Mathematical Methods for International Commerce

Week 1/2: Basic Algebra

Iegor Vyshnevskyi, Ph.D.

Sogang University

March 5, 2026

Hello everyone!

I am *Iegor Vyshnevskyi*, your instructor for **Mathematical Methods for International Commerce**.

I hope you are **excited** to start this journey into the *world of mathematics and economics*!

PART 1. Flipped Classroom – Pre-Class Video 1 Slides

Week 1/2: Exchange Rates, Algebra, and Economic Interpretation

Why This Course Looks Different

- This is **not** a pure math course
- This is **not** a theory-only economics course
- This course teaches **mathematical tools through economic problems**

How to Use This Video

Before class, you should:

- watch this video carefully
- pause and replay difficult parts
- write down questions

In class, we will:

- solve problems together
- apply these tools step by step
- practice interpretation, not memorization

What We Cover This Week

In this lecture you will learn:

- how exchange rates affect debt
- how to translate an economic story into equations
- basic algebra, fractions, and inequalities
- how to interpret mathematical results economically

Economic Scenario

A European company:

- borrows **$1,000,000** from a U.S. bank
- earns revenue in **euros**
- must convert dollar debt into euros

This creates **exchange rate risk**.

Key Question

How does a change in the exchange rate affect the company's debt in euros?

What is the economic impact of exchange rate fluctuations on debt repayment?

Use algebra to analyze the exchange rate risk, loan repayment and other questions.

Step black: some important definitions

- Revenue is money coming in from sales.
- Costs are money going out to cover expenses.
- Profit is revenue minus costs.
- Debt is money owed to lenders.
- Interest is the cost of borrowing money.
- Interest rate is the 'price' of borrowed money.
- Exchange rate is the price of one currency in terms of another.
- Risk is the possibility of financial loss.

Exchange Rate Convention (Very Important)

We define:

$$ER = \text{USD per EUR}$$

- Or EUR/USD (Euro/US Dollar): The first currency listed is generally the "base" currency, and the second is the "quote" currency.

Example:

- $ER = 1.10$ means **1 euro buys 1.10 dollars** in EURO/USD exchange rate.

Conversion rule:

$$\text{EUR} = \frac{\text{USD}}{ER}$$

Debt in Euros

Loan amount:

$$L = 1{,}000{,}000 \text{ USD}$$

Initial exchange rate:

$$ER = 1.10$$

Debt in euros:

$$D = \frac{L}{ER} = 909{,}091 \text{ EUR}$$

Exchange Rate Change

One year later:

$$ER = 1.05$$

The euro depreciates.

The loan amount **in dollars stays the same**, but its value **in euros changes**.

When currency depreciates, it loses value relative to another currency. When currency appreciates, it gains value relative to another currency.

New Debt Level

$$D_{\text{new}} = \frac{1{,}000{,}000}{1.05} = 952{,}381 \text{ EUR}$$

Increase in debt:

$$43{,}290 \text{ EUR}$$

Economic Interpretation

- No new borrowing
- No change in interest rate
- No change in repayment schedule

Only the exchange rate changed.

This is why exchange rate risk matters.

Modeling Exchange Rates Over Time

Assume the exchange rate falls steadily:

$$ER(t) = 1.05 - 0.05t$$

This is a **linear function**.

Negative slope (i.e., -0.05) means depreciation over time.

Predicting the Future

After 3 years:

$$ER(3) = 0.90$$

Debt in euros:

$$D = \frac{1{,}000{,}000}{0.90} = 1{,}111{,}111\ \text{EUR}$$

Payments and Fractions

Loan repaid over 5 years:

$$A = 200{,}000 \text{ USD per year}$$

Quarterly payment:

$$Q = 50{,}000 \text{ USD}$$

Inequalities and Risk

Costs:

$$C = 300{,}000\ \mathrm{EUR}$$

Profit condition:

$$R - C \geq 900{,}000$$

Required revenue:

$$R \geq 1{,}200{,}000$$

Why This Matters

This week you learned:

- math describes real economic risks
- equations come from economic logic
- interpretation matters more than formulas

In class: **practice, problems, and applications**.

Check for Understanding

- How does exchange rate affect debt?
- How to convert currencies using exchange rates?
- There is a company in Europe that borrows $1 million from a U.S. bank. The company earns revenue in euros (€), but the loan is in dollars ($). If the exchange rate changes from €1 = $1.15 to €1 = $0.95, how does this affect the company's debt in euros?
- What is the formula for debt in euros given a loan amount in dollars and an exchange rate?

Please answer these questions before our the class!

PART 2. In-Class Presentation Slides

Basic Algebra

Why Does Math Matter in Economics?

- **Mathematics is everywhere** in economics and finance.
- Helps us **analyze markets, optimize decisions, and manage risk**.
- Today, we'll explore **how banks use algebra** in international finance.

