Dissertation Write Up V1

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Introduction

Background and Motivatoin

In 2021 many of Europe's most prominent and successful Association Football teams engaged in discussions that led to the development of the 'European Super League' (ESL). The Super League was spearheaded by it's 12 founding members from 4 of Europe's top 5 leagues including the biggest names in football worth billions of dollars on their own such as Manchester United, Real Madrid, Barcelona. The competition proposed to create a new international standard of Football with aim of providing a more sustainable and lucrative business model for the associated clubs as a direct competitor to the existing UEFA Champions League. The Super League was conceptualised as a closed competition; implying that the founding members would be guaranteed permanent membership and exemption from relegation regardless of their performance in a season. With 20 teams in total, this would mean only the 12 founding members were guaranteed to enjoy the expected revenues from the competition, while the 8 others faced a constant threat of relegation. Another aim of the league was to maintain a more competitive balance, in the sense that the league would only contain games of the highest quality from Europe's Best. The founders believed this would retain a larger global audience, thus increasing the value of broadcasting rights, sponsorship and merchandise sales. The organisers also proposed the reduction of financial disparities with the rest of European football by proposing to share a proportion of the revenue generated with other clubs and investing in grassroots football development.

However, following the announcement April 18th 2021, the organizers and teams faced vast backlash among the various stakeholders of the football community, including fans, football governing bodies, domestic leagues, smaller clubs, and even governments.

The proposed Super League marked a significant departure from the traditional format of European Football, causing concern from European and Global football's governing bodies; UEFA and FIFA, as well as governments including the UK government, with prime minister at the time Boris Johnson threatening to consider what he labelled as a "legislative bomb" . Critics argued the closed format would lead to the erosion of sporting merit and integrity, many fearing the superior ESL would damage the structure of Europe's domestic leagues, given that the resources of the founding clubs would shift to the new competition, devaluing

the domestic leagues as a result and thus widening the financial gap. This concentration of wealth and power was argued to making it more difficult for smaller clubs to compete hence threatening their long term survival. While the reader may not believe this to be of vital concern as other matters, it must be considered that the competition alone was expected to generate Four billion euros , and Europe's top 5 leagues already generating billions of euros alone, notably the Premier League with €5.492 billion. Thus there are billions of euros at stake with the proposition of this new competition, with many subsidiary industries such as sports betting, analytics, and sponsorships that could be affected. So, it is of our interest to investigate what may potentially have occurred as a result of this possible intervention in the world of football, with Juventus, Barcelona, and Real Madrid still yet to withdraw from the competition's establishment, there is still a possibility for the European Super League's fruition into reality. With the 12 founding members leaving their respective leagues, and other clubs such as Paris Saint Germain (Ligue 1, France) potentially leaving theirs, one might consider how this would effect the state of the domestic league should they not participate, who would take crown in each of the top 5 leagues?

Research Question

This project aims to explore and analyse the predicted performance of European teams in their respective domestic leagues by modelling and analyzing a range of factors yet to be discussed. In addition, we will use the resulting methodology and analysis to estimate and discuss the hypothetical outcomes in a scenario where 12 clubs have been excluded from participating in their leagues. This exclusion will have a significant impact on the outcome of the leagues, and in reality would have various potential ripple effects on the teams, competitions, and industries as a whole. By conducting a thorough analysis and exploring the potential outcomes, we hope to provide valuable insights into the current state of European football and the potential impact of excluding certain clubs from participation. In doing this we hope to explore statistical techniques that can be used to model and predict the outcome of match results. We will look at the recent scope of literature on statistical techniques that have been used in the prediction of match results.

Introduction to statistical methods: A Literature Review

A Brief History

The prediction of football match results has been of interest to researchers and statisticians for decades. The use of binomial and negative binomial models were explored as early as 1968 by Reep and Benjamin in their paper "Skill and Chance in Association Football", before the Poisson distribution became the prominent method of modelling these relevant quantities. Many cite and credit Maher (1982) who successfully demonstrated the uses of Poisson Modelling,later addressing the relation between teams in a given match through Bivariate-Poisson modelling. One of the most influential studies in the field came from the work of Dixon and Coles (1997) who proposed a far more effective Poisson Model framework. In their study they introduced further improvements with time-weighting factors and correction terms for low-scoring matches. Their method demonstrated improved accuracy over previous models and has been widely reference in subsequent research.

Bayesian framework:

Bayesian frameworks have gained popularity over recent years due to their ability to incorporate prior knowledge and ability to update probabilities based on newer data, providing promising results with their predictive capabilities. Extensions of the Bayesian framework include the Bayesian generalised linera model of Rue and Salvesen (2000), which has been widely referred to in subsequent research. The framework was later improved by the promising Hierarchical Bayesian Models of Gianluca Baio in his 2010 and 2018 papers (Baio et al.,2010) (Baio et al., 2018). Baio's contributions have been significant to the field of statistical methods in football match modelling. The 2010 paper introduced a multilevel structure in their modelling to estimate team-specific parameters nested within a mixed effects model, applied in the context of the Italian Serie A league. They also later specified a more complex mixture model that aimed to overcome the issue of over shrinkage produced by the Bayesian multilevel model, in order to provide a better fit to the data of previous season. Baio later made further improvements to this area of the Bayesian framework in his 2018 paper incorporating even more complex and effective mixture models accounting for the data available at the time.

His works have been widely cited in later literature, including comprehensive reviews of Bayesian statistical methods in football (Santos-Fernandez et al., 2019) and studies of more modern techniques (Hubáček et al., 2019). The Bayesian Hierarchical Model demonstrates advantages by naturally accounting for relations between variables through the assumption that they come from a common distribution. Overall this approach enables the effective and comprehensive account of team performance while accounting for the hierarchical structure of football league data. More recent research using Bayesian frameworks include that of Razali et al (2017) exploring the use of machine learning methods, in this case Bayesian Networks (BNs) to model, predict, and validate match results for the English premier League. Other recent applications have included Naive Bayes and Tree Augmented Naive Bayes Models in Rahmanet et al. (2018),

Other methods

Although Bayesian methods have become more prominent in the field of predictig football results in recent years, other methods have also been observed. In the past Bradley-Terry models (Bradey and Terry 1952) with comparison modes pairing teams to determine the outcome of a game (Kuk 1995) have been used to estimate the probabiliies of winning, drawing, or losing a match. Related studies in the field include Godin et al's (2014) leveraging of contextual information via "Twitter Microposts" and machine learning techniques in order to comprehensively beat expert and bookmaker predictions, a dynamic approach wit a different end goal to the purpose of our study. Furthermore, other more relevant and modern research cover Machine Learning Methods such as Random Forests (Groll et al. , 2018), Gradient Boosting and Linear Support Vector Machines, notably by Baboota et al. (2018) who's work has been well cited as developments in the use of Artificial Intelligence and Machine Learning methods continue to develop in this field.

As we can observe, the aims of the papers discussed in the literature review vary. Some aim to simply model the outcome of the came like in Fahrmeir and Tutz (1994) using models of paired data with time varying features. Others investigate outcomes by predicting goals scored as seen in Dixon and Coles 1997 and Baio et al. (2010, 2018) while others address other characteristics used to predict outcomes, such as passing movements and shots per game see Reep et al (1968).

Introduction to Bayesian Hierarchical modelling

For the purpose of this study regarding statistical modelling in the prediction of football outcomes, we will be predicting outcomes of the top five European domestic leagues utilising the Bayesian Hierarchical Model (BHM).

