Reference Frames

To perform any type of physical analysis, or create a model of a physical process, the concept of a frame of reference needs to be introduced so as to model and explain the observations of physical systems and the interactions of physical entities that are in motion relative each other. Reference frames are important in that what one observer or entity perceives as one form of physical process or interaction would be different to another when the two observers are in motion, and that the transmission of information by which the interaction or observation is made not instantaneous.

To explain any differences of observations between two observers in motion relative to each other requires a consideration to see things from the perspective of the other observer or entity in respect to ones own. Eg that of a perspective of a fixed location on the surface of the earth vs an object look the moon in a motion of orbiting around it, to that of a location stationary on the axis of the earths rotation. Each have a different observation of perspective of the motion of entities as each will be in a different reference frame of motion.

Thus without taking into context of the reference frame that a human observer is in, a very incorrect and misleading conclusion of what is the true physics that is taking place can be deduced. Examples of defined pseudo forces in certain frames of reference can lead to the wrong models of physical systems being created, and thus leading to a cascading of misunderstanding and incorrect conclusions of the physics that is taking place. The example of the Ptolemaic model of the solar system with the earth as the centre of the universe being one.

And this can even be true in modern physics. Current models of the universe on the macro or micro level may be incorrect if the perspective that the model that is being used is in a reference frame that is incorrect or even not known. Some theories of physics require dimensions of space greater than the three that Is perceived by the human mind. Even the geometry used within a reference frame to create the models may not be sufficient.

With the limitations of what reference frames can be modelled or thought of by the human mind to model and explain the universe, they are thus important and need to exist to do so.

Below is perhaps a different take on an interpretation and model of what reference frames represent and a perspective of how the physics of a moving object may be observed and measured. Some of the results do look different and unexpected. Perhaps the maths and approach may be the reason. No claim that what follows is correct, just that the results of the mathematics gives is what it gives.

01 : Definitions

Definition : DRF01 : Observational frame of reference
In physics, an observational frame of reference is defined, in essence, as the observational properties that are present and "observed" from the perspective of an entity that is in a state of motion and does not interact with other entities, or have physical properties of interaction.

What this means is that more than one entity can share, or be in the same observational frame of reference. Such a shared observational frame of reference is where the physical state of all entities within it do not change. Any entity that has a state of motion continues with that sate of motion. Such as if two entities E_A and E_B are moving with constant velocity in any unchanging direction with respect to each other, and that the direction does not change, then those two entities are in the same observational frame of reference as an "observer" O entity that "observes" their path of motion. If the observer is entity E_A, then from E_A perspective, E_A is stationary and E_B is observed to have motion that does not change. Such an observational frame of reference is defined in physics as an inertial frame of reference

Definition : DRF02 : Observational inertial frame of reference
An observational frame of reference in which the observational physical properties, such as motion, of all entities that are present and "observed" from the perspective of an entity are static and do not change.

The opposite of an observational inertial frame of reference is a non observational frame of reference where changes to the physical state of one or more entities that are "observed" to occur. Such changes are attributed to a force, which most often is attributed as changes in motion such as direction or rate of motion that is defined as acceleration.

Definition : DRF03 : Observational non inertial frame of reference
An observational non inertial frame of reference is a frame of reference in which the observational physical properties, such as motion, of all entities that are present are "observed" from the perspective of an observer to continually change in motion. Observational non inertial frames experience a change of physical properties due to what is perceived as an acceleration that is attributed to some form of perceived force.

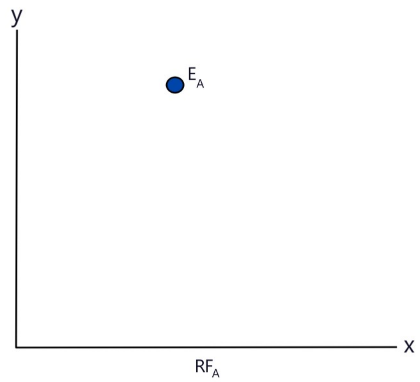


Fig RF01a
Entity E_A in RF_A perspective

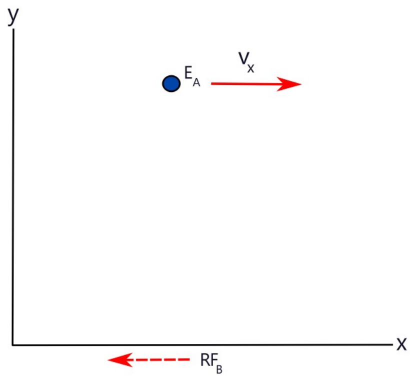


Fig RF01b
Entity E_A in RF_B perspective

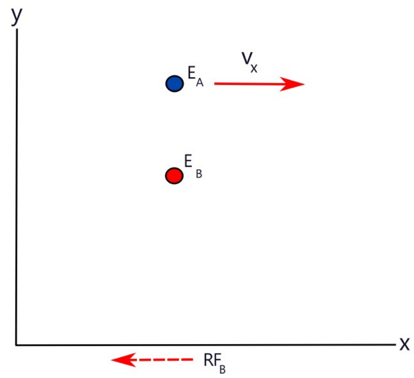


Fig RF01c
Entity E_A and E_B in RF_B perspective

Scenario A

Consider an observer O_A in a reference frame RF_A that releases an entity E_A in this reference frame at a location P in that reference frame such that E_A is observed by O_A to be motionless in respect to that reference frame. Consider that the entity inherits the physical properties that RF_A has such as motion. The observer O_A sees or observes no change in the entity E_A properties that it had prior to being released. E_A can be considered as being in a zero energy state of motion in this reference frame RF_A. Fig RF01a

Consider and observer O_B in a reference frame RF_B that is moving in the -x direction in relation to RF_A, and observes the entity E_A released in RF_A is having motion and properties of that entity that are not observed by O_A . O_B has designated that the entity E_A is changing its coordinates with each period of time in RF_B, and thus has a velocity v_x and other properties such as energy of motion that O_A does not observe or measure. Fig RF01b

Consider that in reference frame RF_B an entity is released by observer O_B at some location in O_B reference frame such that it is designated to have no motion in RF_B by O_B before it is designated to have passed the entity E_A in reference frame RF_A. Both observers O_A and O_B note that the entities in their respective reference frames do not change in any way, either in motion or any other properties they may have. Fig RF01c.

This situation is defined as RF_A and RF_B being independent of one another such that RF_A and RF_B do not interact or influence the physical properties of an entity or events of another reference frame. This can be defined that every entity in a system has it own reference frame, and that the reference frames of each entity does not influence or interact in any way with an entity in any other reference frame such that the properties of that entity in a differing reference frame does not change in that reference frame.

An analogy can be considered as a large flat object lying on a moving platform relative to it, where the platform and object have no friction of contact between them. Each has a frame of reference that can be considered as in motion to each other, but there is no interaction between these reference frames to cause any change in the motion or properties of the object or platform.

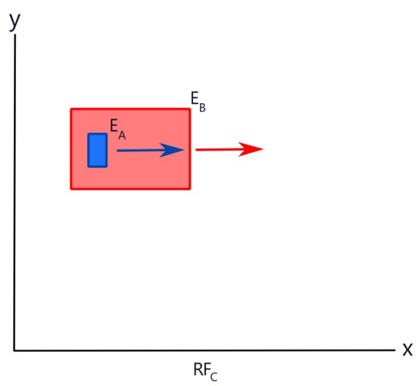


Fig02a

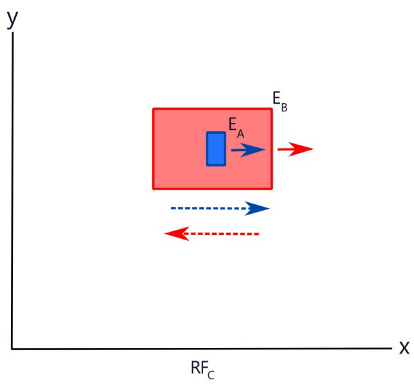


Fig02b

Scenario B

Consider that have a reference frame RF_C that does influence and change the properties of an entity and/or events in RF_A above. Then the observer O_A described above will see a change in the entity properties and thus suggest that some kind of hidden force is at hand. In this scenario, the reference frames are coupled together and that RF_A is embedded as being a part of RF_C . Conversely RF_A must also influence entities and events in RF_C as part of a feedback loop. That is if RF_C has an effect on entities in RF_A,then RF_A needs to be able to have an effect on entities in RF_C.

This situation is as if RF_A and RF_c are being coupled to one another in a way such that RF_A and RF_c interact with each other with some kind of frictional like bonding force between them that kind of sticks the reference frames together such that the events or entity properties appears to change in such a way that each reference frame is part of a single base reference frame RF.

Eg a box on a moving truck that makes up a reference frame of the box RF_A . The moving truck is in a reference frame RF_B and is observed by an observer O_C in RF_C to be moving with the box on the trucks platform that is in the same motion as the truck. (Fig02a) The truck changes its velocity of motion and O_C observes that the previously stationary box moves in the direction that the truck was travelling in, but slows down and eventually stops, reaching an equilibrium with the reference frame of the truck. (Fig02b) The only reason this can occur is if there is some kind of frictional force between the platform inertial frame and that of the truck as it changes its motion, and this frictional force is exerted on the box so as to change its motion. In turn the box exerts a frictional force on the truck trying to prevent it from changing its inertial state of motion from that of the box. These are illustrated as the dashed lines in Fig02b where in this case a slower truck velocity results in the box frictional interaction with the truck (blue dashed line) is wanting to push the truck forward, while the truck frictional interaction with the box (red dashed line) pushes on the box to slow its motion.

This truck - box interaction is an analogy. In a real physical scenario, such an interaction would be between entities that have what may be considered as an interacting field between them, and that these fields are tied to the entities and the entities to a reference frame. In other words, it may be considered that a physical entity, an interacting field associated with it, and the reference frame of that entity are of one, and possibly same single physical property.

Which Scenario

Which of these two scenarios is the correct one to use ?

Or could it be that both are correct and that natural phenomenon involves Scenario A for certain situations, while Scenario B is valid in others.?

If Scenario A is correct and valid for all physical phenomenon, then all forces observed in the universe may very well be one of being in different reference frames and observing the motion of straight lines of motion of entities existing within a universal zero reference frame. Through some form of existence of entities being in a reference frame of some form of complex motion, what appears to be a force is not a force at all.

If Scenario B is correct, then when the reference frames that entities exists in, or when they change reference frames, a force is observed as an interaction of the properties of two entities reference frames. This leads to two possibilities.

i : That there may well be a single field present of which every entity is connected to by a property it has to that field, which each entity interacts with and modifies through its motion.

ii: That each entity generates a field by a field property that generates a net field of that property which it interacts and modifies through its motion.

Both seem valid in a logical sense, but will not be conjectured here if either, both or none will be considered, just yet.

02 : Observation of Reference Frames

02-01 All reference Frames

All observations in all reference frames require an interaction of some kind between the entities of each reference frame so as to register a measurement of some quantity between then. For the following it is considered that the form of interaction is one of photons as at the most basic level of interaction, even of gravity. It is assumed, and from actual experimental observation, that interaction of all kinds occur at the velocity of light. Thus all interactions and physical measurements of all kinds such as distance, energy, and time, has the speed of light at its core, and that from Einsteins' postulate, is always measured to be constant in all frames of reference, be it inertial or non inertial.

Thus any thought experiment or theory needs to take into account this constant measured velocity of light and that it is finite. This means that measured events that any observer in any reference frame will observe and/or interact with an event that, unless the observer is at the same location of that event, is in the past. There is no real simultaneity of events that can be measured as occurring simultaneously with the same time clock that one is observing with. This is unless that event is at the location of that of the observers' clock, or the transmission and interaction with an event is infinite in velocity, and thus requires a zero interval of time for an event of interaction, and interaction to that event to occur by the measurement of all clocks. In this case, there is only one reference frame which all entities exist.

In order to conduct thought experiments so as to gain an understanding of the physical phenomenon that different observers in different reference frames observe, the idea of a universal overview needs to be established Such a universal overview requires a universal observer that can observe all physical phenomenon as if there is only one single reference frame in which all physical properties of all physical entities is known at any instant. That is, there is an instantaneous, zero time, infinite velocity of transmission of information to the observer of all physical phenomenon and properties. This is not physical reality, but a tool for the thought experiments in this documents that require certain definitions to be stated.

