Dissertation Write Up V1

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Introduction

Background and Motivation

In 2021 many of Europe's most prominent and successful Association Football teams engaged in discussions that led to the development of the 'European Super League' (ESL). The Super League was spearheaded by its 12 founding members from 4 of Europe's top 5 leagues including the biggest names in football worth billions of dollars on their own such as Manchester United, Real Madrid, Barcelona. The competition proposed to create a new international standard of Football with aim of providing a more sustainable and lucrative business model for the associated clubs as a direct competitor to the existing UEFA Champions League. The Super League was conceptualised as a closed competition; implying that the founding members would be guaranteed permanent membership and exemption from relegation regardless of their performance in a season. With 20 teams in total, this would mean only the 12 founding members were guaranteed to enjoy the expected revenues from the competition, while the 8 others faced a constant threat of relegation. Another aim of the league was to maintain a more competitive balance, in the sense that the league would only contain games of the highest quality from Europe's Best. The founders believed this would retain a larger global audience, thus increasing the value of broadcasting rights, sponsorship and merchandise sales. The organisers also proposed the reduction of financial disparities with the rest of European football by proposing to share a proportion of the revenue generated with other clubs and investing in grassroots football development.

However, following the announcement April 18th 2021, the organizers and teams faced vast backlash among the various stakeholders of the football community, including fans, football governing bodies, domestic leagues, smaller clubs, and even governments.

The proposed Super League marked a significant departure from the traditional format of European Football, causing concern from European and Global football's governing bodies, UEFA and FIFA. Governments also had a inputs on the matter including the UK government, with Prime Minister at the time Boris Johnson threatening to take "whatever action neccessary" to stop the plans (BBC, 2021) . Critics argued the closed format would lead to the erosion of sporting merit and integrity, many fearing the superior ESL would damage the structure of Europe's domestic leagues, given that the resources of the founding clubs would shift to the new competition, devaluing the domestic leagues as a result and thus widening the financial gap. A report by conultancy firm KPMG revealed La Liga would lose 55% of its revenues (LaLiga Newsletter, 2022) This concentration of wealth and power was argued to making it more difficult for smaller clubs to compete hence threatening their long term survival. While the reader may not believe this to be of vital concern as other matters, it must be considered that the competition alone was expected to generate Four billion euros , and Europe's top 5 leagues already generating billions of euros alone, notably the Premier League with €5.492 billion. Thus there are billions of euros at stake with the proposition of this new competition, with many subsidiary industries such as sports betting, analytics, and sponsorships that could be affected. So, it is of our interest to investigate what may potentially have occurred as a result of this possible intervention in the world of football, with Juventus, Barcelona, and Real Madrid still yet to withdraw from the competition's establishment, there is still a possibility for the European Super League's fruition into reality. With the 12 founding members leaving their respective leagues, and other clubs such as Paris Saint Germain (Ligue 1, France) potentially leaving theirs, one might consider how this would effect the state of the domestic league should they not participate, who would take crown in each of the top 5 leagues?

Research Question

This project aims to explore and analyse the predicted performance of European teams in their respective domestic leagues by modelling and analyzing a range of factors yet to be discussed. In addition, we will use the resulting methodology and analysis to estimate and discuss the hypothetical outcomes in a scenario where 12 clubs have been excluded from participating in their leagues. This exclusion will have a significant impact on the outcome of the leagues, and in reality would have various potential ripple effects on the teams, competitions, and industries as a whole. By conducting a thorough analysis and exploring the potential outcomes, we hope to provide valuable insights into the current state of European football and the potential impact of excluding certain clubs from participation. In doing this we hope to explore statistical techniques that can be used to model and predict the outcome of match results. We will look at the recent scope of literature on statistical techniques that have been used in the prediction of match results.

Statistical methods In Modelling Football Results: A Literature Review

A Brief History

The prediction of football match results has been of interest to researchers and statisticians for decades. The use of binomial and negative binomial models were explored as early as 1968 (Reep and Benjamin, 1968). They modelled "r-pass movements" defined as a series of 'r' successful passes among players before either a shot at goal by the r^{th} or an interception in the $(r+1)^{th}$ pass attemmpt.

$$P(X = r) = \binom{r+k-1}{r} p^r (1-p)^k$$

where:

X is the random variable representing the number of r-pass movements r is the number of r-pass movements we are interested in k is a parameter that represents the number of failures (interceptions) before an r-pass movement is completed p is the probability of a successful r-pass movement $\binom{n}{m}$ denotes the number of combinations of n items taken m at a time (also written as "n choose m" or $C(n,m) = \frac{n!}{m!(n-m)!}$).

The Poisson distribution later became the prominent method of modelling these relevant quantities. Many papers, most notably by Karlis and Ntzoufras (2003), Dixon and Coles (1997), and Hvattum et al. (2010), cite and credit Maher (1982) who successfully demonstrated the uses of Poisson Modelling, also addressing in his later work the relation between teams in a given match through Bivariate-Poisson modelling. Maher considered the following Poisson model: If Team i is playing against team j with observed scored (x_{ij}, y_{ij}), then

$$X_{ij} \sim Poisson(\alpha_i \beta_j)$$

and

$$Y_{ij} \sim Poisson(\gamma_j \delta_i)$$

where α_i and the attacking strength of team i if playing at home, and γ_j for team j if playing away. Likewise β_j is the deefesive weakness of team j when away from home, and δ_i is the defensive weakness of team i at home.

One of the most influential studies in the field came from the work of Dixon and Coles (1997) who proposed a far more effective Poisson Model framework. In their study they introduced further improvements with time-weighting factors and correction terms for low-scoring matches. Their method demonstrated improved accuracy over previous models and has been widely reference in subsequent research.

Bayesian framework:

Bayesian frameworks have gained popularity over recent years due to the increase in computational capabilities making intensive Bayesian analysis more viable, with sufficient computational power not being available until the 1990s . Extensions of the Bayesian framework include the Bayesian generalised linera model of Rue and Salvesen (2000), which has been widely referred to in subsequent research. The framework was later improved by the promising Bayesian Hierarchical Models, abbreviated as BHM (Baio et al.,2010) (Tsokos et al., 2018). The BHM has been a significant addition to the field of statistical methods in football match modelling. The 2010 paper introduced a multilevel structure in their modelling to estimate team-specific parameters nested within a mixed effects model, applied in the context of the Italian Serie A league. They also later specified a more complex mixture model that aimed to overcome the issue of over shrinkage produced by the Bayesian multilevel model, in order to provide a better fit to the data of previous season. Further improvements to the use of BHM were established in the 2018 paper incorporating even more complex and effective mixture models accounting for the data available at the time. These two works in particular have been widely cited in later literature, including comprehensive reviews of Bayesian statistical methods in football (Santos-Fernandez et al., 2019) and studies of more modern techniques (Hubáček et al., 2019). The Bayesian Hierarchical Model uses multiple levels, arranged hierarchically, to estimate parameters of the posterior distribution through Bayesian inference. The BHM links the sub-models for lower levels together propagating uncertainties in each sub model from one level to the next by establishing hyperparameters and hyperpriors, and using data provided in the combined hierarchical to generate predictions. BHM demonstrates advantages by naturally accounting for relations between variables through the assumption that they come from a common distribution. The Bivariate Poisson assumption used by Karlis & Ntzoufras (2003) is not needed to account for correlation, instead two independent Poisson variables are used, with the observable variables being combined at the higher level hence accounting for correlation. Overall this approach enables a flexible, effective and comprehensive analysis of team performance while accounting for the hierarchical structure of football league data. More recent research using Bayesian frameworks include that of Razali et al (2017) exploring the use of machine learning methods, in this case Bayesian Networks (BNs), to model predict, and validate match results for the English premier League. Other recent applications have included Naive Bayes and Tree Augmented Naive Bayes Models in Rahmanet et al. (2018),

