About Space

S01 What is Space

Space, it can be argued, is a natural physical entity that is based upon the human perception of existence and observable experience. Thus much of how space is perceived is how the human mind constructs a model of what is observed with the senses of the human body, and in particular, through vision via the eyes. It is therefore possible for space not to be as it is perceive to be by the human mind, and what is seen may also be an illusion or projection based upon what the human mind can comprehend.

A physical space can be defined by a set of rules which entities and phenomenon can exist, and the behavior that entities and phenomenon must follow. The definition of what space is, is based upon the human observation and comprehension which is the only way that it can be done. Any other method requires imagination and conjecture that goes beyond the human perception, which may also lead to the more correct answer of what physical space truly is. Such methods have lead to the theories of quantum mechanics and general relativity.

One definition of what physical space is

Definition S1a

> Space is the stage upon, or in which interactions of physical entities take place.

This definition implies that space is separable and apart from the entities and phenomenon that exits and are observed. This implies that space is a physical entity in itself and thus has properties without the existence or need to exist of physical entities such as particles of matter or electromagnetic radiation, that is light. This definition implies space is like a sheet of paper where entities are placed on it and interact with each other, and perhaps the space itself.

From the human perspective, distances and angles of rotation are used to create a coordinate system to define a mathematical description of space and in which to perform mathematical calculations. This has worked for most of the time, but even this may not be enough as there are limits on what mathematics can describe. Also, mathematics is a human invention that is used to describe and model in abstract terms what occurs in the natural world, such that the meaning or what is truly occurring can be lost or be completely wrong.

One consideration is that what is perceived as space as a physical entity is that the entirety of the universe is just a space, and that the entities and fields that are observed to be present in the universe are a manifestation of the physical space interacting with itself such that matter "condensates" out of those interactions to form the building blocks and foundations of matter itself, along with the interacting fields that accompany matter such as electromagnetism and gravity.

Another Consideration is that space is created by the interacting fields of matter, and that space is thus not a physical entity in its own right. In other words, the interacting matter and the interacting fields associated with them creates an illusory perception of a space, and that space does not really exist as the human mind has built a model of.

Never the less, a set of basic definitions of the space of the physical world can be more formally defined by what is observable and projected to be true. Using the human perception and experience in the universe we exist in as a basis, a set of rules can be set out for defining space.

Definition S1
Space is a domain where all entities and phenomenon reside and exist in and such that the following properties hold in all situations.

- **i** : Be infinite in extent, but be finite such that there is no starting or end location.
- **ii** : Have no central origin or coordinate reference.
- **iii** : Be scalable to a minimum or maximum extent that can not be exceeded.
- **iv** : Can have any "shape" that satisfies i: -> iii
- **v** : Have no fixed rigid structure that cannot be altered.

S02 Degrees Of Freedom

Space has as a property, a defined a mathematical construct of dimension which is the number of spacial coordinate degrees of freedom in which an entity can be defined to move within, and have a physical form constructed of. Any physical space with any dimension must adhere to all of **Definition S1** properties as any mathematical description.

The human perception of space is restricted to the three dimensions that the human sensors can interact with, and thus the human mind has built a model of. However, that does not mean that the universe does not exist in reality within a space of a different dimension. Speculation and conjecture has been proposed that the universe may exist as a hologram like structure with the three dimensions of space that is perceived is actually a projection from a two dimensional surface.

Another leap of imagination and conjecture can propose that the space that the universe exists in is of a higher dimension, and that the three dimensions that humans perceive is analogous to that of the book flatland, where two dimensional beings exists, and cannot comprehend or perceive of an existence of a higher third dimension. Thus, like a two dimensional being living on a two dimensional surface that makes up the space that it exists within, the same similar condition could be present for that of the universe that humans exist within. The three dimensional space that makes up the perceived space of the universe could be considered as a three dimensional surface existing upon a four dimensional space.

To fit in with **Definition S1**, such a space that the universe exists within needs to be a closed surface. Such a surface must exist on top of a space of one dimension higher. A zero dimension space considered as a point is an exception to this. A line is considered as a form of a one dimension space, that to be closed such as a circle needs a two dimension space to exist within, or indeed creates a two dimension space. A closed line forming the circumference of a circle can be considered as a surface of a two dimensional space. A spherical surface can be considered as a form of a two dimensional closed surface that exists on, or creates a three dimensional volume. Analogous to this is that a three dimensional space can be considered as being a surface of a four dimensional space or hyper sphere.

Thus it can be considered that since no higher dimensions of space that the three dimensions that the human mind can perceive and exist within, no higher dimension of space than four exists, with the three dimensions that is observed of existing upon a four dimensional surface. If **Definition S1** is valid in defining a physical space for the universe to exist within, then a new way of thinking of what is the structure of space is can be conceived so as to build a model of the workings and understanding of the universe.

S03 Constructing a model of space

In creating a model of space that is relevant to the observed physical universe, it is assumes that the mathematics that has been created to describe the geometry that is observed, is the correct and valid mathematics that the universe has as a basis for its functioning. If the universe functions on a level that the human mind has not, or cannot perceive, and the universe observed and comprehended into a model by the human mind is but a projection of the true universe functioning, then the mathematical geometric model of space can be considered as only that. A mathematical model that like the Ptolemaic model of the solar system, may work mathematically within certain limits, but does not reveal the true nature of the workings or fundamentals of the universe and its structure.

Thus what is presented here is but a possible mathematical model of the space that describes the universe and its construction and structure that satisfies all the postulates of **Definition S1**.

The normal mathematical Cartesian coordinate system that is most used in expressing mathematical formulas for space will be used as little as possible. This is because in the following description and modeling of space, space is closed, and this will involve spherical geometry which involve the surfaces of spheres and polar planes that all are best and easiest to describe mathematically using spherical and polar coordinates. ie angles and radials.

S03.1 One dimensional space

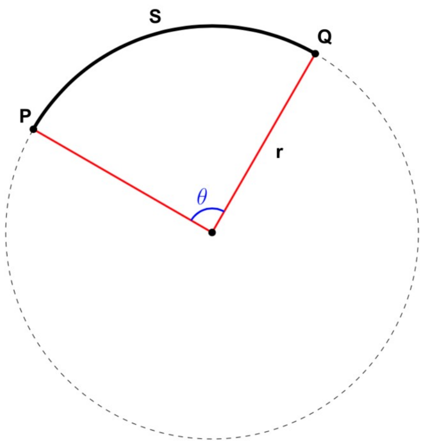


Fig ms01

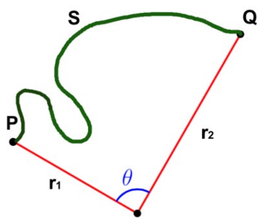


Fig ms02

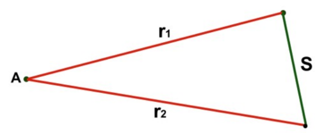


Fig ms03

Consider a closed 1D space being the line on the circumference C of a circle of radius r. A circle or radius r can be expressed mathematically by the equation.

$$r\cos(\theta) + r\sin(\theta) = r \qquad \text{- } \mathbf{MS01}$$

Consider a point at location P on this 1D line to a point Q has a distance S_{PQ} between then defined mathematically as

$$S_{PQ} = r\theta \qquad \text{- } \mathbf{MS02}$$

where θ is the angle of separation between them in radians.

