RLC Notes

Part 1: RC & RL Step and Frequency Response

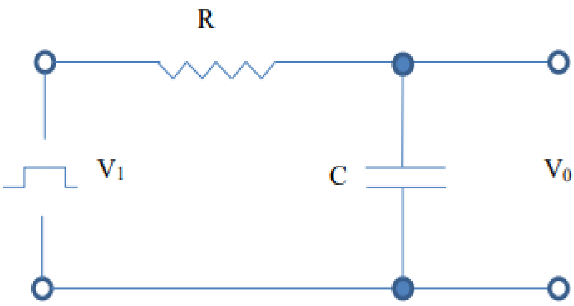
Experiment 1.: RC, DC:

Some electricity equations: On the resistor $V_R = IR$ on the capacitator $V_C = \frac{Q}{C}$ the current follows $I = \dot{Q}$. We know from kirchhoff that $V_R + V_C = V_{in}(t) \rightarrow \frac{dQ(t)}{dt}R + \frac{Q(t)}{C} = V_{in}$ knowing $V_C = \frac{Q}{C}$ we can rewrite the equation in terms of V $RC \cdot \frac{dV(t)}{dt} + V(t) = V_{in}(t)$

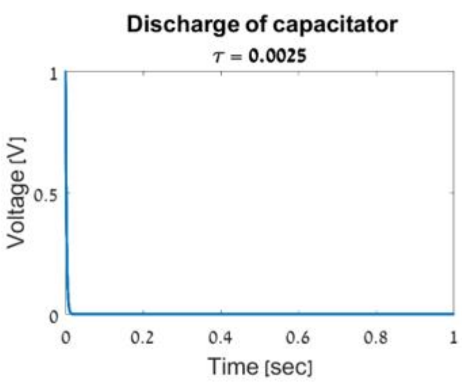
For charging and discharging of capacitator we plug in $V_{in}(0) = 0 \; or \; V_0$ and $V_{in(t)} = 0$ lets us write the equation as: $RC \cdot \frac{dV_c(t)}{dt} + V_c(t) = 0$. Ansatz $V_c = A\, e^{i\omega t} \rightarrow i\omega RC + 1 = 0 \rightarrow \omega = -\frac{1}{iRC} \rightarrow V_c = A * e^{-\frac{t}{RC}}$ where's $A = V_0 \; or \; 0$ depends on the boundary condition (discharge or charge). Substituting $R = 5K\Omega$ and $C = 500nF$ gets us rise time of $\tau = 2.5 * 10^{-2}$.

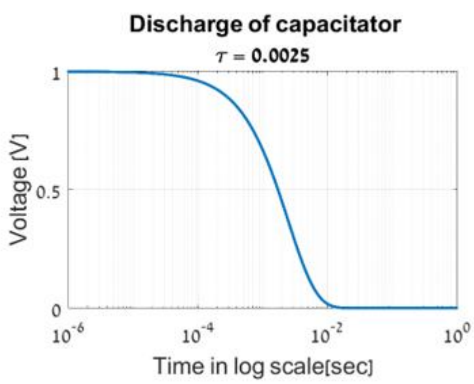
Integrator circuit

On the resistor $I = \frac{V_{in}}{R}$ on the capacitator: $V_c = \frac{Q}{C}$ the relation between the current and the charge is known $I = \frac{dQ}{dt} \rightarrow Q = \int I dt$ using above equation $V_c = \frac{1}{RC}\int V_{in} dt$



Plot VS semilogx





Experiment 2.: RC, AC:

When moving to the frequency domain one can just do the Fourier transform of the time domain or use electric impedances in the following fashion:

Series combination : $Z_{eq} = Z_1 + Z_2 + Z_3 + \cdots + Z_n$

Parallel combination: $\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}$ for n=2 $Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2}$

Where's $Z_C = \frac{1}{i\omega C}$; $Z_R = R$; $Z_L = i\omega L$

Complex numbers:

For $z = a + ib$; $r = \ amp(z) = \sqrt{a^2 + b^2}$ and $\phi = phase(z) = \mathrm{atan}\left(\frac{b}{a}\right)$

