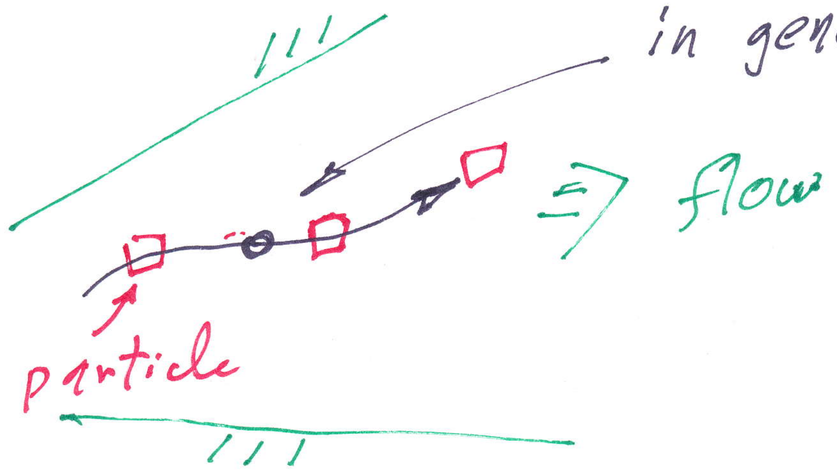
Class # 3 – Stress

Fluid Mechanics = Fluid Dynamics

$\boxed{\text{General Issue of fluids: Follow the particle}}$

$m\underset{\sim}{a} = \sum \underset{\sim}{F}$

on fluid element (particle)



in general, we know $\underset{\sim}{V}$ at spatial point

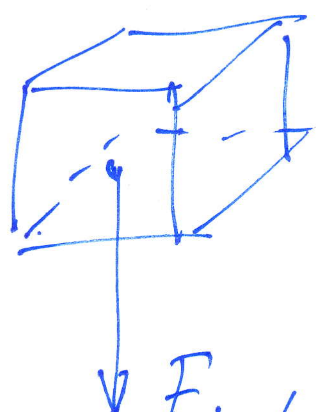
$$\underset{\sim}{a} = \left.\frac{\partial \underset{\sim}{V}}{\partial t}\right|_{\text{element} = \text{const}}$$

$V_x = V_x(x, y, t)$

$\left.\dfrac{\partial V_x}{\partial t}\right|_{x,y \text{ fixed}}$ $\qquad$ $\left.\dfrac{\partial V_x}{\partial t}\right|_{\substack{\text{Particle}\\ \text{element}}}$ fixed

$$m\underset{\sim}{a} = \sum \underset{\sim}{F} \qquad \boxed{\text{Analyze Forces in Fluids}}$$

$$\sum \underset{\sim}{F} = \underbrace{\underset{\sim}{F}_{body}}_{\text{volume}} + \underset{\sim}{F}_{surface}$$



$\underset{\sim}{F}_{body}$

gravity

$$\frac{1}{\forall} \cdot m\,\underset{\sim}{a} = \underset{\sim}{F}$$

$\forall$ volume

$F = m\,a$

$m = F/a$

$= \frac{N\text{-}s^2}{m}$

$$\rho\,\underset{\sim}{a} = \underset{\sim}{f}_g + \underset{\sim}{f}_s$$

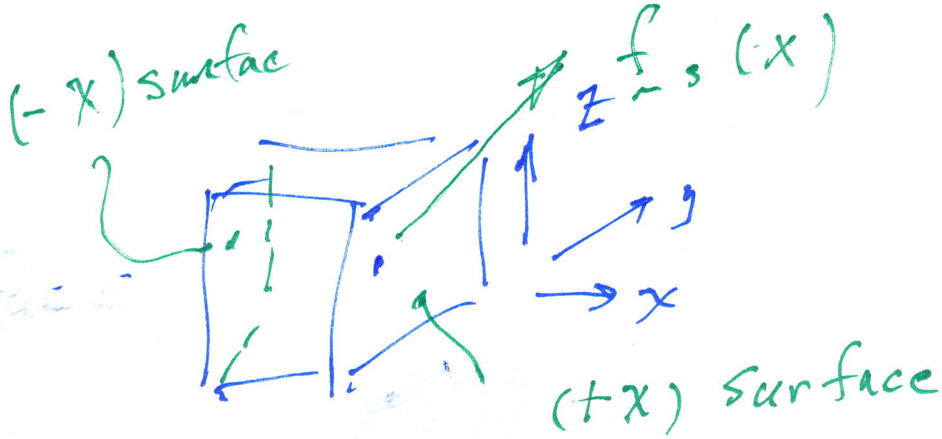
$$\frac{N\text{-}\cancel{s^2}}{m^4}\,\frac{m}{\cancel{s^2}} \qquad \frac{N}{m^3} \qquad \frac{\text{force}}{\text{volume}}$$