In more general terms, if r is expressed as a function of θ or R(θ), then **MS02** would become

$$S_{PQ} = \int_{\theta_1}^{\theta_2} R(\theta)\,d\theta \qquad \text{- } \mathbf{MS03}$$

If r cannot be expressed as a function R(θ), or is not as an integral, then to find the distance between P and Q can be expressed as a summation of all finite distances between P and Q. Such a line may be represented as given in Fig ms02 To find the length S of such an arbitrary line the line segments that make up the line can be represented as in fig ms03. To find the line segment distance S in Fig ms03, the cosine rule can be applied where A is the angle separating the radial distances r1 and r2 to S.

The cosine rule is stated as

$$S^2 = r_1^2 + r_2^2 - 2r_1r_2 \cos(A)$$

=> for each iteration i segment on S in fig ms02

$$S_i^2 = r_{1i}^2 + r_{2i}^2 - 2r_{1i}r_{2i} \cos(\theta_i) \qquad \text{– } \mathbf{MS04}$$

and thus

$$S_{PQ} = \sum_{i=0}^{i=n} S_i = \sum_{i=0}^{i=n} \sqrt{r_{1i}^2 + r_{2i}^2 - 2r_{1i}r_{2i}\cos\theta_i} \qquad \text{- } \mathbf{MS05}$$

=> if $r_{1i} = r_{2i} = r$ as all $r_i = r$

$$S_{PQ} = \sum_{i=0}^{i=n} S_i = \sum_{i=0}^{i=n} \sqrt{2r^2 - 2r^2\cos\theta_i} \;=\; r\sum_{i=0}^{i=n} \sqrt{2(1-\cos\theta_i)} \qquad \text{- } \mathbf{MS06}$$

using the trigonometric identity

$$1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$$

obtain

$$S_{PQ} = r\sum_{i=0}^{i=n} \sqrt{4\sin^2\left(\frac{\theta_i}{2}\right)} \;=\; 2r\sum_{i=0}^{i=n} \sin\left(\frac{\theta_i}{2}\right) \qquad \text{- } \mathbf{MS07}$$

Considering that θ is in radians, for all θ_i being equal and $\theta_i = \theta$, as $n \to \infty$, $\theta \to 0$ and as $\theta \to 0$ $\sin(\theta/2) \to \theta/2$

=> $$\sum_{i=0}^{i=\infty} \sin\left(\frac{\theta_i}{2}\right) \;=\; \sum_{i=0}^{i=\infty} \frac{\theta_i}{2}$$

which is equivalent to an integral

$$\int_{\theta_0}^{\theta_1} \frac{1}{2} d\theta = \frac{\theta_1 - \theta_0}{2}$$

=> if have $\theta_1 - \theta_2 = \theta$ where θ is in radians, then angle of rotation or radius r on a circle circumference, the distance traveled on the circumference between two points P and Q on that circumference using MS07 is

$$S_{PQ} = 2r\frac{\theta}{2} = r\theta \qquad \text{- } \mathbf{MS08}$$

as expected.

Consider that a closed line can be of any shape, and that an expression of a location on that line in terms of a radius and angle coordinate can be derived. The radius is the distance from any location within the boundary of that closed line defined as a point of radial origin , and the angle is an angle of rotation around that point of radial origin from a designated and defined direction of a line from that radial origin that is defined as an origin radial direction.

Thus any coordinate C on a 1D line can be expressed as a function of a radius r, and angle θ where θ is in radians. Ie

$$C = r(u)\theta(v) \qquad \text{- } \mathbf{MS09}$$

where r(u) is a radius value given by a function of variable u, and θ(v) is a value of angle theta given by a function of variable v. u and v can be of any type of variables.

Note that a closed 1D space is a line on the edge of what can be considered to be a 2D space, but it is not part of a 2D space if the only direction of travel or communication is restricted to traversing along that line. Ie traversal or communication can only be performed in the direction of a neighboring 1D spacial coordinate, which is in a positive or negative angular direction, ±θ. Also a 1D space cannot have any rotations as that would add a degree of freedom to the space that the line exists in which as it would invalidate the definition of a 1D space.

S03.2 Two dimensional space

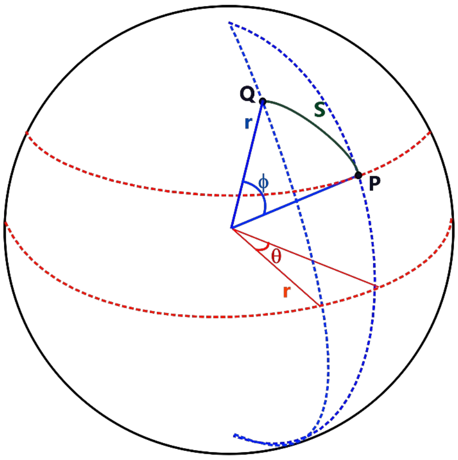


Fig MS04

Consider a closed 2D space being a surface of a sphere or radius r as depicted in fig MS04. A sphere of radius r can be expressed mathematically by the equation.

$$r\cos(\theta)\sin(\varphi) + r\sin(\theta)\sin(\varphi) + r\cos(\varphi) = r \qquad \text{- } \mathbf{MS10}$$

Consider a point at location P on which traverses a 1D path or line to a second point Q. This path or has a distance S_{PQ} between then defined mathematically by the Haversine function as

$$S_{PQ}=2r\arcsin\left[\sqrt{\sin^2\left(\frac{\Delta\phi}{2}\right)+\cos(\phi_1)\cos(\phi_2)\sin^2\left(\frac{\Delta\theta}{2}\right)}\right] \qquad \text{- } \mathbf{MS11}$$

which can be reduced to

$$S_{PQ}=2r\arcsin\left[\sqrt{\frac{1}{2}(1-\cos(\Delta\phi)+\cos(\phi_1)\cos(\phi_2)(1-\cos(\Delta\theta)))}\right] \qquad \text{- } \mathbf{MS12}$$

where $\Delta\theta = \theta_2 - \theta_1$ and $\Delta\varphi = \varphi_2 - \varphi_1$ where θ_1 , φ_1 are the angular coordinates for point P, and θ_2 ,φ_2 angular coordinates for point Q. $\Delta\theta$ and $\Delta\varphi$ are θ and φ in fig ms04.

