```
In[5]:= ClearAll["Global`*"]; (*  F 22 exam #1 *)

In[6]:= vx = Vx (1 - y/H) + A x/L (y/H - y^2/H^2);

In[7]:= p = 6 μ Vy/H L^2/H^2 x^2/L^2;

In[8]:= dvxdx = ∂x vx;

In[11]:= dvydy = -dvxdx;
         vy = ∫dvydy dy + C1
```

(5) $\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0 \qquad V_y = -\int \frac{\partial V_x}{\partial x}\, dy + C_1$

(5)

Out[12]=

$$C1 + \frac{A\left(-\frac{H y^2}{2} + \frac{y^3}{3}\right)}{H^2 L}$$

$V_y = -V_y$ at $y = 0$

```
In[13]:= sol1 = First[Solve[(vy /. y → 0) == -Vy, C1]];
         C1 = Replace[C1, sol1]
```

$\Rightarrow C_1 = -V_y$

Out[14]=

$-Vy$ (5)

```
In[15]:= vy
```

Out[15]=

$$V_y = -Vy + \frac{A\left(-\frac{H y^2}{2} + \frac{y^3}{3}\right)}{H^2 L}$$

```
In[16]:= solA = First[Solve[(vy /. y → H) == 0, A]];
         A = Replace[A, solA]
```

$V_y = 0$ at $y = H$

Out[17]=

$$-\frac{6 L Vy}{H}$$ (5)

-20-

Points per part

1)	20
2)	10
3)	10
4)	10
5)	15
6)	15
7)	20
	100

```
In[18]:= vy (* part 1  *)
```

Out[18]=

$$-Vy - \frac{6 Vy\left(-\frac{H y^2}{2} + \frac{y^3}{3}\right)}{H^3}$$

```
In[19]:= vx
```

Out[19]=

$$Vx\left(1 - \frac{y}{H}\right) - \frac{6 Vy\, x\left(\frac{y}{H} - \frac{y^2}{H^2}\right)}{H}$$

```
In[20]:= v = {vx, vy};

In[21]:= Simplify[∂x vx + ∂y vy] (* check continuity  *)
```

Out[21]=

0

In[22]:=
```
vx /. y → 0 (*  check boundary conditions  *)
vx /. y → H
vy /. y → 0
vy /. y → H
```

Out[22]= Vx

Out[23]= 0

Out[24]= -Vy

Out[25]= 0

In[26]:= `gradv = {{∂x vx, ∂x vy}, {∂y vx, ∂y vy}};`

$$\nabla V = \begin{bmatrix} \partial V_x/\partial x & \partial V_y/\partial x \\ \partial V_x/\partial y & \partial V_y/\partial y \end{bmatrix}$$

(5) In[27]:= `gradvT = {{∂x vx, ∂y vx}, {∂x vy, ∂y vy}};`

In[28]:= `γdot = gradv + gradvT;`

-10

In[29]:= `MatrixForm[γdot] (* part 2 *)`

Out[29]//MatrixForm=

$$\begin{pmatrix} -\frac{12\,Vy\left(\frac{y}{H}-\frac{y^2}{H^2}\right)}{H} & -\frac{Vx}{H}-\frac{6\,Vy\,x\left(\frac{1}{H}-\frac{2y}{H^2}\right)}{H} \\ -\frac{Vx}{H}-\frac{6\,Vy\,x\left(\frac{1}{H}-\frac{2y}{H^2}\right)}{H} & -\frac{12\,Vy\left(-H\,y+y^2\right)}{H^3} \end{pmatrix}$$ (5)

In[32]:= `ωTensor = gradv - gradvT;` (5)

In[33]:= `MatrixForm[ωTensor]`

Out[33]//MatrixForm=

$$\underset{\approx}{\omega} = \begin{pmatrix} 0 & \frac{Vx}{H}+\frac{6\,Vy\,x\left(\frac{1}{H}-\frac{2y}{H^2}\right)}{H} \\ -\frac{Vx}{H}-\frac{6\,Vy\,x\left(\frac{1}{H}-\frac{2y}{H^2}\right)}{H} & 0 \end{pmatrix}$$ — ω_{xy}

(5)

