Fluid Mech HW#5 F22

Solution (Nov 6 2022)

In[22]:= `ClearAll["Global`*"]; (* HW#5 *)`

#1

In[23]:= $h = H\,\mathrm{Exp}\left[m\,\frac{x}{L}\right];$

In[24]:= $\mathrm{dhdx} = \partial_x h$

Reyn Eq

$$\frac{d}{dx}\left(h^3\frac{dp}{dx}\right) = 6\mu V\frac{dh}{dx}$$

Out[24]=

$$\frac{e^{\frac{m x}{L}} H m}{L}$$

In[25]:= $\int \frac{1}{h^2}\,dx$

Out[25]=

$$-\frac{e^{-\frac{2 m x}{L}} L}{2 H^2 m}$$

$$\int d\left(h^3\frac{dp}{dx}\right) = 6\mu V\, dh$$

$$h^3\frac{dp}{dx} = 6\mu V h + C_1$$

In[26]:= $\int \frac{1}{h^3}\,dx$

$$\frac{dp}{dx} = 6\mu V\frac{1}{h^2} + \frac{C_1}{h^3} + C_2$$

Out[26]=

$$-\frac{e^{-\frac{3 m x}{L}} L}{3 H^3 m}$$

In[27]:= $\mathrm{dpdx} = 6\mu V h + C1;$

$$p = 6\mu V\int \frac{1}{h^2}\,dx + C1\int \frac{1}{h^3}\,dx + C2$$

$$p = 6\mu V\int\frac{dx}{h^2} + C_1\int\frac{dx}{h^3} + C_2$$

Out[28]=

$$p = C2 - \frac{C1\, e^{-\frac{3 m x}{L}} L}{3 H^3 m} - \frac{3 e^{-\frac{2 m x}{L}} L V \mu}{H^2 m}$$

BC's

In[29]:= $p\ /.\ x \to 0$

$p(x=0) = P_{atm}$

Out[29]=

$$C2 - \frac{C1\, L}{3 H^3 m} - \frac{3 L V\mu}{H^2 m}$$

In[30]:= $\mathrm{sol1} = \mathrm{Solve}\left[C2 - \frac{C1\, L}{3 H^3 m} - \frac{3 L V \mu}{H^2 m} == \mathrm{pAtm},\ C2\right];$

$\Rightarrow C_2$

`C2 = First[Replace[C2, sol1]]`

Out[31]=

$$\frac{C1\, L + 3 H^3 m\, \mathrm{pAtm} + 9 H L V\mu}{3 H^3 m}$$

In[32]:= $p\ /.\ x \to L$

$p(x=L) = P_{atm}$

Out[32]=

$$-\frac{C1\, e^{-3 m} L}{3 H^3 m} - \frac{3 e^{-2 m} L V\mu}{H^2 m} + \frac{C1\, L + 3 H^3 m\, \mathrm{pAtm} + 9 H L V\mu}{3 H^3 m}$$

$\Rightarrow C_1$

In[33]:= sol2 = Solve[(p /. x → L) == pAtm, C1];
C1 = First[Replace[C1, sol2]]

Out[34]=

C_1

$$-\frac{9\,e^{m}\left(1+e^{m}\right)H\,V\,\mu}{1+e^{m}+e^{2m}}$$

In[35]:= pp = Collect[p, pAtm]

Sub for C_1, C_2; collect simplify

Out[35]=

$$\text{pAtm}+\frac{3\,L\,V\,\mu}{H^2\,m}-\frac{3\,e^{-\frac{2mx}{L}}\,L\,V\,\mu}{H^2\,m}-\frac{3\,e^{m}\left(1+e^{m}\right)L\,V\,\mu}{\left(1+e^{m}+e^{2m}\right)H^2\,m}+\frac{3\,e^{m-\frac{3mx}{L}}\left(1+e^{m}\right)L\,V\,\mu}{\left(1+e^{m}+e^{2m}\right)H^2\,m}$$

Part a

In[36]:= $p=\frac{3\,L\,V\,\mu}{H^2\,m}\left(1-e^{-\frac{2mx}{L}}-\frac{e^{m}\left(1+e^{m}\right)\left(1-e^{-\frac{3mx}{L}}\right)}{1+e^{m}+e^{2m}}\right)+\text{pAtm}$; (* part a *)

in-2-D, comp, no grav.

