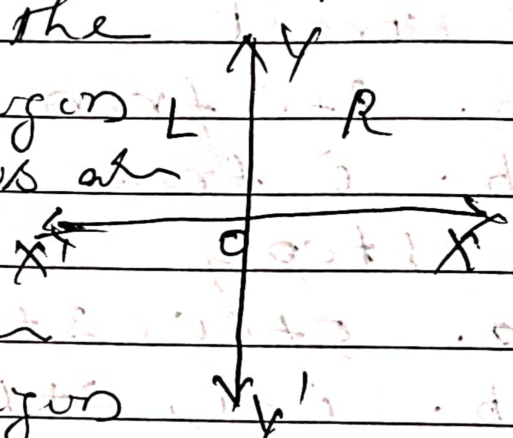
Co-ordinate Geometry

Co-ordinate System

A rectangular co-ordinate system has two axis, XOX' (the x-axis) and YOY' (the y-axis). The axis are mutually ~~perap~~ perpendicular to each other. The intersecting point of the two axis is called the origin of the co-ordinate system. It is denoted by O.

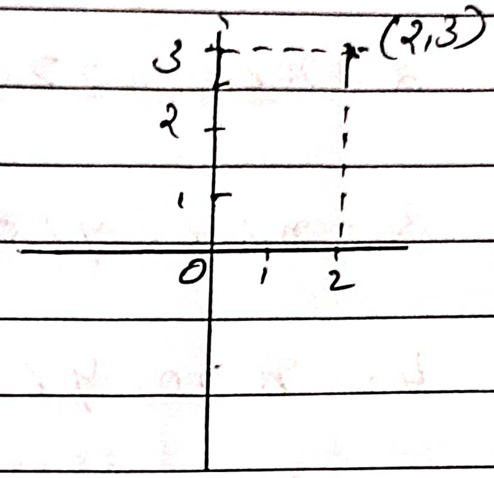
The values in the right of the origin along the x-axis are positive and the values on the left of the origin along the x-axis are negative.



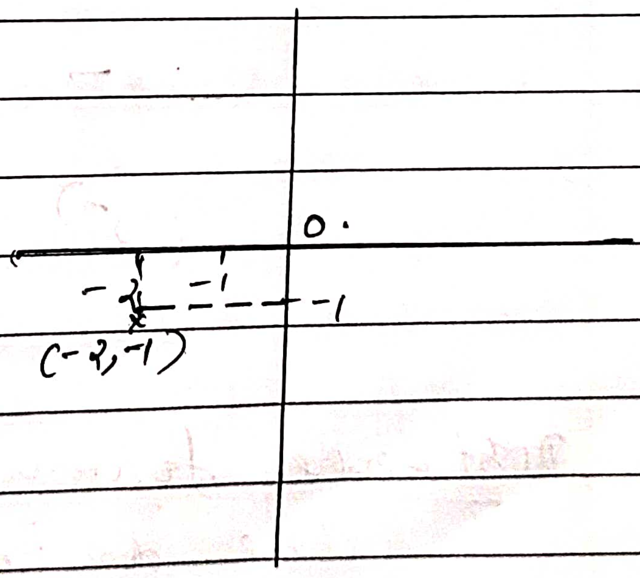
Similarly the values above the original along the y-axis are positive and values below the origin on the y-axis are negative. The x-value is also known as abscissa and y-value is ordinate.

The two value together are called Co-ordinate. At the x-axis y value is zero and the y-axis x-value is zero.

Example(1). Fix (2,3) in a Co-ordinate system.



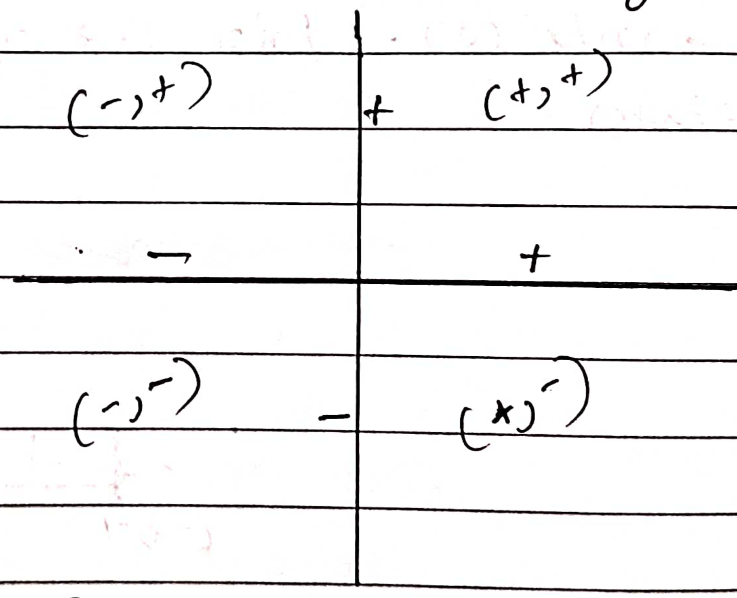
Example (2). Fix (-2,-1) on a Co-ordinate system.



