Hypothesis testing in natural images

Selected topics in statistical signal and image analysis - final project

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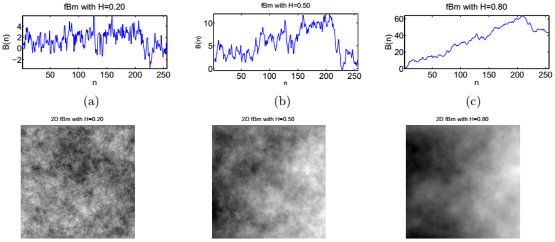
Introduction

- Textured images are observed to be self-similar.
- Proposed model for self-similar textures: fractional Brownian motion (fBm) [1]
- fBm is a Gaussian process with $H \in (0, 1)$ and:

$$E[B_H(t)B_H(s)] = \frac{\sigma_B^2}{2}(|s|^{2H} + |t|^{2H} - |t - s|^{2H})$$



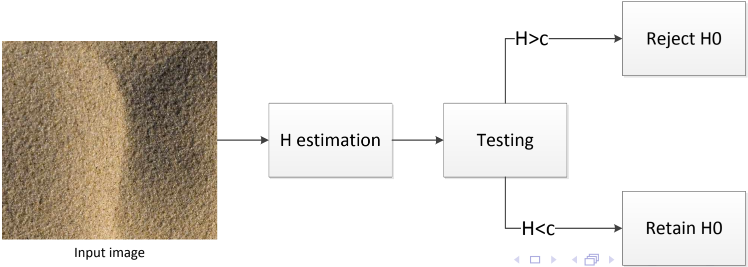
Example for an fBm-based model in image processing

Figure: Low-resolution image

Figure: High-resolution

Hypothesis testing on H

- Goal: Given an image, decide whether it is a stochastic texture.
- Assumption: H values for stochastic textures are low, $H < H_s$.
- Method:
  1. Obtain an estimator, $\hat{H}$, for the Hurst parameter of an image.
  2. Make a decision with hypothesis testing scheme (threshold).
     H0: $H < H_s$.



Method

$$E[B_H(t)B_H(s)] = \frac{\sigma_B^2}{2}(|s|^{2H} + |t|^{2H} - |t-s|^{2H})$$

- The increments of an fBm are stationary, with the following variance:

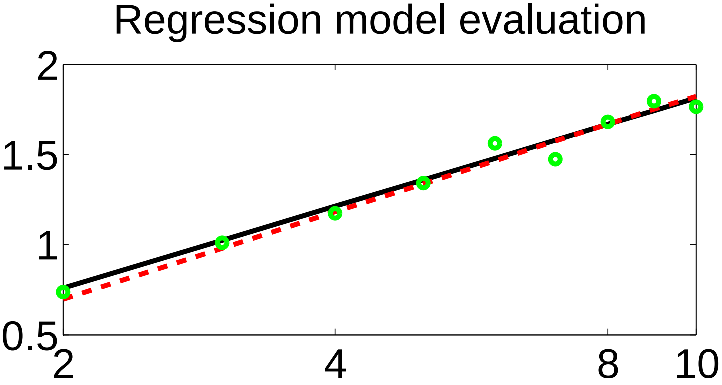
$$\begin{aligned} W_H(\tau) &= B_H(t) - B_H(t+\tau) \\ \Rightarrow \sigma^2_{W_{H,\tau}} &= E[W_H(t)W_H(t+\tau)] = \sigma_B^2 \tau^{2H} \qquad (1) \end{aligned}$$

- Therefore:

$$\begin{aligned} \log \sigma^2_{W_{H,\tau}} &= 2H \log \tau + \log \sigma_B^2 \qquad (2) \\ y_\tau &= \alpha x_\tau + \beta \qquad (3) \\ \Rightarrow Y &= \hat{A}X + \mathcal{E} \qquad (4) \\ \hat{A} &= X^\dagger Y \Rightarrow \hat{H} = \frac{1}{2}\hat{A}(1) \qquad (5) \end{aligned}$$

- Practically, using weighted least squares.

