PRNG & Block cipher

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41900 – Fundamentals of Security

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Pseudorandom Number Generators

A source of random numbers are essential in many occasions:

- Session Keys
- Shuffing of Cards
- Challenges
- Nonces

Computers are inherently deterministic. As a result, true randomness is a difficult thing to come by.

Since we can't get randomness easily, we use **pseudorandom number generator** functions (PRNGs) to generate what appears to be statistically random output.

A **cryptographically secure pseudo random number generator** (CSPRNG) is a type of PRNG whose properties make it suitable for use in cryptography .

Sourcing Randomness

PRNG functions produce the same sequence of seemingly random output when provided with particular "**seed**" data.

Since a computer is deterministic it must extract randomness (entropy) from an external, truly random source. This could be something like:

- Thermal noise of hard drives
- Low-order bit fluctuations of voltage readings
- User input
- Geiger counter click timing

Randomness can actually be *really* hard to come by:

U.S. Patent 5,732,138
Method for seeding a pseudo-random number generator with a cryptographic hash of a digitization of a chaotic system. "**Lavarand**" by Silicon Graphics

Properties of PRNGs

Desirable properties of PRNGs include:

- Repeatability
- Statistical randomness
- Long period / cycle
- Insensitive to seeds

Properties of PRNGs

PRNGs are often broken by:

- Statistical tests that find patterns or biases in the output sequence
- Inferring the state of the internal registers from the output sequence

PRNGs are usually **critically important parts of a cryptosystem**.
They are often a single point of failure.

e.g. Android Bitcoin wallet apps vulnerable to theft

Linear Congruential Generators

An LCG generates a sequence $x_1, x_2, \ldots$ by starting with a *seed* x_0 and using the rule:

$$x_{n+1} = (ax_n + b) \bmod c$$

where a, b, and c are fixed constants.

- The *period* of the PRNG is at most c.
- **Must not** be used for security purposes – it's easily predictable.
- Only two values x_i, x_{i+1} are needed to determine a and b.
- Commonly found in libraries, e.g. the Unix `rand()` function.

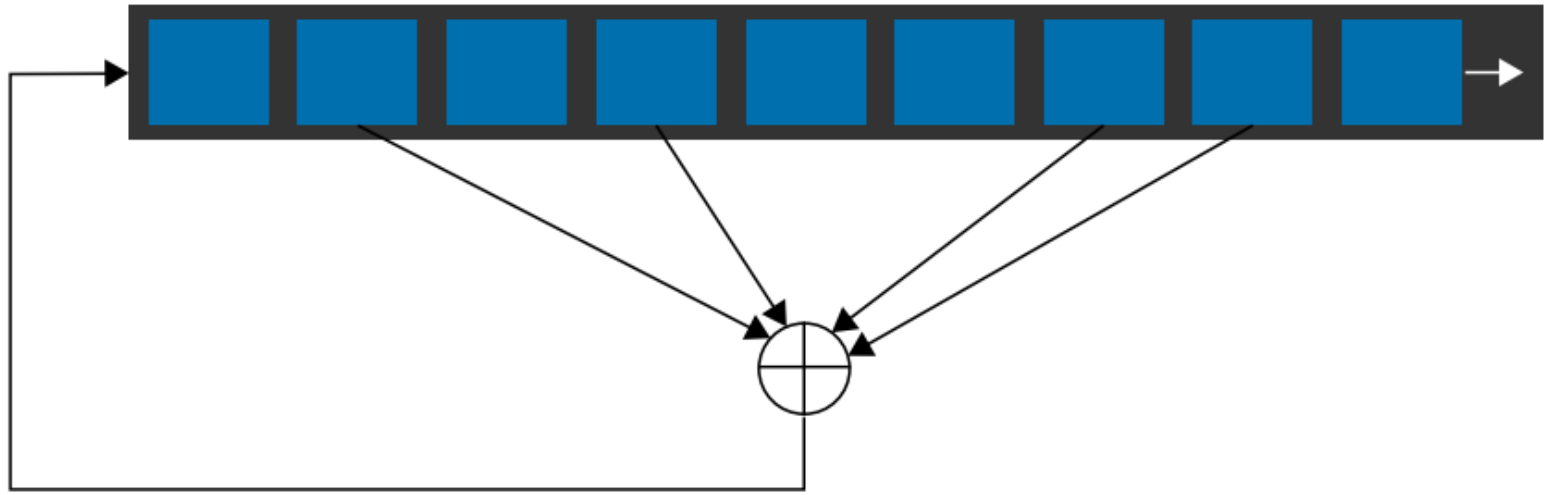
Don't Use LCGs Where Security Matters

An LCG was once used by an online casino who were so sure of their code that they published their algorithms.

