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Definite integrals and applications.

$$\int_a^b f(x)\,dx = \phi(b) - \phi((a).$$

Q. $\int_0^1 (x^2+2x-3)\,dx$

$= \int x^2 dx + 2\int x\,dx - 3\int dx$

$= x^3/3 + 2\cdot\frac{x^2}{2} - 3.x$

$= [x^3/3 + x^2 - 3x]_0^1$

$= (\frac{1}{3} + 1 - 3) - (0)$

$= \frac{1}{3} - 2 = \frac{1-6}{3}$

$= \frac{-5}{3}$ //

H.W. $\int_0^1 x^2(x^3+1)\,dx = 1/2$ //

Q. $\int_0^{\pi/2} \cos x\,dx = [\sin x]_0^{\pi/2}$

$= \sin \pi/2 - \sin 0$

$= 1 - 0 = \underline{1}.$

Q. $\int_0^{\pi/2} \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}\,dx$

$= \frac{1}{2}\int (1-\cos 2x)\,dx$

$= \frac{1}{2}[\int dx - \int \cos 2x\,dx]$

H.W. $\int_0^{\pi/2} \cos^2 x\,dx = \pi/4$ //

$= \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\pi/2}$

$= \frac{1}{2}\left[\left(\pi/2 - \frac{1}{2}\sin 2\times\pi/2\right) - \left(0 - \frac{1}{2}\sin 0\right)\right]$

$= \frac{1}{2}\left[\left(\pi/2 - \frac{1}{2}\sin\pi\right) - 0\right]$

$= \frac{1}{2}\left[\pi/2 - 0\right] = \frac{1}{2}\times\pi/2 = \underline{\frac{\pi}{4}}$

2. $\int_0^{\pi/2} \cos 4x \cos x \, dx$

$= \int_0^{\pi/2} \frac{1}{2}\left[\cos 5x + \cos 3x\right] dx.$

$= \frac{1}{2}\left[\int_0^{\pi/2} \cos 5x \, dx + \int_0^{\pi/2} \cos 3x \, dx\right]$

$= \frac{1}{2}\left[\frac{\sin 5x}{5} + \frac{\sin 3x}{3}\right]_0^{\pi/2}$

$= \frac{1}{2}\left[\frac{1}{5}\left(\sin 5\times\pi/2 - 0\right) + \frac{1}{3}\left[\sin 3\times\pi/2 - 0\right]\right]$

$= \frac{1}{2}\left[\frac{1}{5}\sin 5\pi/2 + \frac{1}{3}\sin 3\pi/2\right]$

$3\times 90 = 270.$ $\quad$ 5×90

$\sin 270 = -1$ $\quad$ $= 450.$

$\sin 450 = \sin(360+90)$

$\frac{1}{2}\left[\frac{1}{5}\times 1 + \frac{1}{3}\times -1\right]$ $\quad$ $= \sin 90 = 1$

$= \frac{1}{2}\left[\frac{1}{5} - \frac{1}{3}\right] = \frac{1}{2}\left[\frac{3-5}{15}\right] = \underline{\frac{-1}{15}}$

H.W $\int_{0}^{\pi/2} \sin 3x \cos x \, dx$.

Q. $\int_{0}^{\pi/2} x.\sin x \, dx$.

$= x.\int \sin x \, dx - \int \left[\frac{d}{dx}(x).\int \sin x \, dx\right] dx$

$= x(-\cos x) - \int 1. -\cos x \, dx$

$= -x\cos x + \int \cos x \, dx$

$= \left[-x\cos x + \sin x\right]_{0}^{\pi/2}$

$= \left(-\pi/2 \cos \pi/2 + \sin \pi/2\right) - \left(-0 \times \cos 0 + \sin 0\right)$

$= \left(-\pi/2 \times 0 + 1\right) - 0$

$= \underline{\underline{1}}$.