In[30]:= `ez = {0, 0, 1};`

-10

In[31]:= `ωVector = ez Curl[v, {x, y}] (* part 3 *)`

$$\underset{\sim}{\omega} = \text{Curl}\,\underset{\sim}{V} = \underset{\sim}{\nabla} \times \underset{\sim}{V}$$

or

in 2-D $\underset{\sim}{\omega} = \hat{e}_z\,\omega_{xy}$

Out[31]=

$$\left\{0,\ 0,\ \frac{Vx}{H}+\frac{6\,Vy\,x\left(\frac{1}{H}-\frac{2y}{H^2}\right)}{H}\right\}$$

ω_z

In[35]:= `(*  ωVector==  *) ez ωTensor[[1,2]]`

Out[35]=

$$\left\{0,\ 0,\ \frac{Vx}{H}+\frac{6\,Vy\,x\left(\frac{1}{H}-\frac{2y}{H^2}\right)}{H}\right\}$$

In[36]:= `τ = μ γdot ;` (5)

-10-

In[37]:= $\sigma = -p\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \tau$ (* part 4 *)

σ_{xx}

Out[37]=

$$\left\{\left\{-\frac{6\,Vy\,x^2\,\mu}{H^3}-\frac{12\,Vy\left(\frac{y}{H}-\frac{y^2}{H^2}\right)\mu}{H},\ \left(-\frac{Vx}{H}-\frac{6\,Vy\,x\left(\frac{1}{H}-\frac{2y}{H^2}\right)}{H}\right)\mu\right\},\right.$$

σ_{xy} etc

(5)

$$\left.\left\{\left(-\frac{Vx}{H}-\frac{6\,Vy\,x\left(\frac{1}{H}-\frac{2y}{H^2}\right)}{H}\right)\mu,\ -\frac{6\,Vy\,x^2\,\mu}{H^3}-\frac{12\,Vy\left(-H\,y+y^2\right)\mu}{H^3}\right\}\right\}$$

In[38]:= n1 = {0, -1}; (* lower, outward from fluid *)

$\hat{n}_1 = -\hat{e}_y$

In[39]:= σ1 = σ /. y → 0 (* stress at lower surface *) (5)

$\underset{\approx}{\sigma}_1 = \underset{\approx}{\sigma}(y=0)$

Out[39]=

$$\left\{\left\{-\frac{6\,Vy\,x^2\,\mu}{H^3},\ \left(-\frac{Vx}{H}-\frac{6\,Vy\,x}{H^2}\right)\mu\right\},\left\{\left(-\frac{Vx}{H}-\frac{6\,Vy\,x}{H^2}\right)\mu,\ -\frac{6\,Vy\,x^2\,\mu}{H^3}\right\}\right\}$$

In[40]:= f1 = n1.σ1 (* traction on fluid at lower surface *) (5)

Out[40]=

$$\left\{-\left(\left(-\frac{Vx}{H}-\frac{6\,Vy\,x}{H^2}\right)\mu\right),\ \frac{6\,Vy\,x^2\,\mu}{H^3}\right\}$$

$\underset{\sim}{f}_1 = \hat{n}_1 \cdot \underset{\approx}{\sigma}_1$

In[41]:= $W\int_0^L f1\,dx$ (* force on fluid at lower surface *)

$\underset{\sim}{F}_1$(on fluid)

Out[41]=

(5)

$$\left\{W\left(\frac{L\,Vx\,\mu}{H}+\frac{3\,L^2\,Vy\,\mu}{H^2}\right),\ \frac{2\,L^3\,Vy\,W\,\mu}{H^3}\right\}$$

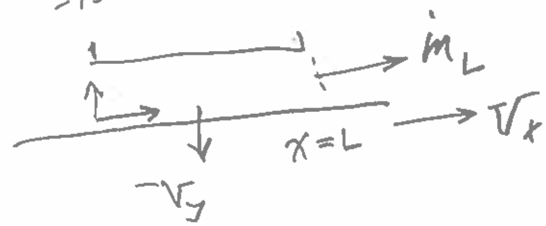
$\underset{\sim}{F}_1$(on surface) $= -F_1$(on fluid)

In[45]:= $F1 = -W\int_0^L f1\,dx$ (* force on lower surface by fluid, part 5 *)

Out[45]=

$$\left\{-W\left(\frac{L\,Vx\,\mu}{H}+\frac{3\,L^2\,Vy\,\mu}{H^2}\right),\ -\frac{2\,L^3\,Vy\,W\,\mu}{H^3}\right\}$$