Other methods

Although Bayesian methods have become more prominent in the field of predictig football results in recent years, other methods have also been observed. In the past Bradley-Terry models (Bradey and Terry 1952) with comparison modes pairing teams to determine the outcome of a game (Kuk 1995) have been used to estimate the probabiliies of winning, drawing, or losing a match. Related studies in the field include Godin et al's (2014) leveraging of contextual information via "Twitter Microposts" and machine learning techniques in order to comprehensively beat expert and bookmaker predictions, a dynamic approach wit a different end goal to the purpose of our study. Furthermore, other more relevant and modern research cover Machine Learning Methods such as Random Forests (Groll et al. , 2018), Gradient Boosting and Linear Support Vector Machines, notably by Baboota et al. (2018) who's work has been well cited as developments in the use of Artificial Intelligence and Machine Learning methods continue to develop in this field.

As we can observe, the aims of the papers discussed in the literature review vary. Some aim to simply model the outcome of the came like in Fahrmeir and Tutz (1994) using models of paired data with time varying features. Others investigate outcomes by predicting goals scored as seen in Dixon and Coles 1997 and Baio et al. (2010, 2018) while others address other characteristics used to predict outcomes, such as passing movements and shots per game see Reep et al (1968).

Introduction to Bayesian Hierarchical modelling

Bayesian Inference

Given the above discussed benefits of Bayesian Hierarchical Modelling over other statistical methods such as Karlis and Ntzoufras (2003) use of Bivariate Poisson modelling, we proceed with the BHM in predicting outcomes of the top five European domestic leagues.

By adding many degrees of hierarchy, BHM enables the representation of complicated relationships and structures within the data. This strategy is based on the Bayesian probability theory discussed above, which offers a methodical manner to revise beliefs or probabilities in light of new information by applying Bayes' theorem.

From Bayesian Inference theory in general applications, we know:

$$P(\theta, y) = P(\theta)P(y \mid \theta)$$

is the joint probability distributions for parameter of interest θ and data y, written as the product of the distributions: * the prior distribution $P(\theta)$, which estimates the parameter θ before data is observed * likelihood $P(y \mid \theta)$, the probability of observing the data conditional on parameter θ.

Using Bayes Theorem:

$$P(\theta \mid y) = \frac{P(\theta, y)}{P(y)} = \frac{P(y \mid \theta)P(\theta)}{P(y)}$$

where $P(\theta \mid y)$ represents the posterior distribution of the parameter θ given observed data. $P(y)$ reflects the marginal likelihood, or model evidence, that is derived as the integral of the joint probability distribution of θ and y:

$$P(y) = \int P(\theta)P(y \mid \theta)d\theta$$

In scenarios where the marginal likelihood is not easily obtained, one may express the posterior distribution as:

$$P(\theta \mid y) \propto P(y \mid \theta)P(\theta)$$

Hierarchical Models:

The extension of the Bayesian framework into hierarchical modelling incorporates two added features used in deriving the posterior distribution: - Hyperparameters: set of parameters that are used to determine prior distributions of other parameters uesd in the model often referred to as lower level parameters. The hierarchical nature of BHM allows for multiple levels, each level having its own distribution. Parameters at the higher levels are used to determine properties of the priors for lower-levels, which are the hyperparameters. - Hyperpriors: these are the prior distributions on the hyperparameters, used to express uncertainty in a hyperparameter.

BHM framework:

We consider the structure of a two level Bayesian Hierarchical Model let \$ **y** \$ be a set of observations $y_1, ..., y_n$ from random variables $Y_1, ..., Y_n$ and θ be the set of parameters from each Y_i; $\theta_1, ..., \theta_n$ from a common population with distribution determined by hyperparameter ϕ

The Likelihood above is $P(y \mid \theta, \phi)$ with $P(\theta, \phi)$ as its prior distribution.

$$\text{Stage 1: } y \mid \theta, \phi \sim P(y \mid \theta, \phi)$$

The prior can be expressed as $P(\theta, \phi) = P(\theta \mid \phi)P(\phi)$ using the definition of conditional probability

$$\text{Stage 2: } \theta \mid \phi \sim P(\theta \mid \phi)$$

The next stage being the hyperparameter: ϕ with prior distribution $P(\phi)$, referred to as the hyperprior.

$$\text{Stage 3: } \phi \sim P(\phi)$$

Using this structure we obtain the posterior distribution using bayes theorem, expressed as:

$$P(\phi, \theta \mid y) \propto P(y \mid \theta, \phi)P(\theta, \phi) = P(y \mid \theta)P(\theta \mid \phi)P(\phi)$$

Using this we can obtain probabilities from the posterior distribution.

Baseline Model

Given the literature in the field of football modelling, we follow suit assuming the number of goals scored by a team in a given fixture follows the Poisson Distriubtion (Maher 1982, Rue and Salvesen 2000, Baio et al. 2010).

This give us our baseline model that we build on later, of the form:

$$Y_i \sim Poisson(\lambda_{1i})$$

$$X_i \sim Poisson(\lambda_{2i})$$

where X_i and Y_i are goals scored by home and away teams respectively in game i, and λ_{2i} and λ_{2i} are the respective parameters that are the measures of the scoring intensities.

In sports modelling the use of log linear random effect modells for the parameters λ_{1i} and λ_{2i} is common (Karlis et al. 2003). We establish a simple baselime log linear effects model:

$$log(\lambda_{1i}) = \beta_1 home + att_{h_i} + def_{a_i}$$

$$log(\lambda_{2i}) = att_{a_i} + def_{h_i}$$

Computation

Markov Chain Monte Carlo (MCMC)

Typically in Bayesian modelling, including hierarchical versions are done using simulations through methods such as MCMC. Various languages used for such techniques include R, Jags, Python, Stan (Hilbe et al., 2017), with popular related studies such as Baio et al.'s paper (2010) addressing the use of WinBugs software. Here they used standard Markov Chain Monte Carlo (MCMC) methods that were used to estimate parameters of the model including hyperparameters and then generate samples from the posterior distribution. The MCMC algorithm in the paper uses Gibbs sampling, iteratively samples from the posterior by updating the values based on the priors and likelihood of observed data, with the generated samples being used to estimate parameters for predicting the new football matches. However this particular method requires a large number of iterations in order to converge, which computationally is incredibly intensive and time consuming as the number of samples needed increases. Although MCMC methods are able to handle very complex and dynamic models that include a large amount of parameters making it flexible in the modelling process, for the purpose of this particular study it raises questions as to whether it is optimal or necessary.

The R-INLA package

We instead consider the R-INLA package, as used in Baio et al.'s 2018 paper for football prediction modelling. The package refers to the Integrated Nested Laplace Approximation (INLA). This method is far more computationally efficient compared to MCMC techniques used in fitting Bayesian Models, particularly those with latent Gaussian Structures such as Gaussian processes and grouped random effect models. Using a combination of analytic approximations and numerical integration with posterior densities, the obtained posteriors can be use to get posterior expectations and quantiles. Thus, in hope of being able to efficiently and effectively generate many models along with their predictions for the different leagues we will use the INLA package going forward.

Report Structure

In the next chapter of the report we will introduce and inspect the data that will be used to determine the outcome of the domestic leagues. We then go onto introduce main concepts of the model building process, before discussing the results obtained along with any limitations and future outlook on the research.

