$y - y_1 = m(x - x_1)$

~~$\frac{\pi}{2} - y_1 = 1$~~

$f(x) = \sin \Rightarrow f'(x) = \cos(x) = -\frac{1}{2}$

$f(x) = e^{2x+1} + e^2$

$\frac{d}{dx}(e^{2x+1}) \cdot \frac{d}{dx}(e^2)$

$= 2e^{2x+1}$

$f(g(4)) \quad f'(g^{-1}(4)) = f'(1) = 2(1) + 6 = \boxed{8}$

$g(4) = 1$

$f(y(x)) = f'(g(x)) \cdot g'(x)$

$f(x) = \log_4(x)$

$f^{-1}(x) = \frac{\ln(x)}{\ln(4)}$

$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$

$f'(x) = \frac{1}{\ln(4)} \cdot \frac{1}{x} = \boxed{\frac{1}{\ln(4)x}}$

$f'(x) = 7 \cdot \frac{d}{dx}(\arctan(x))$

$= 7 \cdot \frac{1}{1+x^2}$

$f'(1) = 7 \cdot \frac{1}{1+1^2} = 7 \cdot \frac{1}{2} = \frac{7}{2}$

$f'(1) \cdot$

~~$2x + 6$~~

$f(x) = \frac{x^4}{\ln(x)} \rightarrow 4x^3 \quad \ln(x) \rightarrow \frac{1}{x}$

~~$\frac{1}{x}$~~

$= \frac{4x^3 \ln(x) - x^3}{(\ln(x))^2} = \frac{x^3(4\ln(x) - 1)}{(\ln(x))^2}$

$= \frac{4x^3 \ln(x) - x^3}{(\ln(x))^2}$

Question 10:

(a) $y^4e^x - 2 = \cos(\pi y)$

$$\frac{d}{dx}(y^4e^x - 2) = \frac{d}{dx}\cos(\pi y)$$

$$y^4\frac{d}{dx}(e^x) + e^x\frac{d}{dx}(y^4) - 0 = \frac{d}{dx}(\cos(\pi y))$$

$$y^4e^x + e^x\frac{d}{dx}(y^4) = -\sin(\pi y)\cdot\pi\frac{dy}{dx}$$

$$y^4e^x = \frac{dy}{dx}(-\pi\sin(\pi y) - e^x\cdot 4y^3)$$

$$\boxed{\frac{dy}{dx} = \frac{y^4e^x}{-\pi\sin(\pi y) - 4y^3e^x}} = (A)$$

(b) $$\frac{dy}{dx} = \frac{(1)^4e^0}{-\pi\sin(\pi(1)) - 4(1)^3e^0} = \frac{1\cdot 1}{-\pi\sin(\pi) - 4\cdot 1\cdot 1} = -\frac{1}{4}$$

$$y - \frac{1}{4}(x - 0) = \boxed{y = -\frac{1}{4}x + 1} = (B)$$

Question 11:

$\frac{dy}{dx}$ if $y = x^{-\sin x}$

$$\ln y = \ln(x^{-\sin x})$$

$$\ln y = -\sin x \ln x$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(-\sin x \ln x)$$

$$\frac{1}{y}\frac{dy}{dx} = -\cos x \ln x - \sin x\cdot\frac{1}{x}$$

$$\frac{dy}{dx} = y\left(-\cos x \ln x - \frac{\sin x}{x}\right)$$

$$\boxed{\frac{dy}{dx} = x^{-\sin x}\left(-\cos x \ln x - \frac{\sin x}{x}\right)} = 11$$