Image denoising using scale mixtures of Gaussians in the wavelet domain

Portilla, J., Strela, V., Wainwright, M., and Simoncelli, E. [1]
A review by Ido Zachevsky

June 25, 2013

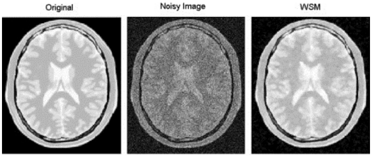
Image denoising

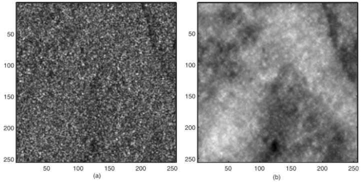
Subject of great interest in image processing

- End-user applications
- Computer vision (recognition)
- Medical images
- Stellar images
- Many more applications

Naïve methods

- Filtering with a low-pass (usually Gaussian) filter.
- Optimal (Wiener) filtering.

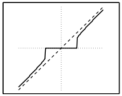




[2]

Wavelets-based denoising

- A dense signal can be sparsely represented in a suitable domain.
- Important coefficients will have high energy.
- Therefore: Wavelet shrinkage - keep only high-energy coefficients.



A thresholding function

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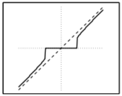

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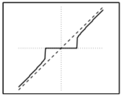

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Improvements

How can wavelet denoising be improved?

- Wavelet shrinkage functions are performed *pointwise*.
- Using orthogonal wavelets causes artefacts.

Possible solutions

- Use a statistical model for images, exploiting *correlations* and performing wiser thresholding.
    - Joint probability model instead of marginal probability.
- Use redundant wavelets.

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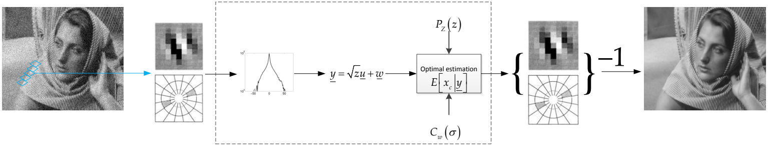
Proposed denoising algorithm



- Use *redundant* wavelets.
- Exploit a suitable conditionally Gaussian *model*.
- Create a *vector* of similar coefficients.
- Perform estimation using *optimal* filtering.

Exploits joint dependencies.

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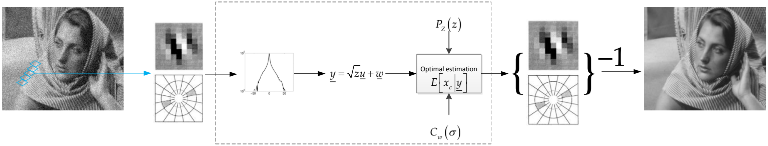

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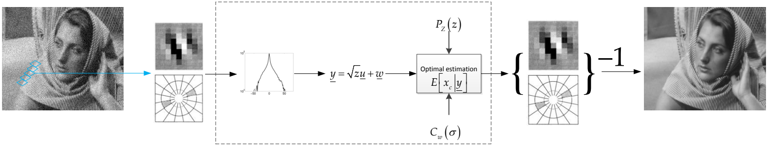

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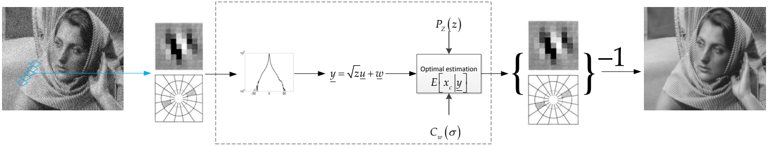

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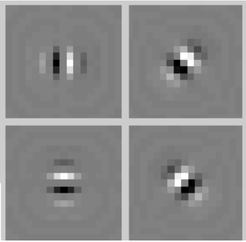
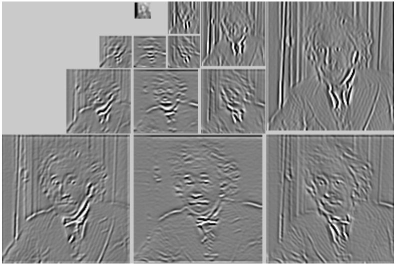
Steerable pyramid wavelets (I)





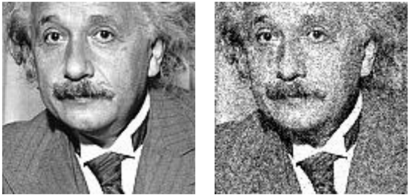
Main properties

- Overcomplete - translation invariant.
- Provides orientation information - rotation invariant.

