# Class #8

Exam #1, Mon Oct 5

Fluid Model - all observers calcute same stress

$$\underset{\approx}{\sigma} = -p\underset{\approx}{\delta} + \underbrace{\underset{\approx}{\tau}}$$

determined by model, extra stress

deviatoric stress

want linear

2-D

$$\underset{\approx}{a} = \underbrace{\text{sym part}}_{\text{deformation}} + \underbrace{\text{anti sym part}}_{\text{rotation}}$$

2-

$$\underset{\sim}{\nabla}\underset{\sim}{V} = \begin{bmatrix} \frac{\partial V_x}{\partial x} & \frac{\partial V_y}{\partial x} \\ \frac{\partial V_x}{\partial y} & \frac{\partial V_y}{\partial y} \end{bmatrix}$$

velocity gradient

transpose

$$(\underset{\sim}{\nabla}\underset{\sim}{V}) = \begin{bmatrix} \frac{\partial V_x}{\partial x} & \frac{\partial V_x}{\partial y} \\ \frac{\partial V_y}{\partial x} & \frac{\partial V_y}{\partial y} \end{bmatrix}$$

sym part $= \frac{1}{2}\left(\underset{\sim}{\nabla}\underset{\sim}{V} + \underset{\sim}{\nabla}\underset{\sim}{V}^T\right)$

anti sym part $= \frac{1}{2}\left(\underset{\sim}{\nabla}\underset{\sim}{V} - \underset{\sim}{\nabla}\underset{\sim}{V}^T\right)$

$\underset{\sim}{\nabla}\underset{\sim}{V} =$ sym + ant-sym

$$\underset{\approx}{\dot{\gamma}} = \text{sym} = \begin{bmatrix} 2\frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \\ \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} & 2\frac{\partial v_y}{\partial y} \end{bmatrix}$$

deformation rate
stretching tensor
strain

$$\underset{\approx}{w} = \underset{\text{sym}}{\text{anti}} = \begin{bmatrix} 0 & -\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \\ +\frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} & 0 \end{bmatrix}$$

spin
tensor
vorticity
tenor

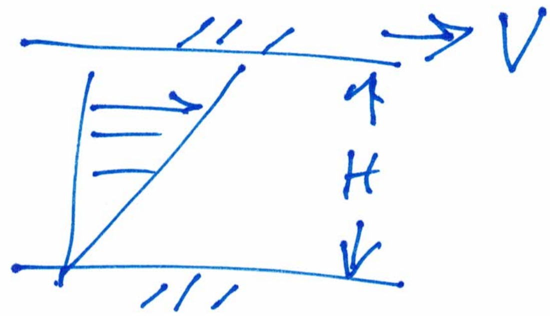
$$\underset{\sim}{w} = \underset{\text{vector}}{\text{vorticity}} = w_z \hat{e}_z$$

$$w_z = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}$$

$$\underset{\sim\sim}{\nabla v} = \underbrace{\frac{1}{2}\underset{\approx}{\dot{\gamma}}}_{\text{deformation}} + \underbrace{\frac{1}{2}\underset{\approx}{w}}_{\text{rotation}}$$

Simple shear flow

Couette flow

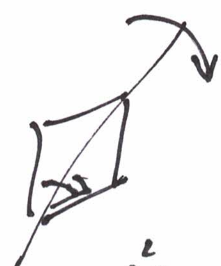



$$V_x = V\frac{y}{H} = \frac{Vy}{H}$$

$$\underset{\approx}{\dot{\gamma}} = \begin{bmatrix} 0 & V/H \\ V/H & 0 \end{bmatrix}$$

fluid element

$$\underset{\approx}{\omega} = \begin{bmatrix} 0 & -V/H \\ +V/H & 0 \end{bmatrix}$$



$$\omega = -\frac{V}{H}$$

$$\dot{\gamma} = +V/H$$

$$\underset{\sim}{\omega} = \underset{\sim}{\nabla} \times \underset{\sim}{V}$$

$$= -\frac{V}{H}$$

$$\underset{\sim}{\nabla} = \frac{\partial}{\partial x}\hat{e}_x + \frac{\partial}{\partial y}\hat{e}_y$$

$$\underset{\sim}{V} = V_x \hat{\imath} + V_y \hat{\jmath}$$

# Model:

$$\underbrace{\nabla \underset{\sim}{V} - \text{rotation}}_{\dot{\underset{\approx}{\gamma}}\ \text{deformation}}$$

