

## First-order Necessary Conditions

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# Solutions to (Smooth) Optimization Problems Satisfy the First-order Conditions

## Theorem: First-order Necessary Conditions [NW06]

If  $x^*$  is a local minimizer and  $f$  is continuously differentiable (in an open neighborhood of  $x^*$ ), then

$$\nabla f(x^*) = 0.$$

## Here is Some Intuition Behind the First-order Conditions

Suppose  $x^*$  is a local minimizer but  $\nabla f(x^*) \neq 0$ .

## Here is Some Intuition Behind the First-order Conditions (cont.)


## What is the Significance of the Word “Necessary”

### Definition: Stationary Point

The point  $\bar{x}$  is a stationary point if  $\nabla f(\bar{x}) = 0$ .

Does  $\nabla f(\bar{x}) = 0$  imply that  $\bar{x}$  is a local minimizer/maximizer?

## References

-  J. Nocedal and S. J. Wright, *Numerical Optimization*, second ed., Springer–Verlag, Berlin, Germany, 2006.