$$\underset{\sim}{f}_g = \rho\,\underset{\sim}{g}$$

$$\underset{\sim}{g} = -g\,\hat{e}_y$$

$$g = 9.81\,\frac{m}{s^2}$$

$\boxed{\text{body force} \quad \text{gravity} = \text{easy}}$



$\boxed{\text{Surface Forces complicated}}$

$\sum \underset{\sim}{f}_{surface}$

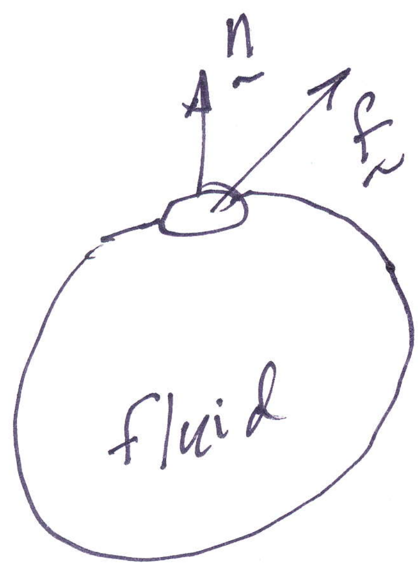
$\underset{\sim}{n}$ outward normal

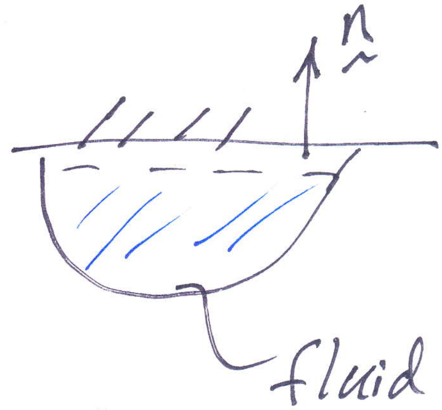
$$\underset{\sim}{f}_s = \underset{\sim}{n} \cdot \underset{\approx}{\sigma}$$

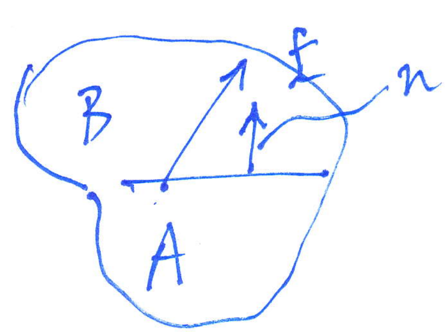
$\underset{\approx}{\sigma}$ stress tensor

