Mathematical Methods for International Commerce

Week 4/2: Geometric Series and Investment Appraisal

Iegor Vyshnevskyi, Ph.D.

Sogang University

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Hello everyone!

I hope you all are doing well and ready for yet another **exciting session** of Mathematical Methods for International Commerce.

PART 1. Flipped Classroom – Pre-Class Video 2 Slides

Week 4/2: Geometric Series and Investment Appraisal

Why it matters (time value of money)

- Regular payments are geometric series
- Compounding grows money forward FV
- Discounting brings money back to today PV
- Finance decisions: loans, projects, bonds
  - Key idea: a dollar today is worth more than a dollar tomorrow

Geometric sequence and finite sum

Sequence: $a, ar, ar^2, \ldots$

Finite sum: $S_n = a\frac{1-r^n}{1-r}$ \$for $r \neq 1$

Quick example: $5 + 10 + 20 + 40 = 75$

Future value of regular savings (ordinary annuity)

Assume deposit at end of each year:

$FV = PMT \cdot \frac{(1+r)^n - 1}{r}$

Example: $PMT = 100,\ r = 0.05,\ n = 5$ Factor: $(\frac{(1.05)^5 - 1}{0.05} \approx 5.526)$

$FV \approx 552.56$

Common pitfall: “saved” vs “balance”

- Total contributions: $100 \times 5 = 500$
- Balance is larger because interest is earned
  - Vocabulary: “accumulated balance” not “saved amount”

Present value of an annuity (loan instalments)

If you repay x each year for n years: $PV = x \cdot \frac{1-(1+r)^{-n}}{r}$

Negative exponent means discounting: $(1 + r)^{-n} = 1/(1 + r)^n$

Loan example

Loan PV = 2000, r = 10%, n = 4

Factor: $(\frac{1-(1.10)^{-4}}{0.10} \approx 3.1699)$

$(x = 2000/3.1699 \approx 631.3)$

Present value: discrete vs continuous

Discrete: $PV = \frac{A}{(1+r)^n}$

Continuous: $PV = Ae^{-rt}$

Example: 1000 in 3 years at 6% Discrete ≈ 839.62 Continuous ≈ 835.27

NPV (project appraisal)

$NPV = -C_0 + \sum_{t=1}^{n} \frac{C_t}{(1+r)^t}$

Example: cost 2000, return 800 for 3 years, r=10%

NPV ≈ -10.52 (slightly negative)

Bond pricing intuition

Bond value = PV(coupons) + PV(face value)

If coupon rate = yield, price = face value

Example: 50 for 10 years + 1000 at end, r=5%

Price = 1000

What you must be ready to do in class

- Compute FV of savings plan (and state timing assumption)
- Compute instalment x from PV annuity formula
- Compute PV/NPV with correct discounting
- Explain results in words (what changes PV/NPV and why)

Why This Topic Matters

Understanding **geometric series** and **investment appraisal** is crucial in economics and finance:

- **Geometric series** model regular payments, compounding, and amortization schedules
- **Investment appraisal** tools help evaluate project viability, return potential, and loan value
- Core to decisions in **banking, government budgeting, business strategy, and personal finance**
- Equips you to analyze time-based value of money, compare alternatives, and manage risk

> "A dollar today is worth more than a dollar tomorrow — let's understand why."

Section 3.3: Geometric Series

What is a Geometric Progression?

A **geometric sequence** has the form:

$$a, ar, ar^2, ar^3, \ldots$$

Where:

- a = first term
- r = common ratio (can be fraction or negative)

Basically, each term is the previous term multiplied by the common ratio.

Example: $2, 4, 8, 16, 32$

$a = 2, r = 2$

Sum of a Finite Geometric Series

$$S_n = a \cdot \frac{1 - r^n}{1 - r}, \quad r \neq 1$$

- Example: $5 + 10 + 20 + 40$
  $a = 5, r = 2, n = 4$

$$S_4 = 5 \cdot \frac{1 - 2^4}{1 - 2} = 5 \cdot \frac{-15}{-1} = 75$$

Example: Regular Savings Plan

Annual deposit: \$100, interest = 5%, duration = 5 years

It means that each year, you save \$100 and earn 5% interest.

$$S_5 = 100 \cdot \frac{1.05^5 - 1}{0.05} \approx 100 \cdot 5.526 = \boxed{\$552.60}$$

In 5 years, you will have saved \$552.60.

Your turn! Calculate the Sum

- Find the sum of the following geometric series:

$$2 + 4 + 8 + 16 + 32$$

$$3 + 6 + 12 + 24 + 48$$

- Find the final Savings Plan amount for $200/year, 4% interest, 10 years

Example: Loan Repayment by Instalments

Loan = \$2000, repaid in 4 annual instalments at 10%

$$x \cdot \left(\frac{1 - (1.10)^{-4}}{0.10} \right) = 2000$$

Solve for x to find equal annual instalments.

$$x = 2000 \cdot \frac{0.4641}{0.10} \approx 928.20$$

Good job: it's a wrong answer!