…the results were as one would expect.

Linear Feedback Shift Registers

A Linear Feedback Shift Register (LFSR) simply combines the bits of a series of registers, and shifts the output onto the register.



- The *seed* is the initial value of the register.
- Easy and fast in hardware (1 bit per clock).
- **Problem:** tap configuration can be determined from 2^n output bits, where n is the length of the LFSR period.

RC4 Stream Cipher

RC4 is a stream cipher which has wide applications in cryptography.

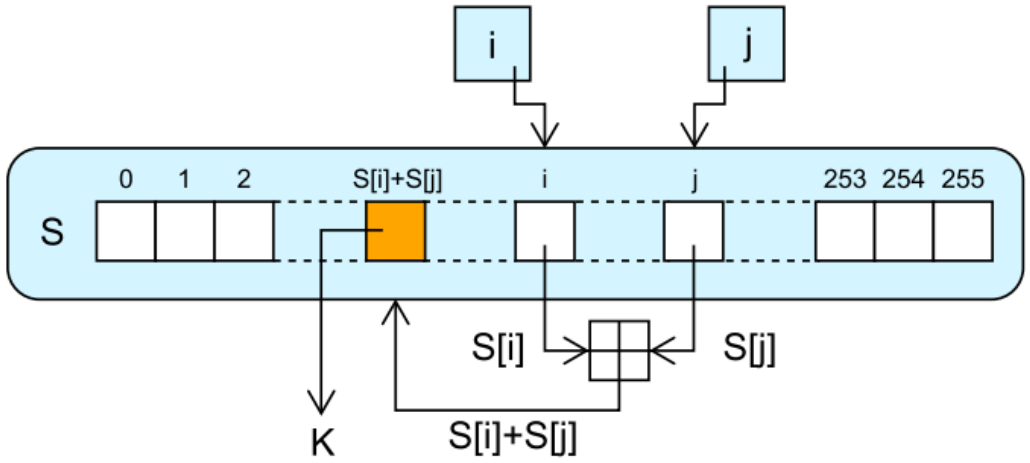
At one point, RC4 was used to encrypt > 50% of all SSL traffic.

It is the core algorithm of Wired Equivalent Privacy (WEP)

RC4 Stream Cipher

Based on permutations of a 256 byte array, the seed is the initial array value. RC4's key scheduling algorithm has known problems (e.g. WEP weakness)

```
1  i := 0
2  j := 0
3  while GeneratingOutput:
4      i := (i + 1) mod 256
5      j := (j + S[i]) mod 256
6      swap values of S[i] and S[j]
7      K := S[(S[i] + S[j]) mod 256]
8      output K
9  endwhile
```



Other PRNGs

ANSI X9.17

Based on 3DES

DSA PRNG

Based on SHA or DES

RSAREF PRNG

Based on MD5 hashing and addition modulo 2128

Tips for using PRNGs

- Be extremely careful with PRNG seeds!
- Hash PRNG inputs with a timestamp or counter
- Reseed the PRNG occasionally
- Use a hash function to protect PRNG outputs if PRNG is suspect

XOR and OTP

What is XOR?

XOR is the “exclusive or” operation: one or the other, but not both.

It is addition modulo 2 and is represented by $\oplus$.

$$a \oplus b = (a + b) \bmod 2$$

a	b	a $\oplus$ b
0	0	0
0	1	1
1	0	1
1	1	0

XOR truth table

XORing Bits

Typically we XOR bits together:

XOR Encryption

```
Plaintext:   011011000110111101101100
Keystream:   011000110110000101110100
=====================================
Ciphertext:  000011110000111000011000
```

Often the plaintext will be XOR'd with a key stream to produce ciphertext.

This is effectively the same as a Vigenere cipher.

Where a XOR is addition modulo 2, a Vigenere cipher is addition modulo 26 since XOR works with bits and not letters.

Interesting XOR Properties

Something **XOR'd with itself is zero**.

$$A \oplus A \equiv 0$$

XOR is also **associative**:

$$A \oplus (B \oplus C) \equiv (A \oplus B) \oplus C$$

and **commutative**:

$$A \oplus B \equiv B \oplus A$$

XOR

XOR (addition modulo 2) is commonly used to provide security in programs. It is very weak by itself, but forms the building block of most crypto primitives.

