Chapter One

Introduction to Statistics

Introduction:

- 'Statistics' derived from the Latin word' status' or the Italian word' statista' or the German word' statistik' each of which means a 'political state'.
- Kautilya's Arthshastra -registration of births and deaths
- Aina,e-Akbari- good records of land and agricultural statistics.

Introduction:

- The theoretical development of the so-called modem statistics came during the mid seventeenth century with the introduction of 'Theory of Probability' and 'Theory of Games and Chance'.
- Sir Ronald A Fisher, known as the **'Father of Statistics'** , placed Statistics on a very sound footing by applying it to various diversified fields, such as genetics, biometry, education; agriculture, etc.

Definitions:

- It is a science which helps us to **collect, analyze** and **present data** systematically.
- It is the process of **collecting, processing, summarizing, presenting, analysing** and **interpreting of data in order to study** and **describe a given problem.**
- Statistics is the **art of learning from data**.
- Statistics may be regarded as (i)the study of populations, (ii) the study of variation, and (iii) the study of methods of the reduction of data.

Definitions:

- The science of Statistics is the method of judging collective, natural or social phenomenon from the results obtained from the analysis or enumeration or collection of estimates.
- Statistics is the science which deals with collection, classification and tabulation of numerical facts as the basis for explanation, description and comparison of phenomenon.

Importance of Statistics:

- It simplifies mass of data (condensation);
- Helps to get concrete information about any problem;
- Helps for reliable and objective decision making;
- It presents facts in a precise & definite form;
- It facilitates comparison(Measures of central tendency and measures of dispersion);
- It facilitates Predictions (Time series and regression analysis are the most commonly used methods towards prediction.);
- It helps in formulation of suitable policies;

Limitation of statistics:

1. Statistics does not deal with **individual items;**
2. Statistics deals only with **quantitatively expressed items**, it does not study qualitative phenomena;
3. Statistical results are **not universally true**;
   - Statistical laws are only approximations and not exact. Of
   - in terms of probability and chance
   - Eg. It has been found that 20 % of-a certain surgical operations by a particular doctor are successful."
4. Statistics is **liable/responsible/ to be misused**.
   - can be moulded and manipulated in any manner to support one's way of argument and reasoning.

Application areas of statistics

➢ Engineering:

Improving product design, testing product performance, determining reliability and maintainability, working out safer systems of flight control for airports, etc.

➢ Business:

Estimating the volume of retail sales, designing optimum inventory control system, producing auditing and accounting procedures, improving working conditions in industrial plants, assessing the market for new products.

➢Quality Control:

Determining techniques for evaluation of quality through adequate sampling, in process control, consumer survey and experimental design in product development etc.

Realizing its importance, large organizations are maintaining their own **Statistical Quality Control Department.**

➢Economics:

Measuring indicators such as volume of trade, size of labor force, and standard of living, analyzing consumer behavior, computation of national income accounts, formulation of economic laws, etc.

Particularly, Regression analysis extensively used in the field of Economics.

- **Health and Medicine:**

  Developing and testing new drugs, delivering improved medical care, preventing diagnosing, and treating disease, etc. Specifically, inferential Statistics has a tremendous application in the fields of health and medicine.

- **Biology:**

  Exploring the interactions of species with their environment, creating theoretical models of the nervous system, studying genetically evolution, etc.

- **Psychology:**

  Measuring learning ability, intelligence, and personality characteristics, creating psychological scales and abnormal behavior, etc.

- **Sociology:**

  Testing theories about social systems, designing and conducting sample surveys to study social attitudes, exploring cross-cultural differences, studying the growth of human population, etc.