Definition : DRF04 : Universal Frame of reference
A universal single frame of reference RF_U where all physical properties of all physical entities can be known at any instant. That is, there is an instantaneous, zero time, infinite velocity of transmission of information from which measurements can be made or deduced from.

Definition : DRF05 : Universal Observer
An observer O_U that can observe in a universal reference frame RF_U, defined in definition **DRF04**, the location, movement, and interactions of all entities and of all physical phenomenon of a universe.

Definition : DRF06 : Universal Time
Universal time T_U is the time that a universal clock measures of the universal frame of reference defined in definition **DRF04** that can measure the passage of all motions within the universe that the universal frame of reference as if there is no delay in observing any entities or physical phenomenon. That is as if the transmission of information in a physical system has infinite velocity.

Definition : DRF07 : Universal Clock
Universal clock C_U is a clock that measures the universal time that a universal observer would observe in a universal fame of reference as defined in **DRF04, DRF05,** and **DRF06.**

With these definitions **DRF04, DRF05, DRF06,** and **DRF07** a construction of thought experiments can be made to postulate and understand the observed physical phenomenon of the real universe.

An important postulate and concept is one of a situation where all entities and their physical properties within a system does not change. Such a system can be said to have a stationary frame of reference and can be defined as in **Postulate : PRF01**.

Postulate : PRF01 : Principle of a stationary inertial frame of reference.
All entities, physical properties and physical phenomenon that are in the same inertial frame of reference such that at all instances of measured time, all observers measure no movement of entity coordinates or changes to an entity or its properties.

Another postulate and concept is one of a situation where all entity locations within a system does not change, but that there are certain events or phenomenon that may or may not be in action or in motion . Such a system can be said to have a stationary frame of reference and can be defined as **Postulate : PRF02**.

Postulate : PRF02 : Principle of a zero inertial frame of reference.

All entities that are in the same inertial frame of reference such that at all instances of measured time, all observers measure no movement of any entity coordinates, but there exists events or physical phenomenon that may or may not be in action or in motion.

02-02 Zero Velocity

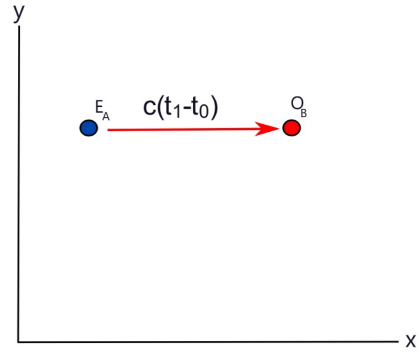


Fig ZRF-01

Consider that the clocks in reference frame FR_A and FR_B have been synchronised in some way that they register the same time value as a universal clock C_U when at some location x_i. Entity E_A emits a photon directly at O_B at a time t_0 equal to the universal time T_0.($t_0 = T_0$) If both FR_A and FR_B were of the same reference frame such that observers O_A and O_B in both observe each other to be motionless, the time measured by O_B in receiving this photon knowing it is measured as t_1, which is equal to the universal time T_1. and the interval would be Δt where $\Delta t = t_1\text{-}t_0$.

Knowing that the emitted photon is at time t_0 , Δt can be calculated. Knowing the velocity of light, c, a distance s_{AB} between E_A and O_B can be determined to be

$$s_{AB} = c\Delta t = c(t_1\text{-}t_0) = c(T_1\text{-}T_0) \qquad \textbf{ZV-E01}$$

Similarly, if the converse was initiated where the observer O_B emits a photon towards E_A, then **ZV-E01** would hold from an observer at the location of E_A.

This is a situation satisfying the postulate of a zero inertial frame of reference **Postulate : PRF02**. Entities initially do not interact with each other or with any other physical property present and universal time intervals and observer measured time intervals and clocks all match with one another. ie universal time equals observer times if all clocks are simultaneously synchronised by a universal observer in a universal frame of reference.

02-03 Constant velocity

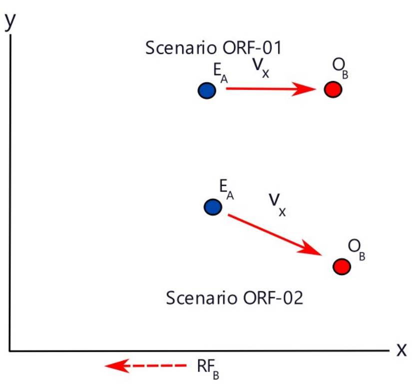


Fig ORF-01a
O_B Observes E_A approaching

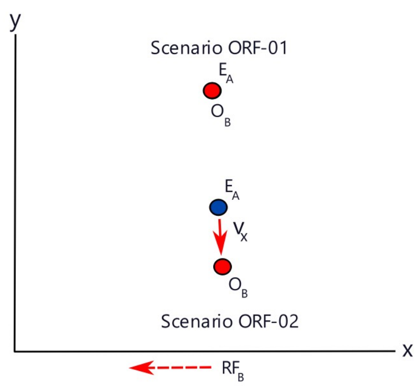


Fig ORF-01b
O_B Observes E_A at minimum distance

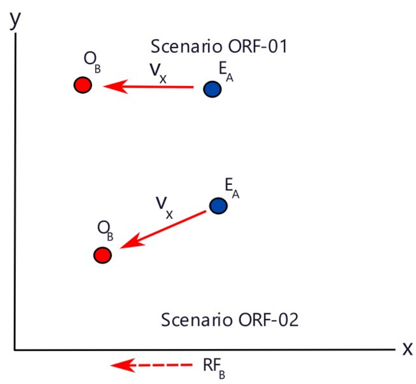


Fig ORF-01c
O_B Observes E_A receding

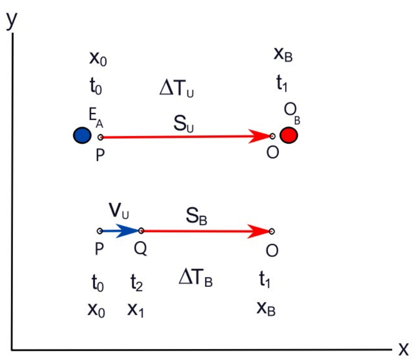
Consider that the situation of **Scenario A** as presented in **section 01 definitions** above is present, and that observer O_B is taking measurements of the distance, velocity and other physical properties of entity E_A from the reference frame FR_B. E_A approaches in a straight line trajectory towards O_B until it reaches a location designated as a measured minimum distance, and then recedes in a straight line trajectory from O_B in the opposite direction from the perspective of FR_B. Consider that O_B "observes" E_A from the interaction of photons emitted by E_A interacting with a photon detector in O_B reference frame.

There are two possible scenarios for this situation as illustrated in Fig ORF-01a, Fig ORF-01b and Fig ORF-01c

Scenario ORF-01: O_B motion is such that it has a zero minimum distance. Ie the location of O_B and E_A coincide at one instant of motion.

Scenario ORF-02: O_B motion is such that it has a non zero minimum distance. Ie the location of O_B and E_A never coincide at one instant of motion.

Analysis of Scenario ORF-01



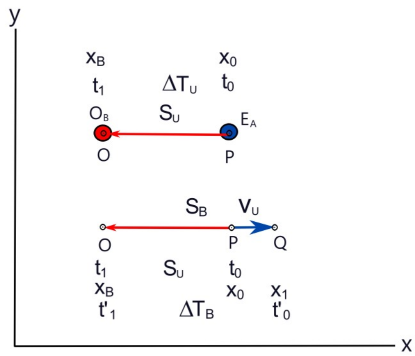


Fig ORF-02a
O_B Observes E_A approaching head on

Fig ORF-02b
O_B Observes E_A receding head on

Motion of approach ORF-01.1

Consider that the clocks in reference frame FR_A and FR_B have been synchronised in some way that they register the same time value as a universal clock C_U in the universal reference frame RF_U, when at some location $P = x_i$. Entity E_A emits a photon directly at O_B at a time t_0 equal to the universal time T_0.($t_0 = T_0$).(Fig ORF-02) If E_A was motionless in respect to an observer O_B, then the time t_1 that O_B would measure to receive the photon would be given by **ZV-E01** to be

$$s_{AB} = c\Delta t = c(t_1\text{-}t_0) = c(T_1\text{-}T_0) = c\Delta T_U \qquad \textbf{CV-E01}$$

However, if E_A is travelling directly at O_B at a universal frame of reference measured velocity of V_U, then when the photon is emitted, by classical physics it will have a net velocity of $c+V_U$. But according to Einsteins postulate and accepted experimental evidence that the speed of light is constant in all reference frames, this photon measured velocity must not change when measured by observer O_B. From the definition of velocity v being

$$v = \frac{\Delta x}{\Delta t} = \frac{(x_1 - x_0)}{(t_1 - t_0)}$$

In the universal frame of reference RF_U , x_0 is the coordinate in the universal reference frame when the photon was emitted by E_A at location P. But in the reference frame of O_B , x_0 is the coordinate in RF_U of E_A when it observes the emitted photon if it is to measure its velocity to be that of light c, and not c+v. Designate this coordinate x_1 in the reference frame of O_B to be

$$x_1 = x_0 + v_x \Delta T_U \qquad \textbf{CV-E02}$$

where v_X is the velocity of entity E_A in the universal reference frame RF_U.
=> distance measured s_B by O_B when the photon was emitted to be

$$s_B = (x_0 + c\Delta T_U) – x_1 \qquad \textbf{CV-E03a}$$

=> $$s_B = (x_0 + c\Delta T_U) - (x_0 + v_x\Delta T_U) \qquad \textbf{CV-E03b}$$

=> $$s_B = c\Delta T_U – v_x\Delta T_U \qquad \textbf{CV-E03c}$$

=> $$s_B = s_U – v_x\Delta T_U \qquad \textbf{CV-E03d}$$

If the distance s_U that the light travelled in RF_U is measured in the reference frame RF_B that O_B measures of a photon emitted from E_A, then to have the distance s_B as measured by O_B from which E_A emitted the photon travelling at a constant velocity v_x towards O_B needs to be equal to s_U. (Fig ORF-02)

Let x_0 be the location P in RF_U of E_A when the photon was emitted by E_A.
Let x_B be the location O in RF_U of E_B when the photon emitted by E_A was received by O_B.
Let x_1 be the location Q in RF_U of E_A that it had translated to through travelling in a direct pat towards O_B at a constant velocity v_x when the photon emitted by E_A was received by O_B. ie

Let ΔT_U be the time interval measured in RF_U that the photon emitted by E_A at location x_0 was received by O_B.
Let ΔT_B be the time interval as measured in RF_U would be if the photon emitted by E_A was at location $Q=x_1$ and was received by O_B.

By Einstein's postulate that the velocity of light, c, measured in all reference frames is constant at all instances. Then the velocity of light emitted at x_0 and observed by O_B at x_B would be equal to the velocity of light emitted at x_1 and observed by O_B at x_B. Thus

$$c = \frac{x_B - x_0}{\Delta T_U} = \frac{x_B - x_1}{\Delta T_B} \qquad \textbf{CV-E04a}$$

=>

$$\Delta T_B = \frac{x_B - x_1}{x_B - x_0} \Delta T_U \qquad \textbf{CV-E04b}$$

=> substituting $x_1 = x_0 + v_X\Delta T_U$ where v_X is the velocity of entity E_A in the universal reference frame RF_U.

$$\Delta T_B = \frac{x_B - (x_0 + v_X \Delta T_U)}{x_B - x_0} \Delta T_U \qquad \textbf{CV-E04c}$$

=>

$$\Delta T_B = \frac{(x_B - x_0) - v_X \Delta T_U}{x_B - x_0} \Delta T_U \qquad \textbf{CV-E04d}$$

=>

$$\Delta T_B = \left(1 - \frac{v_X \Delta T_U}{x_B - x_0}\right) \Delta T_U \qquad \textbf{CV-E04e}$$

=> since $\frac{(x_B - x_0)}{\Delta T_u}$ = c and $\frac{1}{\frac{(x_B - x_0)}{\Delta T_u}} = \frac{1}{(x_B - x_0)} \Delta T_u$ = 1/c have

$$\Delta T_B = \left(1 - \frac{v_X}{c}\right) \Delta T_U \qquad \textbf{CV-E05}$$

CV-E05 gives a measurement of time, ΔT_B, that the photon took to reach O_B from x_0 in the reference frame that O_B exists in with respect to that which a moving E_A exists in. This is equivalent to as if the photon was emitted at location x_1, in the reference frame of O_B. Because O_B exists in the same reference frame as the universal reference frame as it is motionless in the universal reference frame, then it can be said that $RF_B = RF_U$.

