Documentation of Concepts and Formulae for 1D Model

“NUMERICAL MODELLING OF PERMEABILITY HETEROGENEITY IN SLIGHTLY COMPRESSIBLE FLUID RESERVOIR”

Created by: Idris Kagdi

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Abstract

Generally, Reservoir simulation involves partitioning the reservoir into grids or blocks, where properties like permeability, porosity, saturations, viscosity, etc. are defined for each block. In this project, we will be numerically solving diffusivity equation with varying permeability (heterogenous reservoir) and single phase with methods like implicit or explicit solution by Finite Volume Method (FVM) and discretization, using MS Excel and Python, and verifying our solution using CMG software for any number of block sizes.

1. **Introduction:**

   The reservoir heterogeneity is then defined as a variation in reservoir properties as a function of space. In the heterogeneous reservoir properties vary as a function of a spatial location. These properties may include permeability, porosity, thickness, saturation, faults and fractures, rock facies, and rock characteristics. For a proper reservoir description, we need to predict the variation in these reservoir properties as a function of spatial locations.

2. **Conceptualization:**

   2.1 <u>Diffusivity Equation and Constant Permeability Model:</u>

1-D 1-ϕ diffusivity equation describes

$$\frac{\partial^2 P}{\partial x^2} = \frac{1}{\alpha}\frac{\partial P}{\partial t}$$

$$\alpha = \frac{k}{\phi \mu C_t} = Diffusivity\ Coefficient$$

PDE is subject to initial and boundary conditions

Boundary conditions can be:

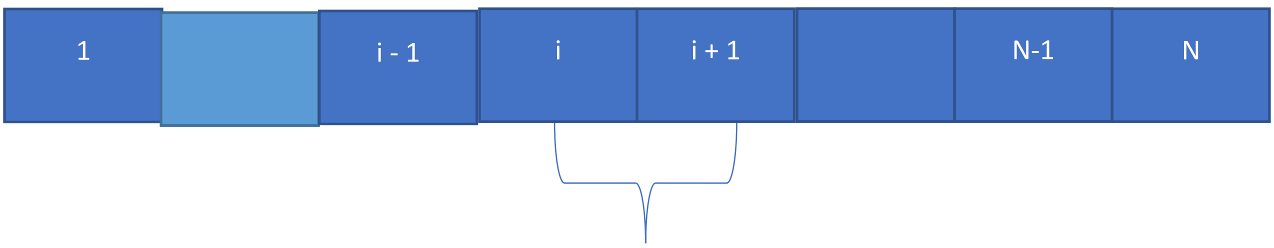
- Dirichlet (constant pressure)
- Neumann (constant flow)

P(x, 0) = P initial

P(0, t) = P_{B1} or $\frac{\partial p}{\partial x}$(0, t) = 0

P(L, t) = P_{B2} or $\frac{\partial p}{\partial x}$(L, t) = 0

Domain in N- Grid block



Δx = $\frac{L}{N}$

- Each block has a finite size Δx. Although it can vary block to block.
- Each block has constant fluid and rock properties
- In a cell centered approach, we solve for discrete pressures, Pi located at the center of each block.

2.1.1 EXPLICIT SOLUTION:

It uses pressure at old time level to calculate the new pressure P_i^{n+1}.

$$\frac{Pi^{n+1} - Pi^{n}}{\Delta t}\frac{1}{\alpha} = \frac{Pi-1^{n} - Pi^{n} - Pi+1^{n}}{\Delta x^2}$$

Rearranging for pressures at n+1 gives,

$$P_i^{n+1} = P_i^n + \frac{\alpha \Delta t}{\Delta x^2}\{ P_{i-1}^n - P_i^n - P_{i+1}^n \}$$

An initial condition is provided, $P(x,0) = [P_1^0, P_2^0, \ldots\ldots P_n^0]$

$$P_i^1 = P_i^0 + \frac{\alpha \Delta t}{\Delta x^2}\{ P_{i-1}^0 - P_i^0 - P_{i+1}^0 \}$$

Now solve for P^1 in every block i = 1 to N using initial condition. Use pressures in P^1 to solve for pressures in P^2. Continue until final time step.

At boundaries

$$P_1^1 = P_1^0 + \frac{\alpha \Delta t}{\Delta x^2}\{ P_0^0 - P_1^0 - P_2^0 \}$$

These grid blocks doesn't exist

$$P_N^{N+1} = P_N^N + \frac{\alpha \Delta t}{\Delta x^2}\{ P_{N-1}^N - P_N^N - P_{N+1}^N \}$$

1. Dirichlet (constant pressure):

   It states that P is known exactly at boundary. But grid pressures are defined at center (not edge) of the block.

   - Approximate pressure in block 1, this is only O (Δx) accurate:

     $$P_1 = P_{B1}$$

   - Better approximation: edge pressures are average of two neighboring grid pressures. $O(\Delta x^2)$ accurate

     $$P_{B1} = \frac{P_0 + P_1}{2}$$

     $$P_0 = 2P_{B1} - P_1$$

   New equation for block 1

$$\frac{P1^{n+1}- P1^{n}}{\Delta t}\frac{1}{\alpha} = \frac{(2P^n_{B1}-P^n_1)-P^n_1- P^n_2}{\Delta x^2}$$

2. Neumann (constant/no flux):
   No flux implies zero pressure gradient at the boundary flux (L,t)= 0
   $\partial P/\partial x$ (L,t) = 0
   Use a finite difference formula for pressure gradient at the boundary: -
   $$\frac{\partial P}{\partial x} = \frac{(P_N - P_{N+1})}{\Delta x}$$
   $$P_N = P_{N+1}$$

Finite difference equation at boundary then becomes:

$$\frac{P^{n+1}_N - P^n_N}{\Delta t}\frac{1}{\alpha} = \frac{(P^n_{N-1}-2P^n_N-P^n_N)}{\Delta x^2} = \frac{P^n_{N-1} - P^n_N}{\Delta x^2}$$

Stability of explicit solution

Explicit method is only stable if (in Field units):

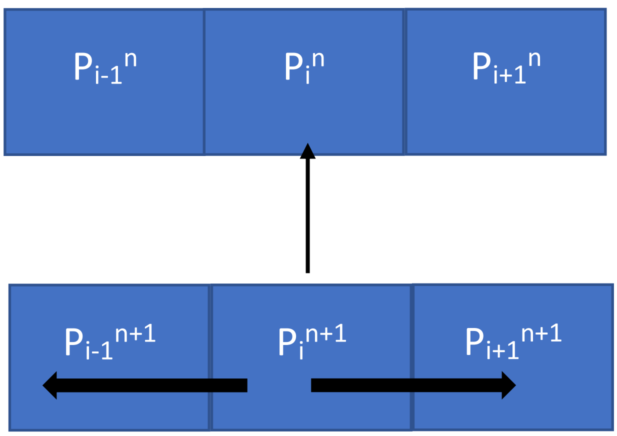
$$\eta = \frac{0.00633\ k\ \Delta t}{\varphi \mu C_t\ \Delta x^2} \leq 0.5$$

2.1.2 IMPLICIT SOLUTION:

$-\eta P_{i-1}^{n+1} + (1+2\eta)P_i^{n+1} - \eta P_{i+1}^{n+1} = P_i^n$

Where n $= \frac{\alpha \Delta t}{\Delta x^2}$ dimensionless

This means that the pressure of block i at the next time level n+1 depends on the pressure at the neighbors i-1 and i+1 also at the time level n+1.