There are two main branches of statistics:

1. Descriptive statistics
2. Inferential statistics

1. Descriptive statistics:

- It is the **first phase** of Statistics;
- involve any kind of data processing designed to the collection, organization, presentation, and analyzing the important features of the data **with out attempting to infer/conclude any thing that goes beyond the known data.**
- In short, descriptive Statistics describes the nature or characteristics of the observed data (usually a sample) **without making conclusion or generalization.**

The following are some examples of descriptive Statistics:

- The daily average temperature range of AA was 25 0c last week .
- The maximum amount of coffee export of Eth. (as observed from the last 20 years) was in the year 2004.
- The average age of athletes participated in London Marathon was 25 years.
- 75% of the instructors in AAU are male.
- The scores of 50 students in a Mathematics exam are found to range from 20 to 90.

2. Inferential statistics (Inductive Statistics):

- It is a **second phase** of Statistics which deals with techniques of making a **generalization** that lie outside the scope of **Descriptive Statistics;**
- It is concerned with the process of **drawing conclusions (inferences)** about specific characteristics of a population based on information obtained from samples;
- It is a process of performing hypothesis testing, determining relationships among variables, and making predictions.
- **The area of inferential statistics entirely needs the whole aims to give reasonable estimates of unknown population parameters.**

The following are some examples of inferential Statistics:

✓The result obtained from the analysis of the income of 1000 randomly selected citizens in Ethiopia suggests that the average monthly income of a citizen is estimated to be 600 Birr.

✓Here in the above example we are trying to represent the income of about entire population of Ethiopia by a sample of 1000 citizens, hence we are making inference or generalization.

✓Based on the trend analysis on the past observations/data, the average exchange rate for a dollar is expected to be 18 birr in the coming month.

✓The national statistical Bereau of Ethiopia declares the out come of its survey as "The population of Eth. in the year 2020 will likely to be 120,000,000."

✓From the survey obtained on 15 randomly selected towns of Eth. it is estimated that 0.1% of the whole urban dwellers are victims of AIDS virus.

Exercise: Descriptive Statistics or Inferential Statistics

1. The manager of quality control declares the out come of its survey as "the average life span of the imported light bulbs is 3000 hrs.
2. Of all the patients taking the drug at a local health center 80% of them suffer from side effect developed.
3. The average score of all the students taking the exam is found to be 72.
4. The national statistical bureau of Eth. declares the out come of its survey for the last 30 years as "the average annual growth of the people of Ethiopia is 2.8%.
5. The national statistical bureau of Eth. declares the out come of its survey for the last 30 years as "the population of Ethiopia in the year 2015 will likely to be 100,000,000.
6. Based on the survey made for the last 10 years 30,000 tourists are expected to visit Ethiopia.
7. Based on the survey made for the last 20 years the maximum number of tourists visited Eth. were in the year 1993.
8. The Ethiopian tourism commission has announced that (as observed for the last 20 years) the average number of tourists arrived Ethiopia per year is 3000.
9. The maximum difference of the salaries of the workers of the company until the end of last year was birr 5000.

Main terms in statistics:

Data: Certainly known facts from which conclusions may be drawn.

Statistical data: Raw material for a statistical investigation which are obtained when ever **measurements or observations are made.**

i. **Quantitative data:** data of a certain group of individuals which is expressed numerically.

Example: Heights, Weights, Ages and, etc of a certain group of individuals.

ii. **Qualitative data:** data of a certain group of individuals that is not expressed numerically.

Example: Colors, Languages, Nationalities, Religions, health, poverty etc of a certain group of individuals.

Primary data and **Secondary data**

Variable: A variable is a factor or characteristic that can take on different possible values or outcomes.

A variable can be qualitative or quantitative (numeric).

Example: Income, height, weight, sex, age, etc of a certain group of individuals are examples of variables.

Population: A complete set of observation (data) of the entire group of individuals under consideration .

A population can be finite or infinite.

Example: The number of students in this class, the population in Addis Ababa etc.

Sample: A set of data drawn from population containing a part which can reasonably serve as a basis for valid generalization about the population.

A sample is a portion of a population selected for further analysis.

Sample size: The number of items under investigation in a sample.

