

The background of the slide features a large, faint, circular watermark of the University of Udine seal. The seal contains a central emblem with a star and a figure, surrounded by the Latin text "SIGILLUM UNIVERSITATIS UDINENSIS".

University of Udine
Department of Mathematics, Computer Science and Physics

NON-WELL-FOUNDED SET BASED MULTI-AGENT EPISTEMIC ACTION LANGUAGE

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1. Multi-Agent Epistemic Planning
2. Kripke Structures
3. Possibilities
4. The action language $m\mathcal{A}^p$
5. Conclusions



Chapter 1

Multi-Agent Epistemic Planning



Introduction



Epistemic Reasoning

Reasoning not only about agents' *perception of the world* but also about agents' *knowledge* and/or *beliefs* of her and others' beliefs.



Introduction



Epistemic Reasoning

Reasoning not only about agents' *perception of the world* but also about agents' *knowledge* and/or *beliefs* of her and others' beliefs.

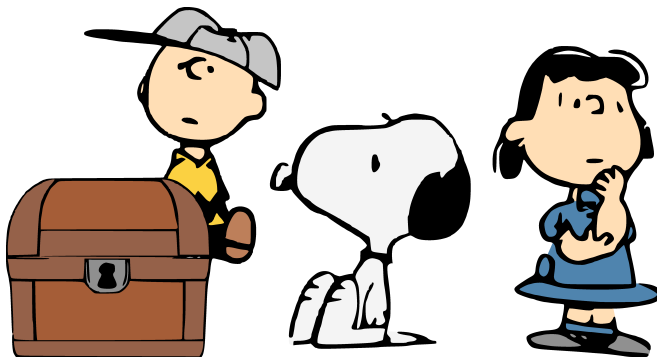
Multi-agent Epistemic Planning Problem **bolander2011epistemic**

Finding *plans* where the goals can refer to:

- the state of the world
- the knowledge and/or the beliefs of the agents



An Example

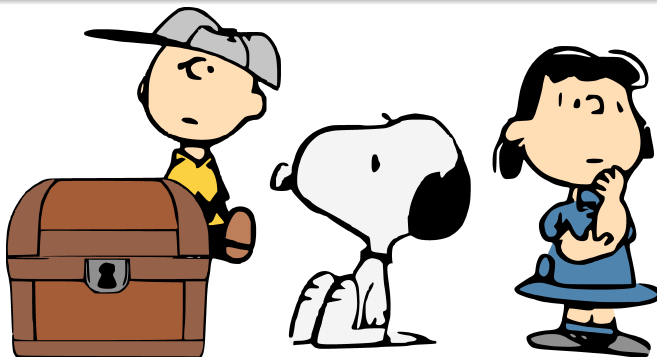


An Example



Initial State

- Snoopy and Charlie are **looking** while Lucy is \neg **looking**
- No one knows the **coin position**.

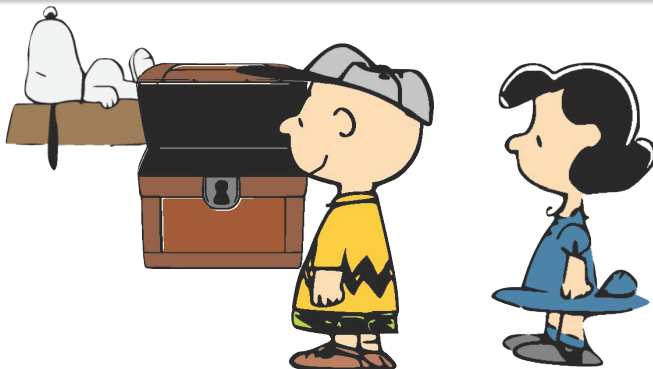


An Example



Goal State

- Charlie knows the **coin position**
- Lucy knows that Charlie knows the **coin position**
- Snoopy does not know anything about the plan execution



Challenges



An agent has to reason about his actions effects on

- The *state of the world*
- The agents' awareness of the environment
- The agents' awareness of other agents' *actions*
- The knowledge of other agents about his own





Given a set of agents \mathcal{AG}

Modal operator B_{ag} where $ag \in \mathcal{AG}$

Models the beliefs of ag about the **state of the world** and/or about the beliefs of other agents.



