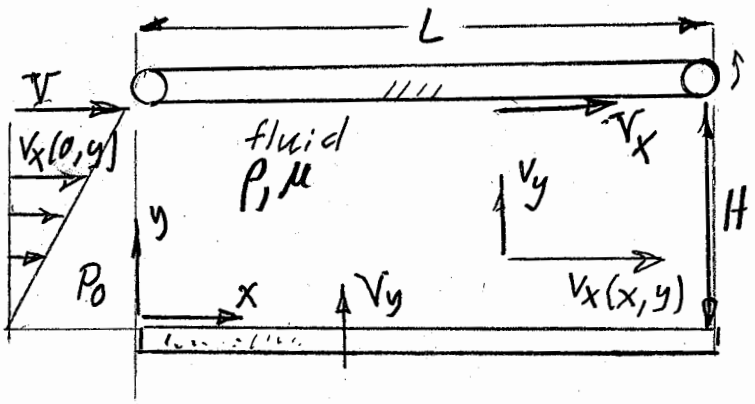
MANE 6520 - Fluid Mechanics
Homework #3 - Thursday, 12 October 2023, due Thursday 19 October



Consider two stationary parallel plates shown. The flow is 2-D, steady, thin, incompressible (with density ρ) and the fluid is Newtonian (constant viscosity μ). The impermeable upper surface slides horizontally at velocity V_x. Fluid passes upwards through the porous lower plate at constant uniform velocity V_y. The length is L, the gap height is $H \ll L$ (thin film) and the width is W into the paper. The pressure is p_0 across the inlet. The inlet velocity at $x = 0$ across the gap $0 \le y \le H$ varies according to $v_x = V_x(y/H)$. Gravity does not play a role. Consider the parameters mentioned in this paragraph to be known and constant. The no-slip conditions are in force. The velocity in the x-direction is $v_x = A(x)y^2 + B(x)y + C(x)$, where A, B, and C are presently unknown functions.

1) Find the velocity v_x in terms of y and the known parameters.
2) Find the velocity v_y in terms of y and the known parameters.
3) For these parameters, what conditions must be met to discard the inertia terms of the Navier Stokes equations?
4) Find the pressure $p(x)$ in terms of the known parameters, with negligible inertia and a thin film.
5) Find the force on the fluid at the upper surface $\mathbf{F}_2$ in terms of the known parameters.
6) Find the mass flow rate out of the right-side open surface $\dot{m}_L$ in terms of the known parameters.
7) Find the mass flow rate out of the top-side porous surface $\dot{m}_H$ in terms of the known parameters.
8) For $H = 5$ mm, $L = 50$ mm, $W = 100$ m, $V_x = 10$ mm/s, $V_y = 1$ mm/s, $p_0 = 1000$ Pa, $\rho = 800$ kg/m^3, and $\mu = 50$ mPa-s; shown that discarding inertia is valid.
9) For these parameter values, find the numerical answer to part 5. Check units.
10) For these parameter values, plot the pressure $p(x)$ for $0 \le x \le L$,