Notation $\quad f(t) = \text{PDF} = e^{-t} \quad (t>0)$, as example,

$$F(s) = \text{L.T. of } f(t) = \frac{1}{1+s} \quad (s>0)$$

We wish to check numerical inversion by Gaver-Stehfest formulae.

For given n (chosen later as, say, 5) define

$$c_k(n) = (-1)^{n+k}\left[\frac{k^n}{k!\,(n-k)!}\right] \quad \text{for } k = 1, 2, \ldots, n$$

Define

$$\hat{f}_k(t) = \left[\frac{\ln(2)}{t}\right]\left[\frac{(2k)!}{k!\,(k-1)!}\right]\left[\sum_{i=0}^{k} \binom{k}{i}\cdot(-1)^i\cdot F\left(\frac{(k+i)\ln 2}{t}\right)\right]$$

$$(1 \le k \le n)$$

for some values of t, e.g. $t = 0.1, 0.2, 0.3, \ldots, 3.0$

Now define

$$f_n(t) = \sum_{k=1}^{n} c_k(n)\cdot\hat{f}_k(t) \quad \text{(for same values of } t)$$

Results. Draw up a table showing:

t	$f(t)$	$f_n(t)$
0.1		
0.2		
...		
3.0		

With luck, $f_n(t)$ will be "approx." $f(t)$

Extensions. (1) Try different values of n, say 5, 6, 7, 8, 9, 10.

(2) Change to CDF, i.e. "$f(t)$" becomes "$1 - e^{-t}$" $(t>0)$

and we define "$F(s)$" = "[L.T. of $f(t)$] / s" $= \frac{1}{s(1+s)}$

We should again find agreement (approx.) between $f(t)$ and $f_n(t)$.

Later. If this works out, we change $F(s)$ to the more complex expression in D-test, i.e.

$$F(s) = \prod_{j,k=1}^{\infty}\left(1 + \frac{2s}{j^2 k^2}\right)^{-1/2}$$

and see if we can find $f_n(t)$ = approximate PDF