In[45]:=
```
ClearAll["Global`*"]; (*  prob 1  *)
```

In[46]:=
```
a = {ax, ay, az};
b = {bx, by, bz};
e1 = {1, 0, 0};
e2 = {0, 1, 0};
e3 = {0, 0, 1};
eps = {{{0, 0, 0}, {0, 0, 1}, {0, -1, 0}}, {{0, 0, -1}, {0, 0, 0}, {1, 0, 0}},
   {{0, 1, 0}, {-1, 0, 0}, {0, 0, 0}}}; (*  permutation symbol  *)
```

In[52]:=
```
aa = a[[1]] e1 + a[[2]] e2 + a[[3]] e3
bb = b[[1]] e1 + b[[2]] e2 + b[[3]] e3
```

Out[52]=
```
{ax, ay, az}
```

Out[53]=
```
{bx, by, bz}
```

In[54]:=
```
((aa[[1]] e1) × (bb[[1]] e1) + (aa[[1]] e1) × (bb[[2]] e2) + (aa[[1]] e1) × (bb[[3]] e3)) +
 ((aa[[2]] e2) × (bb[[1]] e1) + (aa[[2]] e2) × (bb[[2]] e2) + (aa[[2]] e2) × (bb[[3]] e3)) +
 ((aa[[3]] e3) × (bb[[1]] e1) + (aa[[3]] e3) × (bb[[2]] e2) + (aa[[3]] e3) × (bb[[3]] e3))
```

Out[54]=
```
{-az by + ay bz, az bx - ax bz, -ay bx + ax by}
```

In[55]:=
```
a × b
```

Out[55]=
```
{-az by + ay bz, az bx - ax bz, -ay bx + ax by}
```

In[56]:=
```
c = {0, 0, 0};
Do[Do[Do[c[[i]] = c[[i]] + eps[[i,m,n]] a[[m]] b[[n]], {n, 1, 3}], {m, 1, 3}], {i, 1, 3}]; c
```

Out[57]=
```
{-az by + ay bz, az bx - ax bz, -ay bx + ax by}
```

In[58]:=
```
Det[{{ex, ey, ez}, {ax, ay, az}, {bx, by, bz}}]
```

Out[58]=
```
-az by ex + ay bz ex + az bx ey - ax bz ey - ay bx ez + ax by ez
```

In[59]:=
```
Collect [%, {ex, ey, ez}]
```

Out[59]=
```
(-az by + ay bz) ex + (az bx - ax bz) ey + (-ay bx + ax by) ez
```

In[60]:=
```
ClearAll["Global`*"]; (*   Prob 2  *)
```

In[61]:=
```
e1 = {1, 0, 0};
e2 = {0, 1, 0};
e3 = {0, 0, 1};
```

In[64]:=
```
x = {x1, x2, x3};
```

In[65]:=
```
a = {2 x1/L (x2/H)^2, 3 (x1/L)^2 x2/H, 0};
```

In[66]:=
```
d = (x1/L)^2 Cos[3 π x2/H] Exp[2 x3/W];
```

In[67]:=
```
gradd = {∂x1 d, ∂x2 d, ∂x3 d} (*  ∇d, part a  *)
```

Out[67]=
$$\left\{\frac{2\, e^{\frac{2\,x3}{W}}\, x1 \cos\left[\frac{3\,\pi\,x2}{H}\right]}{L^2},\ -\frac{3\, e^{\frac{2\,x3}{W}}\, \pi\, x1^2 \sin\left[\frac{3\,\pi\,x2}{H}\right]}{H\,L^2},\ \frac{2\, e^{\frac{2\,x3}{W}}\, x1^2 \cos\left[\frac{3\,\pi\,x2}{H}\right]}{L^2\,W}\right\}$$

In[68]:=
```
Sqrt[a.a] (*  Sqrt[a·a,  part b  *)
```

Out[68]=
$$\sqrt{\frac{9\,x1^4\,x2^2}{H^2\,L^4}+\frac{4\,x1^2\,x2^4}{H^4\,L^2}}$$

In[69]:=
```
x1 x2/(H L) Sqrt[9 x1^2/L^2 + 4 x2^2/H^2]
```

Out[69]=
$$\frac{x1\,x2\,\sqrt{\frac{9\,x1^2}{L^2}+\frac{4\,x2^2}{H^2}}}{H\,L}$$

In[70]:=
```
x1/L x2/H Sqrt[9 (x1/L)^2 + 4 (x2/H)^2];
```

In[71]:=
```
diva = 0; (*  ∇·a, part c *)
Do[diva = diva + ∂x⟦i⟧ a⟦i⟧, {i, 1, 3}]; diva
```