(Need to expand on why?) By adding many degrees of hierarchy, BHM enables the representation of complicated relationships and structures within the data. This strategy is based on the Bayesian probability theory discussed above, which offers a methodical manner to revise beliefs or probabilities in light of new information by applying the Bayes' theorem.

The framework used in BHM adopts a

From Bayesian Inference theory we know:

$$P(\theta, y) = P(\theta)P(y \mid \theta)$$

is the joint probability distributions for θ and y, written as the product of the distributions: - the prior distribution $P(\theta)$, which estimates the parameter θ before data is observed - the sampling distribution $P(y \mid \theta)$, referred to as the Likelihood; the distribution of observed data conditional on θ .

Using Bayes Theorem:

$$P(\theta \mid y) = \frac{P(\theta, y)}{P(y)} = \frac{P(y \mid \theta)P(\theta)}{P(y)}$$

Where $P(\theta \mid y)$ represents the posterior distribution of the parameter θ given observed data .$P(y)$ reflects the marginal likelihood, or model evidence, that is derived as the integral of the joint probability distribution of θ and y:

$$P(y) = \int P(\theta)P(y \mid \theta)d\theta$$

In scenarios where the marginal likelihood is not easily obtained, one may express the posterior distribution

as:

$$P(\theta \mid y) \propto P(y \mid \theta)P(\theta)$$

Hierarchical Models:

The extension of the Bayesian framework into hierarchical modelling incorporates two added features used in deriving the posterior distribution: - Hyperparameters: set of parameters that are used to determine prior distributions of other parameters uesd in the model often referred to as lower level parameters. The hierarchical nature of BHM allows for multiple levels, each level having its own distribution. Parameters at the higher levels are used to determine properties of the priors for lower-levels, which are the hyperparameters. - Hyperpriors: these are the prior distributions on the hyperparameters, used to express uncertainty in a hyperparameter.

The framework of the BHM: We consider the structure of a two level Bayesian Hierarchical Model let y be a set of obesrvations $y_1, ..., y_n$ from random variables $Y_1, ..., Y_n$ and θ be the set of parameters from each Y_i; $\theta_1, ..., \theta_n$ from a common population with distribution determined by hyperparameter ϕ

The Likelihood above is $P(y \mid \theta, \phi)$ with $P(\theta, \phi)$ as its prior distribution.

$$\text{Stage 1: } y \mid \theta, \phi \sim P(y \mid \theta, \phi)$$

The prior can be expressed as $P(\theta, \phi) = P(\theta \mid \phi)P(\phi)$ using the definition of conditional probability

$$\text{Stage 2: } \theta \mid \phi \sim P(\theta \mid \phi)$$

The next stage being the hyperparameter: ϕ with prior distribution $P(\phi)$, referred to as the hyperprior.

$$\text{Stage 3: } \phi \sim P(\phi)$$

Using this structure we obtain the posterior distribution using bayes theorem, expressed as:

$$P(\phi, \theta \mid y) \propto P(y \mid \theta, \phi)P(\theta, \phi) = P(y \mid \theta)P(\theta \mid \phi)P(\phi)$$

Using this we can obtain probabilities from the posterior distribution.

The R-INLA package:

In surrounding literature regarding BHM theory, various languages are used for modelling including R, Jags, Python, Stan (Hilbe et al., 2017), with popular related studies such as Baio et al.'s paper (2010) addressing the use of WinBugs software. Here they used standard Markov Chain Monte Carlo (MCMC) methods that were used to generate samples from the posterior distribution. However this particular method requires a large number of iterations in order to converge, which computationally is incredibly intensive and time consuming as the number of samples needed increases. Although MCMC methods are able to handle very complex and dynamic models that include a large amount of parameters making it flexible in the modelling process, for the purpose of this particular study it raises questions as to whether it is optimal or necessary.

Instead we will consider the R-INLA package, as used in Baio et al.'s 2018 paper for football prediction modelling. The package refers to the Integrated Nested Laplace Approximation (INLA). This method is far more computationally efficient compared to MCMC techniques used in fitting Bayesian Models, particularly those with latent Gaussian Structures such as Gaussian processes and grouped random effect models. Using a combination of analytic approximations and numerical integration with posterior densities, the obtained posteriors can be use to get posterior expectations and quantiles. Thus, in hope of being able to efficiently and effectively generate many models along with their predictions for the different leagues we will use the INLA package going forward.

Report Structure

In the next chapter of the report we will introduce and inspect the data that will be used to determine the outcome of the domestic leagues. We then go onto introduce main concepts of the model building process, before discussing the results obtained along with any limitations and future outlook on the research.

Introducing the Data

Fbref Datasets

Data of the top 5 leagues are available from a vast array of sources, and we select Fbref because of its comprehensive and well structure format of each league, While providing insightful information on characteristics such as individual player statistics, and complex measures of performance including pass progression types and expected goals, which may be considered for more complex models. Fbref provides the data for each league in the same format, and unlike other sources splits the fixtures for each team into rounds, making it very useful when predicting and updating posterior probabilities for each round as the leagues progress in time. Fbref gives access to multiple seasons in time for each league, which also gives rise to the possibility of using these past seasons in the modelling process.

Data Description

The initial Fbref fixture dataframes include fixture lists of 380 games for leagues with 20 teams; Premier Leage, La liga, Ligue 1, and Serie A, while Bundesliga has 18 teams and thus 306 games in a season. They provide information on "Wk"; the round that a particular fixture belongs to, Date, Home Team, Away Team, Score, Venue, and other variables that were not included in the final dataframe, namely; xG, Attendance, Referee, as these would not be able to be reproduced every round of prediction.

Data preperation

We then clean the data handling any missing values and correcting any inconsistencies, particularly in the case of Premier league data. There are 38 rounds of fixtures for each team in all leagues apart from Bundesliga where there are 34. However in some cases there are rounds that are rescheduled thus creating double game weeks and other conflicts, and so in the case of the Premier League the number of rounds has been adjusted so there are 44 rounds to avoid any conflicts making predictions more effective in the long run. We do this and other data preparation steps using the tidyverse set of packages in r, particularly dplyr and tibble. We

first order the data in order of data, adding ID_game indicating the number of the specific fixture out of the 380 that are to be played. The input data is converted into long format by duplicating the rows of each fixture for a home and away team row, adding a binary variable Home, 1 if the team in the row of a given fixture is Home and 0 if away. The Opponent variable was created grouping data by ID_game and assigning each team's opponent as necessary. Venue variable is adjustedinstead of being the nstadium name, to the name of the team it belongs to; "Old Trafford" becomes "Manchester Utd". We then create the following variables that are to be used in the Bayesian Hierachical modelling processing:

Variable	Description
Goal	Number of goals scored by the specific team in a given fixture (this will be our response)
Points Won	Points Gained after each fixture, 3 for Win, 1 for Draw, 0 for Loss
Days Since Last Game	Number of days since the last fixture played by the specific team
Total Points	Cumulative points acquired by each team after accounting for the points won in the specific game
Points Difference	Difference in points between the 2 teams for each fixture
Relative Strength	Weighted value of total points between 2 teams of each fixture
Form	proportion of points won in the last 5 games, i.e x points out of the possible 15
Goals per game (Gpg))	Amount of goals a team has scored per game
Goals Conceded (GC)	Amount of goals the opponent scored against a specific team

Variable	Description
Goals Conceded Per Game(GCpg)	Amount of goals the opponent scored against a specific team per game
Goal Difference (GD)	Difference between the goals scored and concede in a given game
Total Goals Scored (TG)	Cumulative total of goals scored in all games played after given game
Total Goal Difference (TGD)	Total Goal Difference (Total Goals Scored - Total Goals Conceded)
Goal Difference-Difference (TGDDiff)	Difference in the totalgoal difffernece between the teams of a fixture
Rank	The league position of a team at any given game week, (1 to 20), decided by goal difference if tied
Rank Difference	Difference in league position of teams for each fixture

Exploratory Data Analysis

Model Building

Baseline Model

Given the literature in the field of football modelling, we follow suit assuming the number of goals scored by a team in a given fixture follows the Poisson Distriubtion (Maher 1982, Rue and Salvesen 2000, Baio et al. 2010).