## Fluid Model (Newtonian)

$$\boxed{\underset{\approx}{\tau} = \mu\, \dot{\underset{\approx}{\gamma}}}$$

$\frac{N}{m^2}$

$\mu$ = Viscosity

$\frac{N \cdot s}{m^2}$

$$\dot{\gamma} = \frac{V}{H} = \frac{m/s}{m} = \frac{1}{s}$$

air, water } very low viscosity

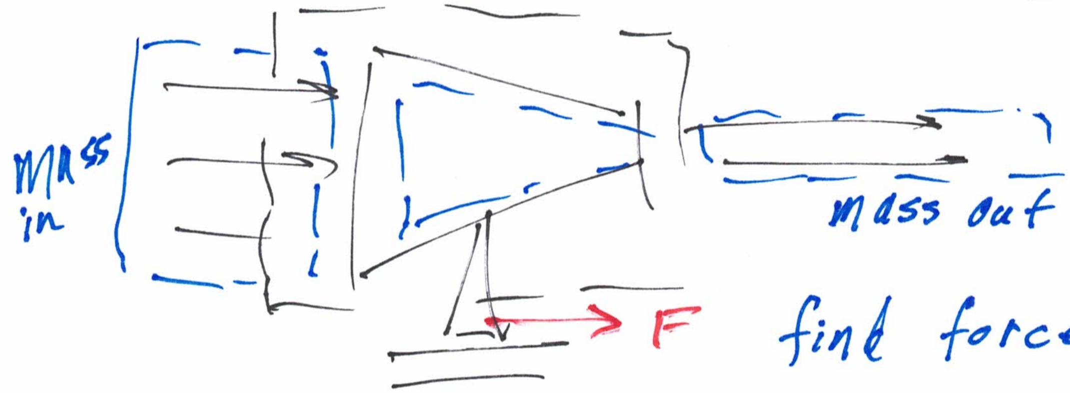
so far

given $\underset{\sim}{V} \Rightarrow \underset{\approx}{\tau} \Rightarrow \underset{\sim}{F}$

Most of course $\underset{\sim}{F} \Rightarrow \underset{\approx}{\tau} \Rightarrow \underset{\sim}{V}$

# Global Approach

nozzle on a stand




find force

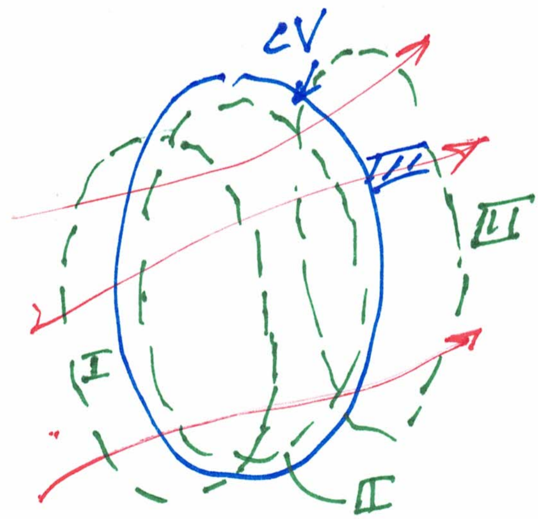
Control volume - any volume of interest

- change size
  shape
  move

$\forall$ = volume

Most commonly $C\forall$ - fixed

# Reynolds Transport Theorem




$$\left.\frac{d\theta}{dt}\right|_{mass} = \left.\frac{d\Phi}{dt}\right|_{cv} + \dot{\Phi}_{out} - \dot{\Phi}_{in}$$

--- ⇒ mass     ——— volume

$\Phi$ = any extensive property

mass, momentum, ~~energy~~ convection

$\theta$ = generation of $\Phi$ in II

$\theta$ = mass     $\frac{d\theta}{dt} = 0$

$\theta$ = momentum $= m\underset{\sim}{V}$

$$\frac{d\theta}{dt} = \frac{d}{dt} m\underset{\sim}{V} = m\underset{\sim}{a} = \underset{\sim}{F}$$

$$\underbrace{\frac{d\Theta}{dt}}_{\text{mass}} = \underbrace{\frac{d\Theta}{dt}}_{CV} + \dot{\underset{\sim}{I}}_{out} - \dot{\underset{\sim}{I}}_{in} \qquad i = \frac{\Theta}{m}$$

mass

$$0 = \frac{d}{dt}\int_{V} \rho \, dV + \dot{m}_{out} - \dot{m}_{in}$$

$$\dot{m} = \int_A \rho \, \underset{\sim}{V} \cdot \underset{\sim}{n} \, dA$$ mass flow rate kg/s