We can write the same equations for all blocks from i = 1 to i = N

Block #1 $-\eta P_0^{n+1} + (1+2\eta)P_1^{n+1} - \eta P_2^{n+1} = P_1^n$

Block #2 $-\eta P_1^{n+1} + (1+2\eta)P_2^{n+1} - \eta P_3^{n+1} = P_2^n$

Block #3 $-\eta P_2^{n+1} + (1+2\eta)P_3^{n+1} - \eta P_4^{n+1} = P_3^n$

Block #i $-\eta P_{i-1}^{n+1} + (1+2\eta)P_i^{n+1} - \eta P_{i+1}^{n+1} = P_i^n$

Block #N $-\eta P_{N-1}^{n+1} + (1+2\eta)P_N^{n+1} - \eta P_{N+1}^{n+1} = P_N^n$

1. Dirichlet (constant pressure):
   - Approximate edge pressure as average of two neighboring grid pressures

$$P_{B1} = \frac{P_0 + P_1}{2}$$

$$P_0 = 2P_{B1} - P_1$$

New equation block #1

$$-\eta P_0^{n+1} + (1+2\eta)P_1^{n+1} - \eta P_2^{n+1} = P_1^n$$

$$-\eta(2P_{B1} - P_1^{n+1}) + (1+2\eta)P_1^{n+1} - \eta P_2^{n+1} = P_1^n$$

$$(1+3\eta)P_1^{n+1} - \eta P_2^{n+1} = P_1^n + 2\eta P_{B1}$$

2. Neumann boundary condition:
   - Use reflection technique at boundary

$$\frac{\partial P}{\partial x} = \frac{(P_N - P_{N+1})}{\Delta x} = 0$$

$$P_N = P_{N+1}$$

New equation block #N

$$-\eta P_{N-1}^{n+1} + (1+2\eta)P_N^{n+1} - \eta P_{N+1}^{n+1} = P_N^n$$

$$-\eta P_{N-1}^{n+1} + (1+2\eta)P_N^{n+1} - \eta P_N^{n+1} = P_N^n$$

$$-\eta P_{N-1}^{n+1} + (1+\eta)P_N^{n+1} = P_N^n$$

2.2 Diffusivity Equation and Permeability HETEROGENEITY Model:

Transmissibility Discretized equation for i^{th} Block:

$T_{i-1,i}\ (P_{i-1} - P_i) + T_{i+1,i}\ (P_{i+1} - P_i) = \frac{B_i}{\Delta t}\ (P_i^{n+1} - P_i^n)$

Where:

$T_{i-1,i}$: Interblock Transmissibility for the i^{th} and $i+1^{th}$ Blocks (Refers to how much fluid flows in or out of the grid block)

B_i : Volume Accumulation for the i^{th} Block (Refers to how much the fluid is expanded/contracted when pressure is increased/decreased)

If in superscript we take

- n+1 -> then implicit form
- n -> then explicit form

In general,

$B_i = \frac{Vi\ \emptyset\ Ct}{B_w}$ $\qquad$ $T = \frac{k\ A}{\mu B_w \Delta x}$

$-TP_{i-1}^{n+1} + (\frac{B_i}{\Delta t} + 2T)\ P_i^{n+1} - TP_{i+1}^{n+1} = \frac{B_i}{\Delta t} P_i^n$

In Field Units:

$T_{i-1,i} = \frac{2*0.001127}{\frac{\Delta x}{K_{i-1}A} + \frac{\Delta x}{K_i A}} = \frac{C\ A}{\Delta x\left(\frac{1}{K_{i-1}} + \frac{1}{K_i}\right)}$

$B_i = \frac{A_i\ \Delta x_i\ \emptyset\ Ct}{B} = \frac{A\ \Delta x\ \emptyset\ Ct}{5.615\ B}$

2.2.1 EXPLICIT SOLUTION:

$T_{i-1,i}\ (P_{i-1}^n - P_i^n) + T_{i+1,i}\ (P_{i+1}^n - P_i^n) = \frac{B_i}{\Delta t}\ (P_i^{n+1} - P_i^n)$

$$T_{i-1,i} = \frac{2*0.001127}{\left(\frac{\Delta x}{K_{i-1}A}+\frac{\Delta x}{K_i A}\right)\mu B} = \frac{2*0.001127\ A}{\mu B \Delta x\left(\frac{1}{K_{i-1}}+\frac{1}{K_i}\right)}$$

$$K_{i-1,i} = \frac{2}{\left(\frac{1}{K_{i-1}}+\frac{1}{K_i}\right)}$$

$$T_{i-1,i} = \frac{0.001127\ K_{i-1,i}\ A}{\Delta x \mu B} \qquad T_{i+1,i} = \frac{0.001127\ K_{i+1,i}\ A}{\Delta x \mu B}$$

$$B_i = \frac{A \Delta X \emptyset\ Ct}{5.615\ B}$$

$$\frac{B_i\ P_i^{n+1}}{\Delta t} = T_{i-1,i}\ P_{i-1}^n + \left\{\left(\frac{B_i}{\Delta t}\right) - T_{i-1,i} - T_{i+1,i}\right\} P_i^n + T_{i+1,i} P_{i+1}^n$$

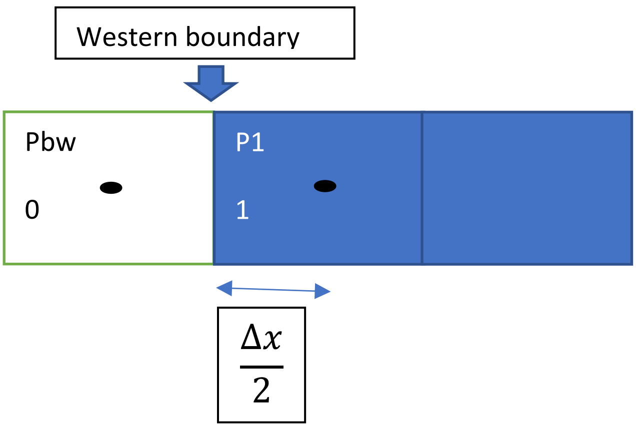
$$P_i^{n+1} = \frac{\Delta t\ T_{i-1,i}}{B_i} P_{i-1}^n + \left(1 - \frac{\Delta t\ T_{i-1,i}}{B_i} - \frac{\Delta t\ T_{i+1,i}}{B_i}\right) P_i^n + \frac{\Delta t\ T_{i+1,i}}{B_i} P_{i+1}^n$$

