An Understanding of The Reality of Mathematics

Mathematics has been a successful tool used by man for accounting, engineering and to describe and predict the workings of nature which in modern times as well as in the past. It is essentially a human invention consisting of abstracting the real world into a system of statements (axioms or proofs) and symbols so as to create models that can be understood by the human mind as a representation of the description of the real world. The simple abstract mathematical relationship or equation represented using the alphabetic symbols in **M01** below that is taught in basic mathematics gives a value represented by C as being the mathematical addition of two values represented by A and B.

$$C = A + B \qquad \textbf{M01}$$

This abstraction of numbers forms the most basic foundation of all mathematics, but it is meaningless unless a context of what A, B and C are is given. C could represent the total population of a country, while A and B the number of Males and Females.

So mathematics in its pure form without a representation in the real world can loose its meaning or relevance. This is the danger of using mathematics as a tool to reflect the real world. If no check is put into place as to what the symbols represents, then what a mathematical result or relationship may be interpreted to represent may be totally incorrect and even lost.

M01.0 : Mathematical Context

In order for mathematics to have meaning in the real physical world, it must have a context associated with the mathematics being used to make sense and be plausible in its use. Take the most basic mathematical statement of

1+1 = 2. What does this mean in the real world? Replace the numbers with an object, say an apple, and the meaning can become obvious. One apple and one apple becomes two apples. But what about if one has one apple and one orange? One could now say that 1+1 ≠ 2 because apples and oranges are not of the same type, thus the mathematical equation **M01** becomes invalid as A and B are not of the same type or same contextual content. However, if an apple and an orange are classified in context as fruit, then one apple and one orange are two types of fruit, and **M01** becomes valid if A, B and C represents fruit as a category, and the mathematical operation of addition is not dependent upon the individual types of fruit that A,B and C represents.

Thus mathematical statements need a context so as to have meaning, relevance and be truthful in interpreting the results that a mathematical statement or equation gives. A more accurate meaning of a mathematical statement such as 1+1=2 could be $^a1+^a1 = {^a2}$ or ${}^a2=\sum_{i=0}^{i=2} {}^a1$ where the superscript a before the number gives an abstract symbol for a physical or real world context of the type of object that the number represents. Thus the mathematical statement $^a1+^b1 \neq {^a2}$, ${}^a2\neq\sum_{i=0}^{i=1} {}^a1+\sum_{i=0}^{i=1} {}^b1$,$^a1+^b1 \neq {^b2}$, ${}^b2\neq\sum_{i=0}^{i=1} {}^a1+\sum_{i=0}^{i=1} {}^b1$ can be written and stated in context.

Of course, if the mathematics represents objects all of the same type or context, this can be negated and the method of writing mathematical equations not be altered. Thus a more abstract statement of this concept could be that the addition of two or more numbers has the relationship

$${}^aC=\sum_{i=0}^{i=n} {}^a c_i \qquad \textbf{EM01.0.1}$$

Where C is the total sum of n numbers, c_i is any number of the same object or number type a. If c_i is not of all of the same object or number type, then **EM01.0.1** becomes invalid. If there is no limit to what value n can take, then C in **EM01.0.1** can take on any value.

M01.1: Context of Mathematical Operations.

The most basic, and probably first use of mathematics by the human mind is to add and subtract objects that eventually became abstract symbols to represent numbers, and then from this, a number system itself was developed. Other concepts such as multiplication developed as a shorthand method of addition was developed and applied, which then invoked the concept of division to group large groups of objects into smaller number objects. More than likely, all this developed as a part of a society that needed some basic maths for the purposes of trade commerce, and the administration and organisation of societies. Maths had real world context and application.

Addition, subtraction, multiplication and division are mathematical operations on numbers. Multiplication can be thought of as an addition of groups of numbers, and division can be thought as the opposite of multiplication and is a form of subtraction of groups of numbers from a larger number to obtain a number of groups of numbers.

With the argument in section **M01.0**, such mathematical operations must also be performed with a context of what the numbers represent to have meaning and relevance.

M01.1.1: Subtraction

The basic mathematical statement of subtraction of two numbers is represented by

$$C = A - B \qquad \textbf{EM01.1.1.0}$$

cannot be performed if A and B are not of the same type of contextual object. One cannot subtract two apples from any number of oranges for example as apples and oranges are different types of incompatible objects. However, if apples and oranges are classified as fruit and are thus equivalent, then **EM01.1.1.0** can be performed only if A,B and C are abstract representations of fruit, not caring what kind of fruit A,B or C represent.

