מעבדה 3 - מבוא לעיבוד ספרתי של אותות ומידע

מגישים: עידו ישראלי (ת"ז - 212432439)

ג'ונתן דרעי (ת"ז - 315856377)

שאלה 1:

א)

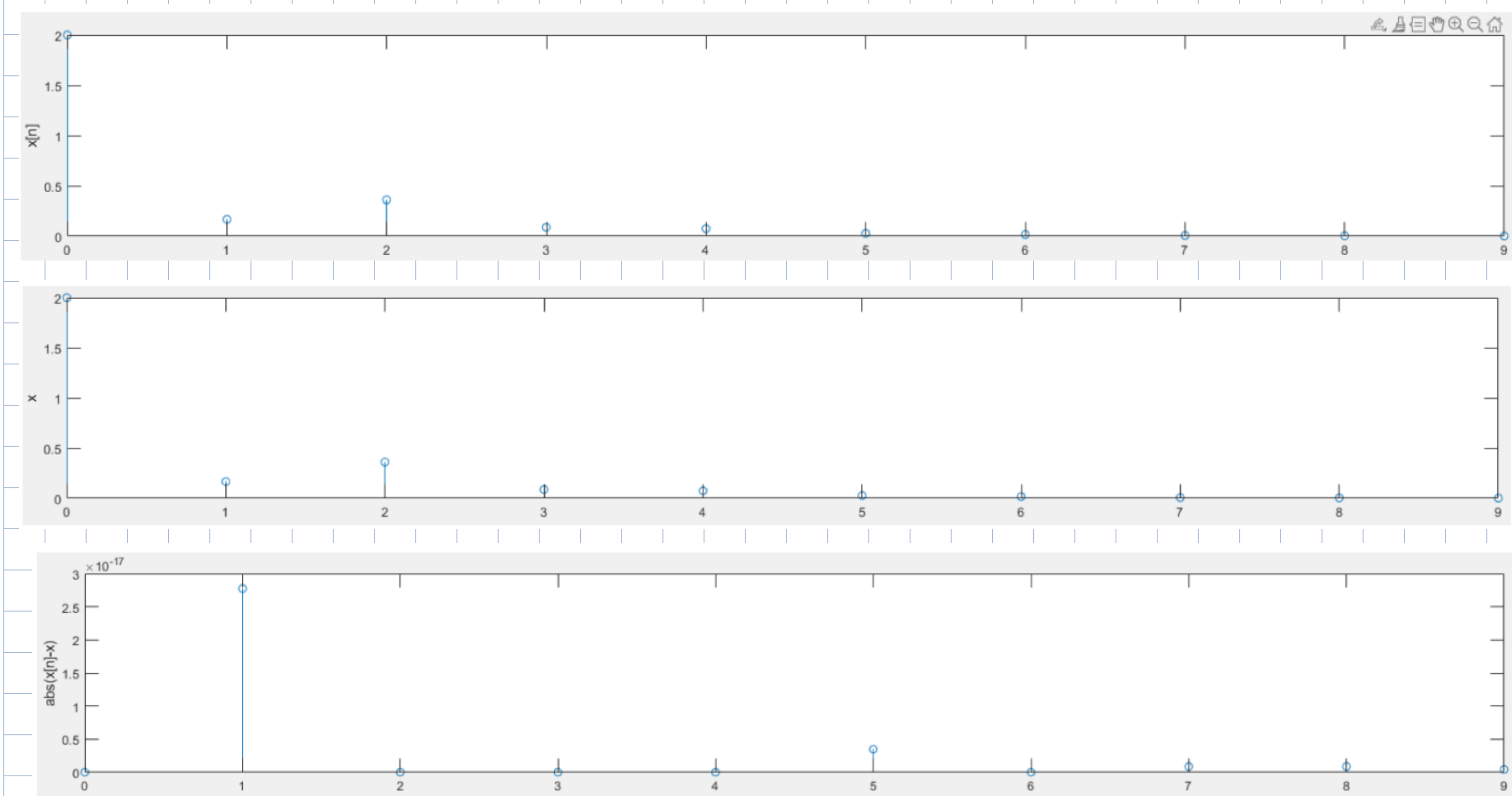
$$x[n]=\left(\frac{1}{2}\right)^n u[n]+\left(-\frac{1}{3}\right)^n u[n]$$

$$X(z)=\frac{1}{1-\frac{1}{2}z^{-1}}+\frac{1}{1+\frac{1}{3}z^{-1}} \qquad |z|>\frac{1}{2}$$

$|z|>\frac{1}{2}$

$$X(z)=\frac{\left(1+\frac{1}{3}z^{-1}\right)+\left(1-\frac{1}{2}z^{-1}\right)}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)}=\frac{2-\frac{1}{6}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)}=\frac{2-\frac{1}{6}z^{-1}}{1-\frac{1}{6}z^{-1}-\frac{1}{6}z^{-2}}$$

ב)



שאלה ②:

א)

$$y[n] = \frac{3}{4}y[n-1] - \frac{1}{8}y[n-2] + x[n]$$

$$x[n] = y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2]$$

$$X(z) = Y(z)\cdot\left[1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} = \frac{1}{z^{-2}\left(z^2 - \frac{3}{4}z + \frac{1}{8}\right)} = \frac{1}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)}$$

נפרק:

$$H(z) = \frac{A_1}{1-\frac{1}{4}z^{-1}} + \frac{A_2}{1-\frac{1}{2}z^{-1}}$$

$$A_1 = \left(1-\frac{1}{4}z^{-1}\right)H(z)\Big|_{z=\frac{1}{4}} = \left.\frac{1-\frac{1}{4}z^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)}\right|_{z=\frac{1}{4}} = \frac{1}{1-\frac{1}{2}\cdot 4} = \boxed{-1}$$

