In[ ]:=
```
ClearAll["Global`*"]; (*  HW#2 F23  *)
```

In[ ]:=
$$h[x\_] := H\,\mathrm{Exp}\left[m\,\frac{x}{L}\right]$$

In[ ]:=
```
f = y - h[x];
```

In[ ]:=
$$\mathrm{gradf} = \{\partial_x f, \partial_y f\}$$

Out[ ]=
$$\left\{-\frac{e^{\frac{m x}{L}} H m}{L}, 1\right\}$$

In[ ]:=
```
n1 = {0, -1};
```
$$\mathrm{n2} = \frac{\mathrm{gradf}}{\sqrt{\mathrm{gradf.gradf}}}\ (*\ \text{part 1}\ \ *)$$

Out[ ]=
$$\left\{-\frac{e^{\frac{m x}{L}} H m}{L\sqrt{1+\frac{e^{\frac{2 m x}{L}} H^2 m^2}{L^2}}}, \frac{1}{\sqrt{1+\frac{e^{\frac{2 m x}{L}} H^2 m^2}{L^2}}}\right\}$$

In[ ]:=
```
n1 = {0, -1};
```
$$\mathrm{n2} = \left\{-\frac{e^{\frac{m x}{L}} H m}{L\sqrt{1+\frac{e^{\frac{2 m x}{L}} H^2 m^2}{L^2}}}, \frac{1}{\sqrt{1+\frac{e^{\frac{2 m x}{L}} H^2 m^2}{L^2}}}\right\};$$

In[ ]:=
$$p[x\_] := p0\left(1 - k\,\frac{x^2}{L^2}\right)$$

In[ ]:=
$$\tau[x\_, y\_] := \tau 0\left(\frac{x}{L}\right)\left(\frac{y}{h[x]}\right)^2$$

In[ ]:=
$$\sigma = \begin{pmatrix} -p[x] & \tau[x, y] \\ \tau[x, y] & -p[x] \end{pmatrix}$$

Out[ ]=
$$\left\{\left\{-p0\left(1-\frac{k x^2}{L^2}\right), \frac{e^{-\frac{2 m x}{L}} x y^2 \tau 0}{H^2 L}\right\}, \left\{\frac{e^{-\frac{2 m x}{L}} x y^2 \tau 0}{H^2 L}, -p0\left(1-\frac{k x^2}{L^2}\right)\right\}\right\}$$

In[ ]:=
```
MatrixForm[σ]
```

Out[ ]//MatrixForm=
$$\begin{pmatrix} -p0\left(1-\frac{k x^2}{L^2}\right) & \frac{e^{-\frac{2 m x}{L}} x y^2 \tau 0}{H^2 L} \\ \frac{e^{-\frac{2 m x}{L}} x y^2 \tau 0}{H^2 L} & -p0\left(1-\frac{k x^2}{L^2}\right) \end{pmatrix}$$

In[ ]:= `MatrixForm[(σ /. y → 0)]`

Out[ ]//MatrixForm=

$$\begin{pmatrix} -p0\left(1-\frac{k\,x^2}{L^2}\right) & 0 \\ 0 & -p0\left(1-\frac{k\,x^2}{L^2}\right) \end{pmatrix}$$

In[ ]:= `MatrixForm[(σ /. y → h[x])]`

Out[ ]//MatrixForm=

$$\begin{pmatrix} -p0\left(1-\frac{k\,x^2}{L^2}\right) & \frac{x\,\tau0}{L} \\ \frac{x\,\tau0}{L} & -p0\left(1-\frac{k\,x^2}{L^2}\right) \end{pmatrix}$$