For a measurement of length to be measured by O_B in respect to that of the moving entity E_A, a second photon is emitted by E_A, and is emitted at the location x_1 as observed by a universal observer O_U in RF_U.

A time t_1 is recorded by O_B of the arrival of the first photon at the location O of O_B.
A time t_2 is recorded by O_B of the arrival at the second photon at the location O of O_B.

The measurement of length s_{B12} between these two photons by O_B is thus measured as

$$s_{B12} = c(t_2 - t_1) = c\Delta t_{B12} \qquad \textbf{CV-E06a}$$

where Δt_{B12} is the measured time interval that O_B observes between photon arrivals.

Given that O_B is in RF_U, and that in RF_U , $s_{U12} = v_x\Delta T_U$, where ΔT_U is the time interval as measured by a universal observer O_U in RF_U. then

$$s_{B12} = c(t_2 - t_1) = v_x\Delta T_U \qquad \textbf{CV-E06b}$$

$t_1 = \Delta T_U$, $t_2 = \Delta T_U + v_x\Delta T_U/c$
=>

$$s_{B12} = c\Delta t_{B12} = v_x\Delta T_U \qquad \textbf{CV-E06}$$

What this indicates is that measurements of length by O_B is the same as that of the universal reference frame. This should be expected as O_B is in the universal reference frame, and $RF_B = RF_U$.

Motion of receding ORF-01.2
If the opposite of the above situation described where E_A was receding from O_B, rather than approaching O_B, then it is not difficult to see that given a negative direction of velocity for E_A in respect to the location of O_B, **CV-E05** would become

$$\Delta T_B = \left(1+\frac{v_X}{c}\right) \Delta T_U \qquad \textbf{CV-E07}$$

and **CV-E06** would be

$$s_{B12} = c\Delta t_{B12} = -v_x\Delta T_U \qquad \textbf{CV-E08}$$

if one considers that length is a vector and is in the same direction as the motion of E_A.

Instant of coincident location ORF-01.3

Consider that on approach of E_A to O_B, at some instant, the location of E_A and O_B coincide and they overlap upon each other. At this instant, it can be considered that there zero time of transmission of any physical properties between them, and as a consequence, zero length of any physical properties such as space measurements. It may be said that either nothing happens between E_A and O_B, or that any interaction of transmission and reaction to transmission of physical phenomenon is truly simultaneous and occurs in zero time. It could be said that at this instance that both reference frames converge into being one common reference frame where all space and time between E_A and O_B coalesce into one state of physical being.

Interval of physical Process – approaching reference frames ORF-01.4

Consider that a physical process, such as the emission of a photon is not instantaneous and takes a period of time $\Delta t_P = t_1 – t_0$ to complete, which is constant as measured in all reference frames. t_0 being the time of initiating the process, t_1 being the time of completing the process.

Consider that this process begins at as performed by an entity E_A and is stationary in the reference frame of O_B. It is quite trivial to deduce that O_B will measure the interval of the process of emission of the photon through measuring its perceived wavelength as Δt_P.

Consider that E_A is in motion on a direct straight line path towards O_B at a velocity v_X, that E_A begins the process of emitting a photon at a location x_0, and as E_A continues moving towards O_B, it continues to emit the photon until it completes the emission process at location

$$x_1= x_0 + v_X \Delta t_P. \qquad \textbf{CV-E09}$$

At all times, the process of photon emission, the photon components are emitted at the velocity of light, c, within the reference frame of E_A.

In the reference frame of O_B, O_B observes the photon at the velocity of light, c and all of its components. The head of the photon travels a time $t_0 = \Delta t_U$ as described above. The end of the photon travels a time $t_1 = \Delta t_U + \Delta t_B$ as described above in deriving **CV-E05**. Consider that Δt_P which is constant as measured in all reference frames.

$$\Rightarrow \qquad \Delta t_P = t_1 – t_0 = \Delta T_U + \Delta T_B - \Delta T_U = \Delta T_B = \left(1-\frac{v_X}{c}\right) \Delta T_U \qquad \textbf{CV-E10}$$

Which is a contraction of the time measured time in the reference frame RF_B that O_B exists within, in respect to the time measured in the reference frame RF_A that the entiry E_A exists within. It is the same as **CV-E05** measuring two instances of motion of E_A travelling at velocity v_X directly towards O_B.

If it is considered that the measurement of length of a photon emitted by E_A travelling at velocity v_X is $c\Delta t_P$ which is equivalent to its physical property of a wavelength λ_P, then using **CV-E10** the measured wavelength of the photon can be deduced to be

$$\Delta\lambda_P = c\Delta t_P = c\Delta T_B = c\left(1-\frac{v_X}{c}\right)\Delta T_U = \left(c-v_X\right)\Delta T_U \qquad \textbf{CV-E11}$$

Which is a contraction of the measured wavelength in the reference frame RF_B that O_B exists within, in respect to the time measured in the reference frame RF_A that the entiry E_A exists within.

Interval of physical Process – receding reference frames ORF-01.5

Consider that E_A is in motion on a direct straight line path receding away from O_B at a velocity v_X, that E_A begins the process of emitting a photon at a location x_0, and as E_A continues moving towards O_B, it continues to emit the photon until it completes the emission process at location

$$x_1 = x_0 - v_X \Delta t_P. \qquad \textbf{CV-E012}$$

At all times, the process of photon emission, the photon components are emitted at the velocity of light, c, within the reference frame of E_A.

In the reference frame of O_B, O_B observes the photon at the velocity of light, c of all its components. The head of the photon travels a time $t_0 = \Delta t_U$ as described above. The end of the photon travels a time $t_1 = \Delta t_U + \Delta t_B$ as described above. Consider that Δt_P which is constant as measured in all reference frames where velocity is in the negative direction, and thus tB is that of **CV-E07**, obtain

$$\Rightarrow \qquad \Delta t_P = t_1 - t_0 = \Delta T_U + \Delta T_B - \Delta T_U = \Delta T_B = \left(1+\frac{v_X}{c}\right)\Delta T_U \qquad \textbf{CV-E13}$$

Which is an expansion of the time measured time in the reference frame RF_B that O_B exists within, in respect to the time measured in the reference frame RF_A that the entiry E_A exists within. It is the same as **CV-E07** measuring two instances of motion of E_A travelling at velocity v_X directly towards O_B.

If it is considered that the measurement of length of a photon emitted by E_A travelling at velocity v_X is $c\Delta t_P$ which is equivalent to its physical property of a wavelength λ_P, then using **CV-E13** the measured wavelength of the photon can be deduced to be

$$\Delta\lambda_P = c\Delta t_P = c\Delta T_B = c\left(1+\frac{v_X}{c}\right)\Delta T_U = \left(c+v_X\right)\Delta T_U \qquad \textbf{CV-E14}$$

Which is an expansion of the measured wavelength in the reference frame RF_B that O_B exists within, in respect to the time measured in the reference frame RF_A that the entiry E_A exists within.

Analysis of Scenario ORF-02

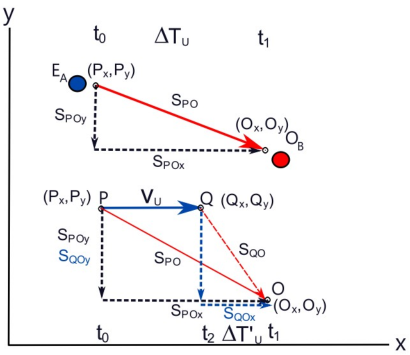


Fig ORF-03a
O_B Observes E_A approaching obliquely

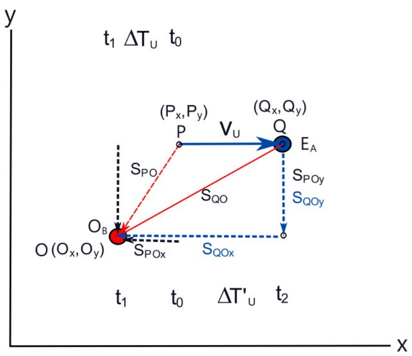


Fig ORF-03b
O_B Observes E_A receding obliquely

Motion of approach ORF-02.1
Consider that the clocks in reference frame FR_A and FR_B have been synchronised in some way that they register the same time value as a universal clock C_U in the universal reference frame RF_U, when at some location x_i. Entity E_A is travelling in a straight line towards and oblique to an observer O_B that does not intercept the location O of O_B, and at a location P emits a photon directly at O_B at a time t_0 equal to the universal time T_0.($t_0 = T_0$). If E_A was motionless in respect to an observer O_B, then the time that O_B would measure to receive the photon would be t_1, and the interval of time between emitting the photon by E_A and O_B receiving it would give a distance of travel from P to O as given by **ZV-E01** to be

$$s_{PO} = c\Delta t = c(t_1\text{-}t_0) = c(T_1\text{-}T_0) = c\Delta T_U \qquad \textbf{CV-E15}$$

Where T_1,T_0 are the universal time that a universal observer wou;d measure and ΔT_U the universal time interval. Designate t_1 to be the measured time t_B of the reference frame RF_B that O_B, exists within.

Now consider that in the time interval ΔT_U, E_A is moves at a constant velocity v_X to a location Q where

$$Q = P + v_X\,\Delta T_U \qquad \textbf{CV-E16}$$

and emits a photon at a universal time T'_0 in a direction that the observer O_B, first detects the emitted photon at a time T'_1, then the universal time interval of transmission and observation would give a distance from Q to O as

$$s_{PO} = c\Delta t' = c(t'_1\text{-}t'_0) = c(T'_1\text{-}T'_0) = c\Delta T'_U \qquad \textbf{CV-E17}$$

if it is considered that the velocity of light as measured by O_B of this photon is c. This would then imply from **CV-E15** and **CV-E17** that

$$c = \frac{S_{PO}}{\Delta T_U} = \frac{S_{QO}}{\Delta T'_U} \qquad \textbf{CV-E18}$$

=>

$$\Delta T'_U = \frac{S_{QO}}{S_{PO}}\Delta T_U \qquad \textbf{CV-E19}$$

Assuming that trigonometry in the universal frame of reference can be used, then

$$(S_{PO})^2 = (S_{POx})^2 + (S_{POy})^2$$

Where S_{POx} and S_{POy} are the x and y components of the distance S_{PO}. And since $S_{PO} = c\Delta t$ then

$$(c\Delta t)^2 = (c_x\Delta t)^2 + (c_y\Delta t)^2 \qquad \textbf{CV-E20}$$

Where c_x and c_y are the x and y components of c.