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Expresses the **common belief** of a group of agents.



Notations



6

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Group operator C_{α} where $\alpha \subseteq \mathcal{AG}$

Expresses the **common belief** of a group of agents.

Belief Formulae

Take into consideration **fluents** and/or agents' beliefs.



Example of Belief Formulae



Given

- $\mathcal{AG} = \{\text{Snoopy, Charlie, Lucy}\}$
- $\mathcal{F} = \{\text{opened, head, looking}_{\text{ag}}\} \text{ ag} \in \mathcal{AG}$



$B_{\text{Snoopy}} B_{\text{Charlie}} \neg \text{opened}$

Snoopy believes that Charlie believes that the box is $\neg \text{opened}$.



Example of Belief Formulae



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$B_{\text{Snoopy}} B_{\text{Charlie}} \neg \text{opened}$

Snoopy believes that Charlie believes that the box is $\neg \text{opened}$.

$C_{\alpha}(\neg B_{\text{Lucy}} \text{heads} \wedge \neg B_{\text{Lucy}} \neg \text{head})$

where $\alpha = \mathcal{AG}$

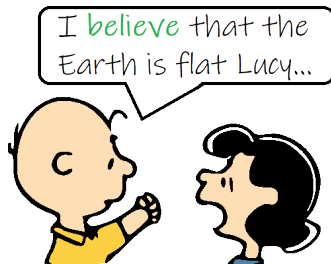
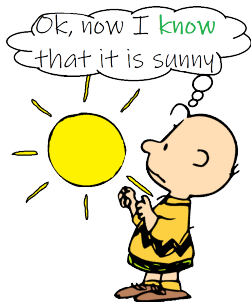
It is common knowledge that Lucy does not know whether the coin lies heads or tails up



Knowledge vs. Belief



- The modal operator B_{ag} represents the worlds' relation
- Different relation's properties imply different meaning for B_{ag}



Knowledge vs. Belief



- The modal operator \mathbf{B}_{ag} represents the worlds' relation
- Different relation's properties imply different meaning for \mathbf{B}_{ag}
- *Knowledge* and *Belief* are characterized by a subset of the following axioms

Serial (D) and S5 (K,T,4,5) Axioms

Given the fluent formulae ϕ, ψ and the worlds i, j

$$D \quad \neg \mathcal{R}_i \perp \quad \text{B K}$$

$$K \quad (\mathcal{R}_i \phi \wedge \mathcal{R}_i (\phi \Rightarrow \psi)) \Rightarrow \mathcal{R}_i \psi \quad \text{B K}$$

$$T \quad \mathcal{R}_i \phi \Rightarrow \phi \quad \text{K}$$

$$4 \quad \mathcal{R}_i \phi \Rightarrow \mathcal{R}_i \mathcal{R}_i \phi \quad \text{B K}$$

$$5 \quad \neg \mathcal{R}_i \phi \Rightarrow \mathcal{R}_i \neg \mathcal{R}_i \phi \quad \text{B K}$$



Chapter 2

Kripke Structures



Pointed Kripke structure

A *Pointed Kripke structure* is a pair $(\langle S, \pi, \mathcal{R}_1, \dots, \mathcal{R}_n \rangle, s_0)$, s.t.:

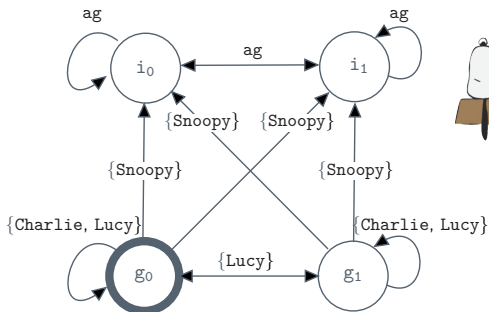
- S is a set of *worlds* and $s_0 \in S$
- $\pi : S \mapsto 2^{\mathcal{F}}$ associates an *interpretation* to each element of S
- for $1 \leq i \leq n$, $\mathcal{R}_i \subseteq S \times S$ is a binary relation over S



