

✓ Note // If $y = (f(x))^n$
$$\frac{dy}{dx} = n (f(x))^{n-1} \cdot \frac{d}{dx} f(x)$$

Q. If $y = (2x+3)^5$, find dy/dx .

$$y = (2x+3)^5$$
$$\frac{dy}{dx} = 5 (2x+3)^{5-1} \cdot \frac{d}{dx} (2x+3)$$

$$\begin{aligned}
 \frac{dy}{dx} &= 5(2x+3)^4 \left(2 \cdot \frac{d}{dx}(x) + \frac{d}{dx}(3) \right) \\
 &= 5(2x+3)^4 (2 \times 1 + 0) \\
 &= 5(2x+3)^4 \times 2 \\
 &= \underline{\underline{10(2x+3)^4}}
 \end{aligned}$$

Q. If $y = \sin^2 x$, find dy/dx

$$\begin{aligned}
 y &= \sin^2 x = (\sin x)^2 \\
 \frac{dy}{dx} &= 2 \cdot (\sin x)^{2-1} \cdot \frac{d}{dx}(\sin x) \\
 &= 2(\sin x) \cdot \cos x \\
 &= \underline{\underline{2 \sin x \cos x}}
 \end{aligned}$$

Q. If $y = \tan^5 x$, find dy/dx

$$\begin{aligned}
 y &= \tan^5 x = (\tan x)^5 \\
 \frac{dy}{dx} &= 5(\tan x)^{5-1} \cdot \frac{d}{dx}(\tan x) \\
 &= 5(\tan x)^4 \cdot \sec^2 x \\
 &= \underline{\underline{5 \tan^4 x \sec^2 x}}
 \end{aligned}$$

Q. If $y = \sqrt{2x-3}$, find dy/dx

$$\begin{aligned} y &= \sqrt{2x-3} = (2x-3)^{1/2} \\ \frac{dy}{dx} &= \frac{1}{2} (2x-3)^{1/2-1} \cdot \frac{d}{dx}(2x-3) \\ &= \frac{1}{2} (2x-3)^{-1/2} \cdot (2 \cdot \frac{d}{dx}(x) - 0) \\ &= \frac{1}{2} (2x-3)^{-1/2} \cdot 2 \\ &= \frac{1}{(2x-3)^{1/2}} = \frac{1}{\sqrt{2x-3}} \end{aligned}$$

Q. If $y = \frac{1}{x^2+x+1}$, find dy/dx

$$\begin{aligned} y &= \frac{1}{x^2+x+1} = (x^2+x+1)^{-1} \\ \frac{dy}{dx} &= -1 (x^2+x+1)^{-1-1} \cdot \frac{d}{dx}(x^2+x+1) \\ &= - (x^2+x+1)^{-2} \cdot \left(\frac{d}{dx}x^2 + \frac{d}{dx}x + \frac{d}{dx}1 \right) \\ &= - (x^2+x+1)^{-2} (2x+1) \\ &= - \frac{(2x+1)}{(x^2+x+1)^2} \end{aligned}$$

a. Result

$$1. \frac{d}{dx} (f(x))^n = n \cdot (f(x))^{n-1} \cdot \frac{d}{dx} f(x).$$

$$2. \frac{d}{dx} (\sin f(x)) = \cos f(x) \cdot \frac{d}{dx} f(x).$$

$$3. \frac{d}{dx} (\tan f(x)) = \sec^2 f(x) \cdot \frac{d}{dx} f(x).$$

$$4. \frac{d}{dx} (\csc f(x)) = -\csc f(x) \cot f(x) \cdot \frac{d}{dx} f(x).$$

$$5. \frac{d}{dx} (\log f(x)) = \frac{1}{f(x)} \cdot \frac{d}{dx} f(x).$$

$$6. \frac{d}{dx} (e^{f(x)}) = e^{f(x)} \cdot \frac{d}{dx} f(x).$$

$$7. \frac{d}{dx} (\sin^{-1} f(x)) = \frac{1}{\sqrt{1-(f(x))^2}} \cdot \frac{d}{dx} f(x).$$

$$8. \frac{d}{dx} (\tan^{-1} f(x)) = \frac{1}{1+(f(x))^2} \cdot \frac{d}{dx} f(x).$$

$$9. \frac{d}{dx} (\cot f(x)) = -\operatorname{cosec}^2 f(x) \cdot \frac{d}{dx} f(x).$$

$$10. \frac{d}{dx} (\sec f(x)) = \sec f(x) \tan f(x) \cdot \frac{d}{dx} f(x).$$

$$\begin{aligned}
 1. \quad \frac{d}{dx}(\sin 2x) &= \cos 2x \cdot \frac{d}{dx}(2x) \\
 &= \cos 2x \times 2 \frac{d}{dx}(x) \\
 &= \cos 2x \times 2 \times 1 \\
 &= \underline{\underline{2 \cos 2x}}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \frac{d}{dx}(\tan 3x) &= \sec^2 3x \cdot \frac{d}{dx}(3x) \\
 &= \sec^2 3x \cdot 3 \frac{d}{dx}(x) \\
 &= \sec^2 3x \cdot 3 \times 1 \\
 &= \underline{\underline{3 \sec^2 3x}}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \frac{d}{dx}(\operatorname{cosec} x^2) &= -\operatorname{cosec}(x^2) \cot(x^2) \cdot \frac{d}{dx}(x^2) \\
 &= -\operatorname{cosec}(x^2) \cot(x^2) \cdot 2x \\
 &= \underline{\underline{-2x \operatorname{cosec}(x^2) \cot(x^2)}}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \frac{d}{dx}(\cot x^4) &= -\operatorname{cosec}^2(x^4) \cdot \frac{d}{dx}(x^4) \\
 &= -\operatorname{cosec}^2(x^4) \cdot 4x^3 \\
 &= \underline{\underline{-4x^3 \operatorname{cosec}^2(x^4)}}
 \end{aligned}$$

$$5. \quad \frac{d}{dx}[\log(\log x)] = \frac{1}{\log x} \cdot \frac{d}{dx}(\log x)$$

$$= \frac{1}{\log x} \cdot \frac{1}{x} = \underline{\underline{\frac{1}{x \log x}}}$$

$$6. \frac{d}{dx} (e^{3x}) = e^{3x} \cdot \frac{d}{dx} (\cancel{3x}) \cdot \frac{d}{dx} (3x)$$

$$= e^{3x} \cdot 3 \cdot \frac{d}{dx} (x)$$

$$= e^{3x} \cdot 3 \cdot 1 = \underline{\underline{3e^{3x}}}$$

$$7. \frac{d}{dx} (\cos(x^2)) = -\sin(x^2) \cdot \frac{d}{dx} (x^2)$$

$$= -\sin(x^2) \cdot 2x$$

$$= \underline{\underline{-2x \sin(x^2)}}$$

$$8. \frac{d}{dx} (\sin^{-1}(x^2)) = \frac{1}{\sqrt{1-(x^2)^2}} \cdot \frac{d}{dx} (x^2)$$

$$= \frac{1}{\sqrt{1-x^4}} \cdot 2x = \underline{\underline{\frac{2x}{\sqrt{1-x^4}}}}$$

$$9. \frac{d}{dx} \tan^{-1}(2x) = \frac{1}{1+(2x)^2} \cdot \frac{d}{dx} (2x)$$

$$= \frac{1}{1+4x^2} \cdot 2 \cdot \frac{d}{dx} (x)$$

$$= \frac{1}{1+4x^2} \cdot 2 \cdot 1 = \underline{\underline{\frac{2}{1+4x^2}}}$$