Survey (experiment): it is a process of obtaining the desired data. Two types of survey:

1. **Census Survey:** A way of obtaining data referring the entire population including a total coverage of the population.
2. **Sample Survey:** A way of obtaining data referring a portion of the entire population consisting only a partial coverage of the population.

STEPS/STAGES IN STATISTICAL INVESTIGATION

1. Collection of Data:

Data collection is the process of gathering information or data about the variable of interest. Data are inputs for Statistical investigation. Data may be obtained either from primary source or secondary source.

2. Organization of Data

Organization of data includes three major steps.

1. ***Editing:*** checking and omitting inconsistencies, irrelevancies.
2. ***Classification :*** task of grouping the collected and edited data.
3. ***Tabulation:*** put the classified data in the form of table.

3. Presentation of Data

The purpose of presentation in the statistical analysis is to display what is contained in the data in the form of Charts, Pictures, Diagrams and Graphs for an easy and better understanding of the data.

4. Analyzing of Data

- In a statistical investigation, the process of analyzing data includes finding the various statistical constants from the collected mass of data such as measures of central tendencies (averages) , measures of dispersions and soon.
- It merely involves mathematical operations: different measures of central tendencies (averages), measures of variations, regression analysis etc. In its extreme case, analysis requires the knowledge of advanced mathematics.

5. Interpretation of Data

- ✓ involve interpreting the statistical constants computed in analyzing data for the formation of valid conclusions and inferences.
- ✓ It is the most difficult and skill requiring stage.
- ✓ It is at this stage that Statistics seems to be very much viable to be misused.
- ✓ Correct interpretation of results will lead to a valid conclusion of the study and hence can aid in taking correct decisions.
- ✓ Improper (incorrect) interpretation may lead to wrong conclusions and makes the whole objective of the study useless.

THE ENGINEERING METHOD AND STATISTICAL THINKING

- An engineer is someone who solves problems of interest to society by the efficient application of scientific principles.

- Engineers accomplish this by either refining an existing product or process or by designing a new product or process that meets customers' expectations and needs.

The steps in the engineering method are as follows:

1. Develop a clear and concise description of the problem.
2. Identify, the important factors that affect this problem or that may play a role in its solution.
3. Propose a model for the problem, using scientific or engineering knowledge of the phenomenon being studied. State any limitations or assumptions of the model.
4. Conduct appropriate experiments and collect data to test or validate the tentative model or conclusions made in steps 2 and 3.

5. Refine the model on the basis of the observed data.
6. Manipulate the model to assist in developing a solution to the problem.
7. Conduct an appropriate experiment to confirm that the proposed solution to the problem is both effective and efficient.
8. Draw conclusions or make recommendations based on the problem solution.

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- ✓ The engineering method features a strong interplay between the problem, the factors that may influence its solution, a model of the phenomenon, and experimentation to verify the adequacy of the model and the proposed solution to the problem.
- ✓ Specifically, statistical techniques can be a powerful aid in designing new products and systems, improving existing designs, and designing, developing, and improving production processes.

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- ✓ Therefore, Engineers must know how to efficiently plan experiments, collect data, analyze and interpret the data, and understand how the observed data are related to the model that have been proposed for the problem under study.

1.2. Data collection and graphical representation of data

Classification of Data based on source:

1. **Primary:** data collected for the purpose of specific study.

   It can be obtained by:

   - Direct personal observation
   - Direct or indirect oral interviews
   - Administrating questionnaires

2. **Secondary:** refers to data collected earlier for some purpose other than the analysis currently being undertaken.

   It can be obtained from:

   - External Secondary data Sources(for eg. gov't and non gov't publications)
   - Internal Secondary data Sources: the data generated within the organization in the process of routine business activities

Qualitative data are nonnumeric. **Quantitative** data are numeric.

Method of data presentation

- The purpose of organizing data is to see quickly some of the characteristics of the data that have been collected.
- **Raw data** is collected numerical data which has not been arranged in order of magnitude.
- **An array** is an arranged numerical data in order of magnitude.