Similarly for S_{QO},

$$(S_{QO})^2 = (S_{QOx})^2 + (S_{QOy})^2$$

Where S_{QOx} and S_{QOy} are the x and y components of the distance S_{QO}. And since $S_{QO} = c\Delta t'$ then

$$(c\Delta t')^2 = (c_x\Delta t')^2 + (c_y\Delta t')^2 \qquad \textbf{CV-E21a}$$

$$S_{QOx} = O_x - Q_x = O_x - (P_x + v_x\Delta T_U) \qquad \textbf{CV-E21b}$$

$$S_{QOy} = O_y – Q_y = O_y – (P_y + v_y\Delta T_U)$$

and since $v_y = 0$

$$S_{QOy} = O_y – Q_y = O_y – P_y \qquad \textbf{CV-E21c}$$

=>

$$(S_{QO})^2 = (O_x – (P_x + v_x\Delta T_U))^2 + (O_y – P_y)^2 \qquad \textbf{CV-E21d}$$

expanding out the right hand side becomes

$$(O_x)^2 + (P_x + v_x\Delta T_U)^2 - 2(O_x(P_x + v_x\Delta T_U)) + (O_y)^2 + (P_y)^2 - 2O_yP_y$$

$$(O_x)^2 + (P_x^2) + (v_x\Delta T_U)^2 + 2(P_xv_x\Delta T_U) - 2(O_xP_x + O_xv_x\Delta T_U) + (O_y)^2 + (P_y)^2 - 2O_yP_y$$

$$(O_x)^2 + (O_y)^2 + (P_x^2) + (P_y)^2 - 2(O_xP_x + O_yP_y) + 2(P_xv_x\Delta T_U - O_xv_x\Delta T_U) + (v_x\Delta T_U)^2$$

$$(O_x)^2 + (O_y)^2 + (P_x^2) + (P_y)^2 - 2(O_xP_x + O_yP_y) + (v_x\Delta T_U)^2 + 2v_x\Delta T_U(P_x - O_x)$$

$$((O_x)^2 + (P_x^2) - 2O_xP_x) + ((O_y)^2 + (P_y)^2 - 2O_yP_y)) + (v_x\Delta T_U)(v_x\Delta T_U + 2(P_x - O_x))$$

$$(O_x - P_x)^2 + (O_y - P_y)^2 + (v_x\Delta T_U)(v_x\Delta T_U + 2(P_x - O_x))$$

and $O_x – P_x = S_{POx}$ and $O_y – P_y = S_{POy}$ =>

$$S_{POx}^2 + S_{POy}^2 + (v_x\Delta T_U)(v_x\Delta T_U + 2(P_x - O_x))$$

and $S_{POx}^2 + S_{POy}^2 = S_{PO}^2$ => **CV-E21d** can be written as

$$(S_{QO})^2 = (S_{PO})^2 + v_X\Delta T_U(v_X\Delta T_U + 2\Delta x_{PO}) \qquad \textbf{CV-E21}$$

where Δx_{PO} is $(P_X – O_X)$, the difference in the x coordinate of the initial emission of a photon from entity E_A at location P, and the observer x coordinate receiving that photon at location O.

Note : **CV-E21** is valid for a photon emitted from P and taking a period of time ΔT_U to reach the observer. Substituting **CV-E21** into **CV-E19**, obtained

$$\Delta T'_U = \frac{S_{QO}}{S_{PO}}\Delta T_U = \frac{\sqrt{S_{PO}^2 + v_x\Delta T_U(v_x\Delta T_U + 2\Delta x_{PO})}}{S_{PO}}\Delta T_U$$

$$= \sqrt{1+\frac{v_x\Delta T_U(v_x\Delta T_U+2\Delta x_{PO})}{S_{PO}^2}}\Delta T_U \qquad \textbf{CV-E22}$$

and since $S_{PO} = c\Delta T_U$ **CV-E22** becomes

$$\Delta T'_U = \frac{S_{QO}}{S_{PO}}\Delta T_U = \sqrt{1+\frac{v_x^2}{c^2}+\frac{2v_x\Delta x_{PO}}{c^2\Delta T_U}}\Delta T_U \qquad \textbf{CV-E23}$$

and $\dfrac{\Delta x_{PO}}{\Delta T_U} = \dfrac{P_x - O_x}{\Delta T_U} = -v_x$ =>

$$\Delta T'_U = \sqrt{1+\frac{v_x^2}{c^2}-\frac{2v_x^2}{c^2}}\Delta T_U \qquad \textbf{CV-E24a}$$

$$= \sqrt{1-\frac{v_x^2}{c^2}}\Delta T_U \qquad \textbf{CV-E24}$$

Which matches the Lorentz transformation.

Receding motion ORF-02.2

For a receding entity E_A moving away from O at a velocity v_x (Fig ORF-03b) , the location of P of the initial emission of a photon is closer to O than the emission of a photon when E_A has moved to location Q at the same time that a photon emitted at P is measured by at O. For this situation with the same arguments leading to **CV-E21d** an expression for the distance S_{QO} becomes

$$(S_{QO})^2 = ((P_x + v_x\Delta T_U) - O_x)^2 + (P_y – O_y)^2 \qquad \textbf{CV-E25}$$

Which when expanded out and evaluated similar to deriving **CV-E21** above gives

$$(S_{QO})^2 = (S_{PO})^2 + v_X\Delta T_U(v_X\Delta T_U + 2\Delta x_{PO}) \qquad \textbf{CV-E26}$$

which is **CV-E21** for an approaching entity emitting a photon, and which will give the same result of a relationship between ΔT_U and $\Delta T'_U$ as **CV-E24**.

Incidence at minimum distance ORF-02.3

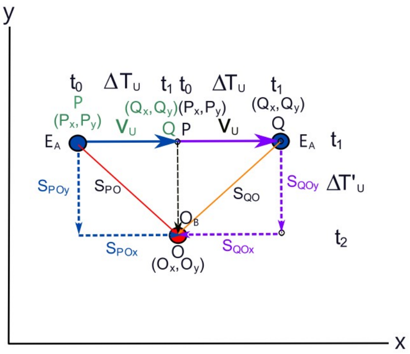


Fig ORF-04
O_B Observes E_A at minimum distance

For the instance of a photon emitted at a minimum distance, an approaching scenario would have $Q_x = O_x$, and a receding scenario would have $P_x = O_x$. For $Q_x = O_x$, $S_{QO} = S_{QOy}$ and for $P_x = O_x$,$S_{PO} = S_{POy}$

This would imply that on approach, a minimum distance of Q would give an expression of relationship between ΔT_U and $\Delta T'_U$ as a Pythagorean trigonometric problem. Distances $S_{QO} = S_{QOy} = c\Delta T'_U$,$S_{PO} = c\Delta T_U$ and $S_{PQ} = v_x\Delta T_U$. =>

$$S_{QO}^2 = S_{PO}^2 - S_{PQ}^2 \qquad \textbf{CV-E27a}$$

=> $$(c\Delta T'_U)^2 = (c\Delta T_U)^2 - (v_x\Delta T_U)^2 \qquad \textbf{CV-E27b}$$

=> $$\Delta T'_U = \sqrt{\Delta T_U^2 - \frac{(v_x \Delta T_U)^2}{c^2}} \qquad \textbf{CV-E27c}$$

=> $$\Delta T'_U = \sqrt{1 - \frac{v_x^2}{c^2}}\, T_U \qquad \textbf{CV-E27}$$

Which agrees with the previous derivation given by **CV-E24**

For entity E_A receding from O at a constant velocity v_x, have

$$S_{PO}^2 = S_{QO}^2 - S_{PO}^2 \qquad \textbf{CV-E28a}$$

=> $$(c\Delta T'_U)^2 = (c\Delta T_U)^2 - (v_x\Delta T_U)^2 \qquad \textbf{CV-E28b}$$

which is the same as **CV-E27b** and which therefore **CV-E27** also holds for an Entity receding away from an observer O.

Interval of physical Process – approaching reference frames ORF-02.4

For an entity E_A emitting a photon oblique to its motion, if the emission process occurs in a given interval such that as it moves the photon is generated, then it has to be assumed that somehow the direction of emission of the photon is always orientated to be in the direct direction towards the observer O. That is, the emission direction changes as the entity moves, otherwise the observer O will only sense the head of the photon unless the absorption process to measure the photon can encapsulate the photon over an area or volume and not at a single point. Whatever process can be made for observer O to observe an incoming photon, it is assumed that for the purposes of this exercise, an observation of a photon that has been generated in a given interval can be made.

Consider that a physical process, such as the emission of a photon is not instantaneous and takes a period of time $\Delta t_P = t_1 - t_0$ to complete, which is constant as measured in all reference frames. t_0 being the time of initiating the process, t_1 being the time of completing the process.

Consider that this process begins at as performed by an entity E_A and is stationary in the reference frame of O_B. It is quite trivial to deduce that O_B will measure the interval of the process of emission of the photon through measuring its perceived wavelength as

$$\lambda_U = c\Delta t_{PO} = c\Delta T_U \qquad \textbf{CV-E29a}$$

Consider that E_A is in motion on an oblique straight line path towards O_B at a velocity v_X, as illustrated in Fig ORF-03a, and that E_A begins the process of emitting a photon at a location P, and as E_A continues moving it continues to emit the photon until it completes the emission process at location Q. The distance

$$S_{QO} = c\Delta T'_U = \lambda'_U = c\sqrt{1 - \frac{v_x^2}{c^2}}\, T_U = \sqrt{c^2 - v_x^2}\, T_U = \sqrt{c^2 - v_x^2}\,\frac{\lambda_U}{c} = \lambda_U \sqrt{1 - \frac{v_x^2}{c^2}} \qquad \textbf{CV-E29}$$

is thus given as the measured wavelength at O of the emitted photon. By the same method, for a receding entity E_A, the same result would occur and **CV-E29** would apply.

Incidence at equal distance ORF-02.5

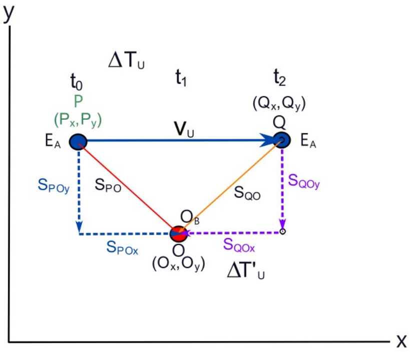


Fig ORF-05

Consider the situation where a photon has emitted from and entity E_A at location P as it approaches an observer O_B at location O at velocity v_x, and it takes a universal measured time ΔT_U to reach O_B. Then consider that a second photon is emitted at location Q where $Q = P + v_U \Delta T_U$ as E_A recedes away from O_B.(Fig ORF-05). The distance from P to O, S_{PO} is

$$S_{PO}^2 = S_{POx}^2 + S_{POy}^2 = (O_x - P_x)^2 + (O_y - P_y)^2 = (c\Delta T_U)^2 \quad \textbf{CV-E30a}$$

and The distance from Q to O, S_{PO} is

$$S_{QO}^2 = S_{QOx}^2 + S_{QOy}^2 = (Q_x - O_x)^2 + (Q_y - O_y)^2$$
$$= ((P_x + v_U\Delta T_U) - O_x)^2 + (Q_y - O_y)^2 = (c\Delta T'_U)^2 \quad \textbf{CV-E30b}$$

=>

$$(c\Delta T'_U)^2 = (P_x + v_U\Delta T_U)^2 + O_x^2 - 2\,O_x\,(P_x + v_U\Delta T_U) + (Q_y - O_y)^2$$
$$= P_x^2 + (v_U\Delta T)^2 + 2\,P_x v_U\Delta T + O_x^2 - 2\,O_x\,(P_x + v_U\Delta T_U) + (Q_y - O_y)^2$$
$$= P_x^2 + O_x^2 - 2\,O_xP_x + (v_U\Delta T_U)^2 + 2P_x v_U\Delta T_U - 2O_xv_U\Delta T_U + (Q_y - O_y)^2$$
$$= (P_x - O_x)^2 + (Q_y - O_y)^2 + (v_U\Delta T_U)^2 + 2v_U\Delta T_U(P_x - O_x)$$

and $(Q_y - O_y) = (P_y - O_y)$ =>

$$(c\Delta T'_U)^2 = (P_x - O_x)^2 + (P_y - O_y)^2 + (v_U\Delta T_U)^2 + 2v_U\Delta T_U(P_x - O_x)$$
$$= S_{PO}^2 + (v_U\Delta T_U)^2 + 2v_U\Delta T_U(P_x - O_x) \quad \textbf{CV-E30d}$$

Consider that $(P_x - O_x) = -(O_x - P_x)$.=>

$$(c\Delta T'_U)^2 = S_{PO}^2 + (v_U\Delta T_U)^2 - 2v_U\Delta T_U(O_x - P_x) \quad \textbf{CV-E30e}$$

Consider that $v_U = \dfrac{O_x - P_x}{\Delta T_{POx}}$ where ΔT_{POx} is the universal time it takes for the entity E_A to travel from location P_x to location O_x. Expressing ΔT_{POx} as a percentage or fraction term in the range of 0.0 to 1.0 of the time ΔT_U for the entity E_A to travel from location P_x to location Q_x.as $T_{POx}^{\%}$ gives $\Delta T_{POx} = T_{POx}^{\%}\,\Delta T_U$.=> $(O_x - P_x) = v_U T_{POx}^{\%}\,\Delta T_U$ and **CV-E30e** can be written as

$$(c\Delta T'_U)^2 = S_{PO}^2 + (v_U\Delta T_U)^2 - 2v_U\Delta T_U\,v_U T_{POx}^{\%}\,\Delta T_U$$

=>

$$(c\Delta T'_U)^2 = S_{PO}^2 + (v_U\Delta T_U)^2 - 2(v_U\Delta T_U)^2\,T_{POx}^{\%} \quad \textbf{CV-E30f}$$

and since $S_{PO} = c\Delta T_U$ then have

$$(c\Delta T'_U)^2 = (c\Delta T_U)^2 + (v_U\Delta T_U)^2 - 2(v_U\Delta T_U)^2\,T_{POx}^{\%}$$
$$= (c\Delta T_U)^2 + (1 - 2T_{POx}^{\%})(v_U\Delta T_U)^2 \quad \textbf{CV-E30g}$$

=>

$$\Delta T'_U = \sqrt{1 + \frac{v_U^2(1 - 2T_{POx}^{\%})}{c^2}}\,T_U \quad \textbf{CV-E30}$$

What **CV-E30** indicates is that depending upon the location of P and Q in respect to the observer at location O as a fraction of the distance that the entity E_A travels in time T_U, the relative measured time of a signal emitted at Q is not constant if P an Q are in opposite coordinate sectors. That is, if P is negative and Q positive or P is positive and Q negative. If P and Q are of the same coordinate sector of being both positive or negative, then $T_{POx}^{\%}$ would be equal to one at all instances, and $\Delta T'_U$ would become as **CV-E24**, the normal relativistic expression with a Lorentz transformation.

