Mathematical Methods for International Commerce

Week 2/2: Supply and Demand Analysis, Transposition of Formulae, National Income Determination

Iegor Vyshnevskyi, Ph.D.

Sogang University

March 12, 2026

Hello everyone!

I hope you all are doing well and ready for another **exciting session** of Mathematical Methods for International Commerce.

PART 1. Flipped Classroom – Pre-Class Video 2 Slides

Week 2/2: Supply and Demand Analysis, Transposition of Formulae, National Income Determination

How to use this video

Before class:

- Watch once straight through

- Rewatch the worked examples and pause to redo steps

- Bring one question to class

- In class: we will solve additional problems in groups

What we cover today

1. Supply & Demand equilibrium (single market)

2. Solving systems (two goods)

3. Transposition of formulae (solve for target variable)

4. National income determination (Keynesian cross with taxes)

Function notation + endogenous/exogenous

- Function notation: $y = f(x)$

  - Example demand: $Q_d = f(P) = 100 - 5P$

- Endogenous: determined inside the model (e.g., P^*, Q^*, Y^*)

- Exogenous: taken as given (parameters, policy variables, shocks)

Linear demand and supply

- Demand: $Q_d = a - bP$ with $b > 0$

- Supply: $Q_s = c + dP$ with $d > 0$

- Economic meaning: demand slopes down, supply slopes up

Market equilibrium (core method)

- Equilibrium condition: $Q_d = Q_s$

- Example: $100 - 5P = 20 + 3P$

- Solve: $P^* = 10$
- Then $Q^* = 50$

Two-commodity equilibrium: idea

- Two goods → two markets → two equilibrium conditions

- You solve a system of linear equations to get P_1^*, P_2^*

- Key skill: put equations into standard form and use elimination/substitution

Two-commodity example (short version)

Demand: From equilibrium you get a system like:

$$-5P_1 + P_2 = -60$$
$$-P_1 - 5P_2 = -55$$

Eliminate one variable → solve the other → back-substitute (Full step-by-step will be in class practice.)

Transposition: what it is and why it matters

- **Goal**: rearrange an equation to isolate a variable you want

- Used constantly in economics: solve for price, income, multipliers, time, elasticities

Transposition example 1: solve for price

- From demand: $Q_d = a - bP$

- Solve for P:

$$P = \frac{a - Q_d}{b}$$

- Example: $Q_d = 100 - 5P \Rightarrow P = \frac{100 - Q_d}{5}$

Transposition example 2: solve for time

- Compound interest: $A = P(1 + r)^t$

$$t = \frac{\ln(A/P)}{\ln(1 + r)}$$

- Interpretation: time needed to reach (A) given (r)

- Because $x^y = z$ implies $y = \log_x(z) = \ln(z)/\ln(x)$

National income determination

- Model: $Y = C + I + G$
- Given: $G = 40, \ I = 55, \ C = 0.8Y_d + 25, \ T = 0.1Y + 10, \ Y_d = Y - T$

Step 1: $Y_d = Y - (0.1Y + 10) = 0.9Y - 10$

Step 2: $C = 0.8(0.9Y - 10) + 25 = 0.72Y + 17$

Step 3: $Y = (0.72Y + 17) + 55 + 40 = 0.72Y + 112$

Step 4: $0.28Y = 112 \Rightarrow Y^* = 400$

What to be ready to do in class

In class you will:

- solve some equilibrium problems (single market)
- solve one 2-good system with elimination
- transpose 2 formulas yourself
- compute Y^* for a new set of (C, T, I, G)

Quick checklist for yourself

Can I:

- set $Q_d = Q_s$ and solve? How?
- identify endogenous vs exogenous? How?
- rearrange to isolate a variable? How?
- solve a 2x2 system by elimination? How?
- substitute $Y_d = Y - T$ correctly and solve for Y^*? How?

Answer via Cyber Campus before class!

Supply and Demand Analysis, Transposition of Formulae, National Income Determination

Why Are These Concepts Essential in Economics?

1. **Supply and Demand Analysis** → Understanding **market equilibrium**.

2. **Transposition of Formulae** → Rearranging equations to **solve for key economic variables**.

3. **National Income Determination** → Finding **equilibrium GDP** in macroeconomic models.

Learning Objectives

At the end of this section, you should be able to:

1. **Use function notation**, $y = f(x)$.

2. **Identify** endogenous and exogenous variables.

3. **Sketch and interpret** a **linear demand function**.

4. **Sketch and interpret** a **linear supply function**.

5. **Determine** equilibrium price and quantity **graphically** and **algebraically**.

6. **Solve simultaneous equations** for multi-commodity equilibrium.

7. **Transpose formulae** to solve for unknown variables in economic models.

8. **National Income Determination**: Understand the concept of national income and its determinants.