Out[72]=
$$\frac{3\,x1^2}{H\,L^2}+\frac{2\,x2^2}{H^2\,L}$$

In[73]:=
```
dela = {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}; (*  ∇a, part d  *)
Do[Do[dela⟦i, j⟧ = ∂x⟦i⟧ a⟦j⟧, {i, 1, 3}], {j, 1, 3}];
dela
```

Out[74]=
$$\left\{\left\{\frac{2\,x2^2}{H^2\,L},\ \frac{6\,x1\,x2}{H\,L^2},\ 0\right\},\ \left\{\frac{4\,x1\,x2}{H^2\,L},\ \frac{3\,x1^2}{H\,L^2},\ 0\right\},\ \{0,\ 0,\ 0\}\right\}$$

In[75]:=
```
MatrixForm[dela]
```

Out[75]//MatrixForm=
$$\begin{pmatrix} \frac{2\,x2^2}{H^2\,L} & \frac{6\,x1\,x2}{H\,L^2} & 0 \\ \frac{4\,x1\,x2}{H^2\,L} & \frac{3\,x1^2}{H\,L^2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

In[79]:=
```
gradd
```

Out[79]=

$$\left\{\frac{2\, e^{\frac{2x3}{W}}\, x1 \cos\left[\frac{3\pi x2}{H}\right]}{L^2},\ -\frac{3\, e^{\frac{2x3}{W}}\, \pi\, x1^2 \sin\left[\frac{3\pi x2}{H}\right]}{H\, L^2},\ \frac{2\, e^{\frac{2x3}{W}}\, x1^2 \cos\left[\frac{3\pi x2}{H}\right]}{L^2\, W}\right\}$$

In[80]:=
```
adotgradd = 0;
Do[adotgradd = adotgradd + a⟦i⟧ gradd⟦i⟧, {i, 1, 3}];  (* a·∇d, part e  *)adotgradd
```

Out[81]=

$$\frac{4\, e^{\frac{2x3}{W}}\, x1^2\, x2^2 \cos\left[\frac{3\pi x2}{H}\right]}{H^2\, L^3} - \frac{9\, e^{\frac{2x3}{W}}\, \pi\, x1^4\, x2 \sin\left[\frac{3\pi x2}{H}\right]}{H^2\, L^4}$$

In[39]:=
```
a.Grad[d, {x1, x2, x3}]
```

Out[39]=

$$\frac{4\, e^{\frac{2x3}{W}}\, x1^2\, x2^2 \cos\left[\frac{3\pi x2}{H}\right]}{H^2\, L^3} - \frac{9\, e^{\frac{2x3}{W}}\, \pi\, x1^4\, x2 \sin\left[\frac{3\pi x2}{H}\right]}{H^2\, L^4}$$

In[82]:=
```
DDdela = 0;  (*  a:a, part f *)
Do[Do[DDdela = DDdela + dela⟦m,n⟧ dela⟦n,m⟧, {m, 1, 3}], {n, 1, 3}];
DDdela
```

Out[82]=

$$\frac{9\, x1^4}{H^2\, L^4} + \frac{48\, x1^2\, x2^2}{H^3\, L^3} + \frac{4\, x2^4}{H^4\, L^2}$$