The message m is XOR'd bitwise with a secret key:

$$c = m \oplus k$$

$$m = c \oplus k$$

XOR

XOR is effectively a Vigenere cypher and easy to break:

- Determine the key length N from index of coincidence
- Shift cyphertext by N and XOR with itself
- This removes the key ($c \oplus c' = m \oplus k \oplus m' \oplus k = m \oplus m'$)
- Results in message XOR'd with a shifted version of itself
- Language is extremely redundant
- Easy to then decrypt

Wizardry

XOR also useful for an old-school assembly / C programming trick

“How do you swap two variables, x and y, without using a third?”

$$x = x \oplus y$$

$$y = x \oplus y$$

$$x = x \oplus y$$

One Time Pad (OTP)

A **one time pad** is using a different substitution cipher for each letter of the plaintext.

Provided that:

- The secret key, k, is truly random
- The plaintext does not repeat
- The keystream does not repeat

a one time pad is **perfectly secure**.

Failure to meet any one of these requirements results in **zero security**.

A-Z mod 26
To encrypt for bits, it is simply the XOR operation, or modulo 2. **When dealing with A-Z, it is equivalent to addition modulo 26.**

One Time Pad (OTP)

The strength comes from the fact that a truly random key added to plaintext, produces a truly random ciphertext.

No amount of computing power can break a one time pad.

brute force would yield each and every possible message that length.

Core Problems: key distribution, key destruction, synchronisation.

- k must be same length as m:
  to encrypt 1GB you need a 1GB shared key.
- Used for ultra-secure, low bandwidth communications
  e.g. military satellites, Moscow-Washington phone line
- Future: Quantum Key Distribution
  secure distribution at a distance.

Perfect Secrecy

Goal of cryptography:

ciphertext reveals nothing about the plaintext.

A cipher has perfect secrecy if, for all m ∈ M, c ∈ C, the plaintext and ciphertext are statistically independent:

$$Pr[m_1 = m_2 | c_1 = c_2] = Pr[m_1 = m_2]$$

Assuming each transmitted message is equally likely, the probability that the transmitted message is m is:

$$Pr[m1 = m2] = |M|^{-1}$$

Now the probability that the transmitted message is m given that the observed ciphertext is c is:

$$\Pr[m_1 = m_2] = \frac{|\{k: E_k(m) = c, k \in K\}|}{|k|}$$

Perfect Secrecy

The key space K must be at least as large as the set of plaintexts:

$$|k| \geqslant |M|$$

For $M = C = \{0, 1\}^n$:

any cipher with perfect secrecy satisfies $|K| \geqslant 2^n$

The one time pad has perfect secrecy as: $M = C = \{0, 1\}^n$

Thus:

$$Pr[m_1 = m_2] = \frac{1}{2^n}$$

$$Pr[m_1 = m_2 | c_1 = c_2] = \frac{1}{2^n}$$

Note: we require $k \in K$ to be as long as the message, which means we need to securely communicating a key as long as the message in advance.

Breaking OTP: Two Time Pad

A **two-time pad** is **perfectly insecure**. Suppose two messages m_1, m_2 are encoded using the same key k:

$$c_1 = m_1 \oplus k$$

$$c_2 = m_2 \oplus k$$

Then the key k may be cancelled by XORing the ciphertexts:

$$c_1 \oplus c_2 = (m_1 \oplus k) \oplus (m_2 \oplus k)$$

$$= m_1 \oplus m_2 \oplus k \oplus k$$

$$= m_1 \oplus m_2$$

$m_1 \oplus m_2$ is easy to separate due to the redundancy in English and in ASCII (for example, bit 6 is set in letters but not most punctuation).

Breaking OTP: Malleability

The OTP and all stream ciphers are **highly malleable**. Suppose plaintext is a one bit vote $v \in 0, 1$

- $v = 0$ is a vote for Labor
- $v = 1$ is a vote for Liberal

Alice encrypts her vote using OTP and sends to Bob:

$$c = v \oplus k \text{ where } k \in 0, 1 \text{ is randomly chosen}$$

Mallory intercepts the ciphertext and sends with bits flipped:

$$C' = c \oplus 1$$

Bob receives c' and decrypts vote:

$$c' \oplus k = c \oplus 1 \oplus k$$
$$= v \oplus k \oplus 1 \oplus k$$

Breaking OTP: Malleability

Malleability Attack Example

A competitor is selling shares of their company by using a "secure" share trading program that encrypts a four byte integer using a four byte key k:

$$c = [b_1, b_2, b_3, b_4] \oplus [k_1, k_2, k_3, k_4]$$

If I wanted to steal a controlling share, I could make him sell a massive number of shares by flipping a high order bit that I was certain he wouldn't use

Block Cipher Modes of Operation

Cipher Modes of Operation

Once a key k is chosen and loaded into a block cipher, E_k only operates on single blocks of data.

