Rotational Dynamics

- A door opens slowly if we push too close to its hinges.

  The more massive the door, the more slowly it opens.

  - The more the force is applied from the pivot, the greater the angular acceleration;

    that angular acceleration is inversely proportional to mass.

These relationships are very similar to the familiar relationships among force, mass, and acceleration embodied in Newton’s second law of motion.

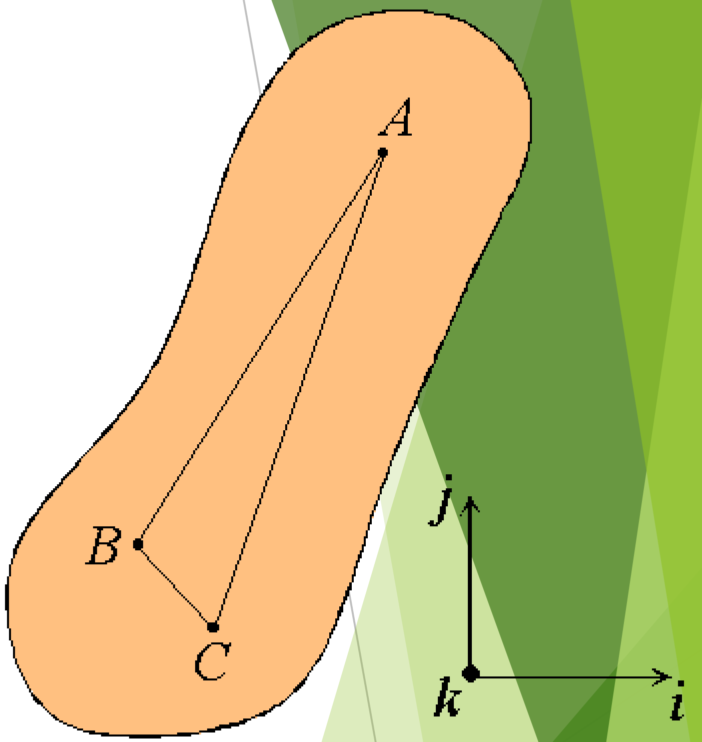
There are precise rotational analogs to both force and mass.

RIGIDBODY

- A solid **body** in which deformation is zero or so small it can be neglected is called a rigid body.

- The distance between any two given points on a **rigid body** remains constant in time regardless of external forces or moments exerted on it.

- Or, particles of the body should always remain in fixed positions relative to one another.



RIGID BODY MOTION

Linear Motion : change in motion only in linear direction
Rotation : the body rotates about an axis'

General Plane motion: consists of both linear and rotational motion- Eg: rolling wheel

Rigid body introduces the concept of Center of Mass

Centre of mass is the point at which all the mass of a body is considered to be concentrated.

Eg: A uniform sphere can be thought of as having all its mass concentrated at its center.

However an unsymmetric object such as a hammer has more mass towards one end and therefore the center of mass is towards the head of the hammer.

Center of Gravity is the average location of the weight distribution of a body. It is the point where all the weight of a body can be considered to be concentrated

Rotational dynamics

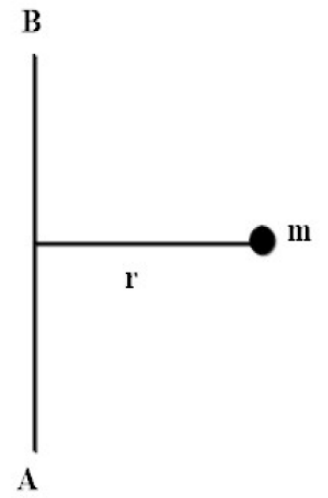
- Study of objects that are rotating or moving in a curved path
- It involves such quantities as torque, moment of inertia/**rotational** inertia, angular displacement (in radians or less often, degrees), angular velocity (radians per unit time), angular acceleration (radians per unit of time squared)
- **Characteristics of rigid body motion:**

All lines on a rigid body have the same angular velocity and the same angular acceleration .

Rigid motion can be decomposed into the translation of an arbitrary **point**, followed by a rotation about the **point**.

Moment of Inertia (Rotational Inertia) of a Particle

- Moment of inertia or angular mass or rotational inertia can be defined w.r.t. rotation axis.
- Consider a particle of mass m capable of rotation about an axis AB. Let r be the perpendicular distance of the particle from AB. The moment of inertia about the axis AB, $I = mr^2$



The SI unit of moment of inertia is kgm^2.

- The moment of inertia of a body depends on

  1. Mass of the body.
  2. The distribution of mass with respect to the axis of rotation
- The moment of inertia of a particle about a given axis is defined as the product of the mass of the particle and the square of the distance of the body from the axis.

Moment of Inertia (Rotational Inertia) of a Rigid Body

Consider a rigid body capable of rotation about an axis AB. Let us consider particles of masses m1, m2, m3,etc. of the body at distances r1, r2, r3,etc. respectively from the axis AB.