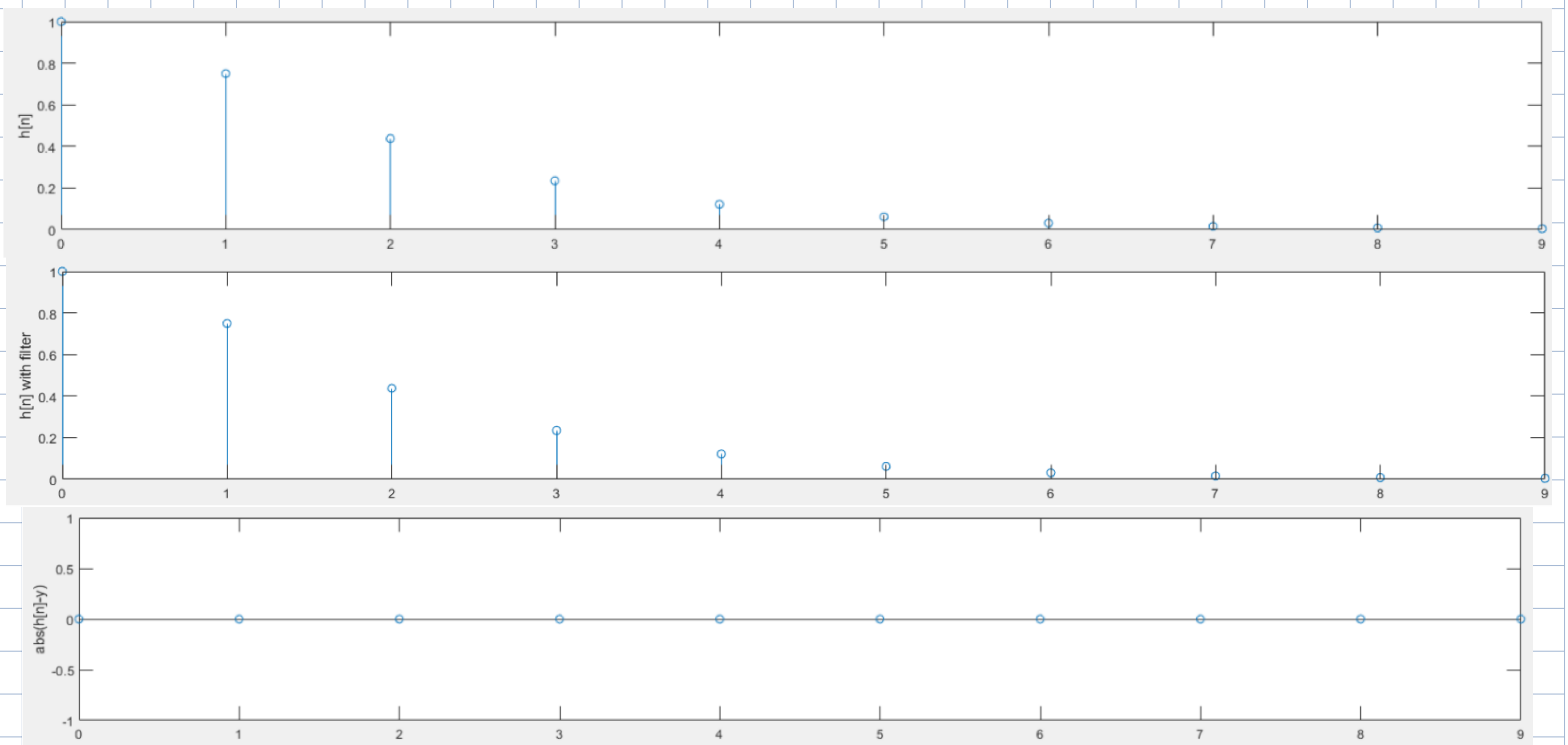
$$A_2 = \left(1-\frac{1}{2}z^{-1}\right)H(z)\Big|_{z=\frac{1}{2}} = \left.\frac{1-\frac{1}{2}z^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)}\right|_{z=\frac{1}{2}} = \frac{1}{1-\frac{1}{4}\cdot 2} = \boxed{2}$$

$$\boxed{H(z) = \frac{-1}{1-\frac{1}{4}z^{-1}} + \frac{2}{1-\frac{1}{2}z^{-1}}, \quad |z| > \frac{1}{2}}$$

לפי טבלת התמרות

$$\boxed{h[n] = 2\cdot\left(\frac{1}{2}\right)^n u[n] - \left(\frac{1}{4}\right)^n u[n]}$$

ב)



שאלה 3:

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$x[n] = u[n]$

$y[n] = 2 \cdot \left(\frac{1}{3}\right)^n \cdot u[n]$

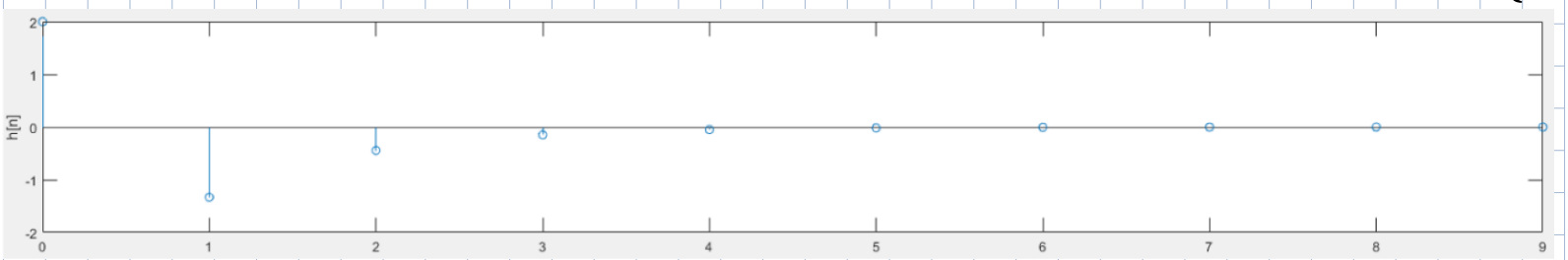
$$X(z) = \frac{1}{1 - z^{-1}} \quad , \quad Y(z) = 2 \cdot \frac{1}{1 - \frac{1}{3} z^{-1}}$$

