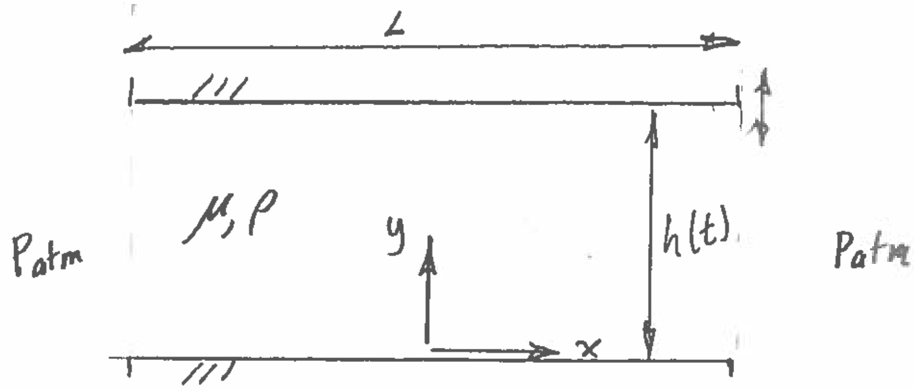
MANE 6520 - Fluid Mechanics, Exam #3 -

Thursday 8 December 2022, due Friday 9 December 12:00 noon.



Consider flow in the gap shown above where the upper surface undergoes sinusoidal normal oscillations: $h = H(1+\varepsilon \sin \omega t)$. The flow is incompressible, Newtonian, *unsteady*, 2-D, laminar, and gravity can be neglected. Both surfaces are solid (nonporous) and neither surface slides in the x-direction. The gap extends $-L/2 \le x \le L/2$ and $0 \le y \le h$. The oscillations are small $\varepsilon \ll 1$, and the flow is thin $H \ll L$. The pressure at the two edges is atmospheric p_{atm}. The y-velocity has the form:

$$v_y = \varepsilon \omega \cos[\omega t]\left(a_0 + a_1 \frac{y}{h} + a_2 \frac{y^2}{h^2} + a_3 \frac{y^3}{h^3} \right), \qquad (1)$$

where the a 's are numerical constants, Consider the following parameters as known: $\rho, \mu, p_{atm}, \varepsilon, \omega, H, L$.

a) Show through order-of-magnitude tests that the convective inertia terms $v_x \frac{\partial v_x}{\partial x}$ and $v_y \frac{\partial v_x}{\partial y}$ are much less than the unsteady term $\frac{\partial v_x}{\partial t}$ and can be neglected.

You should be left with the folllowing governing equation:

$$\rho \frac{\partial v_x}{\partial t} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 v_x}{\partial y^2}$$

b) Show through order-of-magnitude tests what requirement must be met so that the unsteady term is much less than the viscous term. In that case you should be left with: $0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 v_x}{\partial y^2}$

c) Find the general form of the x-velocity v_x in terms of x, y, t, the known parameters, and the as yet unknown a 's of Eq. (1) above.

d) What are the boundary conditions on v_x and v_y on the upper and lower surfaces?

e) Find v_x in terms of (possibly) x, y, t, and the known parameters.
f) Find v_y in terms of (possibly) x, y, t, and the known parameters.
g) Find the stream function ψ for this flow in terms of (possibly) x, y, t, and the known parameters

h) The kinetic energy (per mass) is $K = \frac{1}{2}\mathbf{v}\bullet\mathbf{v}$. The advection of K (transport following the particle) is $\mathbf{v}\bullet\nabla K$. In turbulent flow, the velocities have mean and fluctating components: $v_x = \overline{v}_x + v_x'$, etc. Of the many terms, we have $\phi = v_z \frac{\partial}{\partial z} v_x^2$. Find the mean value $\overline{\phi}$ and the mean of the fluctuation components $\overline{\phi'}$.

i) Use the mixing length model for turbulence near a wall at $y = 0$ with constant shear rate. The turbulent (eddy) viscosity is $\mu_t = \rho \ell^2 \left|\frac{\partial \overline{v}_x}{\partial y}\right|$, with mixing length $\ell = \kappa y\left[1-\exp\left(-\frac{y^+}{A}\right)\right]$, wall friction velocity $v^* = \sqrt{\frac{\tau_w}{\rho}}$, inner variable $y^+ = \frac{y v^*}{\nu}$, constants κ=0.41, A = 25.

For free stream velocity U = 10 m/s, and τ_w=0.2 Pa for air: ρ = 1.2 kg/m^3, and ν= 1.5 10^{-5} m^2/s. Plot the shear stress profile (laminar plus turbulent) $\tau(y)$ for $0 \leq y \leq y_f$, where y_f = 12 mm.