Introducing the Data

Fbref Datasets

Data of the top 5 leagues are available from a vast array of sources, and we select `Fbref` because of its comprehensive and well structure format of each league, While providing insightful information on characteristics such as individual player statistics, and complex measures of performance including pass progression types and expected goals, which may be considered for more complex models. Fbref provides the data for each league in the same format, and unlike other sources splits the fixtures for each team into rounds, making it very useful when predicting and updating posterior probabilities for each round as the leagues progress in time. Fbref gives access to multiple seasons in time for each league, which also gives rise to the possibility of using these past seasons in the modelling process.

Data Description

The initial `Fbref` fixture dataframes include fixture lists of 380 games for leagues with 20 teams; Premier Leage, La liga, Ligue 1, and Serie A, while Bundesliga has 18 teams and thus 306 games in a season. They provide information on “Wk”; the round that a particular fixture belongs to, Date, Home Team, Away Team, Score, Venue, and other variables that were not included in the final dataframe, namely; xG, Attendance, Referee, as these would not be able to be reproduced every round of prediction.

Data preperation

We then clean the data handling any missing values and correcting any inconsistencies, particularly in the case of Premier league data. There are 38 rounds of fixtures for each team in all leagues apart from Bundesliga where there are 34. However in some cases there are rounds that are rescheduled thus creating double game weeks and other conflicts, and so in the case of the Premier League the number of rounds has been adjusted so there are 44 rounds to avoid any conflicts making predictions more effective in the long run. We do this and other data preparation steps using the `tidyverse` (Wickham et al, 2019) set of packages in `R`, particularly `dplyr` (Wickham et al., 2023)and `tibble` (Muller and Wickham, 2022). We first order the data in order of data, adding `ID_game` indicating the number of the specific fixture out of the 380 that are to be played. The input data is converted into long format by duplicating the rows of each fixture for a home and away team row, adding a binary variable Home, 1 if the team in the row of a given fixture is Home and 0 if away. The Opponent variable was created grouping data by ID_game and assigning each team's opponent as necessary. The Venue variable is adjusted instead of being the stadium name, to the name of the team it belongs to; "Old Trafford" becomes "Manchester Utd". We then create the following variables that are to be used in the Bayesian Hierachical modelling processing:

Variable	Description
Goal'	Number of goals scored by the specific team in a given fixture (this will be our response)
Points Won	Points Gained after each fixture, 3 for Win, 1 for Draw, 0 for Loss
Days Since Last Game	Number of days since the last fixture played by the specific team
Total Points	Cumulative points acquired by each team after accounting for the points won in the specific game
Points Difference	Difference in points between the 2 teams for each fixture

Variable	Description
Relative Strength	Weighted value of total points between 2 teams of each fixture
Form	proportion of points won in the last 5 games, i.e x points out of the possible 15
Goals per game (Gpg))	Amount of goals a team has scored per game
Goals Conceded (GC)	Amount of goals the opponent scored against a specific team
Goals Conceded Per Game(GCpg)	Amount of goals the opponent scored against a specific team per game
Goal Difference (GD)	Difference between the goals scored and concede in a given game
Total Goals Scored (TG)	Cumulative total of goals scored in all games played after given game
Total Goal Difference (TGD)	Total Goal Difference (Total Goals Scored - Total Goals Conceded)
Goal Difference-Difference (TGDDiff)	Difference in the totalgoal difffernece between the teams of a fixture
Rank	The league position of a team at any given game week, (1 to 20), decided by goal difference if tied
Rank Difference	Difference in league position of teams for each fixture

Exploratory Data Analysis

Model Building

The Model

Building on our baseline model established earlier, we next aim to address the other covariates in our dataset, adding them into the log linear model estimating the scoring parameter, following the form:

Home Goals:

$$log(\lambda_{1i}) = \beta_1 home + att_{h_i} + def_{a_i} + \sum_j^m \beta_j z_{i1k}$$

where $\sum_j^m \beta_j z_{i1k}$ are the added set of covariates with their effect estimates from the set $\bar{z} = \{z_1, z_2, ..., z_m\}$ Similarly for Away Goals;

$$log(\lambda_{2i}) = att_{a_i} + def_{h_i} + \sum_j^m \beta_j z_{i2k}$$

The set of possible covariates that can be part of the set $\bar{z} = \{z_1, z_2, ..., z_m\}$ are coded as: * Days Since last Game - `days_since_last` * Relative Strength - `rel_strength` * Goals per game - `Gpg` * Goals conceded per game - `GCpg` * Total Goal Difference-Difference - `GDdiff` * Rank Difference - `diff_rank` * Form - `form`

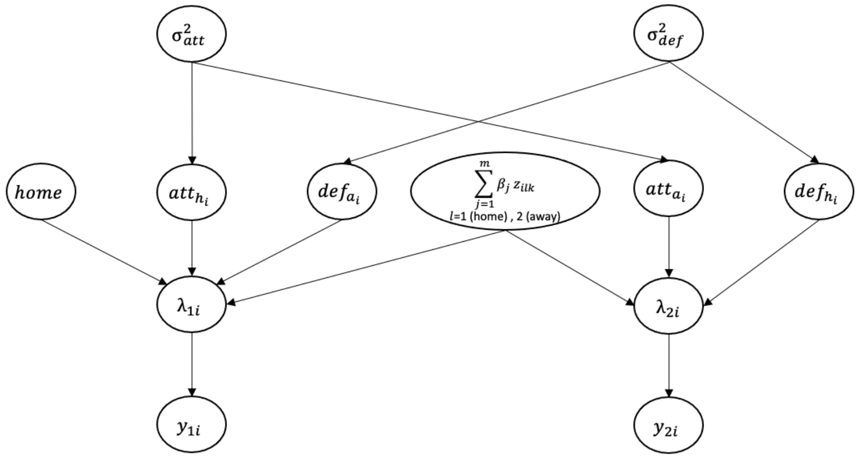
Note some variable from the dataframe are excluded so to reduce multicollinearity between variables as they are derived from them, such as Total Goals Conceded, instead using goals conceded per game as well as total goal difference. Using this variable we aim to select a set that provides a better fit for predictions. We next consider compare different combinations of covariates in finding the best fit through INLA modelling.

Here $i = 1, 2, ..., n$ where n is the number of matches played in a season, i.e. $n = 380$ for Premier League, Ligue 1, La Liga, and Serie A, whereas $n = 306$ for Bundesliga. Home reflects the fixed effect parameter when a team plays at their own Venue, evidently not appearing in the Away scoring parameter model. The `att` and `def` random effects reflect the relative attacking and defensive capabilities of a given team. Thus for the Home scoring Parameter, the model uses the attack effect of the home team and the defensive effect of the away, while for the Away scoring parameter it is modeled using the away attack effect and the home's defense.