-σ

<u>on</u> body

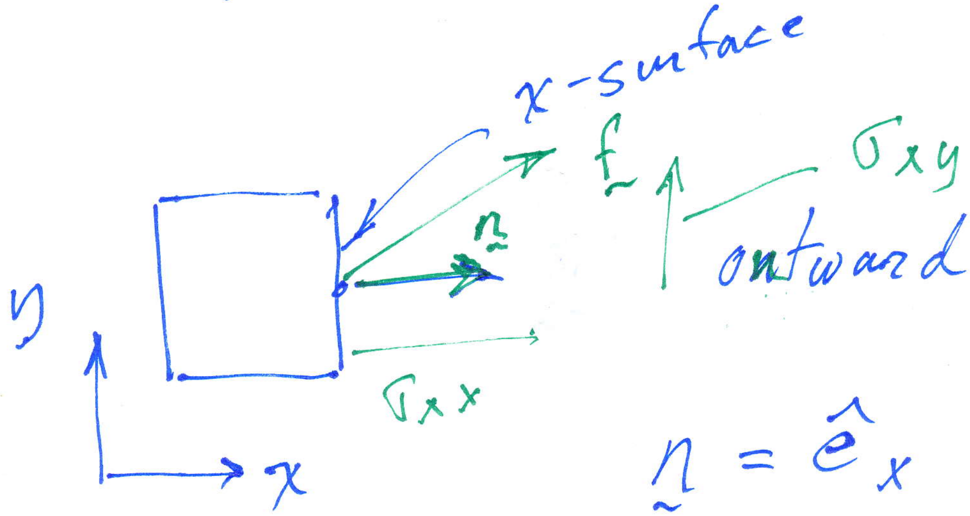






$\underset{\sim}{f}_{A\ on\ B}$

2-D



Stress tensor finds force on a surface

$$\underset{\sim}{n} = \hat{e}_x$$

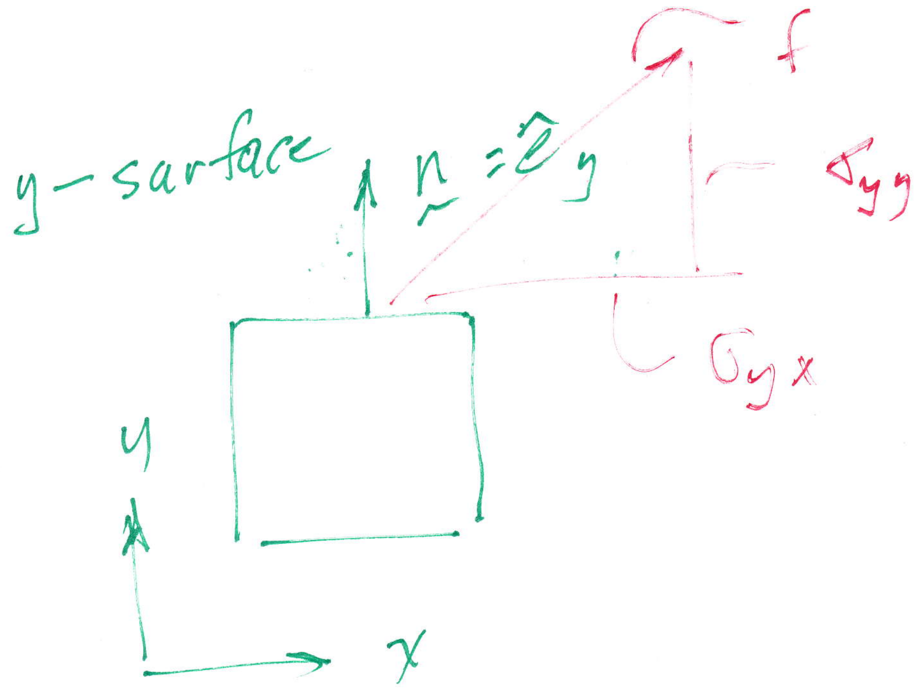
$$\underset{\approx}{\sigma} = \sigma_{xx}\hat{e}_x\hat{e}_x + \sigma_{xy}\hat{e}_x\hat{e}_y + \sigma_{yx}\hat{e}_y\hat{e}_x$$
$$\sigma_{yy}\hat{e}_y\hat{e}_y$$

stress tensor dyadic

$$\underset{\sim}{f} = \underset{\sim}{n} \cdot \underset{\approx}{\sigma}$$

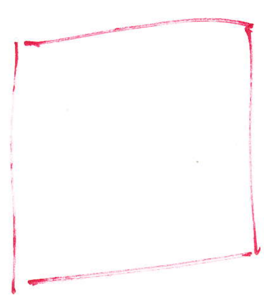
$$= \hat{e}_x \cdot (\sigma_{xx}\hat{e}_x\hat{e}_x + \sigma_{xy}\hat{e}_x\hat{e}_y \ldots\ldots)$$

$$\underset{\sim}{f}_x = \sigma_{xx}\hat{e}_x + \sigma_{xy}\hat{e}_y = \frac{d\underset{\sim}{F}}{dA}$$



$$\underset{\sim}{f} = \underset{\sim}{n} \cdot \underset{\approx}{\sigma} = \hat{e}_y \cdot \left(\sigma_{xx}\hat{e}_x\hat{e}_x + \sigma_{xy}\, e_x\hat{e}_y + \sigma_{yx}\hat{e}_y\hat{e}_x + \sigma_{yy}\hat{e}_y\hat{e}_y \right.$$

