$$\frac{\partial p}{\partial y} = 0 \Rightarrow p(y=0) = p(H)$$

$$p(H) = P_{atm}$$

$$\frac{dp}{dx} = 0$$

```
In[1]:= ClearAll["Global`*"]; (* exam#2 F22 Problem 1 *)

In[2]:= eq = μ ∂y,y vx[y] + ρ g (* part a *)

Out[2]= g ρ + μ vx''[y]

In[3]:= τ == ∂y vx[y]; τ /. y → H == 0;
        vx[0] == 0; (* part b *)
```

no force on edge of film

```
In[5]:= DSolve[{eq == 0, vx[0] == 0, ((∂y vx[y]) /. y → H) == 0}, vx[y], y]
```

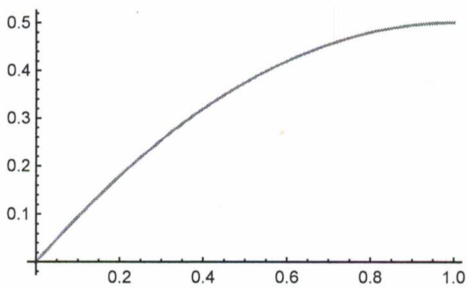
$$\text{Out[5]=}\ \left\{\left\{vx[y] \to \frac{g\,(2 H y - y^2)\,\rho}{2\,\mu}\right\}\right\}$$

$$\text{In[6]:=}\ vx = \frac{g\,(2 H y - y^2)\,\rho}{2\,\mu};\ \text{(* part c *)}$$

```
g = ρ = H = μ = 1; Plot[vx, {y, 0, H}] (* general shape of velocity profile *)
```

$$\frac{V_x}{\rho g/\mu}$$

Out[7]=



y/H

$$\tau = \mu \frac{\partial V_x}{\partial x} = 0$$ no stress at $y = H$

```
In[8]:= ClearAll[g, ρ, H, μ]
```

$$\text{In[9]:=}\ Q == W \int_0^H vx\, dy$$

$$\text{Out[9]=}\ Q == \frac{g H^3 W \rho}{3\,\mu}$$

$$\text{In[ ]:=}\ \text{Solve}\left[Q == \frac{g H^3 W \rho}{3\,\mu}, H\right]$$

$$H == \left(\frac{3 Q \mu}{g W \rho}\right)^{1/3}\ \text{(* part d *)}$$

```
v = {vx, 0}; n = {1, 0}; dA == W dy; gDotX == ρ vx;
v.n
```

$$\text{In[10]:=}\ \text{GdotX} = W \int_0^H ((\rho\, vx)\ (v.n))\, dy$$

Out[10]=

$$\cancel{\frac{g H^3 W \rho^2 v.n}{3 \mu}} \quad \frac{2}{15} \frac{g^2 H^5 W \rho^3}{\mu^2} \quad !?$$

$$\left(\frac{\text{meter}}{\text{sec}^2}\right)^2 \text{meter}^5\,\text{meter}\,\frac{\text{kg}}{\text{meter}^3}\left(\frac{\text{newton sec}^2}{\text{meter}^4}\right)^2\left(\frac{\text{meter}^2}{\text{newton sec}}\right)^2 \quad \text{(* check weird units *)}$$

Out[ ]=

$$\frac{\text{kg meter}}{\text{sec}^2}$$

OK

In[11]:= ClearAll["Global`*"]; (* exam#2 F22 Problem 1 *)

In[ ]:= $eq = -p'[x] + \mu\,\partial_{y,y}vx[y]$ (* part a *)

Out[ ]=

$$-p'[x] + \mu\, vx''[y]$$

In[ ]:= vx[H] == 0;
vx[0] == V; (* part b *)

In[ ]:= DSolve[{eq == 0, vx[0] == V, vx[H] == 0}, vx[y], y]

Out[ ]=

$$\left\{\left\{vx[y] \to \frac{2 H V \mu - 2 V y \mu - H^2 y\, p'[x] + H y^2 p'[x]}{2 H \mu}\right\}\right\}$$

In[ ]:= $vx = \frac{p'[x]\, H^2}{2\mu}\frac{y}{H}\left(-1+\frac{y}{H}\right) + V\left(1-\frac{y}{H}\right)$; (* part c *)

In[ ]:= $Q = W\int_0^H vx\, dy$

Out[ ]=

$$W\left(\frac{H V}{2} - \frac{H^3 p'[x]}{12\mu}\right)$$

In[ ]:= Solve[{Q == 0}, p'[x]]

