Finite Automata

Lecture 6

Content

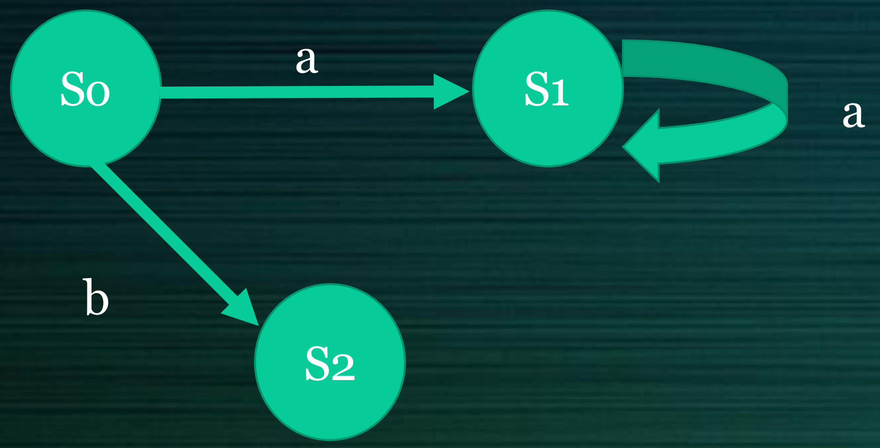
- Introduction to Finite Automata
- Definition of Deterministic Finite Automata (DFA)
- Examples on DFA

Finite Automata

- There are two types of Finite Automata
    1. Deterministic Finite Automata (DFA)
    2. Non – deterministic Finite Automata (NFA)
- DFA is deterministic in nature. Each transition in this automata is uniquely determined on current state and current input
- NFA is non – deterministic in nature. Next state can not be determined uniquely for a given transition with current state and current input

Deterministic Finite Automata (DFA)

- The Finite Automata is called Deterministic Finite Automata (DFA) if there is only one path for a specific input from current state to next state.
- E.g.



- From state So for input 'a' there is only one path going to S1
- Similarly all the transitions can be described

Definition of DFA

Deterministic Finite automata or DFA is defined as

$$M = (Q , \Sigma, \delta , q_o , F)$$

Where

Q is a finite set of internal states

Σ is a finite set of symbols called the input alphabet

$\delta : Q \times \Sigma \rightarrow Q$ is a Total Function called Transition Function

q_o is an initial state $q_o \in Q$

F is a set of final states $F \subseteq Q$

Definition of DFA

- Transition Function accepts two parameters one is current state and other is input symbol
- It returns a state which can be called as next state
- It is described as $\delta : Q \times \Sigma \rightarrow Q$
- For example:

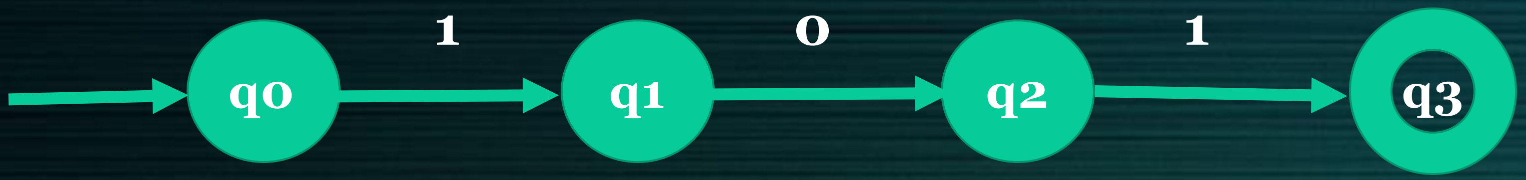
$q_1 = \delta(q_0, a)$ means from current state 'q_0' with input a next state transition is 'q_1'

Examples on DFA

Ex. 1 Design FA which accepts the only string 101 over $\Sigma = \{ 0 , 1 \}$

Solution:

Transition Diagram:



Above DFA can be represented as

$M = (Q , \Sigma , \delta , q_0 , F)$

Where $Q = \{ q0, q1, q2, q3 \}$

$\Sigma = \{ 0, 1 \}$

$q0 = q0$

$F = \{ q3 \}$

Transition Function (δ):

$\delta (q0 , 1) = q1$

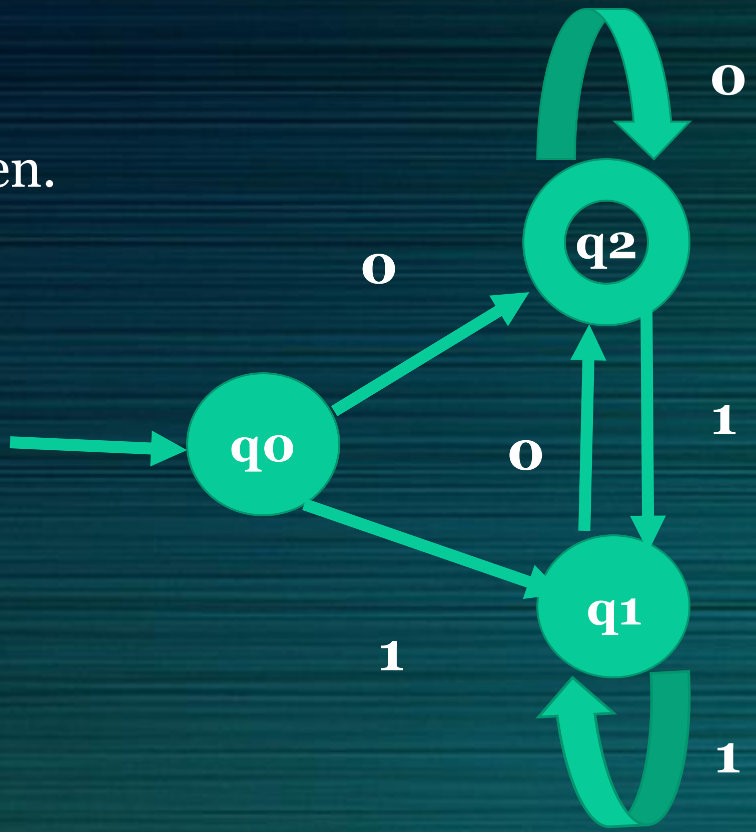
$\delta (q1 , 0) = q2$

$\delta (q2 , 1) = q3$

Transition Table:

Input/ States	**0**	**1**
q0	--	q1
q1	q2	--
q2	--	q3
q3	--	--

Examples on DFA



Ex. 2 Design FA which checks whether the given binary number is even.