$$\underset{\sim}{f} = \sigma_{yx}\hat{e}_x + \sigma_{yy}\hat{e}_y$$

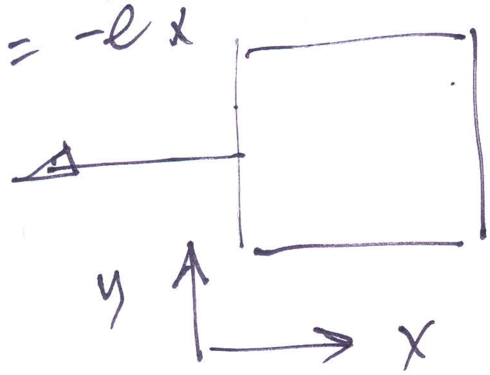


σ_{ij} — i: surface, j: direction

1-

Negative x- surface

$\underset{\sim}{n} = -\hat{e}_x$

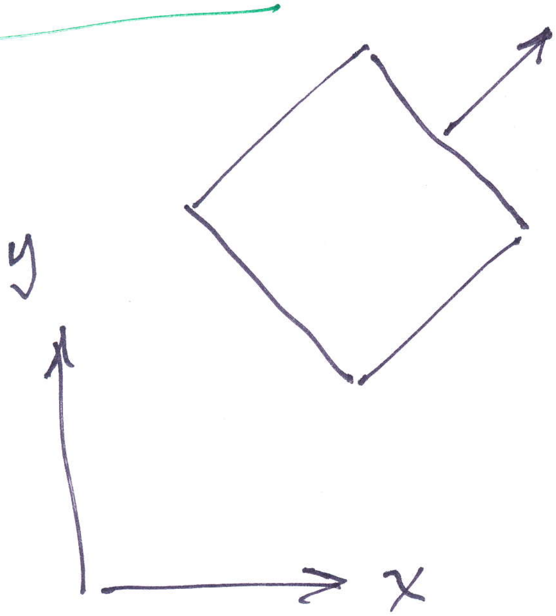


$\underset{\sim}{f} =$

$-\hat{e}_x \cdot (\sigma_{xx} \hat{e}_x \hat{e}_x + \ldots\ldots$

Any surface

$\underset{\sim}{n} = n_x \hat{e}_x + n_y \hat{e}_y$



$$\underset{\sim}{n} \cdot \underset{\approx}{\sigma} = (n_x \hat{e}_x + n_y \hat{e}_y) \cdot (\sigma_{xx} \hat{e}_x \hat{e}_x + \sigma_{xy} \hat{e}_x \hat{e}_y + \sigma_{yx} \hat{e}_y \hat{e}_x + \sigma_{yy} \hat{e}_y \hat{e}_y)$$

$$\underset{\sim}{f} = \left(n_x \sigma_{xx} + n_y \sigma_{yx}\right)\hat{e}_x + \left(n_x \sigma_{xy} + n_y \sigma_{yy}\right)\hat{e}_y$$

