In[ ]:=
```
ClearAll["Global`*"]; (* Fluids F22 HW #7  *)
```

In[ ]:=
$$h = H \sqrt{\frac{x}{L1}} ;$$

In[ ]:=
$$\eta = \frac{y}{h};$$

In[ ]:=
$$\psi = G[x] \times f[\eta];$$

In[ ]:=
$$vx = \partial_y \psi$$

Out[ ]=
$$\frac{G[x]\, f'\left[\frac{y}{H\sqrt{\frac{x}{L1}}}\right]}{H\sqrt{\frac{x}{L1}}}$$

In[ ]:=
```
(*  want vx = V f'  *)
```

In[ ]:=
$$V == \frac{G[x]}{H\sqrt{\frac{x}{L1}}}$$

Out[ ]=
$$V == \frac{G[x]}{H\sqrt{\frac{x}{L1}}}$$

In[ ]:=
$$G == H V \sqrt{\frac{x}{L1}} \quad (*\ \text{part a}\ *)$$

Out[ ]=
$$G == H V \sqrt{\frac{x}{L1}}$$

In[ ]:=
$$\psi = V h f[\eta]$$

Out[ ]=
$$H V \sqrt{\frac{x}{L1}}\ f\left[\frac{y}{H\sqrt{\frac{x}{L1}}}\right]$$

In[ ]:=
$$vx = \partial_y \psi$$
$$vy = -\partial_x \psi$$

Out[ ]=
$$V f'\left[\frac{y}{H\sqrt{\frac{x}{L1}}}\right]$$

Out[ ]=
$$-\frac{H V f\left[\frac{y}{H\sqrt{\frac{x}{L1}}}\right]}{2\, L1\sqrt{\frac{x}{L1}}} + \frac{V y f'\left[\frac{y}{H\sqrt{\frac{x}{L1}}}\right]}{2x}$$

$$vx == V\, f'$$

$$vy == V \frac{H}{2\sqrt{x\, L1}}\ (-f + \eta\, f')\quad (*\ \ part\ b\ \ *)$$

In[ ]:=
```
(*  eq x-NS  *)
dpdx = 0;
Simplify[ρ (vx ∂x vx + vy ∂y vx) + dpdx - μ ∂y,y vx == 0]
```

Out[ ]=

$$\frac{1}{H\,x}\, V \left(H^2\, V\, \rho\, f\left[\frac{y}{H\sqrt{\frac{x}{L1}}}\right] f''\left[\frac{y}{H\sqrt{\frac{x}{L1}}}\right] + 2\, L1\, \mu\, f^{(3)}\left[\frac{y}{H\sqrt{\frac{x}{L1}}}\right] \right) == 0$$

$$H^2\, V\, \rho\, f\left[\frac{y}{H\sqrt{\frac{x}{L1}}}\right] f''\left[\frac{y}{H\sqrt{\frac{x}{L1}}}\right] + 2\, L1\, \mu\, f^{(3)}\left[\frac{y}{H\sqrt{\frac{x}{L1}}}\right] == 0$$

$$H^2\, V\, \rho\, f\left[\frac{y}{H\sqrt{\frac{x}{L1}}}\right] f''\left[\frac{y}{H\sqrt{\frac{x}{L1}}}\right] + 2\, L1\, \mu\, f^{(3)}\left[\frac{y}{H\sqrt{\frac{x}{L1}}}\right] == 0$$

$$\frac{1}{2}\, \frac{\rho\, V\, H^2}{\mu\, L1}\, f\, f'' + f^{(3)} == 0$$

$$\frac{1}{2}\, ReS\, f\, f'' - f^{(3)} == 0$$

$$ReS = \frac{\rho\, V\, H^2}{\mu\, L1}$$

```
(*  BCs   part e   *)
```

$$vx == V\, f';$$

$$vy == V \frac{H}{2\sqrt{x\, L1}}\ (-f + \eta\, f')\quad (*\ \ part\ b\ \ *)$$

$$y == 0;\ \eta == 0;\ vx == 1;\ f'[0] == 1;\ vy = 0$$

$$0 == V \frac{H}{2\sqrt{x\, L1}}\ (-f[0] + 0 \times 1)\ ;\ f[0] == 0$$

$$y == h;\ \eta == 1;\ f'[1] == 0$$