p.1

Solution HW #2 F22

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ClearAll["Global`*"]; (*  HW#2, F22  *)
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In[ ]:= $h[x\_] := h0\left(1+\text{Sin}\left[m\frac{x}{L}\right]\right)$

In[ ]:= $g = y - h[x];$

In[ ]:= $\text{gradg} = \{\partial_x g, \partial_y g\}$

Out[ ]=

$$\left\{-\frac{h0\, m\, \text{Cos}\left[\frac{m x}{L}\right]}{L}, 1\right\}$$

$\hat{n}_1 = 0\,\hat{e}_x - 1\,\hat{e}_y$ etc

In[ ]:= $n1 = \{0, -1\};$

$n2 = \frac{\text{gradg}}{\sqrt{\text{gradg.gradg}}}$ (* part 1 *)

Out[ ]=

$$\left\{-\frac{h0\, m\, \text{Cos}\left[\frac{m x}{L}\right]}{L\sqrt{1+\frac{h0^2 m^2 \text{Cos}\left[\frac{m x}{L}\right]^2}{L^2}}}, \frac{1}{\sqrt{1+\frac{h0^2 m^2 \text{Cos}\left[\frac{m x}{L}\right]^2}{L^2}}}\right\}$$

$\hat{n}_2$

In[ ]:= $p[x\_] := p0\left(1 - k\frac{x}{L}\right)$

In[ ]:= $\tau[x\_, y\_] := \tau 0\left(\frac{x}{L}\right)\left(\frac{y}{h[x]}\right)$

In[ ]:= $\sigma = \begin{pmatrix} -p[x] & \tau[x, y] \\ \tau[x, y] & -p[x] \end{pmatrix}$

Out[ ]=

$$\left\{\left\{-p0\left(1-\frac{k x}{L}\right), \frac{x\, y\, \tau 0}{h0\, L\left(1+\text{Sin}\left[\frac{m x}{L}\right]\right)}\right\}, \left\{\frac{x\, y\, \tau 0}{h0\, L\left(1+\text{Sin}\left[\frac{m x}{L}\right]\right)}, -p0\left(1-\frac{k x}{L}\right)\right\}\right\}$$

In[ ]:= $\sigma\ /.\ y \to 0$

Out[ ]=

$$\left\{\left\{-p0\left(1-\frac{k x}{L}\right), 0\right\}, \left\{0, -p0\left(1-\frac{k x}{L}\right)\right\}\right\}$$

In[ ]:= $f1 = n1.(\sigma\ /.\ y \to 0)$

Out[ ]=

$$\left\{0, p0\left(1-\frac{k x}{L}\right)\right\}$$

In[ ]:= $(\sigma\ /.\ y \to h[x])$

Out[ ]=

$$\left\{\left\{-p0\left(1-\frac{k x}{L}\right), \frac{x\,\tau 0}{L}\right\}, \left\{\frac{x\,\tau 0}{L}, -p0\left(1-\frac{k x}{L}\right)\right\}\right\}$$

In[ ]:= f2 = n2.(σ /. y → h[x])

Out[ ]=

$$\left\{\frac{x\,\tau 0}{L\sqrt{1+\frac{h0^2 m^2 \mathrm{Cos}\left[\frac{m x}{L}\right]^2}{L^2}}}+\frac{h0\, m\, p0\left(1-\frac{k x}{L}\right)\mathrm{Cos}\left[\frac{m x}{L}\right]}{L\sqrt{1+\frac{h0^2 m^2 \mathrm{Cos}\left[\frac{m x}{L}\right]^2}{L^2}}},\ -\frac{p0\left(1-\frac{k x}{L}\right)}{\sqrt{1+\frac{h0^2 m^2 \mathrm{Cos}\left[\frac{m x}{L}\right]^2}{L^2}}}-\frac{h0\, m\, x\,\tau 0\,\mathrm{Cos}\left[\frac{m x}{L}\right]}{L^2\sqrt{1+\frac{h0^2 m^2 \mathrm{Cos}\left[\frac{m x}{L}\right]^2}{L^2}}}\right\}$$

