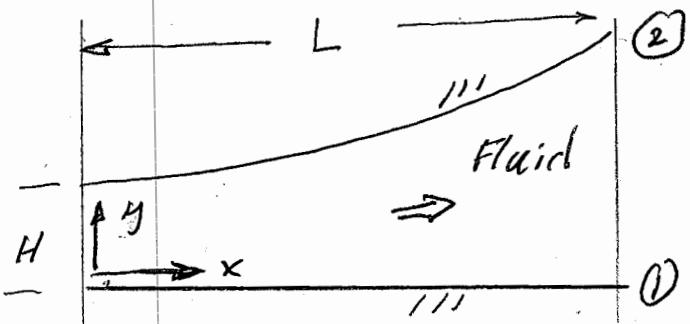
MANE 6520 - Fluid Mechanics, *Homework #2* - Mon 18 Sept 2023, due Mon 21 Sept

Consider flow in a gap $h = H\exp\left(m\frac{x}{L}\right)$ with width W into the plane of the paper, $0 \le x \le L$, and $0 \le y \le h(x)$. The lower surface is 1, the upper surface is 2.



The stress tensor is $\boldsymbol{\sigma} = \begin{pmatrix} -p & \tau \\ \tau & -p \end{pmatrix}$, with $p = p_0\left(1 - k\frac{x^2}{L^2}\right)$ and $\tau = \tau_0 \frac{x}{L}\frac{y^2}{h^2}$, with constants $L, h_0, m, k, p_0, \tau_0$.

1) Find the unit normals $\hat{\mathbf{n}}_1$ and $\hat{\mathbf{n}}_2$ (outward from the fluid).

2) Find the traction vector on the fluid at the two surfaces: $\mathbf{f}_1 = \frac{d\mathbf{F}_1}{dA}$ and $\mathbf{f}_2 = \frac{d\mathbf{F}_2}{dA}$.

3) Find the vector force on the *surface* 2, $\mathbf{F}_2$. (You will have to leave the answer as a definite integral)

4) Find the force on the *surface* 1, $\mathbf{F}_1$, in symbolic form. (In this case you can evaluate the definite integral)

Use the following parameter values: H_0 = 1 mm, L = 10 mm, p_0 = 1 MPa, τ_0 = 100 kPa, m = 0.4, k = 0.5, and W = 1 m.

5) Plot the traction components $f_{2x}(x)$ and $f_{2y}(x)$.

6) Find the force components on the two surfaces F_{1x}, F_{1y}, F_{2x}, and F_{2y}. In the case of F_{2x}, and F_{2y} you will likely need an integration tool such as **NIntegrate** in *Mathematica*.

7) For $h = H\exp\left(m\frac{x}{L}\right)$, $v_x = V\frac{y}{h}\left(1 - \frac{y}{h}\right)$, $v_y = mV\frac{h}{L}\left(\frac{1}{2}\frac{y^2}{h^2} - \frac{2}{3}\frac{y^3}{h^3}\right)$

Find the (2-D) velocity gradient tensor, strain rate tensor, and vorticity (spin) tensor: $\nabla\mathbf{v}, \dot{\boldsymbol{\gamma}}, \boldsymbol{\omega}$