1

Lecture 4

Review Lec ~~4~~ 3

$$F = m a$$ fluid particle (element)

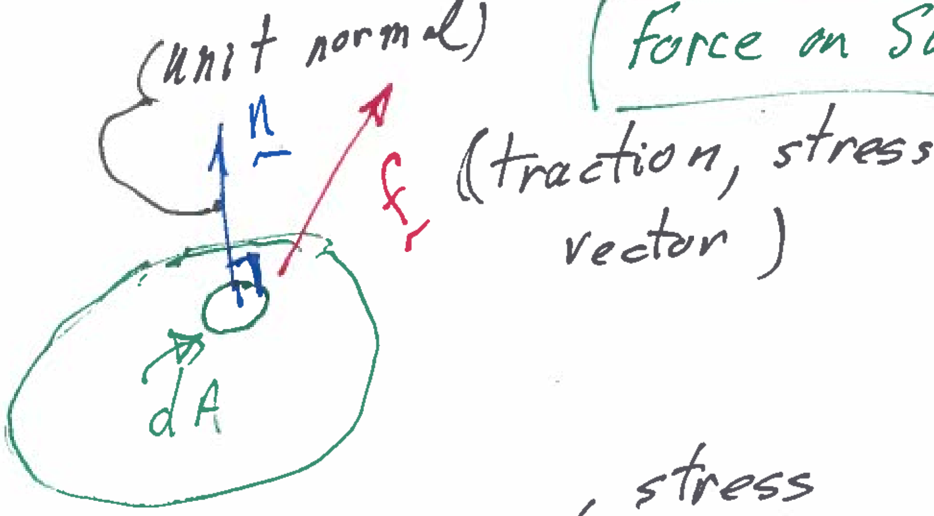
$\Uparrow$ Lec 3 $\Uparrow$ Lec 4

body forces - gravity

surface forces - traction stress vector

Tensor - tool to manipulate vectors

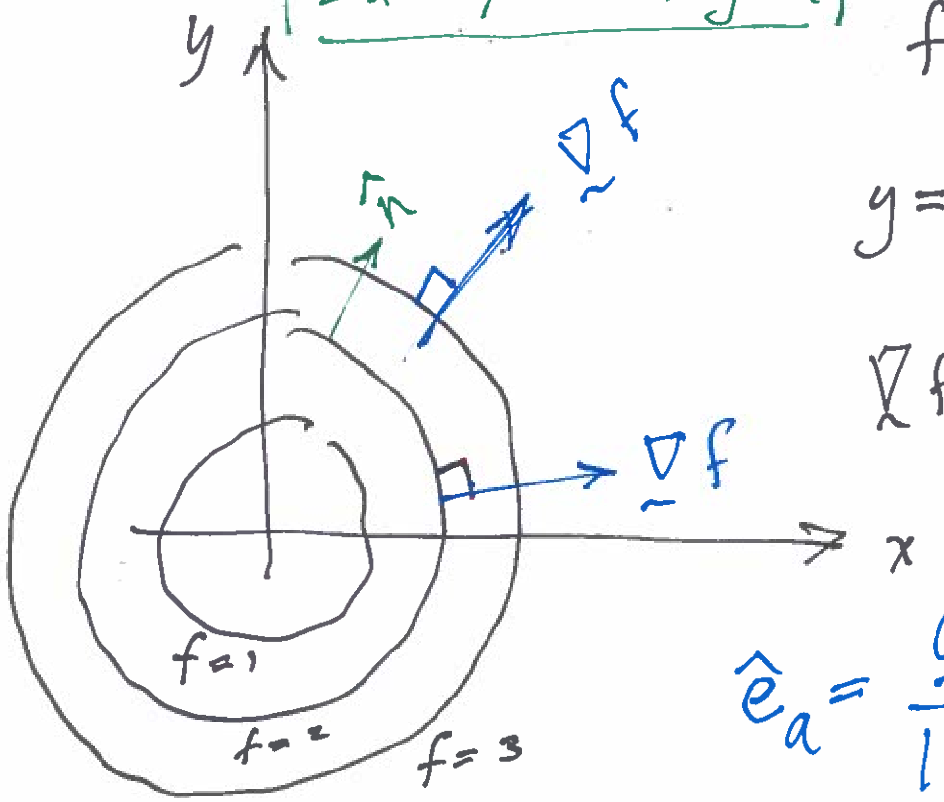
Force on Surface



stress tenor

$$\underset{\sim}{f} = \underset{\sim}{n} \cdot \underset{\approx}{\sigma}$$

Example of grad



equation of circle

$$f = x^2 + y^2$$

$$y = \pm\sqrt{f - x^2}$$

∇f ~ vector

$$\hat{e}_a = \frac{\underset{\sim}{a}}{|\underset{\sim}{a}|} \quad \text{unit vector}$$

$$\underset{\sim}{n} = \frac{\nabla f}{|\nabla f|} \qquad |\underset{\sim}{n}| = 1$$