1. Block size usually small (16 byte blocks for AES)
2. Message to be sent usually large (web page + assets ≈ 500kB)
3. Need a way to repeatedly apply the cipher with the same key to a large message.

By using different *modes of operation*, messages of an arbitrary length can be split into blocks and encrypted using a block cipher.

Each mode of operation describes how a block cipher is repeatedly applied to encrypt a message and each has certain advantages and disadvantages.

Evaluating Block Ciphers & Modes

To evaluate a cipher and a mode of operation, examine:

Key Size: Upper bound on security, but longer keys add costs (generation, storage, etc.)

Block Size: Larger is better to reduce overheads, but is more costly.

Estimated Security Level: Confidence grows the more it is analysed.

Throughput: How fast can it be encrypted/decrypted?
Can it be pre-computed? Can it be parallelised?

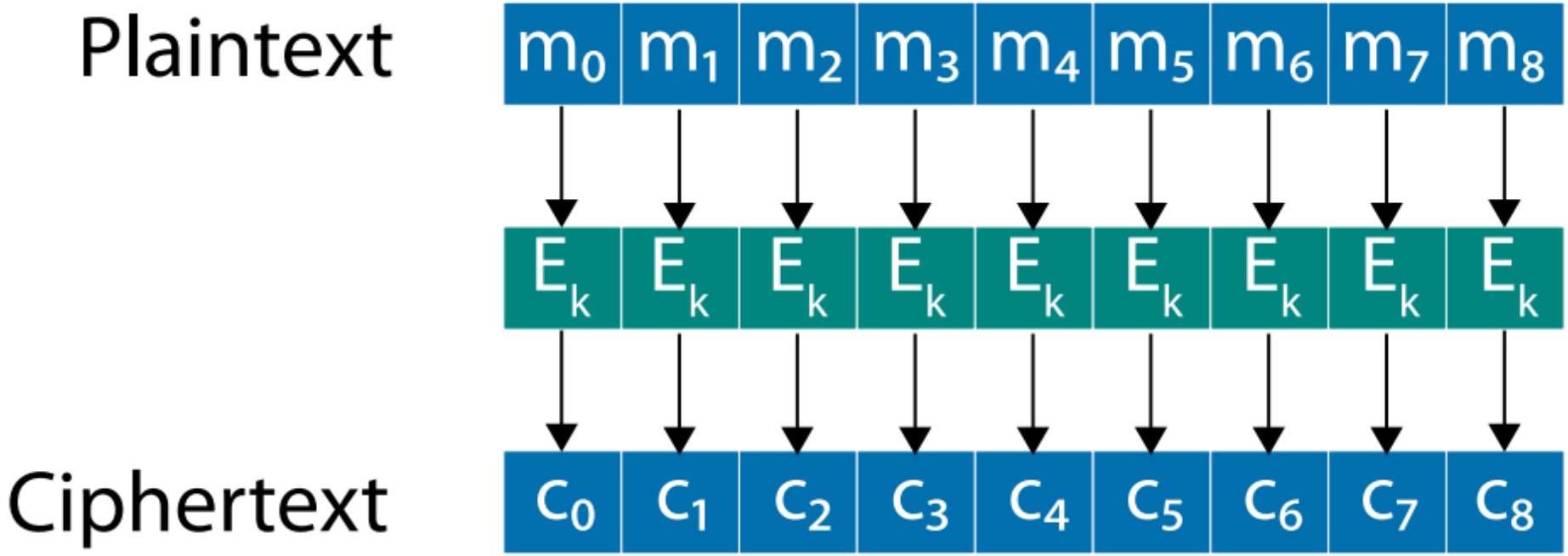
Error Propagation: What happens as a result of bit errors or bit loss?

The first two points above are relevant only to the cipher, while the last three are relevant to both the cipher and a mode of operation.

Electronic Code Book (ECB)

Electronic Code Book (ECB) encrypts each block separately.

