

Part 1: Theoretical Questions

1a) False – procedure's g return value is of type T2, which result in F being called with the wrong type.

1b) False – the procedure forces X to be type T2 in contradiction to the variable's statements of the procedure T1.

1c) True – All types are matching.

1d) True – All types are matching.

Question 2:

2a)

((lambda (f x1) (if x1 (f 1 x1) (f 3 x1))) + #t)	T0
(lambda (f x1) (if x1 (f 1 x1) (f 3 x1)))	T1
(if x1 (f 1 x1) (f 3 x1))	T2
x1	Tx
(f 1 x1)	T3
F	Tf
1	Tnum1
(f 3 x1)	T5
3	Tnum3
+	T plus
#t	T true

((lambda (f x1) (if x1 (f 1 x1) (f 3 x1))) + #t)	T1 = [Tplus x Ttrue -> T0]
(lambda (f x1) (if x1 (f 1 x1) (f 3 x1)))	T1 = [Tf x Tx-> T2]
(if x1 (f 1 x1) (f 3 x1))	T2 = [Tx -> T5]
(f 1 x1)	Tf = [Tnum1 x Tx -> T3]
(f 3 x1)	Tf = [Tnum3 x Tx -> T5]

1	Tnum1 = number
3	Tnum3 = number
+	T plus = [Number X number -> Number]
#t	T true = boolean

Equation	Substitution
$T1 = [Tplus \times Ttrue \rightarrow T0]$	$\{ T1 = [Tplus \times Ttrue \rightarrow T0] \}$
$T1 = [Tf \times Tx \rightarrow T2]$	
$T2 = [Tx \rightarrow T5]$	
$Tf = [Tnum1 \times Tx \rightarrow T3]$	
$Tf = [Tnum3 \times Tx \rightarrow T5]$	
$Tnum1 = number$	
$Tnum3 = number$	
$Tplus = [Number \times number \rightarrow Number]$	
$Ttrue = boolean$	

Equation	Substitution
	$\{ T1 = [Tplus \times Ttrue \rightarrow T0], T2 = [Tx \rightarrow T5] \}$
$Tf = [Tnum1 \times Tx \rightarrow T3]$	
$Tf = [Tnum3 \times Tx \rightarrow T5]$	
$Tnum1 = number$	
$Tnum3 = number$	
$Tplus = [Number \times number \rightarrow Number]$	
$Ttrue = boolean$	
$Tf = Tplus$	
$Tx = Ttrue$	
$T2 = T$	

Equation	Substitution
	$\{ T1 = [Tplus \times Ttrue \rightarrow T0], T2 = [Tx \rightarrow T5],$ $Tf = [Tnum1 \times Tx \rightarrow T3],$ $Tf = [Tnum3 \times Tx \rightarrow T5] \}$
$Tnum1 = number$	
$Tnum3 = number$	
$Tplus = [Number \times number \rightarrow Number]$	
$Ttrue = boolean$	
$Tf = [Number \times number \rightarrow Number]$	
$Tx = Ttrue$	
$T2 = T$	
$Tx = Tx$	
$T3 = T5$	

Equation	Substitution
	{ T1 = [T plus x T true -> T0], T2 =[Tx -> T5], Tf = [number x Tx -> T3], Tf = [number x Tx -> T5], }
T plus = [Number X number -> Number]	
T true = boolean	
Tf = [Number X number -> Number]	
Tx = T true	
T2 = T	
Number = number	
Tx = Tx	
T3 = T5	

Equation	Substitution
	{ T1 = [[Number X number -> Number] x T true -> T0], T2 =[Tx -> T5], Tf = [number x Tx -> T3], Tf = [number x Tx -> T5], }
T true = boolean	
Tf = [Number X number -> Number]	
Tx = T true	
T2 = T	
Number = number	
Tx = Tx	
T3 = T5	

Equation	Substitution
	{ T1 = [[Number X number -> Number] x Boolean -> T0], T2 =[Tx -> T5], [Number X number -> Number] = [number x Boolean -> T3], [Number X number -> Number] = [number x Boolean -> T5], }
Tx = T true	
T2 = T	
Number = number	
Tx = Tx	
T3 = T5	

We got a contradiction with wrong primitive types.