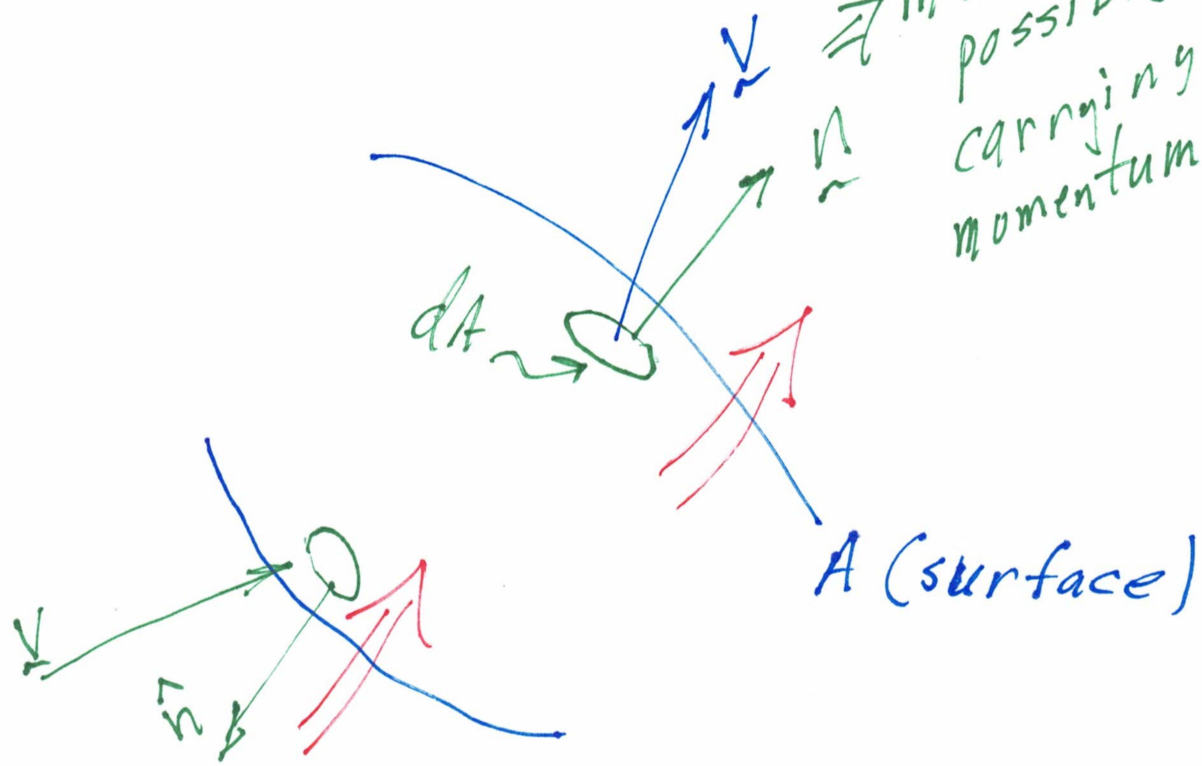
momentum

$$\underset{\sim}{F} = \sum \underset{\sim}{F} = \frac{d}{dt}\int_{CV} (\rho \underset{\sim}{V}) \, dV + \dot{\underset{\sim}{G}}_{out} - \dot{\underset{\sim}{G}}_{in}$$

$\dot{\underset{\sim}{G}}$ = momentum flow rate

$$\dot{\underset{\sim}{G}} = \int_A \rho \underset{\sim}{V}\, \underset{\sim}{V}\cdot\underset{\sim}{n}\; dA = \int_A \rho\, \underset{\sim}{V} (\underset{\sim}{V}\cdot\underset{\sim}{n})\, dA$$

Note $\underset{\sim}{V}\cdot\underset{\sim}{n}$ takes care of sign




out: $\underset{\sim}{V}\cdot\underset{\sim}{n} > 0$ in $\underset{\sim}{V}\cdot\underset{\sim}{n} < 0$

mass $$0 = \frac{d}{dt}\int_{CV} \rho \, dV + \int_{CS} \rho(\underset{\sim}{V} \cdot \underset{\sim}{n})\, dA$$

$\underbrace{\qquad\qquad}_{\text{mass in CV}}$

control surface (surface enclosing control volume)

$$0 = \frac{d}{dt} m_{CV} + \dot{m}_{out} - \dot{m}_{in}$$

mass flow out

momentum

$$\sum \underset{\sim}{F} = \frac{d}{dt}\int_{CV} \rho \underset{\sim}{V}\, dV + \int_{CS} \rho \underset{\sim}{V} (\underset{\sim}{V} \cdot \underset{\sim}{n})\, dA$$

$$= \frac{d\underset{\sim}{G}_{CV}}{dt} + \dot{\underset{\sim}{G}}_{out} - \dot{G}_{in}$$