ECB is generally an insecure and naïve implementation, it is vulnerable to a range of attacks; including dictionary and frequency attacks. *It should never be used.*



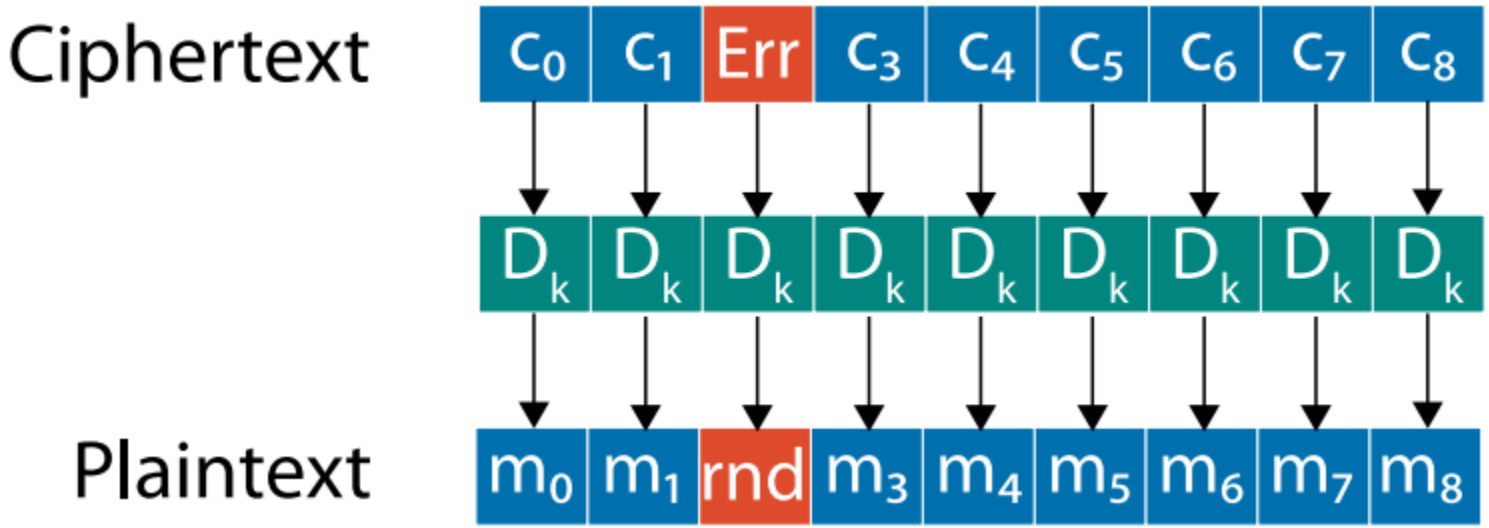
ECB Properties

Identical plaintext blocks result in identical ciphertext blocks

Since blocks are enciphered independently, a reordering of ciphertext blocks results in reordering of plaintext blocks.

ECB is thus not recommended for messages > 1 block in length.

Error propagation: Bit errors only impact the decoding of the corrupted block.