Method of data presentation

- Mechanism for reducing and summarizing data are:

1. Tabular method.
2. Graphical method
3. Diagrammatic method

1. Tabular presentation of data:

- The collected raw data should be put into an ordered array in either ascending or descending order so that it can be organized in to a Frequency Distribution (FD)
- Numerical data arranged in order of magnitude along with the corresponding frequency is called frequency distribution (FD).
- FD is **of two kinds namely ungrouped /and grouped frequency distribution**.

A. Ungrouped (Discrete) Frequency Distribution

- ✓ It is a tabular arrangement of numerical data in order of magnitude showing the **distinct values** with the corresponding frequencies.

Example:

Suppose the following are test score of 16 students in a class, write un grouped frequency distribution.

"14, 17, 10, 19, 14, 10, 14, 8, 10, 17, 19, 8, 10, 14, 17, 14"

Sol: the ungrouped frequency distribution:

Array: 8,8,10,10,10,10,14,14,14,14,14, 17,17,17,19,19.

Then the ungrouped frequency distribution is then grouped:-

Test score	8	10	14	17	19
Frequency	2	4	5	3	2

- The difference between the highest and the lowest value in a given set of observation is called **the range (R)**
  R= L- S, R= 19-8 = 11

B. Grouped (continuous) Frequency Distribution (GFD)

- ✓ It is a tabular arrangement of data in order of magnitude by classes together with the corresponding class frequencies.
- ✓ In order to estimate the number of classes, the ff formula is used:

  Number of classes=1+3.322(log N) where N is the Number of observation.

$$\text{The Class size (class width)} = \frac{\text{Range}}{1+3.322(\log N)} \quad \text{(round up)}$$

Example:

Grouped/Continuous frequency distribution where several numbers are grouped into one class.

e.g.

Student age	Frequency
18-25	5
26-32	15
33-39	10

Components of grouped frequency distribution

1. **Lower class limit:**

   is the smallest number that can actually belong to the respective classes.

2. **Upper class limit:**

   is the largest number that can actually belong to the respective classes.

3. **Class boundaries:**

   are numbers used to separate adjoining classes which should not coincide with the actual observations.

4. **Class mark:**

   is the midpoint of the class.

5. Class width/ Class intervals

is the difference between two consecutive lower class limits or the two consecutive upper class limits. (OR)

can be obtained by taking the difference of two adjoining class marks or two adjoining lower class boundaries.

Class width = Range/Number of class desired.

Where: Number of classes=1+3.322(log N) where N is the Number of observation.

6. Unit of measure

is the smallest possible positive difference between any two measurements in the given data set that shows the degree of precision.

- ✓ **Class boundaries:**

  can be obtained by taking the averages of the upper class limit of one class and the lower class limit of the next class.

- ✓ **Lower class boundaries:**

  can be obtained by subtracting half a unit of measure from the lower class limits.

- ✓ **Upper class boundaries:**

  can be obtained by adding half the unit of measure to the upper class limits.

Example1 :

Suppose the table below is the frequency distribution of test score of 50 students.

Then the frequency table has 6 classes (class intervals).

Test score	Frequency
11-15	7
16-20	8
21-25	10
26-30	12
31-35	9
36-40	4

What is the Unit of Measure, LCLs, UCLs, LCBs, UCBs, CW, and CM

- ✓ The unit of measure is 1
- ✓ The lower class limits are:-11, 16, 21, 26, 31, 36
- ✓ The upper class limits are:- 15, 20, 25, 30, 35, 40
- ✓ The class marks are:- 13((11+15)/2), 18, 23, 28, 33, 38
- ✓ The lower class boundaries are:- 10.5(11-0.5), 15.5, …., 35,5
- ✓ The upper class boundaries are:- 15.5(15+0.5), ….35.5, 40.5
- ✓ Class width (size) is 5.

