In[ ]:= ClearAll["Global`*"]; (* HW#4 F22, prob#1, asll units SI *)

In[ ]:= $Qdot = \dfrac{8.\ (10^3)^3}{365 \times 24 \times 3600}$ (* residential/urban m^3/s *)

Found : Residential/Industrial use 8 km^3

Out[ ]= $\dot{Q}$ = 253.678 m^3/s

In[ ]:= $Dmax = 2.5;\ AiMax = \dfrac{\pi\, Dmax^2}{4};$ (* pipe diametr/cross-section *)

found greater pipe size each

In[ ]:= Vmax = 5.; (* max water flow speed *)

found typical max speed water flow in pipe

In[ ]:= $Areq = \dfrac{Qdot}{Vmax}$

Required total area

Out[ ]= 50.7357

In[ ]:= $\rho = 1000.;\ \mu = 0.01;\ g = 9.81;$ (* properties *)

density, visc, gravity

In[ ]:= $Reyn = \dfrac{\rho\, Vmax\, Dmax}{\mu}$

Reyn No per pipe

Out[ ]= 1.25×10^6 $> 2000 \Rightarrow$ turbulent

In[ ]:= $nP = \dfrac{Areq}{AiMax}$ (* number of pipes *)

Out[ ]= 10.3358

In[ ]:= $L = 2200 \times 1.6 \times 1000$ (* length, Chicago to SanFran *)

miles

Out[ ]= 3.52×10^6 L in meters

In[ ]:= $\epsilon Rough = \dfrac{0.01 \times 12}{40}$ (* pipe roughness *)

Out[ ]= 0.003

In[ ]:= $fDarcy = \dfrac{0.3085}{Log\left[10, \frac{6.9}{Reyn} + \left(\frac{\epsilon Rough}{3.7\ Dmax}\right)^{1.1}\right]^2}$ (* friction factor *)

Out[ ]= 0.0211221 = f

Chi → SF

In[ ]:= $\Delta pTotal = fDarcy \dfrac{L}{Dmax}\left(\dfrac{1}{2}\rho\, Vmax^2\right)$

total pressure loss

Out[ ]= 3.71749×10^8 Pa = 371 MPa ≅ 3700 Bar (huge)

~50000 psi

In[ ]:= $MW = \frac{\Delta pTotal\ Areq\ Vmax}{10^6}$ (* total power required *)

MW

$= \Delta P \cdot \dot{Q}$ $\frac{N}{m^2} \cdot \frac{m^3}{s} = W$

MW

Out[ ]=

94 304.6 ~ 10 x NYC Power

In[ ]:= (* MW ridiculous, Nuke power plant = 100 MW *)

help from elevation change ?

net h ≅ 200m

In[ ]:= h12 = 200.; (* elevation change *)

$\Delta pEl = \rho\, g\, h12$; (* net downhill flow Chi SF, pressure assist *)

$\frac{\Delta pEl}{\Delta pTotal}$

Out[ ]=

0.00527776 fractional contribution by elevation < 1%

In[ ]:= hPumpMax = 200.; $\Delta pPumpMax = \rho\, g\, hPumpMax$

Out[ ]=

1.962×10^6 Let's say each pump 200 m head = 1.96 MPa ≅ 19.6 Bar

In[ ]:= $Lseg = \frac{\Delta pPumpMax\ Dmax}{fDarcy\left(\frac{1}{2}\rho\, Vmax^2\right)}$ (* length of pipe segment at ΔpPumpMax *)

segment length of pipe flow provided by pump

Out[ ]=

18 577.7 = 18.5 km

$nSeg = \frac{L}{Lseg}$ (* number of pump stations *)

Out[ ]=

189.474 Number of pump station

10 pipes total

In[ ]:= $MWseg = \frac{\Delta pPumpMax\ AiMax\ Vmax}{10^6}$

Out[ ]=

48.1547 —— MW per pipe per station

In[ ]:= ClearAll["Global`*"]; (* HW#4 F22, prob#2 *)

In[ ]:= $h = H\left(1 - m\frac{x}{L}\right)$;

In[ ]:= $vx = 6\, V\frac{x}{H}\left(\frac{y}{h} - \frac{y^2}{h^2}\right)$;

In[ ]:= $vy = -\int \partial_x vx\, dy + C1$

Out[ ]=

$$C1 + \frac{6\, L^2\, V\left(-\frac{1}{2} H\, (L - m\, x)\, y^2 + \frac{1}{3}\, (L + m\, x)\, y^3\right)}{H^3\, (L - m\, x)^3}$$

In[ ]:= vy /. y → 0

Out[ ]=

C1 = 0

$$vy = \frac{6 L^2 V \left(-\frac{1}{2} H (L - m x) y^2 + \frac{1}{3} (L + m x) y^3\right)}{H^3 (L - m x)^3};$$ (* part a *)