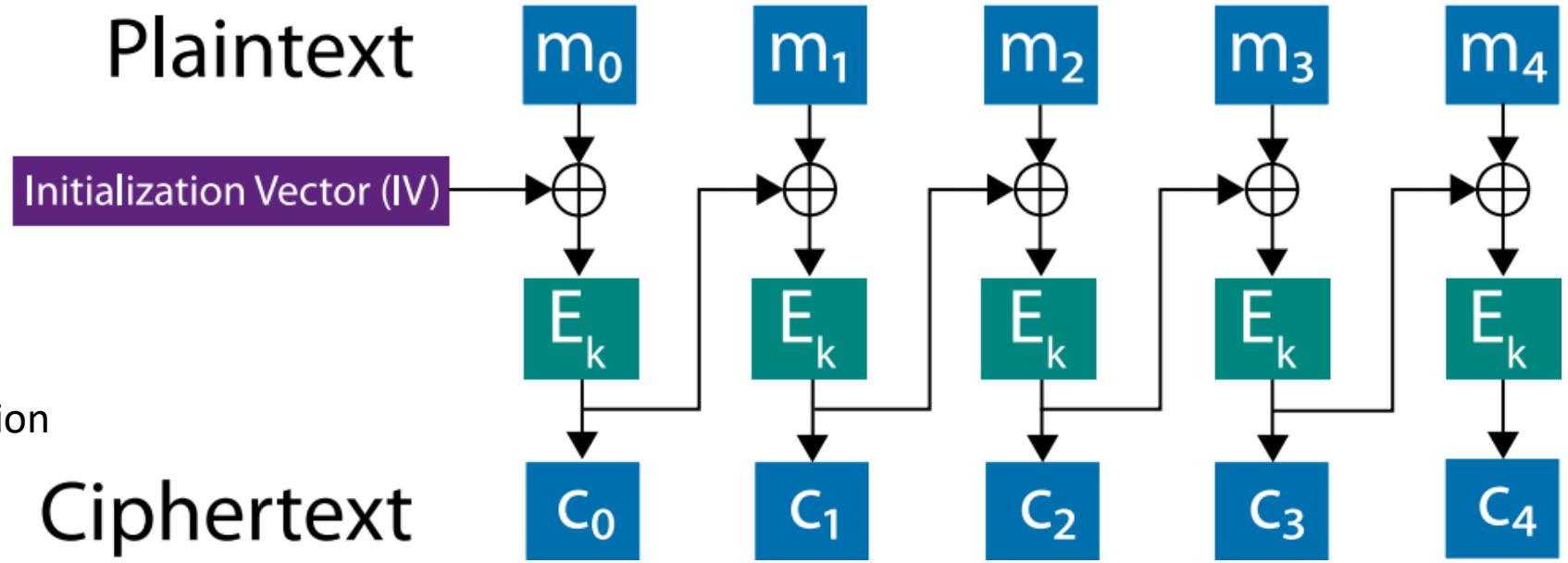


Error propagation in ECB

Cipher Block Chaining (CBC)

In **Cipher Block Chaining (CBC)** blocks are chained together using XOR.

The **Initialisation Vector (IV)** is a random value that is transmitted in the clear that ensures the same plaintext and key does not produce the same ciphertext.



CBC Mode Encryption

CBC Properties

Identical plaintexts result in identical ciphertexts when the same plaintext is enciphered using the same key and IV.

Changing at least one of [k, IV, m_0] affects this.

Rearrangement of ciphertext blocks affects decryption, as ciphertext part c_j depends on all of $[m_0, m_1, \cdots, m_j]$.

Error propagation:

Bit error in ciphertext cj affects deciphering of cj and cj+1.Recovered block m_j typically results in random bits.

Bit errors in recovered block m_{j+1} are precisely where c_j was in error.

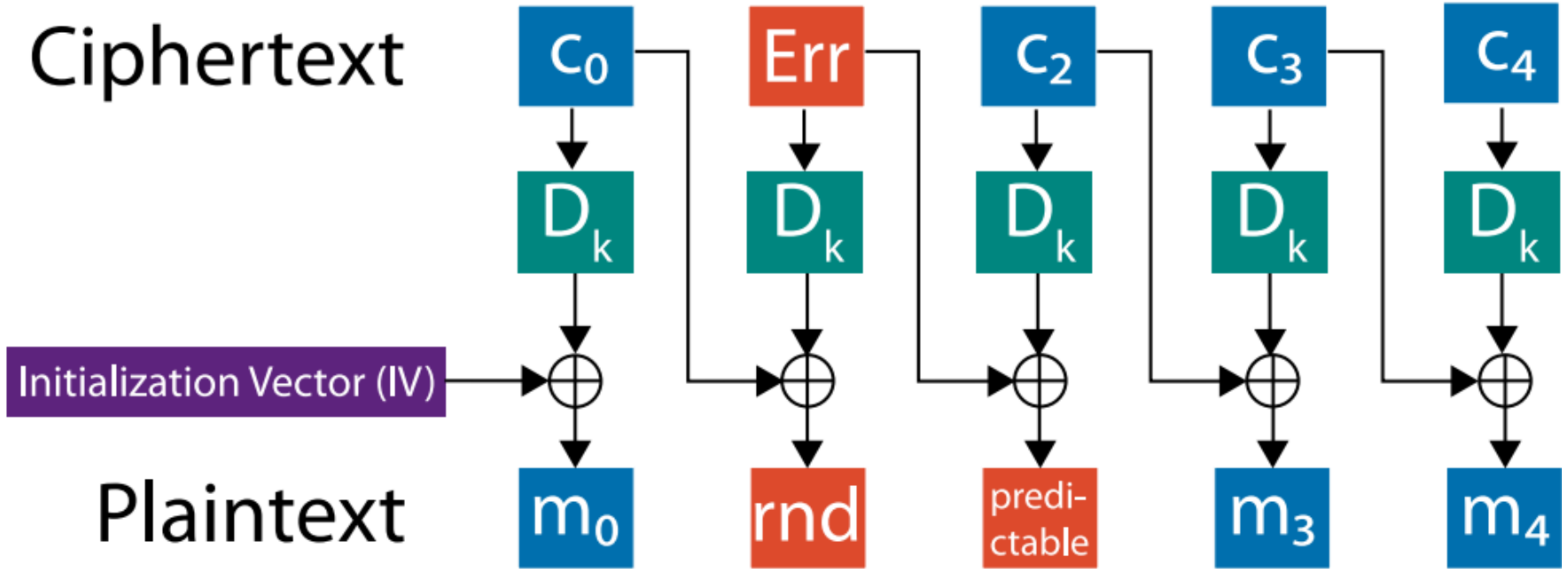
Attacker can cause predictable bit changes in m_{j+1} by altering c_j.