Rules to construct Grouped Frequency Distribution (GFD):

i. Find the **unit of measure** of the given data;
ii. Find the **range**;
iii. Determine the **number of classes** required;
iv. Find **class width (size)**;
v. Determine a **lowest class limit** and then find the **successive lower and upper class limits** forming non over lapping intervals such that each observation falls into exactly one of the class intervals;
vi. Find the **number of observations** falling into each class intervals that is taken as **the frequency of the class (class interval)** which is best done using a tally.

Exercise:

Construct a GFD of the following aptitude test scores of 40 applicants for accountancy positions in a company with

a. 6 classes b. 8 classes

96	89	58	61	46	59	75	54
41	56	77	49	58	60	63	82
66	64	69	67	62	55	67	70
78	65	52	76	69	86	44	76
57	68	64	52	53	74	68	39

Types of Grouped Frequency Distribution

1. Relative frequency distribution (RFD)
2. Cumulative Frequency Distribution (CFD)
3. Relative Cumulative Frequency Distribution (RCFD)

Types of Grouped Frequency Distribution

1. Relative frequency distribution (RFD):

- A table presenting the ratio of the **frequency of each class to the total frequency of all the classes**.
- Relative frequency generally expressed **as a percentage**, used to show the percent of the total number of observation in each class.

For example

Test score	F	RFD	PFD
37.5-47.5	4	4/40=0.1	10%
47.5-57.5	8	8/40=0.2	20%
57.5-67.5	13	13/40=0.325	32.5%
67.5-77.5	10	10/40=0.25	25%
77.5-87.5	3	3/40=0.075	7.5%
87.5-97.5	2	2/40=0.05	5%

2. Cumulative Frequency Distribution (CFD):

- It is applicable when we want to know how many observations lie **below or above a certain value/class boundary.**

- **CFD** is of two types, **LCFD and MCFD:**
  - ✓ **Less than Cumulative Frequency Distribution (LCFD):** shows the collection of cases lying **below the upper class boundaries of each class.**
  - ✓ **More than Cumulative Frequency Distribution (MCFD):** shows the collection of cases lying **above the lower class boundaries of each class.**

Test score	CF
Less than 37.5	0
Less than47.5	4
Less than 57.5	12
Less than 67.5	25
Less than 77.5	35
Less than 87.5	38
Less than 97.5	40

Test score	CF
more than 37.5	40
more than 47.5	36
more than 57.5	28
more than 67.5	15
more than 77.5	5
more than 87.5	2
more than 97.5	0

3. Relative Cumulative Frequency Distribution (RCFD)

It is used to determine the ratio or the percentage of observations that lie below or above a certain value/class boundary, to the total frequency of all the classes. These are of two types: The LRCFD and MRCFD.

- **Less than Relative Cumulative Frequency Distribution (LRCFD):** A table presenting the ratio of the cumulative frequency **less than upper class boundary of each class to the total frequency of all the classes**
- **More than Relative Cumulative Frequency Distribution (MRCFD):** A table presenting the ratio of the cumulative frequency **more than lower class boundary of each class to the total frequency of all the classes.**

LRCFD

Test score	LCF	LRCF	LPCF
Less than 37.5	0	0/40=0	0%
Less than 47.5	4	4/40=0.1	10%
Less than 57.5	12	12/40=0.3	30%
Less than 67.5	25	25/40=0.625	62.5%
Less than 77.5	35	35/40=0.875	87.5%
Less than 87.5	38	38/40=0.95	95%
Less than 97.5	40	40/40=1	100%

MRCFD

Test score	MCF	MRCF	MPCF
More than 37.5	40	40/40=1	100%
More than 47.5	36	36/40=0.9	90%
More than 57.5	28	28/40=0.7	70%
More than 67.5	15	15/40=0.375	37.5%
More than 77.5	5	5/40=0.125	12.5%
More than 87.5	2	2/40=0.05	5%
More than 97.5	0	0/40=0	0%

