[Intro to AI] HW 2

We want to confirm the theory behind the bias-variance decomposition with an empirical experiment that measures the bias and variance for polynomial models.

In our experiment, we will reuse python codes in HW1 and repeatedly fit our hypothesis model to a random training set. We then find the expectation and variance of the fitted models generated from these training sets. Follow the instruction below.

1. Repeat the following steps by changing the polynomial degree d from 0 to 9.
2. i. Make a training data $\mathbf{x}_{\text{train}}$, which is evenly spaced 21 numbers over $[-\pi, \pi]$, and $\mathbf{y}_{\text{train}} = f(\mathbf{x}_{\text{train}}) + \epsilon$ where $\epsilon \sim \mathcal{N}(0, 0.5^2)$ is i.i.d. samples from Gaussian distribution.

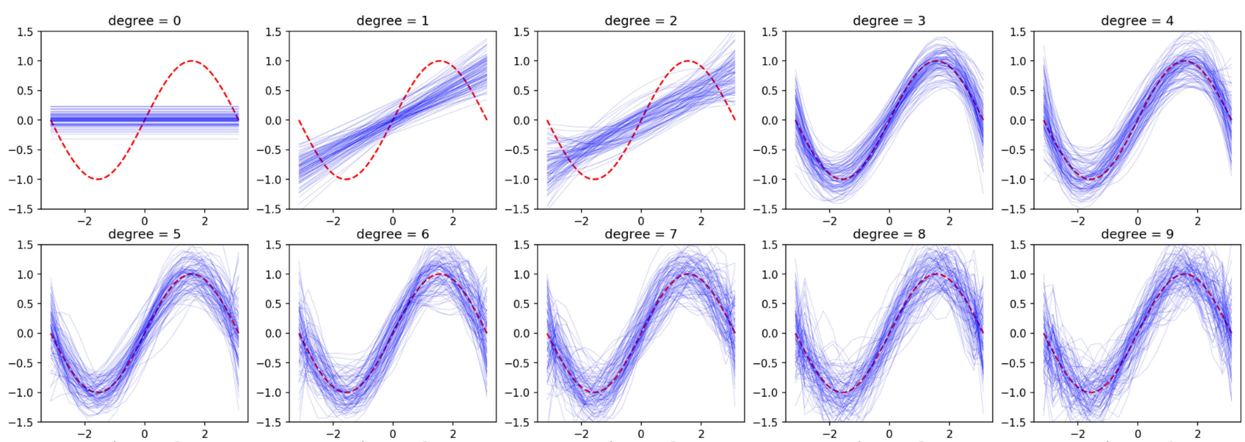
   ii. Fit d-th order polynomial to the training data and estimate the optimal model parameters/coefficients $\mathbf{w}^*$.

   iii. Repeat [steps 2i-2ii] 100 times to obtain 100 different model parameters $\{\mathbf{w}_i\}_{i=1:100}$ and model predictions $\{\hat{\mathbf{y}}_{\text{train},i}\}_{i=1:100}$.

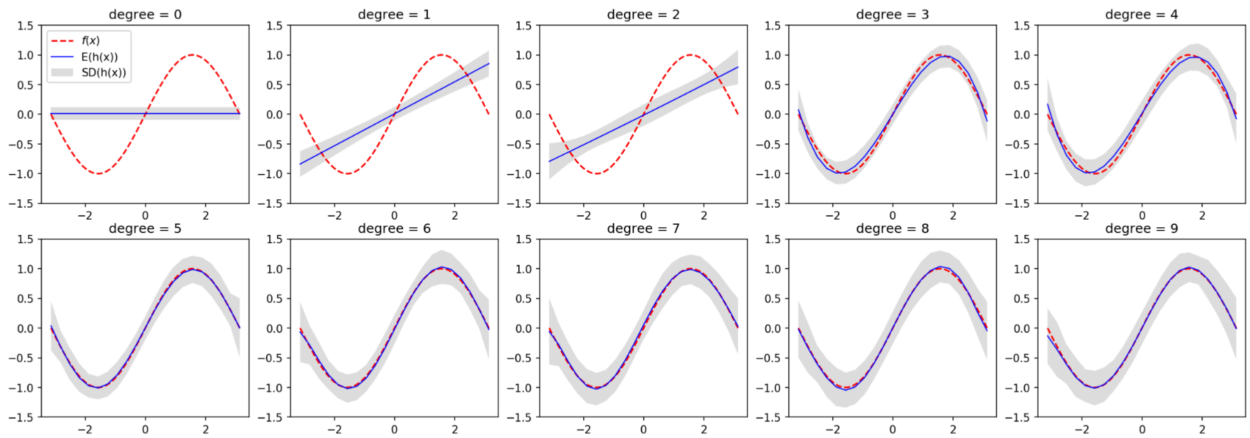
   iv. Compute the sample mean μ and standard deviation σ of $\{\hat{\mathbf{y}}_{\text{train},i}\}_{i=1:100}$. Note that μ, σ is a vector of length 21. Use `np.mean` & `np.std`.

   v. Compute $\text{bias}^2 = (\mu - \mathbf{y}_{\text{true}})^2$ and $\text{variance} = \sigma^2$. Note that bias^2 and variance is a vector of length 21.

   vi. Make a subplot of the true model and 100 model predictions as follows:

   vii. Make a subplot of the true model and μ, σ as follows. Use `fill_between` to fill the area between $\mu \pm \sigma$.



3. Make a bar plot (use `bar`) that takes an average of bias^2 and $\mathrm{variance}$ over all values of $\mathbf{x}_{\mathrm{train}}$, together with the sum of two.