Solution:

Binary no is made up of 1's and 0's. Hence $\Sigma = \{ 0 , 1 \}$

When binary number is ended with 1 $\rightarrow$ Odd Number

When binary number is ended with 0 $\rightarrow$ Even Number

Above DFA can be represented as

$M = (Q , \Sigma , \delta , q_0 , F)$

Where $Q = \{ q0, q1, q2 \}$

$\Sigma = \{ 0, 1 \}$

$q0 = q0$

$F = \{ q2 \}$

Transition Function (δ):

$\delta (q0 , 0) = q2$

$\delta (q0 , 1) = q1$

$\delta (q1 , 0) = q2$

$\delta (q1 , 1) = q1$

$\delta (q2 , 0) = q2$

$\delta (q2 , 1) = q1$

Transition Table:

Input/ States	**0**	**1**
q0	q2	q1
q1	q2	q1
q2	q2	q1

Examples on DFA

Ex. 2 Design FA which checks whether the given binary number is even.

Solution:

The simulation to check whether given binary number is even or not.

Suppose input number is 11010

δ (q0 , 1 1 0 1 0) |-- δ (q1 , 1 0 1 0)

|-- δ (q1 , 0 1 0)

|-- δ (q2 , 1 0)

|-- δ (q1 , 0)

|-- δ (q2 , ε)

= q2

q2 is a final state.

Hence given number 11010 is accepted by given DFA

Transition Table:

Input/ States	**0**	**1**
q0	q2	q1
q1	q2	q1
q2	q2	q1

Examples on DFA

Ex. 2 Design FA which checks whether the given binary number is even.

Solution:

The simulation to check whether given binary number is even or not.

Suppose input number is 10101

$$\delta(q0,\ 10101) \vdash \delta(q1,\ 0101)$$
$$\vdash \delta(q2,\ 101)$$
$$\vdash \delta(q1,\ 01)$$
$$\vdash \delta(q2,\ 1)$$
$$\vdash \delta(q1,\ \varepsilon)$$
$$= q1$$

q1 is not a final state.

Hence given number 10101 is not accepted by given DFA

Transition Table:

Input/ States	**0**	**1**
q0	q2	q1
q1	q2	q1
q2	q2	q1

Examples on DFA

Ex. 3 Design DFA which accepts only those strings which starts with 1 and ends with 0 for $\Sigma = \{ 0, 1 \}$.

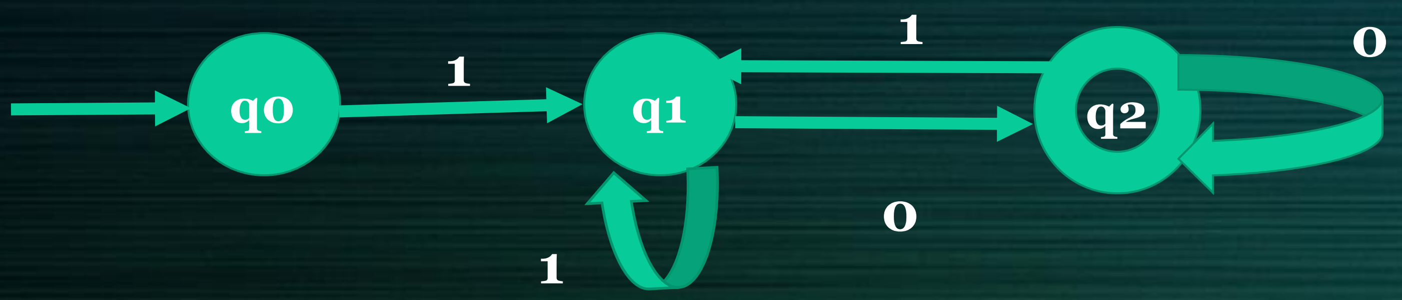
Solution:

Regular expression = $1 . (0 + 1)^* . 0$

Here, q1 represents strings start with 1 and end with 1

State q2 which represents strings start with 1 and end with zero

Transition Diagram:



Transition Function (δ):

$\delta (q0 , 1) = q1$

$\delta (q1 , 0) = q2$

$\delta (q1 , 1) = q1$

$\delta (q2 , 0) = q2$

$\delta (q2 , 1) = q1$

Above DFA can be represented as

$M = (Q , \Sigma , \delta , q_0 , F)$

Where $Q = \{ q0, q1, q2 \}$

$\Sigma = \{ 0, 1 \}$

$q0 = q0$

$F = \{ q2 \}$

Transition Table:

Inputs/ States	**0**	**1**
q0	--	q1
q1	q2	q1
q2	q2	q1

Examples on DFA

Ex. 3 Design DFA which accepts only those strings which starts with 1 and ends with 0 for $\Sigma = \{ 0, 1 \}$.

Solution:

Simulation for the string 10010

$\delta (q0 , 10010)$ |-- $\delta (q1 , 0010)$

|-- $\delta (q2 , 010)$

|-- $\delta (q2 , 10)$

|-- $\delta (q1 , 0)$

|-- $\delta (q2 , \varepsilon)$

= q2

q2 is a final state.

Hence given string 10010 is accepted by given DFA

Transition Table:

Input/ States	**0**	**1**
q0	--	q1
q1	q2	q1
q2	q2	q1

Examples on DFA

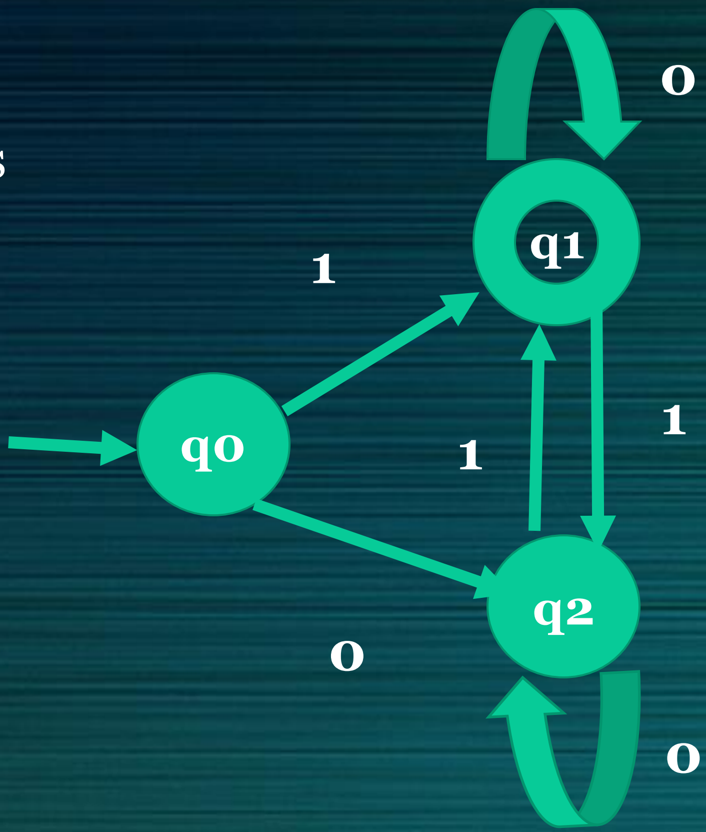
Ex. 4 Design DFA which accepts odd number of 1's and any number of 0's