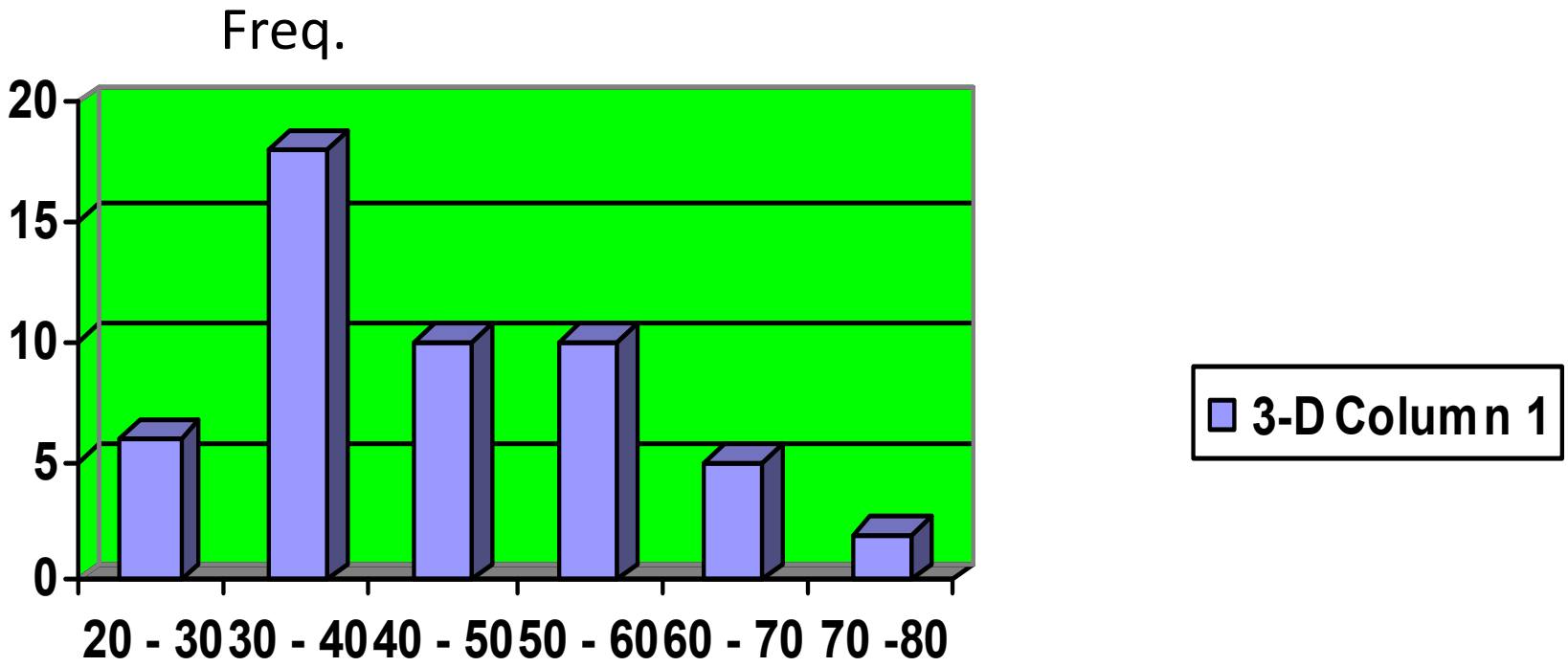
Graphic Methods of Data presentation

1. Histogram
2. Frequency Polygon (Line graph)
3. Cumulative frequency curve (o-give)

1. Histogram:

- ✓ A graphical presentation of grouped frequency distribution consisting of a series of adjacent rectangles whose bases are the class intervals specified in terms of class boundaries (equal to the class width of the corresponding classes) shown on the x-axis and whose heights are proportional to the corresponding class frequencies shown on the y-axis.

Histogram: E.g.