Bit recovery:

CBC is self-synchronising in that if a bit error occurs in c_j but not c_{j+1}, then c_{j+2} correctly decrypts to m_{j+2}.

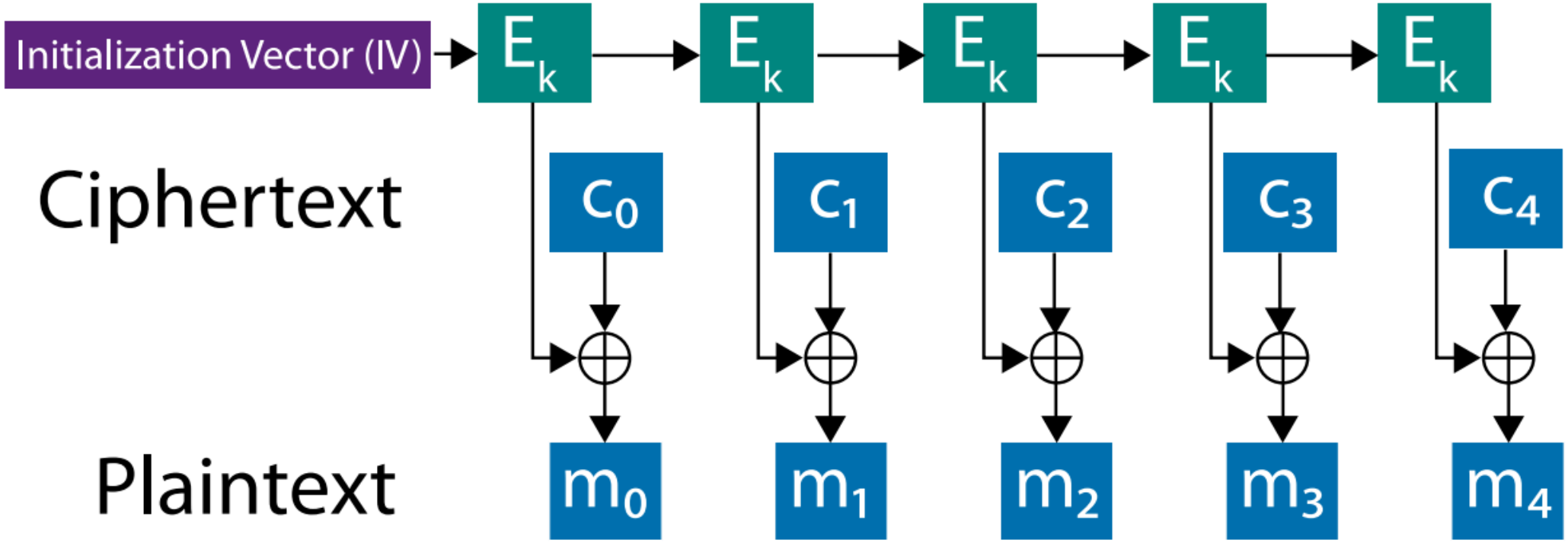
CBC Decryption

- Ciphertext errors only affect two plaintext blocks, one in a predictable way.
- Encryption must be done sequentially.
- Decryption can be random-access and is fully parallelisable.



Output Feedback Mode (OFB)

Output Feedback Mode (OFB) effectively turns a block cipher into a synchronous stream cipher.



OFB Properties

Identical plaintext results in identical ciphertext when the same plaintext is enciphered using the same key and IV.

Chaining Dependencies: (*Same as a stream cipher*) The key stream is plaintext independent.

Error propagation: (*Same as a stream cipher*) Bit errors in ciphertext blocks cause errors in the same position in the plaintext.