As

$$\frac{\Delta t\ T_{i-1,i}}{B_i} = \frac{\Delta t*0.001127\ \ K_{i-1,i}\ A*5.615\ B}{B\ \Delta x\ \mu\ A\ \Delta x\ \emptyset\ Ct} = \frac{0.00633\ \Delta t\ K_{i-1,i}}{\emptyset \mu Ct\ (\Delta x)^2} = \eta_{i-1,i}$$

$$\eta_{i-1,i} P_{i-1}^n + \left(1 - \left(\eta_{i-1,i} + \eta_{i+1,i}\right)\right) P_i^n + \eta_{i+1,i} P_{i+1}^n = P_i^{n+1}$$

- For first block, using Dirichlet (constant pressure):



Here $P_{bw} = P_{b1}$

$$T_{bw,1}(P_{bw}^n - P_1^n) + T_{1,2}(P_2^n - P_1^n) = \frac{B_1}{\Delta t}(P_1^{n+1} - P_1^n)$$

$$T_{bw,1} = \frac{0.001127\ K_{0,1}\ A}{\mu\ B\ \frac{\Delta x}{2}} \qquad \text{Where } K_{0,1} = \frac{2}{\left(\frac{1}{K_1}+\frac{1}{K_1}\right)} = K_1$$

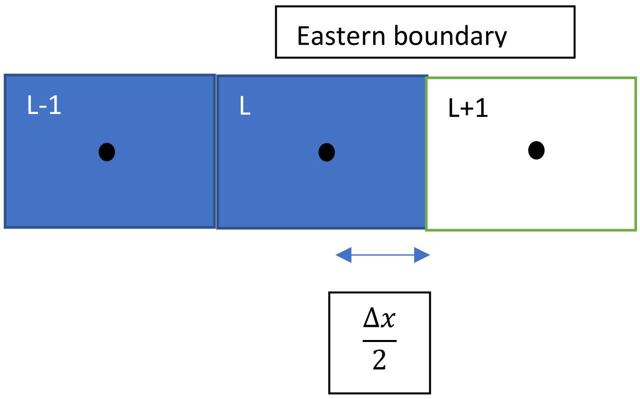
$$P_1^{n+1} = \eta_{0,1} P_{b1}^n + \left(1 - (\eta_{0,1} + \eta_{1,2})\right) P_1^n + \eta_{1,2} P_2^n$$

Where $\eta_{0,1} = \frac{0.00633\ \Delta t\ 2K_1}{\phi \mu Ct\ (\Delta x)^2}$

- Intermediate block:

$$\eta_{i-1,i} P_{i-1}^n + \left(1 - (\eta_{i-1,i} + \eta_{i+1,i})\right) P_i^n + \eta_{i+1,i} P_{i+1}^n = P_i^{n+1}$$

- Last block – Using Neuman condition:



$$\frac{\partial P}{\partial x} = \frac{(P_{bc} - P_l)}{\Delta x} = 0$$

$$P_{bc} = P_l$$

$$K_{bc,l} = \frac{2}{\left(\frac{1}{K_{bc}} + \frac{1}{K_l}\right)}$$

$$T_{bc,l}(P_{bc}^n - P_l^n) + T_{l-1,l}(P_{l-1}^n - P_l^n) = \frac{B_l}{\Delta t}(P_l^{n+1} - P_l^n)$$

$$P_l^{n+1} = P_l^n + \eta_{l-1,l}\,(P_{l-1}^n - P_l^n)$$

2.2.2 IMPLICIT SOLUTION:

$$-\eta_{i-1,i} P_{i-1}^{n+1} + \left(1 + (\eta_{i-1,i} + \eta_{i+1,i})\right) P_i^{n+1} - \eta_{i+1,i} P_{i+1}^{n+1} = P_i^n$$

- First block – Dirichlet condition:

$$\left(1 + (\eta_{0,1} + \eta_{1,2})\right) P_1^{n+1} - \eta_{1,2} P_2^{n+1} = P_1^n + \eta_{0,1} P_{b1}^n$$

- Intermediate block:

$$-\eta_{i-1,i}P_{i-1}^{n+1} + \left(1+\left(\eta_{i-1,i}+\eta_{i+1,i}\right)\right)P_i^{n+1} - \eta_{i+1,i}P_{i+1}^{n+1} = P_i^n$$

- Last block –Neuman condition:

$$-\eta_{l-1,l}P_{l-1}^{n+1} + \left(1+\eta_{l-1,l}\right)P_l^{n+1} = P_l^n$$

Tri-diagonal Matrix Equation:

$$\begin{bmatrix} (1+\eta_{0,1}+\eta_{1,2}) & -\eta_{1,2} & 0 & 0 & 0 & \dots & 0 \\ -\eta_{1,2} & (1+\eta_{1,2}+\eta_{2,3}) & -\eta_{2,3} & 0 & 0 & \dots & 0 \\ 0 & -\eta_{2,3} & (1+\eta_{2,3}+\eta_{3,4}) & -\eta_{3,4} & 0 & \dots & 0 \\ \vdots & \vdots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots\ -\eta_{i-1,i} & (1+\eta_{i-1,i}+\eta_{i+1,i}) & -\eta_{i+1,i} & & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & -\eta_{l-1,l} & (1+\eta_{l-1,l}) \end{bmatrix} .$$

$$\begin{bmatrix} P_1^{n+1} \\ P_2^{n+1} \\ P_3^{n+1} \\ \dots . \\ P_i^{n+1} \\ \dots \\ P_l^{n+1} \end{bmatrix} = \begin{bmatrix} P_1^n \\ P_2^n \\ P_3^n \\ \dots \\ P_i^n \\ \dots \\ P_l^n \end{bmatrix} + \begin{bmatrix} \eta_{0,1}P_{b1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

So,

$\eta * P^{n+1} = P^n + M$

$M = \left[\eta_{0,1}\ P_{b1} \quad 0 \quad 0 \quad 0 \quad \dots\ 0\right]^T$

2.3 Quantifying Permeability Heterogeneity in a reservoir :

The following are the three most widely used descriptors of the heterogeneity of the formation:

- Dykstra-Parsons permeability variation V
- Lorenz coefficient L
- Crossflow Index CI

In our study, we will be using Dykstra-Parsons Coefficient for estimating extent of Permeability Heterogeneity. Dykstra and Parsons (1950) introduced the concept of the permeability variation coefficient V, which is a statistical measure of non-uniformity of a set of data. It is generally applied to the property of permeability but can be extended to treat other rock properties. It is generally recognized that the permeability data are log-normally distributed. That is, the geologic processes that create permeability in reservoir rocks appear to leave permeabilities distributed around the geometric mean. Dykstra and Parsons recognized this feature and introduced the permeability variation that characterizes a particular distribution.

Dykstra-Parsons Heterogeneity Coefficient (V) is estimated as:

$$V = \frac{k_{50} - k_{84.1}}{k_{50}} = \frac{k_{\mu} - k_{1\sigma}}{k_{\mu}}$$

$$k_{50}\ or\ k_{\mu} = Permeability\ value\ at\ 50\%\ CPD\ data\ or\ mean\ k$$

$$k_{84.1}\ or\ k_{1\sigma} = Permeability\ value\ at\ 84.1\%\ CPD\ data\ or\ one\ standard\ deviation\ k$$

CPD data is Cumulative Probability distributed Data.

