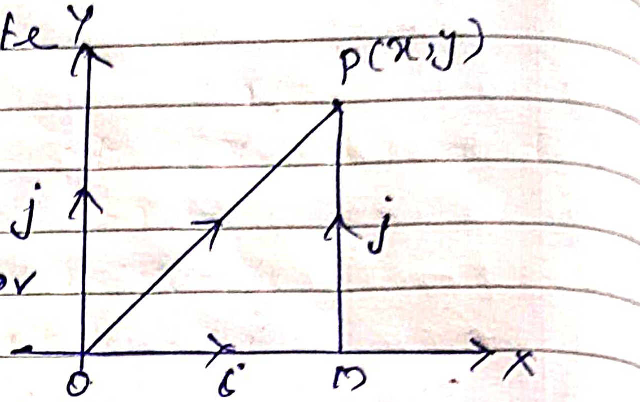
orthogonal cartesian axis and unit vector



Consider the co-ordinate plane XOY and let i be the unit vector along x axis, OM = $x\hat{i}$ and j be the unit vector along y-axis, MP = yj (OY parallel to MP)

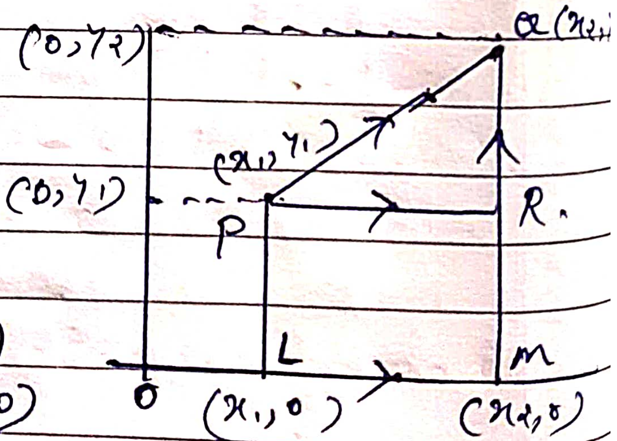
By triangle law of addition

$$OP = OM + MP = xi + yj$$

The position vector of a point with respect to the ~~origin~~ origin O is given by $\vec{OP} = xi + yj$ (Any vector in XY plane which has initial point (0,0) & terminal point (x,y))

$|\vec{OP}| = \sqrt{x^2+y^2}$ (length of the vector or magnitude of the vector)

Note: (1). A vector with initial point (x_1, y_1) and terminal point (x_2, y_2).



By triangle law of addition:

$\vec{PQ} = \vec{PR} + \vec{RQ}$.

$= LM + (MQ - MR)$

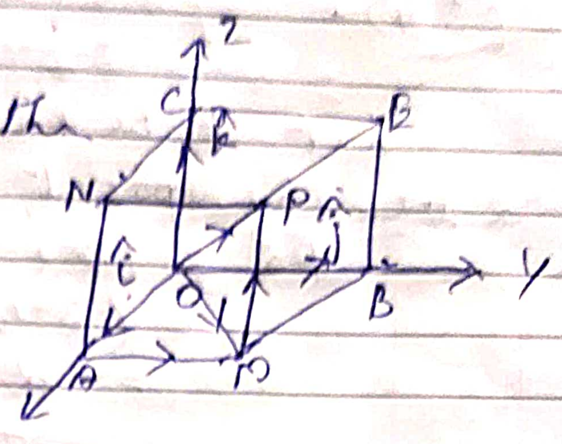
$= (OM - OL) + (MQ - MR)$

$= (x_2 - x_1)i + (y_2 - LP)$

$\vec{PQ} = (x_2 - x_1)i + (y_2 - y_1)j$

$|\vec{PQ}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

(2) Any vector in XYZ plane having initial point (0,0,0) and terminal point (x,y,z) is given by



$\vec{OA}$, $\vec{OB}$ and $\vec{OC}$ along the axis.

