MANE 6520 - Fluid Mechanics

Homework #4 - Thursday 13 October 2022, due Thursday 20 October

1) It has been proposed that a giant pipeline (or many pipelines in parallel) can transport water from the Great Lakes to California to relieve the water shortage which is foreseen for many years to come. Is this reasonable? "Design" a system meaning specify the number and diameter of pipelines, and number of pumping stations. You will have to find some information on the internet and make some gross assumptions. There is no one right answer, or maybe no answer at all that makes sense.

I assume you will need pumping stations along the route so that the pressures can be reasonable. I am guessing a large industrial machine can pump $\dot{Q} = 1$ m^3/s at $\Delta p = 10^6$ Pa, power $P = \Delta p \dot{Q} = 1$ MW maximum power. Thus a station at a given power can pump high flow rate and low pressure or vice versa and anything in between. For comparison, the average power usage of NYC = 8000 MW. The average power usage of a house is about 1 kW. Let's say the largest pipe size available is D = 1 m.

Some limits would be total power consumption, and that the pipe pressure should not fall below atmospheric (or the flow will cavitate).

I will send you Mathematica formulas for pipe friction factors.

2) Work the problem below in symbols but consider that the problem will have the following parameter values in an order of magnitude sense. The flow is 2-D steady and incompressible, without body force (gravity).

$H = 1$ mm, $L = 1$ m, $W = 1$ m, $V = 1$ mm/s, $\rho = 1000$ kg/m^3, g = 10 m/s^2, $\mu = 0.010$ Pa-s, $p0 = 10$ MPa.

Consider configuration shown ~~above~~ below

The x-velocity profile is

$$h = H\left(1 - m\frac{x}{L}\right)$$

$$v_x = 6V\frac{x}{H}\left(\frac{y}{h(x)} - \frac{y^2}{h(x)^2}\right).$$

$$0 \le m < 1$$

a) Find the y-velocity field, v_y, $v_y(y=0)=0$

b) Find the material derivative $\dfrac{D\mathbf{v}}{Dt}$, $\mathbf{v} = v_x\mathbf{e}_x + v_y\mathbf{e}_y$

c) Find $\nabla\mathbf{v}$ and $\nabla\cdot\nabla\mathbf{v} \equiv \nabla^2\mathbf{v}$

d) Derivatives in the x-direction can be neglected relative derivatives in the y-direction. Show this is true by order of magnitude analysis.

e) An argument to be made that inertia can be neglected. Show that is reasonable by order of magnitude analysis.

f) Using d) and c) (i.e., throw away the small terms) and show what is now left of the x- and y-Navier-Stokes equations.

g) Using the Navier Stokes equations, there is an order of magnitude argument to be made that $p = p(x)$, meaning pressure doesn't vary in the y-direction. Please do so.

h) Using the Navier Stokes equations, find the order of magnitude of the pressure generated by viscous forces.

i) Using order of magnitude analysis show that pressure change due to gravity (= ρ g H) can be neglected relative to pressue generated by viscous forces; thus justifying the assumtion to ignore gravity.

j) Making use of d) through i) and the x-Navier Stokes equation, find $p(x)$