Second-Order Necessary Conditions

The Second-Order Sufficient Conditions Are Not Necessary

As the example in the previous video revealed, the second-order sufficient conditions cannot identify all local minimizers.

Unfortunately, we cannot find conditions that are necessary and sufficient for local minimizers[1].

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[1]Except for some special functions

But, We Can Find Further Conditions That Local Minimizers Must Satisfy

Theorem: Second-order Necessary Conditions [NW06]

If x^* is a local minimizer of f and $\nabla^2 f$ exists and is continuous in an open neighborhood of x^*, then $\nabla f(x^*) = 0$ and $p^T \nabla^2 f(x^*) p \geq 0$, for all $p \neq 0$.

Example

Use the second-order necessary conditions to classify the stationary points of the following 1D functions:

1. $f(x) = -(x - 2)^2$
2. $f(x) = x^3$

References

📄 J. Nocedal and S. J. Wright, *Numerical Optimization*, second ed., Springer–Verlag, Berlin, Germany, 2006.