Nabla "del" operator

$\boxed{\text{Use of } \underset{\sim}{\nabla}}$

$\underset{\sim}{\nabla}$ Vector 1st derivative

$\frac{\partial f}{\partial x}$

$\underset{\sim}{\nabla} f$ slope in direction of largest change

Use of $\underset{\sim}{\nabla}$

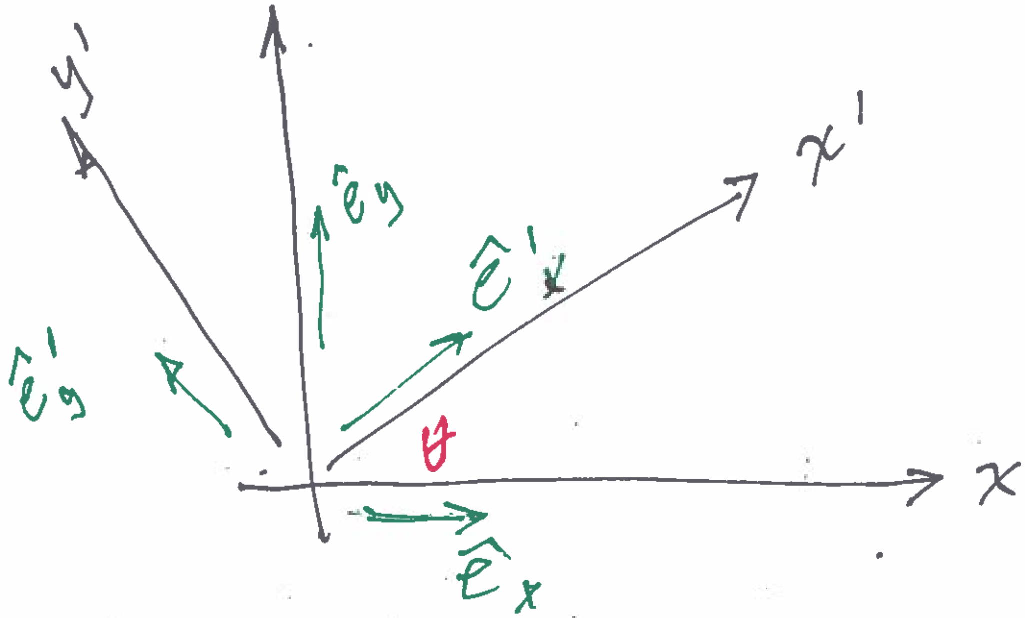
derive governing eq.
in (x, y) replace $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}$

by $\underset{\sim}{\nabla}$

Equation is good in any system

A'

Find unit vectors (Cartesian)



