

# SYNTAX ANALYSIS



BOTTOM-UP  
PARSING

# Operator Precedence Parser

$E \rightarrow E + T \mid T$

$T \rightarrow T * V \mid V$

$V \rightarrow a \mid b \mid c \mid d$

Input:  $a + b * c * d$

	a	b	c	d	+	*	\$
a					>	>	>
b					>	>	>
c					>	>	>
d					>	>	>
+	<	<	<	<	>	<	>
*	<	<	<	<	>	>	>
\$	<	<	<	<	<	<	A

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$V \rightarrow a \mid b \mid c \mid d$

Input:  $a + b * c * d$

Stack	Relation	Input	Comment
\$	<	$a + b * c * d$ \$	Push a

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$V \rightarrow a \mid b \mid c \mid d$

Input:  $a + b * c * d$

Stack	Relation	Input	Comment
\$	<	$a + b * c * d \$$	Push a
\$ a	>	$+ b * c * d \$$	Pop a

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$V \rightarrow a \mid b \mid c \mid d$

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\$	<	$a + b * c * d \$$	Push a
\$ a	>	$+ b * c * d \$$	Pop a
\$	<	$+ b * c * d \$$	Push +

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\$	<	$a + b * c * d \$$	Push a
\$ a	>	$+ b * c * d \$$	Pop a
\$	<	$+ b * c * d \$$	Push +
\$ +	<	$b * c * d \$$	Push b

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\$ a	>	$+ b * c * d \$$	Pop a
\$	<	$+ b * c * d \$$	Push +
\$ +	<	$b * c * d \$$	Push b
\$ + b	>	$* c * d \$$	Pop b

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$T \rightarrow T * V \mid V$

$V \rightarrow a \mid b \mid c \mid d$

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Stack	Relation	Input	Comment
\$	<	$a + b * c * d \$$	Push a
\$ a	>	$+ b * c * d \$$	Pop a
\$	<	$+ b * c * d \$$	Push +
\$ +	<	$b * c * d \$$	Push b
\$ + b	>	$* c * d \$$	Pop b
\$ +	<	$* c * d \$$	Push *



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\$	<	$a + b * c * d \$$	Push a
\$ a	>	$+ b * c * d \$$	Pop a
\$	<	$+ b * c * d \$$	Push +
\$ +	<	$b * c * d \$$	Push b
\$ + b	>	$* c * d \$$	Pop b
\$ +	<	$* c * d \$$	Push *
\$ + *	<	$c * d \$$	Push c

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$V \rightarrow a \mid b \mid c \mid d$

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Stack	Relation	Input	Comment
\$	<	$a + b * c * d \$$	Push a
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\$	<	$+ b * c * d \$$	Push +
\$ +	<	$b * c * d \$$	Push b
\$ + b	>	$* c * d \$$	Pop b
\$ +	<	$* c * d \$$	Push *
\$ + *	<	$c * d \$$	Push c
\$ + * c	>	$* d \$$	Pop c

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Input:  $a + b * c * d$

Stack	Relation	Input	Comment
\$	<	$a + b * c * d \$$	Push a
\$ a	>	$+ b * c * d \$$	Pop a
\$	<	$+ b * c * d \$$	Push +
\$ +	<	$b * c * d \$$	Push b
\$ + b	>	$* c * d \$$	Pop b
\$ +	<	$* c * d \$$	Push *
\$ + *	<	$c * d \$$	Push c
\$ + * c	>	$* d \$$	Pop c
\$ + *	>	$* d \$$	Pop *

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$E \rightarrow E + T \mid T$

$T \rightarrow T * V \mid V$

$V \rightarrow a \mid b \mid c \mid d$

Input:  $a + b * c * d$

Stack	Relation	Input	Comment
\$	<	$a + b * c * d \$$	Push a
\$ a	>	$+ b * c * d \$$	Pop a
\$	<	$+ b * c * d \$$	Push +
\$ +	<	$b * c * d \$$	Push b
\$ + b	>	$* c * d \$$	Pop b
\$ +	<	$* c * d \$$	Push *
\$ + *	<	$c * d \$$	Push c
\$ + * c	>	$* d \$$	Pop c
\$ + *	>	$* d \$$	Pop *
\$ +	<	$* d \$$	Push *