2B)

$((\lambda (f\ x\ y)\ (f\ x\ y))\ *\ 1\ 3) \rightarrow ((\lambda (f\ x\ y)\ (f\ x\ y))\ *\ 1\ 3)$

$((\lambda (f\ x\ y)\ (f\ x\ y))\ *\ 1\ 3)$	T0
$(\lambda (f\ x\ y)\ (f\ x\ y))$	T1
$(f\ x\ y)$	T2
F	T f
X	Tx
Y	Ty
*	Tmul
1	T num1
3	T num3

$((\lambda (f\ x\ y)\ (f\ x\ y))\ *\ 1\ 3)$	$T1 = [Tmul\ x\ T\ num1\ x\ T\ num3 \rightarrow T0]$
$(\lambda (f\ x\ y)\ (f\ x\ y))$	$T1 = [Tf\ x\ Tx\ x\ Ty \rightarrow T2]$
$(f\ x\ y)$	$Tf = [Tx\ X\ Ty \rightarrow T2]$

1	$Tnum1 = number$
3	$Tnum3 = number$
Tmul	$T\ mul = [Number\ X\ number \rightarrow Number]$

Equation	Substitution
$T1 = [Tmul\ x\ T\ num1\ x\ T\ num3 \rightarrow T0]$	{}
$T1 = [Tf\ x\ Tx\ x\ Ty \rightarrow T2]$	
$Tf = [Tx\ X\ Ty \rightarrow T2]$	
$Tnum1 = number$	
$Tnum3 = number$	
$T\ mul = [Number\ X\ number \rightarrow Number]$	

Equation	Substitution
	$\{ T1 = [Tmul\ x\ T\ num1\ x\ T\ num3 \rightarrow T0]$ $, T1 = [Tf\ x\ Tx\ x\ Ty \rightarrow T2]$
$Tf = [Tx\ X\ Ty \rightarrow T2]$	
$Tnum1 = number$	
$Tnum3 = number$	
$T\ mul = [Number\ X\ number \rightarrow Number]$	
$Tf = Tmul$	
$Tx = T\ num1$	
$Ty = T\ num\ 3$	

Equation	Substitution
	{ T1 = [Tmul x T num1 x T num3 -> T0], Tf =[Tx X Ty -> T2] }
Tnum1 = number	
Tnum3 = number	
T mul = [Number X number -> Number]	
Tf = Tmul	
Tx = T num1	
Ty = T num 3	

Equation	Substitution
	{ T1 = [Tmul x number x T num3 -> T0], Tf =[Tx X Ty -> T2], T1 = [Tf x Tx x Ty -> T2] Tnum1 = number }
Tnum3 = number	
T mul = [Number X number -> Number]	
Tf = Tmul	
Tx = T num1	
Ty = T num 3	

Equation	Substitution
	{ T1 = [Tmul x number x number -> T0], Tf =[Tx X Ty -> T2], T1 = [Tf x Tx x Ty -> T2] Tnum1 = number, Tnum3 = number }
T mul = [Number X number -> Number]	
Tf = Tmul	
Tx = T num1	
Ty = T num 3	

Equation	Substitution
	{ T1 = [[Number X Number -> Number] x number x number -> T0], Tf =[Tx X Ty -> T2], T1 = [Tf x Tx x Ty -> T2] Tnum1 = number, Tnum3 = number Tmul = [Number X Number -> Number] }
Tf = Tmul	
Tx = T num1	
Ty = T num 3	

Equation	Substitution
	{ T1 = [[Number X Number -> Number] x number x number -> T0], Tf = [Tx X Ty -> T2], T1 = [Tf x Tx x Ty -> T2] Tnum1 = number, Tnum3 = number Tmul = [Number X Number -> Number], }
Tx = T num1	
Ty = T num 3	
Tx = number	
Ty = number	
T2 = number	

Equation	Substitution
	{ T1 = [[Number X Number -> Number] x number x number -> T0], Tf = [T num1 X Ty -> T2], T1 = [Tf x Tx x Ty -> T2] Tnum1 = number, Tnum3 = number Tmul = [Number X Number -> Number], Tx = T num1 }
Ty = T num 3	
Tx = number	
Ty = number	
T2 = number	

Equation	Substitution
	{ T1 = [[Number X Number -> Number] x number x number -> T0], Tf = [number X number -> T2], T1 = [Tf x Tx x Ty -> T2] Tnum1 = number, Tnum3 = number Tmul = [Number X Number -> Number], Tx = T num1, Ty = T num 3 }
T2 = number	

Equation	Substitution
	{ T1 = [(Number X Number -> Number) x number x number -> T0], Tf = [number X number -> number], T1 = [Tf x Tx x Ty -> T2] Tnum1 = number, Tnum3 = number Tmul = [Number X Number -> Number], Tx = T num1, Ty = T num 3, T2 = number }
T2 = number	

Equation	Substitution
	{ T1 = [(Number X Number -> Number) x number x number -> number], Tf = [number X number -> number], T1 = [Tf x Tx x Ty -> T2] Tnum1 = number, Tnum3 = number Tmul = [Number X Number -> Number], Tx = T num1, Ty = T num 3, T2 = T0 T2 = number }

((lambda (f1 x1 y1) (f1 x1 y1)) * 1 3) -> ((lambda (f x y) (f x y)) * 1 3) ->

((lambda ([f : (Number X number -> Number)]) [x : Number] [Y: number])) : Number