MANE 6520 - Fluid Mechanics

Homework #6 - Monday Nov 14 2022, due Monday Nov 21 2022

Regarding plotting below, your job isn't done just because you hit "plot" and a graph is produced. You need labels and units (by hand is OK) and see if the results make sense (BCs satisfied, reasonable numerical values, etc.)

#1- For the stream function $\psi = VH\frac{x^2}{L^2}\sin\left(\pi\frac{y}{H}\right)$, determine the velocity field. Show that this velocity field satisfies the incompressible 2-D continuity equation. Plot stream lines for V=1 m/s, $L = H$ = 1 m. ψ = 1 and 2 m²/s, $L \le x \le 2L$.

#2 – Find an approximate regular perturbation for small ε to the nonlinear equation:
$\frac{d^2v}{dy^2} + \varepsilon\frac{1}{V^2H^2}v^3 = 0$, with boundary conditions $y = 0 : v = 0$; $y = H : v = V$

Plot results $v(y)$ for $H = V = 1$, ε=0, 0.1, 1 (the three cases on the same graph).

#3 - Perform a laminar boundary layer integral analysis of flow of velocity U over a flat plate of length L. The flow is steady, Newtonian (viscosity μ) incompressible (density ρ), two-dimensional, the width is W into the paper, and gravity can be neglected. The assumed velocity profile is $v_x = U\sin\left[\pi y/\delta(x)\right]$. Find the boundary layer thickness $\delta(x)$, the displacement thickness $\delta^*(x)$, the momentum thickness $\theta(x)$, and the skin friction coefficient $C_f(x)$. The solution in all cases will involve $Re_x = \rho U x/\mu$. For ρ = 1000 kg/m³, μ = 0.01 Pa-s, U = 1 m/s, plot $\delta(x)$ for $0\ \delta < x < L = 1$ m.