$$f_i = \sum_{m=1}^{3} n_m \sigma_{mi}$$

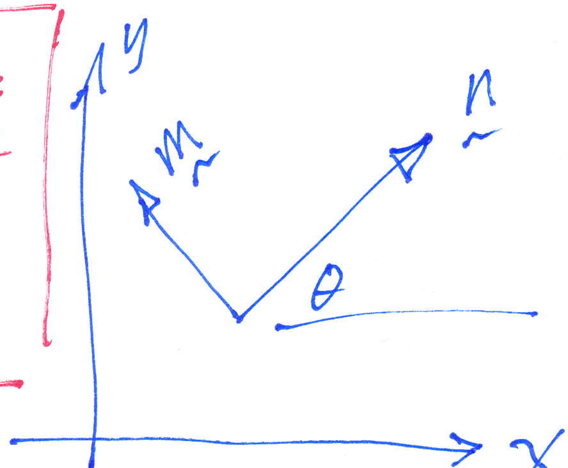
summation index l, m, n

free index i, j, k

$$f_i = n_1 \sigma_{1i} + n_2 \sigma_{2i} + n_3 \sigma_{3i}$$

$$=$$

$$\boxed{\text{Miscellaneous stuff about } \underset{\sim}{n}}$$

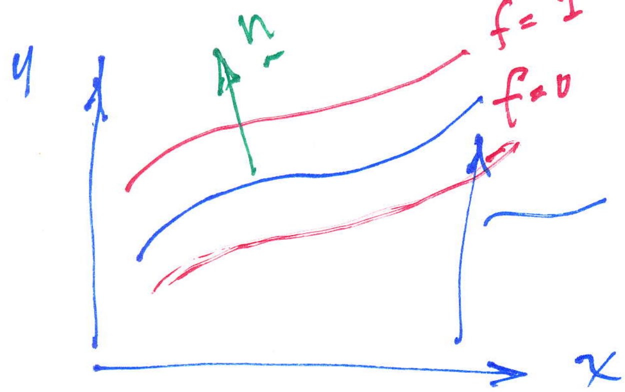


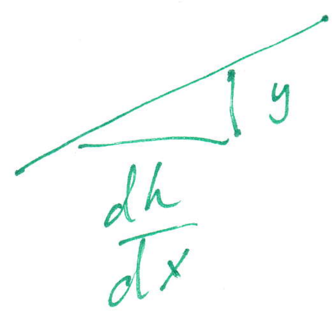
$\underset{\sim}{m} = -\sin\theta\, \hat{e}_x + \cos\theta\, \hat{e}_y$

$$\underset{\sim}{n} = \cos\theta\, \hat{e}_x + \sin\theta\, \hat{e}_y$$

$$\underset{\sim}{m} = \cos(\theta + 90^\circ)\hat{e}_x + \sin(\theta + 90^\circ)\hat{e}_y$$