Such a model takes the form:

$$Y_i \sim Poisson(\lambda_{1i})$$

$$X_i \sim Poisson(\lambda_{2i})$$

where X_i and Y_i are goals scored by home and away teams respectively in game i, and λ_{2i} and λ_{2i} are the respective parameters that are the measures of the scoring intensities.

In sports modelling the use of log linear random effect modells for the parameters λ_{1i} and λ_{2i} is common (Karlis et al. 2003). We establish a simple baselime log linear effects model:

$$log(\lambda_{1i}) = \beta_0 + \beta_1 home + att_{h_i} + def_{a_i}$$

$$log(\lambda_{2i}) = B_0 + att_{a_i} + def_{h_i}$$

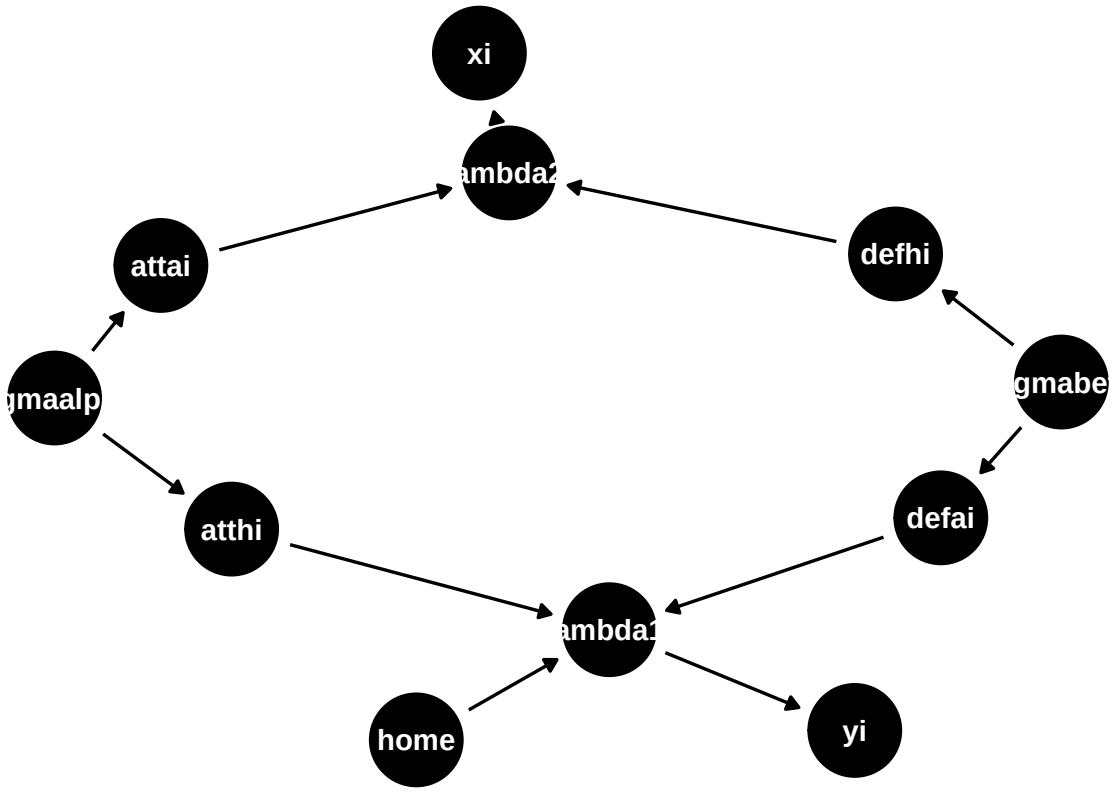
Here i=1,2,. . . ,n where n is the number of matches played in a season, i.e. n=380 for Premier League, Ligue 1, La Liga, and Serie A, whereas n = 306 for Bundesliga. Home reflects the fixed effect parameter when a team plays at their own Venue, evidently not appearing in the Away scoring parameter model. The 'att' and 'def' random effects reflect the relative attacking and defensive capabilities of a given team. Thus for the Home scoring Parameter, the model uses the attack effect of the home team and the defensive effect of the away, while for the Away scoring parameter it is modeled using the away attack effect and the home's defense.

In this model we make the assumption that the random effects of individual teams attacking and defending effects are in an exchangeable structure, the order in which the teams appear in the data does not have an impact on the inferences of their individual attacking and defending strengths. For instance the attacking strength of a team A and the defending strength of team B are considered to be drawn from a similar underlying distribution, regardless of the order they appear in the dataset. The exchangeable structure helps simplify the analysis and pool information across different teams during the season, and the assumption implicitly applies there is some correlation between the attack strength of a given team, and the defense of their opponent, arising since they are in the same game drawn from a similar underlying distribution. In doing this we imply that teams on average are similar in offensive and defensive capabilities, and that any differences are attributed to random variations arising from the common distribution. In our model we assume the effects to be distributed as

$$att_i \mid \sigma_\alpha \; Normal(0, \sigma_\alpha^2) and def_i \mid \sigma_\beta \; Normal(0, \sigma_\beta^2)$$

Below is a representation of the hierarchical structure of the model at hand:

try something similar to DAG representation that will show a hierarchical strucutre.



Having established the baseline model's formula, we fit the model using the INLA package, again specifying the poisson distribution for he response variable; number of Goals. Before this we prepare the data initially on a single season for each of the leagues. We first fit the model up to round r for prediction, with rounds 1,. . . , r-1 having been played already. The starting round number r was determined as the next game week at the time of modelling, in the case of the Premier league: 28, La Liga: 23, Ligue 1: 25, Serie A: 24, and Bundesliga: 22.

