NAO Robot Summary

Agenda

1 Introduction

Game Theory focuses its attention on the interaction among independent and self-interested agents, trying to study their behaviors mathematically; all the emerging concepts have been successfully applied to different kind of disciplines, such as Economics, Biology and Computer Science.

In order to understand the following sections, a brief introduction to some useful concepts and a recall of the notations is needed.

In Game Theory, a *game* is formally defined by a pair (M,σ), where M is called *mechanism*, representing how agents interact, and σ is the set of *strategies*, that describes how agents will play. There exist three different types of mechanisms:

- Strategic-form game
- Extensive-form game
- Bayesian-form game

These lecture notes focus on the first type of game, the *normal* or *strategic* form, which is the most familiar representation of strategic interactions and is formally defined as (Eq. **??**):

$$M = (N, \{A\}_i, X, f, \{U\}_i) \tag{1}$$

where N is the set of agents, $\{A\}_{i \in N}$ is the set of action for agent i, X is the set of outcomes, $f : A_1 \times A_2 \times \cdots \times A_n \to X$ is a function that maps actions of agents to an outcome, and $U_i : X \to \mathbb{R}$ describes the utility function of agent i.

A particular kind of game is called *Constant-Sum game*; in this kind of game the following hold (Eq. **??**):

$$\sum_{i=1}^{N} u_i(x) = const \qquad \forall x \tag{2}$$

If a game does not respect this property, it is called *Bimatrix game*.

A natural way to represent constant-sum games is via an n-dimensional matrix, where n is the number of players; as an example, Table **??** reports the matrix that describes the "Rock-Paper-Scissor" game.

		Agent 2		
		R	**P**	**S**
Agent 1	**R**	T	W2	W1
	P	W1	T	W2
	S	W2	W1	T

Table 1: The Rock-Paper-Scissor game; T = tie, W_1 = Agent 1 wins, W_2 = Agent 2 wins.

On this game, it is possible to define two utility functions U_1 and U_2 for the two agents respectively.

$$U_1 = \begin{cases} 1, & x = W_1 \\ -1, & x = W_2 \\ 0, & x = T \end{cases} \qquad\qquad U_2 = \begin{cases} -1, & x = W_1 \\ 1, & x = W_2 \\ 0, & x = T \end{cases}$$

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A *strategy* σ_i for agent i is defined as a probability distribution over the set of actions A_i; if we denote the probability with which the agent i plays an action j with $x_{i,j}$, the two followings property must holds:

1. $\mathbf{x_i} \geq \mathbf{0}$
2. $\mathbf{1^T x_i} = 1$

A *strategy profile* σ is the collection of one strategy per agent (Eq. **??**).

$$\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_{|N|}) \tag{3}$$

As an example, for the Rock-Paper-Scissor strategies for the two players can be:

$$x_1 = \begin{cases} x_{1,R} = 0.4 \\ x_{1,P} = 0.4 \\ x_{1,S} = 0.2 \end{cases} \qquad x_2 = \begin{cases} x_{2,R} = 0.5 \\ x_{2,P} = 0.1 \\ x_{2,S} = 0.4 \end{cases}$$

A *stategy* can be:

- pure, when all actions but one have a zero probability, i.e. the players will just select that action and play it;
- mixed, if the players will choose randomly over the set of available actions according to a probability distribution;
- fully mixed, if it is mixed for each player and each action has a non-zero probability.

The *belief* $\hat{\sigma_i}$ agent i has over strategy σ_j of agent j describes how agent i beliefs the agent j will play.

It is interesting to compute the *expected utility* of each agent, i.e how much a player will get playing its strategy. If the game has a pure strategy, the expected utility is computed simply applying the strategy to the mechanism; otherwise for a (fully) mixed strategy game (Eq. **??**):

$$E(u_i) = \mathbf{x}_1^T \mathbf{U}_i \prod_{k \in N, k \neq i} \mathbf{x}_k \tag{4}$$

where $\mathbf{U}_i$ is the matrix of utilities of agent i.

A particular case are games with two players (Eq. **??**):

$$\begin{aligned} E(u_1) = \mathbf{x}_1^T \mathbf{U_1 x_2} \\ E(u_2) = \mathbf{x}_1^T \mathbf{U_2 x_2} \end{aligned} \tag{5}$$

It is possibile now to compute the expected utility for both agent involved in the the Rock-Paper-Scissor game:

$$U_1 = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \qquad \mathbf{x}_1 = \begin{bmatrix} 0.4 \\ 0.4 \\ 0.2 \end{bmatrix} \qquad \mathbf{x}_2 = \begin{bmatrix} 0.5 \\ 0.1 \\ 0.2 \end{bmatrix}$$

$$E(u_1) = \begin{bmatrix} 0.4 & 0.4 & 0.2 \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.1 \\ 0.2 \end{bmatrix} = 0.24$$

Now, given a game, the strategy profile σ and the belief μ_i of each agent, how is it possible to reason about that game?

In a single-agent decision theory, it is simply necessary to search for the optimal strategy for the agent, i.e. a list of actions that maximizes the agent's expected utility; with multiple agents this reasoning is more difficult, as the optimal strategy depends also on others' choices. It is thus necessary to identify a certain subset of outcomes, called *solution concepts*, that are interesting in some sense. As two examples, the *Pareto optimality* and the *Nash equilibrium* have been defined.

Given a solution concept, it is then interesting to study its *complexity* and the *best algorithms* to find it, as well as to characterize all the possible *instances*.

With all these definitions in mind, these lecture notes are organized as follows:

- Topic **??** will be about complexity;
- Topic **??** will describe the Pareto Optimality solution concept;
- Topic **??** will concern dominations of strategies;
- Topic **??** will discuss the MinMax/MaxMin solution concept;
- Topic **??** will present some advanced topics: ...