In this model we make the assumption that the random effects of individual teams attacking and defending effects are in an exchangeable structure, the order in which the teams appear in the data does not have an impact on the inferences of their individual attacking and defending strengths. For instance the attacking strength of a team A and the defending strength of team B are considered to be drawn from a similar underlying distribution, regardless of the order they appear in the dataset. The exchangeable structure helps simplify the analysis and pool information across different teams during the season, and the assumption implicitly applies there is some correlation between the attack strength of a given team, and the defense of their opponent, arising since they are in the same game drawn from a similar underlying distribution. In doing this we imply that teams on average are similar in offensive and defensive capabilities, and that any differences are attributed to random variations arising from the common distribution. In our model we assume the effects to be distributed as

$$att_i \mid \sigma_\alpha \ Normal(0, \sigma_\alpha^2) \text{ and } def_i \mid \sigma_\beta \ Normal(0, \sigma_\beta^2)$$

Below is a visual representation of the hierarchical structure for the model at hand:



Using the formula established for the log linear model, we then fit the model using the function`inla()` as so;

```
inla(formula = ModelFormula,
                data=LeagueData,
                family="poisson",
                control.predictor=list(compute=TRUE,link=1),
                control.compute=list(config=TRUE,dic=TRUE,waic=TRUE))‘
```

The `family` argument is specified asthe poisson distribution for he response variable; number of Goals. The `data` argumnt uses `LeagueData`, which is prepared via our created `datprep()` function, producing a dataset of fixtures up to the prediction round r, with rounds $1, ..., r-1$ having been played already. The intial simulation round number `r` was determined as the next unplayed gameweek at the time of modelling, in the case of the Premier league: $r = 28$, La Liga: $r = 23$, Ligue 1: $r = 25$, Serie A: $r = 24$, and Bundesliga: $r = 22$. The argument `control.predictor` is a list specifying the predictor variables such as `link` the link function of the model, with `link=1` used to ensure the log link is used, and `compute` which is a boolean variable which in this case is `TRUE` to make sure marginal densities for the linear predictor are also be computed. The code also specifies the `control.compute` argument to include values in the summary for `WAIC` and `DIC` that will be discussed further in the model selection section.

The INlA package's methodology (Rue et al, 2009) focuses on the posterior density of hyperparameters $\pi(\lambda \mid$

y) and on the conditional posterior for the latent $\pi(x_i \mid \lambda, y_i)$ A Laplace approximation for marginal posterior density of random effects' hyperparamters $\tilde{\pi}(\lambda \mid y)$ and Taylor approximation for conditional posterior of latent $\tilde{\pi}(x_i \mid \lambda, y)$ From these approximations, marginal posteriors are obtained, where integrations are carried out numerically.

$$\tilde{\pi}(x_i \mid y_i) = \int \tilde{\pi}(\lambda \mid y)\tilde{\pi}(x_i \mid \lambda, y)$$

Thus, using our model the predictive distribution will have a probability mass function:

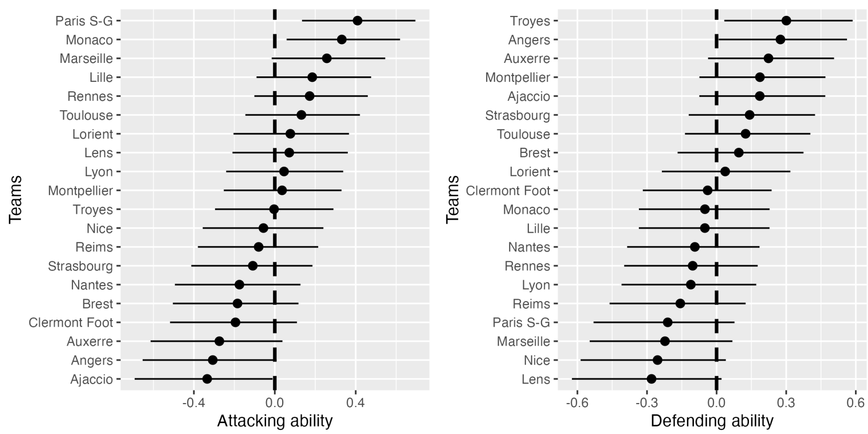
$$\tilde{\pi}(\hat{y_1}, \hat{x_1} \mid y_1, x_1, z_1, z_2) = \int \tilde{\pi}(\hat{y_1}, \hat{x_1} \mid z_1, z_2, v)\tilde{\pi}(v \mid y_1, y_2, z_1, z_2,)$$

Where $\hat{y_1}, \hat{x_1}$ are the predicted goals scored in a future match, z_1, z_2 are the features used, and v is the vector of all paramaters of the model. Then cases are considered based on the predicted goals scored:

- $\hat{y_1} > \hat{x_1} \therefore$ Home Team wins
- $\hat{y_1} < \hat{x_1} \therefore$ Away Team wins
- $\hat{y_1} = \hat{x_1} \therefore$ Draw

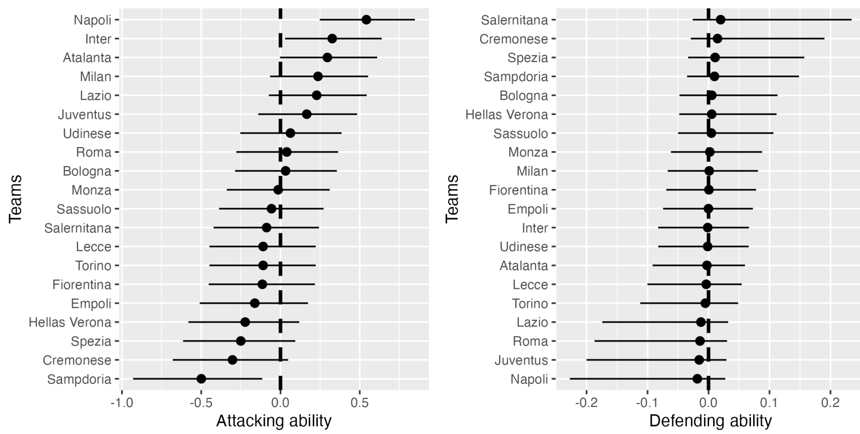
Using values from `inla.summary.random` which outputs the INLA model summary random effect estimates, we are able to extract the quantiles measuring the teams' attacking and defensive strength.

Below are visual representations for the quantiles team attack and defence effects for some of the leaguesusing only data from the 2022/23 seasons. We will discus later how this is impacted when adding more data from previous seasons. Consider the first plot for the random effect quantiles of Ligue 1:



For each team the range of values represent the 95% confidence interval for attack and defence effect estimates. The more positive values are for the attack effect, the stronger the team's offensive capabilities, while for the defence effect a more negative value is desirable as it shows the team's opponents will have lower scoring propensity. PSG has the greatest offensive capability while Ajaccio has the weakest, as for defence Lens can be considered the best defensive team while Troyes are the worst.

However we notice that for some leagues for example Serie A, the values for the defence effect are considerably closer centred around the mean 0, indicating the model is not able to account for the variability in the `Opponent` variable, due to overfitting since we only use the 2022/23 season at this stage. We discuss this later in the model building section, and how we deal with this.



If we plot tttack and defence effects on the same grid, we gain an insight of how teams proficient specific teams are at both defending and attacking overall. Consider the case below for La Liga in the 2022/23 season showing by far how Barcelona and Real Madrid are the most outstanding teams demonstrated by their attack effect noticeable above the rest indicating they are more proficient at scoring goals, and defence effects that are a lot lower than the other teams, indicating they are not as prone to conceding goals. They are followed by a cluster of “next contenders”; Atletico Madrid, Real Sociedad, and Athletic Club, leading us to suspect they would be the new “outstanding” clubs should Barelona and Real Madrid leave to join the Super League. Elche are noticeably both the worst offensive and defensive team.