For a subtraction to occur in the real physical world, physical objects need to exit and be present. Thus the value of A in **EM01.1.1.0** must be a number that in the mathematical domain is positive, and so must also be the case with B. By taking B away from A, C is derived. If A is greater than B, then C has some objects remaining over. However, if B is of a greater number than A, then zero objects remain. Therefore, in a real world physical sense of objects, subtraction has a lowest limit values of zero. In other words, in the physical real world of objects, negative objects do not exist. One cannot have negative apples on a tree if one wants to pick more apples than exist on a tree. In this context of real world objects, negative numbers do not exist and have no relevance.

In the same rational as used in section **M01.0** to create **EM01.0.1** obtain a concept for subtraction

$$^{a}C = {}^{a}A - {}^{a}B \qquad \textbf{EM01.1.1.1}$$

where A, B, C ≥ 0 and are all of the same objects of type a.

M01.1.2: Multiplication

The basic mathematical statement of multiplication of two numbers is commonly represented by

$$C = A \times B \qquad \textbf{EM01.1.2.0}$$

Multiplication in essence is a summation of the number A, performed B times. That is if A = 3 and B = 2, then **EM01.1.2.0** is equivalent to C = A+A = 3+3 = 6. Conversely, can have C = B + B + B = 2 + 2 + 2 = 6. Thus multiplication can be regarded as equivalent to the expression for a summation of n numbers given by **EM01.0.1**. Using the same arguments that the numbers represented by A and B must be of the same contextual type as for the addition of two numbers in section **M01.0** to create **EM01.0.1**, an expression of the multiplication of two numbers can be expressed as

$$^{a}C = {}^{a}A \times {}^{a}B = \sum_{0}^{B} {}^{a}A \qquad \textbf{EM01.1.2.1}$$

Where C is the total sum of objects of type a, A is the number of objects of type a, and B, also a number of object type a is the number of times A has the addition operator applied to form a total. Conversely as demonstrated above and proven in many mathematical derivations.

$$^{a}C = {}^{a}B \times {}^{a}A = \sum_{0}^{A} {}^{a}B \qquad \textbf{EM01.1.2.2}$$

=> $$^{a}C = {}^{a}A \times {}^{a}B = {}^{a}B \times {}^{a}A \qquad \textbf{EM01.1.2.3}$$

=> the order in which two numbers representing the same object type gives the same resultant total and is said to be commutative. A basis of definition of multiplication in mathematics.

M01.1.3: Division

The basic mathematical statement of division of two numbers is commonly represented by

$$B = C / A \qquad \textbf{EM01.1.3.0}$$

Division in essence is the opposite of multiplication, and the number represented by B is how many groups of objects of number A are present within the number C. For example, if have C = 12 and A = 4, then the result B would be 3. Thus representing the division being the opposite of multiplication can be represented by

EM01.1.3.1

Using the same arguments that the numbers represented by A and B must be of the same contextual type as for the addition of two numbers in section **M01.0** to create **EM01.0.1**, an expression of the division of two numbers can be expressed as

$$^{a}B = \frac{^{a}C}{^{a}A} = ({}^{a}B \times {}^{a}A)^{inv} \qquad \textbf{EM01.1.3.2}$$

Where C is the total sum of objects of type a, A is the number of objects of type a to find that is in C that gives the result represented by B. Thus if swapping both A for B in the division side of the expression to give the number of times B can occur in C, the same swap needs to be done in the multiplication side. In other words in the case of division, to conform to the definition of multiplication as given in **EM01.1.2.1** and **EM01.1.2.2**. Thus division of two numbers that can be multiplied together can b said to be strictly not commutative.

M01.1.4 : Mathematical Context of Generating a Number Representing of a New Contextual Nature.

In many cases in mathematics, numbers of different object contextual types have operations performed on them that generate a number that has a different or no contextual meaning to that of the individual numbers on their own. An example can be that of graphical Cartesian coordinates, where two designated coordinate types x and y on a flat two dimensional plane that represent a length or distance in different directions are multiplied together. The resultant number of this multiplication is interpreted and designated in the context of an area. This is different from the multiplication of a coordinate, say x with a number representing the same coordinate type as a length that gives a resultant number representing an increase in x or the length of the coordinate. An area can be interpreted as having a totally different meaning and context to a length or distance, even if it is defined using the same units as the length, eg meters squared.

Continuing with the example of area, mathematical operations of numbers representing different units or of contextual objects are often performed to give a result that is then interpreted as a new quantity of a new type of object, concept, or simply to create an equation of relationship between two or more properties of an object or objects to gain an understanding of how certain properties interact or are related to each other. This may be logical or even desired, but the danger is that in doing so, a fundamental understanding of the physical process or processes of the interaction and relationship between objects, their properties, and other factors may get lost or be misinterpreted.