Referencing [Congdon, 2019] below; The INlA alogrithm focuses on the posterior density of hyperparameters $\pi(\lambda \mid y)$ and on the conditional posterior for the latent $\pi(x_i \mid \lambda, y_i)$ A Laplace approximation formarginal posterior density of random effects' hyperparamters $\tilde{\pi}(\lambda \mid y)$ and Taylor approximation for coonditional posterior of latent $\tilde{\pi}(x_i \mid \lambda, y)$ From these approximations, marginal posteriors are obtatined, where integrations

are carried out numerically.

$$\tilde{\pi}(x_i \mid y_i) = \int \tilde{\pi}(\lambda \mid y)\tilde{\pi}(x_i \mid \lambda, y)$$

Thust using our model the predictive distribution will have a probability mass function [Baio et al., 2018]:

$$\tilde{\pi}(\hat{y_1}, \hat{x_1} \mid y_1, x_1, z_1, z_2) = \int \tilde{\pi}(\hat{y_1}, \hat{x_1} \mid z_1, z_2, v)\tilde{\pi}(v \mid y_1, y_2, z_1, z_2,)$$

Where $\hat{y_1}, \hat{x_1}$ are the predicted goals scored in a future match, z_1 z_2 are the feature ests used, and v is the vector of all paramaters of the model. Then cases are considered based on the predicted goals scored:

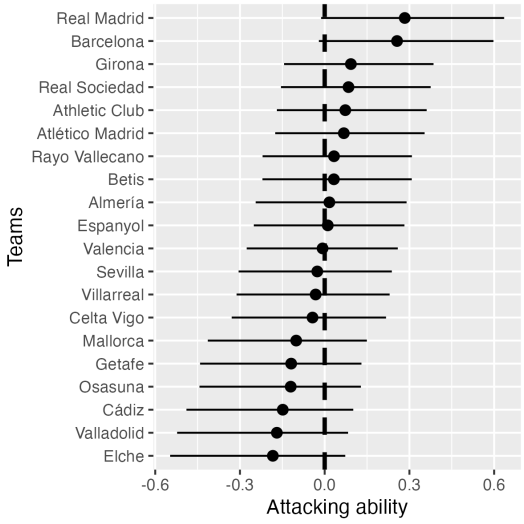
-$\hat{y_1} > \hat{x_1} \therefore$ Home Team wins -$\hat{y_1} < \hat{x_1} \therefore$ Away Team wins -$\hat{y_1} = \hat{x_1} \therefore$ Draw

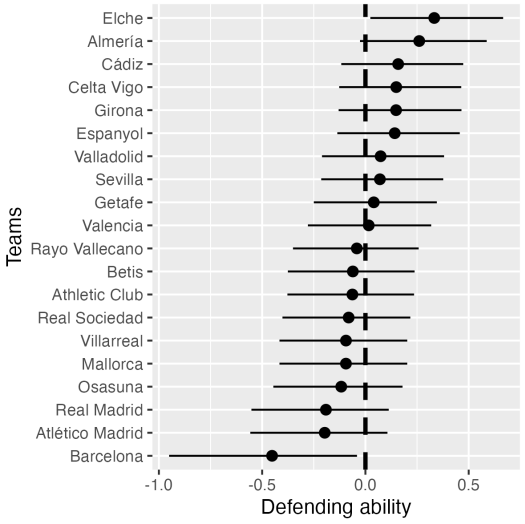
Table 2: Example: La Liga Team Attack and Defense Random Effect Quantiles

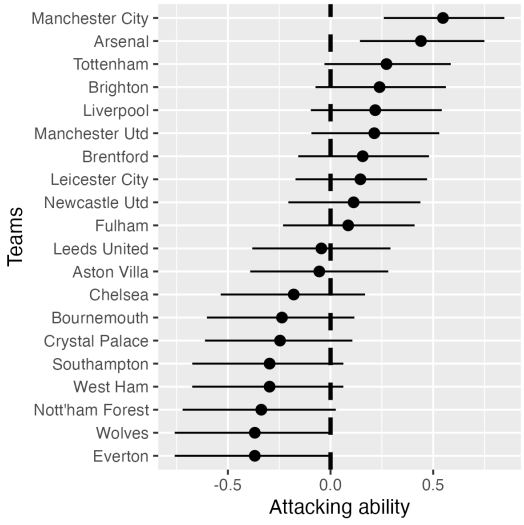
Team	Attack Mean	Attack 2.5%	Attack Median	Attack 97.5%	Defense Mean	Defense 2.5%	Defense Median	Defense 97.5%
Almería	0.0165858	-0.2442866	0.0119552	0.2900798	0.2605182	-0.0263106	0.2550529	0.5872316
Athletic Club	0.0730128	-0.1692706	0.0573021	0.3612349	-0.0630958	-0.3777389	-0.0583591	0.2355749
Atlético Madrid	0.0678000	-0.1751912	0.0528079	0.3537545	-0.1970099	-0.5575115	-0.1861125	0.1066024
Barcelona	0.2562312	-0.0206139	0.2544078	0.5975623	-0.4520438	-0.9504112	-0.4408945	-0.0407376
Betis	0.0325754	-0.2204437	0.0240692	0.3083492	-0.0609481	-0.3755189	-0.0563188	0.2380369
Cádiz	-0.1484229	-0.4895383	-0.1280458	0.1012194	0.1585272	-0.1167629	0.1504560	0.4736459
Celta Vigo	-0.0430518	-0.3291654	-0.0326270	0.2173067	0.1492726	-0.1269781	0.1411888	0.4639464
Elche	-0.1835821	-0.5472418	-0.1643777	0.0728908	0.3332316	0.0239312	0.3313689	0.6667913
Espanyol	0.0109749	-0.2509528	0.0076827	0.2823582	0.1419334	-0.1359064	0.1338731	0.4568310
Getafe	-0.1186305	-0.4412232	-0.0984994	0.1301768	0.0405140	-0.2497939	0.0365224	0.3458216
Girona	0.0927870	-0.1437525	0.0755216	0.3853525	0.1484430	-0.1298679	0.1402905	0.4650535
Mallorca	-0.1008751	-0.4138545	-0.0816367	0.1493625	-0.0943192	-0.4161951	-0.0875291	0.2024995
Osasuna	-0.1202544	-0.4433080	-0.1001325	0.1282259	-0.1170141	-0.4457641	-0.1089552	0.1796874
Rayo Vallecano	0.0329594	-0.2199748	0.0243647	0.3088858	-0.0417472	-0.3506895	-0.0387154	0.2577195
Real Madrid	0.2835537	-0.0125882	0.2857220	0.6354251	-0.1912204	-0.5519073	-0.1802186	0.1130000
Real Sociedad	0.0843772	-0.1547532	0.0675793	0.3753758	-0.0811191	-0.4018317	-0.0749700	0.2175607
Sevilla	-0.0262526	-0.3049172	-0.0198600	0.2377308	0.0701400	-0.2138898	0.0642331	0.3768571
Valencia	-0.0074862	-0.2761453	-0.0060062	0.2589239	0.0160662	-0.2784914	0.0140488	0.3186250
Valladolid	-0.1693333	-0.5224280	-0.1496956	0.0828229	0.0735951	-0.2102729	0.0675328	0.3807898
Villarreal	-0.0319555	-0.3119273	-0.0242798	0.2300265	-0.0941439	-0.4161304	-0.0873216	0.2025661

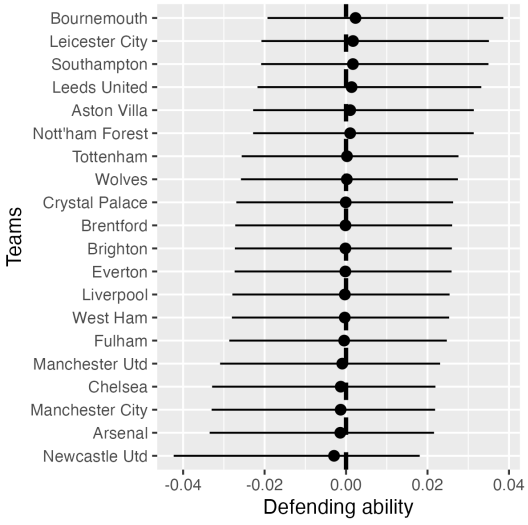
Using values from inla.summary.random we are able to extract the quantiles of the random effects measuring the teams' attacking and defensive strength. Below is a table showing example values of the effects' quantiles:

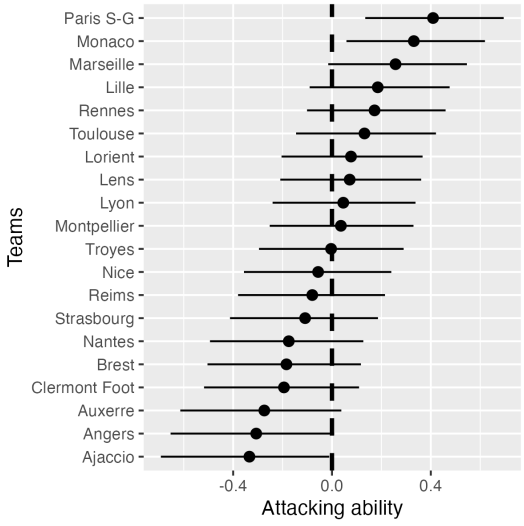
Formatting needed Below are visual representations for the team attack and defence effects for each of the remained of top 5 leagues:


Real Madrid
Barcelona
Girona
Real Sociedad
Athletic Club
Atlético Madrid
Rayo Vallecano
Betis
Almería
Espanyol
Valencia
Sevilla
Villarreal
Celta Vigo
Mallorca
Getafe
Osasuna
Cádiz
Valladolid
Elche
Teams
-0.6
-0.3
0.0
0.3
0.6
Attacking ability


Elche
Almería
Cádiz
Celta Vigo
Girona
Espanyol
Valladolid
Sevilla
Getafe
Valencia
Rayo Vallecano
Betis
Athletic Club
Real Sociedad
Villarreal
Mallorca
Osasuna
Real Madrid
Atlético Madrid
Barcelona
Teams
-1.0
-0.5
0.0
0.5
Defending ability


Manchester City
Arsenal
Tottenham
Brighton
Liverpool
Manchester Utd
Brentford
Leicester City
Newcastle Utd
Fulham
Leeds United
Aston Villa
Chelsea
Bournemouth
Crystal Palace
Southampton
West Ham
Nott'ham Forest
Wolves
Everton
Teams
-0.5
0.0
0.5
Attacking ability


Bournemouth
Leicester City
Southampton
Leeds United
Aston Villa
Nott'ham Forest
Tottenham
Wolves
Crystal Palace
Brentford
Brighton
Everton
Liverpool
West Ham
Fulham
Manchester Utd
Chelsea
Manchester City
Arsenal
Newcastle Utd
Teams
-0.04
-0.02
0.00
0.02
0.04
Defending ability


Paris S-G
Monaco
Marseille
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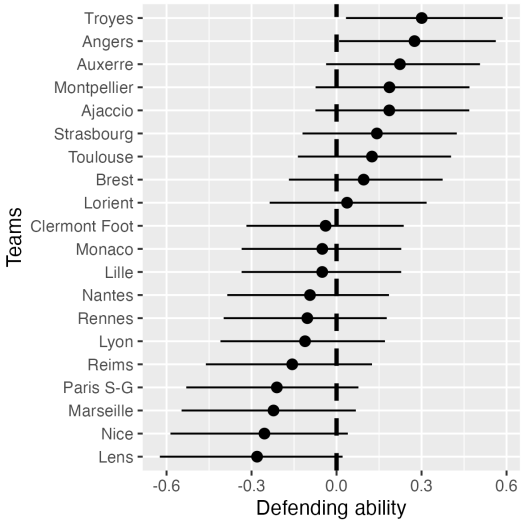
Rennes
Toulouse
Lorient
Lens
Lyon
Montpellier
Troyes
Nice
Reims
Strasbourg
Nantes
Brest
Clermont Foot
Auxerre
Angers
Ajaccio
Teams
-0.4
0.0
0.4
Attacking ability


