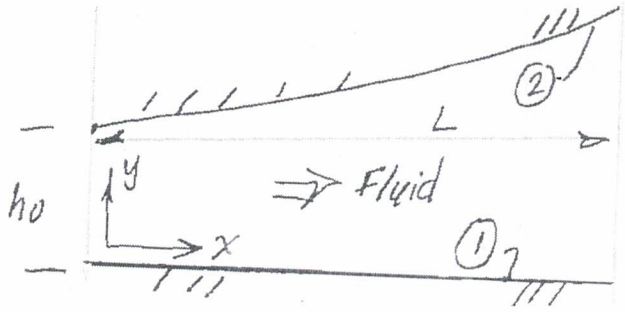
MANE 6520 - Fluid Mechanics, Fall 2022

Homework #2 - Thurs 15 Sept 2022, due Thurs 22 Sept 2022
Be precise about your notation.

Consider flow in a gap $h = h_0\left[1+\sin\left(m\frac{x}{L}\right)\right]$ with width W into the plane of the paper, $0 \le x \le L$, and $0 \le y \le h(x)$. The sine argument is in radians. The lower surface is 1, the upper surface is 2.



The stress tensor is $\boldsymbol{\sigma} = \begin{pmatrix} -p & \tau \\ \tau & -p \end{pmatrix}$, with $p = p_0\left(1-k\frac{x}{L}\right)$ and $\tau = \tau_0 \frac{x}{L}\frac{y}{h}$, with constants $L, h_0, m, k, p_0, \tau_0$.

1) Find the unit normals $\hat{\mathbf{n}}_1$ and $\hat{\mathbf{n}}_2$ (outward from the fluid).

2) Find the traction vector on the fluid at the two surfaces: $\mathbf{f}_1 = \frac{d\mathbf{F}_1}{dA}$ and $\mathbf{f}_2 = \frac{d\mathbf{F}_2}{dA}$.

3) Find the vector force on the *surface* 2, $\mathbf{F}_2$. (You will have to leave the answer as a definite integral)

4) Find the force on the *surface* 1, $\mathbf{F}_1$, in symbolic form. (In this case you can evaluate the definite integral)

Use the following parameter values: h_0 = 1 mm, L = 10 mm, p_0 = 1 MPa, τ_0 = 100 kPa, m = 0.2, k = 0.5, and W = 1 m.

5) Plot the traction components $t_{1x}(x)$ and $t_{1y}(x)$.

6) Find the force components on the two surfaces F_{1x}, F_{1y}, F_{2x}, and F_{2y}. In the case of F_{2x}, and F_{2y} you will likely need an integration tool such as **NIntegrate** in *Mathematica*.