$$\hat{e}_x' = \cos\theta\,\hat{e}_x + \sin\theta\,\hat{e}_y$$

$$\hat{e}_y' = -\sin\theta\,\hat{e}_x + \cos\theta\,\hat{e}_y$$

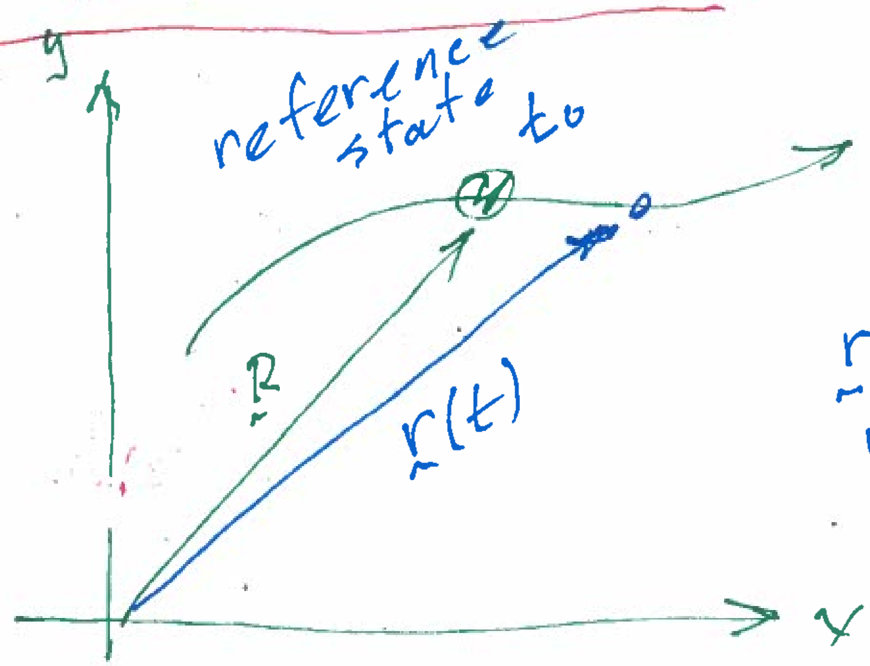
$$\hat{e}_x = \qquad = \hat{e}_x'\cos\theta - \hat{e}_y'\sin\theta$$

$$\hat{e}_y = \hat{e}_x'\sin\theta + \hat{e}_y'\cos\theta$$

Lec 4

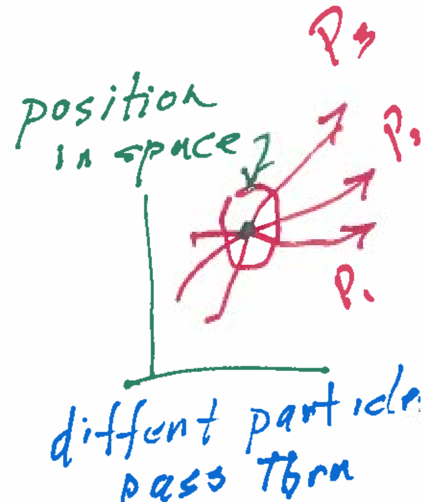
acceleration for fluids

$\underset{\sim}{F} = m \underset{\sim}{a}$



$\underset{\sim}{r}$ position vector of particle

Want $\quad \underset{\sim}{a} = \left.\dfrac{d\underset{\sim}{V}}{dt}\right|_{P=\text{const}}$



likely to have $\quad \left.\dfrac{\partial \underset{\sim}{V}}{\partial t}\right|_{x,y,z \text{ const}}$

want

$$\underset{\sim}{r} = \underset{\sim}{r}(\underset{\sim}{R}, t)$$

$\underset{\sim}{R}$ names particle

$\underset{\sim}{R}$ at $t = t_0$ "names the particle"

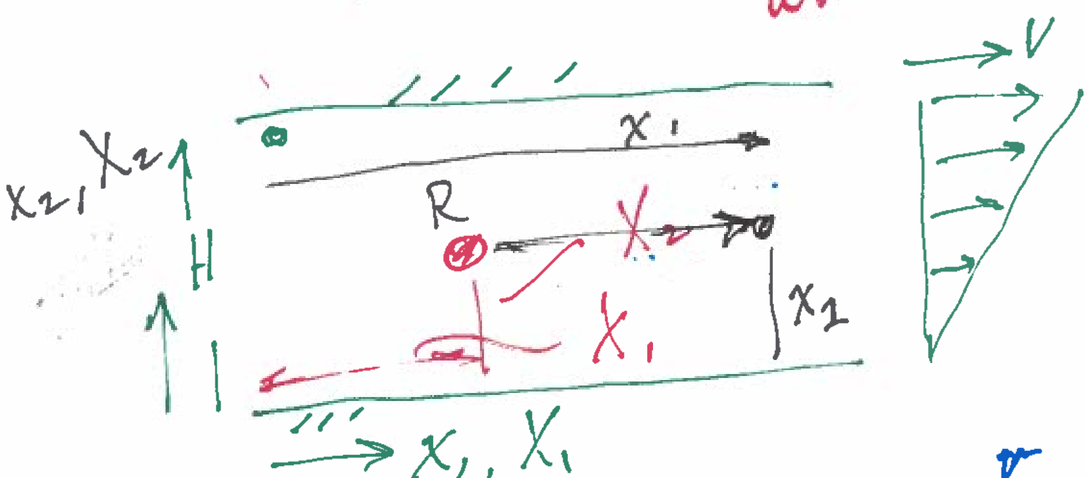
We want

$$\underset{\sim}{a} = \left.\frac{d\underset{\sim}{v}}{dt}\right|_{R \text{ fixed}}$$

