51 56, 52, 53, 54, 55, 57, 61, 59

Limits

Variable: A variable is a varying quantity, whose value changes during any mathematical investigation.

eg 1. the atmospheric temperature

2. The angle between the hour hand and minute hand of a working clock.

Dependent and Independent variables

A variable whose value is chosen arbitrarily is called independent variable.

A dependent variable depends on independent variable.

eg : $y = f(x)$.

$y = x^2$

Constant is a fixed quantity whose value remains unchanged throughout a mathematical investigation

eg : $\pi = 3.14$

Function: If a variable y ~~depends~~ depends on a variable x in such a way that each value of x determines exactly one value of y, then we say that y is a function of x, it can be written as $y = f(x)$

(1) $y = f(x)$ are called explicit functions

eg: $y = e^x$, $y = x^2 + x + 1$

(2) If a function in x and y is given in such a way that x and y can not be seperated, then the function is called implicit function.

eg: $x^2 + y^2 = 25$, $x^2 + xy + y^2 = 0$

Parametric function

~~If x~~

If two variables x and y are expressed in terms of a third variable (θ, t) then the variable is called the parameter and the function containing the parameter is known as parametric function.

eg: (1) $x = at^2$, $y = bt^2$

parametric function with parameter t.

(2) $x = a\cos\theta$, $y = b\sin\theta$

with parameter θ.

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Limits

Consider a variable x ~~tak~~ taking an infinite number of values according to a certain rule, as x approaches 'a' after some time, we say that x tends to a or limit of x equal to a

$Lt\ x = a$ or $\lim x = a$

Limits of a function

The sequence of numbers $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ As x increases, the value of $\frac{1}{x}$ becomes smaller and smaller. Or the value approaches to "zero",

$$\lim_{x \to \infty} \frac{1}{x} = 0$$

The numbers $1, \frac{1}{0.1}, \frac{1}{0.01}, \ldots$ As x decreases, the value of $\frac{1}{x}$ become larger and larger or the value approaches to infinity.

$$\lim_{x \to 0} \left(\frac{1}{x}\right) = \infty$$

Properties

1. $\lim (u+v) = \lim u + \lim v$
2. $\lim (u-v) = \lim u - \lim v$
3. $\lim (uv) = \lim u \cdot \lim v$
4. $\lim (u/v) = \frac{\lim u}{\lim v}, \quad v \neq 0$

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1. Calculate $\lim_{x \to \infty} \frac{x^2+2x+1}{x^2+x-3}$

$\lim_{x \to \infty} \frac{\cancel{x^2}(1+2/x+1/x^2)}{\cancel{x^2}(1+1/x-3/x^2)}$

$\lim_{x \to \infty} \frac{(1+2/x+1/x^2)}{(1-1/x-3/x^2)}$ $\quad \lim_{x \to \infty} \frac{1}{x} = 0$

$= 1/1 = 1$

$= \frac{x^2+2x+1}{x^2}$

$= \frac{x^2}{x^2} + \frac{2x}{x^2} + \frac{1}{x^2}$

$= 1 + \frac{2}{x} + \frac{1}{x^2}$

$= \frac{1+0+0}{1+0-0} = \frac{1}{1}$

Similarly

$\lim_{x \to \infty} 1/x^2 = 0.$

2. Evaluate $\lim_{x \to \infty} \frac{x^2-2x+8}{4x^3-3}$

$\lim_{x \to \infty} \frac{x^2(1-2/x+8/x^2)}{x^3(4-3/x^3)}$

$\lim_{x \to \infty} \frac{1}{x} \left(\frac{1-2/x+8/x^2}{4-3/x^3}\right)$

$\frac{1}{4} \lim_{x \to \infty} 1/x = \frac{1}{4} \times 0 = 0$

$\frac{x^2-2x+8}{x^2}$

$= \frac{x^2/x^2 - \frac{2x}{x^2} + \frac{8}{x^2}}{\frac{4x^3}{x^3} - \frac{3}{x^3}}$

$= \frac{1-2/x+8/x^2}{4-3/x^3}$

3. Evaluate $\lim_{x \to \infty} \frac{3x^2+5}{x^2-2}$

$\lim_{x \to \infty} \frac{\cancel{x^2}(3+5/x^2)}{\cancel{x^2}(1-2/x^2)}$

$\lim_{x \to \infty} \left(\frac{3+5/x^2}{1-2/x^2}\right) = \frac{3+0}{1-0} = 3/1 = 3$

classmate

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4. Evaluate $\lim_{x\to\infty} \frac{x^2-2x+3}{3x^2-2x}$.