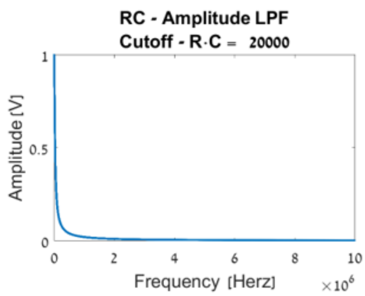
$$Z = re^{i\phi}$$

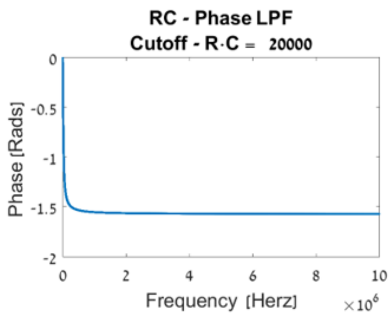
Bode plot are used to visualize data better. We use the transfer function which is $H(\omega) = \frac{V_{out}}{V_{in}}$ The gain (or magnitude) plot is semilogx of $20\log_{10}\big(H(\omega)\big)$ and the phase plot is $angle\big(H(\omega)\big)$

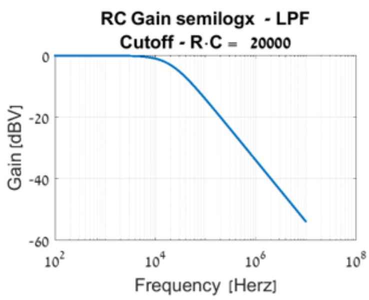
LPF:

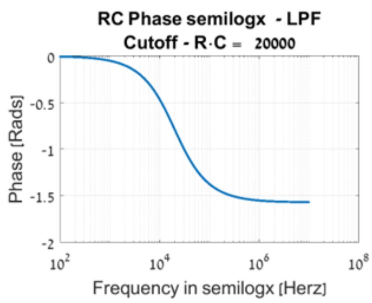
$$\frac{V_{out}}{V_1} = \frac{Z_c}{Z_R + Z_C} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + (\omega RC)^2} + j\frac{\omega RC}{1 + (\omega RC)^2}$$

In out LPF $amp\left(\frac{V_{out}}{V_1}\right) = \sqrt{\frac{1}{1+(\omega RC)^2}}$ and $phase\left(\frac{V_{out}}{V_1}\right) = \mathrm{atan}(\omega RC)$

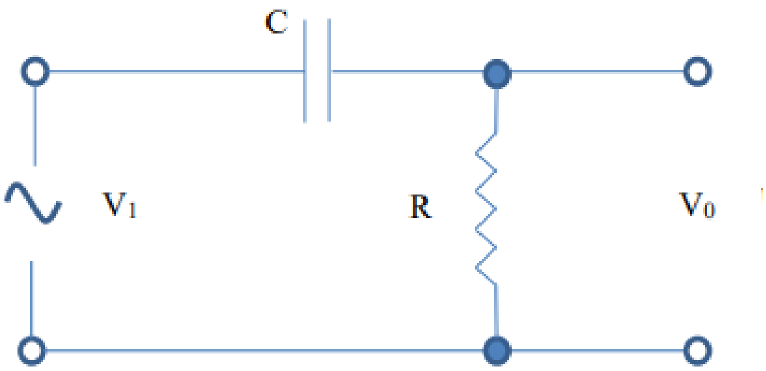








HPF:

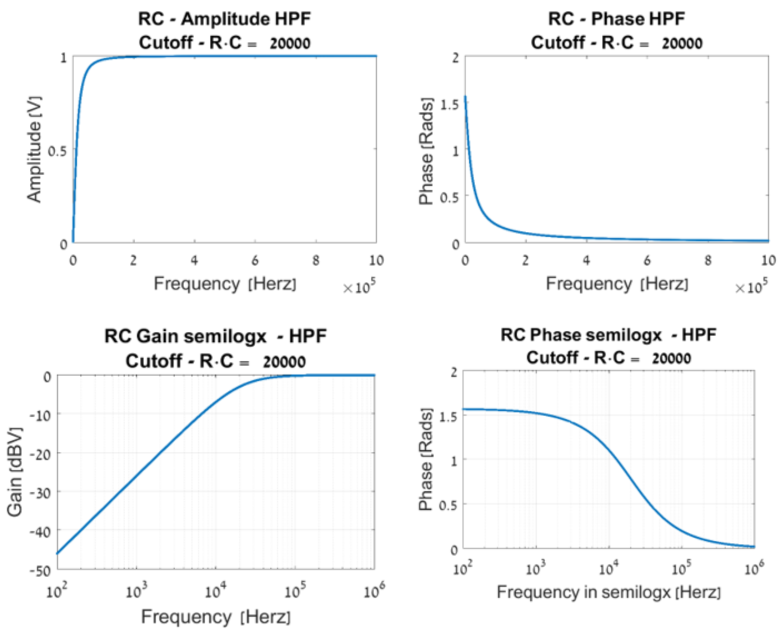


The voltage on $\frac{V_0}{V_1}$ is $\frac{Z_R}{Z_R+Z_C} = \frac{R}{R+\frac{1}{j\omega C}} = \frac{j\omega RC}{1+j\omega RC} = \frac{-(\omega RC)^2}{1+(\omega RC)^2} + \frac{j\omega RC}{1+(\omega RC)^2}$