In some instances, it makes no sense, or has not ability (ie is invalid) to perform a mathematical operation on two numbers of different object types. Eq if one number A, represents a number of apples, and B represents a number of oranges, the equation C = A x B makes no sense and is invalid as apples and oranges are not alike in any manner, do not share any of the same properties, and cannot be merged together. Thus the concept of multiplying apples with oranges being invalid can be translated into mathematical tools used to model the real physical world that only operations on numbers that represent objects or properties that have the same or compatible physical context of form can be valid to use.

With this in mind, a way of thinking of understanding the universe by using mathematical models needs to have a consideration that the model needs to include a context that the mathematical operations performed within the model needs to have a consistency of like with like objects or properties to produce a new physical concept of contextual meaning. The example of area is that of an enclosed space in which physics can happen and be measured, and even equations of relationships derived to that area, even if area in reality has no fundamental physical process attached to it, only the physical processes at locations within that area.

Take the simple physical equation of

$$PV = nRT \qquad \textbf{EM01.1.4.1}$$

giving the relationship of properties for an ideal gas, where P is the measured pressure of a gas within a volume V, and T the measured temperature. R is a constant of proportionality and n the number of molecules of gas. Pressure is a defined quantity of force per unit area on a surface. This force is dependent upon the kinetic energy of the gas molecules impacting upon a surface area and how many molecules are impacting upon a surface at the instant of measurement. This kinetic energy is determined by how fast the molecules of gas are travelling, and since the energy of any gas is defined by a measurement of its temperature one should expect a relationship between pressure and temperature. The number of gas molecules impacting upon a surface at any instance can be increased without increasing the total number of gas molecules by increasing the frequency of gas molecules hitting a closed surface within volume and rebounding in a continuous motion from one portion of the closed surface to the other. This frequency of impacting can be determined to be a function of volume as decreasing the volume increases the frequency of impact and hence pressure.

But **EM01.1.4.1** does not take into account that the gas molecules are interacting with each other that can and do give results such that this equation fails when certain temperatures, volumes or pressures arise as the interactions between gas molecules create conditions that do not honour the basis upon which this equation is observed or derived.

EM01.1.4.1 is an equation derived from observation that holds in certain physical conditions for certain types of gas, but if a mathematical equation is derived from a pure mathematical derivation and based upon certain axioms and or assumptions, then such equations can end up being incorrect or even meaningless without limits and conditions applied. One may say that even if limits and conditions are applied, mathematical equations may be so abstract that the origin and relevance of an equation is lost, and that what is the result has no function in the real world of a physical universe. This can be the case even if mathematical predictions give accurate results. These results may only be accurate for a limited set of physical circumstances, and the mathematics in deriving those results may be so abstract that the real physical processes that are being modelled are lost and misinterpreted, and thus so is a real understanding of the physics involved.

This may be the case in attempts to unify the standard model of the quantum world with general relativity for gravitation. The mathematics of each satisfy the observation each theory with prediction, but there is within the mathematical equations, a loss of meaning of one or more real world physical processes that will give a greater understanding of the processes of the universe.

M01.1.5 : Mathematical Context of Non Natural numbers.

In the evolution of mathematics, and used widely in modern times, the mathematical concept and construct of negative and imaginary numbers and number systems has arisen.

M01.1.5.1 : negative numbers and number system

Negative numbers has three meanings of context in mathematics as related to a physical object and physical systems.

i. To represent a coordinate on a coordinate axis that is below some coordinate of origin which has a value of zero for that coordinate.

ii. To represent a mirror or opposite property of an object or object type. Eg electric charge of an electron and proton, or anti particle to a particle.

iii. To represent a quantity of direction of a coordinate or property within a coordinate system that is opposite to a defined positive direction.

In accounting and other applications of negative numbers, negative numbers represent a deficit or debit of a resource or money that is owed and needs to be payed back. In the real world of physical objects, negative numbers, it can be argued not to exist as one cannot have negative quantities of an object or properties as negative numbers represent a real physical object or property of an object, not an accounting deficit of a non existent object or property that needs to be payed back.

Thus when used in mathematical models of a physical system, negative numbers need to be treated differently to that of a pure mathematical operation with different and even new mathematical rules and operations that may be valid from a pure mathematical perspective, but are invalid from the perspective of a real world physical system. One such rule has already been explained and demonstrated in section **M01.1.1** for subtraction. No negative number of apples can be extracted from an apple tree.

In respect of context of negative numbers being used in models of a coordinate system, those negative numbers can become zero or positive by simply moving or translating the location of the origin of that coordinate system. Some coordinate systems such as polar or spherical do not have negative values for a measurement of radial distance, but can have a negative value of orientation angle that indicate a direction of rotation of an angel in those coordinate systems. However, a negative angular coordinate in this system is equal to a positive angular coordinate which then creates a dilemma of which is the correct number to use. Eg $+\pi$ radians = $-\pi$ radians in a polar or spherical coordinate system. Same applies to $+3\pi/4$ radians = $-\pi/4$ radians. Which one should one use? Perhaps it does not matter if one has knowledge and understanding of the concept and context of what a negative coordinate number represents and means. A positive radial coordinate is equivalent to a negative coordinate $+2\pi$ radians.

