In[ ]:=
```
ClearAll["Global`*"];

(*  prob 1  *)
```

Note $\epsilon_{111} = 0$

In[ ]:=
```
a = {ax, ay, az};
b = {bx, by, bz};
e1 = {1, 0, 0};
e2 = {0, 1, 0};
e3 = {0, 0, 1};
eps = {{{0, 0, 0}, {0, 0, 1}, {0, -1, 0}}, {{0, 0, -1}, {0, 0, 0}, {1, 0, 0}},
   {{0, 1, 0}, {-1, 0, 0}, {0, 0, 0}}}; (*  permutation symbol  *)
```

$[a] = [a_x\ a_y,\ a_z]$

$\hat{e}_1 = \hat{\imath}$ $\hat{e}_2 = \hat{\jmath}$

ϵ_{111}

In[ ]:=
```
aa = a[[1]] e1 + a[[2]] e2 + a[[3]] e3
bb = b[[1]] e1 + b[[2]] e2 + b[[3]] e3
```

$\underset{\sim}{a} = a_x \hat{\imath} + a_y \hat{\jmath} + a_z k$

Out[ ]=
```
{ax, ay, az}
```

Out[ ]=
```
{bx, by, bz}
```

$\underset{\sim}{b} = b_x \hat{\imath} + b_y \hat{\jmath} + b_z \hat{k}$

In[ ]:=
```
((aa[[1]] e1) × (bb[[1]] e1) + (aa[[1]] e1) × (bb[[2]] e2) + (aa[[1]] e1) × (bb[[3]] e3)) +
 ((aa[[2]] e2) × (bb[[1]] e1) + (aa[[2]] e2) × (bb[[2]] e2) + (aa[[2]] e2) × (bb[[3]] e3)) +
 ((aa[[3]] e3) × (bb[[1]] e1) + (aa[[3]] e3) × (bb[[2]] e2) + (aa[[3]] e3) × (bb[[3]] e3))
```

Out[ ]=
```
{-az by + ay bz, az bx - ax bz, -ay bx + ax by}
```

e.g.

$a_z \hat{e}_z \times b_x \hat{e}_x$

$a_z b_x \underbrace{(\hat{e}_z \times \hat{e}_x)}_{+\hat{e}_y}$

In[ ]:=
```
a × b
```

Out[ ]=
```
{-az by + ay bz, az bx - ax bz, -ay bx + ax by}
```

In[ ]:=
```
c = {0, 0, 0};
Do[Do[Do[c[[i]] = c[[i]] + eps[[i,m,n]] a[[m]] b[[n]], {n, 1, 3}], {m, 1, 3}], {i, 1, 3}]; c
```

Out[ ]=
```
{-az by + ay bz, az bx - ax bz, -ay bx + ax by}
```

In[ ]:=
```
cx = c[[1]]
cy = c[[2]]
cz = c[[3]]
```

Out[ ]=
```
-az by + ay bz
```

Out[ ]=
```
az bx - ax bz
```

Out[ ]=
```
-ay bx + ax by
```

$$\underset{\sim}{c} = (-a_z b_y + a_y b_z)\hat{e}_x + (a_z b_x - a_x b_z)\hat{e}_y + (-a_y b_x + a_x b_y)\hat{e}_z$$

In[ ]:=

$$\text{Det}\left[\begin{pmatrix} ex & ey & ez \\ ax & ay & az \\ bx & by & bz \end{pmatrix}\right]$$

Out[ ]=

```
-az by ex + ay bz ex + az bx ey - ax bz ey - ay bx ez + ax by ez
```

In[ ]:=

```
Collect [%, {ex, ey, ez}]
```

$\underset{\sim}{C} =$

Out[ ]=

```
(-az by + ay bz) ex + (az bx - ax bz) ey + (-ay bx + ax by) ez
```

In[ ]:=

```
DD = Outer[Times, {ax, ay, az}, {cx, cy, cz}]
```

$D = \underset{\sim}{a}\,\underset{\sim}{c}$

Out[ ]=

```
{{ax (-az by + ay bz), ax (az bx - ax bz), ax (-ay bx + ax by)},
 {ay (-az by + ay bz), ay (az bx - ax bz), ay (-ay bx + ax by)},
 {az (-az by + ay bz), az (az bx - ax bz), az (-ay bx + ax by)}}
```