Troyes
Angers
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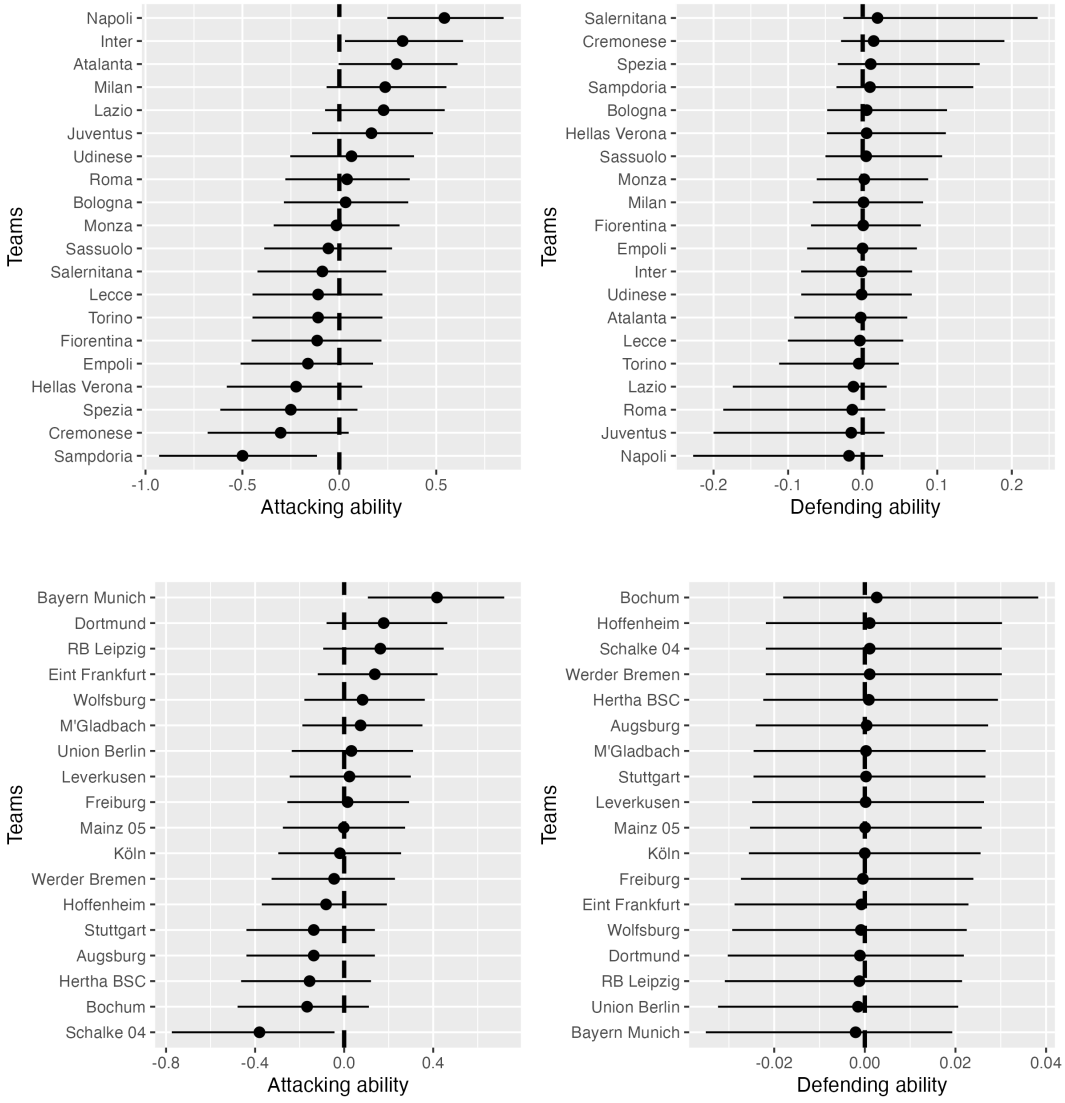
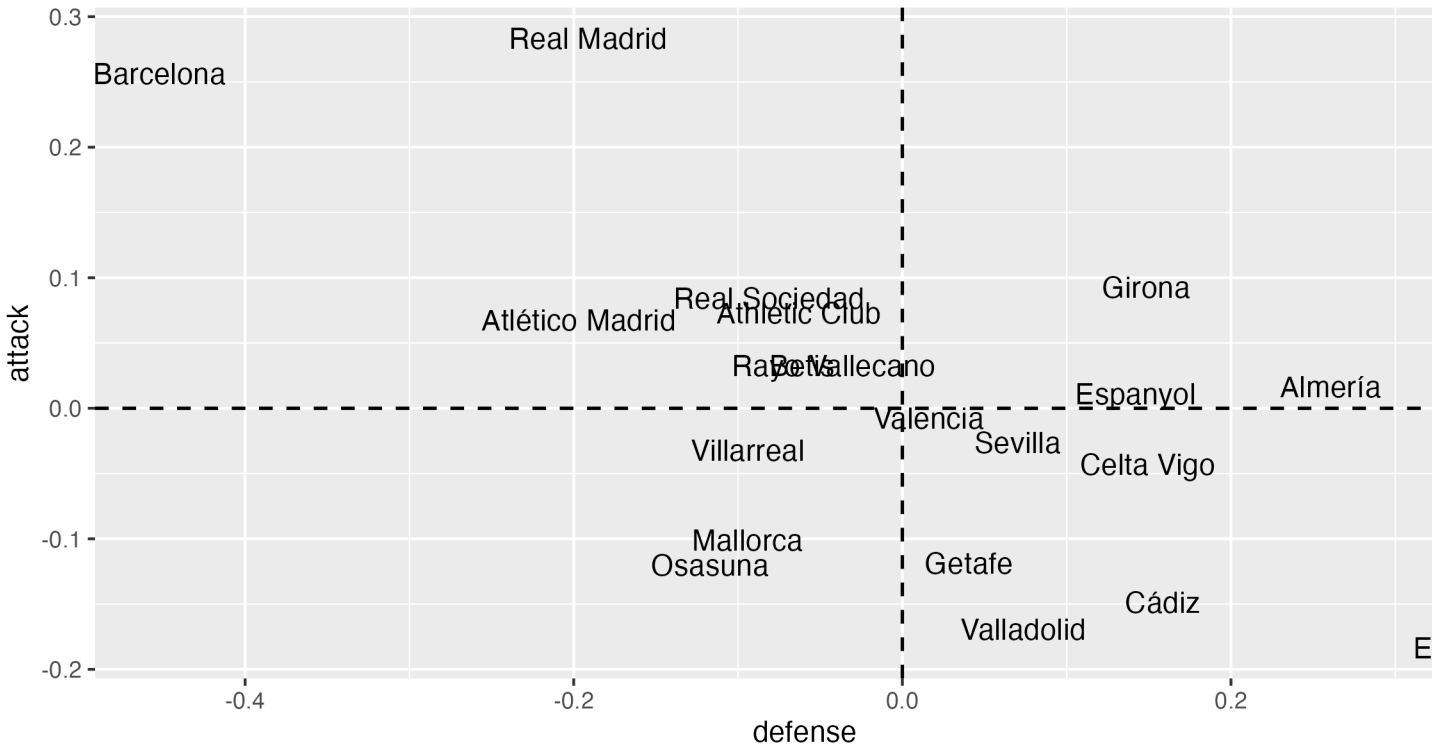
Strasbourg
Toulouse
Brest
Lorient
Clermont Foot
Monaco
Lille
Nantes
Rennes
Lyon
Reims
Paris S-G
Marseille
Nice
Lens
Teams
-0.6
-0.3
0.0
0.3
0.6
Defending ability



We can also put the attack and defense effects on the same plot to gain an idea of how teams proficient specific teams are at both defendg and attacking, showing by far how Barcelona and Real Madrid are the most outstanding teams, followed by a cluster of "next contenders"; Atletico Madrid, Real Sociedad, and Athletic Club.



Post Processing

make_scored():

Having run the model through INLA, We are then able to generate the predictions for goals scored, by simulating from the posterior distribution for the specific round of matches. Sampling was completed using R-INLA's method: inla.posterior.sample. Taking the input round number, data, model, and number of simulations (default 1000) as inputs, we created a function `make_scored()` that will get the index of corresponding rows. Then the posterior samples of latent variables are obtained using inla.posterior.sample, say $\hat{att}_{hi}, \hat{def}_{aj}$ and $\hat{home}$, storing the exponentiated posterior sample values. Then we compute the predicted scoring intensity paramter by each team in the specific round:

$$\hat{\lambda_{ij}} = \exp(\hat{att}_{hi} + \hat{def}_{aj} + \hat{home} + \text{offset})$$

Finally the function generates the predictions for the number of goals by simulating from a Poisson Distribution using rpoois(), with the predicted scoring intensity parameter:

$$goals_{ij} \sim \text{Poisson}(\hat{\lambda_{ij}})$$

Outcome predictions:

Running inla.summary we are able to extract coefficients for estimates of our models fit. For example in the case of baseline model applied to Serie A;

```
##              mean    sd 0.025quant 0.5quant 0.975quant  mode kld
## (Intercept) 0.088 0.092     -0.096    0.089      0.265 0.091   0
## Home        0.213 0.083      0.050    0.213      0.376 0.213   0
```

```
##                                    mean      sd 0.025quant 0.5quant 0.975quant
## Precision for factor(Team)       15.378   7.807      5.391   13.696     35.255
## Precision for factor(Opponent) 205.856 405.976     18.487   99.446   1083.293
##                                    mode
## Precision for factor(Team)       10.891
## Precision for factor(Opponent) 39.830
```

We see mean values for the intercept and Home fixed effect are 0.088 and 0.213 respectively. Thus the model equation will take the form:

$$\log(\lambda_{ij}) = 0.088 + 0.213home_j + att_{hi} + def_{ai}$$

where

for game g, where: - 0.088 is the intercept - $0.213home_i$ is the home advantage efffect for home team in game i, 0.213 increase in the log of the scoring paramater used in the poisson model predicting goals scored. - att_{hi} is the effect for home team of game i - def_{ai} is the effect for away team of game i

We notice from the model output that the precision for Team factor has a significantly lower precision than opponent factor, indicating noticeable variability in the attack effects, while very little hy variation for defence effects. Thus the effect of an opponent on a teams goal scoring ability (i.e. a teams ability to defend against a team) is less impactful than the teams innate attacking ability itself.

