Problems

1. Find the values of x, y, z so that the vectors $\vec{a} = x\hat{i} + 4\hat{j} + z\hat{k}$ and $\vec{b} = 2\hat{i} + y\hat{j} + \hat{k}$ are equal

The vectors are equal

$$\bar{a} = \bar{b}$$

$$x\hat{i} + 4\hat{j} + z\hat{k} = 2\hat{i} + y\hat{j} + \hat{k}$$

$$x = 2, \quad y = 4, \quad z = 1$$

2. Find the sum of the vectors $\bar{a} = \hat{i} - 2\hat{j}$, $\bar{b} = 2i - 3j$, $\bar{c} = 2i + 3k$.

$$\begin{aligned} \bar{a} + \bar{b} + \bar{c} &= i - 2j + 2i - 3j + 2i + 3k \\ &= (1 + 2 + 2)i + (-2 - 3)j + 3k. \\ &= 5i - 5j + 3k \end{aligned}$$

3. Find the unit vector in the direction of $\bar{a} = 2i + 3j + k$

The unit vector in the direction of $\bar{a}$ is $= \dfrac{\bar{a}}{|\bar{a}|}$

$\therefore$ The unit vector in the direction of $2i + 3j + k$ is

$$\frac{2i + 3j + k}{\sqrt{2^2 + 3^2 + 1^2}} = \frac{2i + 3j + k}{\sqrt{4 + 9 + 1}}$$

$$= \frac{2i + 3j + k}{\sqrt{14}}$$

Q. Find the length of the vector $3i - j + 2k$

$\bar{a} = 3i - j + 2k$.

The length of the vector $\bar{a}$ is $|\bar{a}|$

$$|\bar{a}| = \sqrt{3^2 + (-1)^2 + 2^2}$$

$$= \sqrt{9 + 1 + 4} = \sqrt{14}$$

Q. If P represent a point (2,3) and Q represents a point (5,2), express the vector PQ in terms of the unit vector $\hat{i}$ and $\hat{j}$, also find its length

$$\overline{PQ} = (5-2)i + (2-3)j$$

$$= 3i + -j = 3i - j$$

$$|\overline{PQ}| = \sqrt{3^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

Q. Find the vector whose initial point is the origin and terminal point is (2,1,3)

P be the origin (0,0,0) and Q be the point (2,1,3).

$$\overline{PQ} = 2i + j + 3k$$

Q. If the position vector of A is $2i - j - k$ and position vector of B is $-i - 3j + 2k$. Find $\overline{AB}$

$\overline{AB}$ = position vector of B − position vector of A

$= -i - 3j + 2k - (2i - j - k)$

$= -i - 3j + 2k - 2i + j + k$

$= (-1 - 2)i + (-3 + 1)j + (2 + 1)k$

$= -3i - 2j + 3k$

Q. If A is the point (1,2,3) and B is the point (0,4,1). Find the unit vector along $\overline{BA}$

A is the point (x_1, y_1, z_1) is $(1,2,3)$ and B is the point (x_2, y_2, z_2) is $(0,4,1)$

$\therefore \overline{BA} = (x_1 - x_2)i + (y_1 - y_2)j + (z_1 - z_2)k$

$= (1 - 0)i + (2 - 4)j + (3 - 1)k$

$= i - 2j + 2k$

Product of two vectors

The scalar or dot product of two vectors a and b is defined as the scalar $ab\cos\theta$, where θ is the angle between the vectors $\bar{a}$ and $\bar{b}$ and ab is the product of lengths of vectors $\bar{a}$ and $\bar{b}$

Thus $\bar{a}\cdot\bar{b} = ab\cos\theta$.

$$\cos\theta = \frac{\bar{a}\cdot\bar{b}}{ab}$$

$ab = \sqrt{a_1^2+a_2^2+a_3^2} \times \sqrt{b_1^2+b_2^2+b_3^2}$

$$\theta = \cos^{-1}\left(\frac{\bar{a}\cdot\bar{b}}{ab}\right)$$

$i\cdot i = 1$, $i\cdot j = 0$.

Note

Scalar product of two vectors is equal to the sum of the products of their corresponding components.

Consider two vectors $\bar{a} = a_1 i + a_2 j + a_3 k$ and $\bar{b} = b_1 i + b_2 j + b_3 k$.

$$\bar{a}\cdot\bar{b} = (a_1 i + a_2 j + a_3 k)\cdot(b_1 i + b_2 j + b_3 k)$$
$$= a_1 b_1 + a_2 b_2 + a_3 b_3$$

Q. If $\bar{a} = i + 2j - 3k$, $\bar{b} = 3i - j + 2k$ find $\bar{a}\cdot\bar{b}$

$$\bar{a}\cdot\bar{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$
$$= 1\times 3 + 2\times -1 + -3\times 2$$
$$= 3 - 2 - 6 = 3 - 8 = -5$$

Q. If $\bar{a} = i + j + k$, $\bar{b} = 2i - j + 3k$, find $\bar{a}\cdot\bar{b}$

$$\bar{a}\cdot\bar{b} = a_1b_1 + a_2b_2 + a_3b_3$$
$$= 1\times 2 + 1\times -1 + 1\times 3$$
$$= 2 - 1 + 3 = 5 - 1 = 4 //$$

Q. Find the angle between the vectors $i - 2j + 3k$ and $3i - 2j + k$.

$\bar{a} = i - 2j + 3k$, $\bar{b} = 3i - 2j + k$.

$$\theta = \cos^{-1}\left(\frac{\bar{a}\cdot\bar{b}}{ab}\right) = \cos^{-1}\left(\frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}\cdot\sqrt{b_1^2 + b_2^2 + b_3^2}}\right).$$

$$\bar{a}\cdot\bar{b} = a_1b_1 + a_2b_2 + a_3b_3$$
$$= 1\times 3 + -2\times -2 + 3\times 1$$
$$= 3 + 4 + 3 = 10 //$$

$$ab = \sqrt{1^2 + (-2)^2 + 3^2} \times \sqrt{3^2 + (-2)^2 + 1^2}$$
$$= \sqrt{1+4+9} \times \sqrt{9+4+1}$$
$$= \sqrt{14} \times \sqrt{14}$$
$$= 14 //.$$

$$\theta = \cos^{-1}(10/14)$$
$$= \cos^{-1}(5/7)$$

Q. Find the angle between the vectors $i - j + k$ and $-i + j + 2k$

$$\theta = \cos^{-1}\left(\frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}\cdot\sqrt{b_1^2 + b_2^2 + b_3^2}}\right)$$

$$\theta = \cos^{-1} \frac{(-1-1+2)}{\sqrt{1^2+(1)^2+1^2} \cdot \sqrt{(-1)^2+(1)^2+(1)^2}}$$

$$= \cancel{\cos^{-1}(} \quad = \cos^{-1} \frac{0}{3}$$

$$= \cos^{-1} 0$$

$$\theta = 90^\circ \text{ or } \underline{\underline{\pi/2^c}}$$