# Operator Precedence Parser

$E \rightarrow E + T \mid T$

$T \rightarrow T * V \mid V$

$V \rightarrow a \mid b \mid c \mid d$

Input:  $a + b * c * d$

Stack	Relation	Input	Comment
\$	<	$a + b * c * d \$$	Push a
\$ a	>	$+ b * c * d \$$	Pop a
\$	<	$+ b * c * d \$$	Push +
\$ +	<	$b * c * d \$$	Push b
\$ + b	>	$* c * d \$$	Pop b
\$ +	<	$* c * d \$$	Push *
\$ + *	<	$c * d \$$	Push c
\$ + * c	>	$* d \$$	Pop c
\$ + *	>	$* d \$$	Pop *
\$ +	<	$* d \$$	Push *
\$ + *	<	$d \$$	Push d

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Input:  $a + b * c * d$

Stack	Relation	Input	Comment
\$	<	$a + b * c * d \$$	Push a
\$ a	>	$+ b * c * d \$$	Pop a
\$	<	$+ b * c * d \$$	Push +
\$ +	<	$b * c * d \$$	Push b
\$ + b	>	$* c * d \$$	Pop b
\$ +	<	$* c * d \$$	Push *
\$ + *	<	$c * d \$$	Push c
\$ + * c	>	$* d \$$	Pop c
\$ + *	>	$* d \$$	Pop *
\$ +	<	$* d \$$	Push *
\$ + *	<	$d \$$	Push d
\$ + * d	>	$\$$	Pop d

# Operator Precedence Parser

$E \rightarrow E + T \mid T$

$T \rightarrow T * V \mid V$

$V \rightarrow a \mid b \mid c \mid d$

Input:  $a + b * c * d$

Stack	Relation	Input	Comment
\$	<	$a + b * c * d \$$	Push a
\$ a	>	$+ b * c * d \$$	Pop a
\$	<	$+ b * c * d \$$	Push +
\$ +	<	$b * c * d \$$	Push b
\$ + b	>	$* c * d \$$	Pop b
\$ +	<	$* c * d \$$	Push *
\$ + *	<	$c * d \$$	Push c
\$ + * c	>	$* d \$$	Pop c
\$ + *	>	$* d \$$	Pop *
\$ +	<	$* d \$$	Push *
\$ + *	<	$d \$$	Push d
\$ + * d	>	$\$$	Pop d
\$ + *	>	$\$$	Pop *

$E \rightarrow E + T \mid T$		$T \rightarrow T * V \mid V$	$V \rightarrow a \mid b \mid c \mid d$	Input: $a + b * c * d$
Stack	Relation	Input		Comment
\$	<	$a + b * c * d$ \$		Push a
\$ a	>	$+ b * c * d$ \$		Pop a
\$	<	$+ b * c * d$ \$		Push +
\$ +	<	$b * c * d$ \$		Push b
\$ + b	>	$* c * d$ \$		Pop b
\$ +	<	$* c * d$ \$		Push *
\$ + *	<	$c * d$ \$		Push c
\$ + * c	>	$* d$ \$		Pop c
\$ + *	>	$* d$ \$		Pop *
\$ +	<	$* d$ \$		Push *
\$ + *	<	$d$ \$		Push d
\$ + * d	>	\$		Pop d
\$ + *	>	\$		Pop *
\$ +	>	\$		Pop +



$E \rightarrow E + T \mid T$		$T \rightarrow T * V \mid V$	$V \rightarrow a \mid b \mid c \mid d$	Input: $a + b * c * d$
Stack	Relation	Input		Comment
\$	<	$a + b * c * d$ \$		Push a
\$ a	>	$+ b * c * d$ \$		Pop a
\$	<	$+ b * c * d$ \$		Push +
\$ +	<	$b * c * d$ \$		Push b
\$ + b	>	$* c * d$ \$		Pop b
\$ +	<	$* c * d$ \$		Push *
\$ + *	<	$c * d$ \$		Push c
\$ + * c	>	$* d$ \$		Pop c
\$ + *	>	$* d$ \$		Pop *
\$ +	<	$* d$ \$		Push *
\$ + *	<	$d$ \$		Push d
\$ + * d	>	\$		Pop d
\$ + *	>	\$		Pop *
\$ +	>	\$		Pop +
\$	Accept	\$		Accept

$E \rightarrow E + T \mid T$

$T \rightarrow T * V \mid V$

$V \rightarrow a \mid b \mid c \mid d$

Input:  $a + b * c * d$

Comment
Push a
Pop a
Push +
Push b
Pop b
Push *
Push c
Pop c
Pop *
Push *
Push d
Pop d
Pop *
Pop +
Accept

