Statistics of Natural Stochastic Textures with application in Image Processing

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Joint work with Prof. Y. Y. Zeevi

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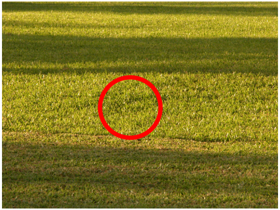
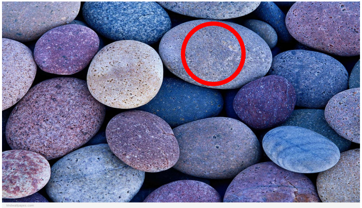
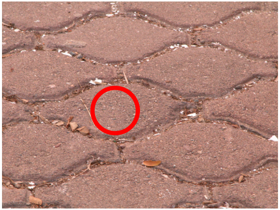
Natural stochastic textures (NST) in natural images

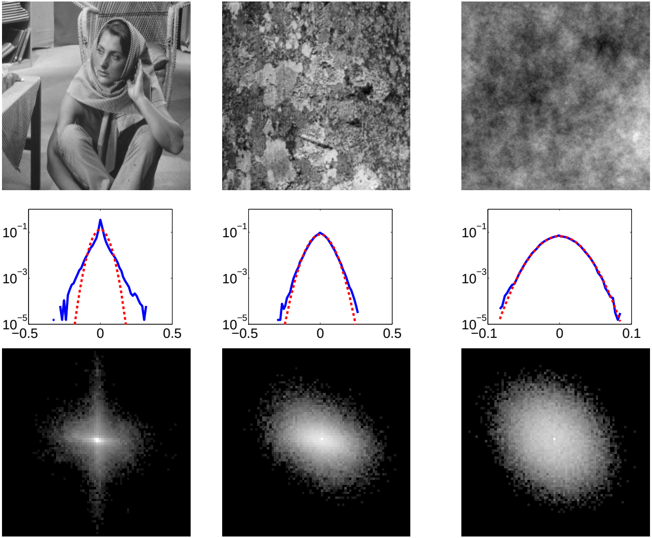
Natural stochastic textures are abundant in natural images.







Statistics of *natural stochastic textures* (NST) (I)



Natural image | Stochastic texture | Synthetic texture (fBm)

- ⋆ Wavelet domain statistics

Statistics of *natural stochastic textures* (NST) (II)

Properties of NSTs include:

- NSTs are considered as realizations of 2D random processes.
- *Gaussian* statistics in marginal and joint distributions, unlike general natural images. [Zachevsky and Zeevi, 2014a, 2016]
  - In image space.
  - In wavelet domain.
- Self similarity – scale invariance [Srivastava et al., 2003].

Main conclusion from current study

Natural stochastic textures have *Gaussian* statistics.

A suitable model should be *Gaussian and self-similar*.

Fractional Brownian motion (fBm)

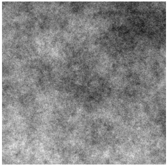
[Mandelbrot and Van Ness, 1968, Kolmogorov, 1940]

FBm $\Rightarrow$ Gaussianity and self-similarity [Samorodnitsky, 2006]:

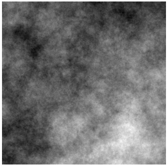
1. $B(\{t_1, t_2, \ldots, t_k\}) \sim \mathcal{N}(0, \Sigma(\{t_i\}))$
2. $B(ct) \overset{d}{=} c^H B(t), c > 0$
3. Stationary increments.

$\Rightarrow$ $B(t)$ is fBm with a self-similarity parameter $H \in (0, 1)$ and autocorrelation:

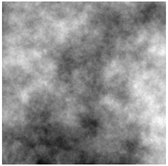
$$E\left[B_H(t)B_H(s)\right] = \frac{\sigma_H^2}{2}\left(|t|^{2H} + |s|^{2H} - |t-s|^{2H}\right)$$



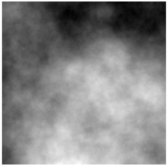
$H = 0.1$



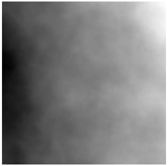
$H = 0.3$



$H = 0.5$



$H = 0.7$



$H = 0.9$

Is fBm sufficient as a prior?

Relevance as a model for NST

+ Gaussian.
+ Statistically self-similar.
- *Random phase* – *no edges* in the usual sense.

NSTs are of a broader class

- Anisotropic/Asotropic.
- *Phase*-based properties.

Conclusion

A suitable model for NST should consist of a generalization of fBm to a more elaborate model.

Isotropic texture

Asotropic texture

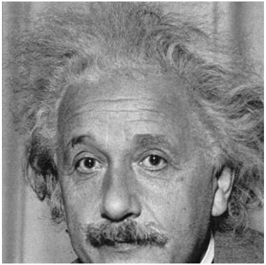
The importance of the phase in natural images

⋆ Phase is important for the visual appearance of images [Oppenheim and Lim, 1981].

