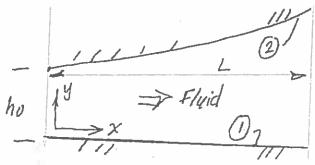
MANE 6520 - Fluid Mechanics, Fall 2022

Homework #2 - Thurs 15 Sept 2022, due Thurs 22 Sept 2022 Be precise about your notation.

Consider flow in a gap $h = h_0 \left[1 + \sin \left(m \frac{x}{L} \right) \right]$ with width W into the plane of the paper, $0 \le x \le L$, and $0 \le y \le h(x)$. The sine argument is in radians. The lower surface is 1, the upper surface is 2.



The stress tensor is $\mathbf{\sigma} = \begin{pmatrix} -p & \tau \\ \tau & -p \end{pmatrix}$, with $p = p_0 \left(1 - k \frac{x}{L} \right)$ and $\tau = \tau_0 \frac{x}{L} \frac{y}{h}$, with constants $L, h_0, m, k, p_0, \tau_0$.

- 1) Find the unit normals $\hat{\mathbf{n}}_1$ and $\hat{\mathbf{n}}_2$ (outward from the fluid).
- 2) Find the traction vector on the fluid at the two surfaces: $\mathbf{f}_1 = \frac{d\mathbf{F}_1}{dA}$ and $\mathbf{f}_2 = \frac{d\mathbf{F}_2}{dA}$.
- 3) Find the vector force on the *surface* 2, F₂. (You will have to leave the answer as a definite integral)
- 4) Find the force on the *surface* 1, \mathbf{F}_1 , in symbolic form. (In this case you can evaluate the definite integral)

Use the following parameter values: $h_0 = 1$ mm, L = 10 mm, $p_0 = 1$ MPa, $\tau_0 = 100$ kPa, m = 0.2, k = 0.5, and W = 1 m.

- 5) Plot the traction components $t_{1x}(x)$ and $t_{1y}(x)$.
- 6) Find the force components on the two surfaces F_{1x} , F_{1y} , F_{2x} , and F_{2y} . In the case of F_{2x} , and F_{2y} you will likely need an integration tool such as **NIntegrate** in *Mathematica*.