And thus when dealing with negative numbers representing physical objects, properties and coordinate systems, a contextual argument needs to be applied that is separate and even counter to the pure mathematical treatment of negative numbers. For example, when modelling an electric charge, if one has a number of A negative electric charge in a system, one cannot just simply multiply that charge by another negative number B since this has no physical meaning if one follows the conventional mathematical statement that a negative number multiplied by a negative number gives a positive number. A physical system will not become positive if the operation of two negative numbers representing electric charge are multiplied together where multiplication is a shorthand method of addition as stipulated in section **M01.1.2** above. The same can be said of two negative numbers representing the direction of vector component in two vectors. Does a mathematical operation applied to two vectors of a particular coordinate component really result in a physical property of that component becoming positive in its direction?

A common place where two negative numbers multiplied together or squared to give a positive value works in the accepted mathematical convention is when calculating areas or lengths. But these are values that are intrinsically always positive as they are pure scalar quantities of magnitude that have no meaning if negative by virtue of the context of what they represent. In calculating a volume in a coordinate system that has negative components to numbers representing length, negative numbers involved can give a negative result for a volume, something that is physically difficult to be contemplated as possible. It could be argued that in evaluating any quantity of scalar magnitude for the real physical world, all negative numbers need to be made positive when performing certain operations to attain results for a defined property. This can be performed by a simple translation in terms of coordinates or change of coordinate system such as what length and area are defined to be part of. In other words, if a negative number that is part of a coordinate system that can be made positive by a translation of coordinates, then performing that translation perhaps should be performed, or a set of rules applied to transform negative numbers into positive numbers before any mathematical operations are applied.

Perhaps a new thinking needs to be incorporated into modelling the physical world through use of the mathematical tool that is so central in trying to create an understanding of how physical systems are constructed and work. The mathematics of the physical world is not fully reflected by the purity of the mathematical world that has developed in with some rules, axioms and postulates that do not exist, or should not be applied in the natural physical world.

M01.1.5.2 : imaginary and complex numbers and number system

An imaginary number, **i**, in mathematics is defined as the square root of negative one. ie **i** = $\sqrt{-1}$. In conventional mathematical theory, the square root of negative numbers is undefined and considered not to exist. But mathematics can create number types and the rules that govern those number types as one wants or wishes hard enough for certain problem solving situations. Such as it is for imaginary numbers that when combined with real numbers creates a new category of number called complex numbers.

However, despite the mathematical convention that the square root of negative numbers is undefined and cannot exist, perhaps in the real physical world the square root of negative numbers is defined and do exist as $\sqrt{-1}$ = -1. But the use of complex or negative numbers yields that the square of a negative number -A is

$$\sqrt{-A} = \sqrt{A}\sqrt{-1} = \sqrt{A}\ \mathbf{i} \qquad \textbf{EM01.1.5.2.0}$$

which can lead to the conclusion that

$$\left(\sqrt{-A}\right)^2 = \left(\sqrt{A}\sqrt{-1}\right)^2 = \sqrt{A}^2\ \mathbf{i}^2 = A(-1) = -A \qquad \textbf{EM01.1.5.2.1}$$

Such a mathematical rule for imaginary numbers can lead to a new rule that negative numbers multiplied by negative numbers is a negative number, and that the square root of a negative number is a negative number. But this is as mentioned in the previous sections, and especially that for negative numbers in section **M01.1.5.1**, is only valid in the context of which such a mathematical operation is being performed.

M01.1.5.3 : Summary

In pure mathematics, various mathematical axioms, postulates, theorems and systems of numbers have been developed with governing rules that have had much success in being applied in accounting, finance, engineering, science and technology. However, as may be hinted and demonstrated with the need for imaginary and complex number systems to compensate for certain situations of solving certain mathematical conundrums involving negative numbers , the current thinking of mathematics may need a modification. That modification is that in the real physical world, conventional mathematical rules do not apply, or in fact are contrary to conventional mathematical thinking as the context in which those rules are applied are not valid for the real world physical system that is at hand.

What is argued here is that in the same context that numbers representing apples and oranges are not able to be merged or mixed together, the same can be said for real world physical systems. When creating mathematical models of real world physical systems, what needs to be taken into account is the context of what the mathematics represent, and thus what mathematical rules and processes can be applied.

And thus is demonstrated by use of imaginary and complex numbers. They are used to compensate the mathematical dogma that square root of negative one is invalid. However, it may be that in the real world, the square root of negative one is valid and is equal to negative one, and all negative numbers have a valid square root value. The only question is, what does that negative number represent. And that negative number represents something that is, again, in the context of physical system that the mathematical model is being used to represent.