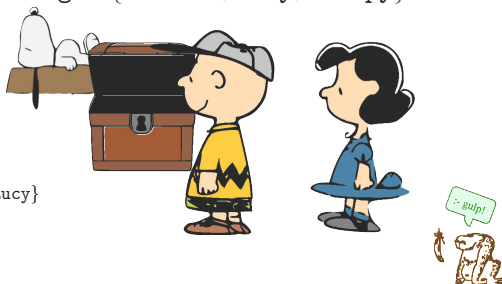
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$ag = \{\text{Charlie, Lucy, Snoopy}\}$



Let φ be a belief formula and (M, s) be a pointed Kripke structure:

Entailment w.r.t. a pointed Kripke structure

- $(M, s) \models \varphi$ if φ is a *fluent formula* and $\pi(s) \models \varphi$;



Let φ be a belief formula and (M, s) be a pointed Kripke structure:

Entailment w.r.t. a pointed Kripke structure

- $(M, s) \models \varphi$ if φ is a *fluent formula* and $\pi(s) \models \varphi$;
- $(M, s) \models \mathbf{B}_{\text{agi}} \varphi$ if $\forall t: (s, t) \in \mathcal{R}_i$ it holds that $(M, t) \models \varphi$;



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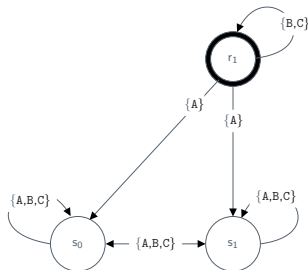
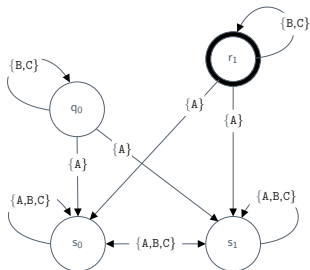
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- $(M, s) \models \mathbf{E}_\alpha \varphi$ if $(M, s) \models \mathbf{B}_{\text{agi}} \varphi$ for all $\text{agi} \in \alpha$;
- $(M, s) \models \mathbf{C}_\alpha \varphi$ if $(M, s) \models \mathbf{E}_\alpha^k \varphi$ for every $k \geq 0$, where $\mathbf{E}_\alpha^0 \varphi = \varphi$ and $\mathbf{E}_\alpha^{k+1} \varphi = \mathbf{E}_\alpha(\mathbf{E}_\alpha^k \varphi)$.

The entailment for the standard operators is defined as usual



- Solvers require high amount of memory
- In literature the states have been represented explicitly
- State comparison needs to find *bisimilar* states



- Heuristics **le2018efp**



- Heuristics **le2018efp**
- Symbolic representation of Kripke structures



- Heuristics **le2018efp**
- Symbolic representation of Kripke structures
- Alternative representations



Chapter 3

Possibilities



- Introduced by Gerbrandy and Groeneveld **Gerbrandy1997**
- Used to represent *multi-agent information change*
- Based on *non-well-founded sets*
- Corresponds with a class of bisimilar Kripke structures **gerbrandy1999bisimulations**

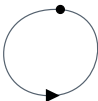


Non-well-founded sets



Non-well-founded set Aczel1989-ACZNS-2

A set is *non-well-founded* (or *extraordinary*) when among its descents there are some which are infinite.



The non-well-founded set $\Omega = \{\Omega\}$



Possibility Gerbrandy1997

Let \mathcal{AG} be a set of agents and \mathcal{F} a set of propositional variables:

- A *possibility* u is a function that assigns to each propositional variable $1 \in \mathcal{F}$ a truth value $u(1) \in \{0, 1\}$ and to each agent $ag \in \mathcal{AG}$ a set of possibilities $u(ag) = \sigma$.