V = 0 => Perfectly Homogeneous Formation
V = 1 => Highly Heterogeneous Formation

3. Methodology and Results:

3.1 Modelling for Constant (Homogeneous) Permeability 4-block 1-D 1-φ reservoir in MS Excel:

3.1.1 EXPLICIT METHOD:

In explicit method, we analyzed 1D block homogeneous permeability by using following values. In this, we took 4 blocks and applied boundary conditions i.e., Dirichlet and Neumann. Dirichlet condition is applied on first block and Neumann condition on last block. Also, Stability Criteria is also taken into account for explicit method.

Reservoir Properties

Length, L(ft)	1000
Porosity, ϕ(fraction)	0.2
Permeability, k md	50
Height H ft	10
Width W ft	100

Numerical Properties

Δt, day	0.002
N	4
Δx	250
α, Diffusivity	2.50E+08
η	0.05064
	<0.5

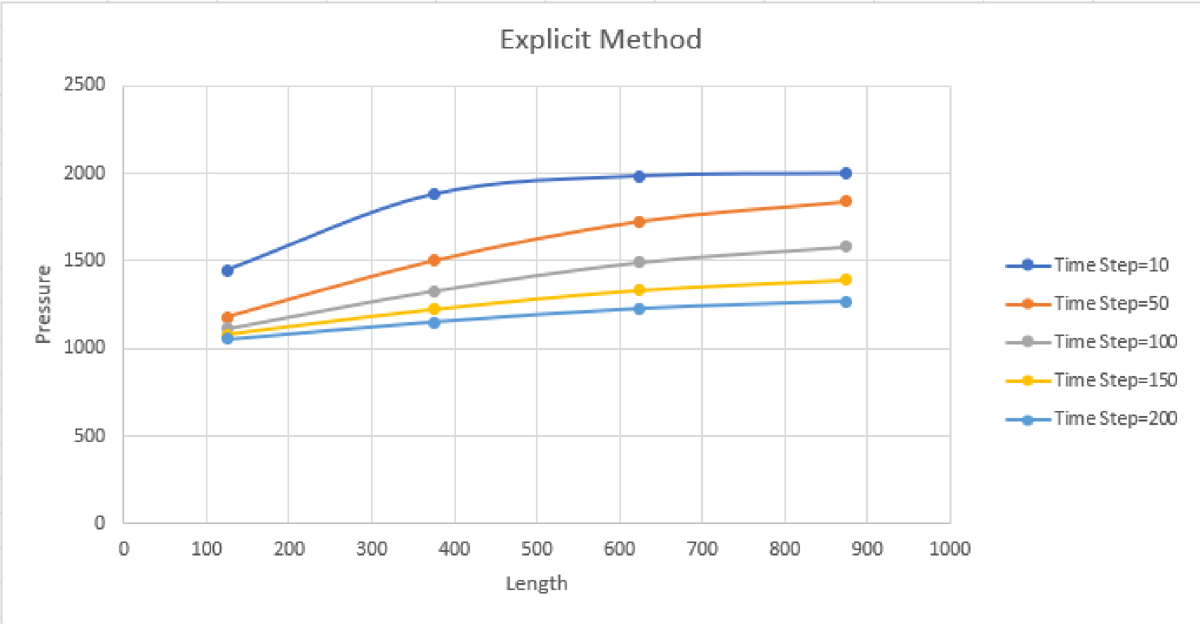
Fluid Properties

Viscosity μ cp	1
Cf, psi-1	0.000001
Bo, RB/STB	1

BCs

L=0 Inlet	1000	psi
		Dirichlet
L=L Outlet	0	ft3/day
		Neumann
t=0 Initial	2000	Psi

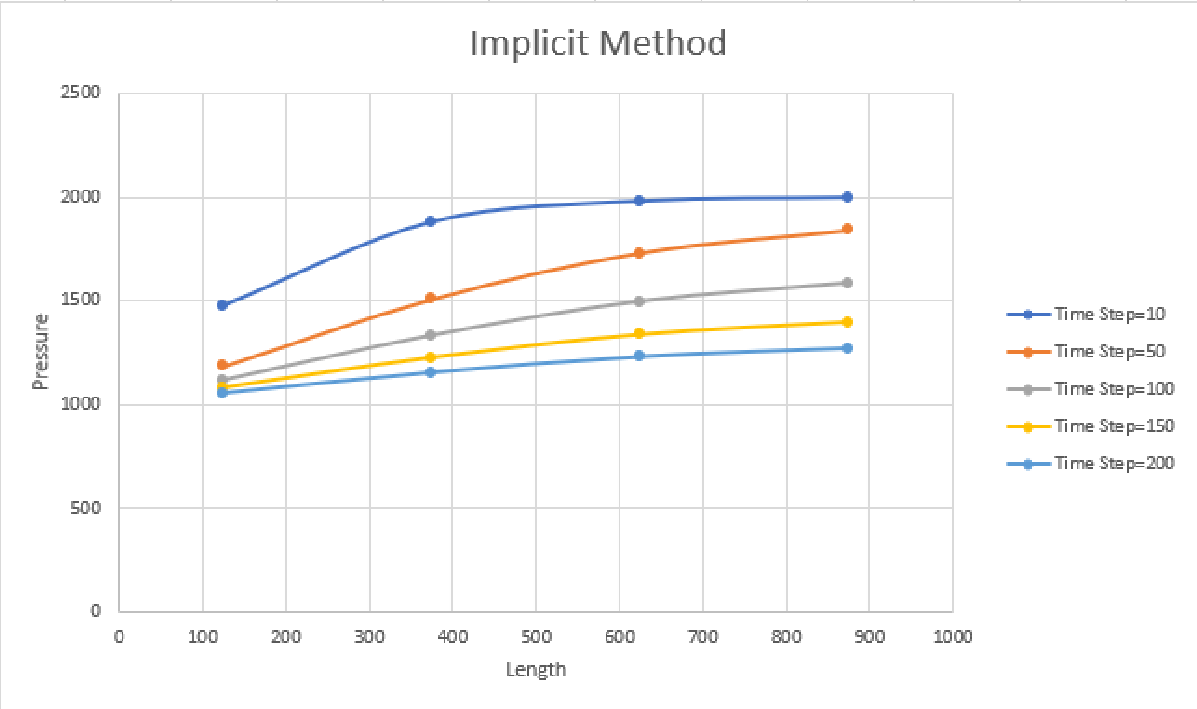
The Pressure profile Plot generated is as shown:



3.1.2 <u>IMPLICIT METHOD:</u>

In implicit method, we analyzed 1D block homogeneous permeability by using same values as in explicit method. In this, we took 4 blocks and applied boundary conditions i.e., Dirichlet and Neumann. Dirichlet condition is applied on first block and Neumann condition on last block. Following excel plot is plotted between pressure and length: -

Tri-Diagonal Matrix: $\begin{pmatrix} 1.15192 & -0.05064 & 0 & 0 \\ -0.05064 & 1.10128 & -0.05064 & 0 \\ 0 & -0.05064 & 1.10128 & -0.05064 \\ 0 & 0 & -0.05064 & 1.05064 \end{pmatrix}$



3.2 Modelling for Variable (Heterogeneous) Permeability 4-block 1-D 1-φ reservoir in MS Excel:

3.2.1 EXPLICIT METHOD:

For permeability heterogeneity case, we varied permeability of 4 blocks and then applied boundary conditions i.e., Dirichlet and Neumann. Dirichlet condition is applied on first block and Neumann condition on last block. Also, Stability Criteria is also taken into account for explicit method.