$$\lim_{x\to\infty} \frac{x^2(1-2/x+3/x^2)}{x^2(3-2/x)}$$

$$\lim_{x\to\infty} \frac{1-2/x+3/x^2}{3-2/x} \qquad \lim_{x\to\infty} \frac{1}{x}=0$$

$$= \frac{1-0+0}{3-0} = 1/3$$

5. $\lim_{x\to\infty} \frac{x^3-2x+3}{2x^3-4x+6}$.

$$\lim_{x\to\infty} \frac{\cancel{x^3}(1-2/x^2+3/x^3)}{\cancel{x^3}(2-4/x^2+6/x^3)}$$

$$\frac{x^3-2x+3}{x^3} = x^3/x^3 - \frac{2x}{x^3} + \frac{3}{x^3} = 1-\frac{2}{x^2}+\frac{3}{x^3}$$

$$\lim_{x\to\infty} \frac{1-2/x^2+3/x^3}{2-4/x^2+6/x^3}$$

$$= \frac{1-0+0}{2-0+0} = 1/2$$

6. Calculate $\lim_{x\to 0} \frac{ax+b}{cx+d}$.

$$\lim_{x\to 0} \frac{ax+b}{cx+d} = \frac{a\times 0+b}{c\times 0+d} = \frac{0+b}{0+d}$$

$$= b/d$$

2. Calculate $\lim_{x \to 1} \frac{2x+3}{4x-1}$

$$\lim_{x \to 1} \frac{2x+3}{4x-1} = \frac{2\times 1+3}{4\times 1-1}$$

$$= \frac{2+3}{4-1} = \frac{5}{3}$$

3. $\lim_{x \to 2} (2x+3)$

$$\lim_{x \to 2} (2x+3) = 2\times 2+3$$

$$= 4+3 = 7$$

4. $\lim_{x \to 2} \frac{x^2-4}{x-2}$

$$\lim_{x \to 2} \frac{x^2-4}{x-2} = \frac{4-4}{2-2} = \frac{0}{0} \neq 0.$$

$$\lim_{x \to 2} \frac{x^2-2^2}{x-2}$$

~~$a^2-b^2=(a-b)($~~

$a^2-b^2=(a-b)(a+b)$

$$\lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$$

$$\lim_{x \to 2} x+2 = 2+2 = 4$$

5. $\lim_{x \to 1} \frac{x^2+4x-5}{x^2+x-2}$

$$\lim_{x \to 1} \frac{x^2+4x-5}{x^2+x-2} = \frac{1^2+4\times1-5}{1^2+1-2} = \frac{1+4-5}{1+1-2}$$

$$= \frac{5-5}{2-2} = 0/0 \neq 0$$

numbers -1 and 5

$$\lim_{x \to 1} \frac{(x-1)(x+5)}{(x-1)(x+2)}$$

pdt $= -5$
sum $= 4$
numbers 1 and 5
pdt $= 1\times5 = 5$
sum $= 1+5 = 6$
But pdt $= 1\times-5 = -5$
sum $= 1-5 = -4$
~~Bdt~~ pdt $= -1\times5 = -5$
sum $= -1+5 = 4$

pdt $= -2$
sum $= 1$
numbers 1 and -2
pdt $= 1\times-2 = -2$.
sum $= 1-2 = -1$
~~pdt~~ numbers -1 and 2
pdt $= -1\times2 = -2$.
sum $= -1+2 = 1$
numbers -1 and 2

$$\lim_{x \to 1} \frac{x+5}{x+2} = \frac{1+5}{1+2} = 6/3 = 2$$

Q. Evaluate $\lim_{x \to 2} \frac{x^2-5x+6}{x^2+x-6}$

$\lim_{x \to 2} \frac{x^2-5x+6}{x^2+x-6} = \frac{2^2-5\times2+6}{2^2+2-6}$

$= \frac{4-10+6}{4+2-6} = \frac{0}{0} \neq 0.$

$\lim_{x \to 2} \frac{\cancel{(x-2)}(x-3)}{(x+3)\cancel{(x-2)}}$

pdt = 6
Sum = 5
numbers
-2 and -3
pdt = $-2\times-3 = 6$
Sum = $-2-3 = -5$

$\lim_{x \to 2} \frac{x-3}{x+3} = \frac{2-3}{2+3}$

$= \frac{-1}{5}$

Q. $\lim_{x \to 1} \frac{x^2+x-2}{x^2+2x-3}$

$\lim_{x \to 1} \frac{x^2+x-2}{x^2+2x-3} = \frac{1^2+1-2}{1^2+2\times1-3} = \frac{1+1-2}{1+2-3}$

$= \frac{2-2}{3-3} = 0/0 \neq 0.$

$\lim_{x \to 1} \frac{(x+2)\cancel{(x-1)}}{\cancel{(x-1)}(x+3)} = \lim_{x \to 1} \frac{x+2}{x+3}$

$= \frac{1+2}{1+3} = 3/4$