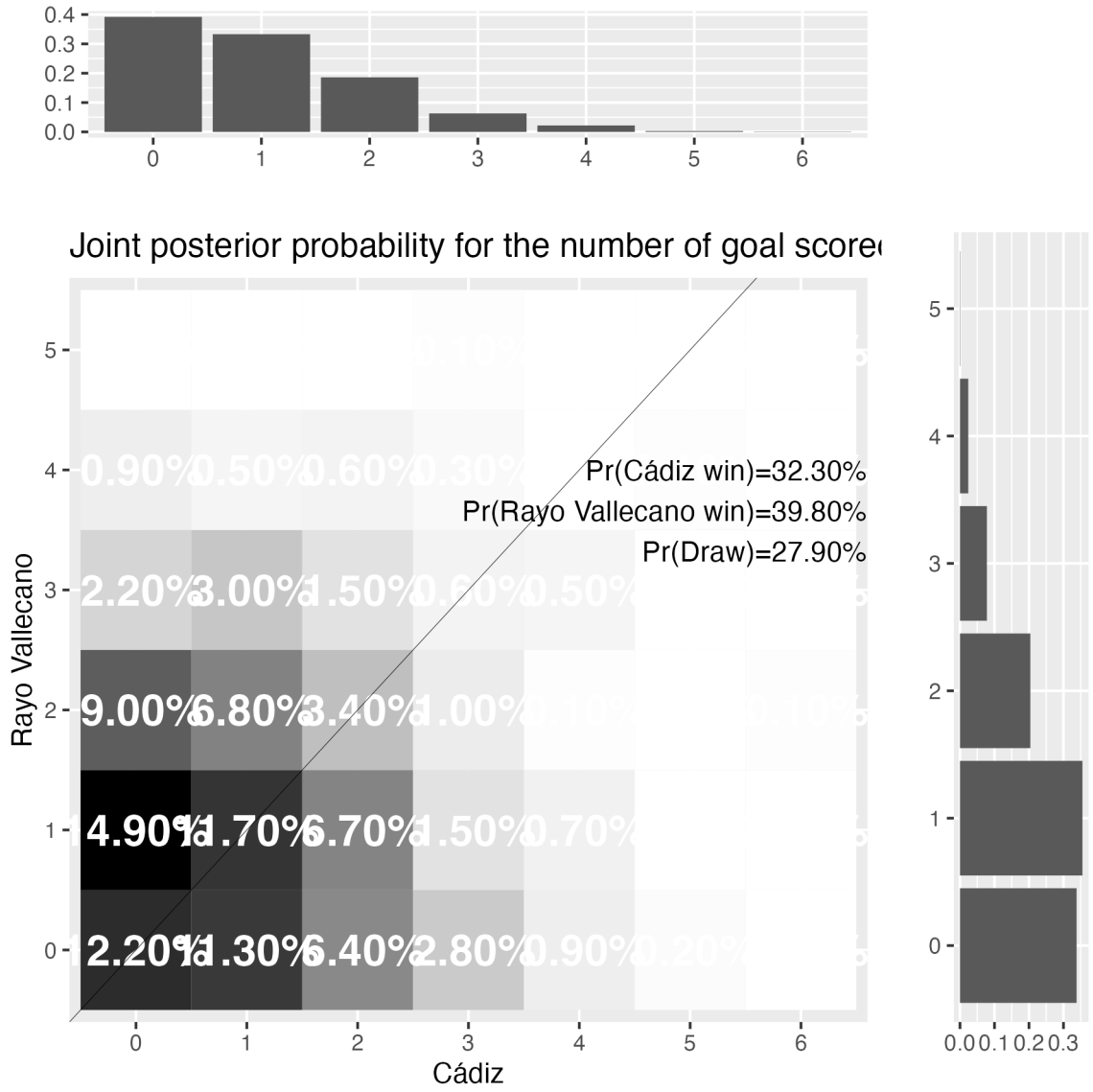
Now that we are able to generate predictions using the inla.posterior.sample method for the number of goals scored for each team after a given round, we are then able to get predictions for the selected fixture of the new round. In similar research the use of posterior means or medians is used for each team, however in our modelling we use the most probable joint outcome of the joint posterior predictive distributions, i.e. the most common outcome after each n simulation is done. In simple terms this is the most common score from the n number of rows, where the two columns are the teams playing. For instance, using the baseline model in round 23 of La Liga, in a match between Cadiz and Rayo Vallecano we have the table:

```
## # A tibble: 42 x 3
##    Cadiz `Rayo Vallecano`  prop
##    <dbl>            <dbl> <dbl>
##  1     0                1 0.149
##  2     0                0 0.122
##  3     1                1 0.117
##  4     1                0 0.113
##  5     0                2 0.09
##  6     1                2 0.068
##  7     2                1 0.067
##  8     2                0 0.064
##  9     2                2 0.034
## 10     1                3 0.03
## # ... with 32 more rows
```

Here 'prop' is the proportion of outcomes with the given result, thus in this case 14.9% of outcomes in the

match see Rayo Vallecano as victors.

We can visually display the joint marginal distribution of goals scored in this specific match as so;



Again we can visually see the probability of outcomes from the joint posterior distribution, along with the probability of winning, drawing, or losing, based on the possible outcomes predicted. However the histograms of the sampled marginal posterior distribution for teams invidiually, with highest density of 0 goal scored for Rayo Vallecano (top), and 1 for Cadiz (right), despit the most probable outcome being a 1-0 win for Rayo Vallecano.

We then update the relevant league dataset's 'Goal' column, as well as the associated variables derived from

it such as Points , Goal Difference, Relative Strength etc. Although as before these are not used in the baseline model.

In the case of bundesliga, we calculate the cumulative points determined from goals scored in each game, and derive the league table at the end of the season: We first show the results when using the Mean of each teams predicted goals after n simulations, compared to using the joint predictive posterior distribution. Using means shows Bayern Munich winning the league, where as the most probable outcome from the joint distributions leads to Dortmund wnning the league

```
## # A tibble: 18 x 6
##    Position Team           total_points total_G total_GC total_GD
##       <int> <chr>                 <dbl>   <dbl>    <dbl>    <dbl>
##  1        1 Bayern Munich            65      84       41       43
##  2        2 Union Berlin             63      56       44       12
##  3        3 RB Leipzig               61      64       45       19
##  4        4 Dortmund                 60      64       48       16
##  5        5 Freiburg                 58      55       52        3
##  6        6 Eint Frankfurt           52      63       52       11
##  7        7 Wolfsburg                51      58       48       10
##  8        8 Mainz 05                 47      54       55       -1
##  9        9 M'Gladbach               46      58       56        2
## 10       10 Leverkusen               44      57       57        0
## 11       11 Werder Bremen            44      51       62      -11
## 12       12 Köln                     41      52       56       -4
## 13       13 Augsburg                 41      46       57      -11
## 14       14 Stuttgart                39      47       56       -9
## 15       15 Bochum                   38      46       72      -26
## 16       16 Hoffenheim               36      49       61      -12
## 17       17 Hertha BSC               36      47       61      -14
```

```
## 18       18 Schalke 04               29      35       63      -28
```

```
## # A tibble: 18 x 6
##    Position Team           total_points total_G total_GC total_GD
##       <int> <chr>                 <dbl>   <dbl>    <dbl>    <dbl>
##  1        1 Dortmund                63      58       41       17
##  2        2 Freiburg                60      47       41        6
##  3        3 Bayern Munich           57      73       34       39
##  4        4 Union Berlin            57      46       37        9
##  5        5 RB Leipzig              55      54       39       15
##  6        6 Eint Frankfurt          50      52       43        9
##  7        7 Wolfsburg               47      51       42        9
##  8        8 M'Gladbach              47      52       49        3
##  9        9 Mainz 05                47      46       46        0
## 10       10 Köln                    46      48       46        2
## 11       11 Werder Bremen           45      43       53      -10
## 12       12 Leverkusen              41      49       48        1
## 13       13 Hertha BSC              40      40       50      -10
## 14       14 Stuttgart               39      44       49       -5
## 15       15 Bochum                  36      39       67      -28
## 16       16 Hoffenheim              33      39       52      -13
## 17       17 Augsburg                33      38       52      -14
## 18       18 Schalke 04              25      25       55      -30
```