Moment of inertia of m_1 about AB = $m_1 r_1^2$

Moment of inertia of m_2 about AB = $m_2 r_2^2$

Moment of inertia of m_3 about AB = $m_3 r_3^2$

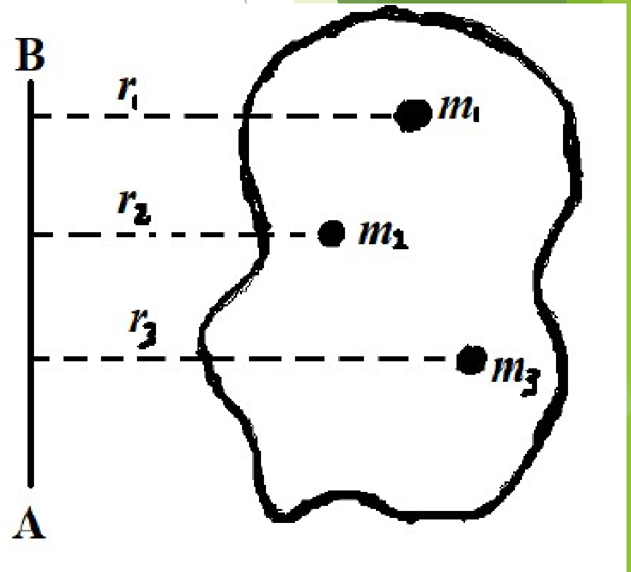
....

Moment of inertia of m_n about AB = $m_n r_n^2$

Therefore, the total moment of inertia of the body about the axis of rotation AB,

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots\ldots m_n r_n^2$$

$$I = \sum_{i=1}^{n} m_1 r_1^2$$



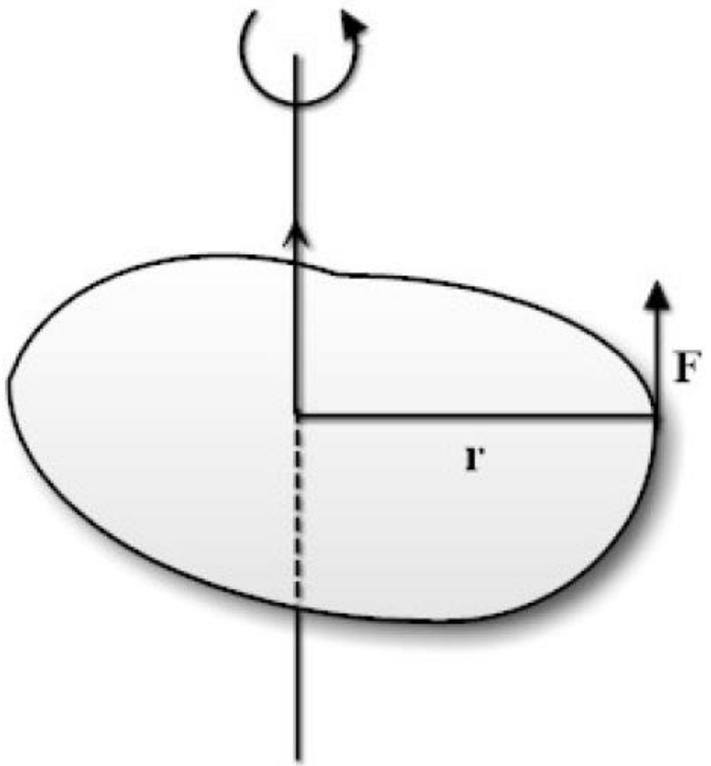
The moment of inertia is the sum of the Moment of inertia of the individual particles of the body about the axis of rotation.

Moment of a force or Torque

- In the case of linear motion, force is required to produce linear acceleration to a body. Similarly in rotational motion torque is required to produce angular acceleration.
- Force tends to change the motion of things. Torque tends to twist or change the state of rotation of things.
- Force should be applied to move a stationary object while torque is applied to rotate a stationary object. Hence, torque is the rotating effect of the force on a body.
- The rotating or turning effect produced by a force is called moment of a force. It is also called Torque or rotating force.

Torque is defined as the product of the force and the perpendicular distance between the line of action of the force and the axis of rotation.

- Consider a rigid body that is free to rotate about an axis. A force is applied to the rigid body at a perpendicular distance r from the axis of rotation as shown in the figure. The rotating effect of force on the rigid body about the axis of rotation depends on the magnitude of the force applied and the perpendicular distance of the point of application of the force from the axis of rotation.



$$\tau = Fr$$

The SI unit of torque is newton meter (Nm)

Torque also depends on the angle between the force and the line joining the point of application of the force and the axis of rotation. If the angle between r and F is θ, the perpendicular distance between the line of action of the force and axis of rotation becomes $r sin\theta$.

Therefore, $\tau = Frsin\theta$

The torque is maximum when $\theta = 90^{\circ}$ or $sin\theta = 1$and $\tau_{max} = Fr$.

Torque is minimum when $\theta = 0^{\circ}$ or $sin\theta = 0$ and $\tau_{min} = 0$.