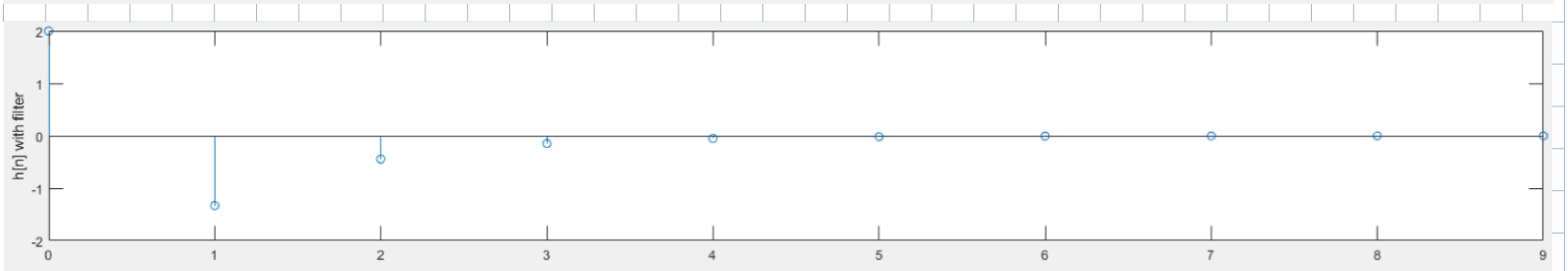
$$H(z) = \frac{Y(z)}{X(z)} = \frac{2 \cdot \frac{1}{1 - \frac{1}{3} z^{-1}}}{\frac{1}{1 - z^{-1}}} = 2 \cdot \frac{1 - z^{-1}}{1 - \frac{1}{3} z^{-1}} = 2 \left(3 + \frac{-2}{1 - \frac{1}{3} z^{-1}}\right) = 6 - \frac{4}{1 - \frac{1}{3} z^{-1}}$$

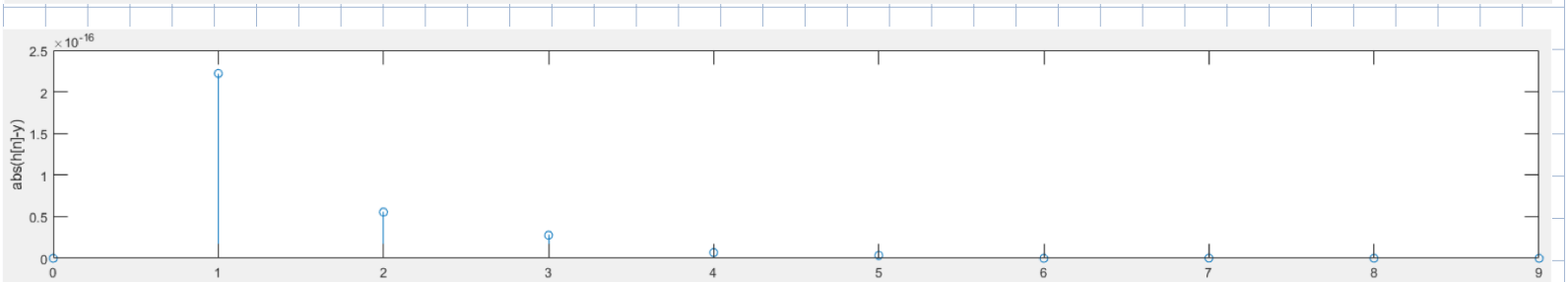
$$\begin{array}{r|l} -z^{-1} + 1 & -\frac{1}{3} z^{-1} + 1 \\ -z^{-1} + 3 & 3 \\ \hline -2 & \end{array}$$

$$\boxed{h[n] = 6\delta[n] - 4 \cdot \left(\frac{1}{3}\right)^n u[n]}$$

(2







שאלה 4:

$$y[n] = \frac{1}{2}y[n-1] + x[n] - \frac{1}{1024}x[n-10]$$

$$y[n] - \frac{1}{2}y[n-1] = x[n] - \frac{1}{1024}x[n-10]$$

(ל

$$Y(z) - \frac{1}{2}\cdot z^{-1}\cdot Y(z) = X(z) - \frac{1}{1024}\cdot z^{-10}X(z)$$