In[ ]:= f1x = f1[[1]]

f1y = f1[[2]]

f2x = Collect[f2[[1]], {p0, τ0}]

f2y = Collect[f2[[2]], {p0, τ0}] (* prob#2 *)

Out[ ]=

$0 = f_{1x}$

$\underset{\sim}{f}_1 = f_{1x}\hat{e}_x + f_{1y}\hat{e}_y$

Out[ ]=

$p0\left(1-\frac{k x}{L}\right) = f_{1y}$

Out[ ]=

$$\frac{x\,\tau 0}{L\sqrt{1+\frac{h0^2 m^2 \mathrm{Cos}\left[\frac{m x}{L}\right]^2}{L^2}}}+\frac{h0\, m\, p0\left(1-\frac{k x}{L}\right)\mathrm{Cos}\left[\frac{m x}{L}\right]}{L\sqrt{1+\frac{h0^2 m^2 \mathrm{Cos}\left[\frac{m x}{L}\right]^2}{L^2}}} = f_{1y}$$

Out[ ]=

$$\frac{p0\left(-1+\frac{k x}{L}\right)}{\sqrt{1+\frac{h0^2 m^2 \mathrm{Cos}\left[\frac{m x}{L}\right]^2}{L^2}}}-\frac{h0\, m\, x\,\tau 0\,\mathrm{Cos}\left[\frac{m x}{L}\right]}{L^2\sqrt{1+\frac{h0^2 m^2 \mathrm{Cos}\left[\frac{m x}{L}\right]^2}{L^2}}} = f_{2y}$$

part 2)

In[ ]:= $F1x = \mathrm{Simplify}\left[-W\int_0^L f1x\, dx\right]$ (* minus sign for on surface, not fluid *)

$F1y = \mathrm{Simplify}\left[-W\int_0^L f1y\, dx\right]$

part 4)

Out[ ]=

$0 = F_{1x}$

Out[ ]=

$\frac{1}{2}(-2+k)\, L\, p0\, W = F_{1y}$

$\underset{\sim}{F}_1 = F_{1x}\hat{e}_x + F_{1y}\hat{e}_y$

In[ ]:= F1 = {F1x, F1y} (* part 4 *)

Out[ ]=

$\left\{0, \frac{1}{2}(-2+k)\, L\, p0\, W\right\}$

Part 3)

In[ ]:= dA2vector = dA2 n2

Out[ ]=

$\underset{\sim}{dA}_2 = dA_2\,\hat{n}_2$

$$\left\{-\frac{dA2\, h0\, m\,\mathrm{Cos}\left[\frac{m x}{L}\right]}{L\sqrt{1+\frac{h0^2 m^2 \mathrm{Cos}\left[\frac{m x}{L}\right]^2}{L^2}}},\ \frac{dA2}{\sqrt{1+\frac{h0^2 m^2 \mathrm{Cos}\left[\frac{m x}{L}\right]^2}{L^2}}}\right\}$$

In[ ]:= ex = {1, 0}; ey = {0, 1};

In[ ]:= dA2y = dA2vector.ey

Out[ ]=
$$\frac{dA2}{\sqrt{1+\frac{h0^2\,m^2\,\mathrm{Cos}\left[\frac{m\,x}{L}\right]^2}{L^2}}}$$

$dA_{2y} = \hat{e}_y \cdot \underset{\sim}{dA_2}$

In[ ]:= sol1 = First[Solve[dA2y == W dx, dA2]];
dA2 = Replace[dA2, sol1]

$dA_2y = \frac{dA_2}{\sqrt{}}$

Out[ ]=
$$dx\,W\,\sqrt{\frac{L^2 + h0^2\,m^2\,\mathrm{Cos}\left[\frac{m\,x}{L}\right]^2}{L^2}} = dA_2$$

scalar magnitude

In[ ]:= dF2x = Simplify[f2x dA2] / dx

Out[ ]=
$$\frac{W\left(L\,x\,\tau0 + h0\,m\,p0\,(L - k\,x)\,\mathrm{Cos}\left[\frac{m\,x}{L}\right]\right)}{L^2}$$

on surface not fluid

as indefinite intgral

In[ ]:= F2x == $\nabla$$\int$dF2x dx

Out[ ]=
$$F2x == -\frac{W\,x^2\,\tau0}{2\,L} + \frac{h0\,k\,p0\,W\,\mathrm{Cos}\left[\frac{m\,x}{L}\right]}{m} - h0\,p0\,W\,\mathrm{Sin}\left[\frac{m\,x}{L}\right] + \frac{h0\,k\,p0\,W\,x\,\mathrm{Sin}\left[\frac{m\,x}{L}\right]}{L}$$

In[ ]:= dF2y = Simplify[f2y dA2] / dx

Out[ ]=
$$-\frac{W\left(L\,p0\,(L - k\,x) + h0\,m\,x\,\tau0\,\mathrm{Cos}\left[\frac{m\,x}{L}\right]\right)}{L^2}$$

In[ ]:= F2y == - $\int$dF2y dx

Out[ ]=
$$F2y == p0\,W\,x - \frac{k\,p0\,W\,x^2}{2\,L} + \frac{h0\,W\,\tau0\,\mathrm{Cos}\left[\frac{m\,x}{L}\right]}{m} + \frac{h0\,W\,x\,\tau0\,\mathrm{Sin}\left[\frac{m\,x}{L}\right]}{L}$$

In[ ]:= h0 = 0.001; L = 0.01; m = 0.2; k = 0.5; p0 = 1. × 10^6; τ0 = 100. × 10^3; W = 1.;

In[ ]:= f1x
f1y

Note units

Out[ ]= 0   f1x

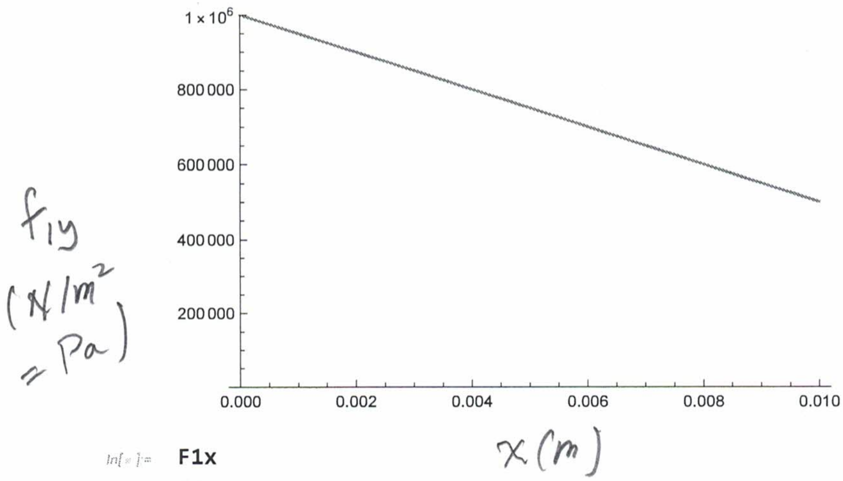
$\frac{x\,\tau_0}{L\sqrt{}} \Rightarrow Pa$

Out[ ]= 1. × 10^6 (1 - 50. x)   f1y

In[ ]:=

```
Plot[f1y, {x, 0, L}, {PlotRange → {0, 10^6}}]
```

Out[ ]=



In[ ]:=

```
F1x
F1y
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Out[ ]=

0

Out[ ]=

-7500. N on surface ⇒ down

In[ ]:=

```
F2x = NIntegrate[-dF2x, {x, 0, L}]
F2y = NIntegrate[-dF2y, {x, 0, L}]
```

N

Out[ ]=

-649.168 x - force to left (-)

on surface 2

Out[ ]=

7509.9 ← y - force up (+)