This is also a very specific and exceptional case where the emission of a photon by E_A at location P travelling at a velocity v_U is such that it is observed at location O at the same universal time another photon is emitted at Q that is of an opposite coordinate sector. In most instances for a single entity this may not be the case and could be quite rare. Consider that $T_{POx}^{\%} = 0.5$ meaning that distance $S_{PO} = S_{QO}$ are equal, and hence $\Delta T'_U = \Delta T_U$ which is expected. This is a potentially even more rare occurrence of such an event where velocities of an entity E_A are not high enough to cross to an opposite coordinate sector in respect to the observer location O.

Along with the derivation of motion of an entity E_A that is on a path incidental to the location of an observer given by **CV-E05** and **CV-E07**, different measurements of relative times and lengths may be made that are not of the Lorentz transformation type, but are of a modified Lorentz transformation as given in **CV-E30**.

03 : Rotational Reference Frames

03.1 Stationary rotational motion

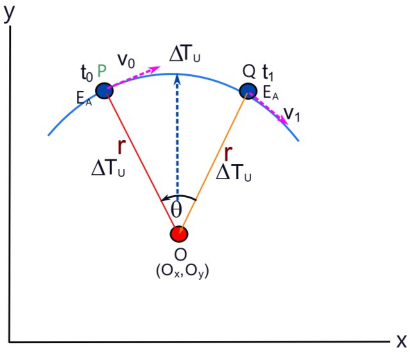


Fig 03-01

Frame of reference as observed by
a rotating observer.

Consider that an observer O exists within a universal reference frame RF_U and is stationary in RF_U. Consider that an entity E_A is also present and is stationary in respect to the reference frame of O. That is, E_A can be considered to be also within the universal reference frame RF_U. Consider that O has a form such that it can observe a rotation about an axis that is perpendicular to a plane between it and E_A, and that it is observed that O that is at the centre of this rotation, and that it is O that is rotating.(fig 03-01)

Consider that the measured distance between O and E_A is r, and that E_A emits a photon at a constant time interval of T_U within its reference frame, and that O rotates at a constant angle θ between emission and observing each photon.

From the perspective of O, if the facing of an observer is fixed and is rotating about an axis, E_A is perceived to be in a rotational motion about O at a constant distance r and angular velocity ω. The entity E_A is thus also perceived to have a changing linear velocity that is tangential to the perceived rotational motion about the observer O. Thus from the perspective of O, O observes a force acting on E_A centred at the location of O.

As illustrated in Fig 03-01, consider that at time t_0, E_A emits a photon that is observed by O after a universal time ΔT_U. After a time interval of ΔT_U, observer O perceives E_A to be at location P, at at this same instant in the universal reference frame that both observer and entity exist within, E_A emits a second photon that is observed by O after a universal time ΔT_U, and during observing the first photon and this second photon, O rotates by an angle θ and perceives that E_A is at location Q. Observer O is rotating anti clockwise in the universal reference frame. If Observer O deduces that E_A had travelled on a circular path as illustrated in Fig03-01, then in a time interval of ΔT_U, E_A has travelled a distance of $\theta r = c\Delta T'_U$, and has undergone a change in velocity according to the perceived velocity vector addition of $\Delta v = v_0 + v_1$. That is this reference frame of O observes an acceleration of E_A despite both having no motion in the reference frame that E_A exists in and which is the universal reference frame RF_U .

What this means is that a rotating entity in itself mathematically, if not physically may have intrinsic relativistic characteristics.

Observer O perception of E_A would be that the distance travelled by E_A would be

$$\theta r = c\Delta T'_U \qquad \textbf{RRF-E01}$$

and $r = c\Delta T_U$ =>

$$\theta\, c\Delta T_U = c\Delta T'_U \qquad \textbf{RRF-E02a}$$

=>

$$\theta\Delta T_U = \Delta T'_U \qquad \textbf{RRF-E02}$$

which would give a perception that the angle θ is an expression of the ratio of relative times between E_A and O. Expressing **RRF-E02** in terms of an angular velocity $\omega = \theta/\Delta T_U$ of O => $\theta = \omega\,\Delta T_U$ have

=>

$$\omega\Delta T_U^2 = \Delta T'_U \qquad \textbf{RRF-E03}$$

The perceived magnitude of rotational velocity of E_A by Observer O would be as defined rotational motion be

$$v = r\omega$$

and $r = c\Delta T_U$, and from => **RRF-E03** $\omega = \Delta T'_U/\Delta T_U^2$ =>

$$v = (c\,\Delta T_U)\frac{(\Delta T'_U)}{\Delta T_U^2} = c\frac{\Delta T'_U}{\Delta T_U} \qquad \textbf{RRF-E04}$$

or rearranging obtain

$$\Delta T'_U = \frac{v}{c}\Delta T_U \qquad \textbf{RRF-E05}$$

What **RRF-E05** seems to be indicating, is that a rotating observer can observe an entity to travel at speeds faster than that of light, and that any rate of rotation is possible. An implication is that with a fast enough rotation, an observer can observe E_A in a periodic pattern that is dependent upon the distance of O from E_A and rotational velocity ω.

However this is considering that O is a point object. If O is not a point object and is not an entity that can be of some structure that is impervious to rotational motion without limits, then it can be considered that **RRF-E05** has a limit according to Einstein's postulate that v < c, and that all observers have a limit on their rotational motion.

Change in velocity $\Delta\mathbf{v}$ of E_A is expressed as

$$\Delta\mathbf{v} = \mathbf{v_0} + \mathbf{v_1} = (v \cos \theta_1 + v \sin \theta_1) - (v \cos \theta_0 + v \sin \theta_0)$$

where v is the velocity magnitude which is constant and =>

$$\Delta\mathbf{v} = \mathbf{v_0} + \mathbf{v_1} = v\,(\cos \theta_1 - \cos \theta_0 + \sin \theta_1 - \sin \theta_0) \qquad \textbf{RRF-E06}$$

Consider That $\theta_0 = 0$ and using **RRF-E04** then have

$$\Delta\mathbf{v} = c\frac{\Delta T'_U}{\Delta T_U}(\cos \theta_1 - 1 + \sin \theta_1)$$

$$= c\frac{\Delta T'_U}{\Delta T_U}(\cos \theta_1 + \sin \theta_1 - 1) \qquad \textbf{RRF-E07}$$

Discrepancy between head on and oblique motion of entities

There seems to be a difference in the measurements that an observer O_B would measure of time and length that an entity E_A would emit as a photon if the motion of E_A was oblique, as opposed to a direct path that would have E_A and O_B “collide” with each other.

CV-E05 and **CV-E24** are expressions of the same quantity of measurement of the ratio of the time measurement that a moving entity would measure compared to a “stationary” entity. However they give different results as **CV-E05** looks more Doppler like, and **CV-E24** more special relativity like. Looking the derivations of **CV-E05** and **CV-E24**, this can only be explained that in deriving **CV-E05**, it has no y component of the velocity of light compared to **CV-E24** that does.

But it may also be considered that **CV-E24** is not realistic in that for a photon to be observed, the emission direction needs to rotate, at just the right angle so as to keep its form, and not have its components skim past the observer as the entity moves at a velocity v_x.

An analogy would be a bomber having all its bombs dropped one at a time having each bomb hitting the ground in a straight line and not all at one single location.

For all the bombs to hit at a single location, each successive bomb would need to have its velocity upon release reduced in a constant deceleratory manner. That would mean successive components of the photon need to travel as lower x component velocities. To maintain an overall constant speed of light, the y component would need to increase in the reference frame it was emitted in which would just simply re create the initial problem.

Thus an observer measuring such a photon would only observe the first bomb, as it were, and not any of the others as they pass by.

Does this mean that measurements of physical phenomenon differ not only in what reference frame that one is in, but also the relative direction of interaction, and that an observer is taking measurements of physical phenomenon?

Reference Frame as a source of observation of a Physical Force

non-embedded reference frame

Consider an observer O_r is rotating about a central axis at a radius r from an axis of rotation z, and rotates about this central axis with a constant angular velocity ω. The observer O_r can be considered to be in a rotational frame of reference RF_R. Consider that the observer O_r does not experience any restraint or "force" of movement away from, or towards the central axis, and r does not change. In other words, O_r is in a rotational reference frame RF_R that is an inertial frame of reference, and if an object was freely placed in this frame of reference by an observer O_r, it would be seen not to have any motion. Such a frame of reference can thus be considered as self contained and exist as an independent and non embedded reference frame RF_{ne}. It can be considered that such a reference frame is a universal reference frame RF_U.

embedded reference frame

Consider that O_r does experience a restraint or "force" of movement away from, or to the central axis and r does change in time. O_r is in a rotational reference frame RF_R that is a non inertial frame of reference, and if an object was freely placed in this frame of reference by an observer O_r, it would be seen to have motion. Such a frame of reference RF_e can thus be considered as non self contained and exists embedded within a lager reference frame RF_U.

Thus if the observer releases an object o at some instant that is free and not bound to the rotational motion reference frame that the observer is bound to. (ie there is no frictional force between object o and system that the observer is bound to.) it would be observed that the object o has its own inertial frame of reference RF_o, and gains the properties of motion based upon the location and motion of inertial frame RF_R of the observer when it was released.

Eg consider that the object was released at radius r from the center of rotation. It would then gain the properties of linear velocity of the point of rotation at the moment it was released and move at a constant velocity and direction relative to the reference frame that the rotational reference exists within. In other words, it travels in a straight line in the reference frame that the rotational reference frame exists in. From the point of view of the rotating reference frame, the released object moves away from the center of rotation according to the following.

At time of release at radius r from center of rotation, object o has linear velocity

$$v_0 = r\omega$$

consider that in the external reference frame that RF_e is embedded in, the direction of motion is in the +y axis.

Thus as the object moves in the +y direction, the observer will see it move in a spiral like motion away from point of release around the axis of rotation.

In time t, the object has moved distance v_0t in the +y direction and the change in radius at which the object is seen to change in the rotational reference frame is given by

$$r=\sqrt{\Delta y^2+r_0^2}$$

since $\Delta y = v_0\Delta t$

$$r=\sqrt{(v_0\Delta t)^2+r_0^2}$$

and $v_0\Delta t = r_0\omega$

$$r=\sqrt{(r_0\omega_0)^2+r_0^2}$$

$$r=r_0\sqrt{\omega_0^2+1} \qquad \textbf{ERF-01}$$

And thus from the perspective of the observer in the rotational reference frame, the object moves in a spiral motion away from the center axis of rotation and radius given by **ERF-01**.