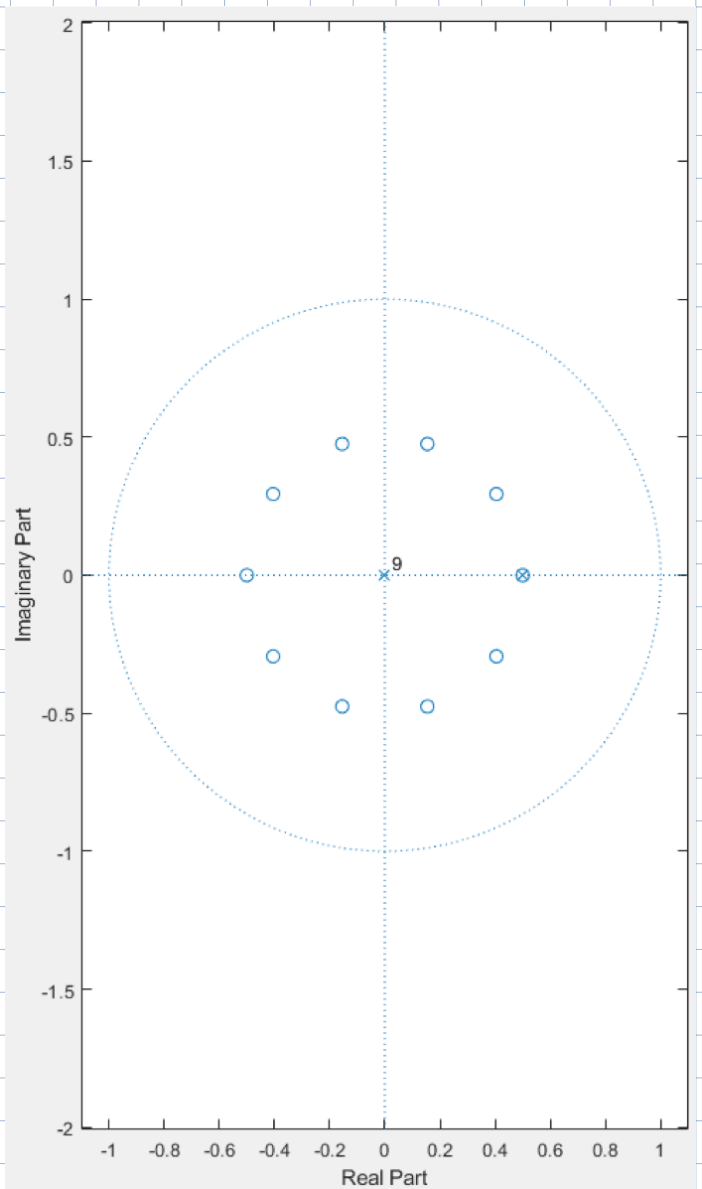
$$Y(z)\left(1 - \frac{1}{2}z^{-1}\right) = X(z)\cdot\left(1 - \frac{1}{2^{10}}z^{-10}\right)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - \frac{1}{2^{10}}z^{-10}}{1 - \frac{1}{2}z^{-1}} = \frac{1}{2^9}z^{-9} + \frac{1}{2^8}z^{-8} + \frac{1}{2^7}z^{-7} + \frac{1}{2^6}z^{-6} + \frac{1}{2^5}z^{-5} + \frac{1}{2^4}z^{-4} + \frac{1}{2^3}z^{-3} + \frac{1}{2^2}z^{-2} + \frac{1}{2}z^{-1} + 1 = \sum_{n=0}^{9}\frac{1}{2^n}z^{-n}$$

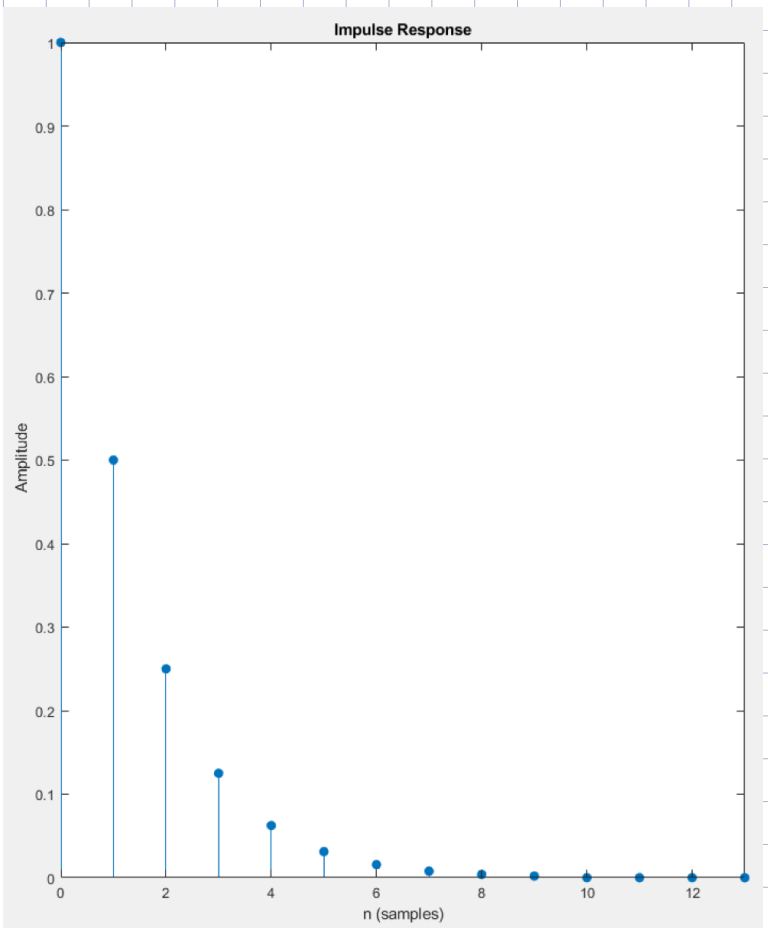
$$\begin{array}{r|l}
-\frac{1}{2^{10}}z^{-10} + 1 & -\frac{1}{2}z^{-1} + 1 \\
\hline
-\left(-\frac{1}{2^{10}}z^{-10} + \frac{1}{2^9}z^{-9}\right) & \\
\hline
-\frac{1}{2^9}z^{-9} + 1 & \\
-\left(-\frac{1}{2^9}z^{-9} + \frac{1}{2^8}z^{-8}\right) & \\
\hline
-\frac{1}{2^8}z^{-8} + 1 & \\
-\left(-\frac{1}{2^8}z^{-8} + \frac{1}{2^7}z^{-7}\right) & \\
\hline
\vdots & \\
\hline
-\frac{1}{2^2}z^{-2} + 1 & \\
-\left(-\frac{1}{2^2}z^{-2} + \frac{1}{2}z^{-1}\right) & \\
\hline
-\frac{1}{2}z^{-1} + 1 & \\
-\left(-\frac{1}{2}z^{-1} + 1\right) & \\
\hline
0 &
\end{array}$$