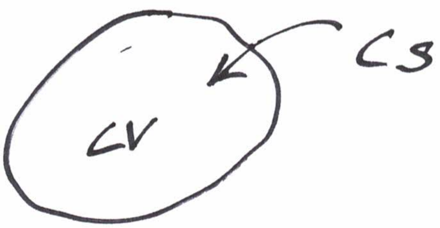
LHS:

mass - $\left.\frac{dm}{dt}\right|_{m=const.} = 0$

momentum - $\left.\frac{dm\underset{\sim}{V}}{dt}\right|_{m=const} = \underset{\sim}{F}$

# Divergence Theorem

changes $\int_{C\forall} \Longleftrightarrow \int_{CS}$




$d\underset{\sim}{A} = \underset{\sim}{n}\, dA$

$$(1) \quad \int_{CS} \rho \phi \underbrace{\underset{\sim}{V} \cdot d\underset{\sim}{A}}^{V \cdot n\, dA} = \int_{\forall} \underset{\sim}{\nabla} \cdot (\rho \phi \underset{\sim}{V})\, d\forall$$

flow out $\qquad$ divergence inside

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Back to Reynold Transport Th
for mass - fixed CV

$$0 = \frac{d}{dt}\int_{\forall} \rho\, d\forall + \underbrace{\int_{S} \rho (\underset{\sim}{V} \cdot \underset{\sim}{n})\, dA}$$

(1) for $\phi = 1$ $\qquad \int_{\forall} \underset{\sim}{\nabla} \cdot \rho \underset{\sim}{V}\, d\forall$

2D, incompressible

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0$$

$V_x = \ldots$

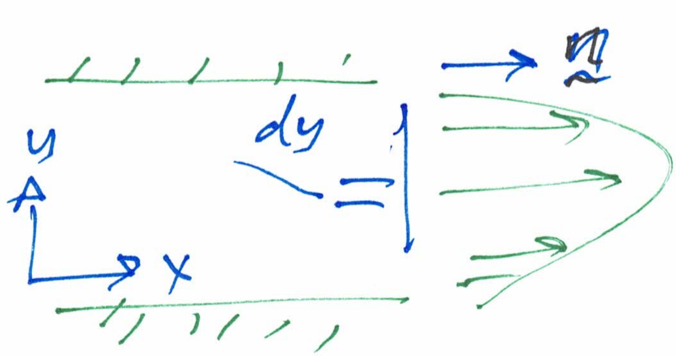
know $V_x \Rightarrow$ Find $V_y$

know $V_y$ find $V_x$ $\quad \frac{\partial V_x}{\partial x} = -\frac{\partial V_y}{\partial y}$

e.g. know $V_y$

$$V_x = \int \frac{\partial V_y}{\partial y}\, dx + \text{const} \leftarrow \text{BC}$$

$$\dot{m} = \text{mass flow} = \int_A \rho (\underset{\sim}{V} \cdot \underset{\sim}{n})\, dA$$




$$\dot{m} = \int_A \rho V_x\, dA$$

$$dA = W\, dy$$

W into paper

$$0 = \int_{\forall} \frac{\partial \rho}{\partial t}\, d\forall + \int_{\forall} (\underset{\sim}{\nabla} \cdot \rho \underset{\sim}{V})\, d\forall$$

$$0 = \int \Big( \overbrace{\frac{\partial \rho}{\partial t} + \underset{\sim}{\nabla} \cdot \rho \underset{\sim}{V}}^{=0} \Big)\, d\forall$$

$$0 = \frac{\partial \rho}{\partial t} + \underset{\sim}{\nabla} \cdot \rho \underset{\sim}{V}$$ differential continuity equation

$$0 = \frac{d}{dt} \int_{c\forall} \rho\, d\forall + \int_{cs} \rho (\underset{\sim}{V} \cdot n)\, dA$$ global cont equation

$\rho$ = density = const
incompressible

$$0 = \cancel{\frac{\partial \rho}{\partial t}} + \cancel{\rho} (\underset{\sim}{\nabla} \cdot \underset{\sim}{V}) \Rightarrow \underset{\sim}{\nabla} \cdot \underset{\sim}{V} = 0$$

$\operatorname{div} \underset{\sim}{V} = 0$