Spherical Surface Field

Consider the concept of a field **S** that is defined to originate from some location in space that is designated as a point of origin such that the field is of a spherical shape, and that it can be modelled and represented as a surface or volume of vectors. At any given distance from the central point of origin, the individual vectors representing the field are all of the same magnitude and pointing outwards from the central point of origin, and thus are perpendicular to a spherical surface at a given radius r from this central point. Consider that the number of vectors represents a density or flux of the field, and that the total number of these vectors over a given spherical surface at radius r from the origin is constant. Thus over a given area of the spherical surface, a representation a field intensity or strength can be defined as the number of these field vectors per unit area. If evenly distributed, this would mean that as the area increases, the field intensity decreases as fewer field vectors per unit area are present, and that this intensity or flux of the field at radius r from the origin σ_r is decreased by a factor of

$$\frac{1}{area}=\frac{1}{4\pi r^2}$$

for a sphere where r is the radius of the surface of a surrounding sphere, and for a given total flux or charge Q over the entire surface

$$\sigma_r=\frac{Q}{area}=\frac{Q}{4\pi r^2}$$

Consider a Cartesian coordinate system of axis designated as i, j, k forms the basis of the spherical coordinate system such that the coordinates of a sphere is defined as

$$1=\hat{r_{Si}}\cos(\theta)\cos(\phi)+\hat{r_{Sj}}\sin(\theta)\cos(\phi)+\hat{r_{Sk}}\sin(\phi)$$

where $\hat{r_{Si}},\hat{r_{Sj}},\hat{r_{Sk}}$ represents the radius unit vectors that define the direction of the radius in the i, j , k axis.

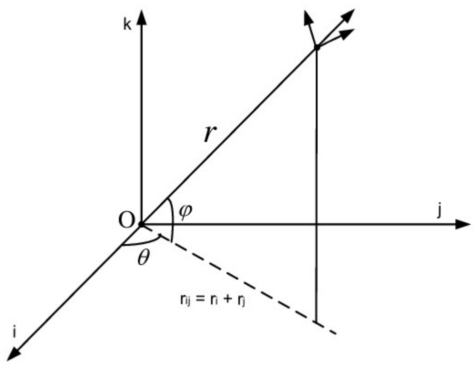


Fig SSF-01

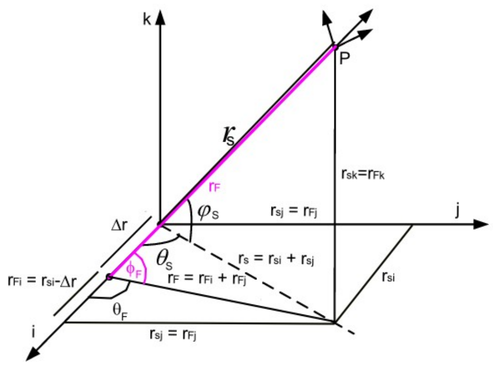


Fig SSF-02

The spherical coordinate convention used here are of the form as given in fig SSF-01 where θ is the angle on the i-j plane from the positive i direction axis in an anti clockwise direction looking perpendicular to this plane from above or in the direction of the -k vector. Φ is the angle from the i-j plane to the +k axis. This convention is used to have consistency and common to the i-j plane having its angle of rotation or vectors expressed as θ and to have angles Φ measured from this plane.

Consider that this field model origin is displaced by an amount Δr_i in the designated i axis direction. Now consider that the vector field as described above of this displaced field **F** is now overlapped upon the surface of the previous field surface at some radius r_s from the origin. What is given below is a mathematical treatment and description of this field **F** on the surface of the original field **S** at some radius r_s from the origin.

To help with this mathematical description, refer to Fig SSF-02 that gives a description of the relationship of **F** with **S**. From vector mathematics, it can be seen that the location of the origin of field **F** is (Δri,0j,0k), and the radius or distance from this location to point P on the surface of the surface S is

$$r_F=\sqrt{r_{Fi}^2+r_{Fj}^2+r_{Fk}^2}=\sqrt{(r_{Si}-\Delta r)^2+r_{Sj}^2+r_{Sk}^2}$$

Other relationships of angles and distances are given below.

Spherical surface vector

$$S=\hat{r_{Si}}\cos(\theta_S)\cos(\phi_S)+\hat{r_{Sj}}\sin(\theta_S)\cos(\phi_S)+\hat{r_{Sk}}\sin(\phi_S) \quad \text{- SSF-01}$$

Where

$\hat{r_{Si}},\hat{r_{Sj}},\hat{r_{Sk}}$ unit radial surface vector in the i, j, k vector axis direction such that unit radius of spherical surface

$$\hat{r_S}=\sqrt{\hat{r_{Si}}^2+\hat{r_{Sj}}^2+\hat{r_{Sk}}^2} \quad \text{- SSF-02}$$

θ_S,ϕ_S the spherical coordinate surface angels that define a coordinate on the spherical surface

Spherical field vector F

$$F=\hat{r_{Fi}}\cos(\theta_F)\cos(\phi_F)+\hat{r_{Fj}}\sin(\theta_F)\cos(\phi_F)+\hat{r_{Fk}}\sin(\phi_F) \quad \text{- SSF-03}$$

Where

$\hat{r_{Fi}}, \hat{r_{Fj}}, \hat{r_{Fk}}$ unit radial spherical field vector in the i, j, k vector axis direction such that unit radius of spherical field is given by

$$\hat{r_F} = \sqrt{\hat{r_{Fi}}^2 + \hat{r_{Fj}}^2 + \hat{r_{Fk}}^2} \qquad \text{- SSF-04}$$

θ_F, ϕ_F the spherical coordinate surface angels that define a coordinate on the spherical field.

Spherical field vector F coordinate relationship to spherical surface

$$\hat{r_{Fi}} = \frac{r_{Si} - \Delta r_i}{|r_F|}\hat{i} = \frac{r_{Si} - \Delta r_i}{|r_S - \Delta r|}\hat{i} \qquad \text{- SSF-05a'}$$

Which is the unit vector from the origin of the spherical field vector F in the i axis direction. r_{Si} is the spherical surface radial i component and Δr_i is the change in the radial component of the spherical surface origin in the i axis direction to the center of the spherical field vector F. | r_S - Δr | is the distance between the centers of S and F.

Now

$$r_F = |r_S - \Delta r| = \sqrt{(r_{Si} - \Delta r_i)^2 + (r_{Sj} - \Delta r_j)^2 + (r_{Sk} - \Delta r_k)^2}$$

and have $\Delta r_j = 0$, $\Delta r_k = 0$ =>

$$r_F = |r_S - \Delta r| = \sqrt{(r_{Si} - \Delta r_i)^2 + (r_{Sj})^2 + (r_{Sk})^2} \qquad \text{- SSF-05}$$

thus SSF-05a' becomes

$$\hat{r_{Fi}} = \frac{r_{Si} - \Delta r_i}{|r_S - \Delta r_i|}\hat{i} = \frac{r_{Si} - \Delta r_i}{\sqrt{(r_{Si} - \Delta r_i)^2 + (r_{Sj})^2 + (r_{Sk})^2}}\hat{i} \qquad \text{- SSF-05a}$$

Similarly

$$\hat{r_{Fj}} = \frac{r_{Sj} - \Delta r_j}{|r_S - \Delta r|}\hat{j} = \frac{r_{Sj}}{\sqrt{(r_{Si} - \Delta r_i)^2 + (r_{Sj})^2 + (r_{Sk})^2}}\hat{j} \qquad \text{- SSF-05b'}$$

$$\hat{r_{Fk}} = \frac{r_{Sk} - \Delta r_k}{|r_S - \Delta r|}\hat{k} = \frac{r_{Sk}}{\sqrt{(r_{Si} - \Delta r_i)^2 + (r_{Sj})^2 + (r_{Sk})^2}}\hat{k} \qquad \text{- SSF-05c'}$$

SSF-05b' and SSF-05c' are for r_F of a spherical field of radius $r_F = r_S - \Delta r_i = \sqrt{r_{Fi}^2 + (r_{Fj})^2 + (r_{Fk})^2}$. For r_F to be the radius to the spherical surface S from center location (Δr,0,0) in respect to the sphere S located at (0,0,0), by just looking at the geometric diagram, Fig SSF-02

$$\hat{r_{Fj}} = \hat{r_{Sj}} \qquad \text{- SSF-05b}$$

$$\hat{r_{Fk}} = \hat{r_{Sk}} \qquad \text{- SSF-05c}$$

From definition of a spherical surface

$$\hat{r_{Si}} = \hat{r_S}\cos(\theta_S)\cos(\phi_S) \qquad \text{- SSF-06a}$$

$$\hat{r_{Sj}} = \hat{r_S}\sin(\theta_S)\cos(\phi_S) \qquad \text{- SSF-06b}$$

$$\hat{r_{Sk}} = \hat{r_S}\sin(\phi_S) \qquad \text{- SSF-06c}$$

=>

$$\hat{r_{Fj}} = \hat{r_S}\sin(\theta_S)\cos(\phi_S) \qquad \text{- SSF-07a}$$

$$\hat{r_{Fk}} = \hat{r_S}\sin(\phi_S) \qquad \text{- SSF-07b}$$

and thus rF is related to RS by SSF-05a, SSF-07a and SSF-07b

in addition

$$\hat{r_F} = r_S - \Delta r = r_S - \Delta r_i \qquad \text{- SSF-07c}$$

From the rule of sines and using geometric diagram,

$$\frac{r_S}{\sin(\pi - \theta_F)} = \frac{r_F}{\sin(\theta_S)}$$

=> $$\sin(\pi - \theta_F) = \frac{r_S}{r_F}\sin(\theta_S)$$

and since from trigonometric relations $\sin(\pi-\theta_F)=\sin(\theta_F)$ have

$$\sin(\theta_F)=\frac{r_S}{r_F}\sin(\theta_S) \qquad \text{- SSF-08}$$

To find cos(θ$_F$), can use the vector relationship

$$\cos(\theta_F)=\frac{r_F\cdot\Delta r}{|r_F||\Delta r|}=\frac{r_{Fi}\cdot\Delta ri}{|r_F||\Delta r|}$$

now $\Delta r_i=\Delta r=1$ =>

$$\cos(\theta_F)=\frac{r_{Fi}}{|r_F|}=\frac{r_{Si}-\Delta r_i}{\sqrt{(r_{Si}-\Delta r_i)^2+(r_{Sj})^2+(r_{Sk})^2}}=\hat{r_{Fi}} \qquad \text{- SSF-09}$$

Similarly to deriving SSF-08

$$\sin(\phi_F)=\frac{r_S}{r_F}\sin(\phi_S) \qquad \text{- SSF-10}$$

To find cos(ϕ$_F$) need to consider that the projection from S onto the i-j plane has i_s defined by the vector rFi + rFj as in fig XXX thus from the definition of cos(angle) = adjacent/hypotenuse

$$\cos(\phi_F)=\frac{(r_{Fi}+r_{Fj})}{|r_F|}=\frac{(r_{Fi}+r_{Fj})}{\sqrt{r_{Fi}^2+r_{Fj}^2+r_{Fk}^2}}=\frac{(r_{Si}-\Delta r_i+r_{Sj})}{\sqrt{(r_{Si}-\Delta r_i+r_{Sj})^2+r_{Sj}^2+r_{Sk}^2}} \qquad \text{- SSF-11}$$

Divergence of F onto S Div$_{F\to S}$

Div$_{F\to S}$ =

F●S = $\hat{r_{Fi}}\cos(\theta_F)\cos(\phi_F)+\hat{r_{Fj}}\sin(\theta_F)\cos(\phi_F)+\hat{r_{Fk}}\sin(\phi_F) \bullet \hat{r_{Si}}\cos(\theta_S)\cos(\phi_S)+\hat{r_{Sj}}\sin(\theta_S)\cos(\phi_S)+\hat{r_{Sk}}\sin(\phi_S)$

$$= [\hat{r_{Fi}}\cos(\theta_F)\cos(\phi_F)\hat{r_{Si}}\cos(\theta_S)\cos(\phi_S)]+[\hat{r_{Fj}}\sin(\theta_F)\cos(\phi_F)\hat{r_{Sj}}\sin(\theta_S)\cos(\phi_S)]+[\hat{r_{Fk}}\sin(\phi_F)\hat{r_{Sk}}\sin(\phi_S)] \qquad \text{SDF-01}$$

substituting for $\sin(\theta_F),\cos(\theta_F),\sin(\phi_F),\cos(\phi_F)$ from SSF-08 to SSF-11 obtain