Solution:

Here, $\Sigma = \{ 0 , 1 \}$

Here, q1 represents strings with odd number of 1's

State q2 represents strings with even number of 1's



Above DFA can be represented as

$M = (Q , \Sigma , \delta , q_0 , F)$

Where $Q = \{ q0, q1, q2 \}$

$\Sigma = \{ 0, 1 \}$

$q0 = q0$

$F = \{ q1 \}$

Transition Function (δ):

$\delta (q0 , 1) = q1$

$\delta (q0 , 0) = q2$

$\delta (q1 , 0) = q1$

$\delta (q1 , 1) = q2$

$\delta (q2 , 0) = q2$

$\delta (q2 , 1) = q1$

Transition Table:

Input/ States	0	1
q0	q2	q1
q1	q1	q2
q2	q2	q1

Examples on DFA

Ex. 4 Design DFA which accepts odd number of 1's and any number of 0's

Solution:

Simulation for the string 11010

$$\delta(q0, 11010) \vdash \delta(q1, 1010)$$
$$\vdash \delta(q2, 010)$$
$$\vdash \delta(q2, 10)$$
$$\vdash \delta(q1, 0)$$
$$\vdash \delta(q1, \varepsilon)$$
$$= q1$$

q1 is a final state.

Hence given string 11010 is accepted by given DFA

Transition Table:

Input/ States	0	1
q0	q2	q1
q1	q1	q2
q2	q2	q1

Examples on DFA

Ex. 5 Design DFA to accept the string which always ends with 00 for $\Sigma = \{ 0, 1 \}$.

Solution:

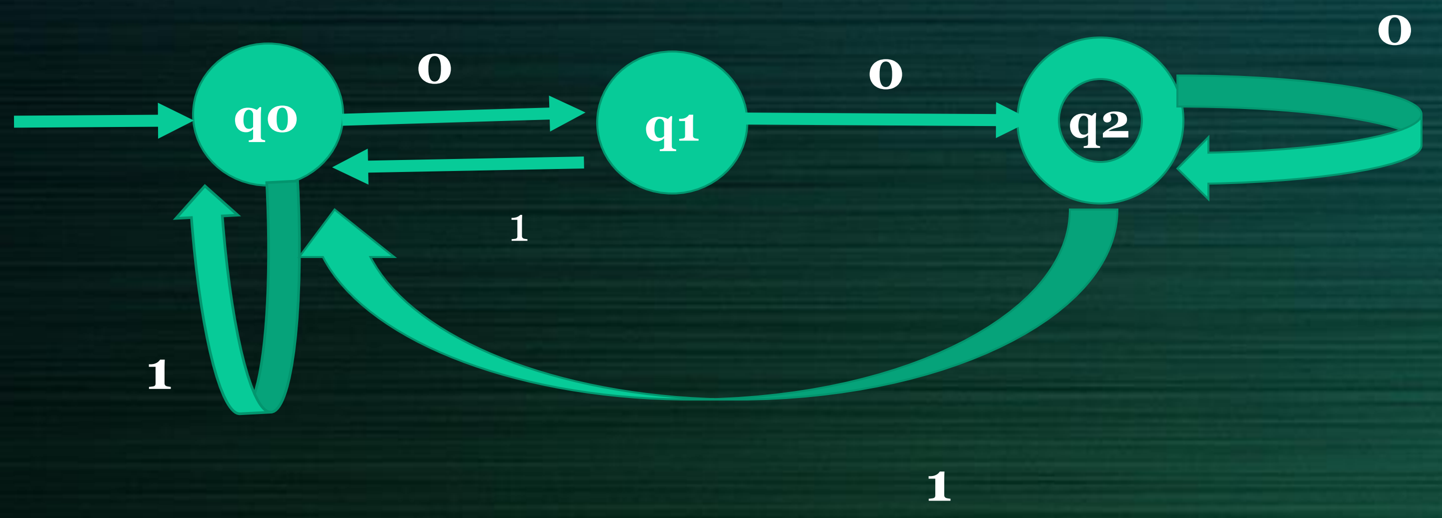
Regular expression = $(0 + 1)^* . 00$

Here, q1 represents strings end with 0

q2 represents strings end with 00

q0 represents strings other than cases of q1 and q2

Transition Diagram:



Above DFA can be represented as

$M = (Q , \Sigma , \delta , q_0 , F)$

Where $Q = \{ q0, q1, q2 \}$

$\Sigma = \{ 0, 1 \}$

$q0 = q0$

$F = \{ q2 \}$

Transition Function (δ) :

$\delta (q0 , 0) = q1$

$\delta (q0 , 1) = q0$

$\delta (q1 , 0) = q2$

$\delta (q1 , 1) = q0$

$\delta (q2 , 0) = q2$

$\delta (q2 , 1) = q0$

Examples on DFA

Ex. 5 Design DFA to accept the string which always ends with 00 for $\Sigma = \{ 0, 1 \}$.

Solution:

Simulation for the string 10100

$$\delta (q0 , 1 0 1 0 0) \vdash \delta (q0 , 0 1 0 0)$$
$$\vdash \delta (q1 , 1 0 0)$$
$$\vdash \delta (q0 , 0 0)$$
$$\vdash \delta (q1 , 0)$$
$$\vdash \delta (q2 , \varepsilon)$$
$$= q2$$

q2 is a final state.

Hence given string 10100 is accepted by given DFA

Transition Table:

Input/ States	**0**	**1**
q0	q1	q0
q1	q2	q0
q2	q2	q0

Examples on DFA

Ex. 6 Design DFA to accept the strings of a's and b's ending with abb over $\Sigma = \{ a, b \}$.

