Contours and Level Curves

Contours, or Level Curves, Are Like Topographic Maps

Definition: Contours/Level Curves

A level curve (or contour) of $f : \mathbb{R}^n \to \mathbb{R}$ is a set of points $\mathcal{C}_k = \{x \mid f(x) = k\}$ for some constant $k \in \mathbb{R}$.

Contours Are Closely Related to the Gradient

Theorem

Suppose that $f : \mathbb{R}^n \to \mathbb{R}$ has continuous partial derivatives and that the gradient is nonzero at some point y, i.e. $\nabla f(y) \neq 0$. Then $\nabla f(y)$ is orthogonal (perpendicular) to the level curve $f(x) = k$ that passes through the point y.

Contours Are Closely Related to the Gradient (cont.)