MS11 and **MS12** are only valid for a spherical surface of constant radius r and will find the shortest distance only. More general distance S on a path according to a function between two points requires a different method.

If one were to consider that the distance S is extremely small compared to the radius r, and thus θ and φ are also very small, then one can consider that one has a Euclidean right angle triangle. Thus it can be considered that

$$S^2 \approx (r\theta)^2 + (r\varphi)^2 \approx r^2(\theta^2 + \varphi^2) = S^2(\theta) + S^2(\varphi) \qquad \text{- } \mathbf{MS13}$$

=> for a total distance S_{pq} between P an Q on a closed 2D surface can be expressed as a sum of infinitesimal small Euclidean right angle triangles

$$S_{PQ}^2=\sum_{i=0}^{i=n} S_i^2=\sum_{i=0}^{i=n} S_{PQ}^2(\theta_i)+\sum_{i=0}^{i=n} S_{PQ}^2(\phi_i) \qquad \text{- } \mathbf{MS14}$$

If consider that radius r_1 at P not equal r_2 at Q, then have a similar situation as illustrated in fig MS02 and MS03 for a closed 1D line. Thus a more general form of MS14 using the analogy of being able to use the cosine rule to find the length of $S(\theta)$ and $S(\varphi)$ for infinitesimal small values of θ and φ for very large relative values of r_1 and r_2

$$S_i^2(\theta) = r_{1i}^2 + r_{2i}^2 - 2r_{1i}r_{2i}\cos(\theta_i)$$

and

$$S_i^2(\varphi) = r_{1i}^2 + r_{2i}^2 - 2r_{1i}r_{2i}\cos(\varphi_i)$$

By following the same arguments for obtaining a 1D spacial distance between two 1D points P and Q on a 1D closed line with different radius r_1 at location P not equal to r_2 at Q, obtain

$$S_{PQ}^2 = \sum_{i=0}^{i=n} S_i^2 = \sum_{i=0}^{i=n} r_{1i}^2 + r_{2i}^2 - 2 r_{1i} r_{2i} \cos\theta_i + \sum_{i=0}^{i=n} r_{1i}^2 + r_{2i}^2 - 2 r_{1i} r_{2i} \cos\phi_i$$

=>

$$S_{PQ}^2 = 2\left(\sum_{i=0}^{i=n} r_{1i}^2 + r_{2i}^2 - \sum_{i=0}^{i=n} r_{1i} r_{2i} (\cos\theta_i + \cos\phi_i)\right) \qquad \textbf{- MS15}$$

which implies as $i \to \infty$, $r_1 \to r_2$, $\theta \to 0$ $\varphi \to 0$. and

$\cos(\theta)$ as $\theta \to 0$ = 1 => integral of derivative of $f(\theta) = \int f'(\theta) = 1$ => $f'(\theta) = d\theta$ as $\int_0^1 d\theta = 1$

=> $\sum \cos(\theta) = \int d\theta$

similarly for φ, $\sum \cos(\varphi) = \int d\varphi$

For radius, $r_1 \to r_2$ as $i \to \infty$ => $\sum r^2 =$ $\int_0^r f'(r) dr = r^2$

=> $f'(r) = 2r$

thus expressing MS14 in integral form is

$$S_{PQ}^2 = 2\left[\int_{r1}^{r2} 2r\, dr - \int_{r1}^{r2} \int_{\theta 1}^{\theta 2} \int_{\phi 1}^{\phi 2} 2r (d\theta\, dr + d\phi\, dr)\right] \qquad \textbf{- MS16}$$

$$S_{PQ}^2 = 2\left[\int_{r1}^{r2} 2r\, dr - \left(\int_{r1}^{r2} \int_{\theta 1}^{\theta 2} 2r\, d\theta\, dr + \int_{r1}^{r2} \int_{\phi 1}^{\phi 2} 2r\, d\phi\, dr\right)\right] \qquad \textbf{- MS17}$$

or more generally as

$$S_{PQ}^2 = 2\left[\int_{r1}^{r2} 2 f_r\, dR - \left(\int_{r1}^{r2} \int_{\theta 1}^{\theta 2} 2 f_r f_\theta\, d\Theta\, dR + \int_{r1}^{r2} \int_{\phi 1}^{\phi 2} 2 f_r f_\phi\, d\Phi\, dR\right)\right] \qquad \textbf{- MS18}$$

where f_r, f_θ and f_φ are functions that give the values for r, θ and φ on a 3D surface, and dR, dΘ, and dΦ are the derivative dependencies of those functions.

Such functions of f_r, f_θ and f_φ are equivalent to creating a non even closed 2D surface that can be of any shape similar to that of a 1D line given in fig MS02, This is a topological issue and needs a topological approach to deal with such a closed 2D surface.

If follow same logic arguments as steps **MS04** to **MS08** for a 1D line on the circumference of circle, **MS15** becomes

$$S_{PQ} = \sum_{i=0}^{i=n} S_i = \sum_{i=0}^{i=n} \sqrt{r_{1i}^2 + r_{2i}^2 - 2 r_{1i} r_{2i} \cos\theta_i + r_{1i}^2 + r_{2i}^2 - 2 r_{1i} r_{2i} \cos\phi_i} \qquad \textbf{- MS19}$$

which for $r_1 = r_2 = r$ becomes

$$S_{PQ} = \sum_{i=0}^{i=n} \sqrt{2r^2 - 2r^2 \cos\theta_i + 2r^{¿} - 2r^2 \cos\phi_i}$$

$$= \sum_{i=0}^{i=n} \sqrt{2r^2(1-\cos\theta_i) + 2r^2(1-\cos\phi_i)}$$

using the trigonometric identity

$1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$ have

$$= \sum_{i=0}^{i=n} \sqrt{4r^2(\sin^2\frac{\theta_i}{2}) + 4r^2(\sin^2\frac{\phi_i}{2})}$$

$$= 2r\sum_{i=0}^{i=n} \sqrt{\sin^2\frac{\theta_i}{2} + \sin^2\frac{\phi_i}{2}} \qquad \text{- MS20}$$

and as n→∞, θ →0 sin(θ/2) -> θ/2 and, as φ → 0, sin(φ/2) -> φ/2 =>

$$S_{PQ} = 2r\sum_{i=0}^{i=\infty} \sqrt{\left(\frac{\theta_i}{2}\right)^2 + \left(\frac{\phi_i}{2}\right)^2}$$ where θ_i and $\varphi_i \approx 0$ but $\neq 0$

which can be argued, is is equivalent to

$$= 2r\sqrt{\frac{\theta^2}{4} + \frac{\phi^2}{4}} = r\sqrt{\theta^2 + \phi^2} \qquad \text{- MS21}$$

MS21 will only be valid if r is very much larger compared to angles θ and φ and is effectively a flat surface. Ie θ and φ are infinitesimal compared to r.

