The Step-Size Dilemma

What Went Wrong in the Example?

The truncation error in the forward-difference approximation, $L_{\mathrm{FD}}h$, suggests that we make h small, but this failed in the example.

Can We Find h That Balances Truncation Error and Round-Off Error?

The following analysis is based on [NW06, pg. 196].

- Let $\tilde{f}(x)$ denote the computed value of $f(x)$ $\leftarrow$ the exact value.
- Let $\tilde{f}(x + he_j)$ the computed value of $f(x + he_j)$ $\leftarrow$ the exact value.

Can We Find h That Balances Truncation Error and Round-Off Error? (cont.)

We want the value of h that minimizes this error; we know how to do that!

The Finite-Difference Error is a Balancing Act

The analysis shows the error in the finite-difference approximation cannot be made zero.

Higher-Order Finite-Difference Methods Can Help...A Bit

Definition: Central-Difference Approximation

The central-difference approximation of the partial derivative $\partial f/\partial x_j$ of the function $f : \mathbb{R}^n \to \mathbb{R}$ is given by

$$\frac{f(x + he_j) - f(x - he_j)}{2h} = \frac{\partial f}{\partial x_j} + \underbrace{L_{\mathrm{CD}} h^2}_{\text{error}}$$

where $h > 0$ is the step size, e_j is the j^{th} Cartesian basis vector, and L_{CD} is a constant that does not depend on h.

References

J. Nocedal and S. J. Wright, *Numerical Optimization*, second ed., Springer–Verlag, Berlin, Germany, 2006.