In[ ]:= `f1 = n1.(σ /. y → 0)`

Out[ ]=

$$\left\{0,\ p0\left(1-\frac{k\,x^2}{L^2}\right)\right\}$$

In[ ]:= `f2 = n2.(σ /. y → h[x])`

Out[ ]=

$$\left\{\frac{e^{\frac{m\,x}{L}}\,H\,m\,p0\left(1-\frac{k\,x^2}{L^2}\right)}{L\sqrt{1+\frac{e^{\frac{2\,m\,x}{L}}\,H^2\,m^2}{L^2}}}+\frac{x\,\tau0}{L\sqrt{1+\frac{e^{\frac{2\,m\,x}{L}}\,H^2\,m^2}{L^2}}},\ -\frac{p0\left(1-\frac{k\,x^2}{L^2}\right)}{\sqrt{1+\frac{e^{\frac{2\,m\,x}{L}}\,H^2\,m^2}{L^2}}}-\frac{e^{\frac{m\,x}{L}}\,H\,m\,x\,\tau0}{L^2\sqrt{1+\frac{e^{\frac{2\,m\,x}{L}}\,H^2\,m^2}{L^2}}}\right\}$$

In[ ]:=
```
f1x = f1[[1]]
f1y = f1[[2]]
f2x = Collect[f2[[1]], {p0, τ0}]
f2y = Collect[f2[[2]], {p0, τ0}]
```

Out[ ]=

$$0$$

Out[ ]=

$$p0\left(1-\frac{k\,x^2}{L^2}\right)$$

Out[ ]=

$$\frac{e^{\frac{m\,x}{L}}\,H\,m\,p0\left(1-\frac{k\,x^2}{L^2}\right)}{L\sqrt{1+\frac{e^{\frac{2\,m\,x}{L}}\,H^2\,m^2}{L^2}}}+\frac{x\,\tau0}{L\sqrt{1+\frac{e^{\frac{2\,m\,x}{L}}\,H^2\,m^2}{L^2}}}$$

Out[ ]=

$$-\frac{p0\left(1-\frac{k\,x^2}{L^2}\right)}{\sqrt{1+\frac{e^{\frac{2\,m\,x}{L}}\,H^2\,m^2}{L^2}}}-\frac{e^{\frac{m\,x}{L}}\,H\,m\,x\,\tau0}{L^2\sqrt{1+\frac{e^{\frac{2\,m\,x}{L}}\,H^2\,m^2}{L^2}}}$$

In[ ]:=

```
dA1 = W dx;
```

$$dA2 = \sqrt{1 + \frac{e^{\frac{2 m x}{L}} H^2 m^2}{L^2}};$$

In[ ]:=

$$F1x = \text{Simplify}\left[-W \int_0^L f1x \, dx\right] \text{(* minus sign for on surface, not fluid *)}$$

$$F1y = \text{Simplify}\left[-W \int_0^L f1y \, dx\right]$$

Out[ ]=

0

Out[ ]=

$$\frac{1}{3}(-3 + k) \, L \, p0 \, W$$

In[ ]:=

$$\text{(* } F2xA = -W \int_0^L \left(t2x \sqrt{1 + \frac{e^{\frac{2 m x}{L}} H^2 m^2}{L^2}} \right) dx \text{ *)}$$

In[ ]:=

$$\text{(* } F2yA = -W \int_0^L \left(t2y \sqrt{1 + \frac{e^{\frac{2 m x}{L}} H^2 m^2}{L^2}} \right) dx \text{ *)}$$

In[ ]:=

$$H = 0.001;\ L = 0.01;\ m = 0.4;$$
$$k = 0.5;\ p0 = 1. \times 10^6;\ \tau0 = 100. \times 10^3;\ W = 1.;$$

In[ ]:=

```
F1x = NIntegrate[-W f1x, {x, 0, L}]
F1y = NIntegrate[-W f1y, {x, 0, L}]
```

NIntegrate: Integral and error estimates are 0 on all integration subregions. Try increasing the value of the MinRecursion option. If value of integral may be 0, specify a finite value for the AccuracyGoal option.

Out[ ]=

0.