1. Supply and Demand Analysis

Function Notation in Economics

Economic models often use **function notation**:

$$y = f(x)$$

where:

- x is the **independent variable** (e.g., price, income).
- y is the **dependent variable** (e.g., quantity demanded, total revenue).

Note: **Functions** describe **relationships** between variables.

Example: Demand Function
If demand depends on price:

$$Q_d = f(P) = 100 - 5P$$

where:

- Q_d is **quantity demanded**.
- P is **price**.
- The function shows that as **price increases, demand decreases**.

Endogenous vs. Exogenous Variables

- **Endogenous Variables** → **Determined within the model** (e.g., equilibrium price and quantity).
- **Exogenous Variables** → **Determined outside the model** (e.g., government policies, income levels).

Example:

$$Q_d = 100 - 5P$$

$$Q_s = 20 + 3P$$

Here:

- P (price) and Q (quantity) are **endogenous.**
- **Shocks like taxes or subsidies** are **exogenous.**

Demand and Supply Functions

Linear Demand Function

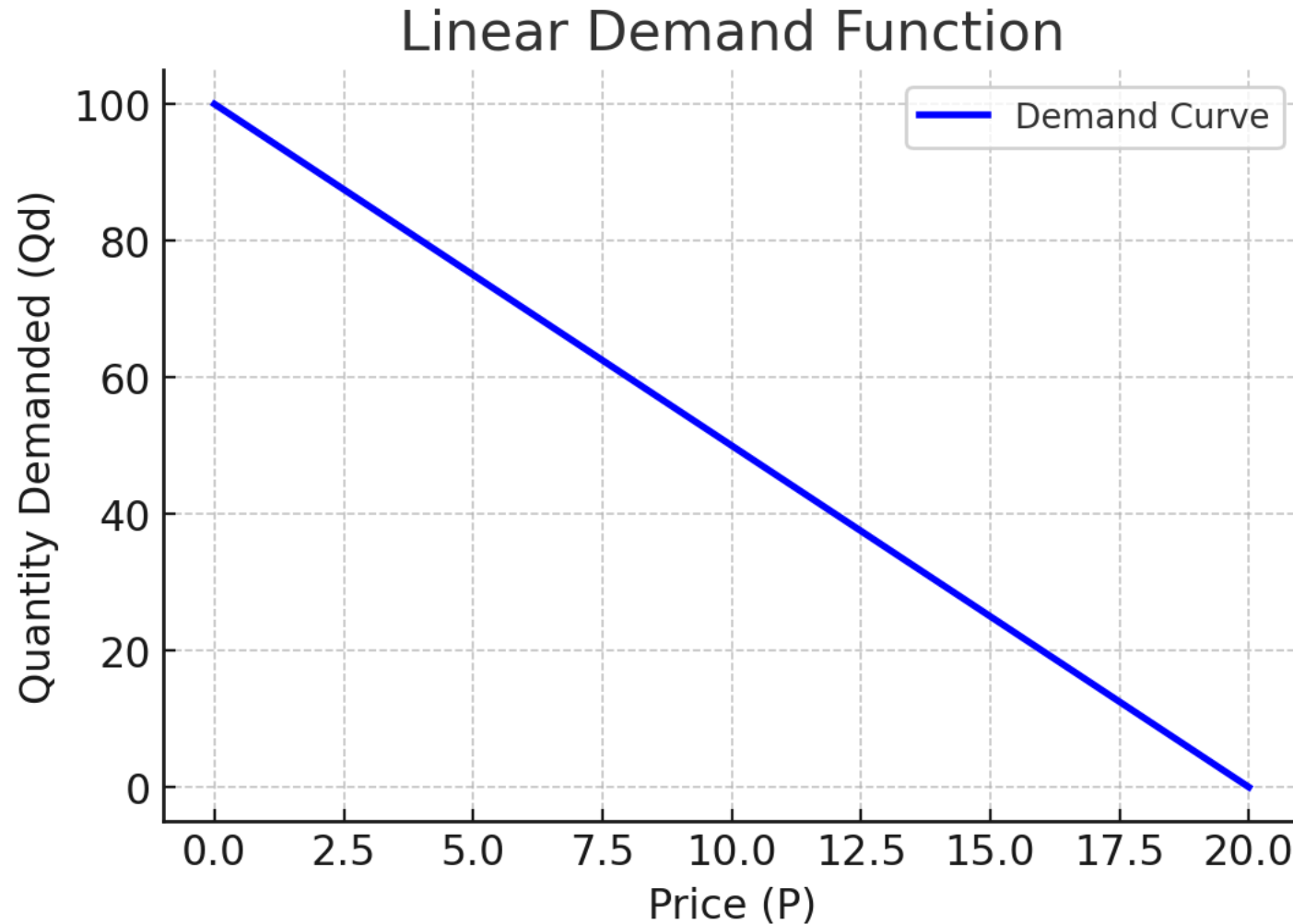A linear demand function follows:

$$Q_d = a - bP$$

where:

- a is the **intercept** (max demand at price = 0).
- b is the **slope** (rate at which demand falls when price rises).