$E \rightarrow E + T \mid T$  $T \rightarrow T * V \mid V$  $V \rightarrow a \mid b \mid c \mid d$ Input:  $a + b * c * d$ **Comment**

Pop a (1)

Pop b (2)

Pop c (3)

Pop \* (4)

Pop d (5)

Pop \* (6)

Pop + (7)

Accept

Sequence to be processed:

 $a, b, c, *, d, *, +$

$E \rightarrow E + T \mid T$

$T \rightarrow T * V \mid V$

$V \rightarrow a \mid b \mid c \mid d$

Input:  $a + b * c * d$

Sequence to be processed:

$a, b, c, *, d, *, +$

Stack
c
b
a
\$

$E \rightarrow E + T \mid T$

$T \rightarrow T * V \mid V$

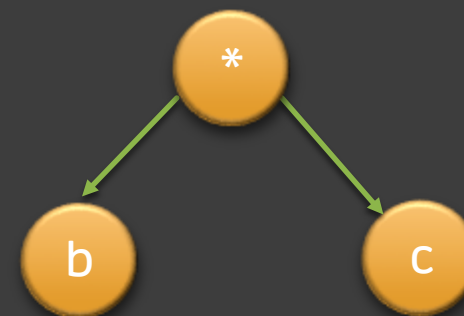
$V \rightarrow a \mid b \mid c \mid d$

Input:  $a + b * c * d$

Sequence to be processed:

$a, b, c, *, d, *, +$

Stack
a
\$



Sub Tree 1

$E \rightarrow E + T \mid T$

$T \rightarrow T * V \mid V$

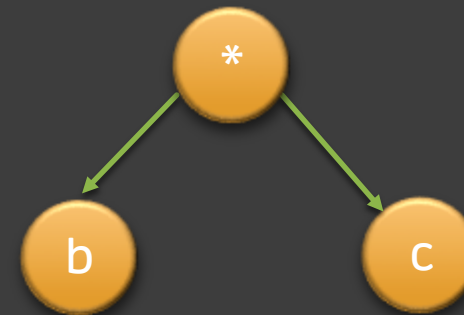
$V \rightarrow a \mid b \mid c \mid d$

Input:  $a + b * c * d$

Sequence to be processed:

$a, b, c, *, d, *, +$

Stack
Sub Tree 1
a
\$



Sub Tree 1

$E \rightarrow E + T \mid T$  $T \rightarrow T * V \mid V$  $V \rightarrow a \mid b \mid c \mid d$ Input:  $a + b * c * d$ 

Sequence to be processed:

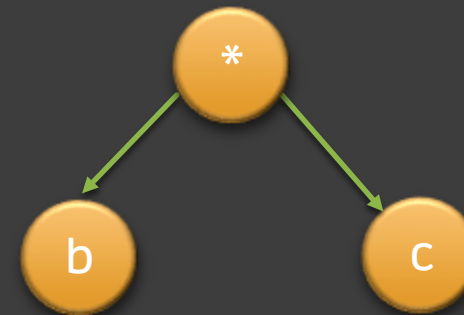
 $a, b, c, *, d, *, +$ **Stack**

d

Sub Tree 1

a

\$



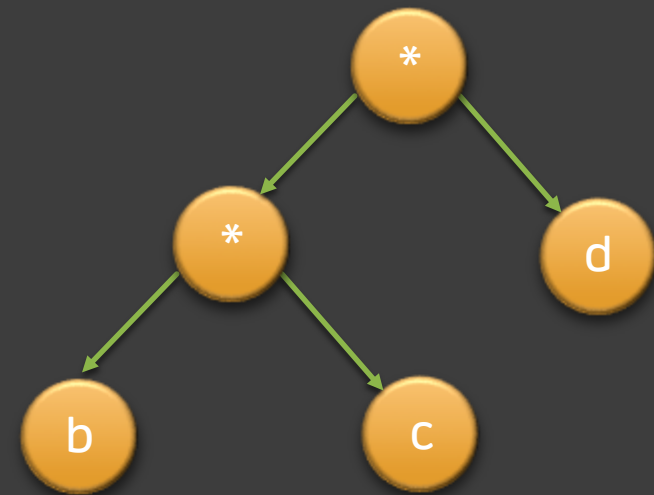
Sub Tree 1

$E \rightarrow E + T \mid T$  $T \rightarrow T * V \mid V$  $V \rightarrow a \mid b \mid c \mid d$ Input:  $a + b * c * d$ 

Sequence to be processed:

 $a, b, c, *, d, *, +$ 

Stack
a
\$



Sub Tree 2



$E \rightarrow E + T \mid T$

$T \rightarrow T * V \mid V$

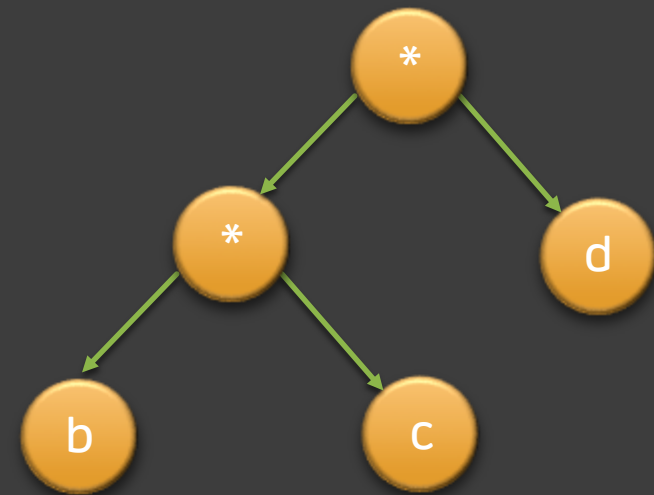
$V \rightarrow a \mid b \mid c \mid d$

Input:  $a + b * c * d$

Sequence to be processed:

$a, b, c, *, d, *, +$

Stack
Sub Tree 2
a
\$



Sub Tree 2

$E \rightarrow E + T \mid T$

$T \rightarrow T * V \mid V$

$V \rightarrow a \mid b \mid c \mid d$

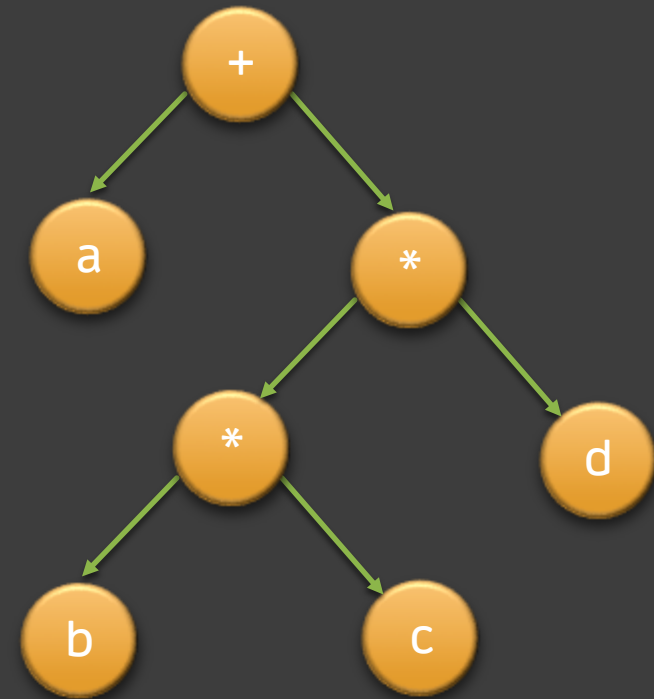
Input:  $a + b * c * d$

Sequence to be processed:

$a, b, c, *, d, *, +$

Stack

\$



Sub Tree 3

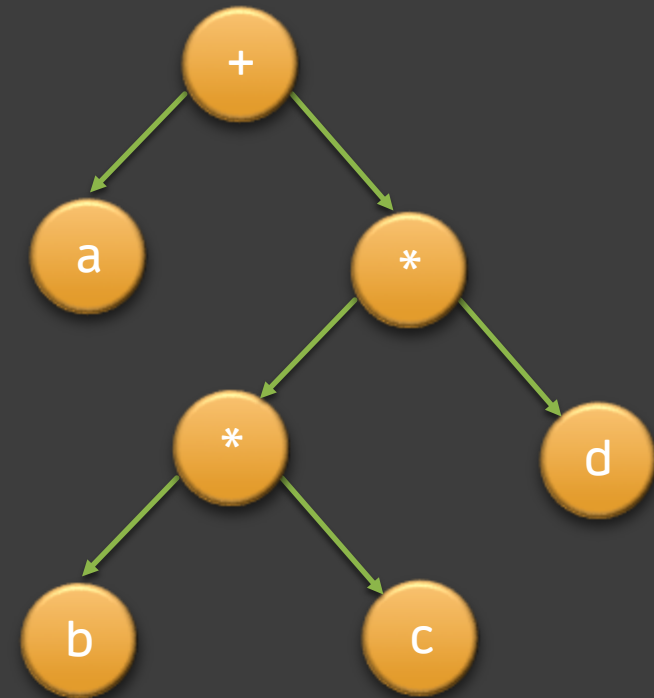
$E \rightarrow E + T \mid T$  $T \rightarrow T * V \mid V$  $V \rightarrow a \mid b \mid c \mid d$ Input:  $a + b * c * d$ 

