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Class #10

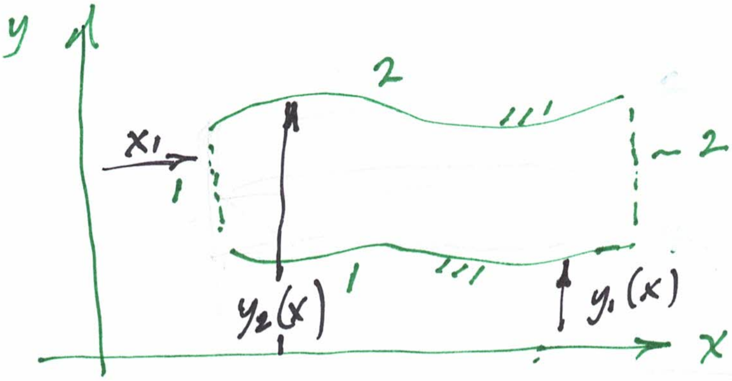
Boundary Conditions

N-S equations (2-D, incompressible, Newtonia)

$$0 = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}$$

$$\rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} \right)$$

$$\rho \left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} \right) - \rho g$$



IC

$t=0$ $v_x =$

$v_y =$

BC $y = y_1(x)$ $v_x =$ $v_y =$ $y = y_2(x)$ $v_x =$ $v_y =$

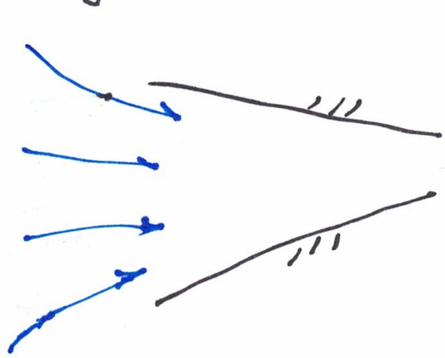
$x = x_1$ $v_x =$ $v_y =$ $p =$ $x = x_2$ $v_x =$ $v_y =$

mathematically

$\frac{\partial v_x}{\partial y} = 0$?

$v_y = 0$?

$p = p_1$



$p = p_2$?

$\frac{\partial v_x}{\partial y} = 0$?

often these

Order of Magnitude Concept

usually powers of 10

How big are molecules of blood? 10^{-9} m

How far to the sun? 10^{9} m

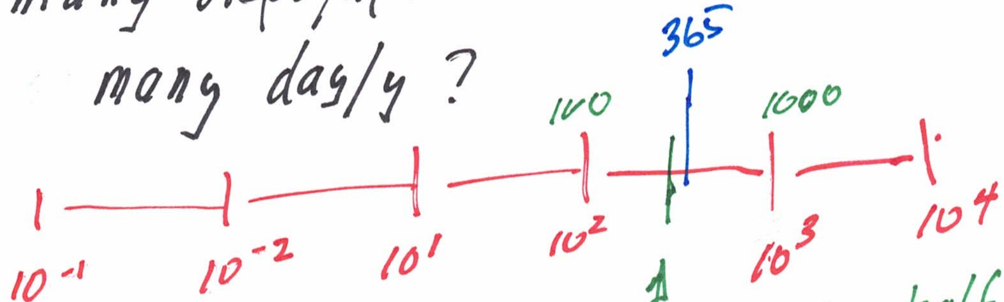
How much volume of material loss from shoe per step?

Eyeball: $1\,cm^3/year? = (10^{-2})^3 = 10^{-6}\,\frac{m^3}{y}$

How many km/day? ~ 1 km $= 1000\,\frac{m}{day}$

How many steps/m? ~ 1 step/m

How many day/y?