F●S

$$= \left[\hat{r_{Fi}}\hat{r_{Fi}}\frac{(r_{Fi}+r_{Fj})}{|r_F|}\hat{r_{Si}}\cos(\theta_S)\cos(\phi_S)\right] + \left[\hat{r_{Fj}}\frac{r_S}{r_F}\sin(\theta_S)\frac{(r_{Fi}+r_{Fj})}{|r_F|}\hat{r_{Sj}}\sin(\theta_S)\cos(\phi_S)\right] + \left[\hat{r_{Fk}}\frac{r_S}{r_F}\sin(\phi_S)\hat{r_{Sk}}\sin(\phi_S)\right]$$

$$= \left[\hat{r_{Fi}}^2\frac{(r_{Fi}+r_{Fj})}{|r_F|}\hat{r_{Si}}\cos(\theta_S)\cos(\phi_S)\right] + \left[\hat{r_{Fj}}\frac{r_S}{r_F}\frac{(r_{Fi}+r_{Fj})}{|r_F|}\hat{r_{Sj}}\sin^2(\theta_S)\cos(\phi_S)\right] + \left[\hat{r_{Fk}}\frac{r_S}{r_F}\hat{r_{Sk}}\sin^2(\phi_S)\right]$$

and substituting SSF-05a, SSF-05b and SSF-05c, have

$$\text{F●S} = \left[\left(\frac{r_{Si}-\Delta r_i}{\sqrt{(r_{Si}-\Delta r_i)^2+(r_{Sj})^2+(r_{Sk})^2}}\right)^2\left(\frac{(r_{Si}-\Delta r_i+r_{Sj})}{\sqrt{(r_{Si}-\Delta r_i+r_{Sj})^2+r_{Sj}^2+r_{Sk}^2}}\right)\hat{r_{Si}}\cos(\theta_S)\cos(\phi_S)\right]$$

$$+ \left[\hat{r_{Sj}}\frac{r_S}{r_F}\left(\frac{(r_{Si}-\Delta r_i+r_{Sj})}{\sqrt{(r_{Si}-\Delta r_i+r_{Sj})^2+r_{Sj}^2+r_{Sk}^2}}\right)\hat{r_{Sj}}\sin^2(\theta_S)\cos(\phi_S)\right]$$

$$+ \left[\hat{r_{Sk}}\frac{r_S}{r_F}\hat{r_{Sk}}\sin^2(\phi_S)\right] \qquad \text{SDF-02}$$

and using SSF-05 for r$_F$ have

$$= \left[\frac{(r_{Si}-\Delta r_i)^2}{(r_{Si}-\Delta r_i)^2+(r_{Sj})^2+(r_{Sk})^2}\left(\frac{(r_{Si}-\Delta r_i+r_{Sj})}{\sqrt{(r_{Si}-\Delta r_i+r_{Sj})^2+r_{Sj}^2+r_{Sk}^2}}\right)\hat{r_{Si}}\cos(\theta_S)\cos(\phi_S)\right]$$

$$+ \frac{r_S}{\sqrt{(r_{Si}-\Delta r_i)^2+r_{Sj}^2+r_{Sk}^2}}\left(\left(\frac{(r_{Si}-\Delta r_i+r_{Sj})}{\sqrt{(r_{Si}-\Delta r_i+r_{Sj})^2+r_{Sj}^2+r_{Sk}^2}}\right)\hat{r_{Sj}}^2\sin^2(\theta_S)\cos(\phi_S)\right]$$

¿

$$+ \quad \left[\frac{r_S}{\sqrt{(r_{Si}-\Delta r_i)^2+r_{Sj}^2+r_{Sk}^2}}\hat{r_{Sk}}^2\sin^2(\phi_S)\right] \qquad \text{SDF-03}$$

and by using relationships SSF-06a, SSF-06b, SSF-06c and SSF-07c and substituting them into SDF-03, the divergence of the spherical vector field F through the spherical surface can be expressed in terms of the spherical surface coordinate and the displacement Δr_i . These substitutions are not expressed here as the resultant expression would be very long and complicated and rather unnecessary as SDF-03 is enough to be able to see this. By use of a simple computer program, the result of F●S can be obtained and graphically displayed.

If $\Delta r_i = 0$ SDF-03 reduces to

$$F\bullet S = \left[\frac{(r_{Si})^2}{(r_{Si})^2+(r_{Sj})^2+(r_{Sk})^2}\left(\frac{(r_{Si}+r_{Sj})}{\sqrt{(r_{Si}-+r_{Sj})^2+r_{Sj}^2+r_{Sk}^2}}\right)\hat{r_{Si}}\cos(\theta_S)\cos(\phi_S)\right]$$

$$+ \frac{r_S}{\sqrt{r_{Si}^2+r_{Sj}^2+r_{Sk}^2}}\left(\left(\frac{(r_{Si}+r_{Sj})}{\sqrt{(r_{Si}+r_{Sj})^2+r_{Sj}^2+r_{Sk}^2}}\right)\hat{r_{Sj}}^2\sin^2(\theta_S)\cos(\phi_S)\right] + \left[\frac{r_S}{\sqrt{r_{Si}^2+r_{Sj}^2+r_{Sk}^2}}\hat{r_{Sk}}^2\sin^2(\phi_S)\right]$$

¿

And since $\frac{r_S}{\sqrt{r_{Si}^2+r_{Sj}^2+r_{Sk}^2}}=1$, $\frac{r_{Si}}{\sqrt{(r_{Si})^2+(r_{Sj})^2+(r_{Sk})^2}}=\hat{r_{Si}}=\cos(\theta_S)$ and $\frac{(r_{Si}+r_{Sj})}{\sqrt{(r_{Si}-+r_{Sj})^2+r_{Sj}^2+r_{Sk}^2}}=\cos(\phi_S)$

have

$$F\bullet S = \hat{r_{Si}}^2\cos^2(\theta_S)\cos^2(\phi_S) + \hat{r_{Sj}}^2\sin^2(\theta_S)\cos^2(\phi_S) + \hat{r_{Sk}}^2\sin^2(\phi_S) = 1$$

As it should be

Curl of F onto S $\text{Curl}_{F\to S}$

$$\text{Curl}_{F\to S} = F \times S = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ F_i & F_j & F_k\\ S_i & S_j & S_k\end{vmatrix} = [F_jS_k-F_kS_j]\hat{i}+[F_iS_k-F_kS_i]\hat{j}+[F_iS_j-F_jS_i]\hat{k} \qquad \text{- SCF-01a}$$

$$= \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \hat{r_{Fi}}\cos(\theta_F)\cos(\phi_F) & \hat{r_{Fj}}\sin(\theta_F)\cos(\phi_F) & \hat{r_{Fk}}\sin(\phi_F)\\ \hat{r_{Si}}\cos(\theta_S)\cos(\phi_S) & \hat{r_{Sj}}\sin(\theta_S)\cos(\phi_S) & \hat{r_{Sk}}\sin(\phi_S)\end{vmatrix} \qquad \text{- SCF-01}$$

Substituting for Fi,Fj,Fk Si,Sj,Sk from SSF-01 and SSF-03 obtain

$$F \times S = \left[(\hat{r_{Fj}}\sin(\theta_F)\cos(\phi_F))(\hat{r_{Sk}}\sin(\phi_S))-(\hat{r_{Sj}}\sin(\theta_S)\cos(\phi_S))(\hat{r_{Fk}}\sin(\phi_F))\right]\hat{i}$$

$$- \left[(\hat{r_{Fi}}\cos(\theta_F)\cos(\phi_F))(\hat{r_{Sk}}\sin(\phi_S))-(\hat{r_{Si}}\cos(\theta_S)\cos(\phi_S))(\hat{r_{Fk}}\sin(\phi_F))\right]\hat{j}$$

$$+ \left[(\hat{r_{Fi}}\cos(\theta_F)\cos(\phi_F))(\hat{r_{Sj}}\sin(\theta_S)\cos(\phi_S))-(\hat{r_{Si}}\cos(\theta_S)\cos(\phi_S))(\hat{r_{Fj}}\sin(\theta_F)\cos(\phi_F))\right]\hat{k}$$

substituting for $\sin(\theta_F), \cos(\theta_F), \sin(\phi_F), \cos(\phi_F)$ from SSF-08 to SSF-11 obtain

$$F \times S = \left[\left(\hat{r_{Fj}}\frac{r_S}{r_F}\sin(\theta_S)\frac{(r_{Fi}+r_{Fj})}{|r_F|}\right)(\hat{r_{Sk}}\sin(\phi_S))-(\hat{r_{Sj}}\sin(\theta_S)\cos(\phi_S))\left(\hat{r_{Fk}}\frac{r_S}{r_F}\sin(\phi_S)\right)\right]\hat{i}$$

$$- \left[\left(\hat{r_{Fi}}(\hat{r_{Fi}})\frac{(r_{Fi}+r_{Fj})}{|r_F|}\right)(\hat{r_{Sk}}\sin(\phi_S))-(\hat{r_{Si}}\cos(\theta_S)\cos(\phi_S))\left(\hat{r_{Fk}}\frac{r_S}{r_F}\sin(\phi_S)\right)\right]\hat{j}$$

$$+ \left[\left(\hat{r_{Fi}}\hat{r_{Fi}}\frac{(r_{Fi}+r_{Fj})}{|r_F|}\right)(\hat{r_{Sj}}\sin(\theta_S)\cos(\phi_S))-(\hat{r_{Si}}\cos(\theta_S)\cos(\phi_S))\left(\hat{r_{Fj}}\frac{r_S}{r_F}\sin(\theta_S)\frac{(r_{Fi}+r_{Fj})}{|r_F|}\right)\right]\hat{k} \qquad \text{- SCF-02}$$

and substituting SSF-05a, SSF-05b and SSF-05c, have

$$F \times S = \left[\left(\hat{r_{Sj}}\frac{r_S}{r_F}\sin(\theta_S)\frac{(r_{Fi}+r_{Sj})}{|r_F|}\right)(\hat{r_{Sk}}\sin(\phi_S))-(\hat{r_{Sj}}\sin(\theta_S)\cos(\phi_S))\left(\hat{r_{Sk}}\frac{r_S}{r_F}\sin(\phi_S)\right)\right]\hat{i}$$

$$- \left[\left(\frac{(r_{Si}-\Delta r_i)^2}{(r_{Si}-\Delta r_i)^2+(r_{Sj})^2+(r_{Sk})^2}\frac{(r_{Fi}+r_{Fj})}{|r_F|}\right)(\hat{r_{Sk}}\sin(\phi_S))-(\hat{r_{Si}}\cos(\theta_S)\cos(\phi_S))\left(\hat{r_{Sk}}\frac{r_S}{r_F}\sin(\phi_S)\right)\right]\hat{j}$$

$$+ \left[\left(\frac{(r_{Si}-\Delta r_i)^2}{(r_{Si}-\Delta r_i)^2+(r_{Sj})^2+(r_{Sk})^2}\frac{(r_{Fi}+r_{Fj})}{|r_F|}\right)(\hat{r_{Sj}}\sin(\theta_S)\cos(\phi_S))-(\hat{r_{Si}}\cos(\theta_S)\cos(\phi_S))\left(\hat{r_{Sj}}\frac{r_S}{r_F}\sin(\theta_S)\frac{(r_{Fi}+r_{Fj})}{|r_F|}\right)\right]\hat{k}$$

$$= \hat{r_{Sj}}\hat{r_{Sk}}\frac{r_S}{r_F}\sin(\theta_S)\sin(\phi_S)\left[\frac{(r_{Fi}+r_{Sj})}{|r_F|}-\cos(\phi_S)\right]\hat{i}$$

$$- \hat{r_{Sk}}\sin\frac{(\phi_S)}{r_F}\left[\left(\frac{(r_{Si}-\Delta r_i)^2}{(r_{Si}-\Delta r_i)^2+(r_{Sj})^2+(r_{Sk})^2}(r_{Fi}+r_{Fj})\right)-(\hat{r_{Si}}\cos(\theta_S)\cos(\phi_S))(r_S)\right]\hat{j}$$

$$+ \hat{r_{Sj}}\frac{(r_{Fi}+r_{Fj})}{|r_F|}\sin(\theta_S)\cos(\phi_S)\left[\left(\frac{(r_{Si}-\Delta r_i)^2}{(r_{Si}-\Delta r_i)^2+(r_{Sj})^2+(r_{Sk})^2}\right)-(\hat{r_{Si}}\cos(\theta_S))\left(\frac{r_S}{r_F}\right)\right]\hat{k} \qquad \text{- SCF-03}$$