In[ ]:= Simplify[∂_xvx + ∂_yvy]

Out[ ]=

0

In[ ]:= vy /. y → H

Out[ ]=

$$\frac{6 L^2 V \left(-\frac{1}{2} H^3 (L - m x) + \frac{1}{3} H^3 (L + m x)\right)}{H^3 (L - m x)^3}$$

In[ ]:= DvxDt = Simplify[vx ∂_xvx + vy ∂_yvx]

for steady

Out[ ]=

$$\frac{6 L^3 V^2 x y^2 \left(3 H^2 (L - m x)^2 - 4 H \left(L^2 - m^2 x^2\right) y + 2 L (L + m x) y^2\right)}{H^6 (L - m x)^5}$$

In[ ]:= DvyDt = Simplify[vx ∂_xvy + vy ∂_yvy] (* part b *)

Out[ ]=

$$\frac{1}{H^6 (L - m x)^6} 6 L^3 V^2 y^3 \left(3 H^2 (L - 2 m x) (L - m x)^2 - H \left(5 L^3 - 14 L^2 m x + 5 L m^2 x^2 + 4 m^3 x^3\right) y + 2 L \left(L^2 - 2 L m x - m^2 x^2\right) y^2\right)$$

In[ ]:= ex = {1, 0}; ey = {0, 1}; (* unit vectors *)

In[ ]:= v = vx ex + vy ey

Out[ ]=

$$\left\{\overbrace{\frac{6 V x \left(\frac{y}{H\left(1-\frac{m x}{L}\right)} - \frac{y^2}{H^2\left(1-\frac{m x}{L}\right)^2}\right)}{H}}^{V_x}, \overbrace{\frac{6 L^2 V \left(-\frac{1}{2} H (L - m x) y^2 + \frac{1}{3} (L + m x) y^3\right)}{H^3 (L - m x)^3}}^{V_y}\right\}$$

r = x ex + y ey (* position vector *)

Out[ ]=

{x, y}

Bird uses $\nabla V_{ij} = \frac{\partial V_j}{\partial x_i}$

gradv = Simplify[Transpose[Grad[v, r]]] (* Mathematica uses gradv[[i,j]] = $\partial_{r[[j]]}$v[[i]] *)

Out[ ]=

$$\left\{\left\{-\frac{6 L^2 V y (-H L + H m x + L y + m x y)}{H^3 (L - m x)^3}, \frac{2 L^2 m V y^2 (-3 H L + 3 H m x + 4 L y + 2 m x y)}{H^3 (L - m x)^4}\right\}, \left\{\frac{6 L V x (H (L - m x) - 2 L y)}{H^3 (L - m x)^2}, \frac{6 L^2 V y (-H L + H m x + L y + m x y)}{H^3 (L - m x)^3}\right\}\right\}$$

Diffence doesn't matter for $\dot{\underline{\underline{\gamma}}} = \underline{\nabla V} + \underline{\nabla V}^T$

```
Simplify[Div[Grad[v, r], r]] (* same as Laplacian *)
```

Out[ ]=

$$\left\{-\frac{12 L^2 V \left(L^2 x - L m \left(2 x^2 + (H - 2 y) y\right) + m^2 x \left(x^2 + y (H + y)\right)\right)}{H^3 (L - m x)^4}, -\frac{1}{H^3 (L - m x)^5} 6 L^2 V \left(-2 y \left(L^3 - L^2 m x - L m^2 \left(x^2 - 3 y^2\right) + m^3 x \left(x^2 + y^2\right)\right) + H (L - m x) \left(L^2 - 2 L m x + m^2 \left(x^2 + 3 y^2\right)\right)\right)\right\}$$

```
Simplify[Laplacian[v, r]] (*  part c  *)
```

check

Out[ ]=

$$\left\{-\frac{12 L^2 V \left(L^2 x - L m \left(2 x^2 + (H - 2 y) y\right) + m^2 x \left(x^2 + y (H + y)\right)\right)}{H^3 (L - m x)^4}, -\frac{1}{H^3 (L - m x)^5} 6 L^2 V \left(-2 y \left(L^3 - L^2 m x - L m^2 \left(x^2 - 3 y^2\right) + m^3 x \left(x^2 + y^2\right)\right) + H (L - m x) \left(L^2 - 2 L m x + m^2 \left(x^2 + 3 y^2\right)\right)\right)\right\}$$

In[ ]:=

```
V = 1.; H = 0.001; L = 1.; W = 1.; μ = 0.010; ρ = 1000.; g = 10.; p0 = 10. × 10^6;
orderDDx = 1 / L
orderDDy = 1 / H (* part d  *)
orderDDx / orderDDy
```

Out[ ]=

0.001

In[ ]:=

$$\text{Reyn} = \frac{\rho V H}{\mu}$$

$$Re^* = \text{ReynS} = \text{Reyn}\,\frac{H}{L} \;\; (\text{* part e *}) \;\; = \frac{\text{inertia}}{\text{viscous}}$$

Note

$$\underbrace{V_x \frac{\partial v_x}{\partial x} + V_y \frac{\partial V_x}{\partial y}}_{\text{always same O-M}}$$

Out[ ]=

100.