M02 Context of Physical Systems

Over the past few centuries, Mathematical theory and practice has become more and more sophisticated, complex and abstract to such and extent that it is quite possible that the real world is no longer represented. What is also possible is that the mathematicians and those that use the mathematical theories and relationships believe that what the mathematics is interpreted to tell them is absolutely true. Even when mathematical results agree with observation or experiment, it does not necessarily mean that the theory or assumptions behind the mathematics is correct.

One example is the case of the Ptolemaic mathematical model of the cosmos that was derived in ancient times. The model had the Earth was the centre of the universe and that the planets, sun and moon revolved around it in perfect circles. A complex mathematical model of circles and epi-circles gave results that at the time fitted with observation and with philosophical and religious dogma. It was, as later discovered by Copernicus, Kepler, Galileo and Newton to be completely incorrect. A sun centred solar system was a more accurate and simpler model. Mathematics was also the tool used to come to this conclusion.

Here the human belief and bias gave the incorrect premise that the earth was the centre of the universe, and therefore, the mathematics was created to fit this belief. This can still be occurring in modern society, science and physics. A prejudice or bias towards a certain belief or theory can lead to an incorrect application of mathematical theory or invention of a new mathematical theory.

One such a possible error in modern physics is the theory of sub atomic particles known as string theory. It is a theory that in a nutshell states that the universe has a multidimensional space, and that all of matter is made up small tiny strings that vibrate in many modes. A change in the mode of vibration causes a change in the physical properties of matter. The mathematics being used to describe this theory is very abstract, complex and un-testable in the real world. New mathematics is being invented to support this theory as well as much of it being made to fit observation, and as well, being inconsistant with observation. Much of the interest in string theory is driven by a quest for a theory or everything (TOE) and conducted by mathematical physicists. That is, a physicist that uses pure mathematics to describe and model nature in pure mathematical terms.

The mathematical results may be able to fit some of the observations or physical laws like gravity, but it may be very likely the and the mathematical workings and theories behind them are incorrect and even biased, and thus how the universe truly is, and its inner workings, are also described incorrectly.

Mathematics is a tool used by physicists to describe the workings of nature. The relationships of certain physical properties give a means to understanding and predicting behaviour of physical bodies, systems and concepts like time and energy. However, just because a mathematical relationship gives a result that agrees with observation or theory does not mean that it is absolute and correct. A mathematical relationship is an abstraction of the real world and the real physical meaning of what it states must be understood.

The physical relationship for an ideal gas of pressure, P, Volume V and temperature T for a number of gas molecules n is given by

$$PV = nRT \qquad \textbf{EM02.1}$$

where R is a constant.

But at certain temperatures, pressures and indeed for certain gasses, this equation no longer is valid. In other words, a mathematical description has been found for relating certain measurements of physical properties of a gas in a certain range of conditions to give a good approximation, but the underlying true nature of physics for a gas is not represented or given by this equation.

Temperature is an expression of energy. In the case of a gas, kinetic energy.

Pressure is an expression of the transfer of that energy or force per unit area from the gas onto a third body, Eg a pressure gauge and is defined as force per unit Area

n is the number of gas particles gives a value for how much gas is involved.

Thus this can be expressed as

$$\frac{\text{Force}}{\text{Area}} \ \text{Volume} = \text{nR Energy}$$

and Volume = Area x displacement gives

$$\frac{\text{Force}}{\text{Area}} \ \text{Area x displacement} = \text{nR Energy}$$

or

$$\text{Force x Displacement} = \text{n R Energy} \qquad \textbf{EM02.2}$$

and if n =1 and removing the constant R obtain

$$\text{Force x Displacement} \propto \text{Energy} \qquad \textbf{EM02.3}$$

The relationship **M04** can be interpreted to say that a force that a particle of gas A exerts on a second stationary body B, and moves it a distance of Displacement before coming to a halt, is proportional to the energy of particle A. If B is part of a matrix of molecules that create a surface that is immovable, or is 100% elastic and energy is transferred back onto the particle A, then the energy that is exerted back onto A changes the direction of motion of A, and hence results in A experiencing a force from B and regaining some or all of its energy.

Thus equation **EM02.1** is really an alternative way of expressing that there is a direct relationship between forces on a body B by the energy of the gas molecule(s) A impacting on that body. Energy would most likely be kinetic energy (KE) defined as

$$KE = \frac{1}{2} m_g v^2 = \frac{1}{2} m_g \left(\frac{ds}{dt}\right)^2$$

Where m_g is the mass of the gas molecule.

and given that force is defined as

$$F = m_w a = m_w \frac{d^2 s}{d^2 t}$$

Where m_w is the mass of the matter that the gas molecule collides with.

