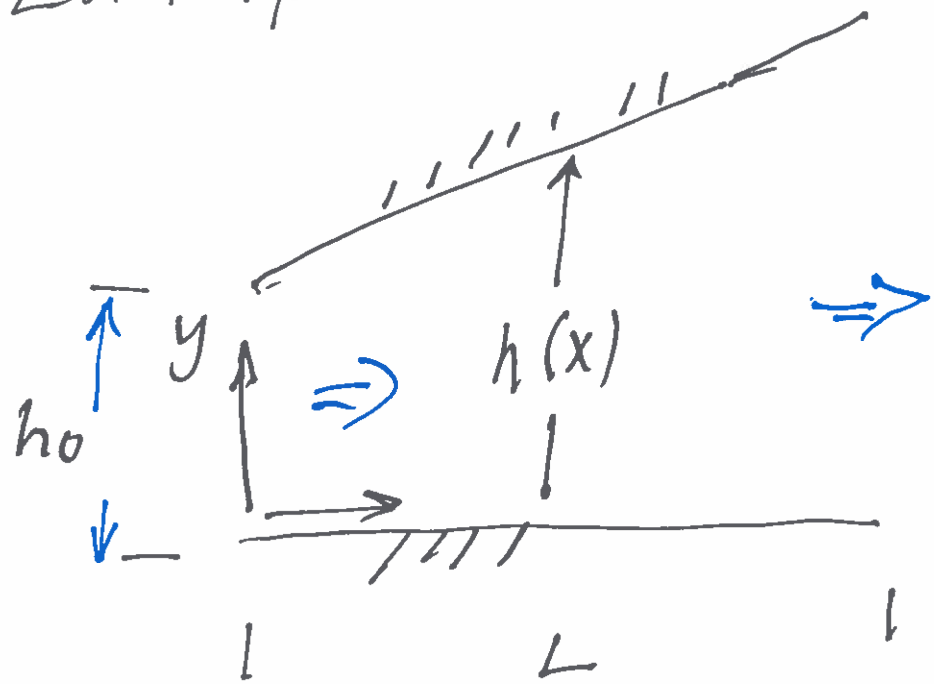
Class # 6

Example - similar to HW



W into paper

$m > 0$

given: $h = h_0 \left(1 + m \frac{x}{L}\right)$

$x = 0 \quad h = h_0 \quad x = L$

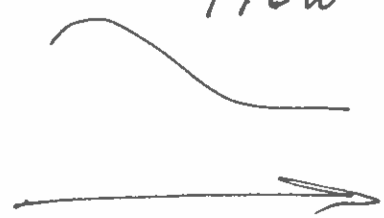
$= h_0 (1 + m)$

$$\underset{\approx}{\sigma} = \begin{bmatrix} -p & \tau \\ \tau & -p \end{bmatrix}$$

Model for shear flow

is $\underset{\approx}{\sigma} = -p\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + \begin{bmatrix}0 & \tau\\ \tau & 0\end{bmatrix}$

shear flow



p = pressure

τ = shear stress

τ depends on motion

$\underset{\sim}{V} \to 0 : \quad \tau \to 0$

$\sigma_{xx} = -p = \sigma_{yy}$

inward/compressive

This problem:

$p = p_0 \qquad \tau = \tau_0 \left(\frac{y}{h}\right)$

HW: $p = p(x) \qquad \tau = \tau(x, y)$

$y = 0 \quad \tau = 0 \qquad y = h(x) \quad \tau = \tau_0$

Find $\underset{\sim}{F}$ on upper surface

Course General:

stress model

$$\underline{\text{Diff Eq}} + \underline{BC} \Rightarrow \underline{\underset{\sim}{V}} = \underset{\approx}{\sigma}$$

$$\Rightarrow \underset{\sim}{F}$$

this example just $\underset{\approx}{\sigma} \Rightarrow \underset{\sim}{F}$

$$\underset{\sim}{t} = \left.\frac{d\underset{\sim}{F}}{dA}\right|_{\text{surface}} = \underset{\sim}{n} \cdot \underset{\approx}{\sigma}$$

$\underset{\sim}{t}$ = traction vector or stress vector

$$F_x = \int_{A \atop \text{surface}} t_x \, dA$$

$$F_y = \int_A t_y \, dA$$

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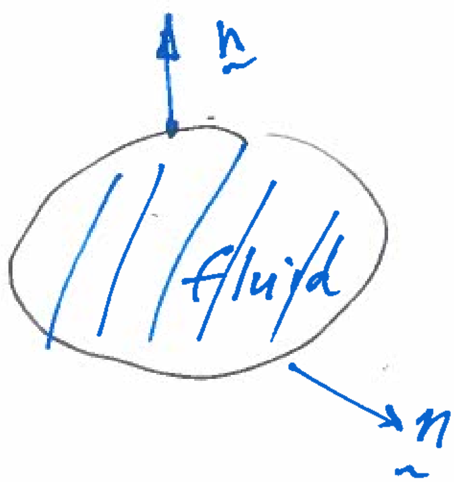
$$\underset{\sim}{F} = F_x \hat{e}_x + F_y \hat{e}_y$$