Out[ ]=

0.1

In[ ]:=

```
ClearAll["Global`*"];
```

x - N-S In[ ]:= $\partial_x p[x, y] == \mu\, \partial_{y,y} vx[x, y]$; (* part f *)

y - N-S $\partial_y p[x, y] == \mu\, \partial_{y,y} \cancel{vy[x}, y]$;

In[ ]:=

```
orderVx = V;
orderVy = V H / L;
orderDpDx = μ V / H^2;
orderDpDy = μ V / L^2; (* part g *)
orderDpDy / orderDpDx
```

$$\frac{N\text{-}s}{m^2}\;\frac{m}{s}\;\frac{1}{m^2} = \frac{N}{m^3}$$

Out[ ]=

$$\frac{H^2}{L^2}$$

In[ ]:= orderP = orderDpDx L

Out[ ]=

$$\frac{L V \mu}{H^2} \qquad m \frac{m}{\cancel{s}} \frac{N\text{-}\cancel{s}}{m^2} \frac{1}{m^2} = \frac{N}{m^2}$$

In[ ]:= orderGrav = ρ g H ;
orderGrav / orderP

$$\frac{N s^2}{m^4}, \frac{m}{\cancel{s^2}}, m \sim \frac{N}{m^2}$$

Out[ ]=

$$\frac{g H^3 \rho}{L V \mu}$$

```
V = 1.; H = 0.001; L = 1.; W = 1.; μ = 0.010; ρ = 1000.; g = 10.;
orderGrav / orderP (* part i *)
```

Out[ ]=

0.001 gravity small

In[ ]:= ClearAll["Global`*"];

In[ ]:= $h = H\left(1 - m\frac{x}{L}\right)$;

In[ ]:= $vx = 6 V \frac{x}{H}\left(\frac{y}{h} - \frac{y^2}{h^2}\right)$;

In[ ]:= dpdx = $\mu\, \partial_{y,y} vx$

$$\frac{dp}{dx} = \mu \frac{d^2 V_x}{dy^2} = \frac{12 V x \mu}{H} \frac{1}{h^2}$$

Out[ ]=

$$-\frac{12 V x \mu}{H^3 \left(1 - \frac{m x}{L}\right)^2}$$

In[ ]:= $p = \int dpdx\, dx + C1$

Out[ ]=

$$C1 - \frac{12 L^2 V \mu \left(\frac{L}{L - m x} + \text{Log}[L - m x]\right)}{H^3 m^2}$$

In[ ]:= p /. x → 0

Out[ ]=

$$C1 - \frac{12 L^2 V \mu \,(1 + \text{Log}[L])}{H^3 m^2}$$

In[ ]:= Solve[(p /. x → 0) == p0, C1]

Out[ ]=

$$\left\{\left\{C1 \to \frac{H^3 m^2 p0 + 12 L^2 V \mu + 12 L^2 V \mu \,\text{Log}[L]}{H^3 m^2}\right\}\right\}$$

$$p = p0 + \frac{12\,L^2\,V\,\mu + 12\,L^2\,V\,\mu\,\mathrm{Log}[L]}{H^3\,m^2} - \frac{12\,L^2\,V\,\mu\left(\frac{L}{L-m\,x} + \mathrm{Log}[L-m\,x]\right)}{H^3\,m^2};$$

$$p = p0 + \frac{12\,L^2\,V\,\mu}{H^3\,m^2}\left(1 + \mathrm{Log}[L] - \left(\frac{L}{L-m\,x} + \mathrm{Log}[L-m\,x]\right)\right);$$

```
(*  part j  *)
```

$$\cancel{M}^2\,\frac{\cancel{M}}{\cancel{s}}\;\frac{N\cdot\cancel{s}}{M^2}\;\frac{1}{\cancel{M^3}} = \frac{N}{M^2}$$

$$1 - \frac{H}{h} + \ln\left(\frac{L}{L-mx}\right)$$

$$\ln\left(\frac{H}{h}\right)$$

$$\frac{L}{L-mx} = \frac{1}{1-\frac{m}{L}x} = \frac{1}{h/H}$$

$$= H/h$$