frac{30}{60}\right)^\circ$

$= \left(75 + \frac{1}{2}\right)^\circ = \left(\frac{151}{2}\right)^\circ$

$= \frac{151}{2} \times \frac{\pi}{180} = \left(\frac{151\pi}{360}\right)^c$

(e) $12^\circ 12' 20'' = \left(12 + \frac{12}{60} + \frac{20}{60 \times 60}\right)^\circ$

$= \left(12 + \frac{1}{5} + \frac{1}{180}\right)^\circ$

$= \left(\frac{61}{5} + \frac{1}{180}\right)^\circ \qquad \left(1'' = \frac{1}{60 \times 60}^\circ\right)$

$= \left(\frac{2197}{180}\right)^\circ$

$= \left(\frac{2197}{180} \times \frac{\pi}{180}\right)^c$

$= \left(\frac{2197\pi}{32400}\right)^c$

$$\begin{array}{r|l} 5 & 180, 5 \\ \hline & 36, 1 \end{array}$$

$$\begin{array}{r} 36 \\ 61 \\ \hline 36 \\ 216 \\ \hline 2196 \end{array}$$

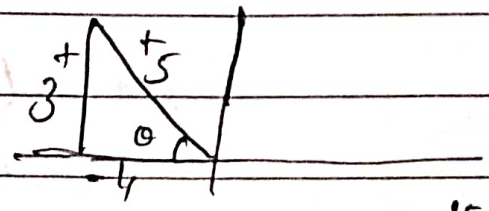
Q. ~~If~~ If $\sin\theta = 3/5$, θ lies in second quadrant, find all other t-functions.

$$\cos^2\theta = 1 - \sin^2\theta$$

$$= 1 - 9/25 = \frac{25-9}{25} = \frac{16}{25}$$

$$\cos\theta = \sqrt{16/25}$$

$$\cos\theta = \pm\frac{4}{5}$$



$25 - 9 = 16$

2nd quadrant $\cos\theta$ is $-ve$.

$$\cos\theta = -4/5.$$

$$\tan\theta = \sin\theta/\cos\theta = \frac{3/5}{-4/5} = -3/4.$$

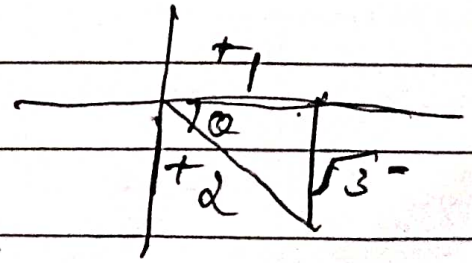
$$\cot\theta = -4/3, \quad \sec\theta = \frac{1}{\cos\theta} = -5/4.$$

$$\operatorname{cosec}\theta = 1/\sin\theta = 5/3$$

Q. If $\cos\theta = 1/2$, θ lies in fourth quadrant, find all other t-functions.

$$\sin^2\theta = 1 - \cos^2\theta$$

$$= 1 - 1/4 = 3/4$$



$\sin\theta = \pm\sqrt{3}/2$

θ lies in fourth quadrant

$\sin\theta$ is $-ve$ in fourth quadrant

$\sin\theta = -\sqrt{3}/2$

$\tan\theta = -\sqrt{3}$, $\cot\theta = -1/\sqrt{3}$

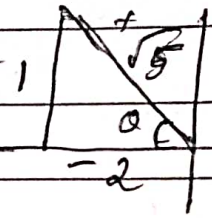
$\operatorname{cosec}\theta = -2/\sqrt{3}$ and $\sec\theta = 2$

Q. ~~If~~ If $\cot\theta = \sqrt{5} - 2$, θ lies in 2nd quadrant find all other t-functions

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\operatorname{cosec}^2\theta = 1 + 4 = 5$

$\operatorname{cosec}\theta = \pm\sqrt{5}$



In 2nd quadrant $\operatorname{cosec}\theta$ is $+ve$.

$\operatorname{cosec}\theta = \sqrt{5}$

$\sin\theta = 1/\sqrt{5}$, $\tan\theta = -1/2$.

$\sec^2\theta = 1 + 1/4 = 5/4$

$\sec\theta = \pm\sqrt{5}/2$

$\sec\theta$ is $-ve$ in 2nd quadrant

$\sec\theta = -\sqrt{5}/2$.

$\therefore \cos\theta = -2/\sqrt{5}$

Trigonometric Ratios of Allied Angles (Related angles).

Table of result.

1. $\sin(-\theta) = -\sin\theta$.
   $\cos\theta(-\theta) = +\cos\theta$.
   $\tan(-\theta) = -\tan\theta$.