$\Rightarrow$ find $\underset{\sim}{n}$ outward unit normal to surface

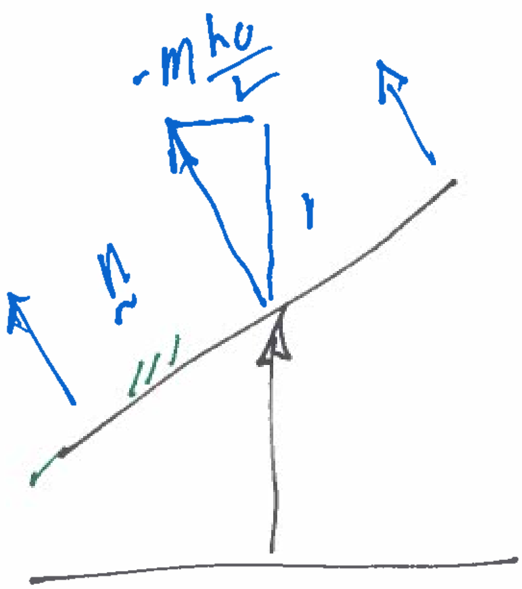


surface defined: $y = h(x)$

$f = y - h(x)$ $\quad f = 0$ surface in question

$$\underset{\sim}{n} = \hat{n} = \frac{\underset{\sim}{\nabla} f}{|\underset{\sim}{\nabla} f|}$$

any vector $\underset{\sim}{a}$ unit vector $= \dfrac{\underset{\sim}{a}}{|\underset{\sim}{a}|}$ in direction of $\underline{a}$



$$h = h_0\left(1 + m\frac{x}{L}\right)$$

$$f = y - h_0\left(1 + m\frac{x}{L}\right)$$

$$\underset{\sim}{\nabla} f = \frac{\partial f}{\partial x}\hat{e}_x + \frac{\partial f}{\partial y}\hat{e}_y$$

$$\frac{\partial f}{\partial x} = -h_0\frac{m}{L} \qquad \frac{\partial f}{\partial y} = 1$$

$$\underset{\sim}{\nabla} f = -m\frac{h_0}{L}\hat{e}_x + \hat{e}_y$$

$$\underset{\sim}{n} = \pm\frac{\underset{\sim}{\nabla} f}{|\underset{\sim}{\nabla} f|} = \frac{-m\frac{h_0}{L}\hat{e}_x + \hat{e}_y}{\sqrt{1 + \left(-\frac{h_0}{L}m\right)^2}}$$

= const (this case, not HW)

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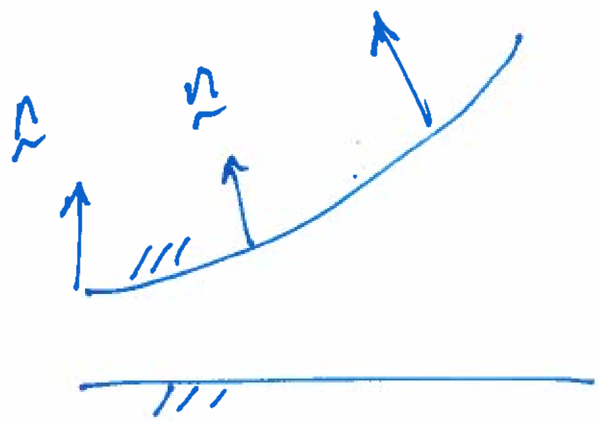
HW: $h \propto x^2$



$$\underset{\sim}{t} = \underset{\sim}{n} \cdot \underset{\approx}{\sigma}$$

$$= \frac{1}{\sqrt{\quad}} \left(-m\frac{h_0}{L}\hat{e}_x + \hat{e}_y \right) \cdot \left(\sigma_{xx}\hat{e}_x\hat{e}_x + \sigma_{xy}\hat{e}_x\hat{e}_y + \dots \right)$$

$$\cdot \left(-p\,\hat{e}_x\hat{e}_x + \tau\,\hat{e}_x\hat{e}_y + \tau\,\hat{e}_y\hat{e}_x - p\,\hat{e}_y\hat{e}_y \right)$$

$$\underset{\sim}{t} = t_x\hat{e}_x + t_y\hat{e}_y =$$

$$= \left(m\frac{h_0}{L}p\,\hat{e}_x - m\frac{h_0}{L}\tau\,\hat{e}_y + \tau\,\hat{e}_x - p\,\hat{e}_y \right) \frac{1}{\sqrt{\quad}}$$

$$= \left[\hat{e}_x \left(\frac{m h_0}{L}p + \tau \right) + \hat{e}_y \left(-\frac{m h_0 \tau}{L} - p \right) \right] \cdot \frac{1}{\sqrt{\quad}}$$

$$F_x = \int_A t_{hx}\, dA \qquad (y=h)$$

$\underset{\sim}{t}_h = \underset{\sim}{t}(y=h)$

$dA = W\,dx$

$$F_x = W\int_0^L \left(\frac{m h_0}{L} P_0 + \tau_0\right) dx$$

$$= WL\left(m\frac{h_0}{L} P_0 + \tau_0\right)$$

$\tau = \tau_0 \left(\frac{y}{h}\right)$

$\tau_h = \tau_0 \left(\frac{h}{h}\right)$

$= \tau_0$

(next page)

$F_y = \dots$

Any ~~case~~ problem — if you can write as def. integral

$$f = \int_0^d f(x;\, \underline{a, b}, c)\, dx$$

know. a, b, c, d

solved!