Out[ ]=

-8333.33

In[ ]:=

$$F2x = \text{NIntegrate}\left[-W \sqrt{1 + \frac{e^{\frac{2 m x}{L}} H^2 m^2}{L^2}} \, f2x, \{x, 0, L\}\right]$$

$$F2y = \text{NIntegrate}\left[-W \sqrt{1 + \frac{e^{\frac{2 m x}{L}} H^2 m^2}{L^2}} \, f2y, \{x, 0, L\}\right]$$

Out[ ]=

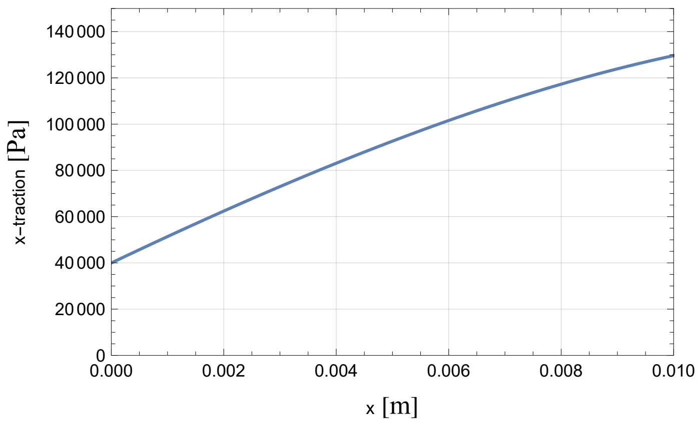
-901.57

Out[ ]=

8359.56

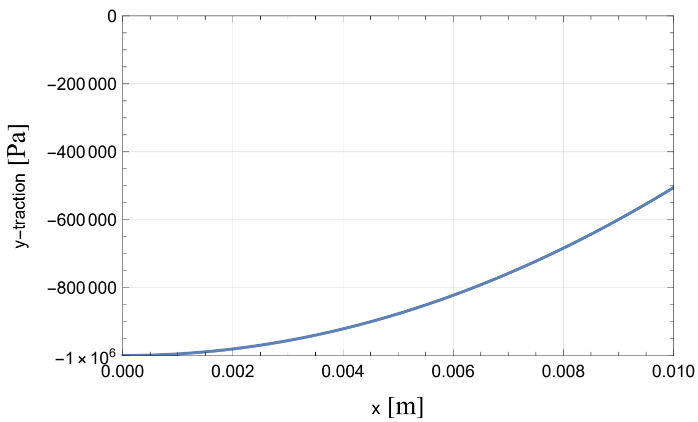
In[ ]:=
```
Plot[{f2x}, {x, 0, L}, Frame → True, GridLines → Automatic,
 FrameLabel → {"x [m]", "x-traction [Pa]"}, PlotRange → {{0, 0.01}, {0, 150000}}]
```

Out[ ]=



In[ ]:=
```
Plot[{f2y}, {x, 0, L}, Frame → True, GridLines → Automatic,
 FrameLabel → {"x [m]", "y-traction [Pa]"}, PlotRange → {{0, 0.01}, {-1000000, 0}}]
```

Out[ ]=



In[ ]:=
```
ClearAll["Global`*"]; (*  HW#2 F23  *)
```

In[ ]:= $h = H\,\mathrm{Exp}\left[m\,\frac{x}{L}\right];\ vx = V\,\frac{y}{h}\left(1-\frac{y}{h}\right);$

In[ ]:= $vy = m\,V\,\frac{h}{H}\,\frac{H}{L}\left(\frac{1}{2}\,\frac{y^2}{h^2}-\frac{2\,y^3}{3\,h^3}\right);$