Steps to draw Histogram

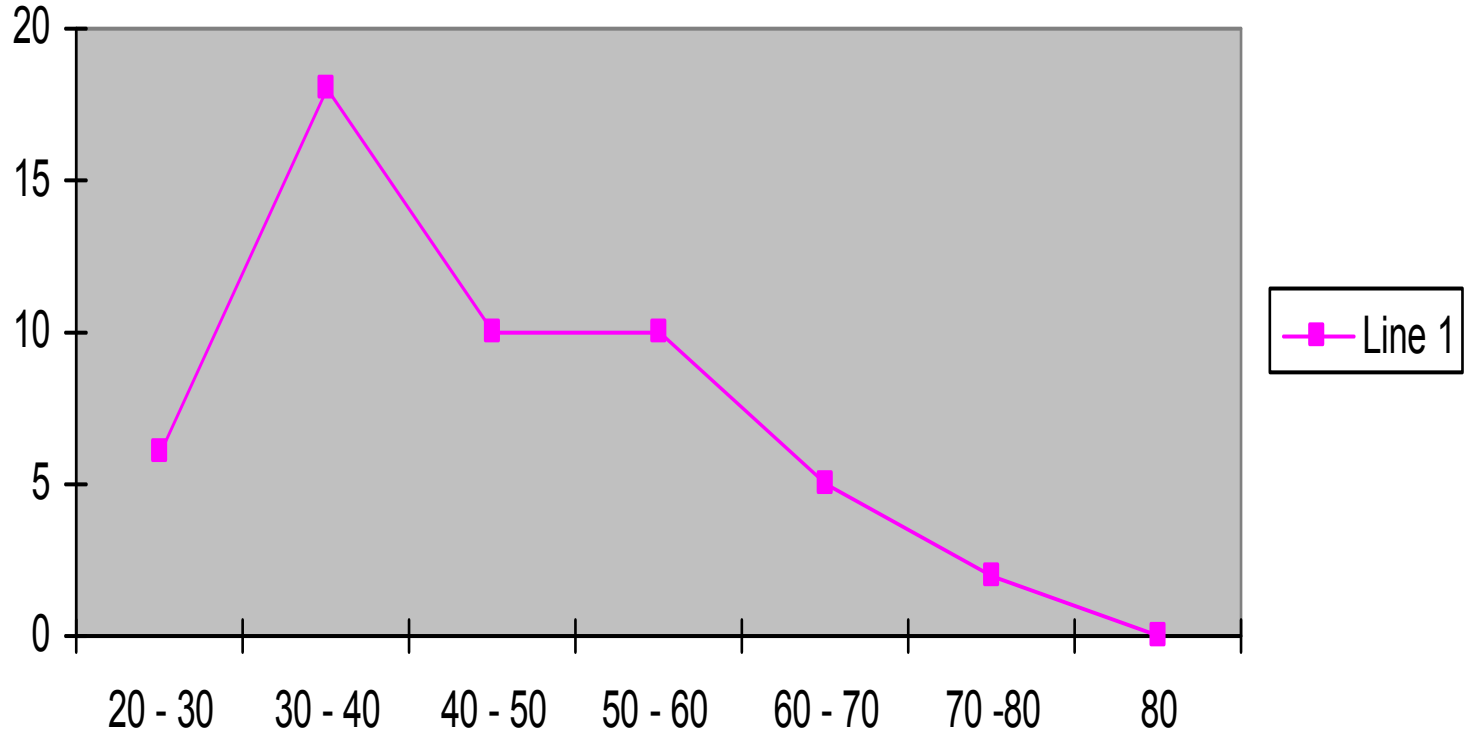
i. Mark the class boundaries on the horizontal axis (x- axis) and the class frequencies along the vertical axis (y- axis) according to a suitable scale.

ii. With each interval as a base draw a rectangle whose height equals the frequency of the corresponding class interval. It describes the shape of the data.