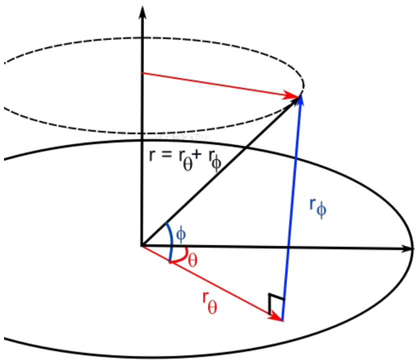


Fig MS05

For a more generalised path on a spherical surface in 3D, consider that the resultant radius from the center of the sphere is of a vector form $\vec{r}$, and that this radius has two vector components based upon the angles θ and φ, such that

$$\vec{r} = \overrightarrow{r_{\theta\phi}} = \vec{r_\theta} + \vec{r_\phi} \qquad \text{- MS22}$$

Fig MS05 illustrates this radial vector addition. Using trigonometry, one can obtain the relationships for r_θ and r_φ as

$r_\theta = r\cos(\varphi)$ - **MS23a**

$r_\varphi = r\sin(\varphi)$ - **MS23b**

The distance of a path on any 3D surface in the θ coordinate of degree of freedom of movement at a constant φ is S_θ is

$S_\theta = r_\theta\theta = r\cos(\varphi)\,\theta = r\,\theta\cos(\varphi)$ - **MS24a**

and The distance of a path on any 3D surface in the φ coordinate of degree of freedom of movement at a constant θ is S_φ is

$S_\varphi = r_\varphi\varphi = r\sin(\varphi)\,\varphi = r\,\varphi\sin(\varphi)$ – **MS24b**

=> the infinitesimal small spherical distance S_i of a path between two points P and Q on a 3D surface of any kind using the cosine rule analogous to that for a 1D closed path on a 2D plane is

$$S_i=\sqrt{r^2_{\theta 1i}+r^2_{\theta 2i}-2\,r_{\theta 1i}r_{\theta 2i}\cos\theta_i+r^2_{\phi 1i}+r^2_{\phi 2i}-2\,r_{\phi 1i}r_{\phi 2i}\cos\phi_i}$$

where $r_{\theta 1i} = r_{1i}\cos(\varphi_{1i})$, $r_{\theta 2i} = r_{2i}\cos(\varphi_{2i})$, $r_{\varphi 1i} = r_{1i}\sin(\varphi_{1i})$, $r_{\varphi 2i} = r_{2i}\sin(\varphi_{2i})$

The total spherical distance S_{PQ} between two points P and Q on a 3D surface of any kind is

$$S_{PQ}=\sum_{i=0}^{i=n} S_i=\sum_{i=0}^{i=n}\sqrt{r^2_{\theta 1i}+r^2_{\theta 2i}-2r_{\theta 1i}r_{\theta 2i}\cos\theta_i+r^2_{\phi 1i}+r^2_{\phi 2i}-2r_{\phi 1i}r_{\phi 2i}\cos\phi_i} \quad \textbf{– MS25}$$

and if the radial coordinates at iteration i are subject to a function Ƒ of one or more variables, then have

$r_\theta = r(Ƒ_r)\cos(\varphi(Ƒ_\varphi))$ - **MS25a**

$r_\varphi = r(Ƒ_r)\sin(\varphi(Ƒ_\varphi))$ - **MS25b**

$\theta = \theta(Ƒ_\theta)$ - **MS25c**

$\varphi = \varphi(Ƒ_\varphi)$ - **MS25c**

where $r(Ƒ_r)$is a function that will give a radial value for r at radial coordinate (θ,φ)

$\varphi(Ƒ_\varphi)$ is a function that will give a radial value for φ at radial coordinate (r,θ)

$\theta(Ƒ_\theta)$ is a function that will give a radial value for θ at radial coordinate (r,φ)

which is a more general, but more complicated version of **MS25** is formed, and can be considered as an expression of the 2D path on a 3D surface. Thus **MS25** can be expressed as

$$S_{PQ}=\sum_{i=1}^{i=n}\sqrt{r^2_{\theta(i-1)}+r^2_{\theta i}-2r_{\theta(i-1)}r_{\theta i}\cos\theta_{i-1,i}+r^2_{\phi(i-1)}+r^2_{\phi i}-2r_{\phi(i-1)}r_{\phi i}\cos\phi_{i-1,i}} \quad \textbf{- MS25}$$

where the subscripts i and i -1 indicate the value for the variable at the current iteration of the 2D path i , and the previous iteration of the 2D path respectively. $\theta_{i,i-1}$ and $\varphi_{i,i-1}$ are the angles subtended between the previous radial vectors of r_θ and r_φ and their current radial vectors of r_θ and r_φ..

No attempt will be made to evaluate or create an integral of **MS25** on a regular sphere of constant radius r.

S03.3 Three dimensional space

Consider that similar to the definition of a closed 1D and 2D space as given in the description of sections **S03.1 One dimensional space**, and **S03.2 two dimensional space** above , for a closed 3D space to exist, there needs to be a forth dimensional 4D space on which a closed 3D space is but a surface of that 4D space. Analogous to a 1D or 2D space, an observer would see that surface appear as all their space in all directions that they can observe. In other words, the space one can observe in any dimension is the surface of a dimension one degree above that, that can be observed and that the observe exists within. Thus if one could observe the point in 3D space that is directly opposite their own position on that 4D surface, they would observe the entire limit of their observation. ie the spherical edge of their 3D space being a different observation of that 3D location.

Similar to a 2D surface, by rotating π radians on any 2D plane, one would see the same 3D point on the 4D surface that their 3D space exists on from the opposite direction. A rotation of any amount would result in observing an antipodal point in 3D space from a different direction or vector upon that point similar to observing a 2D antipodal point on the surface of a 3D sphere. Such an observation can be considered as the next dimensional view above ones own. That is, if one were able to observe in all the directions at one instance in their own space unobstructed to an antipodal location of a closed surface that they exist in, they would be able to observe what an observer in the next higher dimension could observe.

To define a closed 3D surface in a 4D space, consider that analogous to defining a closed 1D and 2D space that a point P in 3D space has a 4D spacial radius r to point P, and that in addition to the angle coordinates θ and φ that define a location of P in 3D space, a third, γ is required to define a location P in 4D space. Thus a coordinate P in 4D space can be expressed as.

