The Sufficient Decrease Condition

How Far Should We Step in the Direction p_k

At each iteration of the steepest descent method, we move in the direction of the negative gradient: $p_k = -\nabla f(x_k)$.

But, how far should we step in the direction p_k?

How Far Should We Step in the Direction p_k (cont.)

Rather than solving $\min_\alpha \phi(\alpha)$ exactly, which can be computationally expensive, we seek an inexact solution that makes progress toward the solution x^*.

The Sufficient Decrease Gives Us a Basic Condition to Satisfy

Definition: Sufficient Decrease Condition [NW06]

A step length $\alpha > 0$ satisfies the sufficient decrease, or Armijo, condition, if

$$\phi(\alpha) \leq \phi(0) + c_1 \phi'(0)\alpha,$$

for some constant $c_1 \in (0, 1)$.

Illustration of Sufficient Decrease

The Sufficient Decrease Ain't Enough

The sufficient decrease condition is an important requirement, but it is not a "strong" enough condition to ensure good steps.

References

J. Nocedal and S. J. Wright, *Numerical Optimization*, second ed., Springer–Verlag, Berlin, Germany, 2006.