Conclusion

If a point (x, y) lies on a rectangular co-ordinate system, we can see that:

1. $x > 0, y > 0$ means (x, y) lies on 1^{st} quadrant
2. $x < 0, y > 0$ means (x, y) lies on 2^{nd} quadrant
3. $x < 0, y < 0$ means (x, y) lies on 3^{rd} quadrant
4. $x > 0, y < 0$ means (x, y) lies on 4^{th} quadrant



Distance formula

The distance between two points.

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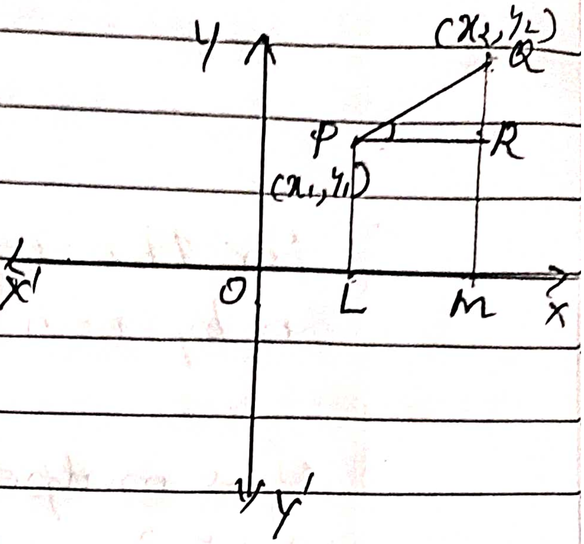


We have $OL = x_1$, $LP = y_1$

$OM = x_2$, $MQ = y_2$.

$PR = LM = OM - OL$

$= x_2 - x_1$.

$QR = QM - MR$

$= QM - LP$

$= y_2 - y_1$.

From ΔPQR

$PQ^2 = PR^2 + QR^2$ (pythagorus theorem)

$= (x_2 - x_1)^2 + (y_2 - y_1)^2$

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\therefore$ Distance between two points (x_1, y_1) and (x_2, y_2) is

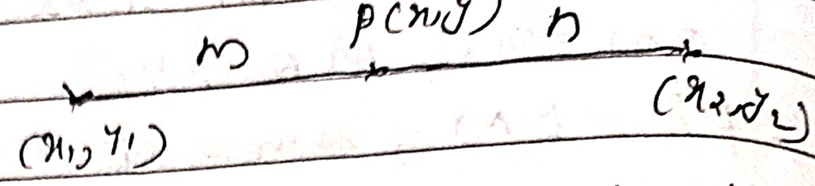
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Note : The distance of a point (x, y) from the origin is $\sqrt{x^2 + y^2}$

Section formula

If a point $P(x, y)$ divides the segment joining (x_1, y_1) and (x_2, y_2) in the ratio $m:n$ internally the co-ordinates of P are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

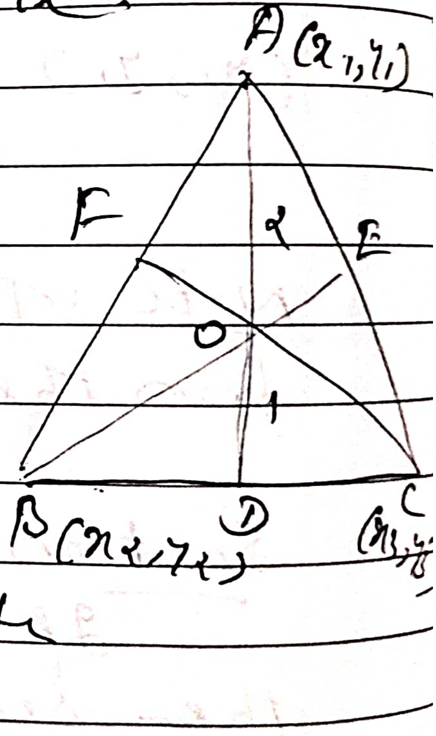


(For the external division, replace n by $(-n)$.

The midpoint of a segment joining (x_1, y_1) and (x_2, y_2) is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Centroid of a triangle



The centroid of a triangle is the point of intersection of the three medians (median is the segment drawn from a vertex to the midpoint of the opposite side. The centroid divides each median in the ratio 2:1

Consider AD. The centroid ~~of~~ O divides AD in the ~~rat~~

ratio 2:1. The co-ordinates of D are $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

Since D is the mid point BC
O is the point which divides
the segment $\overline{AD}$ in the ratio 2:1
∴ The co-ordinates of O

$$\left(\frac{2\cdot\frac{x_2+x_3}{2}+1\cdot x_1}{2+1}, \frac{2\cdot\frac{y_2+y_3}{2}+1\cdot y_1}{2+1}\right)$$

$$=\left(\frac{x_2+x_3+x_1}{3}, \frac{y_2+y_3+y_1}{3}\right)$$

i.e. $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$ is

the centroid of $\triangle ABC$.