Q. $\int_{0}^{2} x^2 \log x \, dx$.

$= \int \log x . x^2 \, dx$.

$= \log x \int x^2 dx - \int \left[\frac{d}{dx}(\log x) \int x^2 dx\right] dx$

$= \log x . x^3/3 - \int \frac{1}{x} \times x^3/3 \, dx$

$= x^3/3 \log x - \int x^2/3 \, dx$

$= x^3/3 \log x - 1/3 \left(x^3/3\right)$

$= \left[x^3/3 \log x - \frac{1}{3} \frac{x^3}{3}\right]_{0}^{2}$

$$\left(\frac{8}{3}\log 2-\frac{8}{9}\right)-(0-0)$$

$$= 8/3 \log 2 - 8/9.$$

2. $\int_1^e \log x \, dx$

$$= \int \log x \cdot 1 \, dx.$$

$$= \log x \int dx - \int\left[\frac{d}{dx}(\log x)\cdot \int dx\right] dx$$

$$= \log x \cdot x - \int \frac{1}{x} \cdot x \, dx$$

$$= x \log x - \int dx = [x \log x - x]_1^e$$

$$= (e \log e - e) - (1 \cdot \log 1 - 1)$$

$$= (e \log e - e) - (0-1)$$

$$= (e \log e - e) - (-1)$$

$$= e \log e - e + 1$$

$$= e \log e - e + 1$$

$$= e - e + 1 = 1.$$

$\log e = 1$

$\log 1 = 0.$

Q. $\displaystyle\int_0^{\pi/4} \frac{\sec^2 x \, dx}{(1+\tan x)}$

$u = 1 + \tan x$

$\frac{du}{dx} = \sec^2 x$

$du = \sec^2 x \, dx$

$\int \frac{du}{u} = \log u = \log \left[1+\tan x\right]_0^{\pi/4}$

$= \left[\log(1+\tan \pi/4) - \log(1+\tan 0)\right]$

$= \log(1+1) - \log(1+0)$

$= \log 2 - \log 1$

$\log \frac{a}{b}$

$= \log a - \log b$

$= \log(2/1)$

$= \underline{\log 2}$

Q. $\displaystyle\int_0^{\pi} \frac{1-\sin x}{x+\cos x} \, dx.$

$u = x + \cos x$

$\frac{du}{dx} = 1 - \sin x.$

$du = (1-\sin x) \, dx.$

$\int \frac{du}{u} = \log u + C$

$= \left[\log(x+\cos x)\right]_0^{\pi}$

$= \log(\pi + \cos\pi) - \log(0 + \cos 0)$

$= \log(\pi - 1) - \log(1)$

$= \log\left(\frac{\pi - 1}{1}\right) = \underline{\underline{\log(\pi - 1)}}$

Q. $\int_0^1 \frac{1}{1+x^2}\,dx$.

$= \left(\tan^{-1}x\right)_0^1$

$= \tan^{-1}1 - \tan^{-1}0$

$= \underline{\underline{\pi/4}}$

Q. $\int_0^{\pi} \frac{1}{1+\sin x}\,dx$.

$= \int \frac{1-\sin x}{(1+\sin x)(1-\sin x)}\,dx$

$= \int \frac{1-\sin x}{1-\sin^2 x}\,dx$.

$= \int \frac{1-\sin x}{\cos^2 x}\,dx$

$= \int \sec^2 x - \tan x \sec x \cdot dx$

$= \tan x - \sec x$.

$\int_0^{\pi} \left(\frac{1}{1+\sin x}\right)dx = \left(\tan x - \sec x\right)_0^{\pi}$

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$\int_0^{\pi/4} \tan x \, dx$

$= (\log \sec x)_0^{\pi/4}$.

$= \log \sec \pi/4 - \log \sec 0$.

$= \log \sqrt{2} - \log 1$

$= \log (\sqrt{2}/1) = \log \sqrt{2}$