P = (r, ,θ, φ, γ) - **MS26**

Taking the construction of a closed 1D and 2D space in sections **S03.1 One dimensional spac**e and **S03.2 Two dimensional Space** as an analogy, A 4D space would have a path length between two points P and Q in 4D space defined as

$$S_{PQ}^2 = \sum_{i=1}^{i=n} r_{\theta(i-1)}^2 + r_{\theta i}^2 - 2 r_{\theta(i-1)} r_{\theta i} \cos\theta_{i-1,i} + r_{\phi(i-1)}^2 + r_{\phi i}^2 - 2 r_{\phi(i-1)} r_{\phi i} \cos\phi_{i-1,i}$$

$$+ r_{\gamma(i-1)}^2 + r_{\gamma i}^2 - 2 r_{\gamma(i-1)} r_{\omega i} \cos\gamma_{i-1,i} \quad \textbf{- MS27}$$

Where r_γ is the radial vector component that makes up the total 4D radius to a point P on a 4D surface. For all intents and purpose, this can be interpreted as the component that gives a 3D space its volume similarly to how a 2D surface on a 3D sphere has an area defined by its r_Φ radial vector component.

If the radial coordinates at iteration i are subject to a function Ƒ of one or more variables, then have

$r_\theta = r(Ƒ_r)\cos(\varphi(Ƒ_\varphi))$	- **MS28a**
$r_\varphi = r(Ƒ_r)\sin(\varphi(Ƒ_\varphi))$	- **MS28b**
$r_\gamma = r(Ƒ_r)\sin(\gamma(Ƒ_\gamma))$	- **MS28c**
$\theta = \theta(Ƒ_\theta)$	- **MS28d**
$\varphi = \varphi(Ƒ_\varphi)$	- **MS28e**
$\boldsymbol{\gamma} = \gamma(Ƒ_\gamma)$	- **MS28f**

where $r(Ƒ_r)$is a function that will give a radial value for r at radial coordinate (θ,φ,γ)

$\varphi(Ƒ_\varphi)$ is a function that will give a radial value for φ at radial coordinate (r,θ,γ)

$\theta(Ƒ_\theta)$ is a function that will give a radial value for θ at radial coordinate (r,φ,γ)

$\gamma(Ƒ_\gamma)$ is a function that will give a radial value for γ at radial coordinate (r,θ,φ)

The consequence of all equations for a path or line in 1D, 2D and 3D space given by **MS05**, **MS25**, and **MS27** is that they can all be considered as equivalent as a path in 3D space that can be reduced to a path in 2D space, and then to a path in 1D space, or vise versa. Thus, 1D space is a subset of 2D space, and both 1D and 2D space are subsets of 3D space.

For 1D space to be a subset of 2D space, φ = constant => r_φ = constant =>

$$S_{PQ} = \sum_{i=1}^{i=n} \sqrt{r_{\theta(i-1)}^2 + r_{\theta i}^2 - 2 r_{\theta(i-1)} r_{\theta i} \cos\theta_{i-1,i} + 2 r_\phi^2 (1-\cos\phi)} \quad \textbf{- MS29}$$

For 1D space to be a subset of 3D space,γ = constant => r_γ = constant and φ = constant => r_φ = constant =>

$$S_{PQ} = \sum_{i=1}^{i=n} \sqrt{r_{\theta(i-1)}^2 + r_{\theta i}^2 - 2 r_{\theta(i-1)} r_{\theta i} \cos\theta_{i-1,i} + 2 r_\phi^2 (1-\cos\phi) + 2 r_\gamma^2 (1-\cos\gamma)} \quad \textbf{- MS30}$$

For 2D space to be a subset of 3D space,γ = constant => r_γ = constant =>

$$S_{PQ} = \sum_{i=1}^{i=n} \sqrt{r_{\theta(i-1)}^2 + r_{\theta i}^2 - 2 r_{\theta(i-1)} r_{\theta i} \cos\theta_{i-1,i} + r_{\phi(i-1)}^2 + r_{\phi i}^2 - 2 r_{\phi(i-1)} r_{\phi i} \cos\phi_{i-1,i} + 2 r_\gamma^2 (1-\cos\gamma)} \quad \textbf{- MS31}$$

If a closed line does not have all its coordinates existing on a single 2D plane in 3D space , then have a closed 2D line on a closed 3D surface and do not have a 1D line as a subset of 2D or 3D space.

If a closed line does not have all its coordinates existing on a single 3D surface in 4D space, then have a closed 3D line on a closed 4D surface and do not have a 1D or 2D line as a subset of 3D space.

Area

An area is a region of 2D or 3D space that is enclosed by any closed line on any dimension of the same or lower dimension than the space it exists in.

In the same definition of a closed path or line being a conglomeration of 1D points in space, an area can be considered as a conglomeration of closed paths or lines. By definition, this means that an area can also be considered as a conglomeration of 1D points all bunched up together and forming a matrix of neighbors, where the edge of the area is considered as a location where in one or more radial directions, no neighbors exist.

Thus, a definition of an area A would be the summation of neighboring closed paths of length S, or in mathematical form,

$$A=\sum_{j=0}^{j=m} S_j \qquad \text{- } \mathbf{MS32}$$

$$A=\sum_{j=0}^{j=m} \sum_{i=0}^{i=n(j)} S_{i,j} \qquad \text{- } \mathbf{MS33}$$

The length of a closed path S_j is defined in a 3D space on a 4D surface by **MS27** where each S_i of is also a function of j. That is

$$S_{i,j}^2=r_{\theta(i-1),j}^2+r_{\theta i,j}^2-2r_{\theta(i-1),j}r_{\theta i,j}\cos\theta_{(i-1,i),j}+r_{\phi(i-1),j}^2+r_{\phi i,j}^2-2r_{\phi(i-1),j}r_{\phi i}\cos\phi_{(i-1,i),j}$$

$$+r_{\gamma(i-1),j}^2+r_{\gamma i,j}^2-2r_{\gamma(i-1),j}r_{\gamma i,j}\cos\gamma_{(i-1,i),j} \qquad \text{- } \mathbf{MS34}$$

In **MS33**, i=n(j) refers to the number of iterations for n may be dependent upon the value of j, and the subscripts with j refer to the values or functions of radial coordinates for (r,θ,φ,γ) for iteration value of j, where j refers to the jth closed line that makes up the area A If it is considered that space is continuous, then m and n(j) → ꝏ and have **MS33** evaluated as an integral.

If radial coordinates for φ = constant and γ = constant for each iteration i,j , then have an area on a 2D plane surface.

If radial coordinates for φ ≠ constant and γ = constant for each iteration i,j , then have an area on a 3D surface.

If γ ≠ constant for each iteration i,j , then have a volume in 3D space, and not an area.

Volume

A volume is a region of 3D space that is enclosed by any closed surface. Volumes exist in 4D space as well but are a region on the surface of a 4D space.