In[ ]:=

```
MatrixForm[DD]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} ax\,(-az\,by + ay\,bz) & ax\,(az\,bx - ax\,bz) & ax\,(-ay\,bx + ax\,by) \\ ay\,(-az\,by + ay\,bz) & ay\,(az\,bx - ax\,bz) & ay\,(-ay\,bx + ax\,by) \\ az\,(-az\,by + ay\,bz) & az\,(az\,bx - ax\,bz) & az\,(-ay\,bx + ax\,by) \end{pmatrix}$$

```
(*   Prob 2   *)
```

In[ ]:=

```
ClearAll["Global`*"];
```

$$T = \begin{pmatrix} Txx & Txy & 0 \\ Tyx & Tyy & 0 \\ 0 & 0 & Tzz \end{pmatrix};$$

from Prob 1

In[ ]:=

```
c = {-az by + ay bz, az bx - ax bz, -ay bx + ax by};
```

In[ ]:=

```
DDP = 0;  (*  double dot product  *)Do[Do[DDP = DDP + T[[m,n]] T[[n,m]], {m, 1, 3}], {n, 1, 3}]; DDP
```

Out[ ]=

$$Txx^2 + 2\,Txy\,Tyx + Tyy^2 + Tzz^2$$

In[ ]:=

```
σ = c.T
```

short way

Out[ ]=

```
{(-az by + ay bz) Txx + (az bx - ax bz) Tyx,
 (-az by + ay bz) Txy + (az bx - ax bz) Tyy, (-ay bx + ax by) Tzz}
```

same

```
σ = {0, 0, 0};  (*  dot (inner) product  *)
Do[Do[σ[[i]] = σ[[i]] + c[[m]] T[[m,i]], {m, 1, 3}], {i, 1, 3}];
σ
```

Out[ ]=

```
{(-az by + ay bz) Txx + (az bx - ax bz) Tyx,
 (-az by + ay bz) Txy + (az bx - ax bz) Tyy, (-ay bx + ax by) Tzz}
```

```
(*   Prob 3   *)

ClearAll["Global`*"];

x = {x1, x2, x3};
v = {v1[x1, x2, x3], v2[x1, x2, x3], v3[x1, x2, x3]};
τ = {{τ11[x1, x2, x3], τ12[x1, x2, x3], τ13[x1, x2, x3]}, {τ21[x1, x2, x3], τ22[x1, x2, x3],
    τ23[x1, x2, x3]}, {τ31[x1, x2, x3], τ32[x1, x2, x3], τ33[x1, x2, x3]}};
MatrixForm[τ]
```

Out[5]//MatrixForm=

$$\underset{\approx}{\tau} = \begin{pmatrix} \tau11[x1, x2, x3] & \tau12[x1, x2, x3] & \tau13[x1, x2, x3] \\ \tau21[x1, x2, x3] & \tau22[x1, x2, x3] & \tau23[x1, x2, x3] \\ \tau31[x1, x2, x3] & \tau32[x1, x2, x3] & \tau33[x1, x2, x3] \end{pmatrix}$$

$\underset{\sim}{\nabla}\underset{\approx}{\tau} = (\nabla \tau)_{ijk}$

```
delτ = {{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}},
   {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}, {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}};
(*   initialize; note third order, has 3 indices   *)
```

suppress output

```
Do[Do[Do[delτ[[i, m, n]] = ∂x[[i]] τ[[m, n]], {i, 1, 3}], {m, 1, 3}], {n, 1, 3}];
```

1st index indicates derivative