Data used is given as, along with different permeability values for each block and Inter-Block:

Reservoir Properties	
Length, L(ft)	1000
Porosity, ϕ(fraction)	0.2
Permeability, k md	**Var**
Height H ft	10
Width W ft	100

k1	k2	k3	k4
30	60	40	100

Fluid Properties	
Viscosity μ cp	1
Cf, psi-1	0.000001
Bo, RB/STB	1

k12	k23	k34
40	48	57.14286

eta12	eta23	eta34
0.040512	0.048614	0.057874

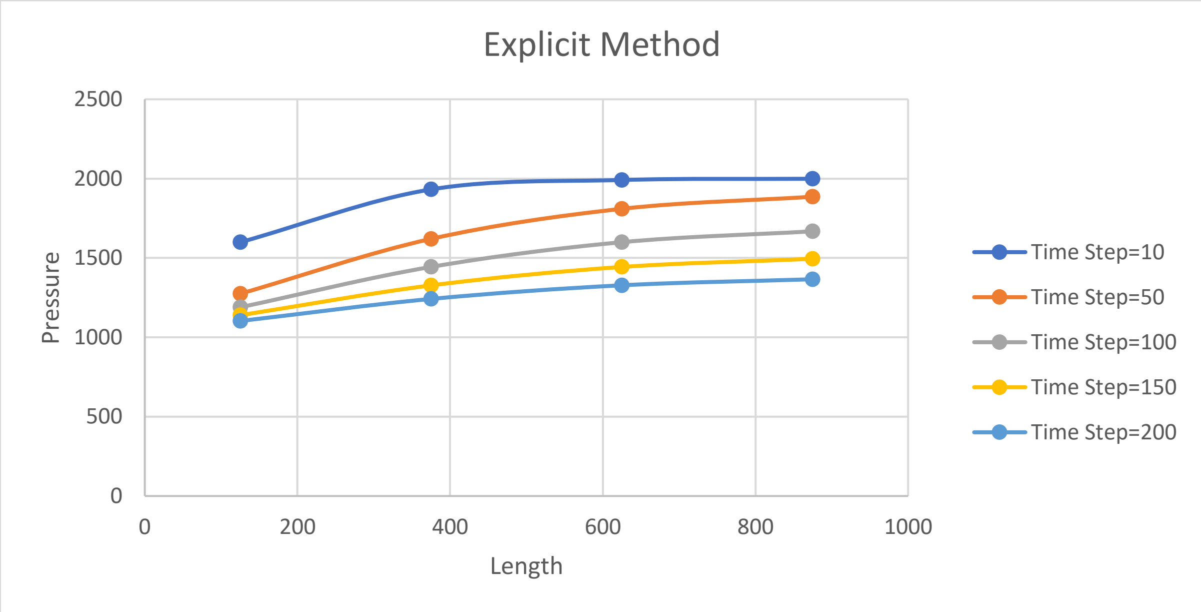
Numerical Properties	
Δt, day	0.002
N	4
Δx	250
α/k, Diffusivity/k	5000000
η/k	0.001013
	<0.5

BCs		
L=0 Inlet	1000	psi
		Dirichlet
L=L Outlet	0	ft3/day
		Neumann
t=0 Initial	2000	Psi

Dykstra-Parsons Heterogeneity Coefficient (V) is estimated as:

$$V = \frac{k_{50} - k_{84.1}}{k_{50}} = \frac{57.5 - 30.96}{57.5} = 0.462$$

The Pressure profile Plot generated is as shown:



3.2.2 IMPLICIT METHOD:

In Implicit method, we analyzed 1D block heterogeneous permeability by using same data as in explicit method.

k1	k2	k3	k4
30	60	40	100

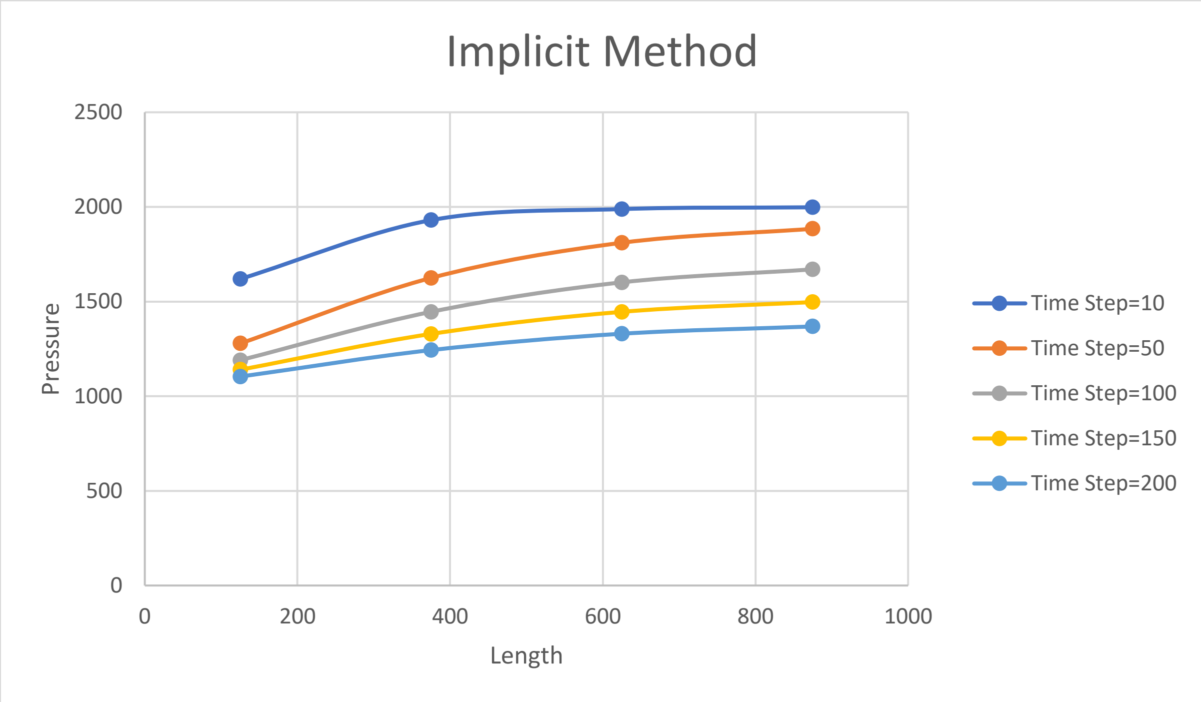
k12	k23	k34
40	48	57.14286

eta12	eta23	eta34
0.040512	0.048614	0.057874

Generated Tri-Diagonal Matrix:

1.10128	-0.04051	0	0
-0.04051	1.089126	-0.04861	0
0	-0.04861	1.106489	-0.05787
0	0	-0.05787	1.057874

Pressure Profile Plot:



3.3 Modelling for Variable (Heterogeneous) Permeability for any number of blocks for 1-D 1-φ reservoir in Python:

3.3.1 EXPLICIT METHOD:

For heterogeneity case, we varied permeability of 10 blocks and then applied boundary conditions i.e., Dirichlet and Neumann. Dirichlet condition is applied on first block and Neumann condition on last block. Data was input as an excel file, with ‘sheet 1’ as Reservoir data and BCs and ‘sheet 2’ as Heterogeneous Permeability data, and fed into the python program. D-P Coefficient is calculated in the program and Output Pressure Profile Plot is also given (Pressure response table is in Annexure). Also, the plot is smoothened using Basis-Spline interpolation. Output excel file obtained has:

- Sheet 1 – Each Block eta
- Sheet 2 – Inter- Block k and eta
- Sheet 3 – Pressure Response

Input – Sheet 1:

Property	Annotation	Value
Porosity in fraction:	phi	0.5
Reservoir Length in ft:	L	1000
Reservoir Width in ft:	W	100
Reservoir Height in ft:	H	10
Viscosity in cp:	mu	0.8
Total Compressibility in psi^-1:	Ct	0.000005
Fluid Formation Volume Factor in rb/stb:	Bo	1.1
Time Step in days:	Del_t	0.003
Number of Time Steps:	Nt	180
Number of Blocks:	N	10
Reservoir Pressure in psi at t=0:	Pi	2000
BHP in psi at x=0 for all t:	Pwb	1000

Input – Sheet 2:

Property	Value
Permeability for Block 1 in md:	50
Permeability for Block 2 in md:	100
Permeability for Block 3 in md:	20
Permeability for Block 4 in md:	60
Permeability for Block 5 in md:	145
Permeability for Block 6 in md:	75
Permeability for Block 7 in md:	20
Permeability for Block 8 in md:	55

Permeability for Block 9 in md:	120
Permeability for Block 10 in md:	90

Output – Each Block eta:

Blocks	eta
Block 1	0.047475
Block 2	0.09495
Block 3	0.01899
Block 4	0.05697
Block 5	0.1376775
Block 6	0.0712125
Block 7	0.01899
Block 8	0.0522225
Block 9	0.11394
Block 10	0.085455

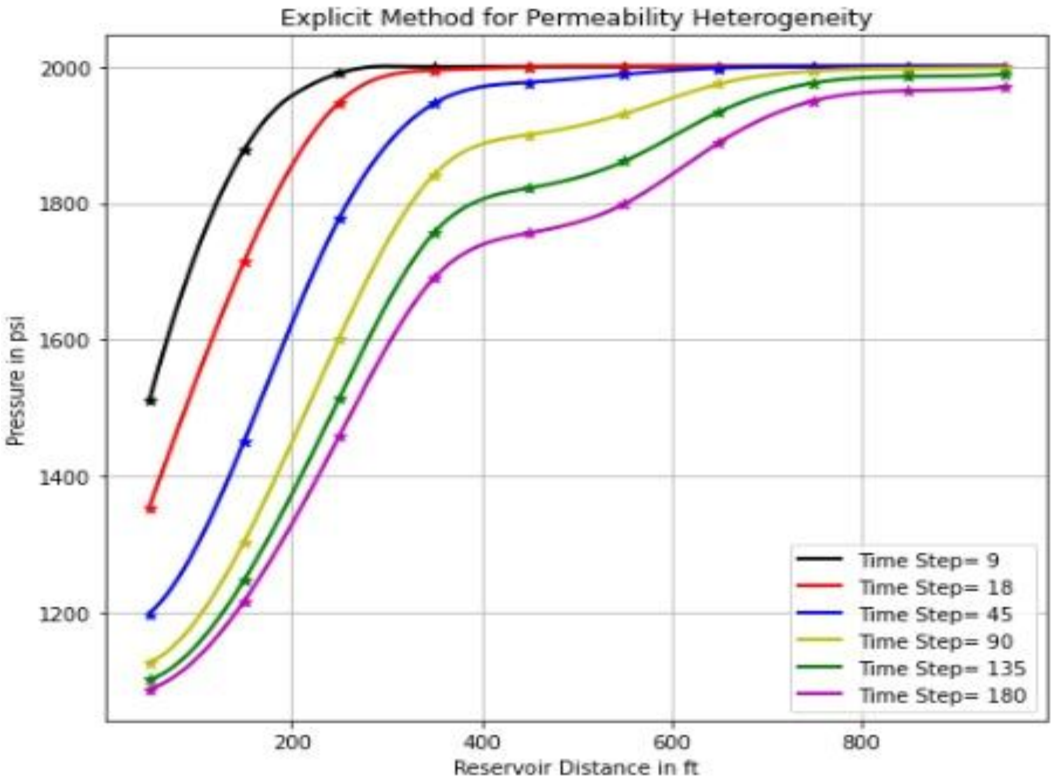
Output – Inter Block k and eta:

Blocks	InterBlock k in md	InterBlock eta
Block 0,1	100	0.09495
Block 1,2	66.66667	0.0633
Block 2,3	33.33333	0.03165
Block 3,4	30	0.028485
Block 4,5	84.87805	0.080592
Block 5,6	98.86364	0.093871
Block 6,7	31.57895	0.029984
Block 7,8	29.33333	0.027852
Block 8,9	75.42857	0.071619
Block 9,10	102.8571	0.097663

From the given Permeability data, K_{50} = 73.5 md and $K_{84.1}$ = 40.89621009335706 md

So, V = (k50-k84)/k50 = 0.4435889783216726

Output Pressure profile Plot at different time-steps:



3.3.2 IMPLICIT METHOD:

In implicit method, we analyzed 1D block heterogeneous permeability by using same data as in explicit method. Same Input and Output files are given as in explicit, with Extra sheet in Output file for Tri-Diagonal Matrix.

Input Files:

Property	Annotation	Value
Porosity in fraction:	phi	0.5
Reservoir Length in ft:	L	1000
Reservoir Width in ft:	W	100
Reservoir Height in ft:	H	10
Viscosity in cp:	mu	0.8
Total Compressibility in psi^-1:	Ct	0.000005
Fluid Formation Volume Factor in rb/stb:	Bo	1.1
Time Step in days:	Del_t	0.003
Number of Time Steps:	Nt	180
Number of Blocks:	N	10
Reservoir Pressure in psi at t=0:	Pi	2000
BHP in psi at x=0 for all t:	Pwb	1000

Property	Value
Permeability for Block 1 in md:	50
Permeability for Block 2 in md:	100
Permeability for Block 3 in md:	20
Permeability for Block 4 in md:	60
Permeability for Block 5 in md:	145

Permeability for Block 6 in md:	75
Permeability for Block 7 in md:	20
Permeability for Block 8 in md:	55
Permeability for Block 9 in md:	120
Permeability for Block 10 in md:	90