Sequence to be processed:

 $a, b, c, *, d, *, +$ **Stack**

Sub Tree 3

\$



Sub Tree 3

$E \rightarrow E + T \mid T$

$T \rightarrow T * V \mid V$

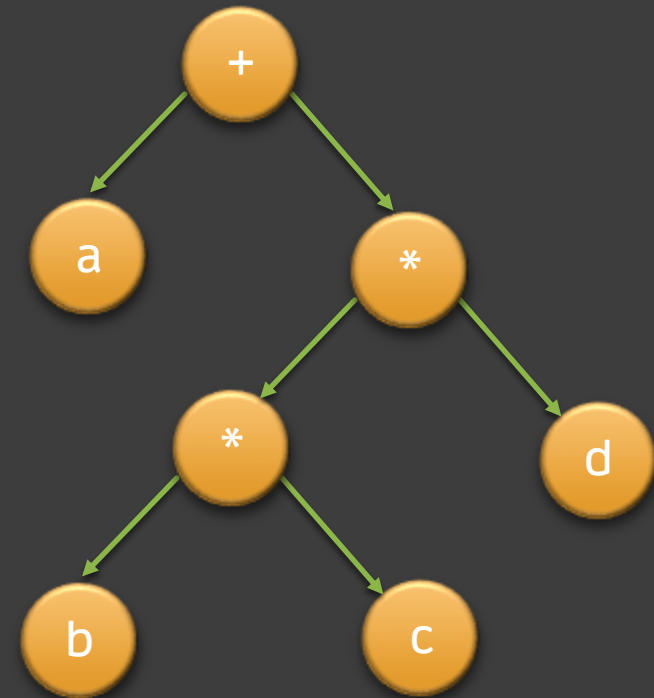
$V \rightarrow a \mid b \mid c \mid d$

Input:  $a + b * c * d$

Sequence to be processed:

$a, b, c, *, d, *, +$

Stack
\$



Final Tree

$E \rightarrow E + T \mid T$  $T \rightarrow T * V \mid V$  $V \rightarrow a \mid b \mid c \mid d$ Input:  $a + b * c * d$ **Comment**

Pop a (7)

Pop b (6)

Pop c (5)

Pop \* (4)

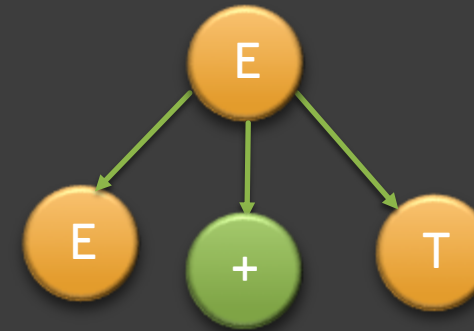
Pop d (3)

Pop \* (2)

Pop + (1) :  $E \rightarrow E + T$ 

Accept

Construction of Entire Tree:



$$E \rightarrow E + T \mid T$$
$$T \rightarrow T * V \mid V$$
$$V \rightarrow a \mid b \mid c \mid d$$

Input:  $a + b * c * d$

## Comment

Pop a (7)

Pop b (6)

Pop c (5)

Pop \* (4)

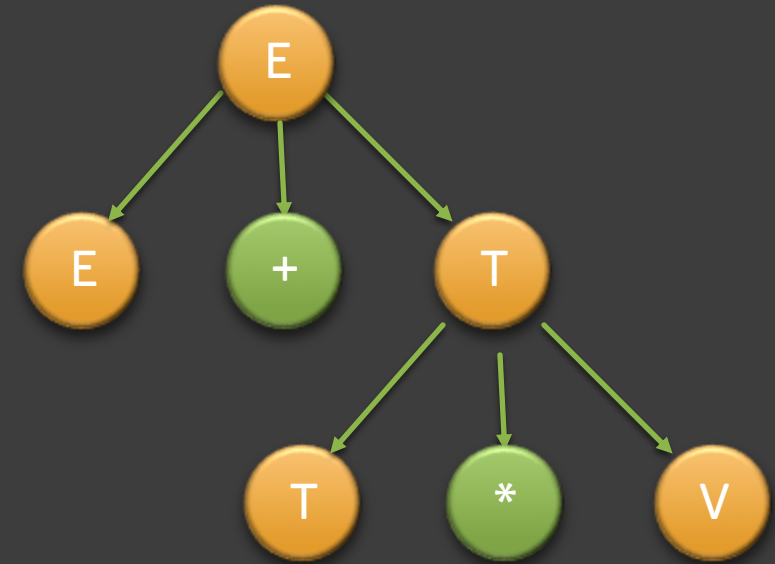
## Pop d (3)