Possibility Gerbrandy1997

Let \mathcal{AG} be a set of agents and \mathcal{F} a set of propositional variables:

- A *possibility* u is a function that assigns to each propositional variable $\mathbf{1} \in \mathcal{F}$ a truth value $u(\mathbf{1}) \in \{0, 1\}$ and to each agent $\mathbf{ag} \in \mathcal{AG}$ a set of possibilities $u(\mathbf{ag}) = \sigma$.

Intuitively ...

- The possibility u is a possible interpretation of the world and of the agents' beliefs
- $u(\mathbf{1})$ specifies the truth value of the literal $\mathbf{1}$
- $u(\mathbf{ag})$ is the set of all the interpretations the agent \mathbf{ag} considers possible in u

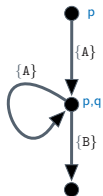


From Possibilities to Kripke Structures



Considering a possibility

A possibility

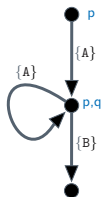


From Possibilities to Kripke Structures



Considering a possibility
 → Can be expressed as a *system of equations*

A possibility



Its system of equation

$$\begin{cases} w(p) = 1 & w(q) = 0 \\ v(p) = 1 & v(q) = 1 \\ u(p) = 0 & u(q) = 0 \\ w(A) = \{v\} & w(B) = \{\emptyset\} \\ v(A) = \{v\} & v(B) = \{u\} \\ u(A) = \{\emptyset\} & u(B) = \{\emptyset\} \end{cases}$$



From Possibilities to Kripke Structures

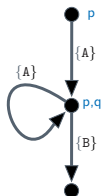


Considering a possibility

Can be expressed as a *system of equations*

Systems of equations have unique solutions

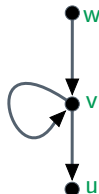
A possibility



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The solution

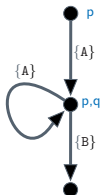


From Possibilities to Kripke Structures



- Considering a possibility
- Can be expressed as a *system of equations*
 - Systems of equations have unique solutions
 - The solution decorates a Kripke structure

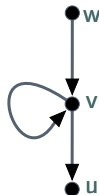
A possibility



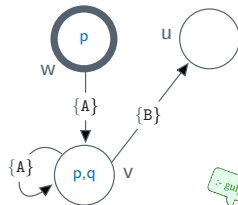
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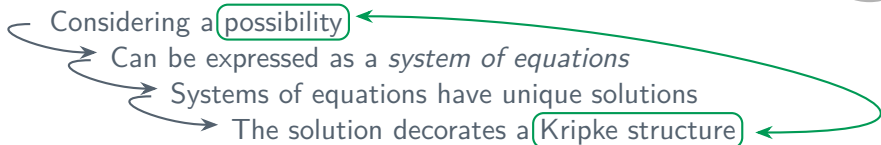
The solution



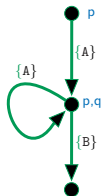
Relative Kripke Structure



From Possibilities to Kripke Structures



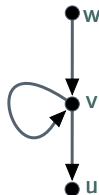
A possibility



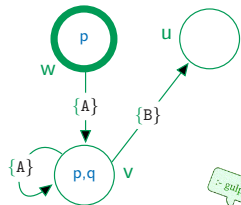
Its system of equation

$$\begin{cases} w(p) = 1 & w(q) = 0 \\ v(p) = 1 & v(q) = 1 \\ u(p) = 0 & u(q) = 0 \\ w(A) = \{v\} & w(B) = \{\emptyset\} \\ v(A) = \{v\} & v(B) = \{u\} \\ u(A) = \{\emptyset\} & u(B) = \{\emptyset\} \end{cases}$$

The solution



Relative Kripke Structure



Chapter 4

The action language $m\mathcal{A}^\rho$



Overview



We introduce the action language $m\mathcal{A}^p$

- Used to describe MEP problems
- Same syntax of the action language $m\mathcal{A}+$ **baral2015action**
- As expressive as $m\mathcal{A}+$
- Uses possibilities as states



