Fluid Mechanics Lec 12

Thin film $\frac{H}{L} \ll 1$, steady 2-D
incompressible

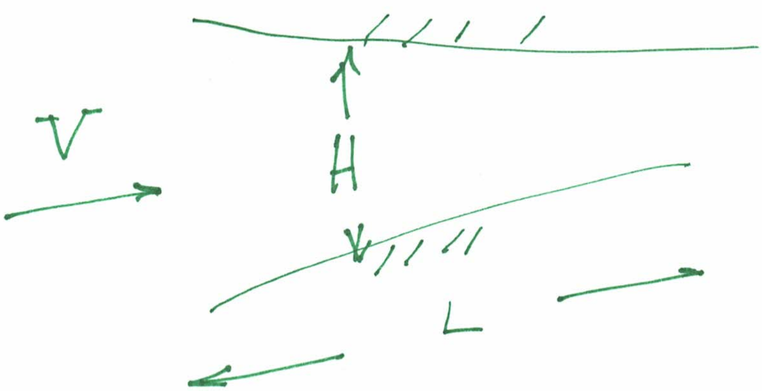
$$\rho\left(V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 V_x}{\partial x^2} + \frac{\partial^2 V_x}{\partial y^2}\right)$$

$$\rho\left(V_x \frac{\partial V_y}{\partial x} + V_y \frac{\partial V_y}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 V_y}{\partial x^2} + \frac{\partial^2 V_y}{\partial y^2}\right) - \rho g$$

$$0 = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y}$$

$V_x \sim V$

$V_y \sim V\, H/L$



$$\frac{\text{Inertia-}y}{\text{Inertia-}x}$$

$$\frac{\cancel{\rho} \cancel{V} \cancel{V} \frac{H}{L} \cancel{\frac{1}{L}}}{\cancel{\rho} \cancel{V} \cancel{V} \cancel{\frac{1}{L}}} \sim \frac{H}{L} \;\|$$

$$\frac{H}{L} \sim \frac{\text{Visc Shear - y}}{\text{Visc Shear x}}$$

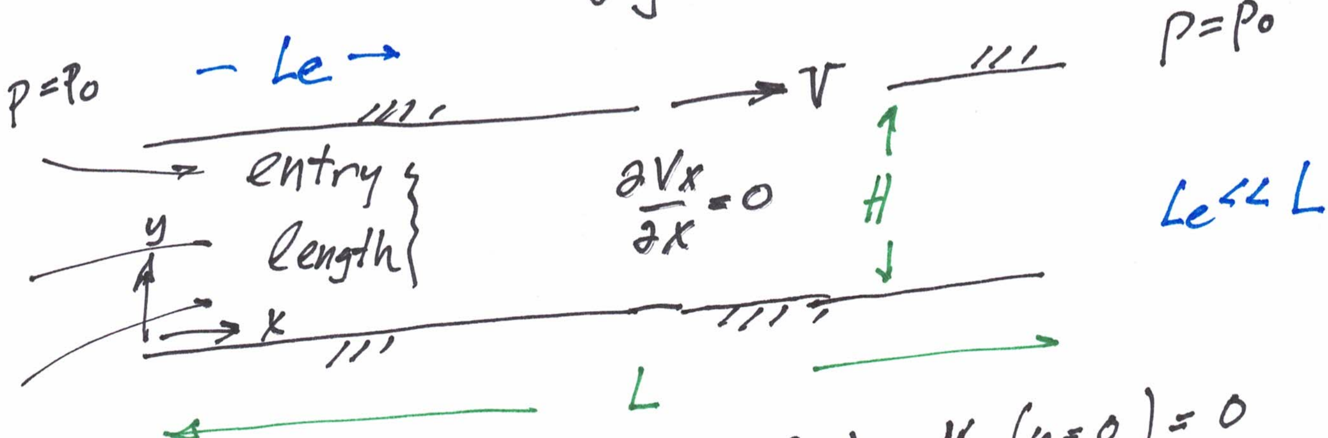
possibly:
$V_x = V_x(x,y)$
$V_y = V_y(x,y)$
$p = p(x,y)$

(1) $$0 = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y}$$

(2) $$\rho\left(V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 V_x}{\partial y^2}$$

Couette Flow

(3) $$0 = \frac{\partial p}{\partial y} - \rho g$$



continuity $\frac{\partial V_y}{\partial y} = 0$ $\quad V_y = f(x)$ not const! $\quad V_y(y=0) = 0$ $\quad \underline{V_y = 0}$

$V_x = V_x(y)$
$V_y = 0$

(3) $\rightarrow$ $\frac{\partial p}{\partial y} = \rho g \qquad p = \rho g y + f(x)$

(2) $$\underbrace{\underbrace{\frac{\partial p}{\partial x}}_{fn\,(x)} = \underbrace{\mu \frac{\partial^2 V_x}{\partial y^2}}_{fn\,(y)}}_{const.} \qquad \frac{\partial p}{\partial x} = f'(x)$$