what particle? the one that was at location $\underset{\sim}{R}$ at time t_0

Example

at t_0



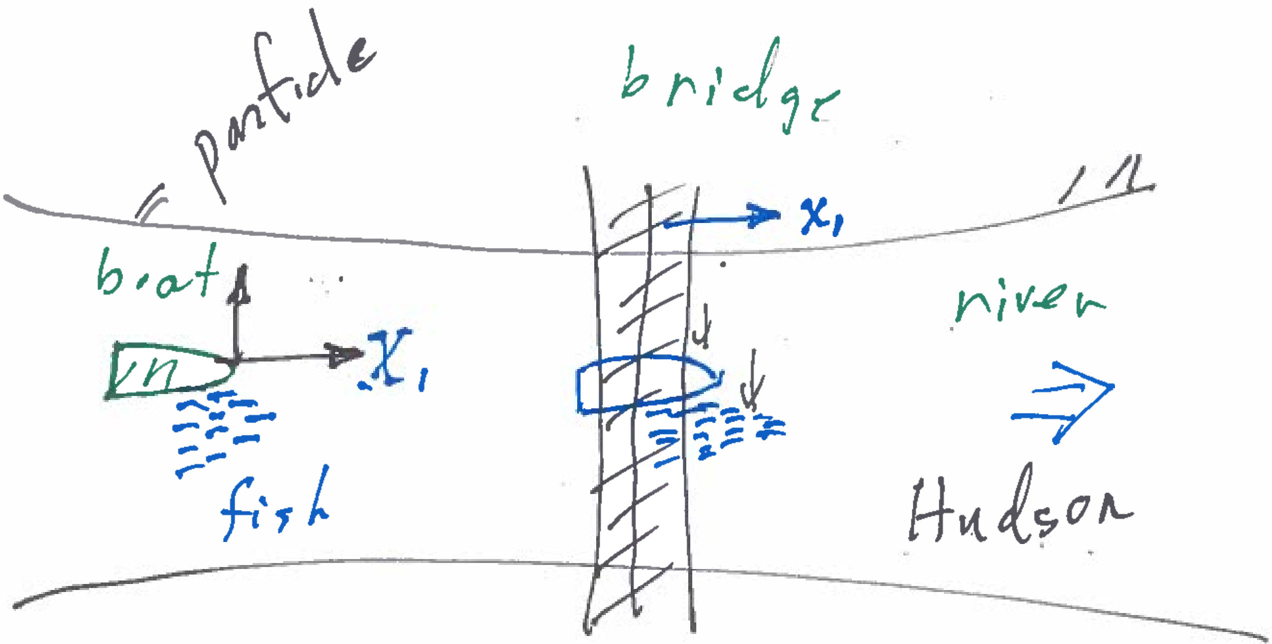
$$x_1 = X_1 + V(t - t_0)\frac{X_2}{H}$$

$$x_2 = X_2$$

$$v_1 = \left.\frac{\partial x_1}{\partial t}\right|_{X_1, X_2 \text{ const}}$$

$$v_2 = \left.\frac{\partial x_2}{\partial t}\right|_{X_1, X_2} = 0$$

$$v_1 = \frac{V}{H} X_2$$



boat drifts with current

$$\frac{\#\,\text{fish}}{m^2} = f$$

$$\left.\frac{df}{dt}\right|_{\substack{\text{boat}\\ X_1=\text{const}}} = 0$$

$$\left.\frac{df}{dt}\right|_{\substack{\text{bridge}\\ x_1=\text{const}}} \neq 0 \qquad \left.\frac{df}{dt}\right|_{\text{bridge}} \simeq \frac{f(t_2)-f(t_1)}{t_2-t_1}$$

$$t_2 \to t_1$$

$$\frac{Df}{Dt} = \left.\frac{df}{dt}\right|_{\cancel{X}\ \text{fixed particle}}$$

material derivative

substantial "

Lagrangian "

$$f = f\left[x_1(\underset{\sim}{X}, t), x_2(\underset{\sim}{X}, t), t\right]$$

$$\left.\frac{\partial f}{\partial t}\right|_{X\ \text{const}} = \frac{Df}{Dt}$$

$$= \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x_1}\overbrace{\frac{\partial x_1}{\partial t}}^{V_x} + \frac{\partial f}{\partial x_2}\overbrace{\frac{\partial x_2}{\partial t}}^{V_y}$$