Now $r_{Fi}=r_{Si}-\Delta r_i$, $r_{Fj} = r_{Sj}$ $r_{Fk} = r_{Sk}$ and using SSF-05 for r_F have

F **x** S = $$\frac{\hat{r_{Sj}}\hat{r_{Sk}}r_S\sin(\theta_S)\sin(\phi_S)}{\sqrt{(r_{Si}-\Delta r_i)^2+(r_{Sj})^2+(r_{Sk})^2}}\left[\frac{(r_{si}-\Delta r_i+r_{Sj})}{\sqrt{(r_{Si}-\Delta r_i)^2+(r_{Sj})^2+(r_{Sk})^2}}-\cos(\phi_S)\right]\hat{i}$$

$$- \frac{\hat{r_{Sk}}\sin(\phi_S)}{\sqrt{(r_{Si}-\Delta r_i)^2+(r_{Sj})^2+(r_{Sk})^2}}\left[\left(\frac{(r_{Si}-\Delta r_i)^2}{(r_{Si}-\Delta r_i)^2+(r_{Sj})^2+(r_{Sk})^2}(r_{Si}-\Delta r_i+r_{Sj})\right)-(\hat{r_{Si}}\cos(\theta_S)\cos(\phi_S))(r_S)\right]\hat{j}$$

$$+ \frac{\hat{r_{Sj}}(r_{Si}-\Delta r_i+r_{Sj})\sin(\theta_S)\cos(\phi_S)}{\sqrt{(r_{Si}-\Delta r_i)^2+(r_{Sj})^2+(r_{Sk})^2}}\left[\left(\frac{(r_{Si}-\Delta r_i)^2}{(r_{Si}-\Delta r_i)^2+(r_{Sj})^2+(r_{Sk})^2}\right)-(\hat{r_{Si}}\cos(\theta_S))\left(\frac{r_S}{\sqrt{(r_{Si}-\Delta r_i)^2+(r_{Sj})^2+(r_{Sk})^2}}\right)\right]\hat{k}$$

$$= \frac{\hat{r_{Sj}}\hat{r_{Sk}}r_S\sin(\theta_S)\sin(\phi_S)}{\sqrt{(r_{Si}-\Delta r_i)^2+(r_{Sj})^2+(r_{Sk})^2}}\left[\frac{(r_{si}-\Delta r_i+r_{Sj})}{\sqrt{(r_{Si}-\Delta r_i)^2+(r_{Sj})^2+(r_{Sk})^2}}-\cos(\phi_S)\right]\hat{i}$$

$$- \frac{\hat{r_{Sk}}\sin(\phi_S)}{\sqrt{(r_{Si}-\Delta r_i)^2+(r_{Sj})^2+(r_{Sk})^2}}\left[\left(\frac{(r_{Si}-\Delta r_i)^2}{(r_{Si}-\Delta r_i)^2+(r_{Sj})^2+(r_{Sk})^2}(r_{Si}-\Delta r_i+r_{Sj})\right)-(\hat{r_{Si}}\cos(\theta_S)\cos(\phi_S))(r_S)\right]\hat{j}$$

$$+ \frac{\hat{r_{Sj}}(r_{Si}-\Delta r_i+r_{Sj})\sin(\theta_S)\cos(\phi_S)}{(r_{Si}-\Delta r_i)^2+(r_{Sj})^2+(r_{Sk})^2}\left[\left(\frac{(r_{Si}-\Delta r_i)^2}{\sqrt{(r_{Si}-\Delta r_i)^2+(r_{Sj})^2+(r_{Sk})^2}}\right)-(\hat{r_{Si}}\cos(\theta_S))(r_S)\right]\hat{k} \qquad \text{- SCF-04}$$

If $\Delta r_i = 0$ SCF-04 reduces to

F **x** S = $$\frac{\hat{r_{Sj}}\hat{r_{Sk}}r_S\sin(\theta_S)\sin(\phi_S)}{\sqrt{(r_{Si})^2+(r_{Sj})^2+(r_{Sk})^2}}\left[\frac{(r_{si}+r_{Sj})}{\sqrt{(r_{Si})^2+(r_{Sj})^2+(r_{Sk})^2}}-\cos(\phi_S)\right]\hat{i}$$

$$- \frac{\hat{r_{Sk}}\sin(\phi_S)}{\sqrt{(r_{Si})^2+(r_{Sj})^2+(r_{Sk})^2}}\left[\left(\frac{(r_{Si})^2}{(r_{Si})^2+(r_{Sj})^2+(r_{Sk})^2}(r_{Si}+r_{Fj})\right)-(\hat{r_{Si}}\cos(\theta_S)\cos(\phi_S))(r_S)\right]\hat{j}$$

$$+ \frac{\hat{r_{Sj}}(r_{Si}+r_{Sj})\sin(\theta_S)\cos(\phi_S)}{(r_{Si})^2+(r_{Sj})^2+(r_{Sk})^2}\left[\left(\frac{(r_{Si})^2}{\sqrt{(r_{Si})^2+(r_{Sj})^2+(r_{Sk})^2}}\right)-(\hat{r_{Si}}\cos(\theta_S))(r_S)\right]\hat{k}$$

now $\frac{(r_{si}+r_{Sj})}{\sqrt{(r_{Si})^2+(r_{Sj})^2+(r_{Sk})^2}}=\cos(\phi_S)$, $\frac{(r_{Si})^2}{(r_{Si})^2+(r_{Sj})^2+(r_{Sk})^2}$ = $\hat{r_{Si}}^2$ = $\hat{r_{Si}}\cos(\theta_S)$, $\frac{(r_S)}{\sqrt{(r_{Si})^2+(r_{Sj})^2+(r_{Sk})^2}}$ = 1

thus have

F **x** S = $$0\hat{i} - \hat{r_{Sk}}\sin(\phi_S)\left[(\hat{r_{Si}}\cos(\theta_S)\cos(\phi_S))-(\hat{r_{Si}}\cos(\theta_S)\cos(\phi_S))\right]\hat{j}$$

$$+ \hat{r_{Sj}}\cos(\phi_S)\sin(\theta_S)\cos(\phi_S)\left[(\hat{r_{Si}}\cos(\theta_S))-(\hat{r_{Si}}\cos(\theta_S))\right]\hat{k}$$

= $0\hat{i} - 0\,\hat{j} + 0\hat{k}$

as it should be

Cartesian coordinates

Using Cartesian coordinates where x_S, y_S, z_S are the Cartesian coordinates on the surface of a sphere that the initial field vectors of a field **S** are defined from a field that has as its center the origin and a radius of this sphere r_S where

$$r_S = \sqrt{r_{Si}^2 + r_{Sj}^2 + r_{Sk}^2} \qquad \text{- SCF-05}$$

where r_{Si} = x_S, r_{Sj} = y_S, r_{Sk} = z_S then the radius to this surface coordinate from a field **F** displaced by coordinate (Δx,0,0) as described above is r_F where

$$r_F = \sqrt{r_{Fi}^2 + r_{Fj}^2 + r_{Fk}^2} \qquad \text{- SCF-06a}$$

where r_{Fi} = r_{Si} - Δx and r_{Fi} = r_{Si}, r_{SFi} = r_{Sk} => r_{Fi} = x_S - Δx and r_{Fi} = y_S, r_{Fk} = z_S and radius r_F from center of field **F** to surface of field **S** at radius r_S from center of field **S** is

$$r_F = \sqrt{(x_S - \Delta x)^2 + y_S^2 + z_S^2} \qquad \text{- SCF-06}$$

Thus

$\text{Div}_{F\to S}$ = F●S = $((x_S - \Delta x), y_S, z_S) \;\bullet\; (x_S, y_S, z_S)$

$$= (x_S(x_S - \Delta x) + y_S^2 + z_S^2) \qquad \text{- SCF-07}$$

And

$\text{Curl}_{F\to S}$ = F **x** S = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ F_i & F_j & F_k \\ S_i & S_j & S_k \end{vmatrix}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ (x_S - \Delta x) & y_S & z_S \\ x_S & y_S & z_S \end{vmatrix} \qquad \text{- SCF-08a}$$

$$= 0\hat{i} - z_S[(x_S - \Delta x) - x_S]\hat{j} + y_S[(x_S - \Delta x) - x_S]\hat{k} \qquad \text{- SCF-08}$$

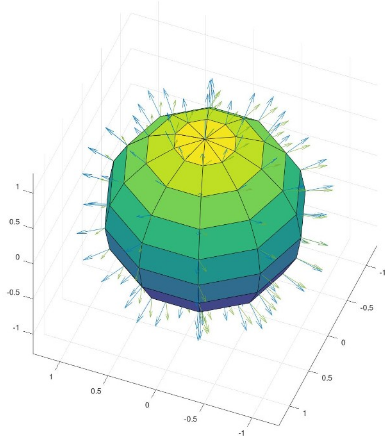
As Δx → x_S or Δx → 0 $\text{Div}_{F\to S}$ = F●S → r_S^2 and $\text{Curl}_{F\to S}$ = F **x** S → $0\hat{i} - 0\,\hat{j} + 0\hat{k}$

as they should.

Illustration of change in field location

(to be edited and modified)

In **Fig SCF01a** and **Fig SCF01b** is illustrated a resultant field on an initial spherical surface S of a field S centered at the origin (0,0,0), and the vector representation of this field given by the blue arrows. If the Field is displaced to a location (0.5,0,0) the field **F** on this surface at the same location on this surface is represented by the green arrows. The divergence of **F** on this surface **S** is given by SCF-07.



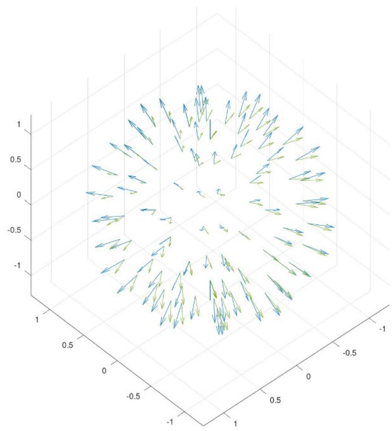
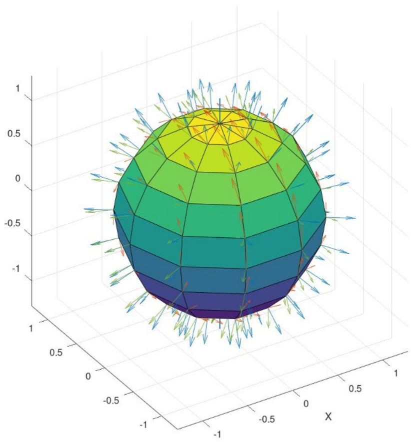


Fig SCF01a **Divergence of field F from S** **Fig SCF01b**



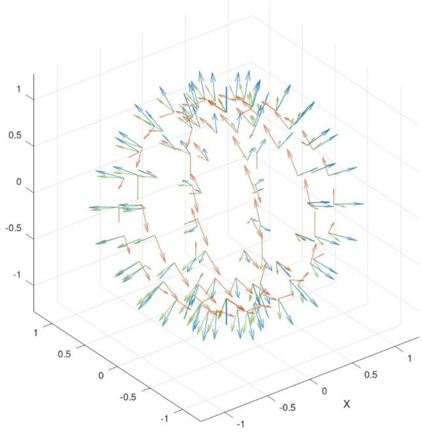


Fig SCF02a **Curl of field F and S** **Fig SCF02b**

In **Fig SCF02a** and **Fig SCF02b** has illustrated the resultant curl of the vectors representing the fields **S** and **F** as given by use of equation **SCF-08**, which are displayed as red colored arrows. As can be seen, and expected, these arrows rotate about the x axis and on the surface of the initial field **S**. The length of these vectors increases with the distance from the x axis that they rotate about and are not an indication of anything other than the direction of the vector cross product or curl operation. What this suggests is that all spherical fields have this curl property, at least as a mathematical product if it is in motion and the displaced field **F** can interact with the initial Field **S**.