המשך שאלה 4:

ג)



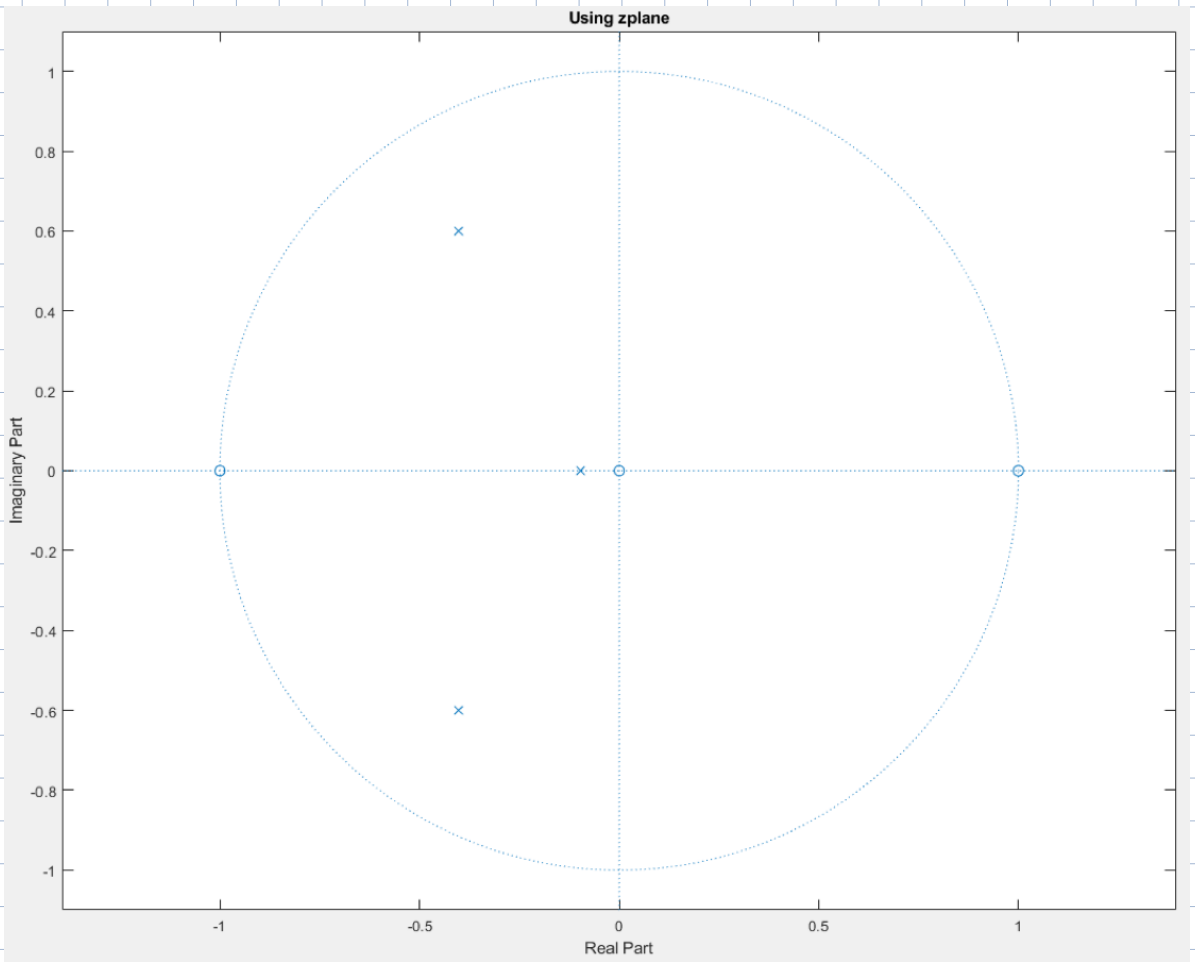
ב)



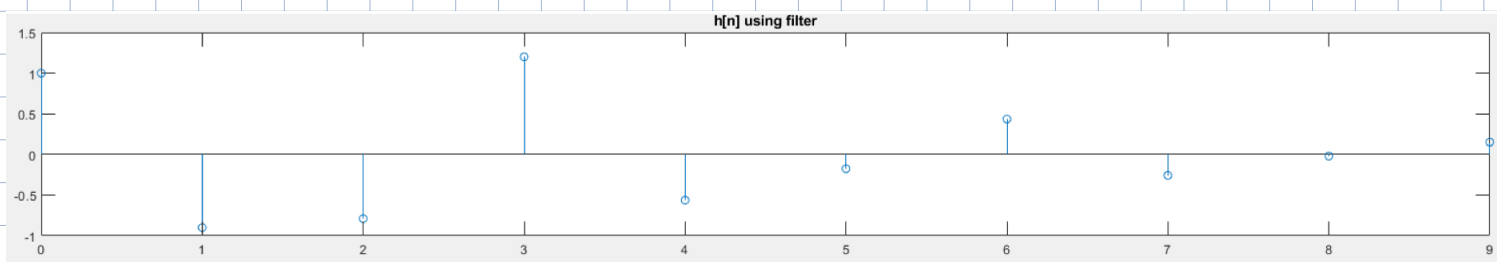
ד) מכיוון שהקוטב שלנו מעל הציר המדומה, ושאר הקטבים נמצאים בתוך מעגל היחידה, המערכת יציבה ולכן אורך תגובת ההלם הוא סופי.