Solution:

Regular expression = $(a + b)^* . abb$

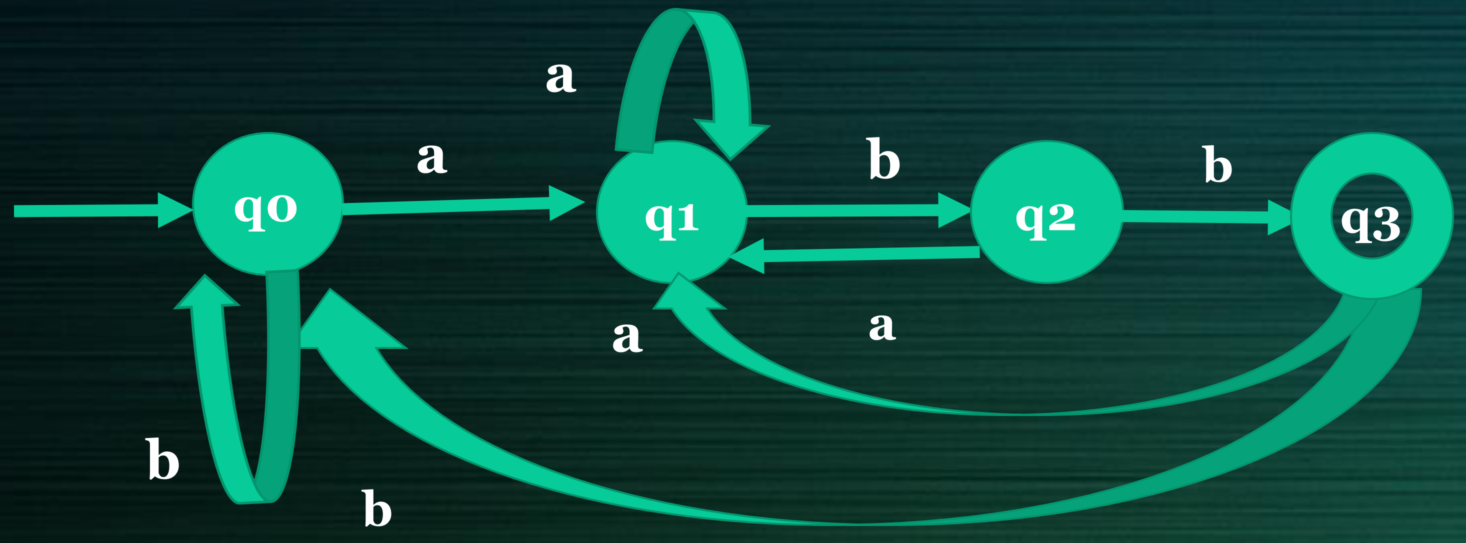
Here, q1 represents strings end with a

q2 represents strings end with ab

Q3 represents strings end with abb

q0 represents strings other than cases of q1, q2 and q3

Transition Diagram:



Above DFA can be represented as

$M = (Q , \Sigma , \delta , q_0 , F)$

Where $Q = \{ q0, q1, q2, q3 \}$

$\Sigma = \{ a, b \}$

$q0 = q0$

$F = \{ q3 \}$

Transition Function (δ) :

$\delta (q0 , a) = q1$ $\delta (q0 , b) = q0$

$\delta (q1 , a) = q1$ $\delta (q1 , b) = q2$

$\delta (q2 , a) = q1$ $\delta (q2 , b) = q3$

$\delta (q3 , a) = q1$ $\delta (q3 , b) = q0$

Examples on DFA

Ex. 6 Design DFA to accept the strings of a's and b's ending with abb over $\Sigma = \{ a, b \}$

Solution:

Simulation for the string baabb

$$
\begin{aligned}
\delta(q0,\ b\,a\,a\,b\,b) &\vdash \delta(q0,\ a\,a\,b\,b) \\
&\vdash \delta(q1,\ a\,b\,b) \\
&\vdash \delta(q1,\ b\,b) \\
&\vdash \delta(q2,\ b) \\
&\vdash \delta(q3,\ \varepsilon) \\
&= q3
\end{aligned}
$$

q3 is a final state.

Hence given string baabb is accepted by given DFA

Transition Table:

Input/ States	**a**	**b**
q0	q1	q0
q1	q1	q2
q2	q1	q3
q3	q1	q0

Examples on DFA

Ex. 7 Construct DFA that accepts all the strings on $\Sigma = \{ 0, 1 \}$ except those containing the substring 101

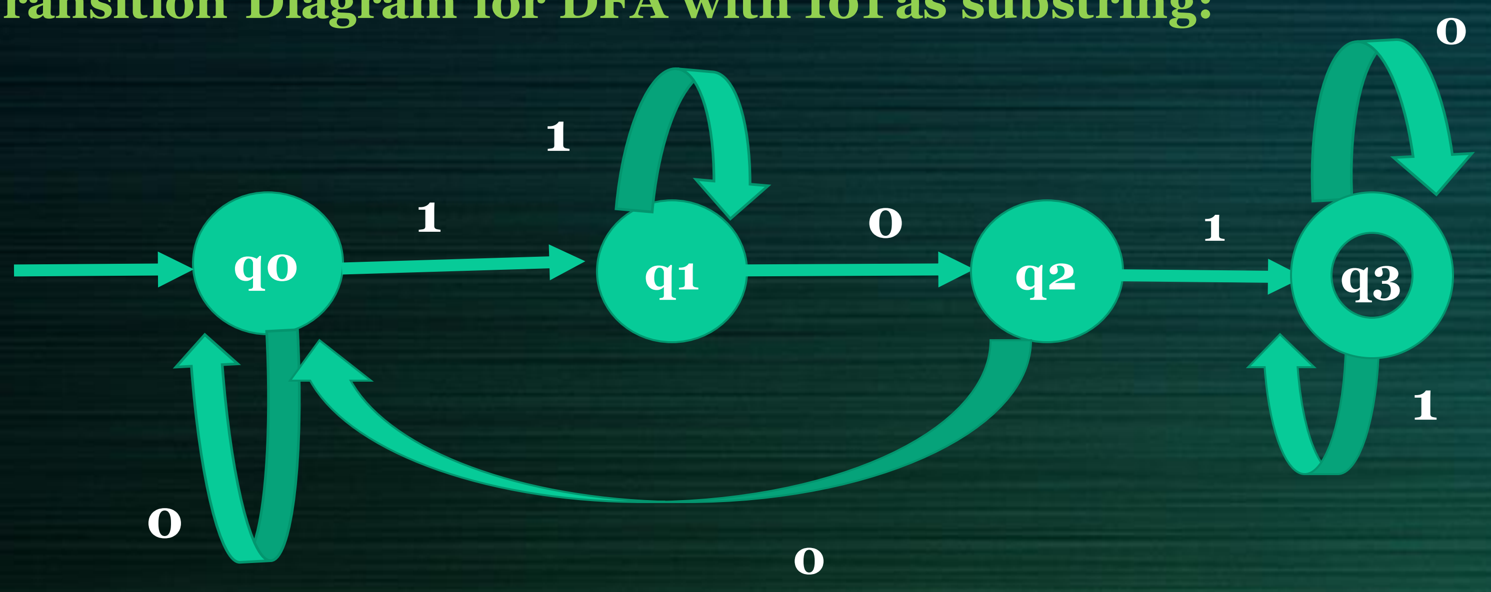
Solution:

Design DFA for accepting strings which has 101 as substring.

Final state is representing state which accepts strings having 101 as substring. All other states representing other cases.