$$\text{Pop}^*(2) : T \rightarrow T^*V$$

Pop + (1) :  $E \rightarrow E + T$

Accept

## Construction of Entire Tree:



$E \rightarrow E + T \mid T$  $T \rightarrow T * V \mid V$  $V \rightarrow a \mid b \mid c \mid d$ Input:  $a + b * c * d$ **Comment**

Pop a (7)

Pop b (6)

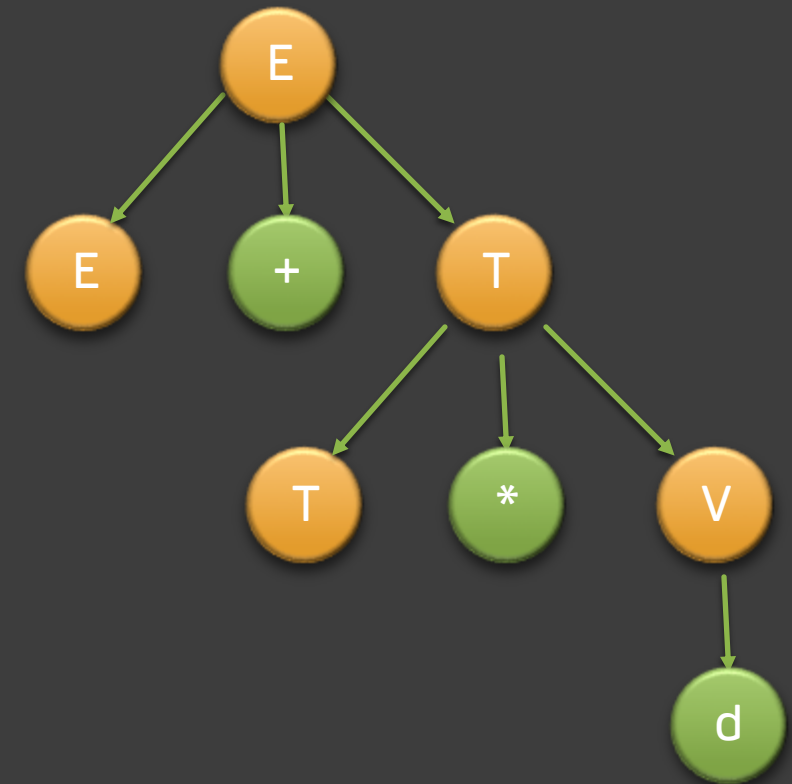
Pop c (5)

Pop \* (4)

Pop d (3) :  $V \rightarrow d$ Pop \* (2) :  $T \rightarrow T * V$ Pop + (1) :  $E \rightarrow E + T$ 

Accept

Construction of Entire Tree:



$E \rightarrow E + T \mid T$  $T \rightarrow T * V \mid V$  $V \rightarrow a \mid b \mid c \mid d$ Input:  $a + b * c * d$ **Comment**

Pop a (7)