שאלה ⑤:

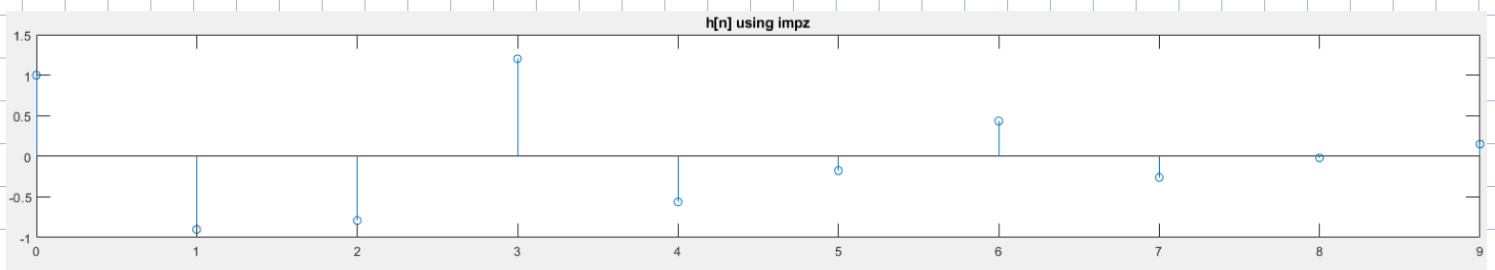
(1c

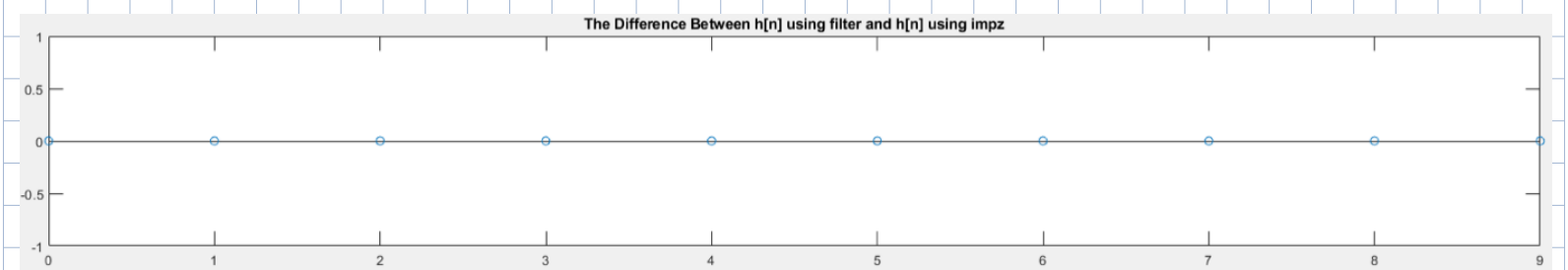

Using zplane
Imaginary Part
Real Part
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.5
0
0.5
1

(2


h[n] using filter
1.5
1
0.5
0
-0.5
-1
0
1
2
3
4
5
6
7
8
9

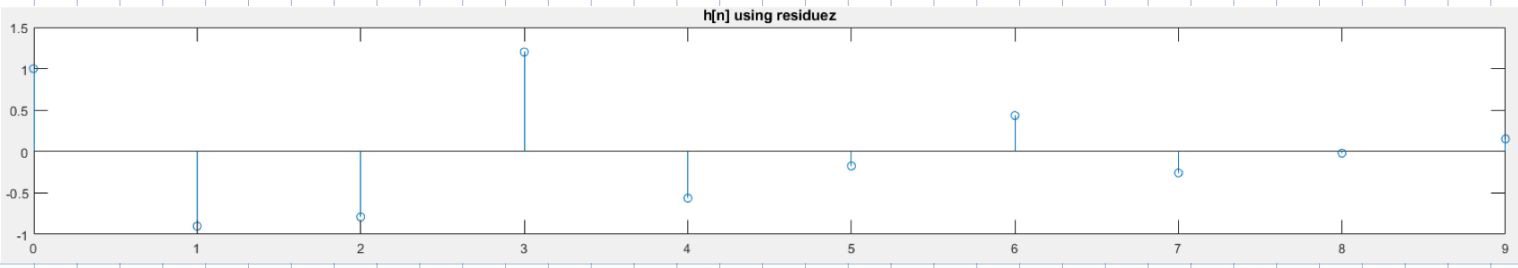
(3


h[n] using impz
1.5
1
0.5
0
-0.5
-1
0
1
2
3
4
5
6
7
8
9

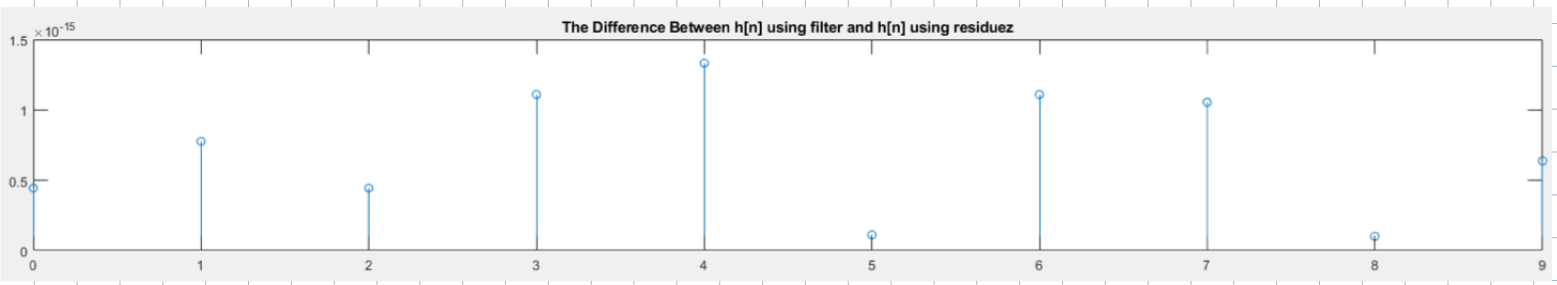

The Difference Between h[n] using filter and h[n] using impz
1
0.5
0
-0.5
-1
0
1
2
3
4
5
6
7
8
9

