Forward Difference Approximation

We Need Derivatives for Gradient-Based Optimization

The steepest descent method relies on $-\nabla f$ to locate a minimum, and it is not alone:

- Newton's method, quasi-Newton methods, and the nonlinear Conjugate-Gradient method are all derivative-based methods that need. . . ,well, derivatives!

Unfortunately, few engineering analysis codes provide ∇f, so it is up to us to compute the gradient.

We Can Approximate the Gradient Using the Forward-Difference Method

The class of finite-difference approximations use Taylor's theorem to construct difference formulae that approximation the derivative.

- The forward-difference approximation is the simplest and most commonly used finite-difference method in optimization.

The Forward-Difference Method is Easy to Derive

Forward-Difference Approximation: Summary

Definition: Forward-Difference Approximation

The forward-difference approximation of the partial derivative $\partial f / \partial x_j$ of the function $f : \mathbb{R}^n \to \mathbb{R}$ is given by

$$\frac{f(x + he_j) - f(x)}{h} = \frac{\partial f}{\partial x_j} + \underbrace{L_{\mathrm{FD}} h}_{\text{error}}$$

where $h > 0$ is the step size, e_j is the j^{th} Cartesian basis vector, and L_{FD} is a constant that does not depend on h.