Two dimensional Poisson equation
Computational project No. 2

Ido Schaefer

February 12, 2013

1 Iterative smoothing

The iterative smoothing procedure is implemented in the function `smooth.m`.

The efficiency of the relaxation for several ω values is tested in the procedure `test_smooth`. The root mean square of the residual:

$$r_{rms} = \sqrt{\frac{\sum_{i,j} r_{i,j}^2}{(N-1)^2}} = \frac{|\mathbf{r}|}{(N-1)} \tag{1}$$

is plotted Vs. the number of iterations ($N-1$ grid points in each dimension participate in the computational process). The procedure was employed for two N values: $N = 8$ and $N = 16$. The results are shown in Figs. 1, 2. It is shown that the relaxation is faster for the values $\omega = 0.5$, $\omega = 1$. The relaxation is faster for $N = 8$, as expected.

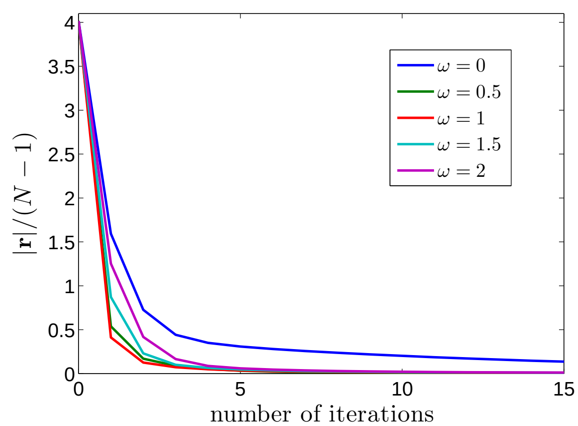

Figure 1: r_{rms} Vs. the number of iterations for $N = 8$, for several ω values.

The smoothing of the diagonal error function, $\epsilon(x, x) = \bar{\phi}(x, x) - \phi(x, x)$, during the iterative process is demonstrated by the procedure `smth_diag`. The error function is plotted Vs. x for various iteration numbers in Fig. 3. It is evident from the figure that the shape of $\epsilon(x, x)$ is getting smoother with the propagation of the iterative process.

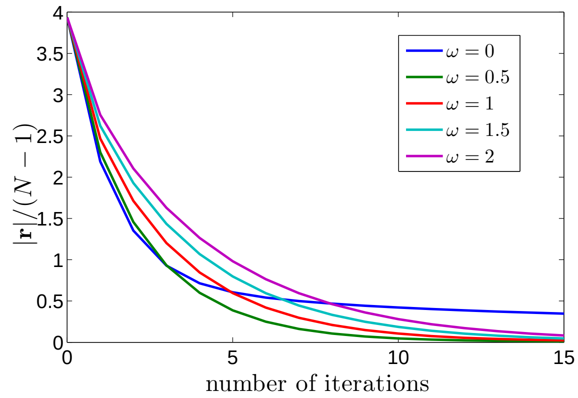

Figure 2: r_{rms} Vs. the number of iterations for $N = 16$, for several ω values.

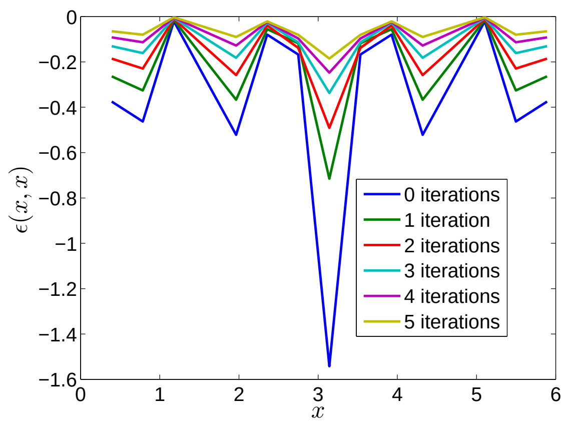

Figure 3: $\epsilon(x, x)$ Vs. x after various iteration numbers, from 0 to 5.

2 The multigrid method

The multigrid method is implemented in the procedure `test_multigrid.m`. The function `multigrid.m` performs a single V-cycle in a recursive process. In `test_multigrid.m`, the multigrid method is tested for $N = 16$, $\omega = 0.5$, and for 10 smoothing iterations in each call to `smooth.m`.