NS thin:

In[37]:= Simplify[p - pp]

check

Out[37]=

0

In[38]:= dpdx = ∂_x p

Find dp/dx

$$0=-\frac{dp}{dx}+\mu\frac{d^2V_x}{dy^2}$$

Out[38]=

$$\frac{dp}{dx}=\frac{3\,L\left(\frac{2\,e^{-\frac{2mx}{L}}\,m}{L}-\frac{3\,e^{m-\frac{3mx}{L}}\left(1+e^{m}\right)m}{\left(1+e^{m}+e^{2m}\right)L}\right)V\,\mu}{H^2\,m}$$

$y=0$ $V_x=V$

from N-S eqs

$y=h(x)$ $V_x=0$

In[39]:= $vx=\text{dpdx}\,\frac{h^2}{2\,\mu}\left(\frac{y^2}{h^2}-\frac{y}{h}\right)+V\left(1-\frac{y}{h}\right)$ (* part b *)

← you already have this

Out[39]=

part b

$$V\left(1-\frac{e^{-\frac{mx}{L}}\,y}{H}\right)+\frac{1}{2\,m}3\,e^{\frac{2mx}{L}}\,L\left(\frac{2\,e^{-\frac{2mx}{L}}\,m}{L}-\frac{3\,e^{m-\frac{3mx}{L}}\left(1+e^{m}\right)m}{\left(1+e^{m}+e^{2m}\right)L}\right)V\left(-\frac{e^{-\frac{mx}{L}}\,y}{H}+\frac{e^{-\frac{2mx}{L}}\,y^2}{H^2}\right)$$

In[40]:= dpdx == 0

$\frac{dp}{dx}=0$

no need to derive

Out[40]=

$$\frac{3\,L\left(\frac{2\,e^{-\frac{2mx}{L}}\,m}{L}-\frac{3\,e^{m-\frac{3mx}{L}}\left(1+e^{m}\right)m}{\left(1+e^{m}+e^{2m}\right)L}\right)V\,\mu}{H^2\,m}==0$$

Solve for $x=x_m$ where $\frac{dp}{dx}=0$

In[41]:=
```
sol3 = Solve[dpdx == 0, x]; (* part c *)
xm = FullSimplify[First[Replace[x, sol3]]]
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[42]=

$$\frac{L\,\mathrm{Log}\left[\frac{3\,(1+e^{m})}{2+4\,\mathrm{Cosh}[m]}\right]}{m}$$

part c $= x_m$

In[48]:=
```
τ = μ ∂y vx;
τW = Simplify[τ /. y → 0]
```

$\tau_{xy}(y) = \mu \frac{\partial v_x}{\partial y}$

Out[49]=

$$-\frac{e^{-\frac{2mx}{L}}\left(-9\,e^{m}-9\,e^{2m}+8\,e^{\frac{mx}{L}}+8\,e^{m\left(2+\frac{x}{L}\right)}+8\,e^{m+\frac{mx}{L}}\right)V\mu}{2\,(1+e^{m}+e^{2m})\,H}$$

Part d

$\tau_w = \tau_{xy}(y=0) =$

In[50]:=

$$Fx = W\int_0^L \tau W\,dx$$

Out[50]=

$$-\frac{(-1+e^{m})\,L\,V\,W\,\mu\,(-1+7\,\mathrm{Cosh}[m])}{2\,(1+e^{m}+e^{2m})\,H\,m}$$

$$Fy = W\int_0^L (p - pAtm)\,dx \quad (*\ \text{part d}\ *)$$

Out[ ]=

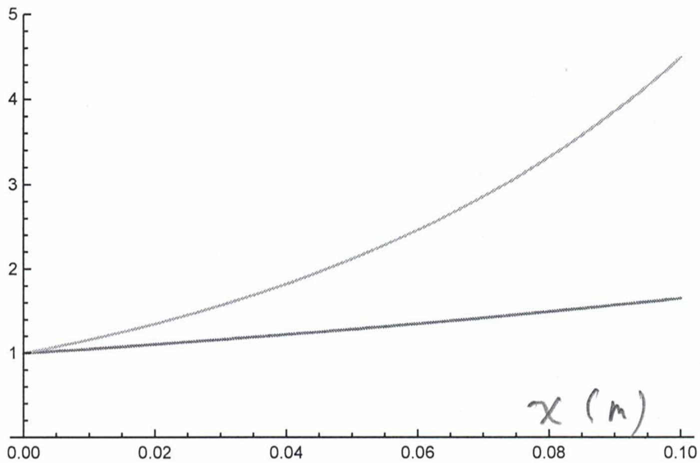
$$-\frac{e^{-m}\,L^2\,V\,W\,\mu\,(-3\,m+\mathrm{Sinh}[m]+\mathrm{Sinh}[2\,m])}{H^2\,m^2\,(1+2\,\mathrm{Cosh}[m])}$$

In[ ]:=
```
ρ = 850.; μ = 0.02; V = -1.; L = 0.1; W = 1.; H = 0.001 ; g = 9.81; pAtm = 0.1 × 10^6;
```