Actions

Three types of actions:

- *Ontic*: modifies some fluents of the world

Charlie *opens* the box



Actions

Three types of actions:

- 0.5Ontic: modifies some fluents of the world
Charlie *opens* the box
- *Sensing*: senses the true value of a fluent
Charlie *peeks* inside the box



Actions

Three types of actions:

- 0.5 Ontic: modifies some fluents of the world
 - Charlie *opens* the box
- 0.5
 - Sensing: senses the true value of a fluent
 - Charlie *peeks* inside the box
 - **Announcement**: announces the fluent to other agents
 - Charlie *announces* the coin position



Observability Relations



An *execution* of an action might change or not an agents' belief accordingly to her degree of awareness

Action type	Full observers	Partial Observers	Oblivious
Ontic	✓		✓
Sensing	✓	✓	✓
Announcement	✓	✓	✓



Possibility as a state



In $m\mathcal{A}^p$ a state is encoded by a possibility where

- (agent, σ) represent the possibilities believed by agent
- If $f \in \mathcal{F}$ is present then it is true

$w = \{(\text{ag}, \{w, w'\}), (C, \{v, v'\}), \text{look}(\text{ag}), \text{key}(A), \text{opened}, \text{heads}\}$

0.8 where $\text{ag} \in \{A, B\}$



Possibility as a state



In $m\mathcal{A}^p$ a state is encoded by a possibility where

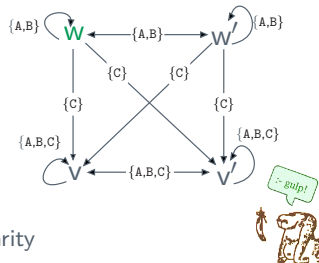
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$$\begin{cases} w &= \{(\text{ag}, \{w, w'\}), (C, \{v, v'\}), \text{look}(\text{ag}), \text{key}(A), \text{opened}, \text{heads}\} \\ w' &= \{(\text{ag}, \{w, w'\}), (C, \{v, v'\}), \text{look}(\text{ag}), \text{key}(A), \text{opened}\} \\ v &= \{(A, \{v, v'\}), (B, \{v, v'\}), (C, \{v, v'\}), \text{look}(\text{ag}), \text{key}(A), \text{heads}\} \\ v' &= \{(A, \{v, v'\}), (B, \{v, v'\}), (C, \{v, v'\}), \text{look}(\text{ag}), \text{key}(A)\} \end{cases}$$

where $\text{ag} \in \{A, B\}$.



0.8 Possibility w expanded for clarity

State equality



- Possibilities captures classes of bisimilar Kripke structures
- Possibilities equality considers bisimilarity
- This help for the *visited states* problem in MEP

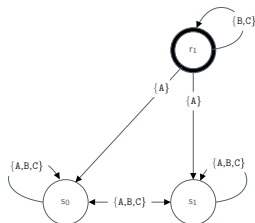
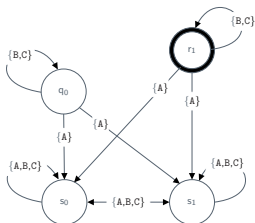


State equality



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$r_1 = \{(A, \{s_0, s_1\}), (B, \{r_1\}), (C, \{r_1\}), \text{look}(C), \text{key}(A), \text{opened}, \text{heads}\}$



Entailment



Let $\varphi, \varphi_1, \varphi_2$ be beliefs formula and u be a possibility

Entailment w.r.t. possibilities

- $u \models 1$ if $u(1) = 1$;



Entailment



Let $\varphi, \varphi_1, \varphi_2$ be beliefs formula and u be a possibility

Entailment w.r.t. possibilities

- $u \models \mathbf{1}$ if $u(\mathbf{1}) = 1$;
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The entailment for the standard operators is defined as usual