Example: Loan Repayment by Instalments (continued)

Correct Calculation

We use the **Present Value of an Ordinary Annuity** formula:

$$PV = x \cdot \frac{1 - (1 + r)^{-n}}{r}$$

PS: we have $-n$ power because we are calculating the present value of future payments.

$$(1 + r)^{-n} = \frac{1}{(1 + r)^n}$$

This tells us:

- **Raising to a negative power** means **discounting** — calculating the **present value** of a future amount.
- It reflects how **$1 received in the future is worth less today**.

Substitute known values:

- $PV = 2000$
- $r = 0.10$
- $n = 4$

Step-by-Step Solution

Step 1: Plug values into the formula:

$$2000 = x \cdot \frac{1 - (1.10)^{-4}}{0.10}$$

$$(1.10)^{-4} = \frac{1}{1.4641} \approx 0.6830$$

$$\Rightarrow \frac{1 - 0.6830}{0.10} = \frac{0.3170}{0.10} = 3.170$$

Step 2: Solve for x

Now solve:

$$2000 = x \cdot 3.170 \Rightarrow x = \frac{2000}{3.170} \approx \boxed{631.23}$$

So your **annual repayment** is **$631.23**.

Your turn! Calculate the Instalments

- A loan of \$5000 is repaid in 5 annual instalments at 8%. Find the annual instalment.
- A loan of \$3000 is repaid in 3 annual instalments at 6%. Find the annual instalment.

Section 3.4: Investment Appraisal

Investment Appraisal

Stands for evaluating the financial viability of investments.

Present Value (Discrete)

- Present value PV is the current value of future cash flows.

$$PV = \frac{A}{(1+r)^n}$$

- 1000 in 3 years at 6%:

$$PV = \frac{1000}{1.06^3} \approx 839.62$$

Present Value (Continuous)

$$PV = A \cdot e^{-rt}$$

- 1000 in 3 years at 6%:

$$PV = 1000 \cdot e^{-0.18} \approx 836.00$$

What's the difference between discrete and continuous?

- **Discrete**: Values occur at specific intervals (e.g., yearly, monthly).
  Example: Annual interest compounding.

- **Continuous**: Values change smoothly over time, modeled using exponential functions.
  Example: Continuous compounding of interest.

Net Present Value (NPV)

- Net present value NPV is the sum of present values of cash flows.

$NPV = -C_0 + \sum_{t=1}^{n} \frac{C_t}{(1+r)^t}$

- Example: Cost = $2000, Returns = $800/year for 3 years, $r = 10\%$:

$$NPV \approx -2000 + \frac{800}{1.10} + \frac{800}{1.10^2} + \frac{800}{1.10^3} \approx 75.13$$

Good job - this answer is wrong!

Correct Calculation

$$= -2000 + 727.27 + 661.16 + 601.06 = \boxed{-10.51}$$

So, the project has a **negative NPV** of about **–$10.51**, meaning **it's not financially viable** at a 10% discount rate.

Present Value of an Annuity

- Annuity is a series of equal payments over time.

$$PV = PMT \cdot \frac{1-(1+r)^{-n}}{r}$$

- \$1000/year for 5 years at 6%:

$$PV = 1000 \cdot \frac{1-(1.06)^{-5}}{0.06} \approx 4212.40$$

Internal Rate of Return (IRR)

- IRR is the rate at which NPV = 0.
- We calculate IRR to compare investment options.

Find r such that $NPV = 0$:

$$0 = -C_0 + \sum \frac{C_t}{(1+r)^t}$$

Solve numerically or using software.

Present Value of Government Bonds

Bond pays \$50/year for 10 years, plus \$1000 at end.

$$PV = \sum_{t=1}^{10} \frac{50}{(1.05)^t} + \frac{1000}{(1.05)^{10}} \approx 925.81$$

Good job - this answer is wrong!

Correct Calculation

$$PV = \sum_{t=1}^{10} \frac{50}{(1.05)^t} + \frac{1000}{(1.05)^{10}} \approx \boxed{1000}$$

This makes sense: at a 5% rate, the bond is worth **its face value**.

Practice Problems

1. Sum of: $3 + 6 + 12 + 24$
2. \$200/quarter, 4% quarterly compounding for 3 years
3. PV of \$1500 in 2 years at 8% (continuous)
4. NPV: \$3000 cost, \$1200/year for 3 years, 7% rate
5. PV of \$500 annuity for 6 years at 5%
6. IRR: \$2000 cost, \$500/year for 5 years

Summary

- Geometric series = finance modeling
- Investment tools = PV, NPV, IRR
- Practical for decision-making in public/private sectors

Any QUESTIONS?

Thank you for your attention!

Next Class

- (March 31) The Derivative of Functions (4.1), Rules of Differentiation (4.2)

Next Friday (April 2) - Quiz #1

- Please review your Home Work #1, in-class problems and problems from the textbook.