```
Plot[{(h /. m → 0.5)/H, (h /. m → 1.5)/H}, {x, 0, L}, PlotRange → {0, 5}]
 (* part e *)
```

Out[ ]=

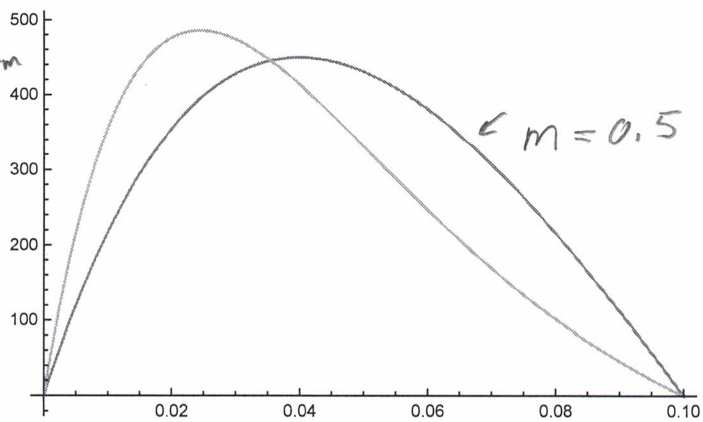
part e)




```
Plot[{(p - pAtm) /. m → 0.5, (p - pAtm) /. m → 1.5}, {x, 0, L}](* part f *)
```

Out[ ]=

part f



In[ ]:= m = 1.5;

In[ ]:= h0 = h /. x → 0;
hL = h /. x → L;

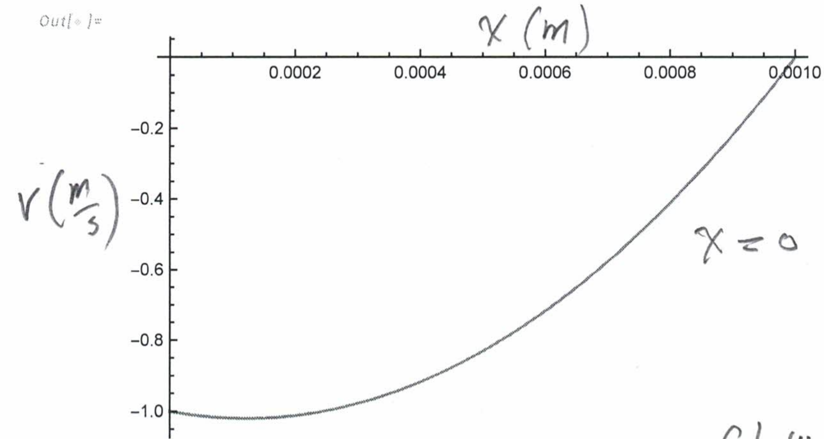
In[ ]:= dpdx0 = dpdx /. x → 0;
dpdxL = dpdx /. x → L;

In[ ]:= $vx0 = dpdx0 \frac{h0^2}{2\mu} \left(\frac{y^2}{h0^2} - \frac{y}{h0}\right) + V \left(1 - \frac{y}{h0}\right);$

$vxL = dpdxL \frac{hL^2}{2\mu} \left(\frac{y^2}{hL^2} - \frac{y}{hL}\right) + V \left(1 - \frac{y}{hL}\right);$

```
Plot[vx0, {y, 0, h0}] (*  plot g *)
Plot[vxL, {y, 0, hL}]
```

Out[ ]=

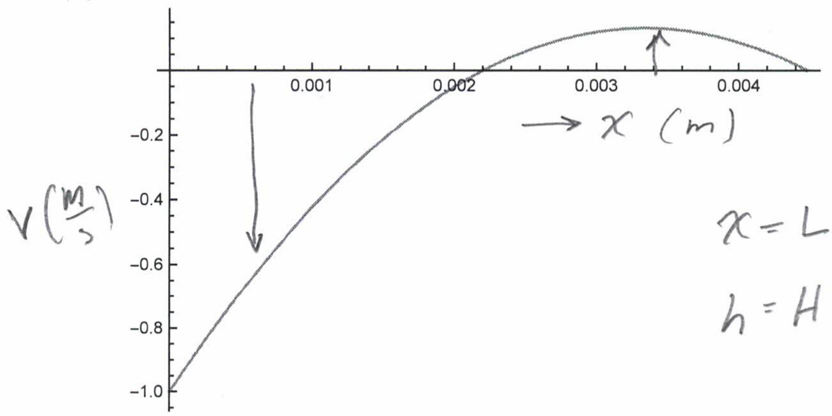


x = 0

x = 0
h = H

flow reversal

Out[ ]=



x = L
$h = He^{mL}$