It is assumed that these mathematical definitions are known and understood by the reader. Thus equation **EM02.3** can be re-written as

$$m_w \frac{d^2 s}{d^2 t} \; s = \frac{1}{2} m_g \left(\frac{ds}{dt}\right)^2 \qquad \textbf{EM02.4}$$

If a given constant time interval is used in equation **EM02.4**, then for larger values of displacement for the gas molecule to travel in the same period of time, it must have its energy and hence rate of displacement per unit time (ie velocity) also increased if the force exerted by the gas molecule on a third body is to be maintained. It must be noted that this is only valid for a gas in a confined space. In free space the displacement value would be irrelevant, and the relationship would be that Force $\propto$ Energy.

What is trying to be expressed here is that a mathematical expression given by equation **EM02.1** with measurements defined and taken by humans, can loose the real nature of the underlying physics of the real world, and thus mislead or misinterpret reality. Likewise if equation **EM02.1** were used in another mathematical expression without context or understanding of what it represents, then a greater cascading misinterpretation and/or error can occur in the results given.

And likewise with all mathematical theories and expressions. If the application of abstraction has no relationship with the real world, then the result can or will be misinterpreted or wrong, and the meaning of what the mathematics represents in the real world is lost.

03 Context of Mathematical limits

Mathematical theories and statements have limitations and boundaries to their application and accuracy of results. One foundation of mathematics, calculus can be considered as a prime example of the limitations of mathematics. Differential and integral calculus have one prime condition or principle that is crucial to what most, if not all of calculus is based that could be considered as a flaw.

Differential calculus is based upon the definition of the differential or slope of a line or surface between two coordinates as expressed for a mathematical function $f(x)$ at a coordinate as in Equation **EM02.5**.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} \qquad \textbf{EM02.5}$$

It is assumed that the reader is familiar with this fundamental definition. What this definition says is that the small value Δx can become smaller and smaller and get so close to zero that a derivative for the function exists if equation **EM02.5** is bounded, or in other words does not go to infinity.

The limitation of this definition is that if there is an integer value at which Δx can not become smaller than, then the definition of a derivative is an approximation, not an absolute value for the function $f(x)$. That is, Δx cannot become infinitesimally small and close to zero, so there is a limitation to the use of and accuracy of using equation **EM02.5**.

If the order of magnitude of the values being used for function $f(x)$ is of a certain value much greater than Δx, then equation **EM02.5** would be able to be used as any errors would be small enough to be negligible and unnoticed.

That may be true for much of every day physics and engineering calculations in what is termed the macro world of the large.

However, in the micro world where the values that the function $f(x)$ is of a certain critical order or below, the definition of a derivative in equation **EM02.5** needs to be modified to be.

$$f'(x) = \lim_{\Delta x \to \delta x} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} \qquad \textbf{EM02.6}$$

Were δx is the smallest value of difference between two successive coordinates that the function $f(x)$ can have. If

Equation **EM02.5** were used instead of equation **EM02.6**, then inaccurate results would occur. In the case of using definition **EM02.5** in deriving other theories and mathematical relationships, misleading or incorrect conclusions and representations can result.

Mathematicians and those that apply it such as physicists do not want to use equation **EM02.6** as it is messy and complicated to deal with as additional terms are involved that do not exist using definition equation **EM02.5**.

What needs to be found is the limitation of each situation in which the definition of a derivative given by definition equation **EM02.6** can be used. If this is not done, then inaccurate and false mathematics is applied, and incorrect interpreted results are derived.

A similar situation is present in integral calculus, where integration of a function over an interval a to b is given by adding very small areas below a curve or surface as given in the definition in equation **EM02.7**.

$$\int_a^b f(x)dx = \lim_{\Delta x \to 0} \sum_{i=1}^{n} f(x+i\Delta x)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} f(a+i\frac{b-a}{n})\frac{b-a}{n} \quad \text{- } \mathbf{EM02.7}$$

As before, if there is a limit top how small a value Δx can be, then there is a limit to the accuracy, and of being able to use this definition in real situations. If x is of an order larger than some threshold, definition – **EM02.7** can give a good or accurate approximation. Below this threshold, as with derivatives, inaccuracies of increasing magnitude become more present unless the mathematics reflecting the situations at hand is applied. In the case of integration, the definition given by **EM02.7** becomes.

$$\int_a^b f(x)dx = \lim_{\Delta x \to \delta x} \sum_{i=1}^{n} f(x+i\Delta x)\Delta x = \lim_{n\to N} \sum_{i=1}^{n} f(a+i\frac{b-a}{n})\frac{b-a}{n} \quad \text{- } \mathbf{EM02.8}$$

As before, the resultant mathematics may be messy, but needs to be applied.