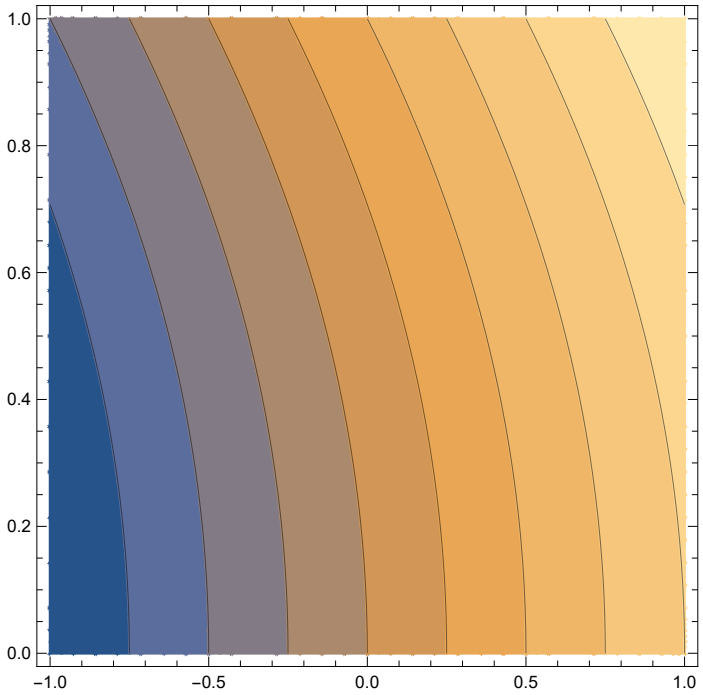
```
(*   Prob 3  *)
```

In[83]:=
```
ClearAll["Global`*"]; (*   Prob 3  *)
```

In[85]:=
```
ϕ = 2 x + y^2;
```

In[86]:=
```
ContourPlot[ϕ, {x, -1, 1}, {y, 0, 1}]
```

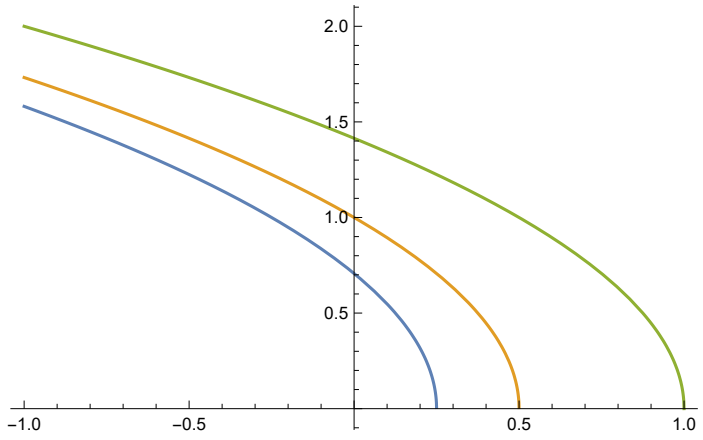
Out[86]=



In[87]:=
```
ClearAll[ϕ]; y = √(ϕ - 2 x);
```

In[88]:=
```
Plot[{y /. ϕ → 0.5, y /. ϕ → 1, y /. ϕ → 2}, {x, -1, 1}] (* part a    *)
```

Out[88]=



In[89]:=
```
ClearAll[y]; ϕ = 2 x + y^2;  gradϕ = Grad[ϕ, {x, y}]
```

Out[89]=

```
{2, 2 y}
```

In[90]:=
$$\mathbf{n = FullSimplify}\left[\frac{\mathbf{grad}\phi}{\sqrt{\mathbf{grad}\phi\mathbf{.grad}\phi}}\right]$$

Out[90]=
$$\left\{\frac{1}{\sqrt{1+y^2}}, \frac{y}{\sqrt{1+y^2}}\right\}$$

In[91]:=
$$\mathbf{ClearAll}[\phi];\ \ n = n /. y \to \sqrt{\phi - 2x}$$

Out[91]=
$$\left\{\frac{1}{\sqrt{1-2x+\phi}}, \frac{\sqrt{-2x+\phi}}{\sqrt{1-2x+\phi}}\right\}$$

In[92]:=
$$n1 = n /. \{\phi \to 1, x \to 0.0\}$$

Out[92]=
$$\{0.707107, 0.707107\}$$

In[93]:=
$$n2 = n /. \{\phi \to 1, x \to 0.5\}$$

Out[93]=
$$\{1., 0.\}$$

In[94]:=
$$\sigma = \sigma 0 \begin{pmatrix} -1 & \frac{x}{L}\left(\frac{y}{H}\right)^{-2} \\ \frac{x}{L}\left(\frac{y}{H}\right)^{-2} & -1 \end{pmatrix};$$

In[102]:=
$$y = H\sqrt{1 - 2\frac{x}{L}};$$

In[103]:=
```
f1 = Chop[(n1.σ) /. x → 0]
f2 = Chop[(n2.σ) /. x → 0.4999 L](*  part  c  *)
```

Out[103]=
$$\{-0.707107\,\sigma 0, -0.707107\,\sigma 0\}$$

Out[104]=
$$\{-1.\,\sigma 0, 2499.5\,\sigma 0\}$$