In the same definition of an area being a conglomeration of 1D closed paths or lines in space, a volume can be considered as a conglomeration of areas. By definition, this means that a volume can also be considered as a conglomeration of 1D points all bunched up together and forming a matrix of neighbors, where the edge of a volume is considered as a location where in one or more radial directions, no neighbors exist.

Thus, a definition of a volume V would be the summation of neighboring closed areas of area A, or in mathematical form,

$$V=\sum_{k=0}^{k=l} A_k \qquad \text{- } \mathbf{MS34}$$

$$=\sum_{k=0}^{k=l} \sum_{m=0}^{j=m(k)} S_{j,k} \qquad \text{- } \mathbf{MS35}$$

$$=\sum_{k=0}^{k=l} \sum_{m=0}^{j=m(k)} \sum_{m=0}^{i=n(j,k)} S_{i,j,k} \qquad \text{- } \mathbf{MS36}$$

where

$$S^2_{i,j,k} = r^2_{\theta(i-1),j,k} + r^2_{\theta i,j,k} - 2r_{\theta(i-1),j,k}\, r_{\theta i,j,k} \cos\theta_{(i-1,i),j,k} \; +$$

$$r^2_{\phi(i-1),j,k} + r^2_{\phi i,j,k} - 2r_{\phi(i-1),j,k}\, r_{\phi i} \cos\phi_{(i-1,i),j,k} \; +$$

$$+ r^2_{\gamma(i-1),j,k} + r^2_{\gamma i,j,k} - 2r_{\gamma(i-1),j,k}\, r_{\gamma i,j,k} \cos\gamma_{(i-1,i),j,k} \quad \textbf{- MS37}$$

In **MS37**, i=n(j,k) refers to the number of iterations for n may be dependent upon the value of j and k, and the subscripts with j,k refer to the values or functions of radial coordinates for (r,θ,φ,γ) for iteration value of j,k, where j refers to the jth closed line of the kth surface that makes up the volume V, If it is considered that space is continuous, then k, m{k) and n(j,k) → oo and have **MS37** evaluated as an integral.

If γ = constant for each iteration i,j ,k then do not have a volume in 3D space or a 3D space on the surface of a 4D space.

S04 : More Space Properties

A further investigation into 1D and 2D space properties that extend into 3D space will help with understanding space .

Fig MSP-01

1: Shape

Considering from **definition S01:iv** that a space can have any "shape", an "observer" in any space cannot determine the true "shape" of that space. That is, a space can take on any "shape". One hypothetical shape could be as in fig MSP-01 for a 1D space. An "observer" only "sees" the "signals" "transmitted" by another entity that travels along the 1D space line. Unless information relating to the 1D space shape is given in this signal, then no determination of the 1D space shape can be made. Topologically though. any 1D space shape can be in its simplest form a circle.

Expanding this argument to 2D and 3D space, the shape of the 2D and 3D surfaces can be of any shape.

2: "Transmitted" "signals" can travel in any space in two possible modes

A: instantaneously:

There is no delay between "transmission" from an "emitter" entity and an "observation" made by an "observing" entity any where in the physical space.=> instantaneous self "observation" is possible.

If the "transmission" and "observation" can be considered as a physical processes, then an infinite number of these physical processes can occur simultaneously with no concept of time existing. That is, can have a system where all possible physical processes in a physical system all occur in one instant. Such a system is where there is no delay between one or more "transmission"-"observation", "observation"-"transmission" pair processes. That is, no change in the physical state of the entities existing in a physical space.

If have a delay between "transmissions" of "signals" or interaction from an entity to an "observer" entity such that the state of the entities changes by the "transmissions" and "observation" process, then a perception of time can be created in respect to each entity individually.

B: Delay:

Compared to the instantaneous condition given in 2A, a delay exists between a "signal" transmitted by entity O and "observed" by entity Q. This signal travels a unit length 1 in a given unit time interval. A definition of time is given in the section "A question of time" This delay creates a situation where physical processes in a physical space are dependent upon the separation between entities, and the perceived time it takes for the "transmitted" "signal" to reach the "observing" entity. If a "transmitted" "signal" were to reach the "observing" entity before an earlier transmitted" "signal" along the same physical path of space, then some further process must have caused this.

A concept of time can be developed for the entire physical space as a "signal" can be considered as being in between entities, and even as an entity itself. The state of the whole physical space changes as a change in the location of the "signal" can be considered as a change in the state of the physical space.

"Signals" transmitted in opposite directions arrive at the "observer" not simultaneously if the "observer" is not directly opposite to the "transmitter", and no interruption of the traveling "signals" occur. That is, if on a circumference of a circle for example, the "observer" was at location $\pi = -\pi$ radians away from a "transmitter" located at 0 radian origin coordinate transmits a signal in all directions simultaneously, then the "observer" would receive the signal simultaneously from all directions.

3: Continuity:

1D space can be considered as continuous if there is no interruption between a “signal” being transmitted and “observed”. If there were, a gap in 1D space will exist and there will be no “joining”, or rather a hole or gap on the physical space surface .

4: Homogeneity:
Any space can be considered as having the same constant properties that govern all physical processes at all locations. If this is not so, then an inconsistency will exist allowing or disallowing one or more physical processes to occur in one or more sections of physical space, or to have one or more physical processes undergoing some form of different transformation such that there is an inconsistency of physical behavior and outcome of results.

5: Physical processes occur at the same rate at all locations in all spaces in respect to an “observer” at that location. This follows from the homogeneity of space.

However, if it is considered that physical processes can occur at different observed rates away from an observers local location in space, but the end result of a physical process is always constant, then it can be argued that homogeneity of physical process still exists, but not the rate of physical processes.

6: A physical Space can change its “shape” or size, but all the properties 1-5 above must hold in this changed “shape”.

Consequences of space properties.

1: Change of shape of physical space can be considered as a change in spacial “density”, and is possible and can be noticed as such by a “stretching” or “squeezing” of a physical space by “observing” an apparent change in a section the spacial properties regarding the time it takes for a “signal” to traverse that section of space, and the observed time any physical processes that occur to take place by an observers clock. The travel times and changes in the physical state of entities take less or more time to occur in one region of space when compared to that in a different region of space with a different spacial “density”.(see about time for a more detailed discussion)

2: If space is “stretched” (“dilated”) in a region of physical space, the physical processes in that “stretched” region must still have the same properties as expressed in section **MSP-1D** above. Thus time to complete a physical process, and for a “signal” to traverse this region is the same as if it were in a relatively “un-stretched” region. This would then infer that a “stretched” region will have time progressing at a faster rate than that in a relatively “un-stretched” region, and the converse would be the case. ie a relatively “squeezed” (“contraction”) region will have time progressing at a slower rate than the regions around it.

Considering that all processes occur at the same rate in all physical spaces, the rate that a “transmitted” “signal” causes a change in the state of an entity remains constant in all regions of space => velocity of “transmitted” “signal” is constant => time slows down in “contracted“ regions and speeds up in “dilated” regions of space.