Therefore all other states will act as final states and final state will act as normal state for given problem statement

Transition Diagram for DFA with 101 as substring:



Regular expression

$= (0 + 1)^* . 101 . (0 + 1)^*$

Here, q1 represents strings end with 1

q2 represents strings end with 10

Q3 represents strings end with 101 and strings with 101 as substring

q0 represents strings other than cases of q1, q2 and q3

Examples on DFA

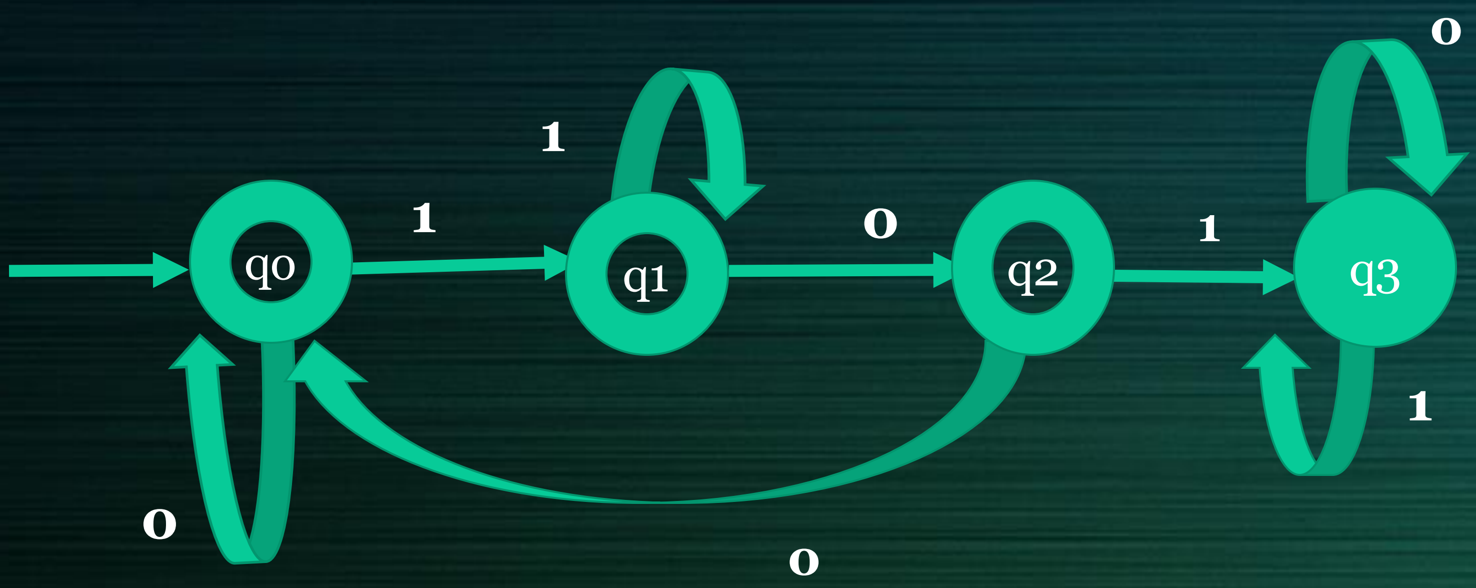
Ex. 7 Construct DFA that accepts all the strings on $\Sigma = \{ 0, 1 \}$ except those containing the substring 101

Solution:

Invert all the Non Final states into Final states and Final state into non final state.

Required transition diagram is as follows:

Transition Diagram:



Above DFA can be represented as

$M = (Q , \Sigma , \delta , q_0 , F)$

Where $Q = \{ q0, q1, q2, q3 \}$

$\Sigma = \{ 0, 1 \}$

$q0 = q0$

$F = \{ q0, q1, q2 \}$

Transition Function (δ) :

$\delta (q0 , 0) = q0$ $\quad$ $\delta (q0 , 1) = q1$

$\delta (q1 , 0) = q2$ $\quad$ $\delta (q1 , 1) = q1$

$\delta (q2 , 0) = q0$ $\quad$ $\delta (q2 , 1) = q3$

$\delta (q3 , 0) = q3$ $\quad$ $\delta (q3 , 1) = q3$

Examples on DFA

Ex. 7 Construct DFA that accepts all the strings on $\Sigma = \{ 0, 1 \}$ except those containing the substring 101

Solution:

Simulation for the string 11001

$$\delta(q0, 11001) \vdash \delta(q1, 1001)$$
$$\vdash \delta(q1, 001)$$
$$\vdash \delta(q2, 01)$$
$$\vdash \delta(q0, 1)$$
$$\vdash \delta(q1, \varepsilon)$$
$$= q1$$

q1 is a final state.

Hence given string 11001 is accepted by given DFA

Transition Table:

Input/ States	**0**	**1**
q0	q0	q1
q1	q2	q1
q2	q0	q3
q3	q3	q3

Examples on DFA

Ex. 8 Design DFA to accept all the string in Language L such that total number of a's in them are divisible by three on $\Sigma = \{ a, b \}$.

Solution:

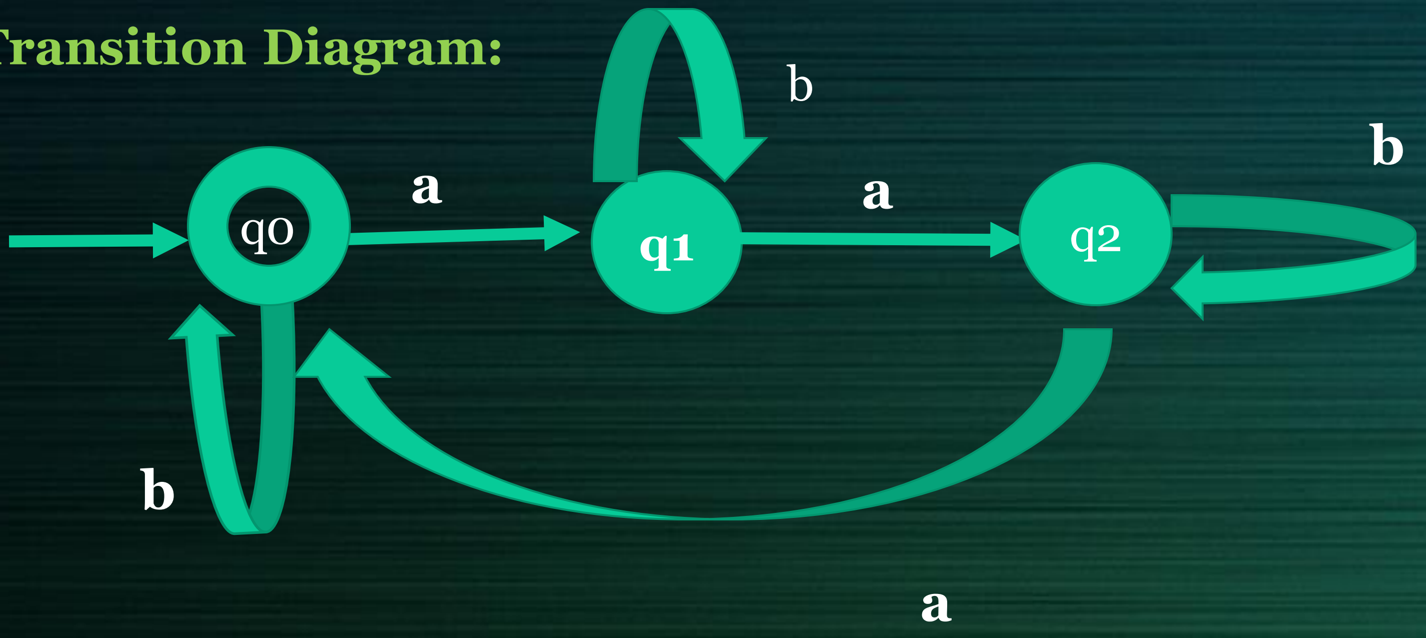
Regular expression: $(b^*ab^*ab^*ab^*)^*$

Here, q0 represents strings with number of a's are divisible by 3

q1 represents strings with number of a's are divisible by 3 plus one

q2 represents strings with number of a's are divisible by 3 plus two

Transition Diagram:



Above DFA can be represented as

$M = (Q , \Sigma , \delta , q_o , F)$

Where $Q = \{ q0, q1, q2 \}$

$\Sigma = \{ a, b \}$

$q0 = q0$

$F = \{ q0 \}$

Transition Function (δ) :

$\delta (q0 , a) = q1$

$\delta (q0 , b) = q0$

$\delta (q1 , a) = q2$

$\delta (q1 , b) = q1$

$\delta (q2 , a) = q0$

$\delta (q2 , b) = q2$

Examples on DFA

Ex. 8 Design DFA to accept all the string in Language L such that total number of a's in them are divisible by three on $\Sigma = \{ a, b \}$.

Solution:

Simulation for the string baaba

$$\delta(q0, baaba) \vdash \delta(q0, aaba)$$
$$\vdash \delta(q1, aba)$$
$$\vdash \delta(q2, ba)$$
$$\vdash \delta(q2, a)$$
$$\vdash \delta(q0, \varepsilon)$$
$$= q0$$

q0 is a final state.

Hence given string baaba is accepted by given DFA

Transition Table:

Input/ States	**a**	**b**
q0	q1	q0
q1	q2	q1
q2	q0	q2

Examples on DFA

Ex. 9 Design DFA which accepts even number of 0's and even number of 1's over $\Sigma = \{ 0, 1 \}$.

Solution:

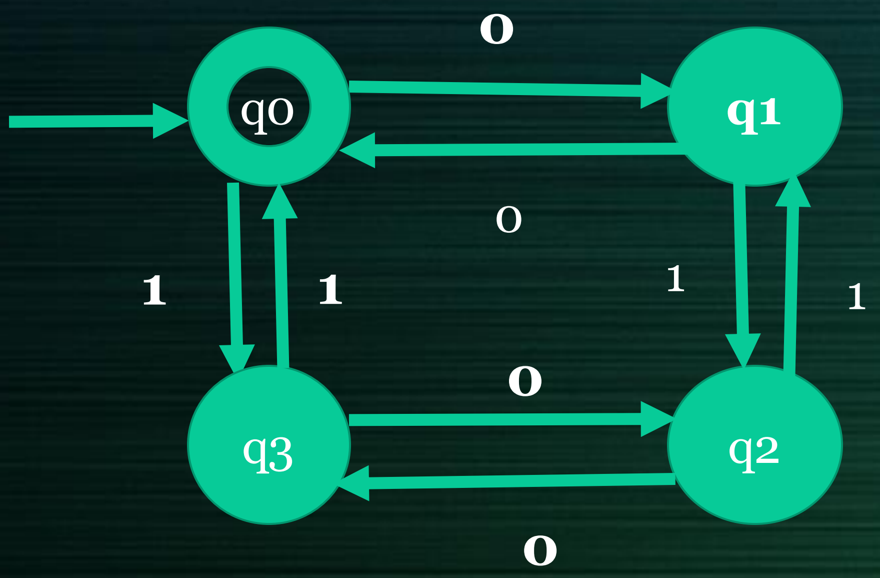
Here, q0 represents strings with even number of 0's and even number of 1's

q1 represents strings with odd number of 0's and even number of 1's

q2 represents strings with odd number of 0's and odd number of 1's

q3 represents strings with even number of 0's and odd number of 1's

Transition Diagram:



Above DFA can be represented as

$M = (Q , \Sigma , \delta , q_0 , F)$

Where $Q = \{ q0, q1, q2, q3 \}$

$\Sigma = \{ 0, 1 \}$

$q0 = q0$

$F = \{ q0 \}$

Transition Function (δ) :

$\delta (q0 , 0) = q1$ $\delta (q0 , 1) = q3$

$\delta (q1 , 0) = q0$ $\delta (q1 , 1) = q2$

$\delta (q2 , 0) = q3$ $\delta (q2 , 1) = q1$

$\delta (q3 , 0) = q2$ $\delta (q3 , 1) = q0$

Examples on DFA

Ex. 9 Design DFA which accepts even number of 0's and even number of 1's over $\Sigma = \{ 0, 1 \}$.

Solution:

Simulation for the string 100010

$$
\begin{aligned}
\delta(q0, 100010) &\vdash \delta(q3, 00010) \\
&\vdash \delta(q2, 0010) \\
&\vdash \delta(q3, 010) \\
&\vdash \delta(q2, 10) \\
&\vdash \delta(q1, 0) \\
&\vdash \delta(q0, \varepsilon) \\
&= q0
\end{aligned}
$$

q0 is a final state.

Hence given string 100010 is accepted by given DFA

Transition Table:

Input/ States	**0**	**1**
q0	q1	q3
q1	q0	q2
q2	q3	q1
q3	q2	q0

Examples on DFA

Ex. 10 Design DFA to check whether given decimal number is divisible by 3

Solution:

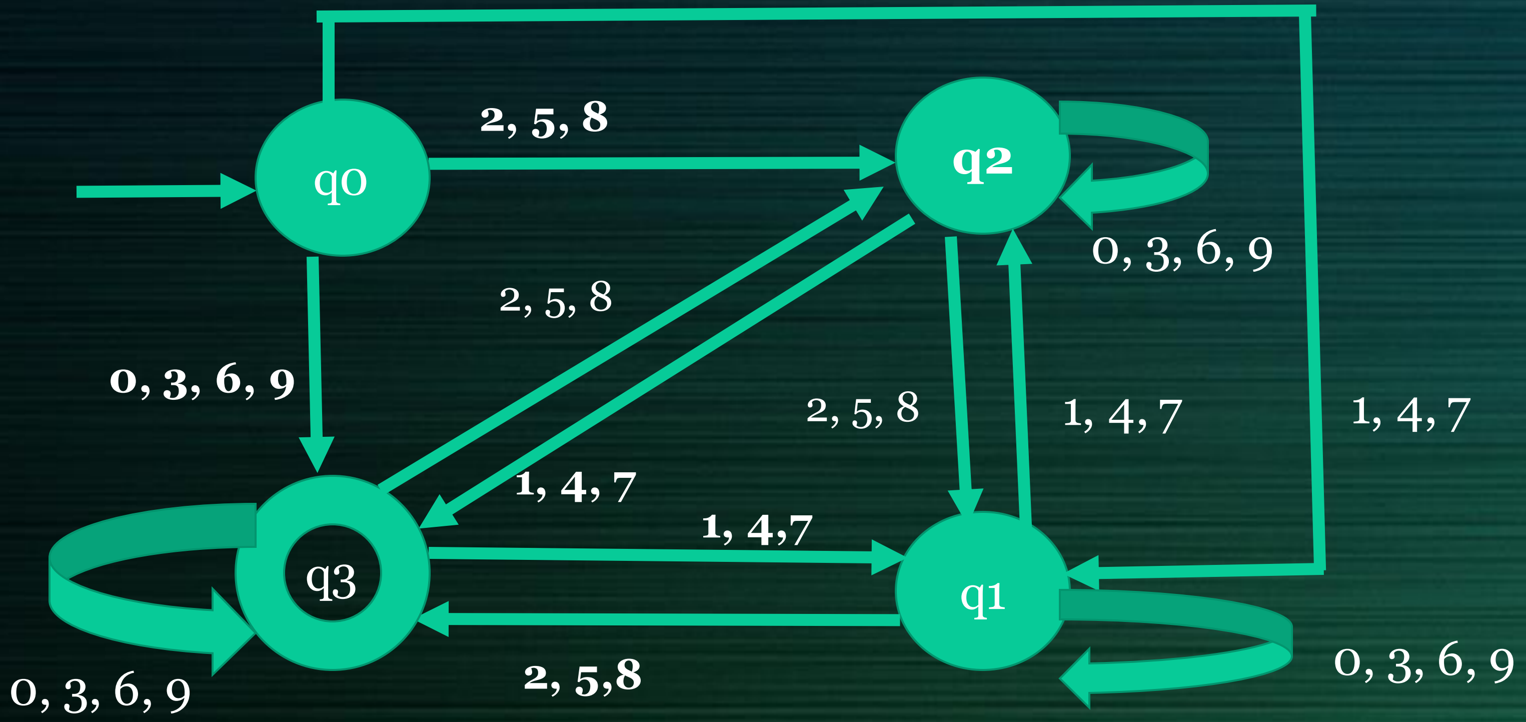
Logic is Divisibility test of 3

Here, q3 represents remainder 0 state

q1 represents remainder 1 state

q2 represents remainder 2 state

Transition Diagram:



Above DFA can be represented as

$M = (Q , \Sigma , \delta , q_o , F)$

Where $Q = \{ q0, q1, q2, q3 \}$

$\Sigma = \{ 0, 1, 2, 3, \ldots , 9 \}$

$q0 = q0$

$F = \{ q3 \}$

Here, we will make group of inputs to represent transitions as they have similar next state on current Input.

Group 1: 0, 3, 6, 9
Group 2: 1, 4, 7
Group 3: 2, 5, 8

Examples on DFA

Ex. 10 Design DFA to check whether given decimal number is divisible by 3

Solution:

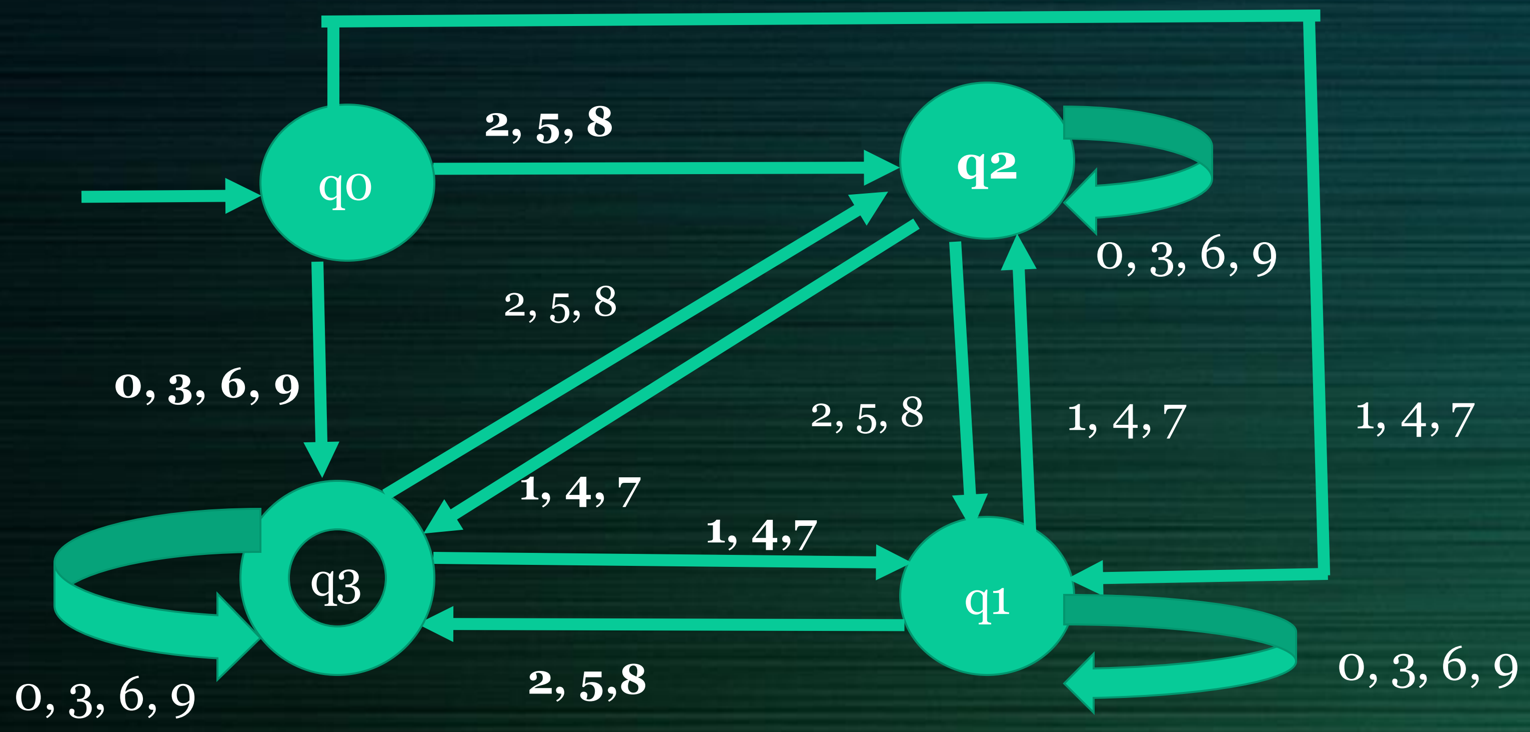
Logic is Divisibility test of 3

Here, q3 represents remainder 0 state

q1 represents remainder 1 state

q2 represents remainder 2 state

Transition Diagram:



Transition Function:

δ (q0 , (0, 3, 6, 9)) = q3

δ (q1 , (0, 3, 6, 9)) = q1

δ (q2 , (0, 3, 6, 9)) = q2

δ (q3 , (0, 3, 6, 9)) = q3

δ (q0 , (1, 4, 7)) = q1

δ (q1 , (1, 4, 7)) = q2

δ (q2 , (1, 4, 7)) = q3

δ (q3, (1, 4, 7)) = q1

δ (q0 , (2, 5, 8)) = q2

δ (q1 , (2, 5, 8)) = q3

δ (q2 , (2, 5, 8)) = q1

δ (q3 , (2, 5, 8)) = q2

Examples on DFA

Ex. 10 Design DFA to check whether given decimal number is divisible by 3

Solution:

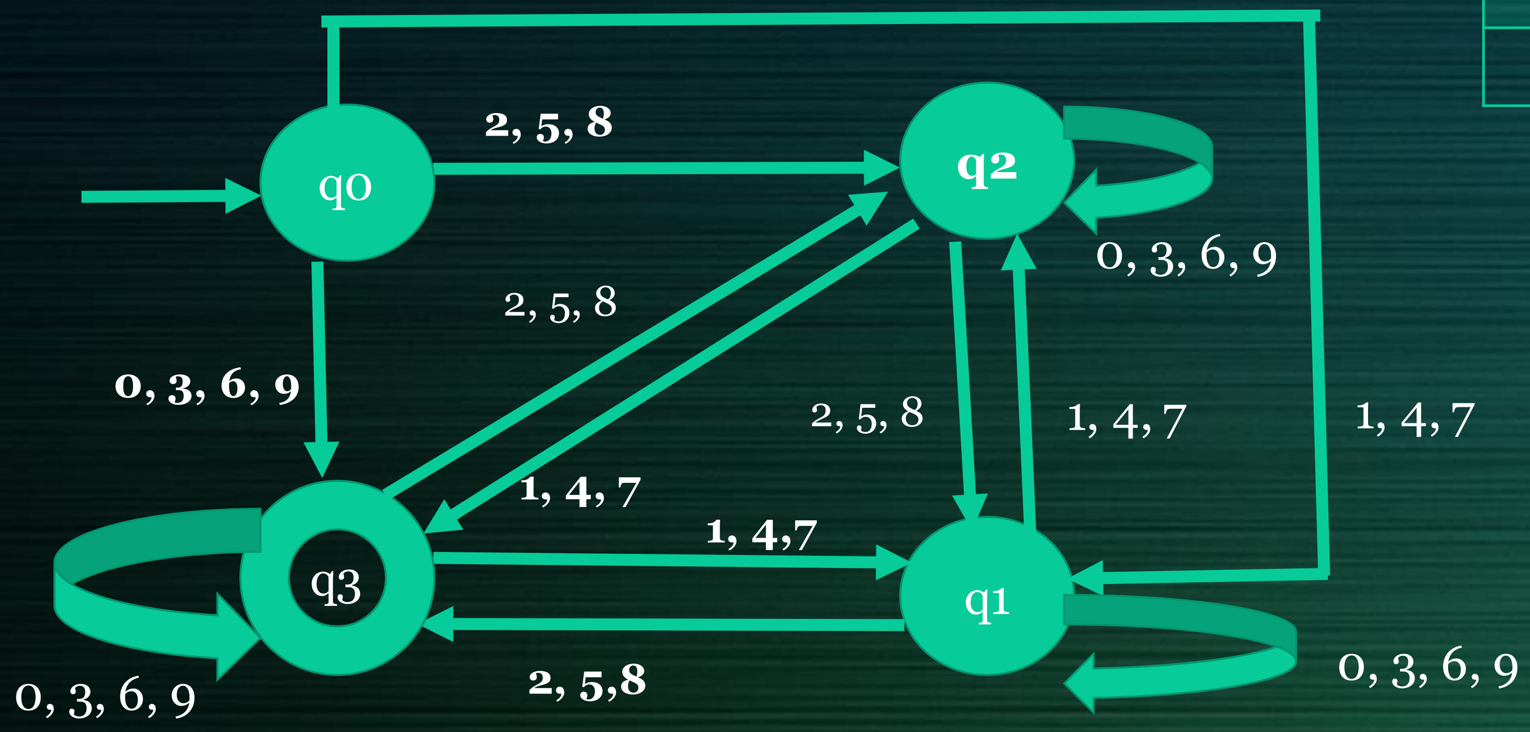
Logic is Divisibility test of 3

Here, q3 represents remainder 0 state

q1 represents remainder 1 state

q2 represents remainder 2 state

Transition Diagram:



Transition Table:

Inputs/ States	0, 3, 6, 9	1, 4, 7	2, 5, 8
q0	q3	q1	q2
q1	q1	q2	q3
q2	q2	q3	q1
q3	q3	q1	q2

Examples on DFA

Ex. 10 Design DFA to check whether given decimal number is divisible by 3

Solution:

Simulation for the string 532

δ (q0 , 5 3 2) |-- δ (q2 , 3 2)

|-- δ (q2 , 2)

|-- δ (q1 , ε)

= q1

q3 is not a final state.

Hence given string 532 is not accepted by given DFA

Transition Table:

Inputs/ States	**0, 3, 6, 9**	**1, 4, 7**	**2, 5, 8**
q0	q3	q1	q2
q1	q1	q2	q3
q2	q2	q3	q1
q3	q3	q1	q2

Examples on DFA

Ex. 10 Design DFA to check whether given decimal number is divisible by 3

Solution:

Simulation for the string 324531

$\delta(q0, 324531) \vdash \delta(q3, 24531)$

$\vdash \delta(q2, 4531)$

$\vdash \delta(q3, 531)$

$\vdash \delta(q2, 31)$

$\vdash \delta(q2, 1)$

$\vdash \delta(q3, \varepsilon)$

$= q3$

q3 is a final state.

Hence given string 324531 is accepted by given DFA

Transition Table:

Inputs/ States	**0, 3, 6, 9**	**1, 4, 7**	**2, 5, 8**
q0	q3	q1	q2
q1	q1	q2	q3
q2	q2	q3	q1
q3	q3	q1	q2