The error decay is presented Figs. 4, 5. We use two measures for the error — the residual, and the relative deviation from the numeric solution (as solved by `MATLAB`), defined as:

$$\epsilon_{rel} = \frac{|\phi - \bar{\phi}|}{|\phi|} \tag{2}$$

In Fig. 4, $\log_{10}(r_{rms})$ is plotted Vs. the number of V-cycles. In Fig. 5, $\log_{10}(\epsilon_{rel})$ is plotted Vs. the number of V-cycles. The error decay rate shows exponential behaviour.

In the procedure `multg_omega.m` the efficiency of the multigrid method is tested for several ω values. We use the same error measures as before. In Fig. 6, $\log_{10}(r_{rms})$ is

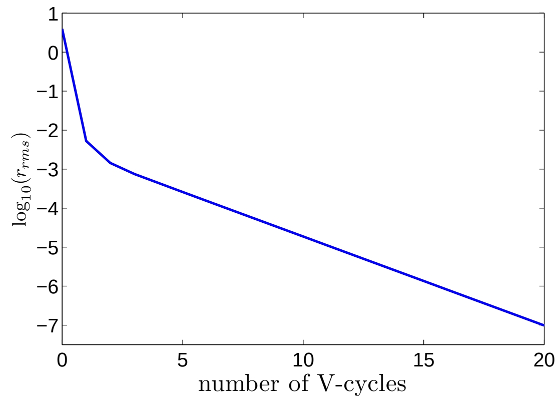

Figure 4: $\log_{10}(r_{rms})$ Vs. the number of V-cycles for $\omega = 0.5$.

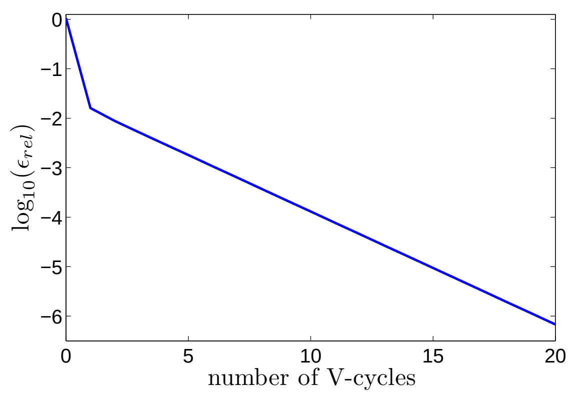

Figure 5: $\log_{10}(\epsilon_{rel})$ Vs. the number of V-cycles for $\omega = 0.5$.

plotted Vs. ω for the sampling points:

$$\omega = 0, 0.5, 1, 1.5, 2$$

In Fig. 7, $\log_{10}(\epsilon_{rel})$ is plotted Vs. ω in the same sampling points. It is shown that in the range of ω that has a relatively small reduction factor for high frequencies, the process is much more efficient.

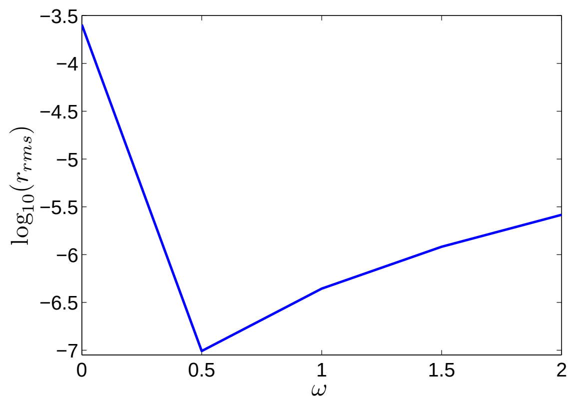

Figure 6: $\log_{10}(r_{rms})$ after 20 V-cycles Vs. the ω value.

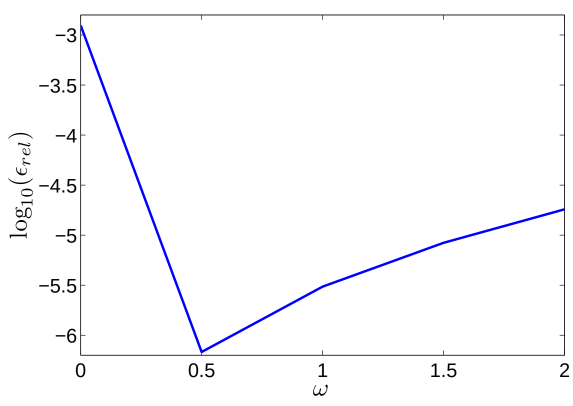

Figure 7: $\log_{10}(\epsilon_{rel})$ after 20 V-cycles Vs. the ω value.