HEAVISIDE UNIT STEP FUNCTION & DIRAC-DELTA FUNCTION

Mathematics Mini Project

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Heaviside Unit Step Function

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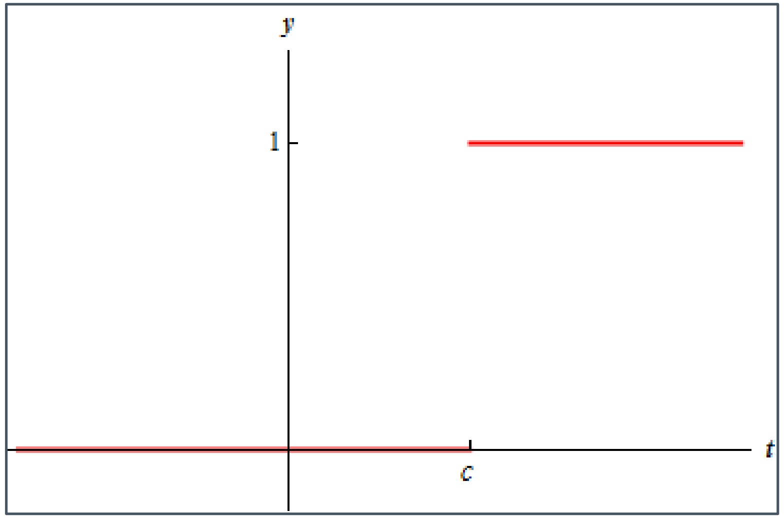


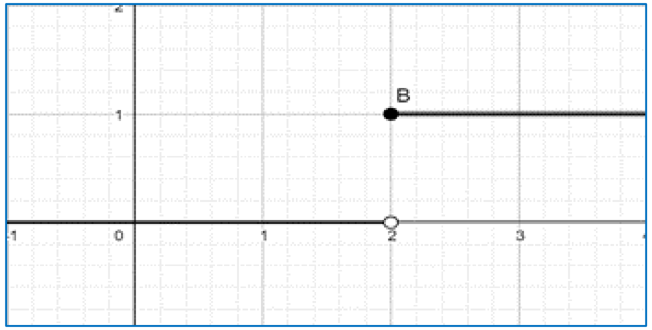
The Heaviside func is defined as $u_c(t) = \begin{cases} 0 & \text{if } t < c \\ 1 & \text{if } t \geq c \end{cases}$

This function can be thought of as a switch that stays off until time $t = c$, at which point it turns on and takes the value of 1

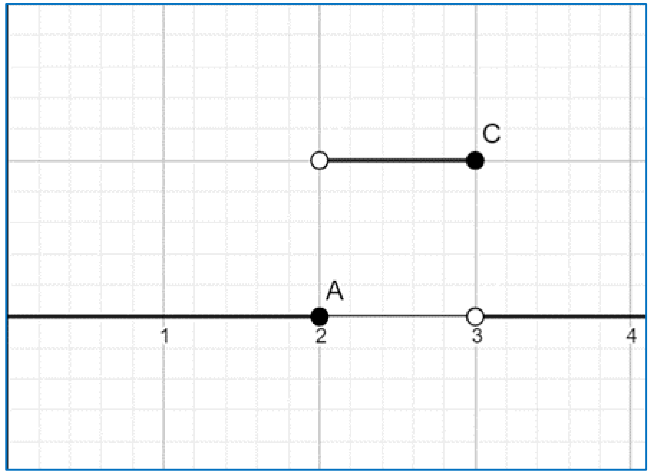
Heaviside functions can only take values of 0 or 1, but they can be used to get other kinds of switches For example, $4u_c(t)$) is a switch that is off until time $t = c$ and then turns on and takes a value of 4. Likewise, $-7u_c(t)$ will be a switch that will take a value of -7 when it turns on.

Heaviside Unit Step Function

1)For the function $H(x-2)$, equating $x-2=0$, we get $x=2$
i.e.at $x=2$ $f(x)$ is 1 and for any value of x less than 2, $f(x)$ is 0.



