A presentation on:

A Spectral Approach to Total Variation Guy Gilboa, SSVM 2013

In the subject of:

Modelling the TV-spectrum of noise

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Background

The Total-Variation (TV)-transform:

$$\phi(x,y;t) = u_{tt}(x,y)\cdot t, \tag{1}$$

where $u(x,y)$ is an image, evolved via TV-flow:

$$u_t(x,y) = \nabla\cdot\left(\frac{\nabla u(x,y)}{|\nabla u(x,y)|}\right). \tag{2}$$

A spectrum, $S(t)$, is defined over $\phi(x,y;t)$:

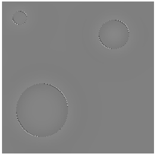
$$S(t) \triangleq \iint_\Omega |\phi(x,y;t)|dxdy \tag{3}$$

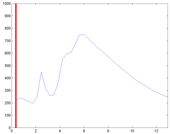
Properties:

- $S(t)|_{t_0}$ represents the "energy" of the object in scale (time) t_0.
- The transformed image can be *filtered*, similar to Fourier analysis.
- The transform is *non-linear*.

Examples

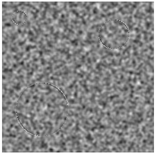
Clean image:

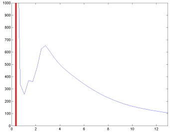




“ Noisy image:

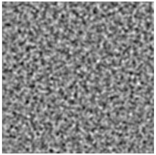
“

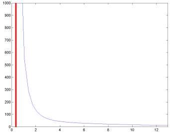




“ Noise:

“





“

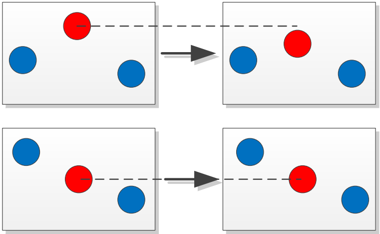
Proposed improvement: Analysis of noise (I)

- Let $X[n]$ be a 1D image, and $X[n] \sim \mathcal{N}(\mathbf{0}, \mathbf{1})$ i.i.d.
- The TV-flow for $X(t)$ is:

$$u_t = \partial_x \frac{u_x}{\sqrt{u_x^2}} = \partial_x \frac{u_x}{|u_x|} = \frac{d}{dx} sign\left(u'\right). \tag{4}$$

- In discrete space, using the explicit scheme:

$$u_{n+1}^m = u_n^m + \Delta t \cdot \underbrace{\left(\mathbf{1}_{u_n^m > u_n^{m-1} \wedge u_n^{m+1} < u_n^m} - \mathbf{1}_{u_n^m < u_n^{m-1} \wedge u_n^{m+1} > u_n^m}\right)}_{\triangleq r_n}. \tag{5}$$



$\star$ Knowing $P(r_n) \Rightarrow$ Knowing $E[\phi_i^T \phi]$.

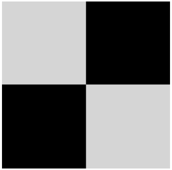
Proposed improvement: Analysis of noise (II)

How is r_n distributed?

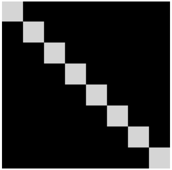
- Assumption: u_n, u_{n-1} are independent.
- Each u_n is smoother than the previous "scale", u_{n-1}.

Model: $u_n = A_n u_0$.

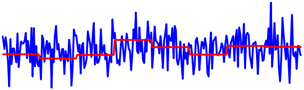
- u_0 is the original i.i.d. noise.
- A_i is a block-diagonal matrix:



$i = 2$



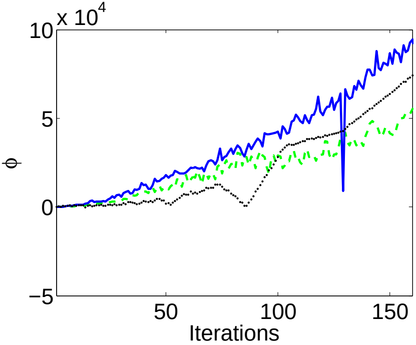
$i = 8$



- Distribution of r_n from u_n's distribution found via Monte-Carlo (WLLN).

Results

True $S(t)$, Simulated and **Analytical** results.



Further work:

1. Incorporate dependencies between u_{n-1}, u_n.
2. Obtain an analytical formula for ϕ_i.