In[ ]:= $\mathrm{Simplify}\left[\partial_x vx+\partial_y vy\right]$

Out[ ]=

0

In[ ]:=
```
v = {vx, vy};
```

In[ ]:=
```
xx = {x, y};
```

In[ ]:=
```
delv = {{0, 0}, {0, 0}};
Do[Do[delv⟦i, j⟧ = ∂xx⟦i⟧ v⟦j⟧, {i, 1, 2}], {j, 1, 2}];
MatrixForm[Simplify[delv]]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} -\frac{e^{-\frac{2mx}{L}} m V \left(e^{\frac{mx}{L}} H-2y\right) y}{H^2 L} & -\frac{e^{-\frac{2mx}{L}} m^2 V \left(3 e^{\frac{mx}{L}} H-8y\right) y^2}{6 H^2 L^2} \\ \frac{e^{-\frac{2mx}{L}} V \left(e^{\frac{mx}{L}} H-2y\right)}{H^2} & \frac{e^{-\frac{2mx}{L}} m V \left(e^{\frac{mx}{L}} H-2y\right) y}{H^2 L} \end{pmatrix}$$

In[ ]:=
```
delvT = {{0, 0}, {0, 0}};
Do[Do[delvT⟦i, j⟧ = delv⟦j, i⟧, {i, 1, 2}], {j, 1, 2}];
MatrixForm[Simplify[delvT]]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} -\frac{e^{-\frac{2mx}{L}} m V \left(e^{\frac{mx}{L}} H-2y\right) y}{H^2 L} & \frac{e^{-\frac{2mx}{L}} V \left(e^{\frac{mx}{L}} H-2y\right)}{H^2} \\ -\frac{e^{-\frac{2mx}{L}} m^2 V \left(3 e^{\frac{mx}{L}} H-8y\right) y^2}{6 H^2 L^2} & \frac{e^{-\frac{2mx}{L}} m V \left(e^{\frac{mx}{L}} H-2y\right) y}{H^2 L} \end{pmatrix}$$

In[ ]:=
```
γdot = {{0, 0}, {0, 0}};
Do[Do[γdot⟦i, j⟧ = delv⟦i, j⟧ + delvT⟦i, j⟧, {i, 1, 2}], {j, 1, 2}];
MatrixForm[Simplify[γdot]]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} -\frac{2 e^{-\frac{2mx}{L}} m V \left(e^{\frac{mx}{L}} H-2y\right) y}{H^2 L} & \frac{e^{-\frac{2mx}{L}} V \left(-12 L^2 y+8 m^2 y^3+3 e^{\frac{mx}{L}} H \left(2 L^2-m^2 y^2\right)\right)}{6 H^2 L^2} \\ \frac{e^{-\frac{2mx}{L}} V \left(-12 L^2 y+8 m^2 y^3+3 e^{\frac{mx}{L}} H \left(2 L^2-m^2 y^2\right)\right)}{6 H^2 L^2} & \frac{2 e^{-\frac{2mx}{L}} m V \left(e^{\frac{mx}{L}} H-2y\right) y}{H^2 L} \end{pmatrix}$$

In[ ]:=
```
ω = {{0, 0}, {0, 0}};
Do[Do[ω⟦i, j⟧ = delv⟦i, j⟧ - delvT⟦i, j⟧, {i, 1, 2}], {j, 1, 2}];
MatrixForm[Simplify[ω]]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & \frac{e^{-\frac{2mx}{L}} \left(-3 e^{\frac{mx}{L}} H V \left(2 L^2+m^2 y^2\right)+4 V y \left(3 L^2+2 m^2 y^2\right)\right)}{6 H^2 L^2} \\ \frac{e^{-\frac{2mx}{L}} V \left(3 e^{\frac{mx}{L}} H \left(2 L^2+m^2 y^2\right)-4 y \left(3 L^2+2 m^2 y^2\right)\right)}{6 H^2 L^2} & 0 \end{pmatrix}$$

In[ ]:=
```
Simplify[γdot⟦1,2⟧] == Simplify[γdot⟦2,1⟧]
```

Out[ ]=
```
True
```

In[ ]:=
```
Simplify[ω⟦1,2⟧ == -ω⟦2,1⟧]
```

Out[ ]=
```
True
```

In[ ]:=
```
curlv = Simplify[Curl[v, {x, y}]]
```
Out[ ]=

$$\frac{e^{-\frac{2 m x}{L}} \left(-3 e^{\frac{m x}{L}} H V \left(2 L^2+m^2 y^2\right)+4 V y \left(3 L^2+2 m^2 y^2\right)\right)}{6 H^2 L^2}$$

In[ ]:=
```
Simplify[ω[[1,2]]]
```
Out[ ]=

$$\frac{e^{-\frac{2 m x}{L}} \left(-3 e^{\frac{m x}{L}} H V \left(2 L^2+m^2 y^2\right)+4 V y \left(3 L^2+2 m^2 y^2\right)\right)}{6 H^2 L^2}$$

In[ ]:=
```
Simplify[curlv - ω[[1,2]]]
```
Out[ ]=

0