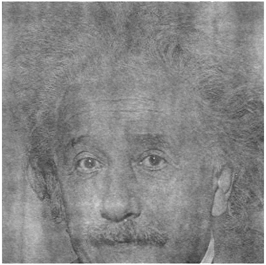
⋆ Global v. Localized

phase [Behar et al.,

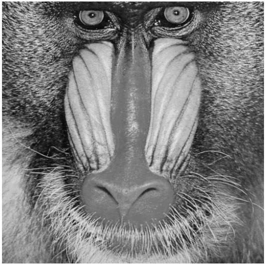
1992]



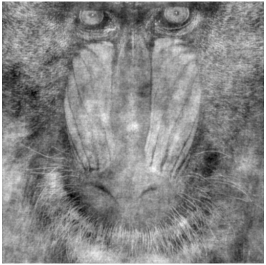
Einstein



$\mathcal{F}^{-1}\{|\mathit{Mandrill}|\exp(j\angle \mathit{Einstein})\}$



Mandrill



$\mathcal{F}^{-1}\{|\mathit{Einstein}|\exp(j\angle \mathit{Mandrill})\}$

The importance of the phase in natural textures
"Unlocking the phase"

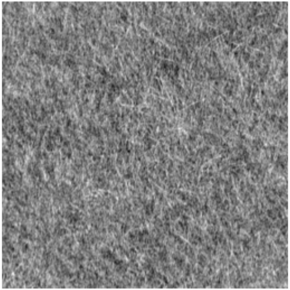
Isotropic texture

Phase with $\sigma = \mathbf{1}$ normal distributed i.i.d noise (14.9dB PSNR)

- *Little* visual change in the case of isotropic textures.
- Isotropic textures – Random phase.
- Asotropic textures – Phase info required.

The importance of the phase in natural textures "Unlocking the phase"

Asotropic texture



Phase with $\sigma = \mathbf{1}$ normal distributed i.i.d noise (18.1dB PSNR)

- ⋆ *Significant* contribution of phase info in the case of anisotropic textures.
- ⋆ Isotropic textures – Random phase.
- ⋆ Asotropic textures – Phase info required.

Some examples

Super-resolution from a single image [Zachevsky and Zeevi, 2014b]

Problem formulation:

$$\hat{X}(\eta_1, \eta_2) = \arg\min_{X} \left\| Y - \mathcal{D}(B * X) \right\|_2^2 + \lambda \phi(X)$$

Solution method:

- Use the fBm averaged spectrum and *retain* phase.
- *Self-similarity*: Use the autocorrelation of the low-resolution image for reconstruction of the high-resolution image.
- Implementation via anisotropic PDE (tensor diffusion), modified to promote recovery of small details:

$$\begin{aligned} X'_t &= 2(DB)^T(DBX' - Y) - 2\bar{H}_{HP}(\hat{X}_{HP} - \bar{H}_{HP}X') + \\ &\quad + \beta\nabla \cdot (\Psi'(|\nabla X' + \gamma\nabla Y_\phi|^2)\nabla(X' + \gamma Y_\phi)) \end{aligned}$$

Super-resolution from a single image [Zachevsky and Zeevi, 2014b]

LR image

Sparseness-based SR [Yang et al., 2008]

Proposed method

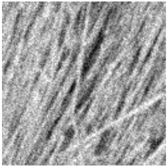
Ground truth

Retaining the phase for anisotropic textures

[Zachevsky and Zeevi, 2016 (submitted)]

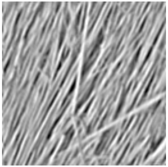
- ⋆ Reconstruction of severely degraded anisotropic textures
- ⋆ Estimation of *magnitudes* while retaining the *phase*
- ⋆ Example: degradation by Gaussian blur, width $\sigma_f = 1$ and AWGN $\sigma_N = 0.1$

Ground truth



Blur+noise (17.47dB, 0.567)

BM3D (20.43dB, 0.713)[Danielyan et al., 2012]



Proposed alg. (20.33dB, 0.841)

Further research: towards a texture manifold

A texture manifold should include:

- Stochastic characteristics (fBm, mBm): Self similarity ($\{\boldsymbol{H}_i\}$), LRD,
- Phase dependency:
  - Local phase.
  - Orientation – θ
  - Coherence – μ

Representation via a manifold [Peyré, 2009]:

$$(\eta_1, \eta_2) \rightarrow (\eta_1, \eta_2, \boldsymbol{H}_i, \theta, \mu, \cdots).$$

Desired properties:

- Natural representation for color textures including depth information.
- A measure of similarity between two textures via the intrinsic metric.

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