$\underset{\sim}{n}$ outward normal to surface





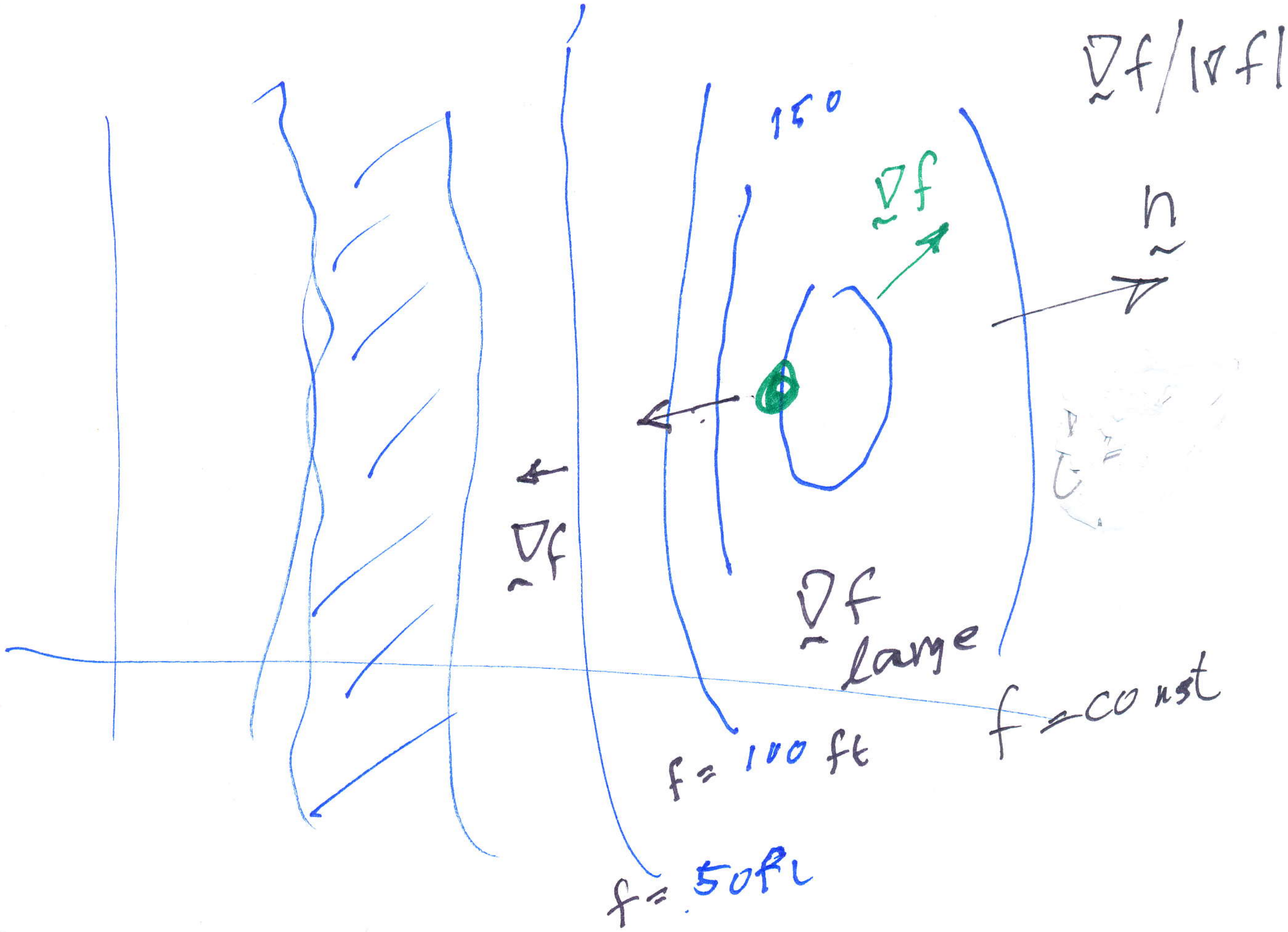
f =

$\underset{\sim}{n}$ = direction $= \dfrac{\underset{\sim}{\nabla} f}{|\underset{\sim}{\nabla} f|}$

$$\underset{\sim}{\nabla} f = \frac{\partial f}{\partial x}\hat{e}_x + \frac{\partial f}{\partial y}\hat{e}_y$$

$$= -\frac{dh}{dx}\hat{e}_x + \hat{e}_y$$

$$\underset{\sim}{n} = \pm \underset{\sim}{\nabla} f / |\underset{\sim}{\nabla} f| = \frac{-\frac{dh}{dx}\hat{e}_x + \hat{e}_y}{\sqrt{1+(dh/dx)^2}}$$