```
delτ[[1, 1, 1]]
delτ[[1, 1, 2]]
delτ[[1, 1, 3]]
delτ[[1, 2, 1]]
delτ[[2, 1, 1]]
```

1st index indicates derivative

Out[7]= $\tau11^{(1,0,0)}[x1, x2, x3]$ $\quad (\nabla\tau)_{111} = \dfrac{\partial \tau_{11}}{\partial x_1}$

Out[8]= $\tau12^{(1,0,0)}[x1, x2, x3]$

Out[9]= $\tau13^{(1,0,0)}[x1, x2, x3]$

Out[10]= $\tau21^{(1,0,0)}[x1, x2, x3]$ $\quad \rightarrow (\nabla\tau)_{121} = \dfrac{\partial \tau_{21}}{\partial x_1}$

Out[11]= $\tau11^{(0,1,0)}[x1, x2, x3]$ $\quad \rightarrow \nabla\tau_{211} = \dfrac{\partial \tau_{11}}{\partial x_2}$

```
vdotdelτ = {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}};

Do[Do[Do[vdotdelτ[[i, j]] = vdotdelτ[[i, j]] + v[[m]] delτ[[m, i, j]], {m, 1, 3}], {i, 1, 3}],
   {j, 1, 3}];

vdotdelτ[[1, 1]]
vdotdelτ[[1, 2]]
```

$(\underset{\sim}{v} \cdot \underset{\sim}{\nabla}\underset{\approx}{\tau})_{11}$

Out[ ]= $v3[x1, x2, x3]\ \tau11^{(0,0,1)}[x1, x2, x3] + v2[x1, x2, x3]\ \tau11^{(0,1,0)}[x1, x2, x3] + v1[x1, x2, x3]\ \tau11^{(1,0,0)}[x1, x2, x3]$

$(\underset{\sim}{v} \cdot \underset{\sim}{\nabla}\underset{\approx}{\tau})_{12}$ etc

Out[ ]= $v3[x1, x2, x3]\ \tau12^{(0,0,1)}[x1, x2, x3] + v2[x1, x2, x3]\ \tau12^{(0,1,0)}[x1, x2, x3] + v1[x1, x2, x3]\ \tau12^{(1,0,0)}[x1, x2, x3]$

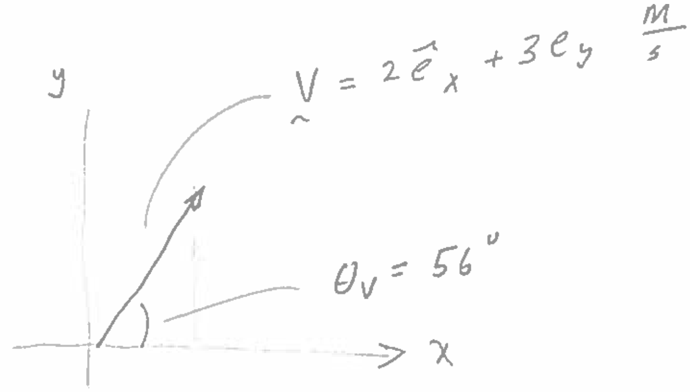
```
(*  Prob 4   *)
ClearAll["Global`*"];
v = {vx, vy}; vx = 2.; vy = 3.;
vMag =   vx^2 + vy^2
```

3.60555

```
θv = ArcTan[vy/vx]; θvDeg = 180. θv/π
```

56.3099

```
θpDeg = 40.;  θp = θpDeg/180 π;
```




```
exp = {Cos[θ], Sin[θ]}
eyp = {-Sin[θ], Cos[θ]}
```

{0.766044, 0.642788}

{-0.642788, 0.766044}

```
vxp = exp.v
vyp = eyp.v
```

3.46045

1.01256

$V_x' = \hat{e}_x' \cdot \underset{\sim}{V}$

```
vpMag =   vxp^2 + vyp^2
```

$|\underset{\sim}{V}'|$

3.60555

```
θvp = ArcTan[vyp/vxp]; θvpDeg = 180. θvp/π
```

16.3099

$\underset{\sim}{V}$ has same mag & dir as $\underset{\sim}{V}'$

```
θvDeg - θvpDeg
```

40.