Ontic Actions



Φ_D for Ontic Actions

$$\Phi_D : \mathcal{AI} \times \Sigma \rightarrow \Sigma \cup \{\emptyset\}$$

Let a be an *ontic action* instance, u a possibility and \mathbf{l} be \mathbf{f} or $\neg\mathbf{f}$

$$\Phi_D(a, u) = \emptyset \quad \text{if } a \text{ is not executable in } u$$

$$\Phi_D(a, u) = v \quad \text{if } a \text{ *modifies* the literals } \in \text{caused}(a)$$

Where v is

$$\begin{cases} v(\mathbf{l}) = u(\mathbf{l}) & \text{if } \mathbf{l} \notin \text{caused}(a) \\ v(\mathbf{l}) = \text{caused}(a)[\mathbf{l}] & \text{if } \mathbf{l} \in \text{caused}(a) \end{cases}$$

and

$$\begin{cases} v(\mathbf{ag}) = u(\mathbf{ag}) & \text{if } \mathbf{ag} \in O_D \\ v(\mathbf{ag}) = \bigcup_{w \in u(\mathbf{ag})} \Phi_D(a, w) & \text{if } \mathbf{ag} \in F_D \end{cases}$$



Sensing Actions



Φ_D for Sensing Actions

$\Phi_D : \mathcal{AI} \times \Sigma \rightarrow \Sigma \cup \{\emptyset\}$

Let a be an *sensing action* instance, and u a possibility and $\mathbf{1}$ be \mathbf{f} or $\neg\mathbf{f}$

$\Phi_D(a, u) = \emptyset$ if a is not executable in u

$\Phi_D(a, u) = v$ if the literal $\mathbf{1}$ is *sensed*

Where v is

$$\begin{cases} \emptyset & \text{if sensed}(a)[\mathbf{1}] \neq u(\mathbf{1}) \\ v(\mathbf{ag}) = u(\mathbf{ag}) & \text{if } \mathbf{ag} \in O_D \\ v(\mathbf{ag}) = \bigcup_{w \in u(\mathbf{ag})} \Phi_D(a, w) & \text{if } \mathbf{ag} \in F_D \\ v(\mathbf{ag}) = \bigcup_{w \in u(\mathbf{ag})} (\Phi_D(a, w) \cup \Phi_D(\neg a, w)) & \text{if } \mathbf{ag} \in P_D \end{cases}$$



Announcement Actions



Φ_D for Announcement Actions $\Phi_D : \mathcal{AI} \times \Sigma \rightarrow \Sigma \cup \{\emptyset\}$

Let a be an *announcement action* instance, and u a possibility.

$\Phi_D(a, u) = \emptyset$ if a is not executable in u

$\Phi_D(a, u) = v$ if the fluent formula ϕ is *announced*

Where v is

$$\begin{cases} \emptyset & \text{if } u \not\models \phi \\ v(\text{ag}) = u(\text{ag}) & \text{if } \text{ag} \in O_D \\ v(\text{ag}) = \bigcup_{w \in u(\text{ag})} \Phi_D(a, w) & \text{if } \text{ag} \in F_D \\ v(\text{ag}) = \bigcup_{w \in u(\text{ag})} (\Phi_D(a, w) \cup \Phi_D(\neg a, w)) & \text{if } \text{ag} \in P_D \end{cases}$$

:- gulp!



Chapter 5

Conclusions



Conclusions



- Exploited an **alternative** to the *Kripke structures* as states representation
- Used *possibilities* to define a stronger concept of states **equality**
- Possibilities helps in **reducing** the search-space dimension
- Defined a new action language for the MEP problem



Future works



- We started *implementing* a planner for $m\mathcal{A}^p$
- Exploit more *set-based operations*: especially for the entailment of group operators
- Formalize the concept of *non-consistent belief* for $m\mathcal{A}^p$
- Consider other *alternatives* to Kripke structures, e.g., *OBDDs*



