```
ClearAll["Global`*"]; (* HW #6, F22 Prob1 *)
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In[ ]:= $\psi = V H \frac{x^2}{L^2} \text{Sin}\left[\pi \frac{y}{H}\right];$

In[ ]:= $vy = -\partial_x \psi$

Out[ ]=

$$-\frac{2 H V x \,\text{Sin}\left[\frac{\pi y}{H}\right]}{L^2}$$

In[ ]:= $vx = \partial_y \psi$

Out[ ]=

$$\frac{\pi V x^2 \,\text{Cos}\left[\frac{\pi y}{H}\right]}{L^2}$$

In[ ]:= $\partial_x vx + \partial_y vy$

Out[ ]=

0

In[ ]:= Solve[ψ0 == ψ, y]

Out[ ]=

$$\left\{\left\{y \to \boxed{\frac{H\left(\pi - \text{ArcSin}\left[\frac{L^2 \psi 0}{H V x^2}\right] + 2 \pi c_1\right)}{\pi} \text{ if } c_1 \in \mathbb{Z}}\right\}, \left\{y \to \boxed{\frac{H\left(\text{ArcSin}\left[\frac{L^2 \psi 0}{H V x^2}\right] + 2 \pi c_1\right)}{\pi} \text{ if } c_1 \in \mathbb{Z}}\right\}\right\}$$

$$y \to \boxed{\frac{H\left(\pi - \text{ArcSin}\left[\frac{L^2 \psi 0}{H V x^2}\right] + 2 \pi c_1\right)}{\pi} \text{ if } c_1 \in \mathbb{Z}}$$

$$y1 = \frac{H\left(\pi - \text{ArcSin}\left[\frac{L^2 \psi 0}{H V x^2}\right]\right)}{\pi};$$

$$y2 = \frac{H\left(\text{ArcSin}\left[\frac{L^2 \psi 0}{H V x^2}\right]\right)}{\pi};$$

In[ ]:= $y1a = \frac{H\left(\pi - \text{ArcSin}\left[\frac{L^2 \psi 0}{H V x^2}\right]\right)}{\pi} \;/.\; \psi 0 \to \frac{V}{H L};$

$$y2a = \frac{H\left(\text{ArcSin}\left[\frac{L^2 \psi 0}{H V x^2}\right]\right)}{\pi} \;/.\; \psi 0 \to \frac{V}{H L};$$

$$y1b = \frac{H\left(\pi - \text{ArcSin}\left[\frac{L^2 \psi 0}{H V x^2}\right]\right)}{\pi} \;/.\; \psi 0 \to \frac{2 V}{H L};$$

$$y2b = \frac{H\left(\text{ArcSin}\left[\frac{L^2 \psi 0}{H V x^2}\right]\right)}{\pi} \;/.\; \psi 0 \to \frac{2 V}{H L};$$

In[ ]:= `V = 1; L = 1; H = 1;`

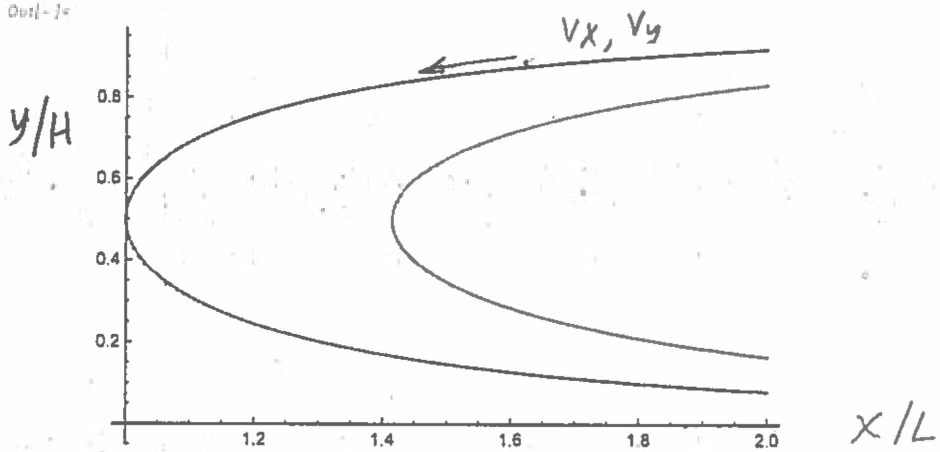
In[-]:= y1a

Out[-]=

$$\frac{\pi - \text{ArcSin}\left[\frac{1}{x^2}\right]}{\pi}$$

In[-]:= Plot[{y1a, y1b, y2a, y2b}, {x, L, 2 L}]

Out[-]=



In[-]:= y1a /. x → 3. L / 2

Out[-]=

0.853401

In[-]:= {vx, vy} /. {x → 3 / 2, y → 0.8534}

Out[-]=

{-6.33207, -1.33334}

In[-]:= ClearAll["Global`*"]; (* HW #6, F22 Prob#2 *)