2) For function $H(x-2)-H(x-3)$, in piecewise form we can say that $H(x-2)$ is one for all $x>2$ and $H(x-3)$ is 1 for all $x>3$ i.e. resultant graph for all $x<2$ will be zero



For, $2<x\leq 3$, $H(x-2)-H(x-3)=1-0=1$
For, $x>3$, $\quad H(x-2)-H(x-3)=1-1=0$

Thus in general we can say that graph of $H(x-a)-H(x-b)$ will show $f(x)=1$ for interval $(a<x\leq b)$

Applications Of Heaviside Unit Step Function

In engineering applications, we frequently encounter functions whose values change abruptly at specified values of time *t*. One common example is when a voltage is switched on or off in an electrical circuit at a specified value of time *t*.

The Heaviside function, H(t), is used to study electrical circuits to represent the sudden surge of current or voltage when the switch is instantly turned on. It is defined as

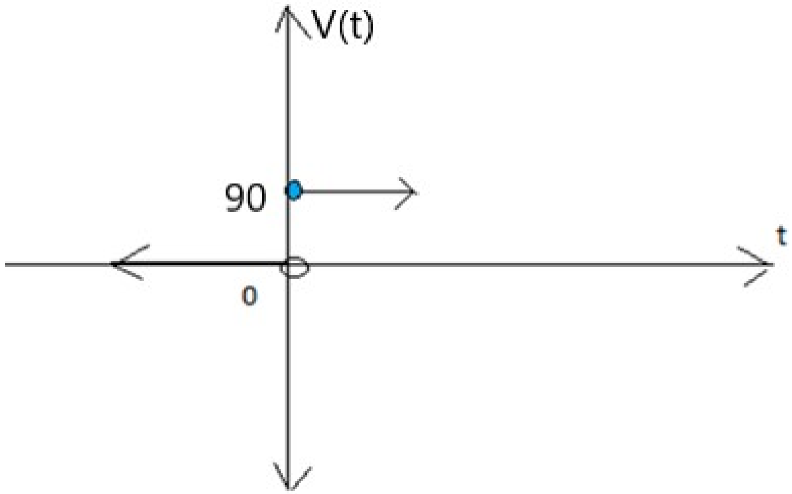
$$u_c(t) = \begin{cases} 0 & \text{if } t < c \\ 1 & \text{if } t \geq c \end{cases}$$

$V(t) = 90\, u_c(t)$,

Here if c = 0,

Case 1 : V(t) = 90v at t ≥ 0

Case 2 : V(t) = 0v at t <0



Applications Of Heaviside Unit Step Function

a) The function is commonly used in the mathematics of control theory and signal processing to represent a signal that switches on at a specified time and stays switched on indefinitely. The Heaviside function is utilised to represent the oscillatory behaviour of nanostructures.

b) Any kind of on-off control such as a solenoid valve can be represented.

c) Approximations to the Heaviside step function are of use in biochemistry and neuroscience, where logistic approximations of step functions (such as the Hill and the Michaelis–Menten equations) may be used to approximate binary cellular switches in response to chemical signals.

Dirac Delta Function

Dirac Delta Function Introduction

When we first introduced **heaviside functions** we noted that we could think of them as switches changing the forcing function, g(t)g(t), at specified times. however, heaviside functions are really not suited to forcing functions that exert a “large” force over a“small” time frame.

Examples of this kind of forcing function would be a hammer striking an object or a short in an electrical system. in both of these cases a large force (or voltage) would be exerted on the system over a very short time frame. At t=a the dirac delta function is sometimes thought of has having an “infinite” value. so, the dirac delta function is a function that is zero everywhere except one point and at that point it can be thought of as either undefined or as having an “infinite” value, somewhat akin to a spike.

This is a very strange function. it is zero everywhere except one point and yet the integral of any interval containing that one point has a value of 1. The dirac delta function is not a real function as we think of them. It is instead an example of something called a **generalized function** or **distribution**.

despite the strangeness of this “function” it does a very nice job of modeling sudden shocks or large forces to a system.