Torque is the rotational equivalent of force. We can relate the torque on a rigid body to the angular acceleration.

$$\tau = Fr$$

From Newton's second law of motion,

$$F = ma$$

$$\therefore \quad \tau = mar$$

The relation between linear acceleration and angular acceleration is given by

$$a = r\alpha$$

$$\therefore \quad \tau = m\, r\alpha r$$

$$\tau = mr^2\alpha$$

Since the moment of inertia, $I = mr^2$

$$\tau = I\alpha$$

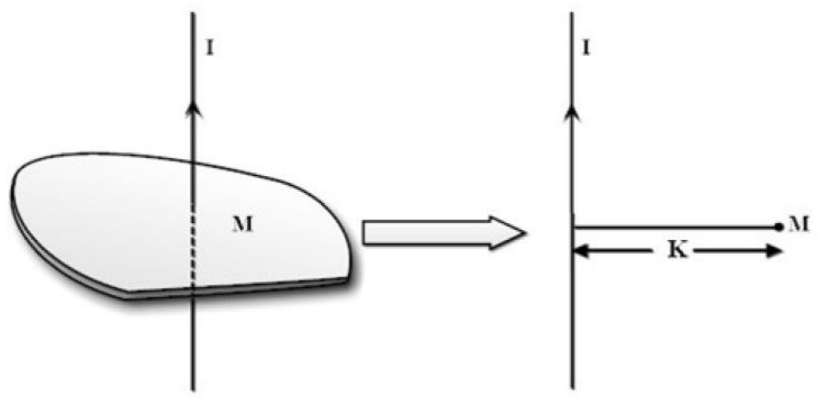
Comparison between linear motion and rotational motion

Linear motion	Rotational motion
Linear displacement(s)	Angular displacement(θ)
Linear velocity(v)	Angular velocity (ω)
Linear acceleration (a)	Angular acceleration(α)
Mass(m)	Moment of inertia(I) $[I = mr^2]$
Linear momentum(p) $[p = mv]$	Angular momentum(L) $[L = I\,\omega]$
Force(F) $[F = ma]$	Torque(τ) $[\tau = I\alpha]$

Radius of Gyration

- Radius of gyration of a body about an axis of rotation is defined as radial distance of a point ,from the axis of rotation at which, if whole mass of any body is assumed to be concentrated, its moment of inertia about the given axis would be same as with its actual distribution of mass.
- Or , it is the effective distance of the particles from its axis of rotation and is denoted by 'k'
- If the whole mass M of a body is supposed to be concentrated at a point of distance k from the axis such that Mk^2 has the same value as the MI about that axis, then k is called radius of gyration.
- Ie.,

$$Mk^2 = \sum mr^2$$



If M is the total mass of the body and K is the radius of gyration of the body about the axis of rotation, then the moment of inertia is given by

$$I = MK^2$$

$$K = \sqrt{\frac{I}{M}}$$

The SI unit of the radius of gyration is meter. The radius of gyration depends on

1. The distribution of mass from the axis of rotation.
2. The position and direction of the axis of rotation.

Theorems on Moment of Inertia

A) Parallel axes Theorem

- *Parallel axes theorem states that the moment of inertia of any rigid body about a given axis is equal to the sum of its moment of inertia about a parallel axis passing through the centre of gravity and the product of the mass of the body and the square of the distance between the axes.*
- Let I be the moment of inertia of a body about an axis AB.

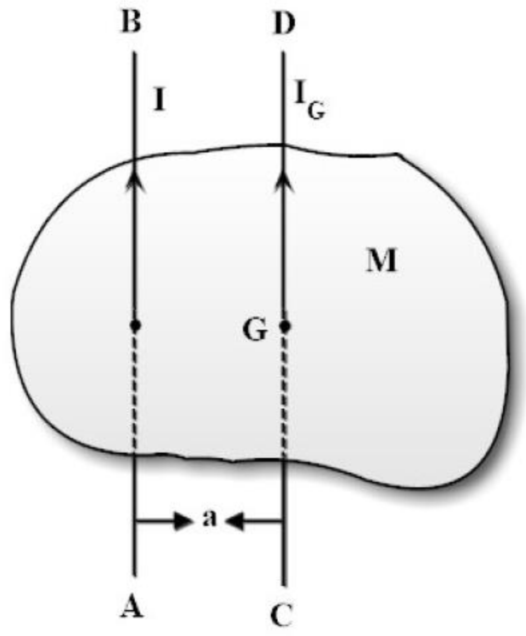
  Let I_G be the moment of inertia about another axis CD which

  is parallel to AB and passing through the centre of gravity G of the body.

  Let M be the mass and ‘a’ be the distance between the two axes.

  Then according to the parallel axes theorem,

$$I = I_G + Ma^2$$



b)Perpendicular Axis theorem

- ***Perpendicular axes theorem states that the sum of the moments of inertia of a plane lamina about two mutually perpendicular axes in its plane is equal to its moment of inertia about a perpendicular axis passing through the intersection of the first two axes.***

Let OX and OY be two mutually perpendicular axes in the plane of the lamina intersecting each other at point O. The axis OZ is perpendicular to both OX and OY. If IX, IY, and IZ are the moment of inertia about the axes OX, OY, and OZ respectively, then by perpendicular axes theorem

$$I_X + I_Y = I_Z$$