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Differential coefficient of inverse t-functions.

(1) Find $\frac{dy}{dx}$, if $y = \sin^{-1}x$.

Let $y = \sin^{-1}x$.

$\Rightarrow x = \sin y$.

Both side differentiating with respect to y.

$$\frac{d}{dy}(x) = \frac{d}{dy}(\sin y).$$

$$\frac{dx}{dy} = \cos y.$$

$$= \sqrt{\cos^2 y} \quad \because \cos^2 y + \sin^2 y = 1$$

$$= \sqrt{1 - \sin^2 y}$$

$$\frac{dx}{dy} = \sqrt{1 - x^2}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}.$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{1}{\sqrt{1-x^2}}.$$

$$\Rightarrow \frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}.$$

Result

1. $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$.
2. $\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$
3. $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$.
4. $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$.

Q. Find the derivative of $e^x \sin^{-1}x$.

$$\frac{d}{dx}(e^x \sin^{-1}x)$$

$$= e^x \cdot \frac{d}{dx}(\sin^{-1}x) + \sin^{-1}x \cdot \frac{d(e^x)}{dx}$$

$$= e^x \cdot \frac{1}{\sqrt{1-x^2}} + \sin^{-1}x \cdot e^x.$$

$$= \frac{e^x}{\sqrt{1-x^2}} + e^x \cdot \sin^{-1}x$$

Q. Find the derivative of $\frac{\sin^{-1}x}{x}$

$$\frac{d}{dx}\left(\frac{\sin^{-1}x}{x}\right) = \frac{x \cdot \frac{d}{dx}(\sin^{-1}x) - \sin^{-1}x \cdot \frac{d(x)}{dx}}{(x)^2}.$$

$$= \frac{x \cdot \frac{1}{\sqrt{1-x^2}} - \sin^{-1}x \cdot 1}{x^2}$$

$$= \frac{x/\sqrt{1-x^2} - \sin^{-1}x}{x^2}$$

Q. Find the derivative of $(x^2+3)\tan^{-1}x$.

$$\frac{d}{dx}\left[(x^2+3)\tan^{-1}x\right]$$

$$= (x^2+3)\frac{d}{dx}(\tan^{-1}x) + \tan^{-1}x \cdot \frac{d}{dx}(x^2+3)$$

$$= (x^2+3)\frac{1}{1+x^2} + \tan^{-1}x\left(\frac{d}{dx}(x^2) + \frac{d}{dx}(3)\right)$$

$$= \frac{x^2+3}{1+x^2} + \tan^{-1}x(2x+0)$$

$$= \frac{x^2+3}{1+x^2} + 2x\tan^{-1}x$$

Q. Find the derivative of $\frac{x\sin^{-1}x}{(1+x^2)}$

$$\frac{d}{dx}\left(\frac{x\sin^{-1}x}{1+x^2}\right)$$

$$= \frac{(1+x^2)\frac{d}{dx}(x\sin^{-1}x) - x\sin^{-1}x\frac{d}{dx}(1+x^2)}{(1+x^2)^2}$$

$$= \frac{(1+x^2)\left(x \cdot \frac{d}{dx}(\sin^{-1}x) + \sin^{-1}x \frac{d}{dx}(x)\right) - x\sin^{-1}x\left(\frac{d}{dx}(1) + \frac{d}{dx}(x^2)\right)}{(1+x^2)^2}$$

$$= \frac{(1+x^2)\left(x\frac{1}{\sqrt{1-x^2}} + \sin^{-1}x \cdot 1\right) - x\sin^{-1}x(2x)}{(1+x^2)^2}$$

$$= \frac{(1+x^2)\left(\frac{x}{\sqrt{1-x^2}} + \sin^{-1}x\right) - 2x^2\sin^{-1}x}{(1+x^2)^2}$$

Q. $\frac{d}{dx}(xe^x \sin^{-1}x) = xe^x \cdot \frac{d}{dx}(\sin^{-1}x) + \sin^{-1}x \frac{d}{dx}(xe^x)$.

$$= xe^x \cdot \frac{1}{\sqrt{1-x^2}} + \sin^{-1}x\left(x\frac{d}{dx}(e^x) + e^x \frac{d}{dx}(x)\right)$$

$$= \frac{xe^x}{\sqrt{1-x^2}} + \sin^{-1}x(xe^x + e^x \cdot 1)$$

$$= \frac{xe^x}{\sqrt{1-x^2}} + (xe^x + e^x)\sin^{-1}x$$