Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

5572680205

FURTHER MATHEMATICS **9231/12**

Paper 1 Further Pure Mathematics 1 **May/June 2023**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Let $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ 1 & 1 \end{pmatrix}$.

(a) Prove by mathematical induction that, for all positive integers n,

$$2\mathbf{A}^n = \begin{pmatrix} 2 \times 3^n & 0 \\ 3^n - 1 & 2 \end{pmatrix}.$$ [5]

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(b) Find, in terms of n, the inverse of $\mathbf{A}^n$. [2]

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2 The cubic equation $x^3 + 4x^2 + 6x + 1 = 0$ has roots α, β, γ.

(a) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. [2]

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(b) Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^{n}\left((\alpha+r)^2 + (\beta+r)^2 + (\gamma+r)^2\right) = n(n^2 + an + b),$$

where a and b are constants to be determined. [6]

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3 **(a)** Use the method of differences to find $\sum_{r=1}^{n} \frac{1}{(kr+1)(kr-k+1)}$ in terms of n and k, where k is a positive constant. [4]

(b) Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{(kr+1)(kr-k+1)}$. [1]

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(c) Find also $\sum_{r=n}^{n^2} \frac{1}{(kr+1)(kr-k+1)}$ in terms of n and k. [2]

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4 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} a & b^2 \\ c^2 & a \end{pmatrix}$, where a, b, c are real constants and $b \neq 0$.

(a) Show that **M** does not represent a rotation about the origin. [2]

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(b) Find the equations of the invariant lines, through the origin, of the transformation in the $x-y$ plane represented by **M**. [5]

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It is given that **M** represents the sequence of two transformations in the $x-y$ plane given by an enlargement, centre the origin, scale factor 5 followed by a shear, x-axis fixed, with $(0, 1)$ mapped to $(5, 1)$.

(c) Find **M**. [3]

(d) The triangle DEF in the $x-y$ plane is transformed by **M** onto triangle PQR.

Given that the area of triangle DEF is $12\,\text{cm}^2$, find the area of triangle PQR. [2]

5 The curve C has polar equation $r^2 = \frac{1}{\theta^2 + 1}$, for $0 \leqslant \theta \leqslant \pi$.

(a) Sketch C and state the polar coordinates of the point of C furthest from the pole. [3]

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(b) Find the area of the region enclosed by C, the initial line, and the half-line $\theta = \pi$. [4]

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(c) Show that, at the point of C furthest from the initial line,

$$\left(\theta + \frac{1}{\theta}\right)\cot\theta - 1 = 0$$

and verify that this equation has a root between 1.1 and 1.2. [5]

6 The curve C has equation $y = \dfrac{x^2 + 2x - 15}{x - 2}$.

(a) Find the equations of the asymptotes of C. [3]

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(b) Show that C has no stationary points. [3]

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(c) Sketch C, stating the coordinates of the intersections with the axes. [3]

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(d) Sketch the curve with equation $y = \left|\dfrac{x^2 - 2x - 15}{x - 2}\right|$. [2]

(e) Find the set of values of x for which $\left|\dfrac{2x^2+4x-30}{x-2}\right| < 15.$ [4]

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7 The plane Π_1 has equation $r = -4\mathbf{j} - 3\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + \mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k})$.

(a) Obtain an equation of Π_1 in the form $px + qy + rz = d$. [4]

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(b) The plane Π_2 has equation $\mathbf{r}.(-5\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}) = 4$.

Find a vector equation of the line of intersection of Π_1 and Π_2. [4]

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The line l passes through the point A with position vector $a\mathbf{i} + a\mathbf{j} + (a-7)\mathbf{k}$ and is parallel to $(1-b)\mathbf{i} + b\mathbf{j} + b\mathbf{k}$, where a and b are positive constants.

(c) Given that the perpendicular distance from A to Π_1 is $\sqrt{2}$, find the value of a. [2]

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(d) Given that the obtuse angle between l and Π_1 is $\frac{3}{4}\pi$, find the exact value of b. [4]

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Additional page

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