Nested derivative rule

-7a-

$u(x) = \sin x$

$f(u) = u^2$

Nested derivatives

$$\frac{df}{dx} = \underbrace{\frac{df}{du}}_{2u} \cdot \underbrace{\frac{du}{dx}}_{\cos x}$$

8-

$$\div \cancel{V} \quad \underset{\sim}{F} = m \underset{\sim}{a} \qquad \cancel{V} \text{ volume}$$

$$\rho = \frac{m}{\cancel{V}} \qquad \Sigma \frac{\underset{\sim}{F}}{\cancel{V}} = \rho \underset{\sim}{a} = \rho \frac{DV}{Dt}$$

$$\frac{D\underset{\sim}{V}}{Dt} = \frac{D}{\cancel{\partial} t}\left(V_x \hat{e}_x + V_y \hat{e}_y\right)$$

$$\frac{DV_x}{Dt} = a_x = \frac{\partial V_x}{\partial t} + V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_{\cancel{x}}}{\partial y}$$

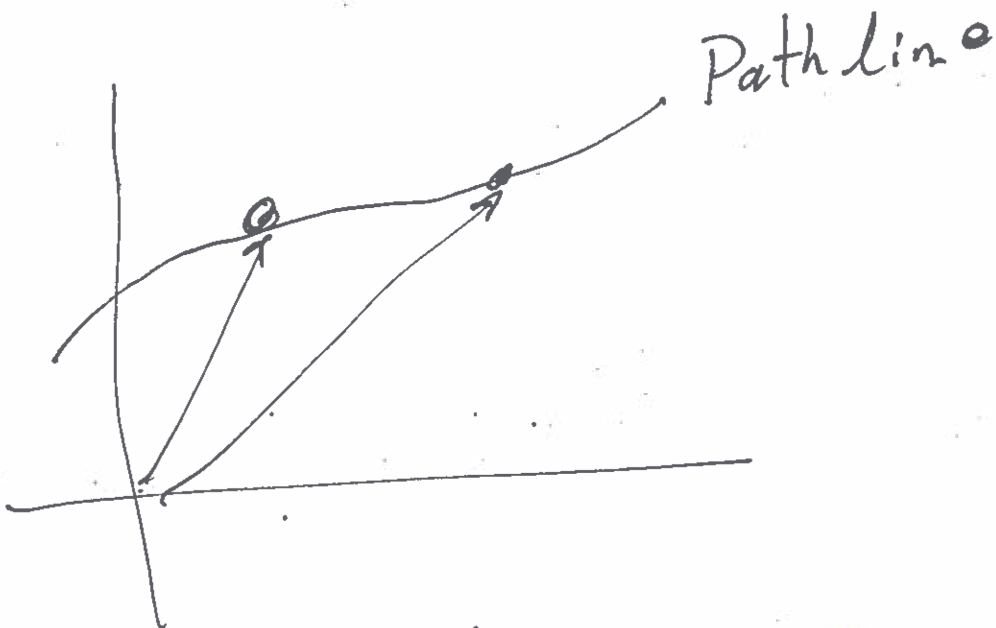
$$a_y = \frac{\partial V_y}{\partial t} + V_x \frac{\partial V_y}{\partial x} + V_y \frac{\partial V_y}{\partial y}$$

