```
ClearAll["Global`*"]; (*  F 22 HW #3 *)
```

In[ ]:= $$vx = 6\,V\,\frac{x^2}{L\,H}\left(\frac{y}{H} - \frac{y^2}{H^2}\right);$$

In[ ]:= $$p = 6\,\mu\,\frac{V}{H}\,\frac{x^2}{L^2};$$

In[ ]:= $$dvxdx = \partial_x vx$$

Out[ ]=

$$\frac{12\,V\,x\left(\frac{y}{H} - \frac{y^2}{H^2}\right)}{H\,L}$$

In[ ]:= $$dvydy = -dvxdx;$$

$$vy = \int dvydy\, dy + C1$$

Out[ ]=

$$C1 - \frac{12\,V\,x\left(\frac{H\,y^2}{2} - \frac{y^3}{3}\right)}{H^3\,L}$$

In[ ]:= Solve[(vy /. y → 0) == 0, C1]

Out[ ]=

{{C1 → 0}}

In[ ]:= $$vy = -\frac{12\,V\,x\left(\frac{H\,y^2}{2} - \frac{y^3}{3}\right)}{H^3\,L};$$ (* part 1 *)

In[ ]:= v = {vx, vy};

In[ ]:= $$\text{Simplify}\left[\partial_x vx + \partial_y vy\right]$$ (* check continuity *)

Out[ ]=

0

In[ ]:= $$gradv = \{\{\partial_x vx, \partial_x vy\}, \{\partial_y vx, \partial_y vy\}\};$$

In[ ]:= $$gradvT = \{\{\partial_x vx, \partial_y vx\}, \{\partial_x vy, \partial_y vy\}\};$$

In[ ]:= MatrixForm[gradv]

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{12\,V\,x\left(\frac{y}{H} - \frac{y^2}{H^2}\right)}{H\,L} & -\frac{12\,V\left(\frac{H\,y^2}{2} - \frac{y^3}{3}\right)}{H^3\,L} \\ \frac{6\,V\,x^2\left(\frac{1}{H} - \frac{2\,y}{H^2}\right)}{H\,L} & -\frac{12\,V\,x\,(H\,y - y^2)}{H^3\,L} \end{pmatrix}$$

In[ ]:= $$\gamma dot = gradv + gradvT;$$

```
MatrixForm[γdot] (* part 2 *)
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{24 V x \left(\frac{y}{H}-\frac{y^2}{H^2}\right)}{H L} & \frac{6 V x^2 \left(\frac{1}{H}-\frac{2 y}{H^2}\right)}{H L}-\frac{12 V \left(\frac{H y^2}{2}-\frac{y^3}{3}\right)}{H^3 L} \\ \frac{6 V x^2 \left(\frac{1}{H}-\frac{2 y}{H^2}\right)}{H L}-\frac{12 V \left(\frac{H y^2}{2}-\frac{y^3}{3}\right)}{H^3 L} & -\frac{24 V x \left(H y-y^2\right)}{H^3 L} \end{pmatrix} = \underset{\approx}{\dot{\gamma}}$$

$= \dot{\gamma}_{xx}$ etc

```
In[ ]:= ω = gradv - gradvT; (* part 3 *)
```

```
In[ ]:= MatrixForm[ω]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & -\frac{6 V x^2 \left(\frac{1}{H}-\frac{2 y}{H^2}\right)}{H L}-\frac{12 V \left(\frac{H y^2}{2}-\frac{y^3}{3}\right)}{H^3 L} \\ \frac{6 V x^2 \left(\frac{1}{H}-\frac{2 y}{H^2}\right)}{H L}+\frac{12 V \left(\frac{H y^2}{2}-\frac{y^3}{3}\right)}{H^3 L} & 0 \end{pmatrix} = \underset{\approx}{W}$$

In[ ]:= $\Omega = \frac{1}{2}$ Curl[v, {x, y}] (* part 4 *)

Out[ ]=

$$\frac{1}{2}\left(-\frac{6 V x^2 \left(\frac{1}{H}-\frac{2 y}{H^2}\right)}{H L}-\frac{12 V \left(\frac{H y^2}{2}-\frac{y^3}{3}\right)}{H^3 L}\right) = \underset{\sim}{\Omega}$$

In[ ]:= τ = μ γdot

Out[ ]=

$$\left\{\left\{\frac{24 V x \left(\frac{y}{H}-\frac{y^2}{H^2}\right) \mu}{H L}, \left(\frac{6 V x^2 \left(\frac{1}{H}-\frac{2 y}{H^2}\right)}{H L}-\frac{12 V \left(\frac{H y^2}{2}-\frac{y^3}{3}\right)}{H^3 L}\right) \mu\right\}, \left\{\left(\frac{6 V x^2 \left(\frac{1}{H}-\frac{2 y}{H^2}\right)}{H L}-\frac{12 V \left(\frac{H y^2}{2}-\frac{y^3}{3}\right)}{H^3 L}\right) \mu, -\frac{24 V x \left(H y-y^2\right) \mu}{H^3 L}\right\}\right\}$$

