[Intro to AI] HW 1

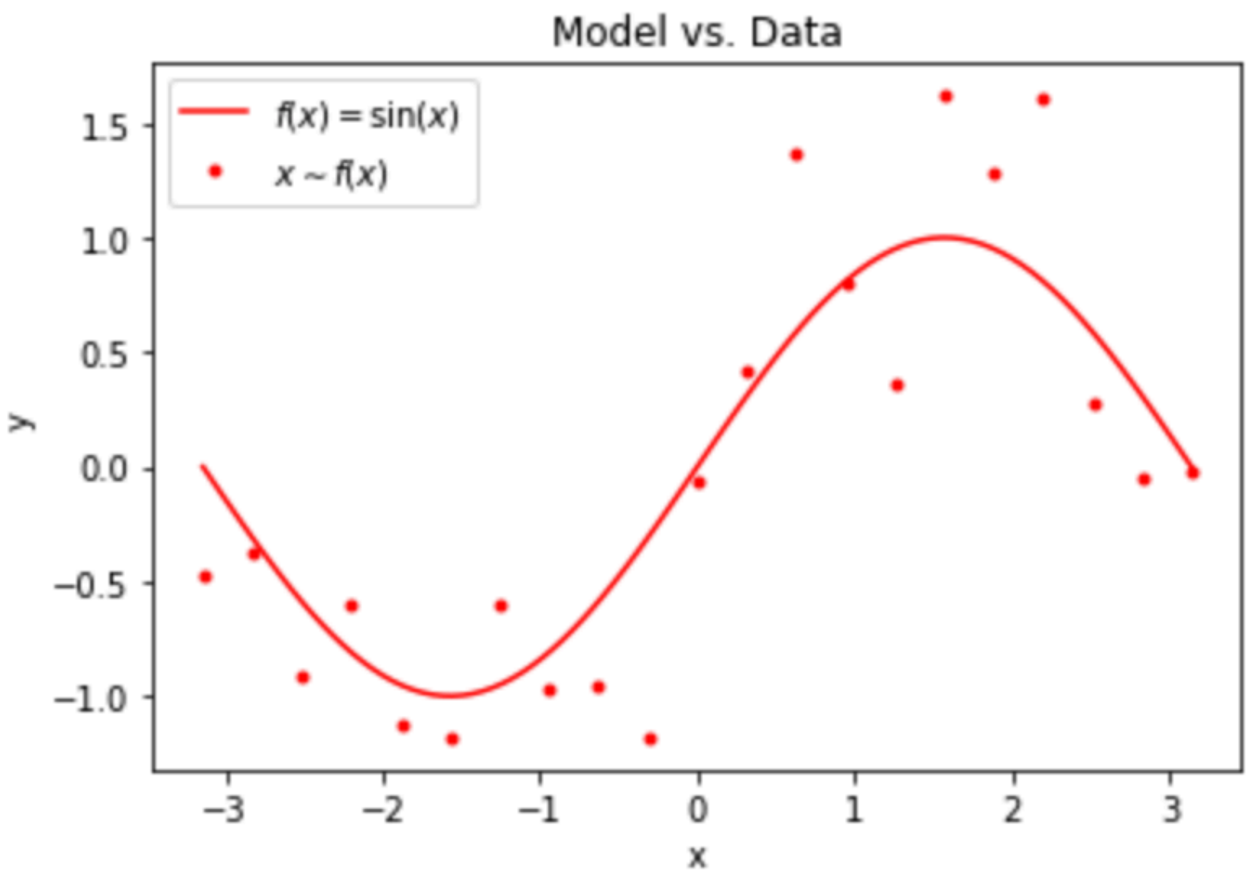
Assuming that the (unknown) true model $f(x) = sin(x)$, we'll be fitting a polynomial model to sampled data with changing the degree d of a polynomial function

$$h(x) = w_0 + w_1 x^1 + w_2 x^2 + \cdots + w_d x^d$$

for understanding the concept of over-fitting as well as model selction. Follow the instruction below.

1. Make a training data $\mathbf{x}_{\text{train}}$, which is evenly spaced 21 numbers over $[-\pi, \pi]$, and $\mathbf{y}_{\text{train}} = f(\mathbf{x}_{\text{train}}) + \epsilon$ where $\epsilon \sim \mathcal{N}(0, 0.5^2)$ is i.i.d. samples from Gaussian distribution.
2. Make a validation data $\mathbf{x}_{\text{val}}$, which is evenly spaced 10 numbers over $[-\pi, \pi]$, and $\mathbf{y}_{\text{val}} = f(\mathbf{x}_{\text{val}}) + \epsilon$ where $\epsilon \sim \mathcal{N}(0, 0.5^2)$ is i.i.d. samples from Gaussian distribution.
3. Plot the true model $f(x)$ and training data as follows:

   Note: Add xlabel, ylabel, title, and legend.



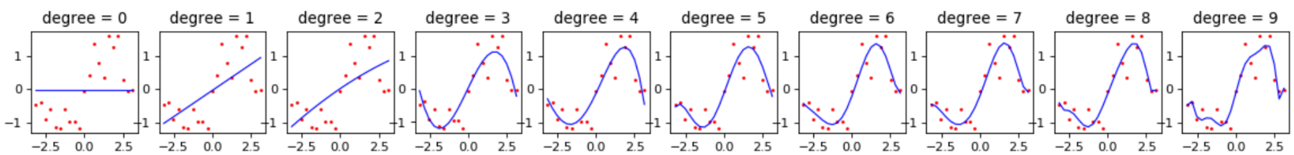
4. Repeat the following steps by changing the polynomial degree d from 0 to 9.

i. Fit d-th order polynomial to the training data and estimate the optimal model parameters/coefficients $\mathbf{w}^*$.

ii. Get your model predictions $\hat{\mathbf{y}}_{\text{train}}$ and $\hat{\mathbf{y}}_{\text{val}}$ from $\mathbf{x}_{\text{train}}$ and $\mathbf{x}_{\text{val}}$.

iii. Compute the mean squared error MSE_{train} and MSE_{val}. The MSE of training and validation data should be stored every iteration in two separate vectors/lists for visualization in step 5.

iv. Make a subplot of the best-fit d-th order polynomial curve along with training data.



5. Plot the MSE as a function of the polynomial degree given the vectors/lists obtained in step 4-iii. Be sure to add xlabel, *ylabel* and *legend* accordingly.