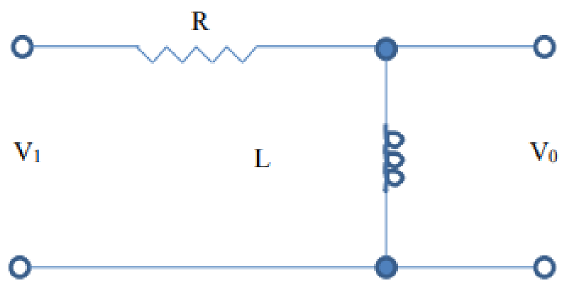
$$r = amplitude\left(\frac{V_{out}}{V_1}\right) = \sqrt{\left(\frac{-(\omega RC)^2}{1+(\omega RC)^2}\right)^2 + \left(\frac{\omega RC}{1+(\omega RC)^2}\right)^2} = \sqrt{\frac{(\omega RC)^2(\omega RC+1)}{(1+(\omega RC)^2)^2}}$$
$$= \sqrt{\frac{(\omega RC)^2}{1+(\omega RC)^2}}$$

$$\phi = angle\left(\frac{V_{out}}{V_1}\right) = \operatorname{atan}\left(\frac{b}{a}\right) = \operatorname{atan}\left(\frac{1}{\omega RC}\right)$$

Computer generated results:



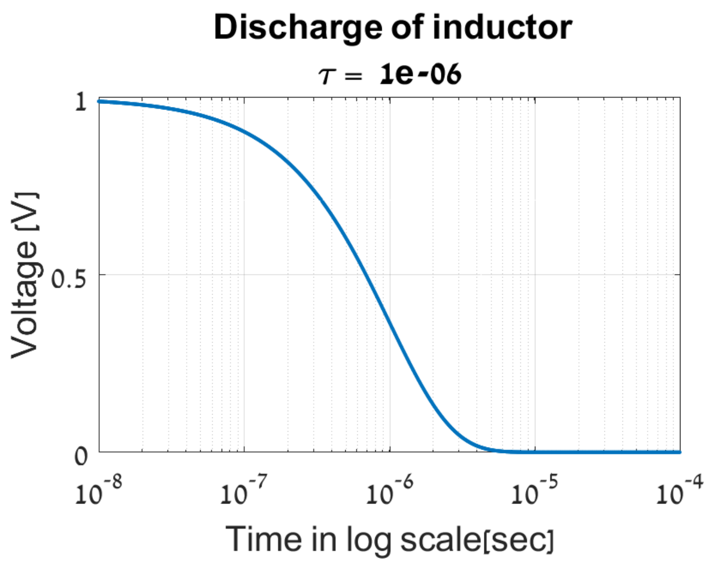
Experiment 3.: RL, DC:



Inductor answers the equation: $V_L = L\frac{dI}{dT}$ for the resistor $V_R = IR$ Kirchhoff gets us $V_1 = RI + \frac{LdI}{dT} \rightarrow I(t)$ given the boundary condition $I(t = 0) = 0$ we can rewrite the equation as: $I(t) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$; $V = IR\ and\ boundary\ conditions \rightarrow$

$$V_L(t) = V_1 e^{-\frac{R}{L}t}$$

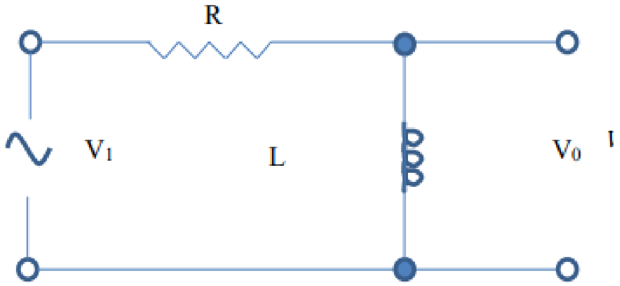
Given $R = 1000\Omega$; $\mathrm{L} = 1mH \rightarrow \tau = \frac{L}{R} = 10^{-6}\ [sec]$