The end



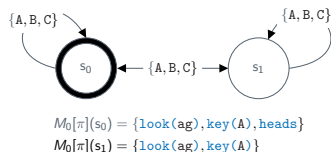
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for the attention*



References I



Side to side Execution



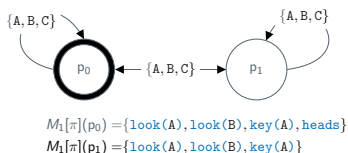
$$\begin{cases} u &= \{(\text{ag}, \{u, u'\}), \\ &\quad \text{look}(\text{ag}), \text{key}(\text{A}), \text{heads}\} \\ u' &= \{(\text{ag}, \{u, u'\}), \\ &\quad \text{look}(\text{ag}), \text{key}(\text{A})\} \end{cases}$$

where $\text{ag} \in \{A, B, C\}$

The initial state



Side to side Execution



$$\begin{cases} v = \{(\text{ag}, \{v, v'\}), \text{look}(A), \\ \text{look}(B), \text{key}(A), \text{heads}\} \\ v' = \{(\text{ag}, \{v, v'\}), \\ \text{look}(A), \text{look}(B), \text{key}(A)\} \end{cases}$$

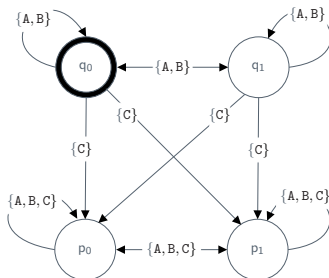
where $\text{ag} \in \{A, B, C\}$

where $\text{ag} \in \{A, B, C\}$

Execution of $\text{distract}(C)\langle A \rangle$



Side to side Execution



$$M_2[\pi](q_0) = \{\text{look}(\text{ag}), \text{key}(\text{A}), \text{opened}, \text{heads}\}$$

$$M_2[\pi](q_1) = \{\text{look}(\text{ag}), \text{key}(\text{A}), \text{opened}\}$$

$$M_2[\pi](p_0) = M_1[\pi](p_0)$$

$$M_2[\pi](p_1) = M_1[\pi](p_1)$$

$$\begin{cases} w &= \{(\text{ag}, \{w, w'\}), (\text{C}, \{v, v'\}), \\ &\quad \text{look}(\text{ag}), \text{key}(\text{A}), \text{opened}, \text{heads}\} \\ w' &= \{(\text{ag}, \{w, w'\}), (\text{C}, \{v, v'\}), \\ &\quad \text{look}(\text{ag}), \text{key}(\text{A}), \text{opened}\} \end{cases}$$

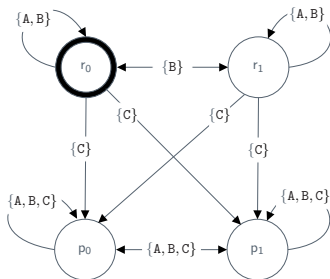
where v, v' , are defined as before.

where $\text{ag} \in \{\text{A}, \text{B}\}$

Execution of $\text{open}(\text{A})$



Side to side Execution



$$M_3[\pi](r_0) = \{\text{look}(\text{ag}), \text{key}(A), \text{opened}, \text{heads}\}$$

$$M_3[\pi](r_1) = \{\text{look}(\text{ag}), \text{key}(A), \text{opened}\}$$

$$M_3[\pi](p_0) = M_1[\pi](p_0)$$

$$M_3[\pi](p_1) = M_1[\pi](p_1)$$

$$\begin{cases} z &= \{(A, \{z\}), (B, \{z, z'\}) (C, \{v, v'\}), \\ &\quad \text{look}(\text{ag}), \text{key}(A), \text{opened}, \text{heads}\} \\ z' &= \{(A, \{z'\}), (B, \{z, z'\}) (C, \{v, v'\}), \\ &\quad \text{look}(\text{ag}), \text{key}(A), \text{opened}, \} \end{cases}$$

where the possibilities v, v' are defined as before.

where $\text{ag} \in \{A, B\}$

Execution of $\text{peek}\langle A \rangle$