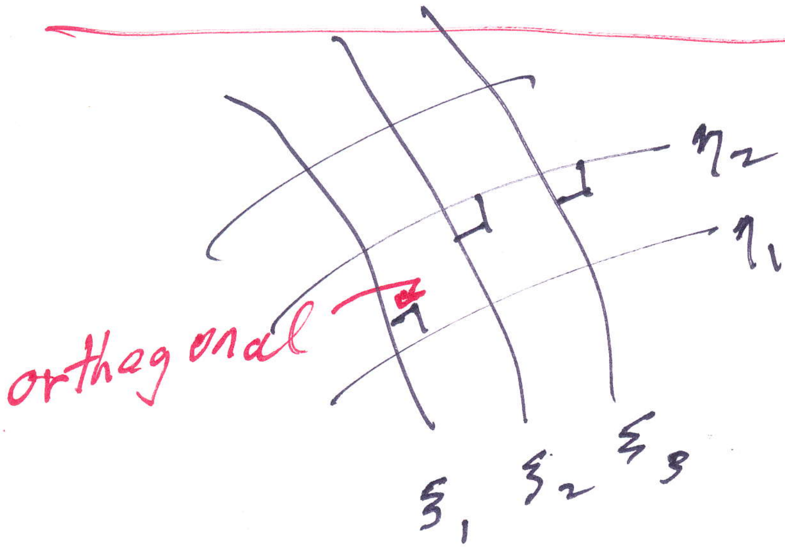
elevation

=

f = elevation

topographical Maps

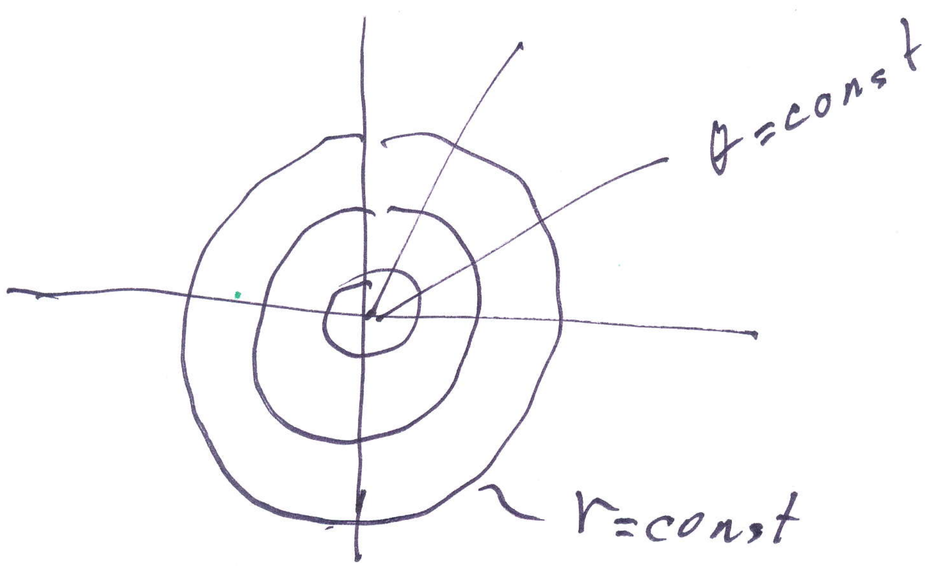
curvelinear coordinates (orthoganal)



ξ = ksi
= greek x

η = eta
= greek y

cylindrical coords



"del" (∇) operator in cyl. coords:

$x = r\cos\theta$ $\qquad r = \sqrt{x^2+y^2}$

$y = r\sin\theta$ $\qquad \theta = \arctan\left(\frac{y}{x}\right)$

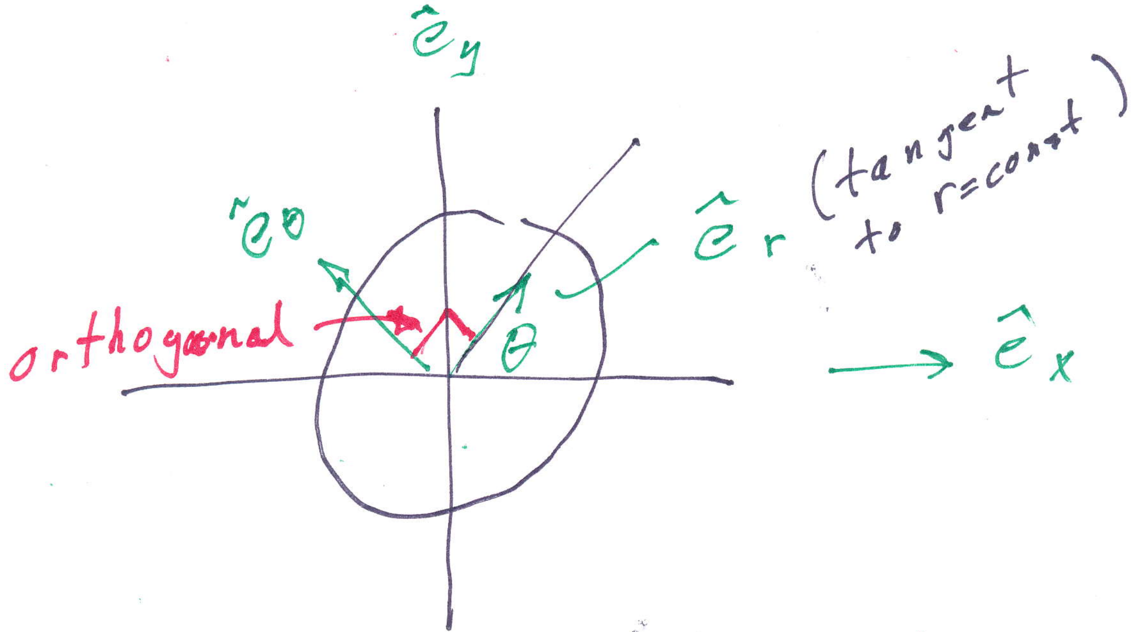
Derive physical equation in cartesian

convert $\frac{d}{dx}$ to $\underset{\sim}{\nabla}$

true in any coord system

$$\underset{\sim}{\nabla} = \hat{e}_x \frac{\partial}{\partial x} + \hat{e}_y \frac{\partial}{\partial y}$$

