Single-image superresolution of self-similar textures

Ido Zachevsky

under the supervision of

Prof. Yehoshua Y. Zeevi

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Introduction

Super-resolution (SR) problem formulation:

$$Y(\eta_1, \eta_2) = \mathcal{D}\left((B * X)(\eta_1, \eta_2)\right) + N(\eta_1, \eta_2)$$

Degradation caused due to both decimation and blur

- Decimation, $\mathcal{D}$, causes aliasing in finitely supported (typically small) PSFs.
- Blur kernel, $B(\eta_1, \eta_2)$: Gaussian, averaging, ...
- Noise, $N(\eta_1, \eta_2)$: iid Gaussian noise, typically with low variance

Original image

Blurred image

BM3D Reconstructed

Blurred+subsampled

BM3D Reconstructed

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Super-resolution

The super-resolution problem

- Multi-frame super-resolution: Given $\{Y_i(\eta_1, \eta_2)\}_{i=1}^{N}$ [Irani and Peleg, 1990, Elad and Feuer, 1997, Yang and Huang, 2010], ...
- Single-frame super-resolution (upscaling): Given a single measurement, $N = 1$ [Freeman et al., 2002, Glasner et al., 2009, Zeyde et al., 2011]
    - *The focus of this research: Restore a high-resolution image from a single blurred and subsampled observation.*

Texture enhancement

Super-resolution as a specific problem in image enhancement

- Cartoon v. textures - non BV space [Gousseau and Morel, 2001]

Methods for texture enhancement

- Differential equations, $I_t = \nabla \cdot (g(|\nabla I|)\nabla I)$ [Gilboa et al., 2002]
- Sparseness-based, $X = D\alpha + V$ [Yang et al., 2010]
- Example (learning)-based superresolution [Freeman et al., 2002]
  - Single-image example-based [Glasner et al., 2009]
- Texture synthesis [Efros and Leung, 1999]

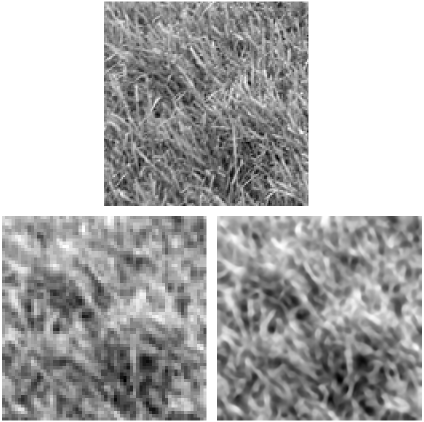
Relevance and contribution

Further research is necessary in textured images

Theoretical model

- Analysis of natural images as realizations of random processes
- Interest in the underlying model

Better models may yield better enhancement algorithms



Example-based SR [Freeman et al., 2002]. Top: Original. Left: Degraded image. Right: Enhanced image.

Texture properties

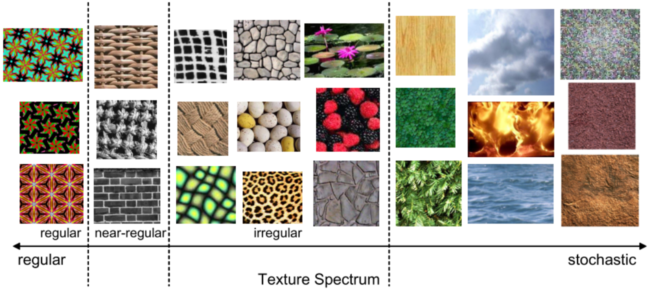
Types of textures [Lin et al., 2004]

- Regular (structured): pattern-based
- Stochastic: based on random processes, $X(\eta_1, \eta_2) = X(\eta_1, \eta_2, \omega)$

Stochastic textures

- Self-similar, $X_c(at) \stackrel{d}{=} |a|^c X_c(t)$
- Long-range dependencies
- Fractal properties [Barnsley, 1988]

For example: fractional Brownian motion.



Background

- Image model:

$$Y(\eta_1, \eta_2) = \mathcal{D}\left((B * X)(\eta_1, \eta_2)\right) + N(\eta_1, \eta_2)$$

- Naive solution: $\hat{X}(\eta_1, \eta_2) = (G * Y)(\eta_1, \eta_2)$, where

$$\tilde{G}(\tilde{\eta}_1, \tilde{\eta}_2) = \tilde{B}^{-1}(\tilde{\eta}_1, \tilde{\eta}_2)$$

  Problem: not practical.