Blocks	eta
Block 1	0.047475
Block 2	0.09495
Block 3	0.01899
Block 4	0.05697
Block 5	0.137678
Block 6	0.071213
Block 7	0.01899
Block 8	0.052223
Block 9	0.11394
Block 10	0.085455

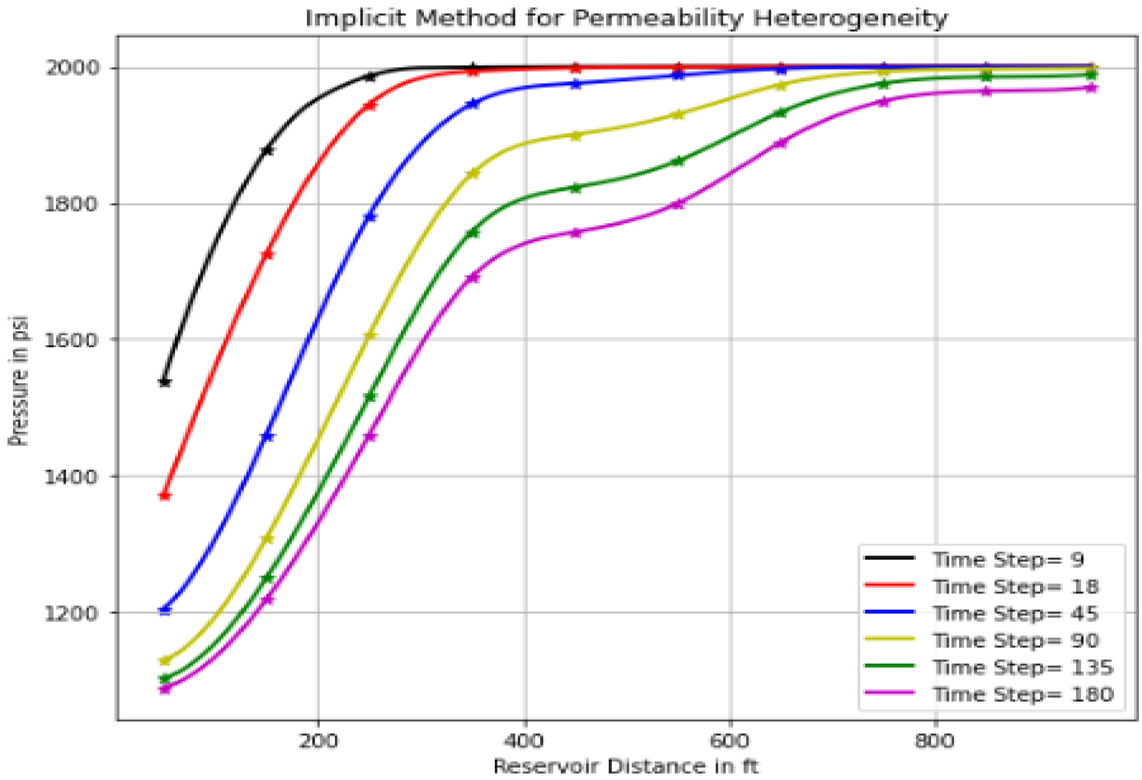
Output Files:

Blocks	InterBlock k in md	InterBlock eta
Block 0,1	100	0.09495
Block 1,2	66.66667	0.0633
Block 2,3	33.33333	0.03165
Block 3,4	30	0.028485
Block 4,5	84.87805	0.080592
Block 5,6	98.86364	0.093871
Block 6,7	31.57895	0.029984
Block 7,8	29.33333	0.027852
Block 8,9	75.42857	0.071619
Block 9,10	102.8571	0.097663

Tridiagonal Matrix:

1.15825	-0.0633	0	0	0	0	0	0	0	0
-0.0633	1.09495	-0.03165	0	0	0	0	0	0	0
0	-0.03165	1.060135	-0.02849	0	0	0	0	0	0
0	0	-0.02849	1.109077	-0.08059	0	0	0	0	0
0	0	0	-0.08059	1.174463	-0.09387	0	0	0	0
0	0	0	0	-0.09387	1.123855	-0.02998	0	0	0
0	0	0	0	0	-0.02998	1.057836	-0.02785	0	0
0	0	0	0	0	0	-0.02785	1.099471	-0.07162	0
0	0	0	0	0	0	0	-0.07162	1.169282	-0.09766
0	0	0	0	0	0	0	0	-0.09766	1.097663

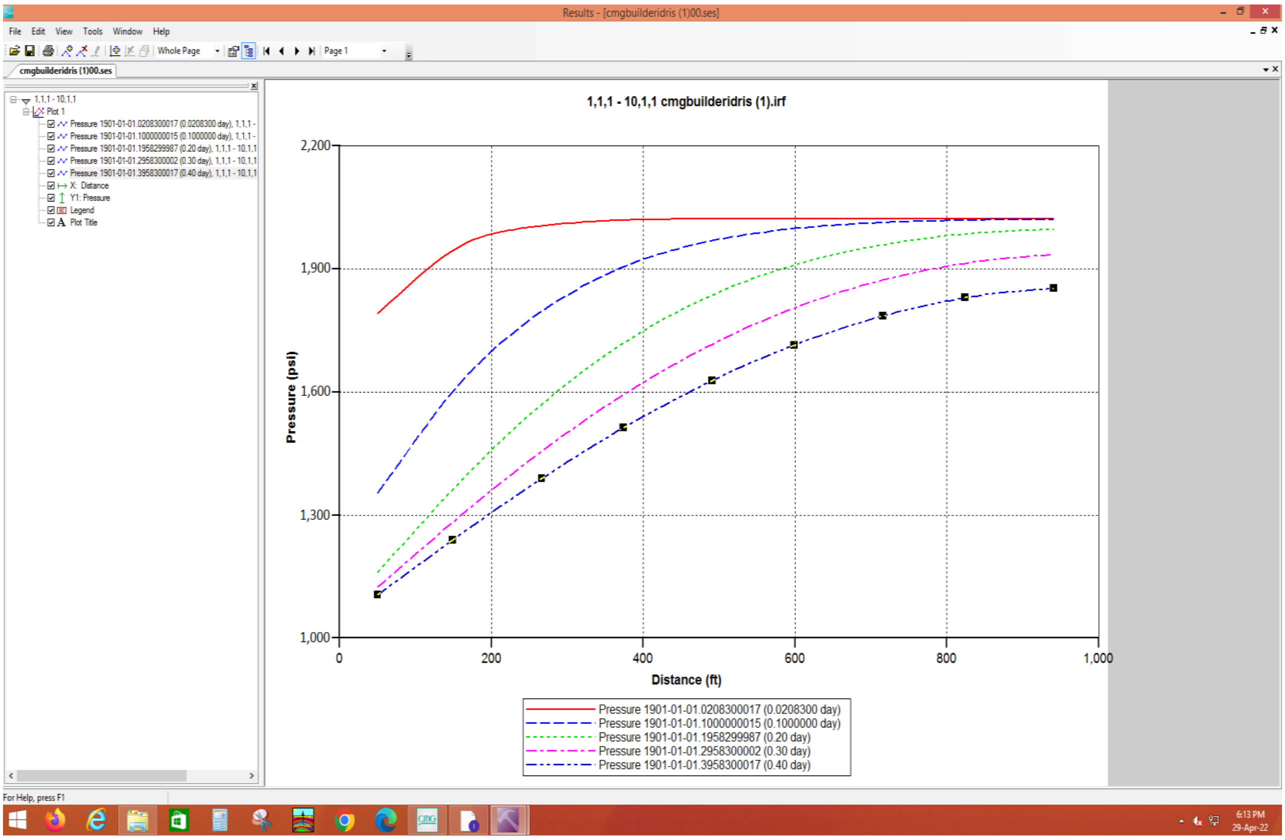
Output Pressure profile Plot at different time-steps:



3.4 Validation of plots and results using CMG Software:

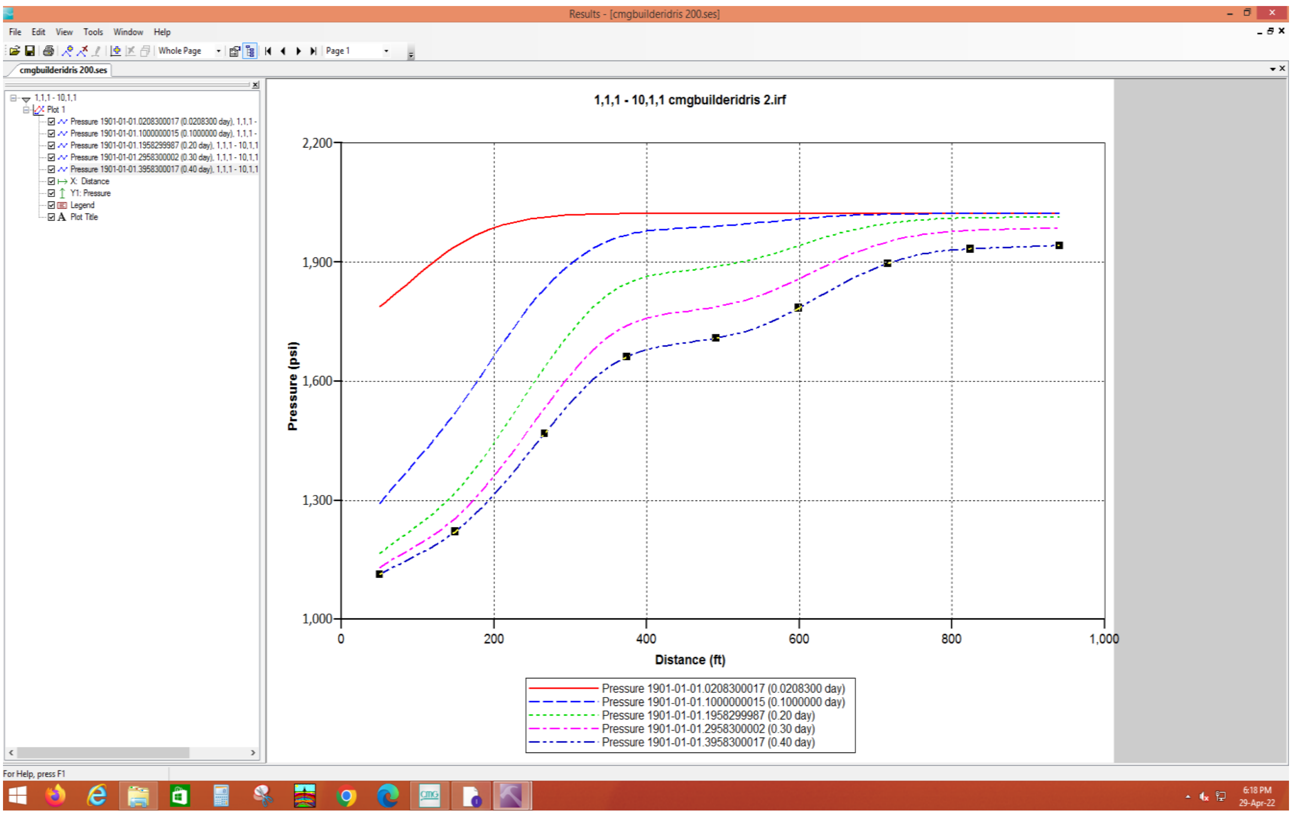
CMG is a sophisticated software used in the Petroleum Industry for Reservoir Simulation of Black Oil (IMEX Suite), Compositional (GEM Suite), Thermal and Chemical EOR (STARS Suite). In my study, IMEX Suite is used for creating simplified Black oil Model in Builder. Two classes of permeability data are used, one for Homogeneous and another for Heterogeneous Reservoir. Input data is the same as the one we used for each class. Generated .dat and .irf file used for Results and Graphs Module. In this module, we have plotted Pressure vs. Distance plot for each no. of time-steps, using Cell-Address as ‘1,1,1’ to ‘10,1,1’ and taking distances at center of each block and using Cubic-Spline Interpolation for plot smoothening. Following are the generated Pressure profile plots:

1. Homogeneous (k = 50 md):



2. Heterogeneous:

Property	K-Value
Permeability for Block 1 in md:	50
Permeability for Block 2 in md:	100
Permeability for Block 3 in md:	20
Permeability for Block 4 in md:	60
Permeability for Block 5 in md:	145
Permeability for Block 6 in md:	75
Permeability for Block 7 in md:	20
Permeability for Block 8 in md:	55
Permeability for Block 9 in md:	120
Permeability for Block 10 in md:	90



4. Conclusions:

- From comparing our excel and python plots and CMG generated plots, our given 1-D 1-ϕ Simulator is Compatible for more applications.
- The simulation is very sensitive towards Transmissibility (or permeability) values, as in the heterogeneous case, the permeability for Block-5 is 145 md (highest among other blocks), therefore the pressure decline for Block-5 is highest.
- The Variation in permeability gives the characteristic undulating (wavy) trend of the pressure profile curve.
- The pressure profile is sensitive towards the Heterogeneity Coefficient (V), as V increases, then the 'Waviness' of the Pressure profile also increases.
- So, it is very much essential to model permeability heterogeneity, as it can highly affect Well tests, Reserve Estimation and Decline Curve Analysis (DCA), for predicting future production.
- Highly Heterogeneous reservoir can also affect Water-flooding operations, as Areal Sweep Efficiency decreases. So, Polymer-flooding can be better alternative.
- This model is only for single-phase (1-ϕ). For more complex, 2-ϕ (water-oil) and 3-ϕ (oil-water-gas), better models, like Black Oil and Compositional Simulation equations should be used.

5. References:

1. *Reservoir Simulation*, by Heriot Watt University Publication.
2. *THE MATHEMATICS OF RESERVOIR SIMULATION*, by Richard E. Ewing.
3. *Basic Applied Reservoir Simulation,* by T. Ertekin, J. H. Abou-Kassem and G. R. King.
4. *Reservoir Simulation,* SPE Monograph 13, by C. C. Mattax and R. L. Dalton.
5. https://petrowiki.spe.org/Reservoir_simulation