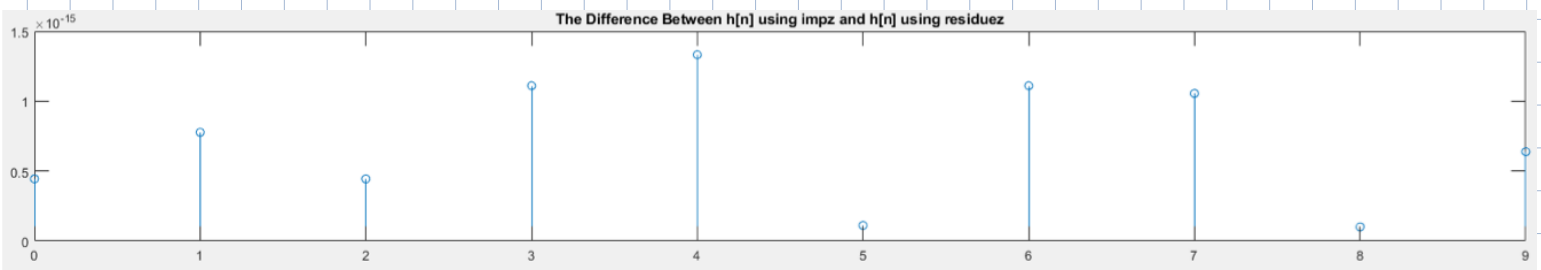
תרגיל שאלה 3:

ב)



ג)





:⑥ λύση

$$y[n] = -\frac{1}{4}y[n-1] + \frac{1}{8}y[n-2] + x[n] + x[n-1]$$

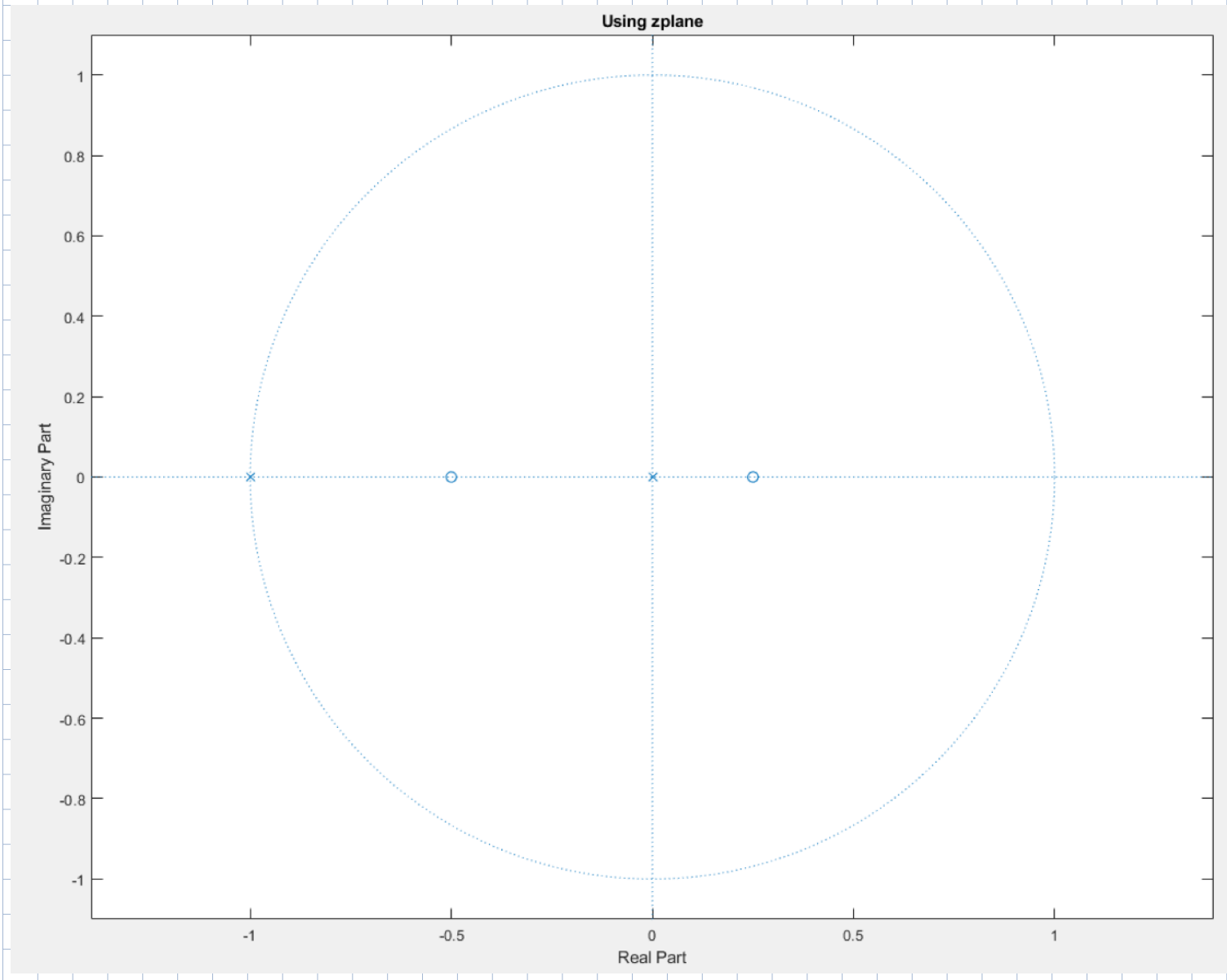
$$y[n] + \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n] + x[n-1]$$

$$Y(z) + \frac{1}{4}z^{-1}\cdot Y(z) - \frac{1}{8}z^{-2}Y(z) = X(z) + X(z)\cdot z^{-1}$$

$$Y(z)\cdot\left(1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}\right) = X(z)\cdot\left(1 + z^{-1}\right)$$