$\vec{OA} = xi$, $\vec{OB} = yj$

$\vec{OC} = zk$

$r = \vec{OP} = OM + MP$

$= OA + AM + MP$

$= OA + OB + OC$ (AM is parallel to OB and MP is parallel to OC)

$= xi + yj + zk$.

$|\vec{OP}| = \sqrt{x^2+y^2+z^2}$ is magnitude of $\vec{OP}$

③ Any vector having initial point (x_1, y_1, z_1) and terminal point (x_2, y_2, z_2).

$\vec{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)k$.

$|\vec{PQ}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Q. If O is the origin and P is (2,3), find $\vec{OP}$ in the XY plane and its magnitude of the vector.

$\vec{OP} = x\hat{i} + y\hat{j}$

$= 2\hat{i} + 3\hat{j}$

$|\vec{OP}| = \sqrt{2^2+3^2} = \sqrt{4+9} = \sqrt{13}$//.

Q. Find the vector XY plane whose initial point (1, 2) and terminal point (3, 4).

Let P be the point (x_1, y_1) (1, 2) and Q be the point (x_2, y_2) (3, 4)

$\therefore \vec{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$

$= (3-1)\hat{i} + (4-2)\hat{j}$

$= 2\hat{i} + 2\hat{j}$

Q. If O is the origin and A is a point (1, -2, 3) in XYZ plane. Find $\vec{OA}$ and its magnitude

A vector having initial point (0,0,0) and terminal point (1, -2, 3).

$\vec{OA} = x\hat{i} + y\hat{j} + z\hat{k}$

$= \hat{i} - 2\hat{j} + 3\hat{k}$

$|\vec{OA}| = \sqrt{1^2 + (-2)^2 + (3)^2}$

$= \sqrt{1+4+9} = \sqrt{14}$

Q. Find the vector having initial point (1, 2, 3) and terminal point (4, 5, 6) and also find its magnitude

Let P be the point (1, 2, 3) and Q be the point (4, 5, 6).

$\vec{PQ} = (4-1)\hat{i} + (5-2)\hat{j} + (6-3)\hat{k}$

$= 3\hat{i} + 3\hat{j} + 3\hat{k}$

$|\vec{PQ}| = \sqrt{3^2 + 3^2 + 3^2} = \sqrt{9+9+9} = \sqrt{27}$

Note (1) The unit vector in the direction of a is given by $\frac{\bar{a}}{|\bar{a}|}$

(2) If OA is the position vector of the point A and OB is the position vector of the point B.

$\overrightarrow{AB}$ = Position vector of B − Position vector of A

= OB − OA.

(3) If two vectors $\bar{a}$ and $\bar{b}$ are parallel (collinear) then $\bar{a} = k\bar{b}$ or $b = k\bar{a}$ where k is a constant.

Q. Find the magnitude of the vector $6i+8j$

$= \sqrt{6^2+8^2} = \sqrt{36+64}$

$= \sqrt{100} = 10$

Q. Find the magnitude of the vector $2i-3j+k$

$= \sqrt{2^2+(-3)^2+1^2} = \sqrt{4+9+1} = \sqrt{14}$

Q. Find the unit vector in the direction of the vector $2i+3j$

The unit vector in the direction of $\bar{a}$ is $\bar{a}/|\bar{a}|$

The unit vector in the direction of $2i+3j$ is

$$\frac{2i+3j}{\sqrt{2^2+3^2}} = \frac{2i+3j}{\sqrt{4+9}} = \frac{2i+3j}{\sqrt{13}}$$

Q. Add the vectors $2i - 3j$ and $5i + 4j$

$2i - 3j + 5i + 4j$
$= (2+5)i + (4-3)j$
$= 7i + j$

Q. Add the vectors $4i - 3j + k$ and $8i - j + 2k$.

$(4+8)i - (3+1)j + (1+2)k$
$= 12i - 4j + 3k$.