2. Frequency Polygon:

It is a line graph of grouped frequency distribution in which the class frequency is plotted against class mark that are subsequently connected by a series of line segments to form line graph including classes with zero frequencies at both ends of the distribution to form a polygon.

Frequency Polygon:



Steps to draw Frequency polygon

i. Mark the class mid points on the x-axis and the frequency on the y-axis.

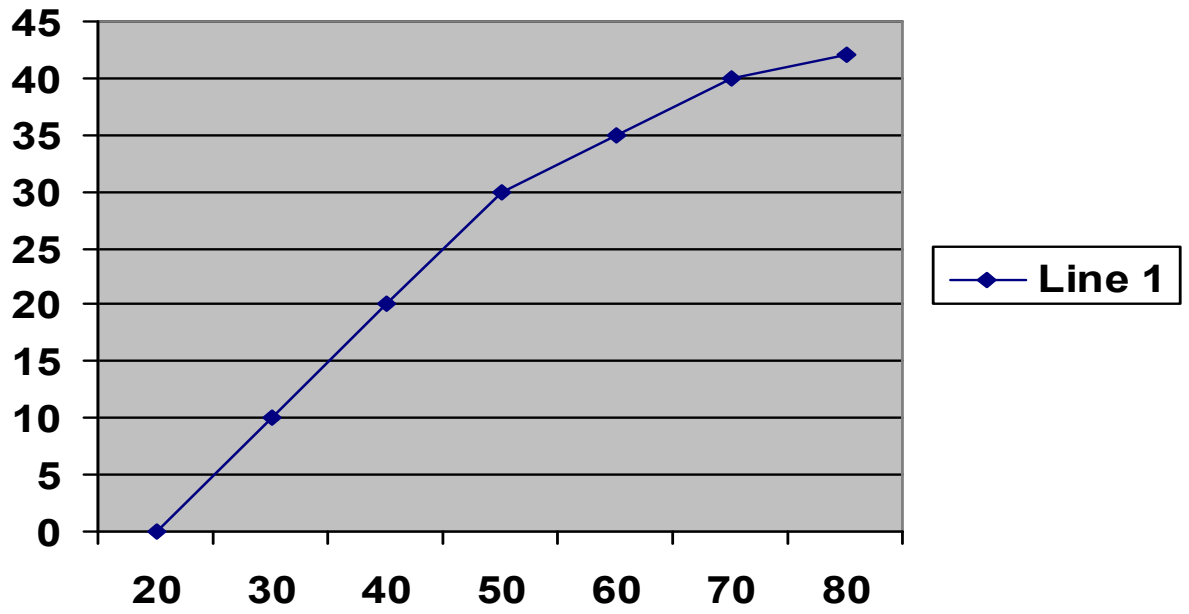
ii. Mark dots which correspond to the frequency of the marked class mid points.

iii. Join each successive dot by a series of line segments to form line graph, including classes with zero frequencies at both ends of the distribution to form a polygon.

3. O-GIVE curve (Cumulative Frequency Curve / percentage Cumulative Frequency Curve)

- ✓ It is a line graph presenting the cumulative frequency distribution.
- ✓ O-gives are of two types: The **Less than O-give** and The **More than O-give**.
- ➢ **The Less than O-give** shows the cumulative frequency less than the upper class boundary of each class; and
- ➢ **The More than O-give** shows the cumulative frequency more than the lower class boundary of each class.

Ogive: E.g.



Steps to draw O-gives

i. Mark class boundaries on the x-axis and mark non overlapping intervals of equal length on the y-axis to represent the cumulative frequencies.

ii. For each class boundaries marked on the x-axis, plot a point with height equal to the corresponding cumulative frequencies.

iii. Connect the marked points by a series of line segments where the less than O-give is done by plotting the less than cumulative frequency against the upper class boundaries

Diagrammatic Presentation Of Data

- Bar charts
- Pie chart
- Pictograph and
- Pareto diagram