Q. The points $(-4,5)$, $(2,-3)$ are at the ends of a diameter of a circle.
Find its radius.

using distance formula.

$$S = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(2+4)^2+(-3-5)^2}$$
$$= \sqrt{36+64} = \sqrt{100}$$
$$= 10$$

diameter = 10

radius = diameter / 2

$= 10/2 = 5$

Q. Find the co-ordinates of the point which divides the segment joining (2,5) and (4,-2) internally in the ratio 3:4.

Co-ordinates pt

$$=\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$(x_1, y_1) = (2,5)$, $(x_2, y_2) = (4,-2)$.

$m:n = 3:4$.

Co-ordinate pt

$$=\left(\frac{3\times4+4\times2}{3+4}, \frac{3\times-2+4\times5}{3+4}\right)$$

$$=\left(\frac{12+8}{7}, \frac{-6+20}{7}\right)$$

$$=\left(\frac{20}{7}, \frac{14}{7}\right) = \left(\frac{20}{7}, 2\right)$$

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Q. Find the centroid of a triangle having vertices $(2,6)$, $(4,0)$ and $(8,2)$.

Centroid of a triangle.

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$=\left(\frac{2+4+8}{3}, \frac{6+0+2}{3}\right)$$

$$=\left(14/3, 8/3\right)$$

Straight lines:

Two mutually ⊥r straight lines $\overleftrightarrow{XOX'}$ and $\overleftrightarrow{YOY'}$ in a coordinate system. The equation of $\overleftrightarrow{XOX'}$ is $y=0$ and the equation of $\overleftrightarrow{YOY'}$ is $x=0$. The equation of any straight line parallel to x-axis is $y=k$ and the equation of any straight line parallel to y-axis is $x=k$.

Slope of a Segment: If a straight line is inclined at an angle α with the x-axis, the slope of that straight line is given by $\tan\alpha$. It is represented by

$$m = \tan\alpha.$$

If two points in any straight line are given, the slope of that straight line is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$, if two points are given

The slope is also called gradient.

Q. Find the slope and the angle of inclination of the pair of points.

1. $(5, -2), (6, 5) \Rightarrow (x_2, y_2) = (6, 5)$
$\Rightarrow (x_1, y_1) = (5, -2)$

$x_1 = 5, \; y_1 = -2.$

$x_2 = 6, \; y_2 = 5$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{5 - (-2)}{6 - 5} = \frac{5 + 2}{1} = 7$$

Slope of tangent

$m = \tan\alpha$

$\tan\alpha = 7$

$\theta = \tan^{-1}(7)$

2. $(-5,2), (9,4) \Rightarrow (x_1,y_1)=(-5,2), (x_2,y_2)=(9,4)$

$x_1=-5, x_2=9.$

$y_1=2, y_2=4.$

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{4-2}{9-(5)}$$

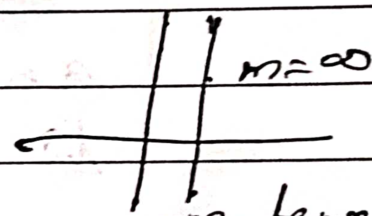
$$= \frac{2}{14} = 1/7.$$

$m = \tan\alpha.$

$\tan\alpha = m = 1/7.$

$\theta = \tan^{-1}(1/7)$

3. $(2,-3), (2,4).$



$(x_1,y_1)=(2,-3)$

$(x_2,y_2)=(2,4).$

$m = \tan\alpha$

$= \tan 90^\circ$

$x_1=2, y_1=-3, x_2=2, y_2=4.$

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{4--3}{2-2} = \frac{4+3}{0}.$$

$= 7/0.$

$m = \infty.$

$\tan\alpha = m = \infty.$

$\therefore$ The line is perpendicular to x-axis.

or

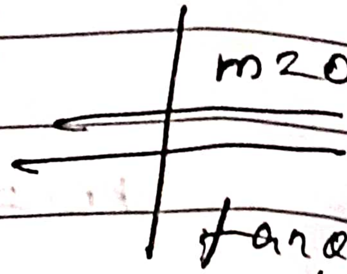
Slope is ∞ for the y-axis and any line parallel to y-axis.

2. $(-3, 2), (4, 2)$.

$(x_1, y_1) = (-3, 2)$

$(x_2, y_2) = (4, 2)$.

$m = 0$.



$\tan\theta = 0$

$m = \tan\theta$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-2}{4 - -3} = \frac{0}{7} = 0.$$

$\tan\theta = 0$.

The line parallel to x-axis

or

Slope is 0 for the x-axis and any line perpendicular to y-axis.