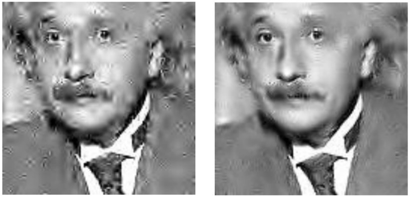
Steerable pyramid wavelets (II)

Advantages over other wavelets

- Redundant
  - No aliasing artefacts
  - Captures different orientations efficiently
- Coefficients with similar orientation and scaling from different locations can be grouped.



Original and noisy image



Restoration by an orthogonal and redundant wavelets [3]

Gaussian scale mixtures (I)

Model (GSM):

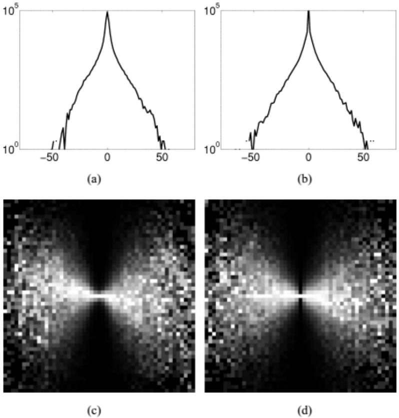
$$Y = \sqrt{z}U + W$$

- U is a Gaussian vector with known statistics.
- z is a positive random scalar.
- W is a Gaussian noise.

Conditionally Gaussian (given z)

Y models wavelet coefficients of similar features.

- Models the kurtotic behaviour of images



(a,c): Natural image. (b,d): GSM

Gaussian scale mixtures (II)

Denoising using GSM:

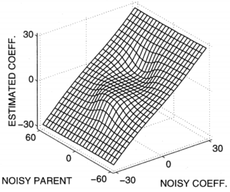
$$Y = \underbrace{\sqrt{z}U}_{X} + W$$

Optimal estimation is given by:

$$E[X_c|Y] = \int_0^\infty p(z|Y)E[X_c|Y,z]dz$$

1. $E[X_c|Y,z]$ - Optimal estimator is linear.
2. $p(z|Y)$ - calculated by Bayes' thm.:

$$p(z|Y) = \frac{p_Z(z)p(Y|z)}{\int da \cdot p(Y|a)p_Z(a)}$$

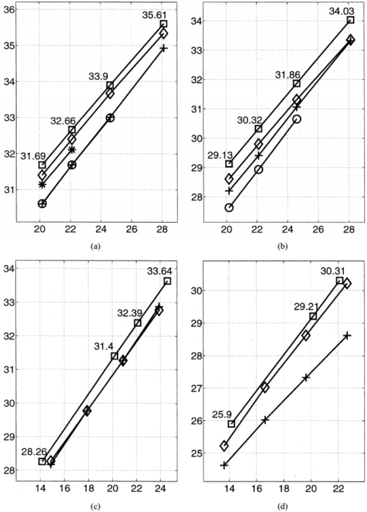


(b)

Results (I)

Noisy PSNR: $20.2dB$. Li et al. PSNR: $28.2dB$. Portilla et al. PSNR: $29.1dB$

Results (II)


35.61
33.9
32.66
31.69
(a)
34.03
31.86
30.32
29.13
(b)
33.64
32.39
31.4
28.26
(c)
30.31
29.21
25.9
(d)

Summary

This method was the state of the art for several years.

- Overthrown by BM3D [4], which uses patch-based 3D-DCT transform.

Main contributions:

- Exploiting sets of related pixels for thresholding
  - Using joint distribution instead of marginal distribution.
  - Using pixels from distant locations, according to features.
- Using an accurate model which also allows simple optimal filtering.
- Using a suitable wavelet representation.

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