301 is half way logarithmically

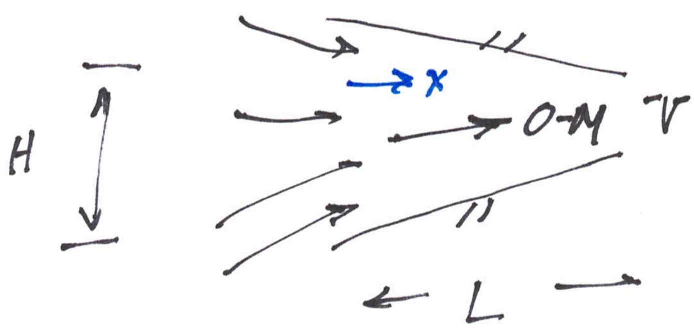
close call say $\frac{10^3\,day}{y}$

$$10^{-6}\,\frac{m^3}{\cancel{y}}\cdot\frac{1\,\cancel{y}}{10^3\,\cancel{day}}\cdot\frac{\cancel{day}}{10^3\,\cancel{m}}\cdot\frac{1\,\cancel{m}}{step} = 10^{-12}\,\frac{m^3}{step} = (10^{-4}\,m)^3/step$$

$$= \frac{0.1\,mm^3}{step}$$

O-M!

Non-dimensional (dimensionless)



parameters

$L, H, \mathcal{V}, \mu, \rho$, etc.

Treat as constant for particular problem

May vary from problem-to-problem

x, y, z, t

independent variables

v, τ, P dependent variables $H \frac{\partial P}{\partial x}$

$V_x^* = \frac{V_x}{\mathcal{V}}$ ← reference $\qquad V_x = V_x^* \mathcal{V} \qquad V_x^* \sim 1$

x^*

$x^* = \frac{x}{L} \qquad \mathcal{V} \sim 1$ m/s not 1 km/s

not 1 mm/s

$y^* = \frac{y}{H} \qquad \mathcal{V}, L, H$ scale factors

usually constant

$$\frac{\partial V_x}{\partial x} = \frac{\partial V_x}{\partial x^*}\frac{\partial x^*}{\partial x} = \frac{\partial}{\partial x^*}(\mathcal{V} V_*)\frac{1}{L} = \frac{\mathcal{V}}{L}\frac{\partial V_x^*}{\partial x^*}$$

scale factors if $L, \mathcal{V}; H$ const

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Continuity (2-D incompressible)

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0$$

order $\sim V$

$O[V]$

$y^* = \frac{y}{H} \sim 1$ $x^* = \frac{x}{L} \sim 1$ $V_x \sim V$

$V_x^* = \frac{V_x}{V}$

$y \sim H$ $x \sim L$

$$\frac{\partial V_x}{\partial x} \sim O\left(\frac{V}{L}\right) = \frac{V}{L} \underbrace{\frac{\partial V_x^*}{\partial x^*}}_{\sim 1}$$

V_r = O-M V_y

= ?

V_y = reference velocity in y-direction

$$\frac{\partial V_y}{\partial y} \sim O\left(\frac{V_r}{H}\right) = \frac{V_r}{H} \underbrace{\frac{\partial V_y^*}{\partial y^*}}_{\sim 1}$$

$$\underbrace{\frac{V}{L}\frac{\partial V_x^*}{\partial x^*}}_{} + \underbrace{\frac{V_r}{H}\frac{\partial V_y^*}{\partial y^*}}_{} = 0$$

same O-M

$$\underline{\underline{V_r = V\frac{H}{L}}}$$

if $H/L << 1$ $V_r << V$

$\underline{\underline{V_x^* \sim V_y^*}}$

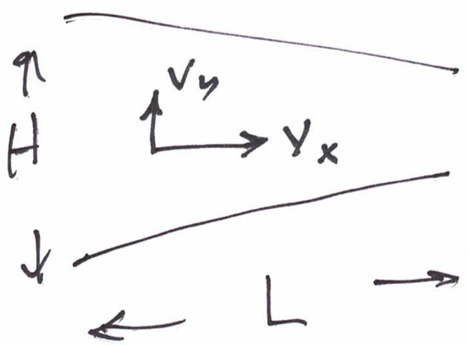
same O-M

$$\underbrace{\frac{\partial V_x^*}{\partial x^*}}_{\sim 1} + \underbrace{\frac{\partial V_y^*}{\partial y^*}}_{\sim 1} = 0$$

$V_y << V_x$

$$\frac{\partial}{\partial x} \sim \frac{1}{L} \qquad \frac{\partial}{\partial y} \sim \frac{1}{H}$$ paramters

$V_x \sim V$ known if possible



$$V_x \sim \frac{H}{L} V_y$$

$$\frac{H}{L} \sim 0.01$$

$$V_y \sim V \frac{H}{L}$$

"Inertia Force" "interia

call $\underline{ma} = \underline{\text{force}}$"