Let's dive in!

What we need to know for today's class?

1. **Basic algebraic operations** (addition, subtraction, multiplication, division).
2. **Negative numbers** and their use in financial calculations.
3. **Algebraic expressions** and their evaluation.
4. **Solving equations** and **inequalities** for financial decision-making.
5. **Fractions** and their application in loan payments.

Diagnostic Warm-Up (Individual)

- A firm borrows *$1,000,000*.
- Exchange rate moves from *1.20* to *1.00* USD/EUR.
- Question 1: Does the euro value of debt increase, decrease, or stay the same?
- Question 2: Why?

Diagnostic Warm-Up (Together)

Exchange rate defined as USD per EUR.

- Initial exchange rate:

$$ER_0 = 1.20$$

- New exchange rate:

$$ER_1 = 1.00$$

- Compute debt in euros at both rates.
- What formula do we use? Do we divide or multiply, why?”

Diagnostic Warm-Up (Together)

- Question 1: Does the euro value of debt increase, decrease, or stay the same?
- Answer: Increase.
- Question 2: Why?
- Answer: Because the euro depreciated, making it more expensive to repay dollar debt in euros.

$$D_0 = \frac{1,000,000}{1.20} = 833,333 \text{ EUR}$$

$$D_1 = \frac{1,000,000}{1.00} = 1,000,000 \text{ EUR}$$

Group Assignment 1: Core Mechanics

Several groups with 3-4 students each.

Group Task 1: Exchange Rate Shock

- A firm borrows $800,000.
- Initial exchange rate: ER = 1.25 USD/EUR
- After 2 years, the exchange rate follows: $ER(t) = 1.25 - 0.10t$
- Tasks:
  - Compute initial debt in euros
  - Compute exchange rate after 2 years
  - Compute debt after 2 years
  - Calculate the increase in euro debt
  - Write one sentence explaining the result economically
- Rules:
  - everyone must write
  - one spokesperson per group

Group Assignment 2: Inequality & Risk

Same groups as before.

Group Task 2: When Is the Loan Risky?

- Given:
  - Loan amount: $1,200,000
  - Annual costs: €250,000
  - Required minimum profit: €150,000
- Tasks:
  - Write the inequality
  - Solve for minimum revenue
  - Interpret the result in words
  - Is higher exchange rate volatility good or bad for this firm? Why?
  - Is this loan risky or safe? Why?

Some other useful algebraic techniques

Simplifying Expressions by Collecting Like Terms

Interest Payments on a Loan

The bank considers **interest payments** when determining the total amount to be repaid.

The formula for total repayment is:

$$T = P + rP$$

Where:

- T = Total repayment
- P = Principal loan amount
- r = Interest rate

Simplifying Expressions by Collecting Like Terms (cont)

Simplifying the Expression

If the interest rate is 5% (r = 0.05), substitute the value into the equation:

$$T = P + 0.05P$$

Now, factor out P (collecting like terms):

$$T = (1 + 0.05)P$$

$$T = 1.05P$$

This simplified formula helps calculate interest payments efficiently in banking.

Practice: Please calculate the total repayment when the principal loan amount is P = 250,000.

Multiplying Out Brackets

- The company signs a **hedging contract** to manage exchange rate risk:

$$C = (D + F)(1 + r)$$

Where:

- C = Cost of the contract
- D = Debt in euros
- F = Fixed fee for hedging. Assumed to be financed by the bank (incl. in loan).
- r = Interest rate

Hedging contract is a financial agreement to reduce risk from exchange rate fluctuations.

Multiplying Out Brackets (cont)

Expanding the Brackets

Using the **distributive property** to expand:

$$C = D(1 + r) + F(1 + r)$$

Expanding each term:

$$C = D \times 1 + D \times r + F \times 1 + F \times r$$

$$C = D + Dr + F + Fr$$

Step 4: Multiplying Out Brackets (cont)

Final Expression: Total Cost Breakdown

$$C = (D + F) + (D + F)r$$

- First part (D + F) represents the original debt and fee.
- Second part (D + F)r represents the additional cost due to interest.

This expanded formula helps in accurately calculating the total cost, including fees and interest.

Note: hedging contract includes forward contract (F) to lock in exchange rate.

Practice: Please calculate the total cost when the debt is D = 1,000,000, fee is F = 50,000, and interest rate is r = 0.05.

Factorizing Expressions

- The bank wants to **simplify the total cost formula** for easier calculations:

$$C = D(1 + r) + F(1 + r)$$

We notice that (1 + r) is a common factor in both terms.

- Since (1 + r) appears in both terms, we can **factor it out**:

$$C = (D + F)(1 + r)$$

This simplifies the formula and makes calculations easier when adjusting values.

Please factorize the expression T = P + 0.05P.

