

1. A mathematical sequence is an ordered list of numbers. This question involves a sequence called a *hailstone sequence*. If  $n$  is the value of a term in the sequence, then the following rules are used to find the next term, if one exists.

- If  $n$  is 1, the sequence terminates.
- If  $n$  is even, then the next term is  $\frac{n}{2}$ .
- If  $n$  is odd, then the next term is  $3n + 1$ .

For this question, assume that when the rules are applied, the sequence will eventually terminate with the term  $n = 1$ .

The following are examples of hailstone sequences.

Example 1: 5, 16, 8, 4, 2, 1

- The first term is 5, so the second term is  $5 * 3 + 1 = 16$ .
- The second term is 16, so the third term is  $\frac{16}{2} = 8$ .
- The third term is 8, so the fourth term is  $\frac{8}{2} = 4$ .
- The fourth term is 4, so the fifth term is  $\frac{4}{2} = 2$ .
- The fifth term is 2, so the sixth term is  $\frac{2}{2} = 1$ .
- Since the sixth term is 1, the sequence terminates.

Example 2: 8, 4, 2, 1

- The first term is 8, so the second term is  $\frac{8}{2} = 4$ .
- The second term is 4, so the third term is  $\frac{4}{2} = 2$ .
- The third term is 2, so the fourth term is  $\frac{2}{2} = 1$ .
- Since the fourth term is 1, the sequence terminates.

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The `Hailstone` class, shown below, is used to represent a hailstone sequence. You will write three methods in the `Hailstone` class.

```
public class Hailstone
{
    /** Returns the length of a hailstone sequence that starts with n,
     *  as described in part (a).
     *  Precondition: n > 0
     */
    public static int hailstoneLength(int n)
    { /* to be implemented in part (a) */ }

    /** Returns true if the hailstone sequence that starts with n is considered long
     *  and false otherwise, as described in part (b).
     *  Precondition: n > 0
     */
    public static boolean isLongSeq(int n)
    { /* to be implemented in part (b) */ }

    /** Returns the proportion of the first n hailstone sequences that are considered long,
     *  as described in part (c).
     *  Precondition: n > 0
     */
    public static double propLong(int n)
    { /* to be implemented in part (c) */ }

    // There may be instance variables, constructors, and methods not shown.
}
```

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- (a) The length of a hailstone sequence is the number of terms it contains. For example, the hailstone sequence in example 1 (5, 16, 8, 4, 2, 1) has a length of 6 and the hailstone sequence in example 2 (8, 4, 2, 1) has a length of 4.

Write the method `hailstoneLength(int n)`, which returns the length of the hailstone sequence that starts with `n`.

```
/** Returns the length of a hailstone sequence that starts with n, as described in part (a).
 * Precondition: n > 0
 */
public static int hailstoneLength(int n)
```

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and fill in the appropriate circle indicating the question number.  
If there are multiple parts to this question, write the part letter with your response.**

Class information for this question

```
public class Hailstone

public static int hailstoneLength(int n)
public static boolean isLongSeq(int n)
public static double propLong(int n)
```

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- (b) A hailstone sequence is considered long if its length is greater than its starting value. For example, the hailstone sequence in example 1 (5, 16, 8, 4, 2, 1) is considered long because its length (6) is greater than its starting value (5). The hailstone sequence in example 2 (8, 4, 2, 1) is not considered long because its length (4) is less than or equal to its starting value (8).

Write the method `isLongSeq(int n)`, which returns `true` if the hailstone sequence starting with `n` is considered long and returns `false` otherwise. Assume that `hailstoneLength` works as intended, regardless of what you wrote in part (a). You must use `hailstoneLength` appropriately to receive full credit.

```
/** Returns true if the hailstone sequence that starts with n is considered long
 * and false otherwise, as described in part (b).
 * Precondition: n > 0
 */
public static boolean isLongSeq(int n)
```

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- (c) The method `propLong(int n)` returns the proportion of long hailstone sequences with starting values between 1 and `n`, inclusive.

Consider the following table, which provides data about the hailstone sequences with starting values between 1 and 10, inclusive.

Starting Value	Terms in the Sequence	Length of the Sequence	Long?
1	1	1	No
2	2, 1	2	No
3	3, 10, 5, 16, 8, 4, 2, 1	8	Yes
4	4, 2, 1	3	No
5	5, 16, 8, 4, 2, 1	6	Yes
6	6, 3, 10, 5, 16, 8, 4, 2, 1	9	Yes
7	7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1	17	Yes
8	8, 4, 2, 1	4	No
9	9, 28, 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1	20	Yes
10	10, 5, 16, 8, 4, 2, 1	7	No

The method call `Hailstone.propLong(10)` returns `0.5`, since 5 of the 10 hailstone sequences shown in the table are considered long.

Write the `propLong` method. Assume that `hailstoneLength` and `isLongSeq` work as intended, regardless of what you wrote in parts (a) and (b). You must use `isLongSeq` appropriately to receive full credit.

```
/** Returns the proportion of the first n hailstone sequences that are considered long,
 * as described in part (c).
 * Precondition: n > 0
 */
public static double propLong(int n)
```

---

**Begin your response at the top of a new page in the Free Response booklet and fill in the appropriate circle indicating the question number.**  
**If there are multiple parts to this question, write the part letter with your response.**

Class information for this question

```
public class Hailstone

public static int hailstoneLength(int n)
public static boolean isLongSeq(int n)
public static double propLong(int n)
```

**GO ON TO THE NEXT PAGE.**