Not really a force

-5- Navier - Stokes Equation

$$\underset{A}{\rho\left(\frac{\partial V_x}{\partial t}\right.} + \overbrace{\underset{a}{V_x \frac{\partial V_x}{\partial x}} + \underset{b}{V_y \frac{\partial V_x}{\partial y}}}^{B} \left.\right) = -\frac{\partial p}{\partial x} + \mu\left(\underset{C}{\frac{\partial^2 V_x}{\partial x^2}} + \underset{D}{\frac{\partial^2 V_x}{\partial y^2}}\right) \underset{E}{+\rho g_x}$$

$\rho \sim ?$ $\quad \frac{\partial}{\partial t} \sim$ say vibration problem

$t \sim$ period $= \underset{(s)}{t_p} \sim \frac{1}{\omega}$ (1/s)

$\frac{\partial}{\partial t} \sim \omega \frac{\partial}{\partial t^*}$

convective inertia (inertia)

A: $\rho \omega V \frac{\partial V_x^*}{\partial t^*}$

unsteady inertia

a. $\rho V \frac{1}{L} V$

b. $V \frac{H}{L} V \frac{1}{H} = \rho \frac{V^2}{L}$

)) same] B $= \rho \frac{V^2}{L}$

viscous elongational stress

C: $\mu \frac{V}{L^2}$

Note: $\frac{\partial^2}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\right) \sim \frac{1}{L^2}$

D: $\mu \frac{V}{H^2}$

viscous shear stress

Note all units $\frac{N}{m^3}$

e.g. a. $\frac{kg}{m^3} \frac{m^2}{s^2} \frac{1}{m}$

$F = ma$

$m = F/a$

$kg = \frac{N\text{-}s^2}{m}$

$\frac{N\text{-}\cancel{s^2}}{\cancel{m}} \frac{1}{m^3} \frac{\cancel{m^2}}{\cancel{s^2}} \frac{1}{\cancel{m}} = \frac{N}{m^3}$

$$\frac{B}{A} = \frac{L\omega}{V} = \frac{\text{unsteady}}{\text{convective}}\text{ inertia} = \text{Strouhal Number}$$

$$\frac{B}{D} = \frac{\rho V H}{\mu} \cdot \frac{H}{L} = \frac{\text{inertia force}}{\text{shear force}} = Re^{*} = \text{reduced Reynolds number}$$

$$= Re \frac{H}{L}$$

flat plate BL   channel

$$\frac{\rho V L}{\mu} \text{ or } \frac{\rho V H}{\mu} = \text{Reynolds number}$$

$$\frac{C}{D} = \frac{H^2}{L^2} = \frac{\text{extension deformation}}{\text{shear deformation}}$$

$\frac{H}{L}$ aspect ratio

$$\frac{B}{E} = \frac{V^2}{g_z L} = \frac{\text{inertia}}{\text{gravity}} = \text{Froude number}$$

Note - these are (governing) dimensionless parameters

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$$p^* = \frac{P}{P_{Ref}}$$ Aero etc $P_{Ref} = \rho V^2$ $\frac{N\text{-}s^2}{m^4}\,\frac{m^2}{s^2}\,\frac{1}{m}$

very viscous flow $P_{ref} = \frac{\mu}{H}\frac{V L}{H}$ $\boxed{\frac{N\text{-}s}{m^2}\,\frac{m}{s}\,\frac{1}{m}}$

$$Re^*\left(\frac{\partial V_x^*}{\partial t^*} St + V_x^*\frac{\partial V_x^*}{\partial x^*} + V_y^*\frac{\partial V_x^*}{\partial y^*}\right)$$

$$= -\frac{\partial p^*}{\partial x^*} + \frac{\partial^2 V_x^*}{\partial y^{*2}} + \left(\frac{H}{L}\right)^2\frac{\partial^2 V_x^*}{\partial x^{*2}}$$

Neglect gravity

start with $St << 1$ non-Vibration

$Re^* << 1$ negligible inerta

$\frac{H}{L} << 1$ thin, neglect extension

$$0 = -\frac{\partial p^*}{\partial x^*} + \frac{\partial^2 V_x^*}{\partial y^{*2}}$$