2. $\sin(90-\theta) = +\cos\theta$.
   $\cos(90-\theta) = +\sin\theta$
   $\tan(90-\theta) = +\cot\theta$.

3. $\sin(90+\theta) = +\cos\theta$.
   $\cos(90+\theta) = -\sin\theta$.
   $\tan(90+\theta) = -\cot\theta$.

4. $\sin(180-\theta) = +\sin\theta$.
   $\cos(18-\theta) = -\cos\theta$.
   $\tan(180-\theta) = -\tan\theta$.

5. $\sin(180+\theta) = -\sin\theta$.
   $\cos(180+\theta) = -\cos\theta$
   $\tan(180+\theta) = +\tan\theta$.

6. $\sin(270-\theta) = -\cos\theta$
   $\cos(270-\theta) = -\sin\theta$
   $\tan(270-\theta) = +\cot\theta$.

7. $\sin(270+\theta) = -\cos\theta$.
   $\cos(270+\theta) = +\sin\theta$
   $\tan(270+\theta) = -\cot\theta$.

8. $\sin(360-\theta) = -\sin\theta$
   $\cos(360-\theta) = +\cos\theta$.
   $\tan(360-\theta) = -\tan\theta$

(3) If θ is an acute angle and $\sin\theta = 0.4$, find the value of $\sec\theta + \tan\theta$.

$\sin\theta = 0.4 = 2/5$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$= 1 - (\sin\theta)^2$

$= 1 - (2/5)^2 = 1 - \frac{4}{25}$

$= \frac{25-4}{25} = \frac{21}{25}$

$\cos\theta = \sqrt{21/25} = \frac{\sqrt{21}}{5}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{5}{\sqrt{21}}$ //.

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2/5}{\sqrt{21}/5} = \frac{2}{5} \times \frac{5}{\sqrt{21}}$

$= \frac{2}{\sqrt{21}}$ //.

$\sec\theta + \tan\theta = \frac{5}{\sqrt{21}} + \frac{2}{\sqrt{21}} = \frac{7}{\sqrt{21}}$

(4) If θ is an acute angle and $\cos\theta = 1/2$, find the value of $3\sin\theta - 4\tan\theta$.

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta = 1 - (\cos\theta)^2$

$$\sin^2\theta = 1 - \frac{1}{4} = 3/4$$

$$\sin\theta = \sqrt{3/4} = \sqrt{3}/2$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{3}/2}{1/2} = \frac{\sqrt{3}}{\cancel{2}} \times \frac{\cancel{2}}{1}$$

$$= \sqrt{3}$$

$$\therefore 3\sin\theta - 4\tan\theta = 3 \times \frac{\sqrt{3}}{2} - 4 \times \sqrt{3}$$

$$= \frac{3\sqrt{3} - 8\sqrt{3}}{2} = \frac{5\sqrt{3}}{2}$$

Prove the following identities.

(1) $\cos A \tan A = \sin A$

$$L.H.S = \cos A \tan A$$

$$= \cancel{\cos A} \frac{\sin A}{\cancel{\cos A}}$$

$$= \sin A = R.H.S.$$

$$L.H.S = R.H.S.$$

(2) $\sin A \tan A \operatorname{cosec}^2 A = \sec A$

$$L.H.S = \sin A \tan A \operatorname{cosec}^2 A$$

$$= \sin A \frac{\sin A}{\cos A} \frac{1}{\sin^2 A} = \frac{1}{\cos A}$$

$= \sec A$

$= R.H.S$

$L.H.S = R.H.S$

(3) $\cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$.

$L.H.S = \cos^2\theta - \sin^2\theta$

$= (1 - \sin^2\theta) - \sin^2\theta$.

$= 1 - \sin^2\theta - \sin^2\theta$.

$= 1 - 2\sin^2\theta$.

(4) $\tan x + \cot x = \sec x \operatorname{cosec} x$.

$L.H.S = \tan x + \cot x$

$= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$

$= \frac{\sin^2 x + \cos^2 x}{\cos x \sin x} = \frac{1}{\cos x \sin x}$

$= \frac{1}{\cos x} \cdot \frac{1}{\sin x} = \sec x \cdot \operatorname{cosec} x$

$= R.H.S$

(5) $\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta \operatorname{cosec}^2\theta$.

$L.H.S = \sec^2\theta + \operatorname{cosec}^2\theta$.

$= \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}$.