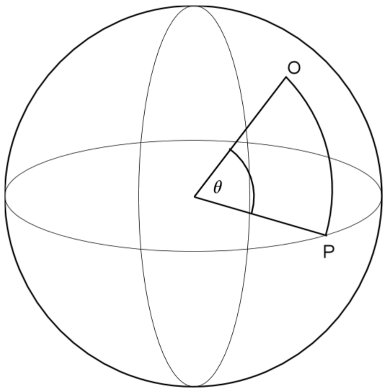
3: The result of this gives another emerging property that space and time are inter woven, and that no definite time clock exists for a physical space, and that an “observer” cannot tell if the region of space they exist in, or if any region of space exists in a natural, un-stretched, or squeezed state. In fact there may not ever be a “natural” state for space.

The discussion in the above sections gives a definition of physical space as if it were a separate physical entity that exists separate to the entities and physical phenomenon. The discussion above is also partly derived from observation of physical phenomenon of the universe and that certain behavior is never observed. Eg instantaneous transmission of electromagnetic waves and the constant velocity of light in all reference frames.

S05.0 Space as a Surface on a Higher Dimensional Space

Consider that there are three points **O,P,Q** that make up a triangle on a 2D surface lying on a sphere in 3D space (fig WP 2b). Consider that there are "2D Beings" that "live" on this surface that would observe the distance $\overline{OP}$, $\overline{OQ}$ and $\overline{PQ}$ as straight lines, where as a "3D being" that lives in 3D space would observe these as curved lines that depend upon the curvature of the sphere that the 2D surface being exist upon. Ie the radius of the sphere $\mathbf{r_s}$.

The distance in $\overline{OP}$ 2D space is the same distance as measured in 3D space over the curvature of the sphere, which is determined by the angle subtended between point P and O (fig WP 2a) (or the angle of rotation to translate point **P** to point **O**) and the radius $\mathbf{r_s}$ of the sphere.



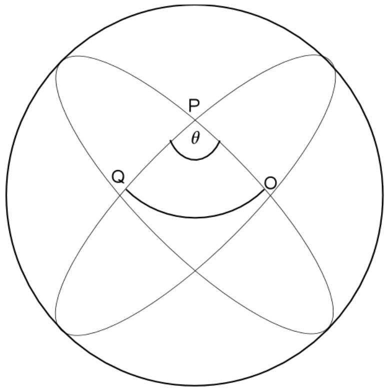


Fig WP 2a Fig WP 2b

If have spheres of different size but the distance $\overline{OP}$ is to be constant, then as

$$r_s \rightarrow \infty \text{ then } \theta \rightarrow 0$$

or as

$$\theta \rightarrow 2\pi \text{ then } r \rightarrow \frac{\overline{OP}}{2\pi}$$

assuming the angle subtended in 3D space cannot > 2π otherwise it folds onto itself and this violates the property of 3D space. So for a given distance $\overline{OP}$ in 2D space has in 3D space, the distance fo $\overline{OP}$ r curvature of 2D space in 3D space of curvature determined by (r_s,θ) being

$$\theta r_s = \overline{OP} \qquad \textbf{WP 3-1}$$

Extrapolating this to a 3D surface in a 4D space, then the distance is $\overline{OP}$ not the distance on a 3D sphere as above in 3D, but the straight line from P to O in 3D space that gives the shortest distance between P and O.

In 4D space li $\overline{OP}$ es on a 4D hyper sphere where 3D space is its surface. The distance in $\overline{OP}$ 3D space = the distance $\overline{OP}$ on a 4D hyper sphere. For an observer that exists in 4D space, that observer will see a $\overline{OP}$ s a curved line, where point **O**, **P** are separated by a hyper sphere angle Φ for a hyper sphere of radius r_h.

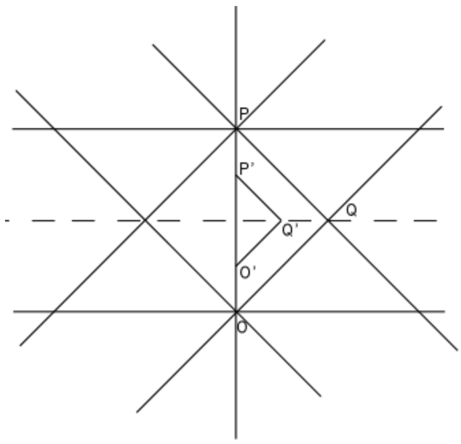


Fig WP 3

To find the radius of curvature, use a property that every point in 3D space is a point where traveling in any direction from it will lead back to that point, which means that for any two points separated by distance t $\overline{OP}$ hey will have these paths crossing over each other even if they start traveling in same direction “parallel” to each other. The distance it takes for this cross over of parallel lines will give a measurement of r_h as the point of cross over or meeting will be

$$\frac{\pi}{2} r_s = \overline{OP} \quad \text{or} \quad r_s = \overline{OP}\frac{\pi}{2} \quad \text{for a 2D surface on a 3D sphere}$$

For a very large $\mathbf{r_s}$, this measurement may be impractical.

For a 3D surface on a 4D space, the line of travel can be any line on a given parallel plane, or rather have planes intersecting each other. For this property to occur, all directions of travel from **O** and **P** travel away on lines forming a “great circle” or geodesic. Easy to assume since the shortest distance between any two points on a spherical surface must have the line joining them be a “great circle” or geodesic

Consider this to measure curvature of a flat 2D surface on a 3D spherical space. The distance from **O** to **Q** on a line of a geodesic midway between those of **P** and **O** has the distance $\overline{OQ}$ = distance $\overline{PQ}$ such that have the angle at **Q** = 90º or π/2 (fig WP 3). Then by Pythagoras theorem

$$\overline{OP}^2 = \overline{OQ}^2 + \overline{PQ}^2$$

$$= \quad 2\overline{OQ}^2$$

Now on a spherical surface $\overline{OQ}^2 = \theta r_s$ and $\overline{OQ} = \overline{PQ} = \Phi r_s$

Thus because Pythagoras theorem does not hold for a spherical surface

$$\overline{Op}^2 \neq 2\overline{OQ}^2$$

On all spheres for distance $\overline{PQ}$ = $\overline{OQ}$ have angles <POQ = <OPQ = π/4 the angle <OQP > π/2 and ≠ $\sqrt{2}\overline{PQ}$. Illustrated in **fig WP 3** from a 2D surface to a 3D sphere is as in **fig WP 4.**