-9- Pathlines – particle (just did) path

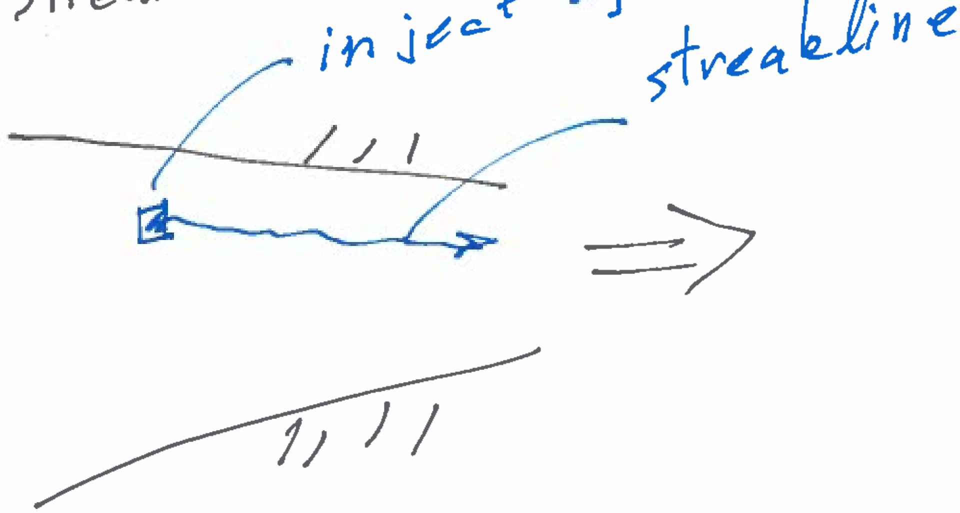
Streak lines

Stream lines

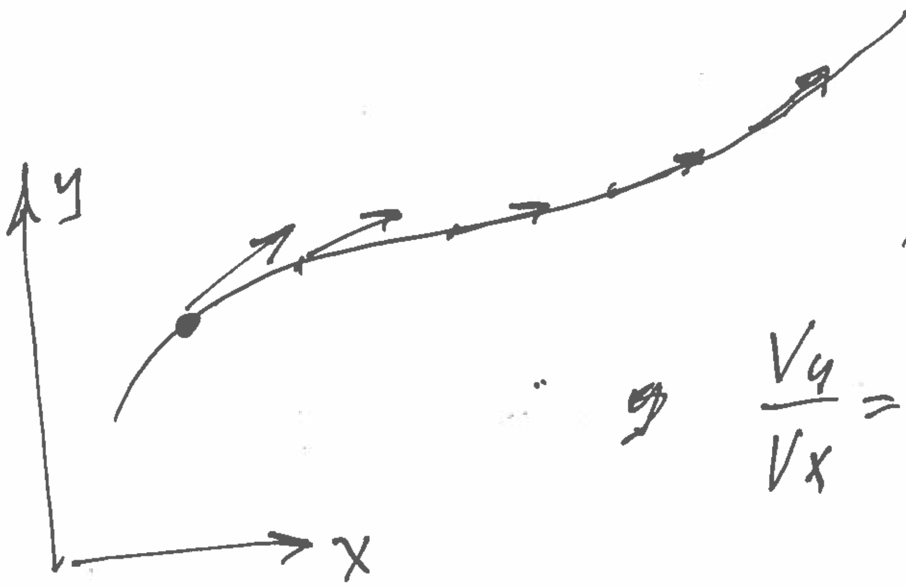
steady flow: all three the same

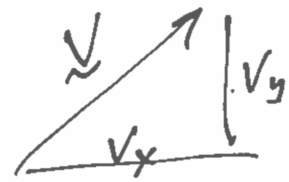


streakline

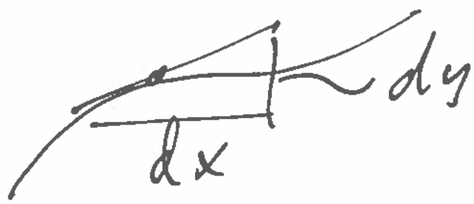


stream line





$$\frac{V_y}{V_x} = \frac{dy}{dx}$$



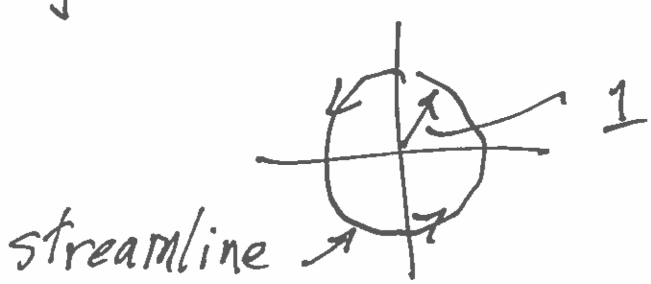
ψ = stream function $\psi(x,y)$

ψ = const. $\Rightarrow$ stream line

example:

$\psi = x^2 + y^2$

set $\psi = 1$



$\psi \Rightarrow$

1) Visualize the flow
2nd

can show: $V_x = \frac{\partial \psi}{\partial y}$ $V_y = -\frac{\partial \psi}{\partial x}$

2) change 2 variable $V_x, V_y \Rightarrow$
2 unknowns

to 1 unknown $\psi(x,y)$