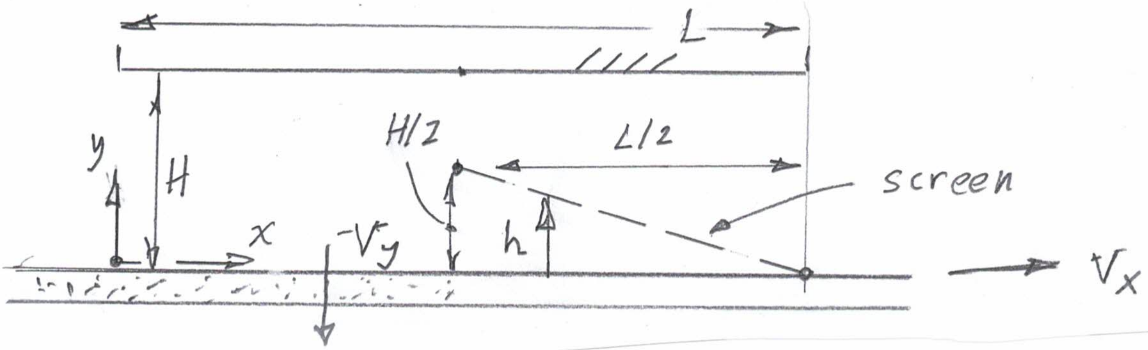
MANE 6520 - Fluid Mechanics

Test #1 - In class – Monday, 3 October 2022



Consider two rigid parallel plates shown. The flow is 2-D, steady, incompressible (with density ρ) and the fluid is Newtonian (constant viscosity μ). The upper plate is stationary, solid (impermeable), and the lower plate slides to the right at velocity V_x. Fluid passes *downwards* through the (permeable) lower plate at constant velocity V_y. The length is L, the gap height is H, and the width is W into the paper. Consider the exterior ambient pressure to be zero. The velocity in the x-direction is $v_x = V_x\left(1-\frac{y}{H}\right)+A\frac{x}{H}\left(\frac{y}{H}-\frac{y^2}{H^2}\right)$ where A is an unknown constant. The pressure $p = 6\mu\frac{V_y}{H}\frac{L^2}{H^2}\frac{x^2}{L^2}$. In parts 5), 6) and 7) you may leave your answer in terms of definite integrals which could be evaluated numerically if all the parameter values ρ, μ, V_x, V_y, L, H, and W were known.

1) Find the velocity v_y in terms of y and the known parameters of the first paragraph above. Along the way you will need to find the constant A.

   If you cannot find A, or you are not confident in the value you found, continue in the parts below using the symbol A, assuming it is now known.

2) Find the strain rate tensor $\dot{\boldsymbol{\gamma}}$ in terms of (possibly) $x, y, \rho, \mu, V_x, V_y, L, H$, and
3) Find the vorticity *vector* $\boldsymbol{\omega}$ in terms of (possibly) $x, y, \rho, \mu, V_x, V_y, L, H$, and W
4) Find the total stress tensor $\boldsymbol{\sigma}$ in terms of (possibly) $x, y, \rho, \mu, V_x, V_y, L, H$, and W
5) Find the force on the lower surface $\mathbf{F}_l$ in terms of (possibly) $\rho, \mu, V_x, V_y, L, H, W$.
6) Find the mass flow rate out of the right-side surface $\dot{m}(x=L)$ in terms of (possibly) $\rho, \mu, V_x, V_y, L, H$, and W.
7) Imagine there is a very thin screen along $h(x)$ (also extending distance W into the paper). Fluid can flow through the screen without resistance. Find the momentum flow rate through this surface $\dot{\mathbf{G}}$. $h=\frac{H}{2}\left(1-2\frac{x-L/2}{L}\right), \ \frac{L}{2}\le x\le L$