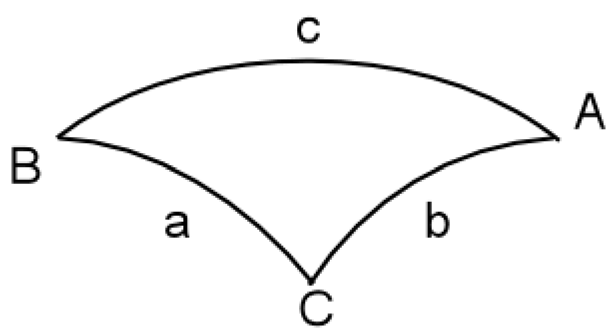


Fig WP 4

The distance a = b and the angle **A** =**B** = π/4. Angle **C** > π/2 and the distance **C** > $\sqrt{2}\,\boldsymbol{a}$

For a spherical surface, have the relationship

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C} \quad \text{WP 3-2}$$

and also

cos (c) = cos (a)cos(b) + sin(a)sin(b)cos(C) WP 3-3

By the use of these relationships, can use numerical methods to find a radius of the curvature of a sphere that this triangle lies on. To do this for a given distance **a**=**b** and measured **c** and **C**, relationship **WP 3-2** is compared to these values to see if there is a violation. If no violation is found then use of **WP 3-3** is used to see if this relationship is also violated. Ie

$\cos(c) = \cos^2(a) + \sin^2(a)\cos(C)$ WP 3-4

since a=b.

By use of computer program code, finding values of a,A a and C that honor relationship **WP 3-2** for various spherical surfaces and in turn also honor **WP 3-4**, it is found that for large spheres, **a** and **c** must become smaller and smaller in value otherwise **WP 3-2** and **WP 3-4** are never honored => for a very large sphere, then **a** and **c** must be exceedingly small and perhaps so small that they cannot be measured and thus have the wrong conclusion that the observer is in a flat space. This is counter intuitive until one considers Girands theorem which states the area for a triangle on a sphere is

$$A = r^2(a+b+c-\pi) \qquad \text{WP 3-5}$$

or

$$\frac{1}{r^2}=\frac{(a+b+c-\pi)}{A} \qquad \text{WP 3-6}$$

which = 0 on a plane. As one takes smaller and smaller areas on a sphere of radius **r**, it is found that **WP 3-6** → $1/r^2$

Equivalent in 4D space curvature most likely would be that

$$\frac{1}{r^3}\propto\frac{f(a,b,c,d)}{Volume} \qquad \text{WP 3-7}$$

where f(a,b,c,d) are angles of a tetrahedron or some function that defines the rules of the angles of a tetrahedron shape or other 3D shape and volume. Similarly expect that

$$\frac{f(a,b,c,d)}{Volume}\rightarrow\frac{1}{r^3} \qquad \text{WP 3-8}$$

for smaller and smaller volumes.

S05.1

If in 4D space have 3D space analogous to a 2D space on a 3D sphere, then 3D space exists on a 4D “hyper sphere”. In 2D space the triangle id the simplest 2D shape. In 3D space the tetrahedron is the simplest 3D shape. In 2D space on a 3D sphere the sides of the triangle each lie on a geodesic of a 3D sphere. In 3D space lying on a 4D hyper sphere the sides of a tetrahedron lay on a 4D geodesic of a 4d Hyper sphere.

S06.0 Most fundamental coordinate system between entities

For any two entities **P** & **Q** on a surface of a sphere (or any surface) that can interact to each other only on this spherical surface, are separated by a distance **OP**. This distance is measured by a 3D observer (fig QS 1a) as

$$\overline{PQ} = \Theta_{\overline{PQ}} r_s \qquad \textbf{- EQS-0}$$

These entities on their own have no preferred orientation in terms of any coordinate system employed. Once one entity is in communication of interaction with another, then a coordinate system can be employed by simply specifying an axis of a coordinate system that passes through the centers of each, establishing a straight line. Ignoring all but the segment of line connecting the these two entities, have only two variables that defines the system as far as coordinates are concerned. Ie a distance of separation and a reference of a coordinates of what is positive and negative in respect to each entity. (**fig QS1b**). Ie each entity shares a distance of separation

$$\overline{PQ} = \overline{QP}$$

Entity **Q** can consider that positive direction of coordinate system is in a direction towards **P**, and vise versa for entity **Q**.

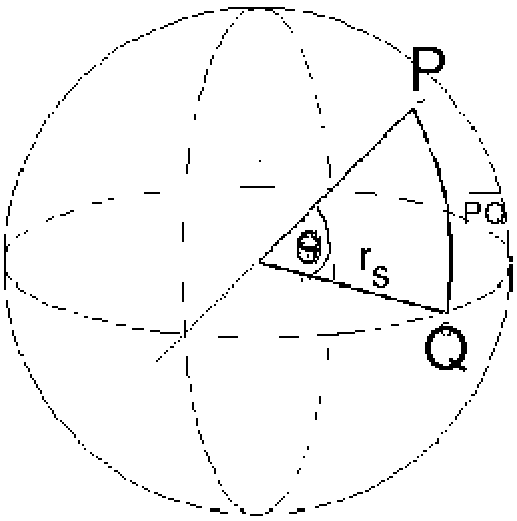


Fig QS 1a

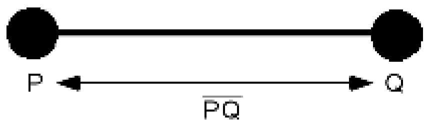


Fig QS1b

Since **Q** "sees" or is in communication with **P**, and **P** "sees" **Q** only and no other entity, then no other than one coordinate system is necessary to consider. Ie have one dimensional action **Q** on **P**, and **P** on **Q**, and thus have a one dimensional coordinate system for this system.

For such a system, any action or influence of **Q** on **P** or **P** on **Q** would be the same function **f** of the distance in one dimension. Ie proportional to $\overline{OP}$ and $\overline{PO}$ and $\Theta_{\overline{PQ}} r_s$ Such a function would have an opposite or negative value in respect from the direction of the measured distance from the perspective of the other entity location to the location of the entity at which the observer is located. That is if the observer was located at entity located at P, then $\overline{PQ} = -\overline{QP}$, and if if the observer was located at entity located at Q, then $\overline{QP} = -\overline{PQ}$. Similarly for function **f**, if the observer was located at entity located at P

$$f(\overline{PQ}) = -f(\overline{QP}) \qquad \textbf{- EQS-1}$$

and

In regards to if an observer was located at entity located at Q

$$f(\overline{QP}) = -f(\overline{PQ}) \qquad \textbf{- EQS-2}$$

Or more general expression for a 3D observer have

$$f_q(\Theta_{QP} r_s) = -f_p(\Theta_{PQ} r_s) \qquad \textbf{- EQS-3}$$

ie each entity sees the other entity as having a negative direction in any coordinate system in respect to itself if the origin of the coordinate system was moved to the other entities perspective, or location in respect of its own.