$\sigma = -p \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \tau$ 0 (*part 5*)

σ_{xx} $\quad$ σ_{xy} etc

Out[ ]=

$$\left\{\left\{-\frac{6 V x^2 \mu}{H L^2}+\frac{24 V x \left(\frac{y}{H}-\frac{y^2}{H^2}\right) \mu}{H L}, \left(\frac{6 V x^2 \left(\frac{1}{H}-\frac{2 y}{H^2}\right)}{H L}-\frac{12 V \left(\frac{H y^2}{2}-\frac{y^3}{3}\right)}{H^3 L}\right) \mu\right\}, \left\{\left(\frac{6 V x^2 \left(\frac{1}{H}-\frac{2 y}{H^2}\right)}{H L}-\frac{12 V \left(\frac{H y^2}{2}-\frac{y^3}{3}\right)}{H^3 L}\right) \mu, -\frac{6 V x^2 \mu}{H L^2}-\frac{24 V x \left(H y-y^2\right) \mu}{H^3 L}\right\}\right\} = \underset{\approx}{\sigma}$$

```
In[ ]:= n1 = {0, -1}; n2 = {0, 1}; (* lower, upper, outward from fluid *)
        n0 = {-1, 0}; nL = {1, 0}; (* left side, right side*)
```

In[ ]:= σ1 = Simplify[σ /. y → 0]
σ2 = Simplify[σ /. y → H]

Out[ ]=
$$\left\{\left\{-\frac{6 V x^2 \mu}{H L^2}, \frac{6 V x^2 \mu}{H^2 L}\right\}, \left\{\frac{6 V x^2 \mu}{H^2 L}, -\frac{6 V x^2 \mu}{H L^2}\right\}\right\} = \underset{\sim}{\sigma}_1 \quad \text{at surface 1}$$

Out[ ]=
$$\left\{\left\{-\frac{6 V x^2 \mu}{H L^2}, -\frac{2 V (H^2 + 3 x^2) \mu}{H^2 L}\right\}, \left\{-\frac{2 V (H^2 + 3 x^2) \mu}{H^2 L}, -\frac{6 V x^2 \mu}{H L^2}\right\}\right\} = \underset{\approx}{\sigma}_2$$

In[ ]:= f1 = n1.σ1
f2 = n2.σ2

Out[ ]=
$$\left\{-\frac{6 V x^2 \mu}{H^2 L}, \frac{6 V x^2 \mu}{H L^2}\right\} = \underset{\sim}{f}_1$$

Out[ ]=
$$\left\{-\frac{2 V (H^2 + 3 x^2) \mu}{H^2 L}, -\frac{6 V x^2 \mu}{H L^2}\right\} = \underset{\sim}{f}_2$$

In[ ]:= $F1 = W \int_0^L f1 \, dx$ (* part 6 *)

$F2 = W \int_0^L f2 \, dx$

Out[ ]=
$$\left\{-\frac{2 L^2 V W \mu}{H^2}, \frac{2 L V W \mu}{H}\right\} = \underset{\sim}{F}_1$$

Out[ ]=
$$\left\{W \left(-2 V \mu - \frac{2 L^2 V \mu}{H^2}\right), -\frac{2 L V W \mu}{H}\right\} = \underset{\sim}{F}_2$$

In[ ]:= v1 = v /. y → 0
v2 = v /. y → H
v0 = v /. x → 0
vL = v /. x → L

Out[ ]=
$$\{0, 0\}$$

Out[ ]=
$$\left\{0, -\frac{2 V x}{L}\right\}$$

Out[ ]=
$$\{0, 0\}$$

Out[ ]=
$$\left\{\frac{6 L V \left(\frac{y}{H} - \frac{y^2}{H^2}\right)}{H}, -\frac{12 V \left(\frac{H y^2}{2} - \frac{y^3}{3}\right)}{H^3}\right\}$$

In[ ]:= mdot1 = $W\rho\int_0^L$ v1.n1 dx

mdot2 = $W\rho\int_0^L$ v2.n2 dx

Out[ ]=

0

Out[ ]=

-L V W ρ $= \dot{m}_1$

mdot0 = $W\rho\int_0^H$ v0.n0 dy

mdotL = $W\rho\int_0^H$ vL.nL dy (* part 7 *)

Out[ ]=

0

Out[ ]=

L V W ρ $= \dot{m}_L$

In[ ]:= mdot0 + mdotL + mdot1 + mdot2 == 0 (* part 7 *)

Out[ ]=

True

In[ ]:= H = 0.005; L = 0.050; W = 1.; V = 1.; μ = 0.050; ρ = 800.; (* part 9 *)

In[ ]:= F1

F2

F_{1x} F_{1y}

Out[ ]=

{-10., 1.} $\underset{\sim}{F}_1$

Out[ ]=

{-10.1, -1.}

In[ ]:=

{mdot0, mdotL, mdot1, mdot2}

Out[ ]=

{0, 40., 0, -40.}