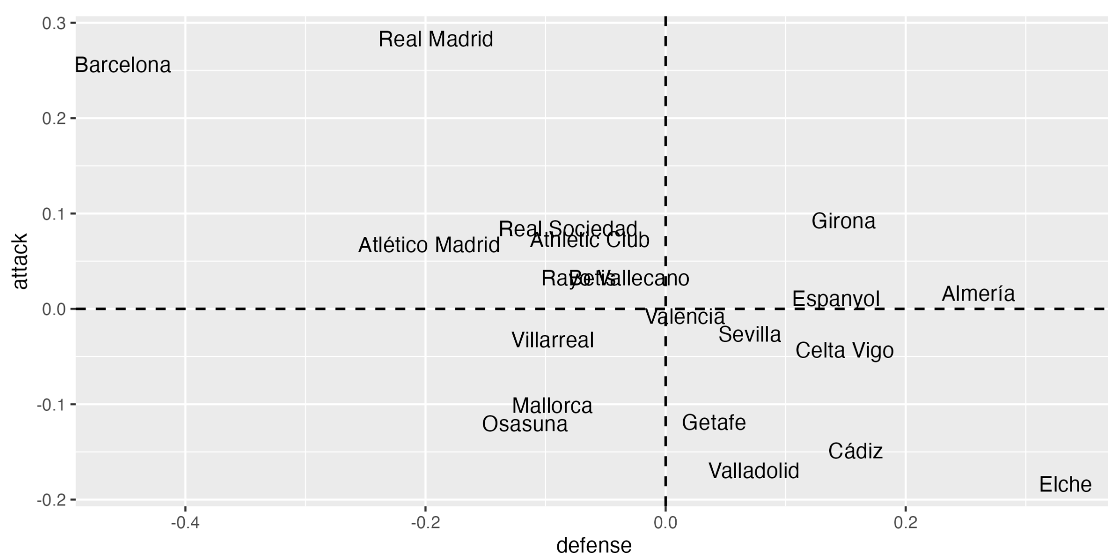


Figure 1: La Liga 2022/23 Plotting Attack v Defence Effects

Post Processing

Having run the model through INLA, We are then able to generate the predictions for goals scored, by simulating from the posterior distribution for the specific round of matches. Sampling was completed using R-INLA's method: `inla.posterior.sample`. This function draws samples from the approximate joint posterior distribution of the latent effects and hyperparameters which are sampled from the configurations used to do the numerical integration. Taking the input round number, data, model, and number of simulations (default 1000) as inputs, we created a function `make_scored()` that will get the index of corresponding rows. Then the posterior samples of latent variables are obtained using inla.posterior.sample, say $\hat{att}_{hi}, \hat{def}_{aj}$ and $\hat{home}$, storing the exponentiated posterior sample values. Then we compute the predicted scoring intensity paramter by each team in the specific round:

$$\hat{\lambda_{ij}} = \exp(\hat{att}_{hi} + \hat{def}_{aj} + \hat{home} + \text{offset})$$

Finally the function generates the predictions for the number of goals by simulating from a Poisson Distribution using `rpois()`, with the predicted scoring intensity parameter:

$$goals_{ij} \sim \text{Poisson}(\hat{\lambda_{ij}})$$

Programmed Functions in R

The code used in the model building process includes utility functions that manipulate, visualise, and simulate from our dataset to generate predictions.

Prediction Processing Functions

- `datprep()` , preps the data of a league to be used in our `runINLA()` function.
- `runINLA()` , runs the INLA model after specifying the formula, data for given round, posterior family,

- `make_scored()` , Processing function create to predict the number of goals scored by a team in a given fixture round.

Visulisation Functions:

- `team_strength()` , Display Only Attack or Defence Defence random effects for teams in a league
- `attack_defense()` , Display Both Attack and Defence effects on the same plot, from the INLA model summary
- `outcome_predict()` , gives the predicted probabilities of either winning, losing, or drawing a match.
- `joint_marginal()` , gives the plot of the joint posterior probabilities for specified teams Plots the joint posterior distributions of all the possible scores between two team given the predicted goals from `make_scored()`

Post-Processing Functions

- `updated_unplayed()` , extracts the unplayed fixtures for a given round and inputs the most common result (i think this should instead be inputting just the sample mean of the column of goals scored for the team rounded?)
- `update_footie()` , updates the premfootie dataset with the goal predictions
- `var_updater()` , updates the other variables in the dataset as a result of the goal predictions
- `roundNinla()` , streamlines the simulating process, by completing all of the above 3 post-processing for a given round N , e.g round 24

Outcome predictions

Running `inla.summary` we are able to extract coefficients for fixed and random effect estimates of our models. For example in the case below we consider a full model with all covariates to examine results in the 2022/23 Serie A season:

Table 2: Serie A Fixed Effects; only 2022-2023 Seasons

	mean
(Intercept)	-0.739
Home	-0.014
diff_point	0.001
diff_rank	0.004
form	0.123
rel_strength	0.303
days_since_last	0.000
Gpg	0.163
GCpg	0.284
GDdiff	0.179

Table 3: Serie A Hyperparameters and Precision; only 2022-2023 Seasons

	mean
Precision for factor(Team)	19027.68
Precision for factor(Opponent)	19118.21

From the model summary's fixed effect table, we see that a few variables namely Rank Difference, Point Difference, and Days Since Last Game, have negligable coefficient means relative to the other variables. We also notice from the output of the model hyperparameter summary that the precision for the random effects are significantly high, indicating very little variation is accounted for both random effects effects.

Both output summaries indicate that the use of some variables, along with the use of only ony season's worth of data may result in the model overfitting the data, leading us to the next section where we aim to mitigate this.

Introduction of Multiple Seasons' Data

As aforementioned in 'The Model' section, upon plotting the defence effect for leagues such as Serie A, we saw effect estimates that were too closely centred around the mean, and the precision for the hyperparameters were suspiciously higher than one would expect. This highlighted the lack of variability accounted for in the model due to overfitting Thus the model may be capturing the noise rather than the underlying pattern, in this case resulting in the lack of variability in th Opponent random effect.

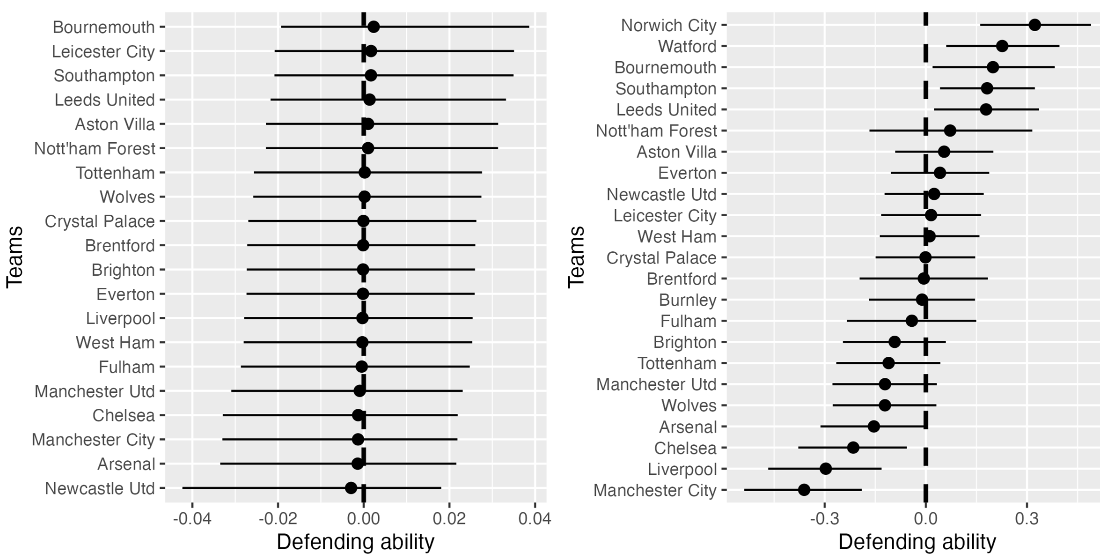
To mitigate this concern we include the use of multiple seasons data as opposed to using a single season, that only provides 760 rows of data. With the inclusion of the 2019/20, 2020/21, and 2021/22 along with the 2022/23 seasons, this now results in dataframes with 3040 rows of data for each league. The effects of overfitting are also demonstrated in the random effect quantile plots, particularly for the Opponent (Defence) factor. Consider the case of the Premier League, where the magnitude of the effects are considerably small and closely centred around the mean;