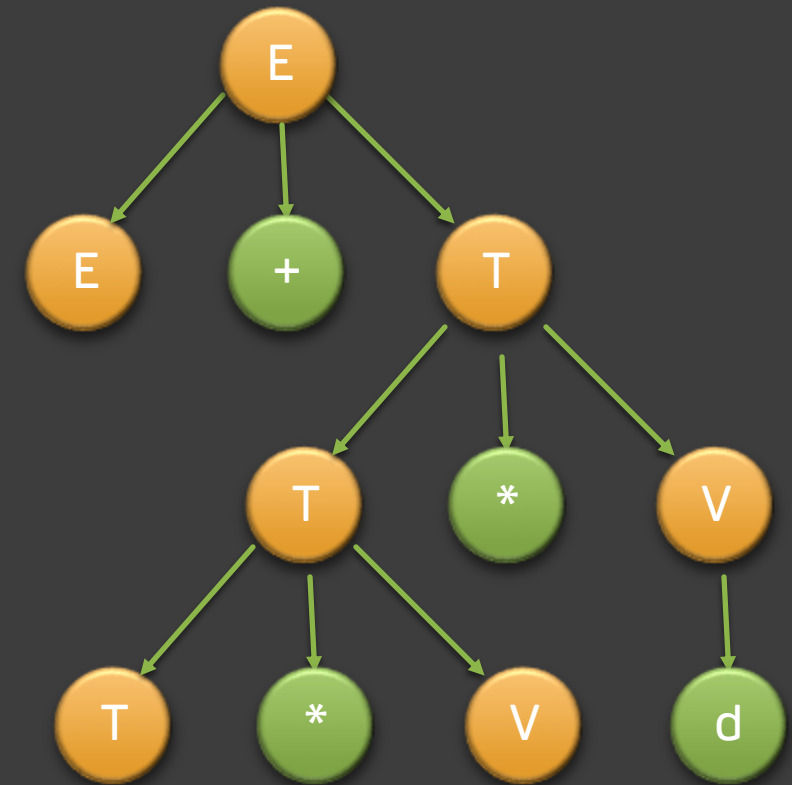
Pop b (6)

Pop c (5)

Pop \* (4) :  $T \rightarrow T * V$ Pop d (3) :  $V \rightarrow d$ Pop \* (2) :  $T \rightarrow T * V$ Pop + (1) :  $E \rightarrow E + T$ 

Accept

Construction of Entire Tree:





$E \rightarrow E + T \mid T$  $T \rightarrow T * V \mid V$  $V \rightarrow a \mid b \mid c \mid d$ Input:  $a + b * c * d$ **Comment**

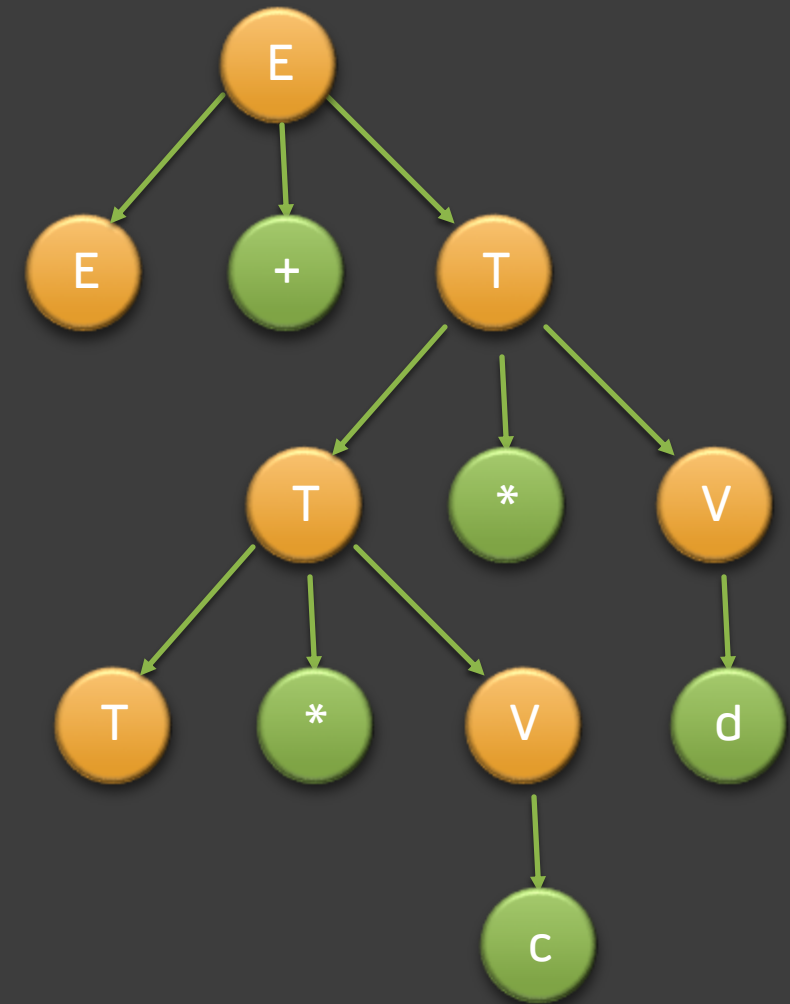
Pop a (7)

Pop b (6)