Experiment



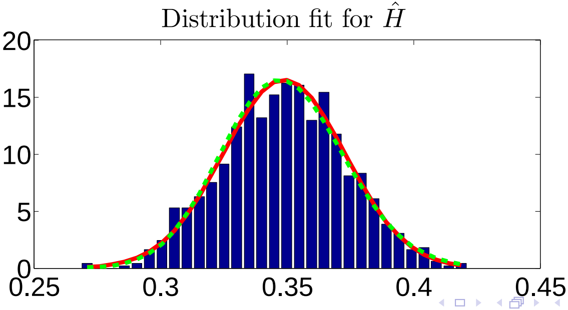
Regression model sample points. True line plotted using the simulated H values. Regression model **result**.

Hypothesis testing

The estimator can be expressed as:

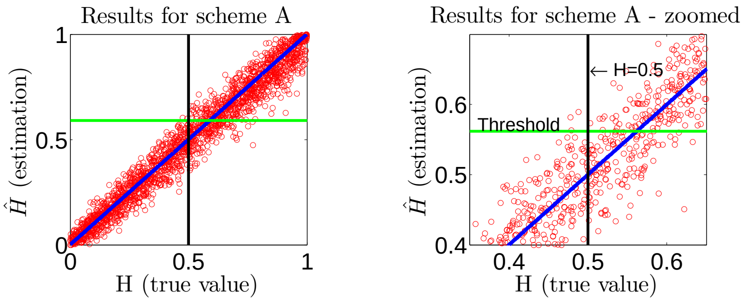
$$\hat{H} \;=\; \frac{1}{2}\sum_{i\in T} x_1^i \log\left(\hat{\sigma}^2_{W_{H,i}}\right) = \frac{1}{2}\sum_{i\in T} x_1^i \log\left(\frac{1}{N-i}\sum_{j=1}^{N-i} W_{H,j}^2\right).$$

- Problem: What is the distribution?
- Proposal: Estimate Gaussian parameters



Hypothesis testing

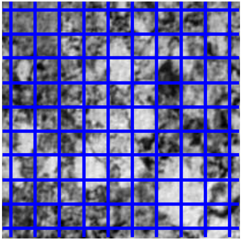
Assuming the estimator is Gaussian, perform a Wald test.

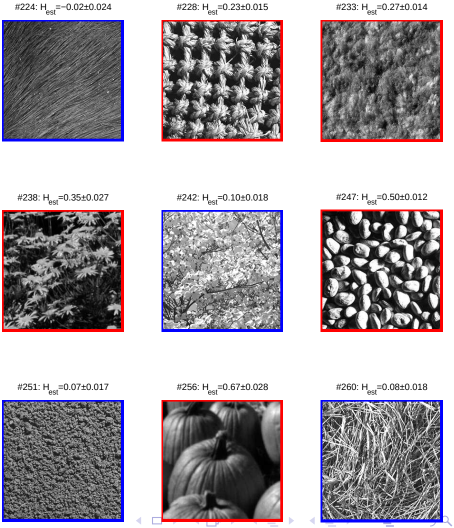


Results on 2D images

Expansion to 2D: By majority of extracted lines

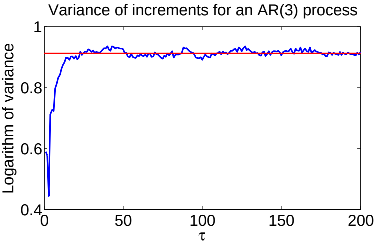
1D extraction grid

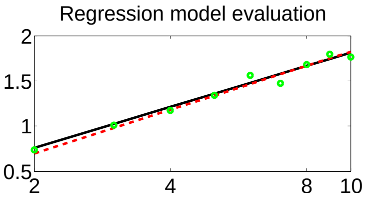




Summary and conclusions

- A hypothesis testing framework was implemented, based on a regression model.
- Estimation of H was performed and used as a test statistic for Wald test.
- Results may improve for a true 2D estimation of H.
- Dependencies between increments need to be incorporated in the model.
- Higher order regression coefficients can be used for deciding between different image models.





Bibliography

[1] B.B. Mandelbrot and J.W. Van Ness. Fractional brownian motions, fractional noises and applications. *SIAM review*, 10(4):422–437, 1968.