Table 4: Hyperparameter Precision using only 2022-2023 Season

	mean
Precision for factor(Team)	11.605
Precision for factor(Opponent)	18194.995

Table 5: Hyperparameter Precision using 2019-2023 Seasons

	mean
Precision for factor(Team)	12.364
Precision for factor(Opponent)	33.254



After adding multiple seasons of data, we see how there is now noticeable variability in the defence effect for all teams. Thus demonstrated how we have mitigated the impact of overfitting by adding more data from multiple seasons. We also see this in the inla.summary output shown below, where the precision for the model hyperparameters are considerably smaller; mean 18194.995 when only using 2022/23 compared to 33.254 for 2019-2023 data.

Result Generation

Now that we are able to generate predictions using the `inla.posterior.sample` method for the number of goals scored for each team after a given round, we are then able to get predicted match outcomes for the selected fixtures. In similar research the use of posterior means or medians is used from both individual team's marginal distribution, however in our modelling we use the most probable outcome from the joint posterior predictive distribution in each fixture, i.e. the most common row outcome between two teams after each n row simulations are complete.

For instance, using the baseline model in round 23 of the 2022/23 La Liga season, in a match between Villarrael and Getafe. We can visually display the joint marginal distribution of goals scored in this specific match, using our created function `joint_marginal()`.

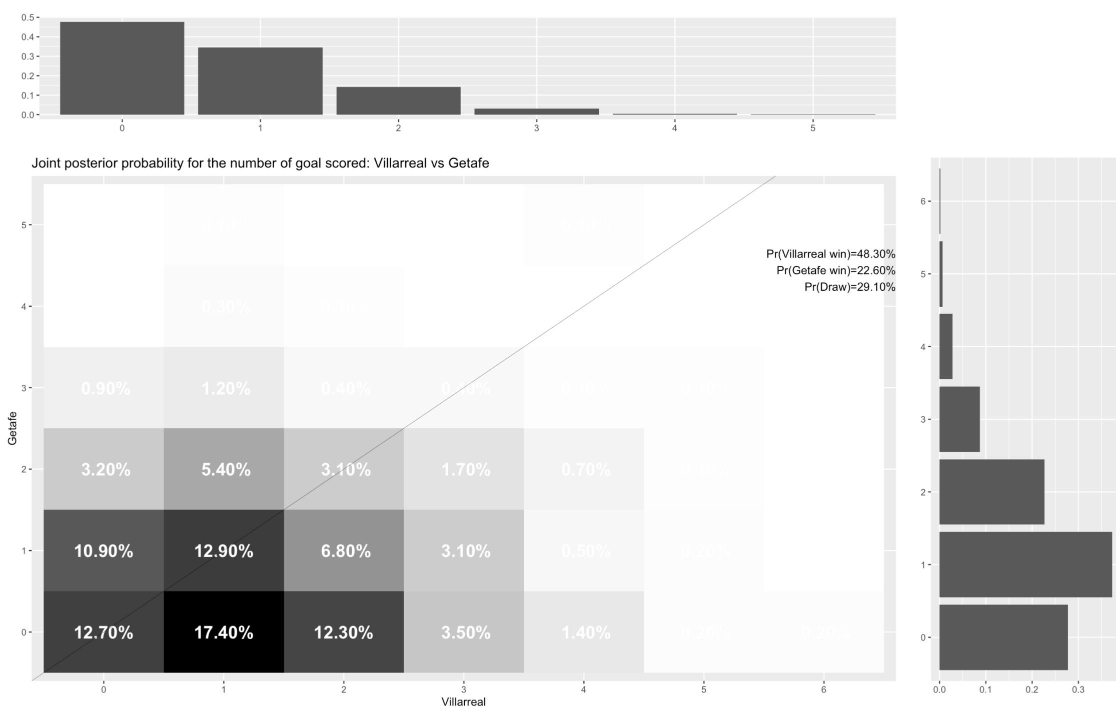


Figure 2: Plotting the Joint Marginal Distriubtion of goals scored between Villarrael and Getafe

Here the percentages represent the proportion of outcomes with the given result, thus in this case 17.4% of outcomes in the match see Villarrael as victors specifically with a score of 1-0.

We see the probability of every possible outcome from the joint posterior distribution, along with the overall probability of winning, drawing, or losing found using our created `outcome_predict()` function. The histograms represents the sampled marginal posterior distribution for teams individually, with highest density of 0 goal scored for Getafe (top), and 1 for Villarrael (Right).

After determining the most likely result of relevant 'Goal' column, as well as the associated variables derived from it such as Points , Goal Difference, Relative Strength etc. Although as before these are not used in the baseline model.

In the case of Bundesliga, we calculate the cumulative points determined from goals scored in each game, and derive the league table at the end of the season: We first show the results when using the Mean of each teams predicted goals after n simulations, compared to using the joint predictive posterior distribution. Using means shows Bayern Munich winning the league, where as the most probable outcome from the joint distributions leads to Dortmund winning the league

Model Selection:

Typically in INLA and hierarchical modelling (Rue et al. 2009), the use of Deviance Information Criterion and Watanabe-Alike-Information-Criterion are use as methods of comparing models in context. DIC is a model that is analogous to the Akaike information criterion (AIC) used in estimating prediction error, instead measuring the trade off between model fit and complexity calculated as follows:

$$DIC(\lambda) = D(\bar{\lambda}) + 2p_D$$

where $D(\bar{\lambda})$ is the deviance at the posterior mean of the parameters and p_D is the effective number of parameters.

Similarly the Watanabe-Akaike Information Criterion is a indicator of singular model performance. As defined by Watanabe himself, (Gelman et al. 2014) WAIC is the "negative of the average log pointwise predictive density and thus is divided by n, and does not have the factor of 2", although in INLA is is scaled by factor of 2 for comparability with DIC and other measures of deviance. DIC makes the assumption that the posterior is approximately multivariate normal, while WAI can be numerically calculated without information on the true distribution. WAIC also an extension of the widely used AIC in the Bayesian context, and also offers the bonus of no need for effective parameters, again unlike DIC. Given the added applicability to general BHM methods, the WAIC is a better indicator of model suitability.

Optimised Model Formulas for Log Linear Models

Premier League

Below is a table showing the top 10 models with the lowest WAIC, indicating top 10 most suitable models given the fact that lower WAIC demonstrates desirable lower deviance. Consider the example below of the Premier League, where the formula that results in the lowest WAIC includes added covariates Form, Goals Per Game (Gpc), Goals Conceded Per Game (GCpg), and Goal Difference-Difference between teams (GDdiff).