$\rightarrow \mu \frac{d^2 V_x}{dy^2}$

$$\frac{\partial p}{\partial x} = C \qquad p = p_0 + C_1 x$$

BC $x = L$ $\quad p = p_0 \quad p(L) = p_0 + \overset{0}{\cancel{C_1}} L = p_0$

$$p = p_0 \qquad 0 \le x \le L$$

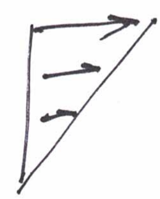
(2) $\quad \cancel{\mu} \dfrac{\partial^2 V_x}{\partial y^2} = 0 \qquad V_x = C_2 y + V_{x0}$

$y = 0 \quad V_x = 0$

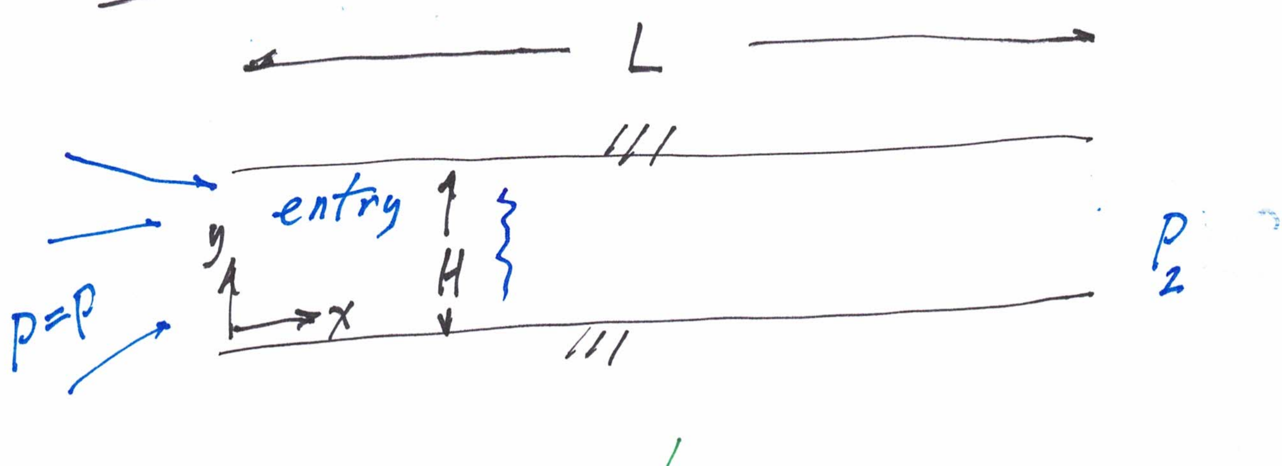
$y = H \quad V_x = V$

$$V_x = V \frac{y}{H}$$

$V_y = 0 \qquad p = p_0$



Poiseuille Flow



same arguments

For incompressible gage pressure

OK $\quad P_{gage} = P_{abs} - P_{atm}$

-4-

$$0 = \cancel{\frac{\partial V_x}{\partial x}} + \frac{\partial V_y}{\partial y} \qquad \ldots\ldots$$

$$V_x = V_x(y) \qquad V_y = 0$$

$$\frac{\partial p}{\partial y} = \rho g \qquad p = \rho g y + f(x)$$

$$\frac{\text{gravity}}{\text{viscous}} = \frac{\rho g}{\mu V L / H^3} << 1$$

neglect gravity

$$p = p(x)$$

$$\underbrace{\frac{dp}{dx}}_{fn(x)} = \mu \underbrace{\frac{d^2 V_x}{dy^2}}_{fn(y)}$$

$$\frac{dp}{dx} = \text{const} \qquad p = P_1 + (P_2 - P_1)\frac{x}{L} \qquad \frac{dp}{dx} = \frac{P_2 - P_1}{L}$$

$$\vdots$$

$$V_x = \frac{P_2 - P_1}{\mu L}\left[\frac{y^2}{H^2} - \frac{y}{H}\right] H^2$$

$$(V_x)_{max}\left(y = H/2\right) = \frac{(P_1 - P_2)H^2}{8\mu L}$$

$$\dot{m} = W\int_0^H \rho\, V_x(y)\, dy = \frac{\rho W (P_1 - P_2) H^3}{12 \mu L}$$

Non-Newtonia

NN
- Generalized Newtonian (purely viscous)
- Viscoelastic

perfect memory - elastic solid

no memory - viscous fluids.

GN $\underset{\approx}{\tau} = \mu(\dot{\gamma})\, \underset{\approx}{\dot{\gamma}}$

invariants

$$\operatorname{tr} \underset{\approx}{\dot{\gamma}} = \dot{\gamma}_{xx} + \dot{\gamma}_{yy}$$

$$\dot{\gamma} = |\underset{\approx}{\dot{\gamma}}| = \sqrt{\frac{1}{2}\, \underset{\approx}{\dot{\gamma}} : \underset{\approx}{\dot{\gamma}}} = \frac{1}{2}\left(\dot{\gamma}_{xx}^2 + 2\dot{\gamma}_{xy}^2 + \dot{\gamma}_{yy}^2\right)$$

Simple shear $\dot{\gamma} = \left|\frac{\partial v_x}{\partial y}\right| = \frac{V}{H}$