- Possible solution: Wiener filtering, Richardson-Lucy deconvolution, L_2-based and TV-based regularization, ...

Regularization

Required for solving ill-posed problems

$$\hat{X}(\eta_1, \eta_2) = \arg\min_X \|Y - \mathcal{D}(B * X)\|_2^2 + \lambda\phi(X)$$

Common regularization functions:

- L_2-based: high gradients are not common, $\phi(x) = \|\nabla X\|_2^2$
- L_1- or TV-based: promotes piecewise-smooth segments

$$\phi(x) = \|\nabla X\|_1$$

- L_p norms, for $p \in (1, 2)$
- L_0 pseudo-norm, promotes sparsity:

$$\phi(\alpha) = \|\alpha\|_0, \; X = D\alpha + V$$

- Reflect an image model (BV space).

Issues

- Restoration results in cartoon images
- Textured details are lost

Plan

Fractional Brownian motion

$$E\left[B_H(t)B_H(s)\right] = \frac{\sigma^2}{2}\left(|t|^{2H} + |s|^{2H} - |t-s|^{2H}\right)$$

- Gaussian, self-similar, fractal process [Mandelbrot and Van Ness, 1968]
- The only self-similar Gaussian process (in 1D)
- Brownian motion for $H = 0.5$
- Non-stationary, with stationary increments
- Exhibits negative correlation for $0 < H < 0.5$
- Can be synthesized *efficiently* in 2D by introducing $\phi(\eta_1, \eta_2)$, autocorrelation of the increments.

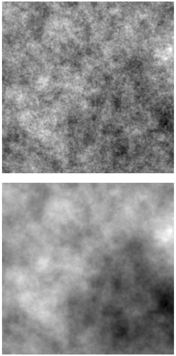


Figure: fBm for $H \in \{0.1, 0.6\}$

Texture model

$$X = X_{LP} + \hat{X}_{HP} + V,$$

where

$$\hat{X}_{HP}(\eta_1, \eta_2) = f\left(\omega, X_{LP}\right)$$

- $X_{LP}(\eta_1, \eta_2)$ is a low-frequency, degraded, image
- $X_{HP}(\eta_1, \eta_2)$ is composed from $\angle X_{LP}$ and an fBm image, $B_H(\eta_1, \eta_2)$
- The high frequencies are an fBm representation of the degraded image

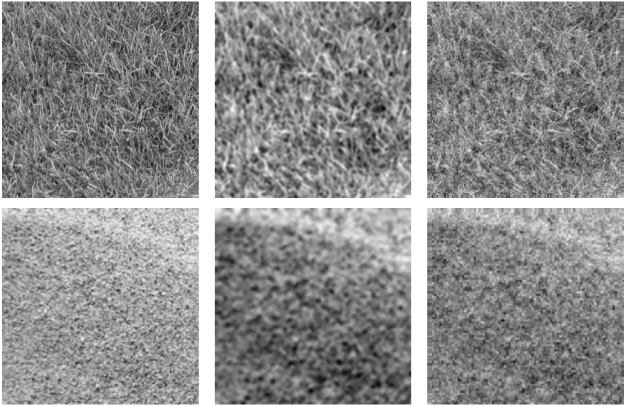


Figure: Example of the texture model.
Left: Original image. Middle: X_{LP}. Right: $X(\eta_1, \eta_2)$

Preliminary algorithm

Perform optimization:

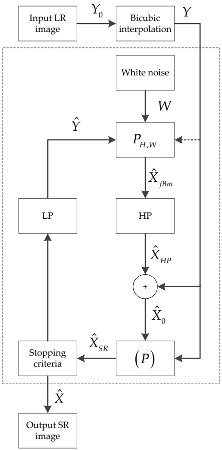
$$\begin{aligned} \hat{X} &= \arg\min_{X} \|Y - BX\|^2 \\ s.t. \quad X &= X_{LP} + \hat{X}_{HP} \\ \hat{X}_{HP} &= f(\omega, X_{LP}) \end{aligned}$$

Solve by iterating:

1. Apply constraint, $\hat{X} = X_{LP} + \hat{X}_{HP}$
2. Solve proximal point problem,

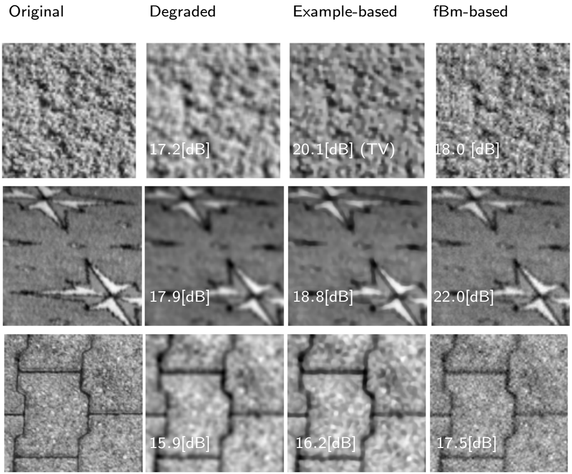
$$\hat{X}_{k+1} = \arg\min_{X} \|Y - BX\|^2 + \alpha \left\| \hat{X}_k - X \right\|^2$$

- May be considered as a POCS problem



Results

- Performs well on isotropic stochastic textures.
- Recovers missing details according to the model.
- PSNR or MSE-based comparison methods not applicable ($PSNR < 20[dB]$)



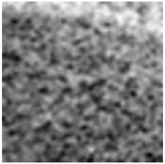
[Zachevsky and Zeevi, 2013]

Plan

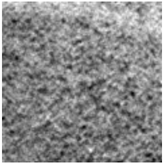
Disadvantages

The previous model does not perform well on anisotropic, oriented, textures.

Original

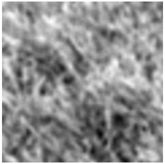


Degraded

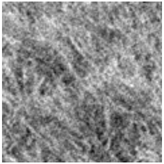


fBm-based

Original



Degraded



fBm-based

A new algorithm is required to handle different orientations in the image.

Tensor diffusion

Basic reaction-diffusion equation for deblurring [Welk et al., 2005]:

$$\begin{aligned}
\hat{X} &= \arg\min_X \|Y - BX\|^2 + \lambda\Phi(\nabla X) \\
X_t &= -B^T * (B * X - Y) + \lambda\nabla\cdot(D(\nabla X)\nabla X) \\
D &= (\omega_1, \omega_2)\begin{pmatrix}\lambda_1 & 0 \\ 0 & \lambda_2\end{pmatrix}\begin{pmatrix}\omega_1 \\ \omega_2\end{pmatrix}
\end{aligned}$$

where

$$\omega_1 \parallel \nabla X, \quad \omega_2 \perp \nabla X$$

- Performs *anisotropic* diffusion, used in various image enhancements tasks
- Applies different diffusion coefficients, according to orientation.
- Can be used for super-resolution.

Tensor diffusion

Adapted for texture enhancement:

- Modified reaction term: $\left(X_{HP} - \hat{X}_{HP}\right)^2$, recovers degraded high frequencies
- Diffusion tensor *modified*: $D\left(\nabla\left(X + \alpha Y_\phi\right)\right)$, preserves texture orientation

New scheme:

$$X_t = 2B^T(BX - Y) - 2H(\hat{X}_{HP} - HX) + \beta\nabla\cdot(\Psi'(|\nabla X + \alpha\nabla Y_\phi|^2)\nabla(X + \alpha Y_\phi))$$

where $Y_\phi(\eta_1, \eta_2)$ is a stochastic image, obtained by a function derived from the degraded image itself.

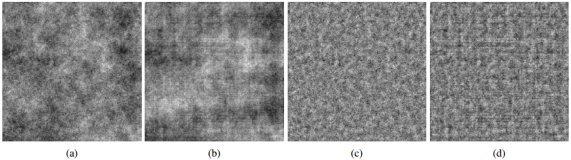
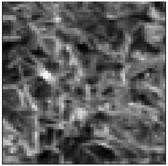


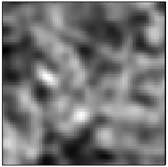
Figure: Original fBm, fBm from $\phi(\eta_1, \eta_2)$ and high-pass versions

Modified diffusion equation

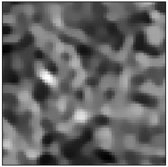
Visualizing the two additions (zoomed-in images)



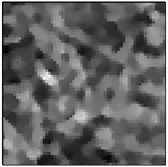
Original image



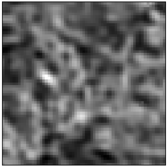
Degraded image



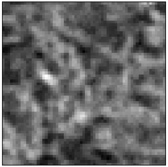
Original PDE



New reaction, $(X_{HP} - \hat{X}_{HP})^2$



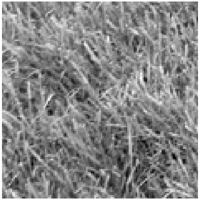
New tensor, $D(\nabla(X + \alpha Y_{\phi}))$