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta \cos^2\theta}$$

$$= \frac{1}{\sin^2\theta \cos^2\theta} = \frac{1}{\cos^2\theta} \cdot \frac{1}{\sin^2\theta}$$

$$= \sec^2\theta \operatorname{cosec}^2\theta.$$

Q. Prove that $\frac{1+\cos A}{\sin A} = \frac{\sin A}{1-\cos A}$.

L.H.S $= \frac{1+\cos A}{\sin A}$

Numerator and denominator multiplied by $1-\cos A$.

$$= \frac{(1+\cos A)(1-\cos A)}{\sin A(1-\cos A)}$$

$$= \frac{1-\cos^2 A}{\sin A(1-\cos A)}.$$

$$= \frac{\sin^2 A}{\sin A(1-\cos A)} \quad (1-\cos^2 A = \sin^2 A)$$

$$= \frac{\sin A}{1-\cos A}$$

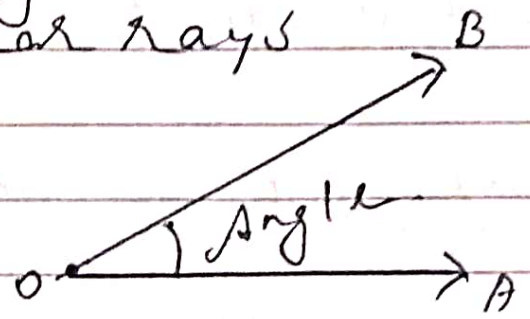
$$= R.H.S$$

Trigonometric functions.

In this chapter, we will generalise the concept of trigonometric ratios to trigonometric functions and study their properties

(1) Angles.

An angle is the union of two non-collinear rays

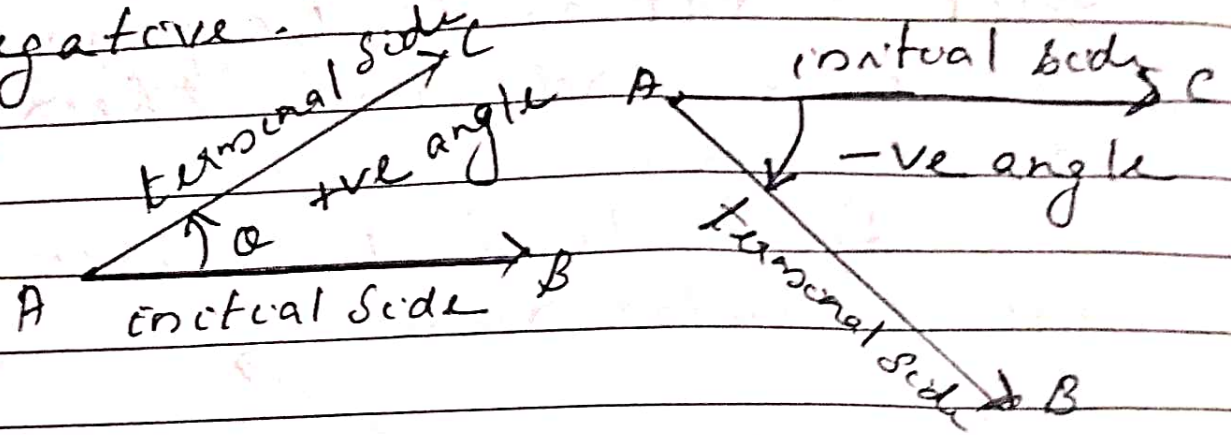


An angle is obtained by rotating a ray from one position to another. In this case the initial position of the rotating ray is called the initial side and the final position of the ray is called the ~~terminal~~ terminal side.

The direction can be taken in two senses: positive and negative.

(1) an angle formed by the rotation of a ray in the counter-clockwise

(anticlockwise) direction is taken as positive and angle formed by the rotation of a ray in the clockwise direction is taken as negative.



Measurement of angles.

There are three systems for measuring angles.

(1) Sexagesimal or English system
(2) Centesimal or French system
(3) Circular system.

(1) Sexagesimal system:

1 right angle = 90 degree (= 90°)
1° = 60 minutes (= 60')
1' = 60 seconds (= 60")

In this system a right angle is divided into 90 equal parts, called degrees.

(2) Centesimal System.

1 right angle = 100 grades ($= 100^g$)

1^g = 100 minutes ($= 100'$)

$1'$ = 100 seconds ($= 100''$)

In this system a right angle is divided into 100 equal parts, called grades.

(3) Circular System.

In this system the unit of measurement of an angle is radian

One radian (1^c), is the measure of an angle subtended by at the centre of a circle by an arc whose length is equal to the radius of the circle.