Error recovery: (*Same as a stream cipher*) Recovers from bit errors, but not bit loss

(misalignment of key stream)

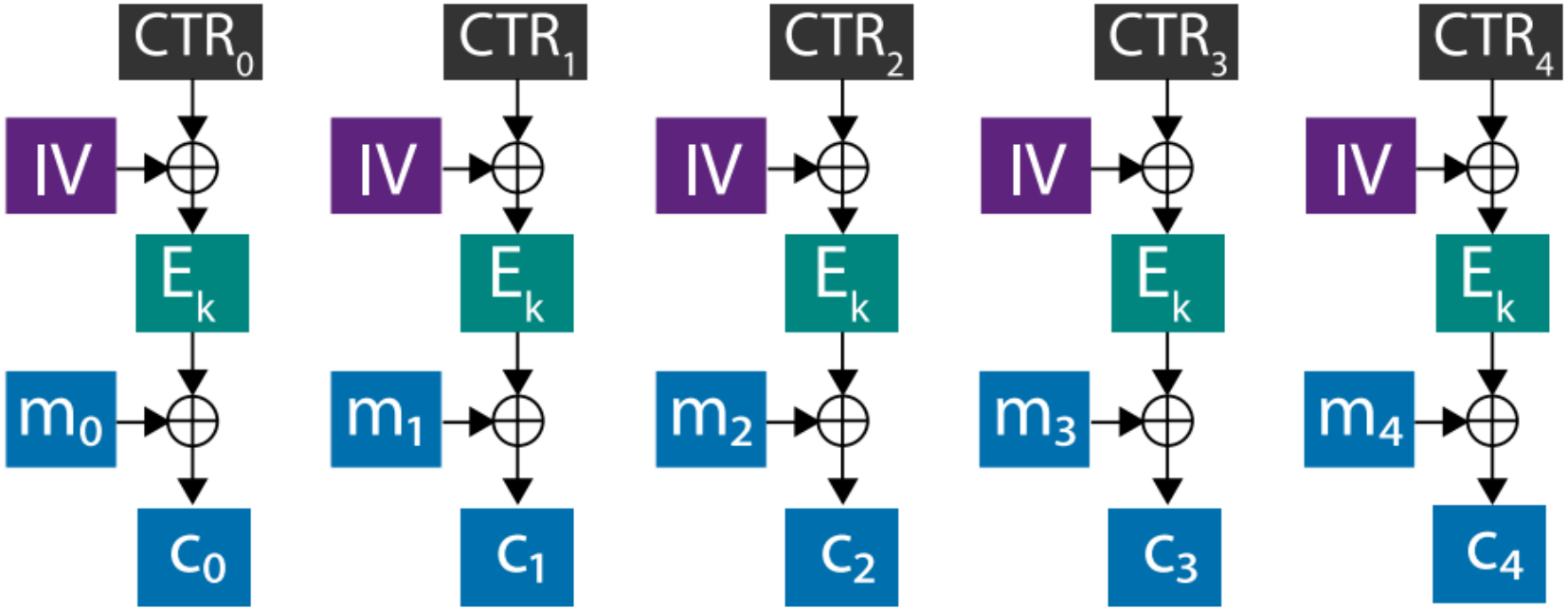
Throughput: Key stream may be calculated independently — e.g. pre-computed — before encryption/decryption become parallelisable.

IV *must* change: Otherwise it becomes a two time pad.

Counter Mode (CTR)

Counter Mode (CTR) modifes the IV for each block using a predictable counter function, turning the block cipher into a stream cipher.

The counter can be any function (e.g. a PRNG), but it is commonly just an incrementing integer.



CTR Mode Encryption

CTR Properties

Identical plaintext results in identical ciphertext when the same plaintext is enciphered using the same key and IV.

Chaining Dependencies: (*Same as a stream cipher*) The key stream is plaintext independent.

Error propagation: (*Same as a stream cipher*) Bit errors in ciphertext blocks cause errors in the same position in the plaintext.

Error recovery: (*Same as a stream cipher*) Recovers from bit errors, but not bit loss

(misalignment of key stream)

Throughput: Both encryption and decryption can be randomly accessed and/or parallelised: the best we could hope for.

IV *must* change: Otherwise it becomes a two time pad.

OFB and CTR share a lot of these properties, because they both make the block cipher act as a stream cipher.

GCM Mode

Galois/Counter Mode (GCM) mode is not strictly a cipher mode of operation since it also provides *authentication*: assurance the ciphertext has not been tampered with.

- An extension of CTR mode.
- While encryption happens, the ciphertext blocks are combined into something like a MAC.
- Unlike HMAC, is parallelisable (you can't combine two HMACs into one larger one).
- Used for low-latency, high-throughput dedicated hardware applications (network packets).

GCM mode is an example of *authenticated encryption*.