$$\underset{\sim}{\nabla} = \hat{e}_r \frac{\partial}{\partial r} + \hat{e}_\theta \frac{1}{r}\frac{\partial}{\partial \theta}$$



$$\hat{e}_r = \hat{e}_x \cos\theta + \hat{e}_y \sin\theta$$

$$\hat{e}_\theta = -\hat{e}_x \sin\theta + \hat{e}_y \cos\theta$$

convert $\quad \dfrac{\partial}{\partial x} \qquad \dfrac{\partial}{\partial r}, \dfrac{\partial}{\partial \theta}$

$$\dfrac{\partial}{\partial y} \quad \Rightarrow \quad \dfrac{\partial}{\partial \theta}, \dfrac{\partial}{\partial r}$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial f}{\partial \theta}\frac{\partial \theta}{\partial x}$$

$$r = (x^2 + y^2)^{1/2} \qquad \theta = \arctan\left(\frac{y}{x}\right)$$

$$\frac{\partial r}{\partial x} = \frac{1}{\cancel{2}}(x^2+y^2)^{-1/2}\cancel{2}x$$

$$= \frac{x}{\sqrt{x^2+y^2}} = \qquad \cos\theta$$

$$\underset{\sim}{\nabla} = \hat{e}_r \frac{\partial}{\partial r} + \hat{e}_\theta \frac{1}{r}\frac{\partial}{\partial \theta}$$

$$\underset{\sim}{V} = V_x \hat{e}_x + V_y \hat{e}_y$$

$$= V_r \hat{e}_r + V_\theta \hat{e}_\theta$$

$$\operatorname{div} V = \underset{\sim}{\nabla} \cdot \underset{\sim}{V} = \left(\frac{\partial}{\partial x}\hat{e}_x + \frac{\partial}{\partial y}\hat{e}_y\right) \cdot (V_x\hat{e}_x + V_y\hat{e}_y)$$

$$= \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y}$$

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{V} = \left(\frac{\partial}{\partial r}\hat{e}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{e}_\theta\right) \cdot \left(V_r \hat{e}_r + V_\theta \hat{e}_\theta\right)$$

$\hat{e}_x$

$\hat{e}_y$

vary with position

$$\hat{e}_r = \cos\theta\, \hat{e}_x + \sin\theta\, \hat{e}_y$$

$$\hat{e}_\theta = -\sin\theta\, \hat{e}_x + \cos\theta\, \hat{e}_y$$

not const

const

$$\frac{\partial \hat{e}_r}{\partial \theta} \neq 0 \qquad \frac{\partial \hat{e}_x}{\partial x} = 0$$

to get grad, need terms like:

$$\frac{1}{r}\frac{\partial}{\partial \theta}\hat{e}_\theta \cdot V_r \hat{e}_r$$

$$\hookrightarrow \underbrace{\frac{1}{r}\hat{e}_\theta \cdot \frac{\partial}{\partial \theta}\left(V_r \hat{e}_r\right)}_{\mathrm{I}} = \frac{1}{r}\hat{e}_\theta \cdot \left(\frac{\partial V_r}{\partial \theta}\hat{e}_r + V_r \underbrace{\frac{\partial \hat{e}_r}{\partial \theta}}_{\hat{e}_\theta}\right)$$

$= 0$

$$\frac{\partial \hat{e}_r}{\partial \theta} = -\sin\theta\, \hat{e}_x + \cos\theta\, \hat{e}_y = \hat{e}_\theta$$

$$\mathrm{I} = \frac{1}{r}\hat{e}_\theta \cdot V_r \hat{e}_\theta = \frac{V_r}{r}$$

$$\underset{\sim}{\nabla} = \frac{\partial}{\partial x}\hat{e}_x + \frac{\partial}{\partial y}\hat{e}_y$$

$$= \frac{\partial}{\partial r}\hat{e}_r + \frac{\partial}{\partial \theta}\hat{e}_\theta$$

div $V =$

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{V} = (\quad) \cdot (V_r \hat{e}_r + V_\theta \hat{e}_\theta)$$

$$\underset{\sim}{\nabla} = \frac{\partial}{\partial r}\hat{e}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{e}_\theta$$

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{V} = \frac{\partial V_r}{\partial r} + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta}$$