Table 6: Top 10 Model Combinations By WAIC: Premier League

Formula	WAIC
Goal ~ Home + form + Gpg + GCpg + GDdiff + Att + Def	6642.466
Goal ~ Home + form + days_since_last + Gpg + GCpg + GDdiff + Att + Def	6643.580
Goal ~ Home + rel_strength + form + Gpg + GCpg + GDdiff + Att + Def	6643.728
Goal ~ Home + diff_rank + form + Gpg + GCpg + GDdiff + Att + Def	6643.918
Goal ~ Home + Gpg + GCpg + GDdiff + Att + Def	6644.575
Goal ~ Home + rel_strength + form + days_since_last + Gpg + GCpg + GDdiff + Att + Def	6644.813
Goal ~ Home + rel_strength + diff_rank + form + Gpg + GCpg + GDdiff + Att + Def	6644.870
Goal ~ Home + diff_rank + form + days_since_last + Gpg + GCpg + GDdiff + Att + Def	6645.032
Goal ~ Home + days_since_last + Gpg + GCpg + GDdiff + Att + Def	6645.622
Goal ~ Home + rel_strength + Gpg + GCpg + GDdiff + Att + Def	6645.838

Thus the optimal formula for the Premier League model after this part of the model building process is of the form:

Home Goals in game i:

$$log(\lambda_{1i}) = att_{h_i} + def_{a_i} + \beta_1 Home_j + \beta_2 form + \beta_3 Gpg + \beta_4 GCpg + \beta_5 GDdiff$$

Away Goals in game i:

$$log(\lambda_{2i}) = att_{h_i} + def_{a_i} + \beta_1 form + \beta_2 Gpg + \beta_3 GCpg + \beta_4 GDdiff$$

As for the other domestic leagues, the optimised formulae for the log linear models are:

La Liga:

Home Team:

$$log(\lambda_{1i}) = att_{a_i} + def_{h_i} + \beta_1 Home + \beta_2 Gpg + \beta_3 GCpg + \beta_4 GDdiff$$

Away Team:

$$log(\lambda_{2i}) = att_{h_i} + def_{a_i} + \beta_1 Gpg + \beta_2 GCpg + \beta_3 GDdiff$$

Serie A:

Home Team:

$$log(\lambda_{1i}) = att_{a_i} + def_{h_i} + \beta_1 Home + \beta_2 Gpg + \beta_3 GCpg + \beta_4 GDdiff + \beta_5 Form$$

Away Team:

$$log(\lambda_{2i}) = att_{h_i} + def_{a_i} + \beta_1 Gpg + \beta_2 GCpg + \beta_3 GDdiff + \beta_4 form$$

Bundesliga:

Home Team:

$$log(\lambda_{1i}) = att_{a_i} + def_{h_i} + \beta_1 Home + \beta_2 Gpg + \beta_3 GCpg + \beta_4 GDdiff$$

Away Team:

$$log(\lambda_{2i}) = att_{h_i} + def_{a_i} + \beta_1 Gpg + \beta_2 GCpg + \beta_3 GDdiff$$

Ligue 1:

Home Team:

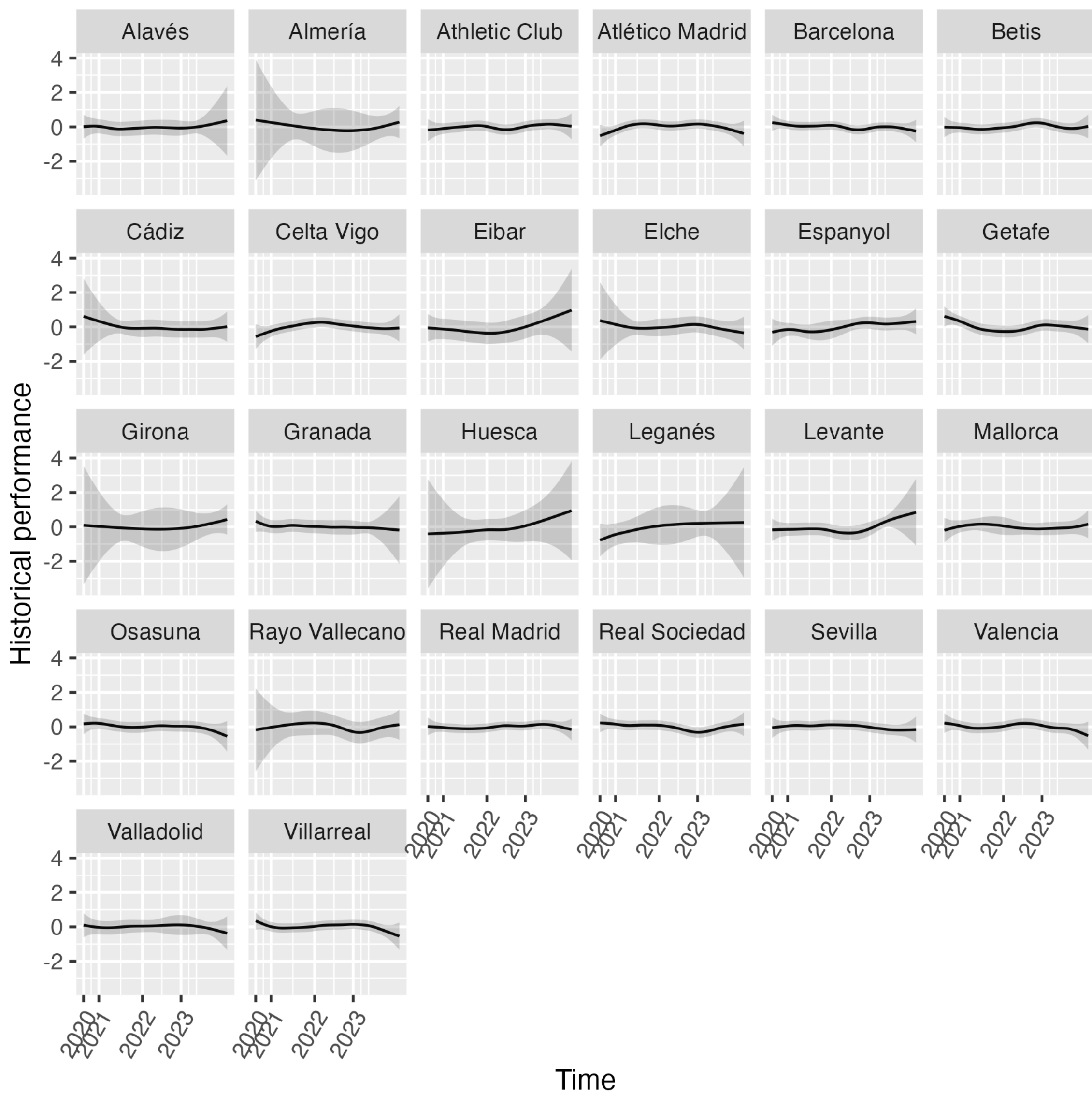
$$log(\lambda_{1i}) = att_{a_i} + def_{h_i} + \beta_1 Home + \beta_2 Gpg + \beta_3 GCpg + \beta_4 GDdiff + \beta_5 diffrank$$

Away Team:

$$log(\lambda_{2i}) = att_{h_i} + def_{a_i} + \beta_1 Gpg + \beta_2 GCpg + \beta_3 GDdiff + \beta_4 diffrank$$

Aditional Covariate Consideration

We also explore the addition of other covariates that may further improve model performance. First, a time component that aims to address team's difference in performance over time. This is added to the INLA model using `f(id_date,model="rw2",replicate=as.numeric(factor(Team)))`. A Random Walk model of order 2 `model="rw2"` based on the fixture date variable representing the time component, is computed separately for each team `replicate=as.numeric(factor(Team))`. The effects from the Random Walk over the course of the dates in our dataset (between the start of the 2019 season and end of the 2023 season) are shown in the plot below:



Another are of Bayesian Hierarchical Modelling to consider is overdispersion, a common feature in count data like goals scored in football. Overdispersion occurs when variance of the observed data is greater than what would be expected under a simple Poisson distribution, thus we aim to capture extra variability that may not be accounted for with other covariates. This is demonstrated in the code by `f(num,model="iid")` which adds another random effect to the INLA model, based on 'num' which is the unique identifier of every row in the data.