-15-



In[46]:= nL = {1, 0}; (5)

In[48]:= vnL = Simplify[(v.nL) /. x → L]

Out[48]=

$$\frac{(H-y)\left(H^2\,Vx-6\,L\,Vy\,y\right)}{H^3}$$

(5)

In[49]:= dAL = W dy;

$\dot{m}_L = \rho\int(\underset{\sim}{V}\cdot\hat{n})\,dA = \rho W\int_0^H \underbrace{v.n}_{f_n(y)}\,dy$ at L

at L

-15

mDotL = $W \rho \int_0^H$ vnL dy (* part 6 *)

Out[51]=

(5) $\left(\frac{H\,Vx}{2} - L\,Vy\right) W \rho = \dot{m}_L$

In[53]:= $h = \frac{H}{2}\left(1 - 2\,\frac{x - L/2}{L}\right)$;

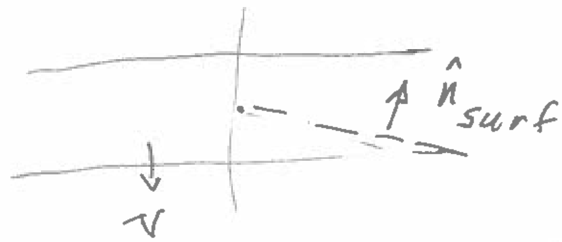
eq. of screen surface

In[54]:= g = -h + y;

$\hat{n} = \frac{\nabla g}{\sqrt{\nabla g \cdot \nabla g}}$

In[55]:= gradg = Grad[g, {x, y}];

In[57]:= $nSurf = \frac{gradg}{\sqrt{gradg.gradg}}$



Out[57]=

(5) $\left\{\frac{H}{\sqrt{1 + \frac{H^2}{L^2}}\,L}, \frac{1}{\sqrt{1 + \frac{H^2}{L^2}}}\right\}$

square root sign doesn't print!?

In[58]:= dAvector = dA nSurf

Out[58]=

$\left\{\frac{dA\,H}{\sqrt{1 + \frac{H^2}{L^2}}\,L}, \frac{dA}{\sqrt{1 + \frac{H^2}{L^2}}}\right\}$

In[59]:= ey = {0, 1};

In[60]:= Wdx == dAvector.ey

Out[60]=

$Wdx == \frac{dA}{\sqrt{1 + \frac{H^2}{L^2}}}$

In[61]:= $dA == W\,dx\,\sqrt{1 + \frac{H^2}{L^2}}$;

(5)

vnSurf = Simplify[(v.nSurf) /. y → h] (* normal velocity at surface *)

Out[62]=

$\frac{x\,(H\,L\,Vx + Vy\,x\,(-9\,L + 8\,x))}{\sqrt{1 + \frac{H^2}{L^2}}\,L^3}$

In[65]:= vxSurf = Simplify[vx /. y → h] (* x-velocity at surface *)

Out[65]=

$\frac{x\,(H\,L\,Vx + 6\,Vy\,x\,(-L + x))}{H\,L^2}$

5

In[67]:= vySurf = Simplify[vy /. y → h] (* y-velocity at surface *)

Out[67]=

$$\frac{Vy\, x^2\, (-3\, L + 2\, x)}{L^3}$$

2pt

In[68]:= $GdotX = W\rho \left(1 \Big/ \sqrt{1 + \frac{H^2}{L^2}}\right) \int_{L/2}^{L} vxSurf\ vnSurf\, dx$

Out[68]=

$$\frac{\left(1960\, H^2\, Vx^2 - 5397\, H\, L\, Vx\, Vy + 3573\, L^2\, Vy^2\right) W \rho}{6720\, H \left(1 + \frac{H^2}{L^2}\right)}$$

5

In[69]:= $GdotY = W\rho \left(1 \Big/ \sqrt{1 + \frac{H^2}{L^2}}\right) \int_{L/2}^{L} vySurf\ vnSurf\, dx$

Out[69]=

$$\frac{Vy\, (-707\, H\, Vx + 1363\, L\, Vy)\, W \rho}{2240 \left(1 + \frac{H^2}{L^2}\right)}$$

Part 7 -20-