Dirac Delta Function Defination

In mathematics, the Dirac delta function (δ function), also known as the unit impulse symbol, is a generalized function or distribution over the real numbers, whose value is zero everywhere except at zero, and whose integral over the entire real line is equal to one.

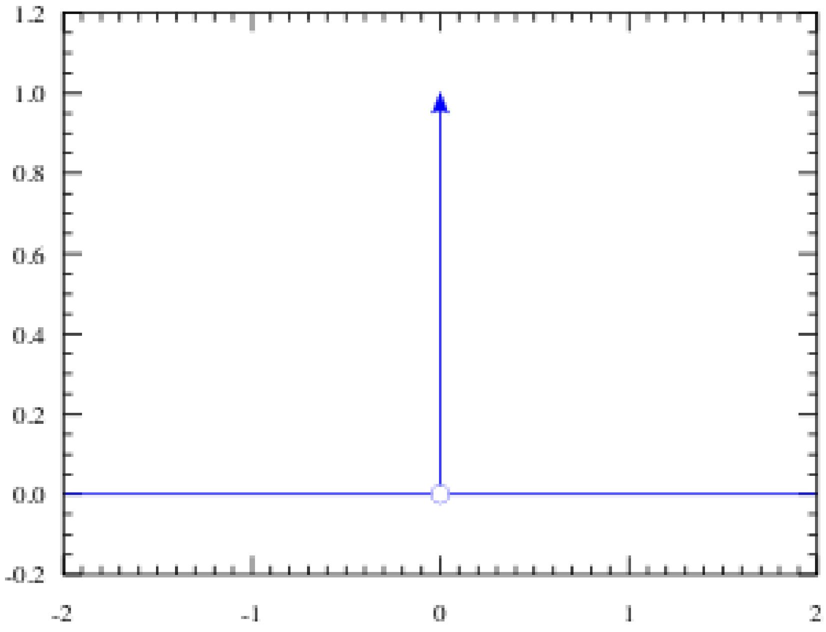
Delta function is denoted by $\delta(x)$.

For, $x<0$, $\delta(x)=0$

$x=0$, $\delta(x)=\infty$

$x>0$, $\delta(x)=0$.

The integral , $\int_{-\infty}^{\infty} \delta(x)\, dx = 1.$



Properties of Dirac Delta Function

1. Dirac delta function is an even function.

$$\delta(y) = \delta(-y)$$

2.Multiplication of x with dirac delta function is always zero.

$x.\delta(x)=0.$

3. Multiplication of a function with dirac delta function is possible only when x=0.

$f(x).\delta(x)=f(0).\delta(x).$

4. $$\int_{-\infty}^{\infty} f(x)\delta(x)\, dx = f(0)$$

Properties of Dirac Delta Function

The Dirac delta has the following properties, which lead one to realize that it is not really a function.

1. $\delta(t - t_0) = \begin{cases} \infty & t = t_0 \\ 0 & t \neq t_0 \end{cases}$
2. $\int_0^{\infty} \delta(t - t_0)\, dt = 1$

Applications of Dirac Delta Function

Probability theory

In probability theory and statistics, the Dirac delta function is often used to represent a discrete distribution, or a partially discrete, partially continuous distribution, using a probability density function (which is normally used to represent absolutely continuous distributions). The delta function is also used to represent the resulting probability density function of a random variable that is transformed by continuously differentiable function.

Applications of Dirac Delta Function

Quantum mechanics

The delta function is expedient in quantum mechanics. The wave function of a particle gives the probability amplitude of finding a particle within a given region of space. Wave functions are assumed to be elements of the Hilbert space L^2 of square-integrable functions, and the total probability of finding a particle within a given interval is the integral of the magnitude of the wave function squared over the interval. The delta function also has many more specialized applications in quantum mechanics, such as the delta potential models for a single and double potential well.

Applications of Dirac Delta Function

Structural mechanics

The delta function can be used in structural mechanics to describe transient loads or point loads acting on structures. The governing equation of a simple mass-spring system excited by a sudden force impulse I at time t= 0 can be written.

As another example, the equation governing the static deflection of a slender beam is, according to Euler-Bernoulli theory.

THANK YOU