Q. Subtract the vector $7i - j$ from $9i + 2j$

$9i + 2j - (7i - j)$
$= 9i + 2j - 7i + j$
$= (9-7)i + (2+1)j$
$= 2i + 3j$

Q. Subtract the vector $i + 2j - k$ from $3i - 2j + k$.

$3i - 2j + k + (i + 2j - k)$
$= 3i - 2j + k - i - 2j + k$
$= (3-1)i - 4j + 2k$
$= 2i - 4j + 2k$

Q. If $\bar{a} = 2i + 3j - k$ find $5\bar{a}$

$5\bar{a} = 5(2i + 3j - k)$
$= 10i + 15j - 5k$.

Q. If $a = 2i + 3j + 4k$, $b = -i + 3j + 2k$. Find the unit vector in the direction of the vector $3a + 4b$.

$$3a = 3(2i + 3j + 4k)$$
$$= 6i + 9j + 12k.$$
$$4b = 4(-i + 3j + 2k)$$
$$= -4i + 12j + 8k.$$
$$3a + 4b = 6i + 9j + 12k + -4i + 12j + 8k$$
$$= (6-4)i + 21j + 20k.$$
$$= 2i + 21j + 20k.$$

The unit vector in the direction of $2i + 21j + 20k$ is

$$\frac{2i + 21j + 20k}{\sqrt{2^2 + (21)^2 + (20)^2}}$$

$$= \frac{2i + 21j + 20k}{\sqrt{4 + 441 + 400}}$$

$$= \frac{2i + 21j + 20k}{\sqrt{845}}$$

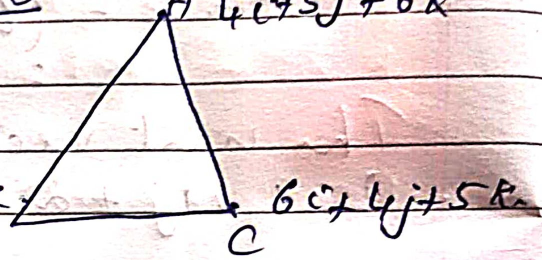
Q. If the position vectors of P, Q, R, S are respectively, $2i + 4k$, $5i + 3\sqrt{3}j + 4k$, $-2\sqrt{3}j + k$ and $2i + k$. Find $\vec{PQ}$ and $\vec{RS}$.

$\vec{PQ}$ = Position vector of Q − Position vector of P.

$$= 5i + 3\sqrt{3}j + 4k - (2i + 4k)$$
$$= 5i + 3\sqrt{3}j + 4k - 2i - 4k$$
$$= 3i + 3\sqrt{3}j$$

$\vec{RS}$ = Position vector of S − Position vector of R
$= 2i + k - (-2\sqrt{3}j + k)$
$= 2i + k + 2\sqrt{3}j - k$
$= 2i + 2\sqrt{3}j$

Q- The position vector of vertices of a triangle are $4i+5j+6k$, $5i+6j+4k$ and $6i+4j+5k$. Find the sides of the triangle and prove that the triangle is equilateral



$\vec{AB}$ = position vector of B − Position vector of A
$= 5i+6j+4k - (4i+5j+6k)$
$= i + j - 2k$
$|AB| = \sqrt{1^2+1^2+(-2)^2} = \sqrt{1+1+4} = \sqrt{6}$

$BC = (6i+4j+5k) - (5i+6j+4k)$
$= i - 2j + k$
$|BC| = \sqrt{1^2+(-2)^2+1^2} = \sqrt{1+4+1} = \sqrt{6}$

$CA = 4i+5j+6k - (6i+4j+5k)$
$= -2i + j + k$
$|CA| = \sqrt{(-2)^2+1^2+1^2} = \sqrt{4+1+1} = \sqrt{6}$

We get three sides are equal.
The triangle is equilateral.