We next aim to add covariates to the log linear model estimating the scoring parameter, which follow the form:

Home Goals:

$$log(\lambda_{1i}) = \beta_0 + \beta_1 home + att_{h_i} + def_{a_i} + \sum_j^m \beta_j z_{i1k}$$

where $\sum_j^m \beta_j z_{i1k}$ are the added set of covariates with their effect estimates from the set $\bar{z} = \{z_1, z_2, ..., z_m\}$ Similarly for Away Goals;

$$log(\lambda_{2i}) = \beta_0 + att_{a_i} + def_{h_i} + \sum_j^m \beta_j z_{i2k}$$

The set of possible covariates that can be part of the set $\bar{z} = \{z_1, z_2, ..., z_m\}$ are coded as: * Days Since last Game - `days_since_last` * Relative Strength - `rel_strength` * Goals per game - `Gpg` * Goals conceded per game - `GCpg` * Total Goal Difference-Difference - `GDdiff` * Rank Difference - `diff_rank` * Form - `form`

Note some variable from the dataframe are excluded so to reduce multicollinearity between variables as they are derived from them, such as Total Goals Conceded, instead using goals conceded per game as well as total goal difference. Using this variable we aim to select a set that provides a better fit for predictions. We next consider compare different combinations of covariates in finding the best fit through INLA modelling.

Model Checking:

Typically in INLA and hierarchical modelling (Rue et al. 2009), the use of Deviance Information Criterion and Watanabe-Alike-Information-Criterion are use as methods of comparing models in context. DIC is a model that is analogous to the Akaike information criterion (AIC) used in estimating prediction error, instead measuring the trade off between model fit and complexity calculated as follows:

$$DIC(\lambda) = D(\bar{\lambda}) + 2p_D$$

where $D(\bar{\lambda})$ is the deviance at the posterior mean of the parameters and p_D is the effective number of parameters.

Similarly the Watanabe-Akaike Information Criterion is a indicator of singular model performance. As defined by Watanabe himself, (Gelman et al. 2014) WAIC is the "negative of the average log pointwise

predictive density and thus is divided by n, and does not have the factor of 2", although in INLA is is scaled by factor of 2 for comparability with DIC and other measures of deviance. DIC makes the assumption that the posterior is approximately multivariate normal, while WAI can be numerically calculated without information on the true distribution. WAIC also an extension of the widely used AIC in the bayesian context, and also offers the bonus of no need for effective paramaters, again unlike DIC. Given the added applicability to general BHM methodology, the WAIC is a better indicator of model suitability.

Below is a table showing the top 10 models with the lowest WAIC, indicating top 10 most suitable models since lowest WAIC demonstrates lower deviance.

Table 3: Top 10 Model Combinations By WAIC

Formulae	WAIC
Home+ diff_rank + Gpg + GCpg + GDdiff + Att + Def	968.456
Home + Gpg + GCpg + GDdiff + Att + Def	969.284
Home + diff_rank + days_since_last + Gpg + GCpg + GDdiff + Att + Def	969.340
Home + diff_rank + form + Gpg + GCpg + GDdiff + Att + Def	969.512
Home + form + Gpg + GCpg + GDdiff + Att + Def	969.619
Home + rel_strength + diff_rank + Gpg + GCpg + GDdiff + Att + Def	969.686
Home + rel_strength + Gpg + GCpg + GDdiff + Att + Def	969.942
Home + rel_strength + form + Gpg + GCpg + GDdiff + Att + Def	970.149
Home + days_since_last + Gpg + GCpg + GDdiff + Att + Def	970.370
Home + diff_rank + form + days_since_last + Gpg + GCpg + GDdiff + Att + Def	970.579

Thus the optimal formula after this point in the model building process is of the form: Home Goals in game i:

$$log(\lambda_i) = \beta_0 + att_{h_i} + def_{a_i} + \beta_1 home + \beta_2 diff_rank + \beta_3 Gpg + \beta_4 GCpg + \beta_5 GDdiff"$$

Away Goals in game i:

$$log(\lambda_i) = \beta_0 + att_{a_i} + def_{h_i} + \beta_2 diff_rank + \beta_3 Gpg + \beta_4 GCpg + \beta_5 GDdiff"$$

Aditional Model Covariate Consideration

We also explore the addition of other covariates that may further improve model performance. First, a time component that aims to address team's difference in performance over time. This is added to the inla model using `f(id_date,model="rw2",replicate=as.numeric(factor(Team)))` This is modelled by a random walk model of order 2 `model="rw2"`on the fixture data variable representing the time component, computed seperately for each tean `replicate=as.numeric(factor(Team))`.

Another feature to consider is overdispersion, a common feature in count data like goals scored in football. Overdispersion occurs when variance of the observed data is greater than what would be expected under a simple Poisson distribution, thus we aim to capture extra variability that may not be accounted for with other covariates. This is demonstrated in the code by `f(num,model="iid")`.

Given the additional covariates we can consider in the formula used for the log linear model to be used in INLA, the next step is to consider validation and comparison methods to determine a suitable model for predicting the remainder of the 2022/23 seasons of the repsective domestic leagues.

However after adding these variables individually and together, we see that they do not improve model fit by WAIC; adding the random walk time component increases deviation to 988.861, while adding the overdispersion parameter neither improves or negatively impacts WAIC. We note the very high in precision in the model hyperparameters for these added random effects via the summary output, demonstrating the lack of variability in the the

Model Validation

In model validation we aim to compare the effectiveness of the more complex model and the baseline model in terms of performance, We execute this using a previous seasons data, for example the Serie A season 2021/22.

Team	Observed_Points	Baseline_Points	Baseline_Difference	Optimized_Points	Optimized_Difference
Atalanta	59	65	6	68	9
Bologna	46	44	2	54	8
Cagliari	30	31	1	30	0
Empoli	41	51	10	60	19
Fiorentina	62	57	5	58	4
Genoa	28	39	11	37	9
Hellas Verona	53	49	4	58	5
Inter	84	68	16	69	15
Juventus	70	60	10	75	5
Lazio	64	57	7	60	4
Milan	86	61	25	64	22
Napoli	79	61	18	66	13
Roma	63	56	7	51	12
Salernitana	31	27	4	21	10
Sampdoria	36	53	17	57	21
Sassuolo	50	44	6	50	0
Spezia	36	32	4	39	3
Torino	50	46	4	44	6
Udinese	47	41	6	46	1
Venezia	27	41	14	55	28

We see from the results and after calcuating the mean square error of points; baseline model 115.55 and optimized model 153.1. In this case the baseline results perform better. This is perhaps because the baseline effects are better at capturing the team effects. . . . . .

Need Gianluca's Help here, i must have approached this part wrong

Thus

Discussion:

- Conclusion
- Limitation

A common probablem of the Bayesian Hierarchical model is overshrinkage where extreme outcomes, like a team scoring 8 goals, are instead pulled towards the grand mean. We notice this in the use of - Future Work

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