As illustrated above, there may be many other mathematical definitions and theories that have assumptions or conditions that do not apply in all situations. This must be noted and recognised, otherwise inaccurate and incorrect results, interpretations of results, and even theories or understanding will most likely emerge. Unfortunately, this is most likely occurring in the applied mathematical world as well as in pure mathematical world.

Mathematics can be considered as an abstraction of the real world, and without the abstract having a context or relationship to reality, misconceptions can easily creep into the mathematics. This can be further compounded by applying blindly mathematical theory and definitions without taking into account the limitations or those theories, or the context of when they can and cannot be used.

Mathematical theories and proofs using a definition or theory that is beyond the limit at which it should be used will most likely give incorrect results and understanding of the physical world. If the results do not give an expected result, a new mathematical theory or method may be created to give a desired result, and the understanding of what are the true workings can be,or is, totally lost.

04 Context of Mathematical Approximation

In physics, systems of entities or bodies have the mathematical description of them simplified to allow an easier mathematical method or equation to be used to model the physical system at hand. One such system are gases and waves. The kinetic theory of gases uses statistics and assumptions of elastic collisions that treat gas molecules as point particles without consideration of the electrical forces that gas molecules have. Such a theory, or those using this theory in other situations, then must be used in context and limitation as explained in the sections above, otherwise there is likely to be erroneous results.

Similarly, the equations for wave motion used in many secondary and undergraduate physics books require that the amplitude of the wave be small in comparison to the wavelength. Without explaining the details, this is to facilitate a simplification of the mathematical description of a travelling sinusoidal wave. As with the kinetic theory of gases, there are limits to its use. The wave motion theory breaks down outside these limits set by the method of simplifying the problem at hand.

And the same is true in mathematics. As mentioned in the section 03 Context of Mathematical limits, derivatives and integrals can be considered as an approximation of calculating gradients and areas, volumes etc if the limit of the difference in variable of the function goes to zero. This approximation, as in the two physics examples, makes the mathematics easier to deal with.

Mathematical theories that use derivatives and integrals as a basis for new theories or methods of mathematical calculations, likewise, must take the limits of using derivatives and integrals into consideration, otherwise they could give a cascade of progressively incorrect results.

And so it must be with all mathematical theories and methods. Any simplification of a system expressed as a pure mathematical construct or theorem will have limits in its use and validity. Without consideration when deriving new theories or mathematical methods from them will most likely give mathematical errors, as well as incorrect interpretations or concepts of understanding in situations outside these limits.

04 Context of Mathematical Modelling

Fractals is a new branch of mathematics that is used to construct a model of the real world by using a combination of geometry, probability and statistics. A cloud, mountain or coastal landscape can be modelled using fractals. So can patterns occurring in biology such as ferns and trees. But fractals do not explain the natural world as forming because of some fractal equation. What fractals give is a method to model the real world through using mathematics.

That model is mostly involves repetitive algorithms run on computers dependent upon the inputs into those algorithms. Many involve feedback loops where the output of one calculation is the input to the next. Thus outcomes can be unpredictable and any interrelating algorithms more unpredictable. However, the structure of the results obtained can mimic the real world such as clouds or trees.

Studies in such mathematical modelling have lead to a greater understanding of how interrelationships of what were thought to be independent systems or entities are dependent upon each other. But these models are mostly not very accurate in prediction, and have limits dependent upon the mathematics, assumptions involved, and the input or data used. Also, unknown elements left out of the model can lead to the wrong conclusion or results.

Many who create and use these computer models know the limits and uncertainty of results they give when compared to the real world. However, there are many who wish to use computer modelling as a case for prediction of events or results of performing certain actions. These are mostly the likes of economists and other believers that seem to hold that the computer model, and the mathematics associated with it and is created, is correct and without any unknowns, or lack of, or incorrect input data used in the modelling. As such, most of the predictions and results of the modelling do not reflect the real world, but the belief in using such modelling is not questioned.

One limitation of computer modelling is that even if all the algorithms, assumptions and input data are correct and all unknowns eliminated, the very nature of using a computer is limited by the precision of using, storing and calculating numbers, especially the very large and the very small. Computer memory and arithmetic can store and calculate numbers within a certain level of accuracy. Numbers can only be stored up to a certain number of decimal points or integer values. As calculation is performed, the lack of ability to have a very high precision for the storage of numbers can, and does yield greater and greater inaccuracies of calculations beyond a certain number of iterations in which feedback loops within computer modelling algorithms are used. The consequence is that a very small inaccuracy is enlarged through each successive iteration, yielding the final result that at best is inaccurate, and worst, is meaningless.

This is one of the basis of what is commonly called chaos theory, and the above effect is commonly called the butterfly effect. The result of this is that beyond a certain level of number of calculations and iterations predicted behaviour or results cannot be derived with a high level of certainty or accuracy.