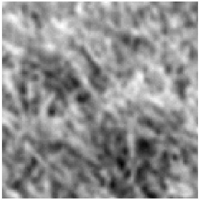
Modified PDE

Comparison with the first algorithm

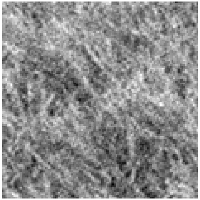
Recall the previous result



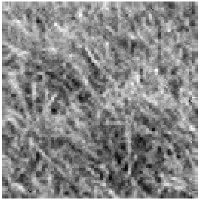
Original



Degraded



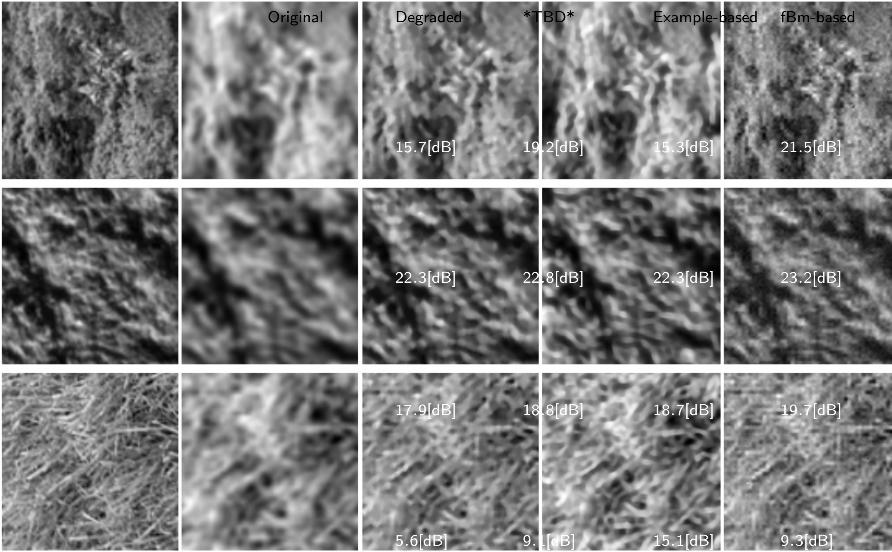
fBm-based



PDE-fBm-based

The current algorithm restores missing details, according to the texture orientation.

Results



Original
Degraded
TBD
Example-based
fBm-based
15.7[dB]
19.2[dB]
15.3[dB]
21.5[dB]
22.3[dB]
22.8[dB]
22.3[dB]
23.2[dB]
17.9[dB]
18.8[dB]
18.7[dB]
19.7[dB]
5.6[dB]
9.1[dB]
15.1[dB]
9.3[dB]

Plan

1. Introduction
2. Self-similar texture model
3. Anisotropic self-similar model
4. Further research

Theoretical tasks

- Creating a unified model which will contain both SR algorithms.
- Geometrical framework
  - Representation in higher spaces and performing color processing [Sochen et al., 1998]

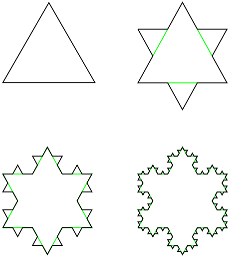
$$C_t = \frac{1}{\sqrt{|g|}} \sum_{\mu,\nu=1}^{2} \partial_\mu \left(\sqrt{|g|} \left(g^{-1}\right)_{\mu,\nu} \partial_\nu C \right)$$

  - Textures as a manifold, embedded in a suitable space, providing an intrinsic metric for comparison [Kimmel et al., 2000].
- Derive a statistical model for textures

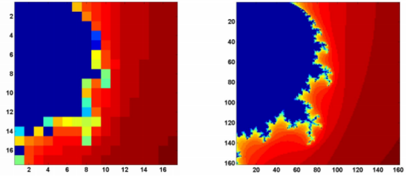
Further research

Geometric fractals

- Geometric fractals appear in various natural images.
- The structure can be exploited for an image model and SR scheme.