Q = 0 is key step

Out[ ]=

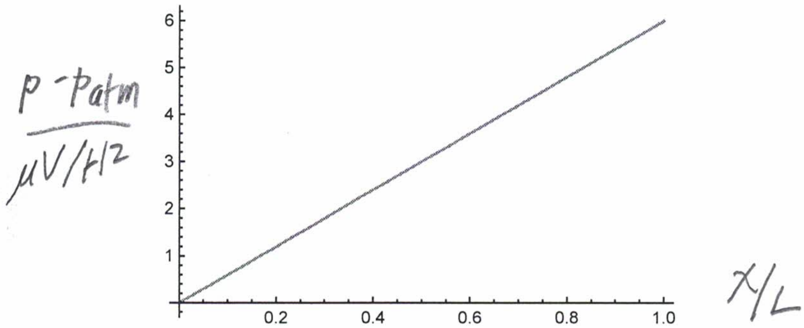
$$\left\{\left\{p'[x] \to \frac{6 V \mu}{H^2}\right\}\right\}$$

In[ ]:= $\text{DSolve}\left[\left\{p'[x] == \frac{6 V \mu}{H^2}, p[0] == pAtm\right\}, p[x], x\right]$

In[16]:= $p = pAtm + \frac{6 V x \mu}{H^2}$; (* part d *)

In[21]:= `pAtm = 0; V = μ = H = L = 1; Plot[p, {x, 0, L}]`

Out[21]=



In[12]:= $dpdx = \frac{6 V \mu}{H^2}$; $vx = \frac{dpdx\, H^2}{2 \mu} \frac{y}{H} \left(-1 + \frac{y}{H}\right) + V \left(1 - \frac{y}{H}\right)$;

In[13]:= $\tau = \text{Simplify}\left[\mu\, \partial_y vx\right]$

Out[13]=

$$-\frac{2 V (2 H - 3 y) \mu}{H^2}$$

In[17]:= `σ = {{-p, τ}, {τ, -p}};`

In[18]:= `σ1 = σ /. y → 0`

Out[18]=

$$\left\{\left\{-pAtm - \frac{6 V x \mu}{H^2}, -\frac{4 V \mu}{H}\right\}, \left\{-\frac{4 V \mu}{H}, -pAtm - \frac{6 V x \mu}{H^2}\right\}\right\}$$

In[19]:= `n = {0, 1}; dA == W dx;  f1 = n.σ1`

Out[19]=

$$\left\{-\frac{4 V \mu}{H}, -pAtm - \frac{6 V x \mu}{H^2}\right\}$$

In[ ]:= $Fx = W \int_0^L f1_{[[1]]}\, dx$

$Fy = W \int_0^L (f1_{[[2]]} + pAtm)\, dx$

Out[ ]=

$$-\frac{4 L V W \mu}{H}$$

Out[ ]=

$$-\frac{3 L^2 V W \mu}{H^2}$$

In[ ]:= $F1 = -\frac{L^2 V W \mu}{H^2} \left\{4 \frac{H}{L}, 3\right\}$ (* part e *)

Fluid Mech MANE 6520 Exam #2

Grading rubric

$\boxed{1.}$ a1) Recognize $p = const = p_{atm}$ $\frac{dp}{dx} = 0$ (-3)

a2) gravity g in $+x$ direction (-3)

b) $\tau = \mu \frac{dv_x}{dy}\Big|_{y=H} = 0$ (-3)

c) Integrate, find constants methodology (-5)

d) $Q = \int v\,dy \Rightarrow H$ (-5)

e) $\dot{G}_x = \int_0^H \rho v_x (\underset{\sim}{v} \cdot \hat{n})\, w\, dy$ (-5)

$\boxed{2.}$ a) $\frac{dp}{dx} = \mu \frac{d^2 v_x}{dy^2}$ (-3) $\frac{\partial p}{\partial y} = 0$ (-2)

b) $y = 0$ $v = V$ (-3) $y = H$ $v = 0$ (-3)

$x = 0$ $p = p_{atm}$

c) integrate find constants methodology (-5)

Recognize $Q = \int v_x\,dy = 0$ (-5)

d) $\frac{dp}{dx} = const$ (-3) $p(x=0) = p_{atm}$ (-2)

(poss. Reyn Eq)

e) $F_x = \int \tau\,dx$ (-3)

$F_y = \int p\,dy$ (-3)