So the use of computer modelling, even if the mathematics, algorithms and data used is flawless, the results given beyond certain limits do not reflect the real world. This should be evident from the real world. After all, a fractal may mathematically be able to expressed a model of a tree branch structure to being infinite in how small or large that tree branch structure is. But it is well known that the branches of a tree have a limit in the number of branches and the size that can be obtained. So the fractal model itself is limited and defined by boundary conditions. More then the mathematics that is needed to model the real world.

04 Context of Mathematical Belief

Many areas of mathematics have no relationship to nature at all, but are a playing with pure numbers and deriving mathematical theories and methods removed from anything real. Imaginary numbers, sets, multidimensional spaces, infinities etc. may exist only in the realm of the human mind rather than in reality. Here, false concepts and conclusions of the real world may emerge if a belief exists that a mathematical result is always true and beyond doubt.

Even when there is a grounding in the physical meaning of the mathematics is involved, the mathematical theories and methods applied can still give results that may still be completely wrong, but are believed to be true because the mathematical doctrine is taken as infallible and that mathematics cannot lie. A mathematical treatment of a real world physical system such as gravity may give a theoretical result that space can be warped such that wormholes can be formed and remain stable. But that does not mean it can be realistically possible to create, or that wormholes exist. Just because the mathematics says it is so, does not mean that it actually so.

Fortunately, mathematical theories involving the physical and real world can and are tested for validity. Those that fail the test are discarded, and those that succeed are retained. But those that are accepted, as explained in all the previous sections, still may be incorrect or incomplete. Newtons gravitational theory is a mathematical model and description of gravity. It does not explain gravity or how it works, but is accurate and valid for everyday use and planetary exploration. Einstein’s theory of general relativity gives a more accurate description of gravity and how it works. Predictions and an understanding of gravity and the universe is given by this theory well beyond what Newtons mathematical treatment. Testing of the mathematics of both have proven both are correct, with the exception that Newtons theory is only valid in low gravitational fields and velocities much less than that of light. That is Newtons mathematical description of gravity is correct within certain limits. If a mathematical belief that Newtons theory is valid outside these limits, then the wrong results, and more importantly, understanding or concept of gravity is derived.

It is the same with a well proven and high integrity theory like General relativity. If the mathematical methods and use of this without concern with which the boundaries or limits of this theory and the mathematics in deriving this theory are taken into account, new theories and mathematical derived results will be just as wrong. This could be the case with the derivation of wormholes from the theory of general relativity. Just because the maths says it is so, does not mean that it is so in the real world.

Conclusion

It is the theme of the discussion above that mathematics is not absolute in its correctness at describing the real world. There are context and limits to its use and the results of from the relationships and theories that it gives. What is mostly forgotten is that mathematics is largely a human invention, and that the natural world, though can be expressed using mathematical equations and expressions, may in fact be very different in what mathematical processes are actually going on. Some mathematical treatments or description of nature involve using imaginary numbers. This may indicate that our understanding of the processes and underlying mathematics is incorrect or needs revision. Do imaginary numbers really exist in nature? Many mathematicians would say no, but that using imaginary numbers is the best we can do to describe and calculate predictions and results of such a real world scenario.

The same must be said of all of mathematical descriptions of the real world. The mathematics involved must be seen as a means to describe and predict the natural world. What makes mathematics a real reflection of the processes and underlying principles of nature is the human mind to come up with a concept and understanding of the real world. Einstein’s theory of relativity, the wave-particle duality of atomic particles and light, Darwin’s theory of natural selection, are a few of these ground breaking concepts. The next task is to express them in an accurate and meaningful way so as to test and gain and expand understanding of these concepts. Mathematics is one powerful tool , and with vigour, can do this to a certain limit.

This mathematical limit, as has been hinted at as above, is the human factor. It may be that a mathematical concept cannot be grasped because it is too complicated or unable to be solved. Again Einstein’s Theory of general relativity has many equations that cannot be solved. So until a new form of mathematics is derived that can describe the concepts of general relativity better, or that the theory itself is not complete and needs some other insight that will give the mathematics to solve these equations.

Some would like to believe that mathematics rules the universe, and that the laws of mathematics is what creates the laws of nature. This is largely false for the reason that mathematics is a human invention and a tool that is used to describe the natural world, and the unnatural and imaginary non existent mathematical universe as well.

There are certain natural phenomenon that are seemingly consistent in appearing in nature and have a mathematical relationship. But these may be due to other factors like natural selection in the case of plants and animals, and one mathematical description or coincidence that fits one, also fits many others. Maths is not a law of the natural world, but the natural world is a law of mathematics. Laws of nature define what mathematics can be applied to it, not the other way around. If that were the case then the universe would be anything that mathematics can imagine, and it well known it is not.