Pop c (5) :  $V \rightarrow c$ Pop \* (4) :  $T \rightarrow T * V$ Pop d (3) :  $V \rightarrow d$ Pop \* (2) :  $T \rightarrow T * V$ Pop + (1) :  $E \rightarrow E + T$ 

Accept

Construction of Entire Tree:



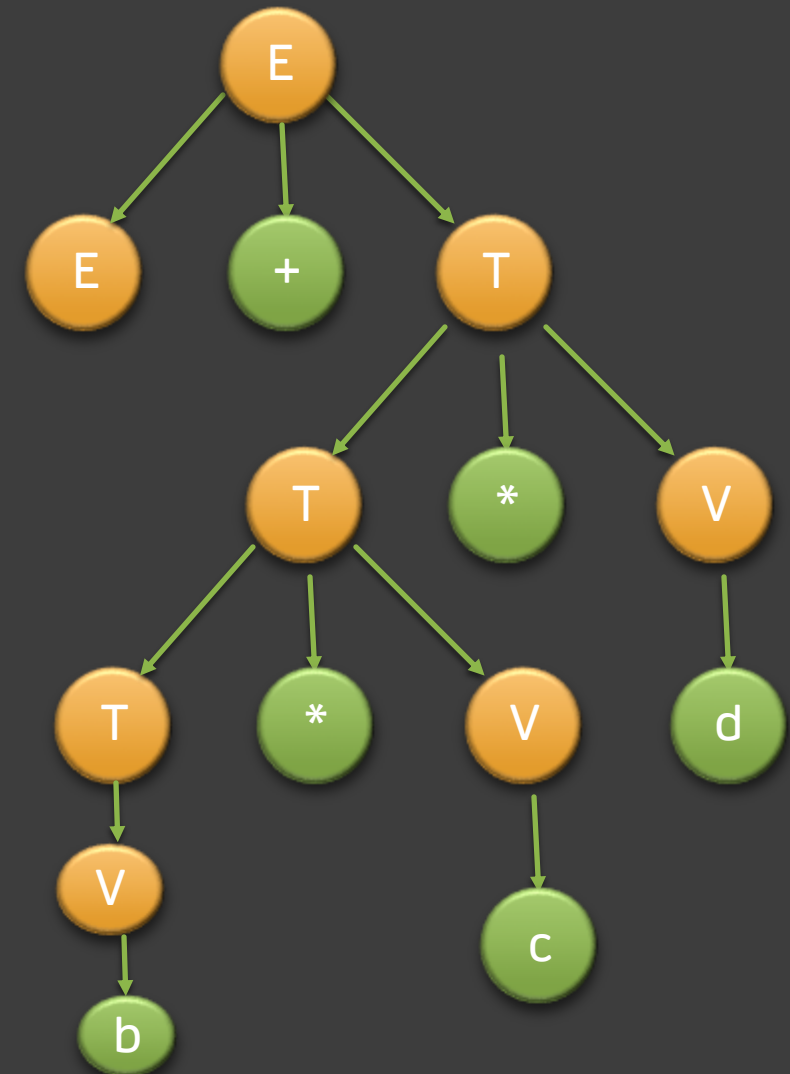
$E \rightarrow E + T \mid T$  $T \rightarrow T * V \mid V$  $V \rightarrow a \mid b \mid c \mid d$ Input:  $a + b * c * d$ **Comment**

Pop a (7)

Pop b (6) :  $T \rightarrow V, V \rightarrow b$ Pop c (5) :  $V \rightarrow c$ Pop \* (4) :  $T \rightarrow T * V$ Pop d (3) :  $V \rightarrow d$ Pop \* (2) :  $T \rightarrow T * V$ Pop + (1) :  $E \rightarrow E + T$ 

Accept

Construction of Entire Tree:



$E \rightarrow E + T \mid T$  $T \rightarrow T * V \mid V$  $V \rightarrow a \mid b \mid c \mid d$ Input:  $a + b * c * d$ **Comment**Pop a (7) :  $E \rightarrow T, T \rightarrow V,$   
 $V \rightarrow a$ Pop b (6) :  $T \rightarrow V, V \rightarrow b$ Pop c (5) :  $V \rightarrow c$ Pop \* (4) :  $T \rightarrow T * V$ Pop d (3) :  $V \rightarrow d$ Pop \* (2) :  $T \rightarrow T * V$ Pop + (1) :  $E \rightarrow E + T$ 

Accept

Construction of Entire Tree:

