Context Free Grammar

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Grammar :

- Grammar is finite set of formal rules for generating syntactically correct sentences.
- Any language requires Grammar , which defines correct statement formats or constructs allowed for that language
- Hence, grammar is called as ***Syntactic Definition*** of the language
- Here, we are to discuss grammar for formal languages , CFG (Context Free Grammar)

Constitutes of Grammar :

- Grammar consists of two types of symbols :
  - Terminals
  - Non Terminals

- Terminals are part of Generated Sentence

- Non Terminals are NOT part of Sentence, but, of generation of Sentence

- Non Terminals are essential for declaring the rules

- These rules are called ***Production Rules***

- A grammar that is based on the Constituent structure is called "***Constituent Structure Grammar***" or "***Phase Structure Grammar***"

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Context Free Grammar :

- A Phase Structure Grammar is defined by a quardruple

G = { V, T, P, S }

Where,

V : Finite set of Non Terminals

T : Finite set of Terminals

S : Start Symbol (S is a Non Terminal)

P : Finite set of Productions

Definition :

Context-free Grammars

Definition 5.1:

A grammar $G = (V, T, S, P)$ is said to be context-free if all production rules in P have the form

$$A \to x$$

where $A \in V$ and $x \in (V \cup T)^*$

A language is said to be context-free iff there is a context free grammar G such that $L = L(G)$.

What are Context Free Grammars?

- In Formal Language Theory , a Context free Grammar(CFG) is a formal grammar in which every production rule is of the form

$$V \longrightarrow w$$

Where V is a single nonterminal symbol and w is a string of terminals and/or nonterminals (w can be empty)

- The languages generated by context free grammars are knows as the context free languages

Practice Context Free Grammars

a) CFG generating alternating sequence of 0's and 1's

b) CFG in which no consecutive b's can occur but consecutive a's can occur

c) CFG for the following language:

$$L(G)= \{a^n b^{2n} \mid n>=0\}$$

Practice Answers

a) $S \rightarrow 0A \mid 1B$

$A \rightarrow 1B \mid 0$

$B \rightarrow 0A \mid 1$

b) $S \rightarrow aS \mid bT \mid a \mid b$

$T \rightarrow aS \mid a$

c) $S \rightarrow aSbb \mid \epsilon$

Example

- $V = \{q, f,\}$
- $\Sigma = \{0, 1\}$
- $R = \{q \rightarrow 11q, \ \ q \rightarrow 00f,$

  $f \rightarrow 11f, \ \ f \rightarrow \varepsilon \ \ \}$
- $S = q$
- $(R = \{q \rightarrow 11q \mid 00f, f \rightarrow 11f \mid \varepsilon \})$

Example

- G = ({S}, {0,1}. {S → 0S1 | ε }, S)
- ε in L(G) because S $\Rightarrow$ ε .
- 01 in L(G) because S $\Rightarrow$ 0S1 $\Rightarrow$ 01.
- 0011 in L(G) because

  S $\Rightarrow$ 0S1 $\Rightarrow$ 00S11 $\Rightarrow$ 0011.
- 0^n1^n in L(G) because S $\Rightarrow^*$ 0^n1^n .
- L(G) = $\{0^n1^n \mid n \geq 0\}$

How do we use rules?

- If $A \rightarrow B$, then $xAy \Rightarrow xBy$ and we say that xAy derivates xBy.

- If $s \Rightarrow \cdots \Rightarrow t$, then we write $s \Rightarrow^* t$.
- A string x in Σ^* is generated by $G=(V,\Sigma,R,S)$ if $S \Rightarrow^* x$.
- $L(G) = \{ x \text{ in } \Sigma^* \mid S \Rightarrow^* x\}$.

Example

- $G = (\{S\}, \{0,1\}. \{S \rightarrow 0S1 \mid \varepsilon\}, S)$
- ε in $L(G)$ because $S \Rightarrow \varepsilon$.
- 01 in $L(G)$ because $S \Rightarrow 0S1 \Rightarrow 01$.
- 0011 in $L(G)$ because

  $S \Rightarrow 0S1 \Rightarrow 00S11 \Rightarrow 0011$.
- $0^n 1^n$ in $L(G)$ because $S \Rightarrow^* 0^n 1^n$.
- $L(G) = \{0^n 1^n \mid n \geq 0\}$