The Wolfe Conditions

We Can Improve Sufficient Decrease if We Also Satisfy the Curvature Condition

Definition: Curvature Condition [NW06]

A step length $\alpha > 0$ satisfies the curvature condition if

$$\phi'(\alpha) \geq c_2 \phi'(0),$$

for some constant $c_2 \in (c_1, 1)$.

Illustration of the Curvature Condition

The Wolfe Conditions Combine Sufficient Decrease and Curvature

Definition: The Wolfe Conditions [NW06]

The Armijo (sufficient decrease) and curvature condition are collectively known as the Wolfe conditions, repeated here in terms of ϕ.

$$\begin{aligned} \phi(\alpha) &\leq \phi(0) + c_1 \phi'(0)\alpha, \\ \phi'(\alpha) &\geq c_2 \phi'(0), \end{aligned}$$

where $\phi(\alpha) = f(x_k + \alpha p_k)$.

The Strong Wolfe Conditions Require More From the Step

Definition: The Strong Wolfe Conditions [NW06]

The Strong Wolfe Conditions are given by

$$\begin{aligned} \phi(\alpha) &\le \phi(0) + c_1\phi'(0)\alpha, \\ |\phi'(\alpha)| &\le c_2|\phi'(0)|, \end{aligned}$$

where $\phi(\alpha) = f(x_k + \alpha p_k)$.

Illustration of the Strong Wolfe Conditions

References

J. Nocedal and S. J. Wright, *Numerical Optimization*, second ed., Springer–Verlag, Berlin, Germany, 2006.