Experiment 4.: RL, AC:

Taking $R = 1000\ [\Omega]$; $\mathrm{L} = 1\ [mH]$; $V_1 = 5\ [V]$ also $Z_L = i\omega L$

HPF

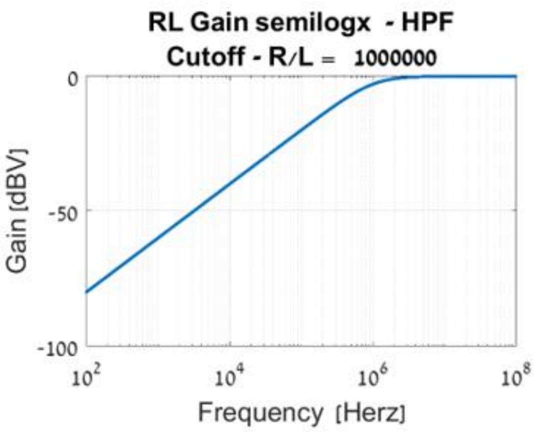


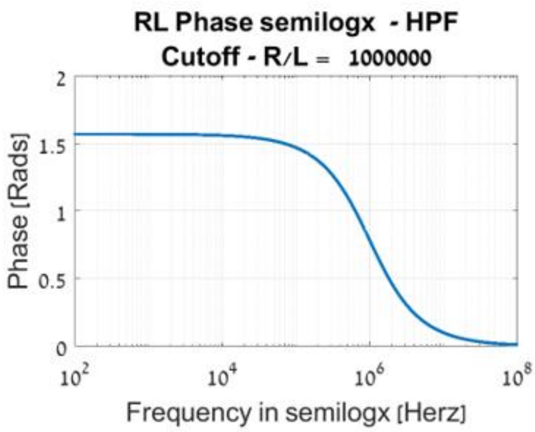
$$\frac{V_{out}}{V_1} = \frac{Z_L}{Z_R + Z_L} = \frac{j\omega L}{R + j\omega L} = \frac{(\omega L)^2}{R^2 + (\omega L)^2} + j\frac{RL\omega}{R^2 + (\omega L)^2}$$

Therefore

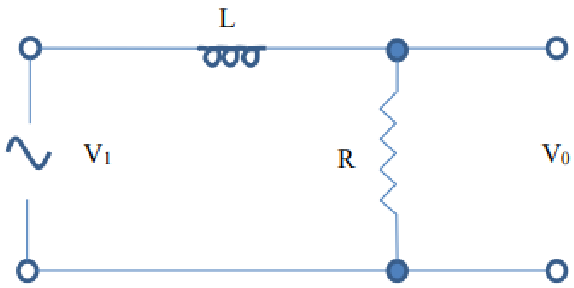
$$Amplitue\left(\frac{V_{out}}{V_1}\right) = \sqrt{\frac{(\omega L)^2(R + (\omega L)^2)}{(R^2 + (\omega L)^2)^2}} = \frac{\omega L}{\sqrt{(R^2 + (\omega L)^2)}} = \frac{\omega \cdot \frac{L}{R}}{\sqrt{1 + \left(\frac{\omega L}{R}\right)^2}}$$

$$Phase\left(\frac{V_{out}}{V_1}\right) = \mathrm{atan}\left(\frac{b}{a}\right) = \mathrm{atan}\left(\frac{R}{L\omega}\right)$$





LPF



$$\frac{V_{out}}{V_1} = \frac{Z_R}{Z_R + Z_L} = \frac{R}{R + j\omega L} = \frac{R^2}{R + (\omega L)^2} - j\frac{RL\omega}{R + (\omega L)^2}$$

Therefore

$$Amplitue\left(\frac{V_{out}}{V_1}\right) = \sqrt{\frac{R^2\,(R + (L\omega)^2)}{(R + (\omega L)^2)^2}} = \frac{R}{\sqrt{R^2 + (\omega L)^2}} = \frac{1}{\sqrt{1 + \left(\frac{\omega L}{R}\right)^2}}$$

$$Phase\left(\frac{V_{out}}{V_1}\right) = \operatorname{atan}\left(\frac{b}{a}\right) = \operatorname{atan}\left(-\frac{L}{R}\cdot\omega\right)$$