

# Algebraical limit

$$\lim_{n \rightarrow \infty} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1} \text{ for all rational values of } n.$$

1. Evaluate  $\lim_{x \rightarrow 2} \frac{x^4 - 16}{x - 2}$

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{x^4 - 2^4}{x - 2} &= 4 \cdot 2^{4-1} \quad n=4, a=2 \\ &= 4 \times 2^3 \\ &= 4 \times 8 = \underline{\underline{32}} \end{aligned}$$

2. Evaluate  $\lim_{x \rightarrow 3} \frac{x^3 - 27}{x^2 - 9}$

$$\lim_{x \rightarrow 3} \frac{x^3 - 3^3}{x^2 - 3^2}$$

Divide the numerator and denominator by  $(x-3)$

$$\lim_{x \rightarrow 3} \frac{(x^3 - 3^3)/x-3}{(x^2 - 3^2)/x-3}$$

$$\begin{aligned} \frac{\lim_{x \rightarrow 3} (x^3 - 3^3)/x-3}{\lim_{x \rightarrow 3} (x^2 - 3^2)/x-3} &= \frac{3 \cdot 3^{3-1}}{2 \cdot 3^{2-1}} = \frac{3 \times 9}{2 \times 3} \\ &= \underline{\underline{\frac{9}{2}}} \end{aligned}$$

3. Evaluate  $\lim_{x \rightarrow 4} \frac{x^3 - 64}{x^2 - 4^2}$

$$\lim_{x \rightarrow 4} \frac{x^3 - 4^3}{x^2 - 4^2}$$

Divide the numerator and denominator by  $(x-4)$

$$\lim_{x \rightarrow 4} \frac{(x^3 - 4^3) / (x-4)}{(x^2 - 4^2) / (x-4)}$$

$$\lim_{x \rightarrow 4} \frac{(x^3 - 4^3) / (x-4)}{(x^2 - 4^2) / (x-4)}$$

$$= \frac{3 \cdot 4^{3-1}}{2 \cdot 4^{2-1}}$$

$$\lim_{x \rightarrow 4} \frac{(x^3 - 4^3) / (x-4)}{(x^2 - 4^2) / (x-4)}$$

$$= \frac{3 \cdot 4^2}{2 \cdot 4} = \underline{\underline{6}}$$

4. Evaluate  $\lim_{x \rightarrow 2} \frac{x\sqrt{x} - 2\sqrt{2}}{x - 2}$

$$\lim_{x \rightarrow 2} \frac{x \cdot x^{1/2} - 2 \cdot 2^{1/2}}{x - 2}$$

$$x^m \cdot x^n = x^{m+n}$$

$$\lim_{x \rightarrow 2} \frac{x^{3/2} - 2^{3/2}}{x - 2} = \frac{3}{2} 2^{3/2-1}$$



$$= \frac{3}{2} \cdot 2^{\frac{1}{2}}$$

$$= \frac{3\sqrt{2}}{2} = \frac{3 \times \sqrt{2} \times \sqrt{2}}{2\sqrt{2}}$$

$$= \frac{3 \times 2}{2\sqrt{2}} = \underline{\underline{3/\sqrt{2}}}$$

### Trigonometrical limit

$$\lim_{\alpha \rightarrow 0} \left( \frac{\sin \alpha}{\alpha} \right) = 1$$

Q. Evaluate  $\lim_{\alpha \rightarrow 0} \frac{\sin m\alpha}{\sin n\alpha}$ .

$$\lim_{\alpha \rightarrow 0} \frac{\sin m\alpha / \alpha}{\sin n\alpha / \alpha}$$

$$\lim_{\alpha \rightarrow 0} \frac{\sin m\alpha}{\alpha} \rightarrow \text{multiply nr and dr by } m$$

$$\lim_{\alpha \rightarrow 0} \frac{\sin n\alpha}{\alpha} \rightarrow \text{multiply nr and dr by } n$$

$$\lim_{\alpha \rightarrow 0} \frac{m \sin \alpha}{m\alpha} = m \cdot \lim_{m\alpha \rightarrow 0} \frac{\sin m\alpha}{m\alpha}$$

$$\lim_{\alpha \rightarrow 0} \frac{n \cdot \sin \alpha}{n\alpha} = n \cdot \lim_{n\alpha \rightarrow 0} \frac{\sin n\alpha}{n\alpha}$$

$$= \frac{n \times 1}{n \times 1} = \underline{\underline{1/n}}$$

Q. Evaluate  $\lim_{x \rightarrow 0} \frac{\tan x}{x}$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\lim_{x \rightarrow 0} \frac{\sin x / \cos x}{x}$$

$$\parallel \frac{a/b}{c} = \frac{a}{bc}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x}{x \cos x}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x}{x} \times \lim_{x \rightarrow 0} \frac{1}{\cos x}$$

$$= 1 \times 1 = \underline{\underline{1}}$$

Q. Evaluate  $\lim_{\alpha \rightarrow 0} \frac{\sin 4\alpha + \sin 2\alpha}{6\alpha}$

$$\lim_{\alpha \rightarrow 0} \left( \frac{\sin 4\alpha}{6\alpha} + \frac{\sin 2\alpha}{6\alpha} \right)$$

$$\frac{1}{6} \lim_{\alpha \rightarrow 0} \left( \frac{\sin 4\alpha}{\alpha} + \frac{\sin 2\alpha}{\alpha} \right)$$

NR + DR lead by 1

NR + DR lead by 2.



$$\frac{1}{6} \lim_{\alpha \rightarrow 0} \left( 4 \frac{\sin 4\alpha}{4\alpha} + 2 \cdot \frac{\sin 2\alpha}{2\alpha} \right)$$

$$= \frac{1}{6} \left[ 4 \lim_{4\alpha \rightarrow 0} \frac{\sin 4\alpha}{4\alpha} + 2 \cdot \lim_{2\alpha \rightarrow 0} \frac{\sin 2\alpha}{2\alpha} \right]$$

$$= \frac{1}{6} (4 + 2) = \frac{6}{6} = 1$$