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$$\underset{\sim}{t} = \frac{1}{\sqrt{\quad}} \left[\underset{=}{t_x} \hat{e}_x + t_y \hat{e}_y \right] = \underset{\sim}{t}(x, y)$$

stress vector any pt in flow

want $\underset{\sim}{t}$ at $y = h(x)$

$\underset{\sim}{t}(x, h(x))$ upper surface

$y = h(x)$ $\quad \tau = \tau_0 \left(\frac{y}{h} \right)$

at $y = h$: $\tau = \tau_0$

at $y = h(x)$

$$\underset{\sim}{t}_{n \, y=h} = \left(\frac{m h_0}{L} p_0 + \tau_0 \right) \hat{e}_x + \left(-\frac{m h_0}{L} \tau - p \right) \hat{e}_y \cdot \frac{1}{\sqrt{\quad}}$$

mathematica

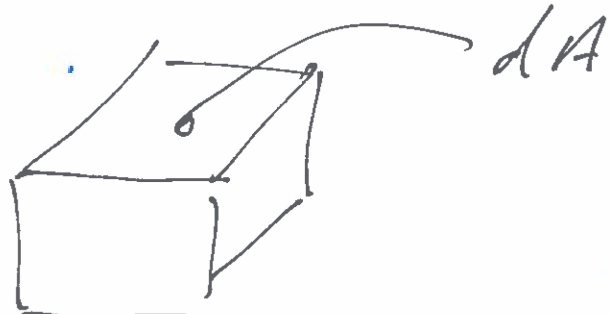
NIntegrate[f(x), {x, 0, L}]

New materia

$$m\underset{\sim}{a} = \sum \underset{\sim}{F}$$

$\underset{\sim}{a}$ of particle, not point in space

$$\sum \underset{\sim}{F} = \underbrace{\underset{\sim}{F}_{body}}_{\text{gravity}} + \underbrace{\underset{\sim}{F}_{surface}}_{\text{stress on surface}}$$



Model for stress

fn($\underset{\sim}{V}$)

$$\underset{\approx}{\sigma} = -p\,\underset{\approx}{\delta} + \underset{\approx}{\tau}$$

$\underset{\approx}{\delta}$ = unit tensor = Kronecker delta

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

want $\underset{\sim}{V} \to 0$ $\quad \underset{\approx}{\sigma} = \begin{bmatrix} -p & 0 \\ 0 & -p \end{bmatrix}$

$\underset{\approx}{\tau} \to 0$

Model: $\underset{\approx}{\tau} = 0$ inviscid flow

rather OK away from body/wall/surface

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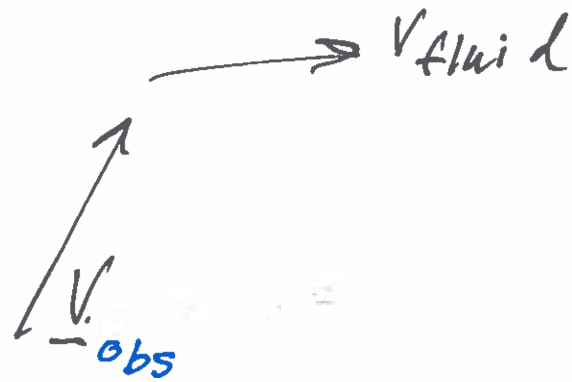
All observers calculate
same stress

observers can translate
rotate

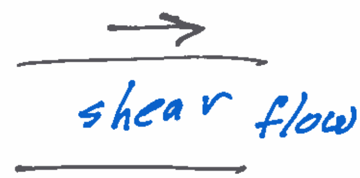
$\underset{\approx}{\tau} = k \underset{\sim}{V}$



$\underset{\sim}{V} =$

$\underset{\sim}{V}_{fluid} - \underset{\sim}{V}_{obs}$

$\tau_{xy} = \mu \frac{\partial v_x}{\partial y}$



linear   viscosity

use $\underset{\sim}{\nabla} \underset{\sim}{V} = \begin{pmatrix} \frac{\partial v_x}{\partial y} & \frac{\partial v_y}{\partial x} \\ \dots & \cdot \end{pmatrix}$

$\underset{\approx}{\tau} \overset{?}{=} \mu \underset{\sim}{\nabla} \underset{\sim}{V}$ why not?

rotating observers calculate
different $\underset{\sim}{\nabla} \underset{\sim}{V}$

$\tau_{xx} = \frac{\partial v_x}{\partial x}$ $\tau_{xy} = \cancel{\mu \frac{\partial v_x}{\partial y}}$ etc