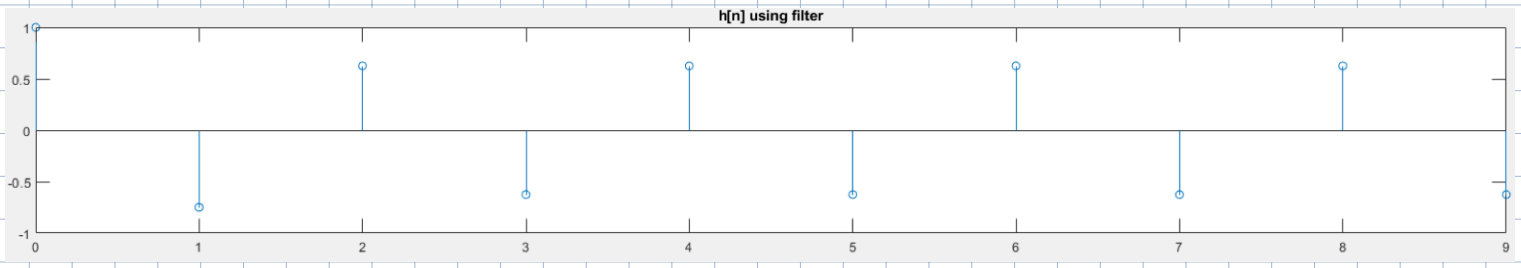
$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}{1 + z^{-1}}$$

(lc

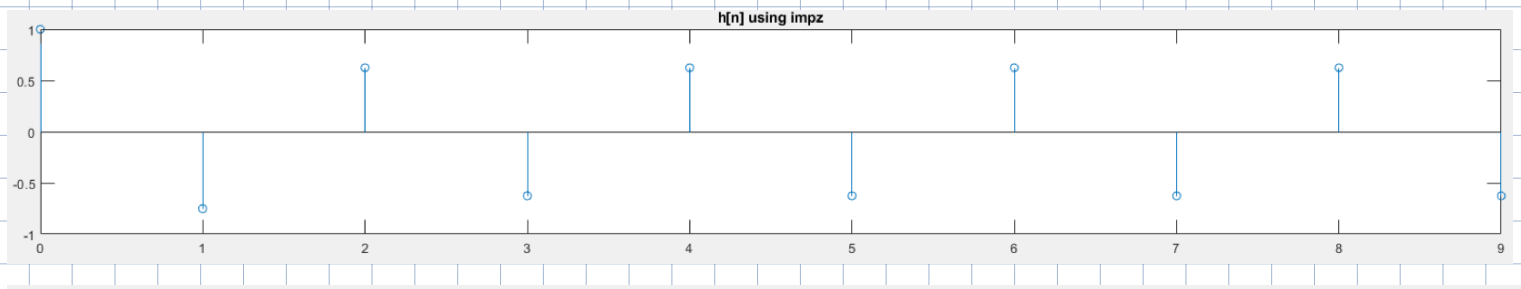


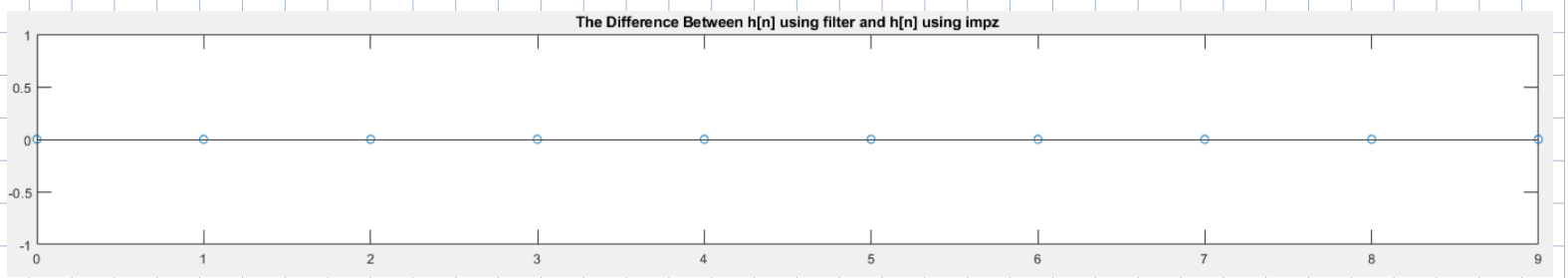
הגשת שאלה ⑨:

ב)

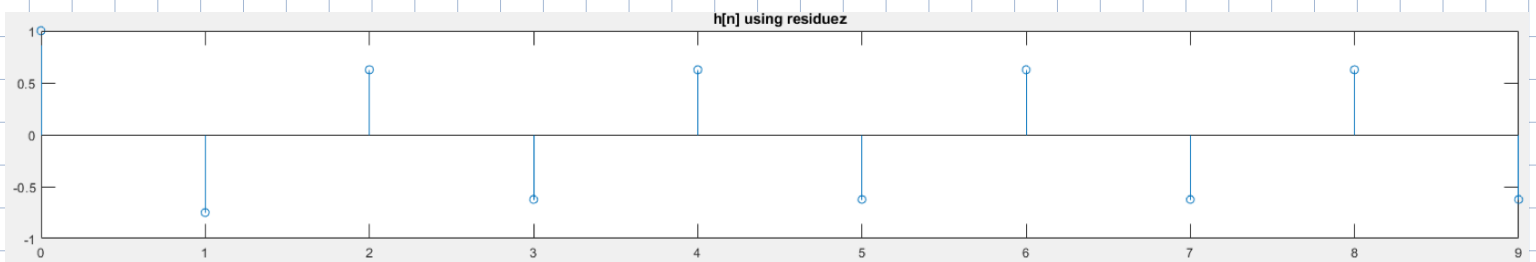


ג)





ד)



ה)