In[-]:= vv = v0[y] + ε v1[y] + ε^2 v2[y]

vyy = $\partial_{y,y}$ vv

Out[-]=

$v0[y] + \epsilon\, v1[y] + \epsilon^2\, v2[y]$

Out[-]=

$v0''[y] + \epsilon\, v1''[y] + \epsilon^2\, v2''[y]$

In[-]:= Collect$\left[\epsilon \frac{1}{V^2 H^2} vv^3 + vyy, \epsilon\right]$

Out[-]=

$$\frac{3 \epsilon^6 v1[y]\, v2[y]^2}{H^2 V^2} + \frac{\epsilon^7 v2[y]^3}{H^2 V^2} + \epsilon^3 \left(\frac{3\, v0[y]\, v1[y]^2}{H^2 V^2} + \frac{3\, v0[y]^2\, v2[y]}{H^2 V^2}\right) + \epsilon^4 \left(\frac{v1[y]^3}{H^2 V^2} + \frac{6\, v0[y]\, v1[y]\, v2[y]}{H^2 V^2}\right) + \epsilon^5 \left(\frac{3\, v1[y]^2\, v2[y]}{H^2 V^2} + \frac{3\, v0[y]\, v2[y]^2}{H^2 V^2}\right) + v0''[y] + \epsilon \left(\frac{v0[y]^3}{H^2 V^2} + v1''[y]\right) + \epsilon^2 \left(\frac{3\, v0[y]^2\, v1[y]}{H^2 V^2} + v2''[y]\right)$$

$$eq == v0''[y] + \epsilon \left(\frac{v0[y]^3}{H^2 V^2} + v1''[y]\right);$$

In[ ]:= $v0 = V \frac{y}{H};$

In[ ]:= $\partial_{y,y} v0$

$v0 \;/.\; y \to 0$

$v0 \;/.\; y \to H$

Out[ ]=

0

Out[ ]=

0

Out[ ]=

V

In[ ]:= $\frac{v0[y]^3}{H^2 V^2} + v1''[y];$

$$\left(V \frac{y}{H}\right)^3 \frac{1}{H^2 V^2}$$

Out[ ]=

$$\frac{V y^3}{H^5}$$

$$v1p = -\frac{V y^4}{4 H^5} + C1$$

$$v1 = -\frac{V y^5}{20 H^5} + C1\, y + C2$$

$$v1 = -\frac{V y^5}{20 H^5} + C1\, y$$

In[ ]:= $Solve\left[0 == -\frac{V H^5}{20 H^4} + C1\, H,\ C1\right]$

Out[ ]=

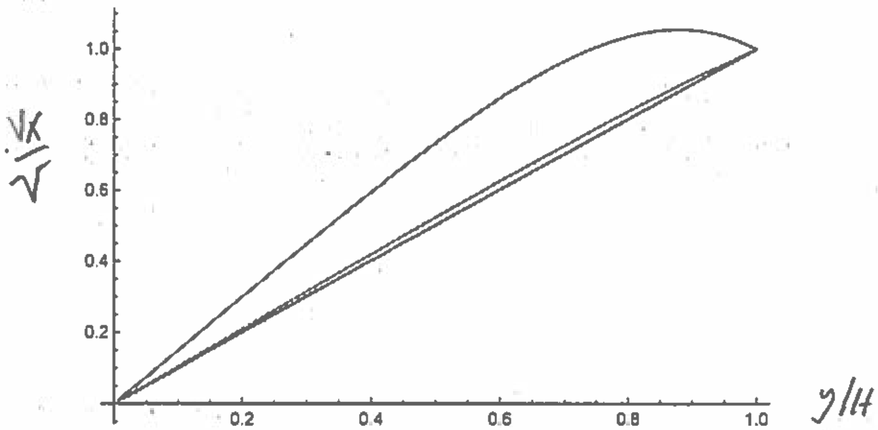
$$\left\{\left\{C1 \to \frac{V}{20}\right\}\right\}$$

$$v1 = -\frac{V y^5}{20 H^5} + \frac{V}{20 H} y;$$

In[ ]:= $vP[y\_, \epsilon\_] := V \left(\frac{y}{H} + \epsilon \frac{1}{20} \left(-\frac{y^5}{H^5} + \frac{y}{H}\right)\right)$

In[ ]:= `V = 1.; H = 1.; Plot[{vP[y, 0.1], vP[y, 1], vP[y, 10]}, {y, 0, 1}]`

Out[ ]=



In[ ]:= `ClearAll["Global`*"]; (* integral boundary layer, HW#6, Prob#3 *)`

In[ ]:= $vx = U \, \mathrm{Sin}\left[\frac{\pi}{2}\frac{y}{\delta[x]}\right];$

In[ ]:= $\tau = \mu \, \partial_y vx$

Out[ ]= $$\frac{\pi U \mu \, \mathrm{Cos}\left[\frac{\pi y}{2\delta[x]}\right]}{2\delta[x]}$$

In[ ]:= $\tau w = \tau \; /. \; y \to 0$