Fractions in Loan Payments & Risk Calculation

Quarterly Loan Payments

If a **$1,000,000 loan** is split into **quarterly payments**, each payment is calculated as:

$$P = \frac{1,000,000}{4}$$

Converting Payments to Euros (€)

If the company **pays in euros**, and the exchange rate is **€1 = $1.10**, the payment in euros is:

$$P_{\text{EUR}} = \frac{250,000}{1.10}$$

Simplifying the fraction:

$$P_{\text{EUR}} = \frac{250,000 \div 10}{1.10 \div 10} = \frac{25,000}{0.11} = 227,273 \text{ EUR}$$

Each quarterly payment costs €227,273 at an exchange rate of 1.10.

Fractions in Loan Payments & Risk Calculation (cont)

Impact of Exchange Rate Drop

If the exchange rate **drops to 1.05**:

$$P_{\text{EUR}} = \frac{250,000}{1.05}$$

Simplifying:

$$P_{\text{EUR}} = \frac{250,000 \times 100}{1.05 \times 100} = \frac{25,000,000}{105} = 238,095 \text{ EUR}$$

Now, each quarterly payment costs €238,095.

Fractions in Loan Payments & Risk Calculation (cont)

Increase in Payment Due to Exchange Rate Change

$$\Delta P_{\text{EUR}} = 238,095 - 227,273$$

$$= 10,822 \text{ EUR}$$

The company must now pay an additional €10,822 per quarter due to currency depreciation.

Fractions in Loan Payments & Risk Calculation (cont)

Multiplying Fractions – Adjusting Loan Payments

If the bank charges a **processing fee of 1/200 (0.5%)** on each quarterly payment, the extra fee per quarter is:

$$F = P_{\text{EUR}} \times \frac{1}{200}$$

$$F = \frac{238,095}{200}$$

- Divide numerator and denominator by 5:

$$\frac{47,619}{40} = 1,190$$

The additional fee per quarter is €1,190.

Practice: Please calculate the processing fee if the quarterly payment is €200,000.

Solving Equations for Loan Repayment

Loan Repayment Over 5 Years

The company wants to **repay the loan over 5 years** with equal annual payments.

The total loan amount is **$1,000,000**, and the **annual payment is P**.

The equation for equal payments

$$1,000,000 = 5P$$

Solving for P (Annual Payment)

To isolate P, **divide both sides by 5**:

$$P = \frac{1,000,000}{5} = 200,000 \text{ USD}$$

Each annual payment is $200,000.

Practice: Please calculate the annual payment if the total loan amount is $500,000 and the repayment period is 3 years.

Linear Inequalities in Risk Management

Recognizing Inequality Symbols

The following symbols are used in **risk management** and **financial decision-making**:

Symbol	Meaning	Example
$<$	Less than	Profit \$ < 100,000 \$ (profit is below €100K)
$>$	Greater than	Revenue \$ > 500,000 \$ (revenue exceeds €500K)
$\leq$	Less than or equal to	Risk score ≤ 70 (acceptable risk level)
$\geq$	Greater than or equal to	Capital reserves $\geq 1,000,000$ (must maintain reserves of €1M or more)

Linear Inequalities in Risk Management (cont)

Bank's Risk Threshold Example

The bank considers a loan **risky** if the company's **annual revenue falls below €900,000**.

Thus, the company's revenue must satisfy the inequality:

$$R - C \geq 900,000$$

Where:

- R = Annual revenue
- $C $ = Annual costs
- $900,000$ = Minimum revenue required to avoid risk

Linear Inequalities in Risk Management (cont)

Solving the Inequality for Minimum Required Revenue

If the company has annual costs of €150,000, we substitute $ C = 150,000 $:

$$R - 150,000 \geq 900,000$$

To isolate R, add 150,000 to both sides:

$$R \geq 1,050,000$$

The company must earn at least €1.05M per year to remain financially stable.

Practice: Please solve the inequality for a minimum revenue of €1,200,000 if costs are €250,000.

Linear Inequalities in Risk Management (cont)

Graphical Representation of the Inequality

The inequality

$$R \geq 1,050,000$$

can be visualized on a number line:

- **Red region**: Revenue below €1.05M (Risky)
- **Green region**: Revenue €1.05M or more (Safe)

The company must be in the green region to meet the bank's stability requirements.

Conclusion: Why This Math Matters

1. Algebra helps predict financial outcomes.

2. Fractions, equations, and inequalities are tools for banking decisions.

3. Math makes economic decision-making precise and reliable.

Next Steps

- Practice **algebra problems from the textbook** (Jacques, Sections 1.1, 1.2).
- Bring any questions to our **next class discussion!**

Math is powerful—and fun!

Any QUESTIONS?

Thank you!

Next Class

- (Mar 10) Graphs of Equations (1.3), Solving Equations (1.4)