However after adding these variables individually and together, they did not improve model fit by WAIC; adding the random walk time component increased deviation, while adding the overdispersion parameter neither improved or negatively impacts WAIC by a noticeable amount. We note the very high in precision in the model hyperparameters for these added random effects via the summary output, demonstrating the lack of variability that is captured by these covariates. Thus we continue without them.

Model Validation

In model validation we aim to compare the effectiveness of the more complex 'optimised' model, based off WAIC, and the baseline model. This is executed using all the prior seasons data, with observed data up until halfway through the 2021/22 season. We then simulate the remainder of the 2021/22 seasons for each league, and compare with the observed results of the final league positions for each team. The mean square error of the optimised and baseline model are compared, and we proceed to the 2022/23 modelling based on which model has lower MSE. Consider the case of the Premier League model validation, the table below shows the predictive performance of the

Using the figures above we are able to calculated the mean squared error for the baseline model and optimised models respectively; 115.55, 100.65 thus demonstrating we have achieved better performance using the optimised formula in the log linear model for the scoring parameters. The improved performance is consistent with results of Bundesliga, Serie A, and La Liga, all of which had lower MSE than their baseline models. However in the case of Ligue 1, as the baseline model achieves lower MSE. This may be due to the reduce number of data points used in modelling. In the 2019/20 seasont the COVID-19 Pandemic resulted in the distruption of football scheduling in the domestic leagues, and while the other four returned to action, Ligue 1 ended halfway through. Thus when modelling using data from 2019-2023, there are missing observations that were removed for the French league. This means that the baseline model, which is stronger at identifying variability in team attack and defence random effects, may better fit the league with fewer data available, since the otimised model may overfit the data.

We next model the 2022/23 seasons using the optimised log linear models for the Premier League, Bundesliga, Serie A, and La liga, whilst using the baseline model for Ligue 1.

Table 7: Comparing Baseline and Optimised models Performance vs Obesrved

Team	Observed_Points	Baseline_Points	Baseline_Difference	Optimized_Points	Optimized_Difference
Arsenal	69	56	13	56	13
Aston Villa	45	61	16	47	2
Brentford	46	35	11	37	9
Brighton	51	36	15	44	7
Burnley	35	33	2	35	0
Chelsea	74	67	7	61	13
Crystal Palace	48	57	9	42	6
Everton	39	37	2	39	0
Leeds United	38	44	6	36	2
Leicester City	52	51	1	47	5
Liverpool	92	65	27	68	24
Manchester City	93	86	7	74	19
Manchester Utd	58	60	2	56	2
Newcastle Utd	49	35	14	35	14
Norwich City	22	34	12	31	9
Southampton	40	41	1	42	2
Tottenham	71	61	10	56	15
Watford	23	32	9	28	5
West Ham	56	62	6	54	2
Wolves	51	56	5	51	0

Final Predictions

Now that we have final models to predict performances in the European domestic leagues, we return to the original questions; How will the teams perform in the 2022/2023 season? How will the results fair with the removal of the Super League Teams?

Predicting with Super League Teams Included

After modelling the 2022/23 seasons with the proposed ‘founding’ members of the super league and the teams likely to join, the league positions are as follows: We notice that (summarise table here)

League Position	Team (Premier League)	Team (La Liga)	Team (Serie A)	Team (Ligue 1)	Team (Bundesliga)
1	Arsenal	Barcelona	Napoli	Paris S-G	Bayern Munich
2	Manchester City	Real Madrid	Inter	Lens	Dortmund
3	Manchester Utd	Real Sociedad	Juventus	Marseille	Union Berlin
4	Newcastle Utd	Atlético Madrid	Milan	Monaco	Eint Frankfurt
5	Tottenham	Betis	Roma	Lille	Freiburg
6	Brentford	Rayo Vallecano	Lazio	Lorient	RB Leipzig
7	Brighton	Athletic Club	Atalanta	Nice	Wolfsburg
8	Fulham	Mallorca	Bologna	Rennes	M’Gladbach
9	Liverpool	Villarreal	Torino	Reims	Köln
10	Aston Villa	Osasuna	Udinese	Toulouse	Mainz 05
11	Nott’ham Forest	Girona	Monza	Clermont Foot	Leverkusen
12	Chelsea	Sevilla	Empoli	Lyon	Werder Bremen
13	Leeds United	Celta Vigo	Lecce	Nantes	Augsburg
14	Crystal Palace	Espanyol	Fiorentina	Montpellier	Bochum
15	Leicester City	Valladolid	Sassuolo	Troyes	Hertha BSC
16	Bournemouth	Almería	Salernitana	Ajaccio	Hoffenheim
17	Everton	Cádiz	Spezia	Auxerre	Stuttgart
18	Wolves	Getafe	Hellas Verona	Brest	Schalke 04
19	West Ham	Valencia	Sampdoria	Strasbourg	NA
20	Southampton	Elche	Cremonese	Angers	NA

Predicting Without Super League Teams

After modelling the 2022/23 seasons without the proposed ‘founding’ members of the super league and the teams likely to join, the league positions are as follows: We notice that (summarise table here)

League Position	Team (Premier League)	Team (La Liga)	Team (Serie A)	Team (Ligue 1)	Team (Bundesliga)
1	Newcastle Utd	Real Sociedad	Atalanta	Marseille	Eint Frankfurt
2	Fulham	Betis	Bologna	Lens	Freiburg
3	Brighton	Athletic Club	Cremonese	Lille	Union Berlin
4	Aston Villa	Osasuna	Empoli	Lorient	RB Leipzig
5	Brentford	Rayo Vallecano	Fiorentina	Monaco	Wolfsburg
6	Crystal Palace	Girona	Hellas Verona	Nice	Mainz 05
7	Nott’ham Forest	Mallorca	Lazio	Rennes	Leverkusen
8	West Ham	Sevilla	Lecce	Reims	M’Gladbach
9	Wolves	Villarreal	Monza	Toulouse	Werder Bremen
10	Everton	Celta Vigo	Napoli	Clermont Foot	Köln
11	Leicester City	Valladolid	Roma	Lyon	Bochum
12	Leeds United	Espanyol	Salernitana	Nantes	Hertha BSC
13	Bournemouth	Getafe	Sampdoria	Montpellier	Augsburg
14	Southampton	Almería	Sassuolo	Ajaccio	Hoffenheim
15	NA	Cádiz	Spezia	Troyes	Stuttgart
16	NA	Valencia	Torino	Auxerre	Schalke 04
17	NA	Elche	Udinese	Brest	NA
18	NA	NA	NA	Strasbourg	NA
19	NA	NA	NA	Angers	NA
20	NA	NA	NA	NA	NA

Discussion:

Conclusions

- talk about what we foud in our results compare the Withoutsuperleague to withoutsuperleague predictions,
- what are the supposed effect of these predictions? the significance?

Limitations

- A common probablem of the Bayesian Hierarchical model is overshrinkage where extreme outcomes, like a team scoring 8 goals, are instead pulled towards the grand mean. We notice this in the use of
- Can improve performance using more data
- Lack of Covariates

Future Work

- 3 Tier Hierarchical Model
- Machine Learning Techniques?
- add more complex covariates ? player specific level perhaps?

etc.

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