Out[ ]= $$\frac{\pi U \mu}{2\delta[x]}$$

In[ ]:=

$\tau\delta = \tau \; /. \; y \to \delta[x]$

$vx \; /. \; y \to 0$

$vx \; /. \; y \to \delta[x]$

Out[ ]= 0

Out[ ]= 0

Out[ ]= U

In[ ]:= $\delta s = \int_0^{\delta[x]} \left(1 - \frac{vx}{U}\right) dy$

Out[ ]= $$\frac{(-2+\pi)\,\delta[x]}{\pi}$$

In[ ]:= $Cf = \frac{\tau w}{\rho U^2 / 2}$

Out[ ]=

$$\frac{\pi \mu}{U \rho \delta[x]}$$

In[ ]:= $\theta = \int_0^{\delta[x]} \frac{vx}{U} \left(1 - \frac{vx}{U}\right) dy$

Out[ ]=

$$-\frac{(-4+\pi)\ \delta[x]}{2\pi}$$

In[ ]:= $2\, \partial_x \theta$

Out[ ]=

$$-\frac{(-4+\pi)\ \delta'[x]}{\pi}$$

In[ ]:= $Cf == 2\, \partial_x \theta$

Out[ ]=

$$\frac{\pi \mu}{U \rho \delta[x]} == -\frac{(-4+\pi)\ \delta'[x]}{\pi}$$

In[ ]:= $\text{DSolve}\left[\left\{\frac{\pi \mu}{U \rho} == -\frac{(-4+\pi)\ \delta[x]\ \delta'[x]}{\pi}, \delta[0] == 0\right\}, \delta[x], x\right]$

Out[ ]=

$$\left\{\left\{\delta[x] \to -\frac{\sqrt{\frac{2}{4-\pi}}\ \pi\ \sqrt{x \mu}}{\sqrt{U}\ \sqrt{\rho}}\right\}, \left\{\delta[x] \to \frac{\sqrt{\frac{2}{4-\pi}}\ \pi\ \sqrt{x \mu}}{\sqrt{U}\ \sqrt{\rho}}\right\}\right\}$$

In[ ]:= $\delta = \left(\sqrt{\frac{2}{4-\pi}}\ \pi\right) x \sqrt{\frac{\mu}{\rho U x}}$

Out[ ]=

$$\sqrt{\frac{2}{4-\pi}}\ \pi\, x \sqrt{\frac{\mu}{U x \rho}}$$

In[ ]:= N[δ]

Out[ ]=

$$4.79533\, x \sqrt{\frac{\mu}{U x \rho}}$$

In[ ]:= $\delta s = \frac{(-2+\pi)\ \delta}{\pi}$

Out[ ]=

$$\sqrt{\frac{2}{4-\pi}}\ (-2+\pi)\ x \sqrt{\frac{\mu}{U x \rho}}$$

In[ ]:= $\theta = -\frac{(-4+\pi)\,\delta}{2\pi}$

Out[ ]=

$$-\frac{(-4+\pi)\ \mathrm{x}\ \frac{\mu}{\mathrm{U\,x}\,\rho}}{2\ (4-\pi)}$$

In[ ]:= N[δ]

Out[ ]=

$$4.79533\ \mathrm{x}\ \frac{\mu}{\mathrm{U\,x}\,\rho}$$

In[ ]:= N[δs]

Out[ ]=

$$1.74253\ \mathrm{x}\ \frac{\mu}{\mathrm{U\,x}\,\rho}$$

In[ ]:= N[θ]

Out[ ]=

$$0.655136\ \mathrm{x}\ \frac{\mu}{\mathrm{U\,x}\,\rho}$$

In[ ]:= U = 1; ρ = 1.225; L = 1.; μ = 1.82 × 10^{-5}; (* properties of air *)

In[ ]:= $\mathrm{ReL} = \frac{\rho\,\mathrm{U\,L}}{\mu}$

Out[ ]=

67 307.7

In[ ]:= Plot[1000 δ, {x, 0, L}]

Out[ ]=