Koch snowflake [von Koch, 1904]



Mandelbrot set [Barnsley, 1988]

Classification and denoising

Classification

- The dominant feature of stochastic textures is self-similarity
- This can be exploited to classify images
- Features: self-similarity index, correlation
- Further features are required
- Segmentation

Image denoising

- Overlap of high-frequency details with that of noise
- The model may prove useful in performing texture denoising

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Backup follows

Fractional Brownian motion synthesis

1. Naive [Hoefer et al., 1992]: Construct $R_B(t,s)$, the autocorrelation function of the fBm:

$$
\begin{aligned}
E\left[B_H(t)B_H(s)\right] &= \frac{\sigma^2}{2}\left(|t|^{2H} + |s|^{2H} - |t-s|^{2H}\right) \\
\sigma_B^2 &= \frac{\sigma_B^2}{2}\frac{\cos(\pi H)}{\pi H}\Gamma(1-2H)
\end{aligned}
$$

   Inefficient due to use of Cholesky decomposition.

2. Via the Fourier domain [Kaplan and Kuo, 1996]:
   - First- and second-order increments are stationary.
   - Their autocorrelations are expressed via a structure function, $\phi(\eta_1, \eta_2)$.
   - Fields with these autocorrelation functions can be efficiently synthesized via the Fourier domain.
   - 2D fBm is obtained by summation of the increments.

Back

Anisotropic diffusion

Non-linear *isotropic* diffusion [Perona and Malik, 1990]:

$$\begin{aligned} X_t &= \nabla \cdot (g(|\nabla X_\sigma|)\nabla X) \\ g(s^2) &= \frac{1}{1 + \left(\frac{s^2}{K^2}\right)} \end{aligned}$$

- Adaptive: diffusion coefficient inversely proportional to norm of gradient
- Done isotropically, regardless of orientation

Weickert anisotropic diffusion [Weickert, 1998]:

$$\begin{aligned} X_t &= \nabla \cdot (D(\nabla X)\nabla X) \\ D &= (\omega_1, \omega_2) \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} \end{aligned}$$

- For edge preserving denoising: λ_1 similar to Perona and Malik, and λ_2=0.1. Enhances edges according to orientation.

Back

Modified diffusion equation

- A nonstationary field with stationary increments can be synthesized using a generating (structure) function, $\phi(\eta_1, \eta_2)$ [Pesquet-Popescu and Larzabal, 1997]:

$$\begin{aligned} \phi(\Delta_1, \Delta_2) &= var\left(F(\eta_1, \eta_2) - F(\eta_1 - \Delta_1, \eta_2 - \Delta_2)\right) \\ \phi_{fBm}(\Delta_1, \Delta_2) &= (\Delta_1^2 + \Delta_2^2)^H \triangleq r^{2H} \end{aligned}$$

- The empirical image, $Y_\phi(\eta_1, \eta_2)$, is obtained by an inverse process using an empirical structure function.

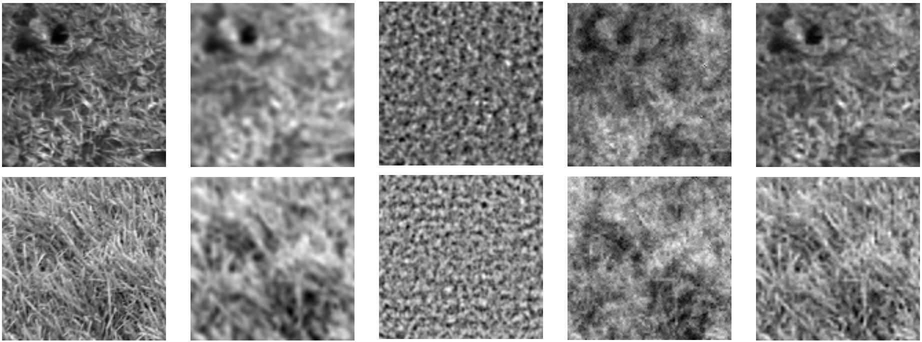
$$\hat{\phi}(\eta_1, \eta_2) = f_1(Y(\eta_1, \eta_2)) \quad \Rightarrow \quad \hat{B}_{\hat{\phi}}(\eta_1, \eta_2) = f_2(\hat{\phi}, \omega)$$

- Due to the self similarity, the correlations are suitable for the superresolution image.
- The inverse process is performed via solving an ill-posed least squares problem:

$$\underline{\phi} = \arg\min_{\underline{x}} \|D\underline{x} - \underline{r}\|_2$$

- Further methods can be explored to find more suitable empirical structure functions. Back

Empirical function in algorithm progress



Original image | Degraded image | $Y_\phi(\eta_1, \eta_2)$ | $\hat{X}(\eta_1, \eta_2)$ | Enhanced image

- Further study necessary for empirical image. Back