Demand and Supply Functions (cont)

Demand Graph

Demand and Supply Functions (cont)

Supply Function

A **linear supply function** follows:
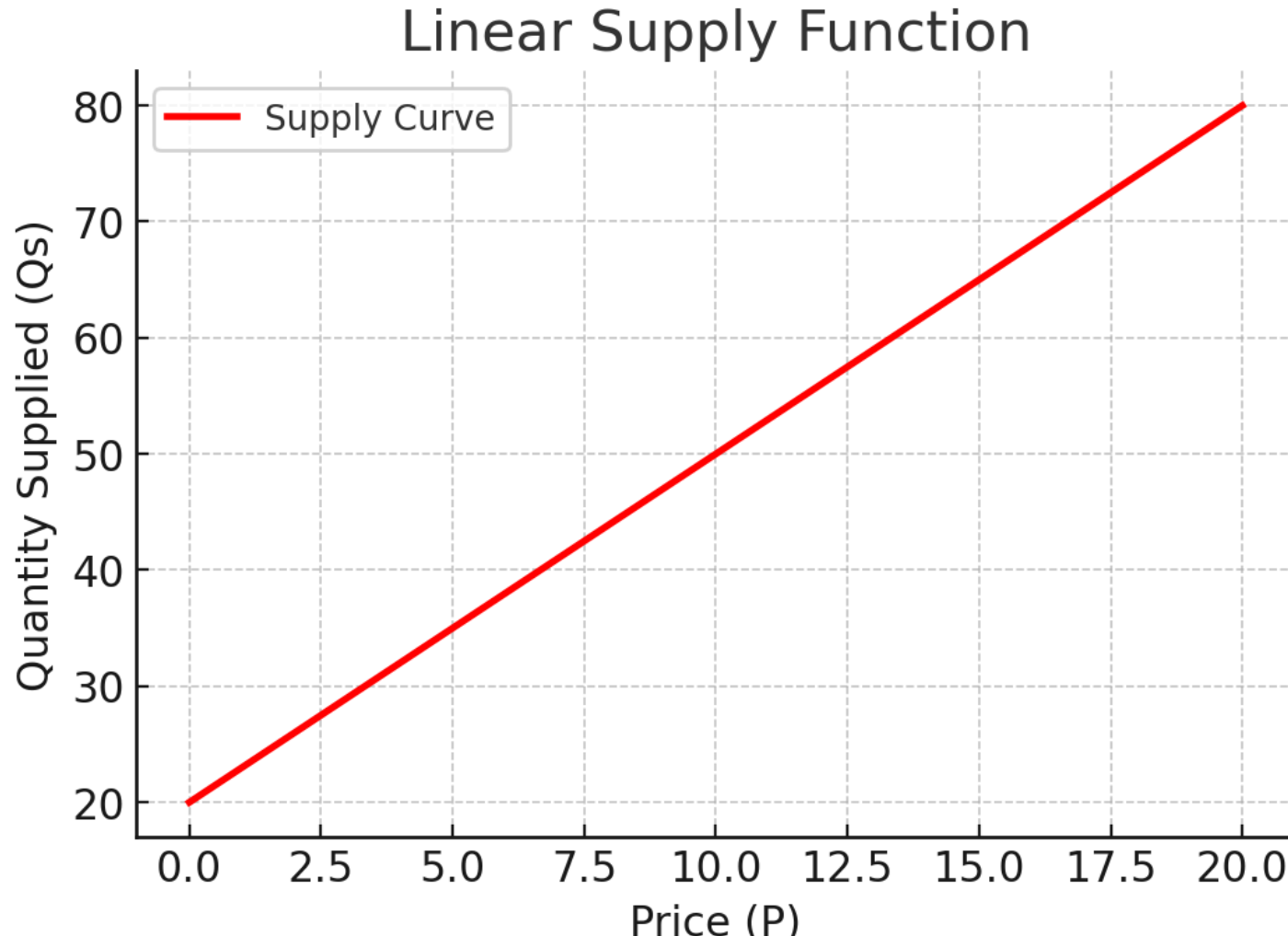
$$Q_s = c + dP$$

where:

- c is the **intercept** (minimum quantity supplied at zero price).
- d is the **slope** (rate at which supply increases as price rises).

Example Supply Function

$$Q_s = 20 + 3P$$

Demand and Supply Functions (cont)

Supply Graph

Linear Supply Function

Market Equilibrium Concept

Market equilibrium occurs where **quantity demanded** equals **quantity supplied**:

$$Q_d = Q_s$$

Using the demand function:

$$Q_d = 100 - 5P$$

And the supply function:

$$Q_s = 20 + 3P$$

Setting them equal:

$$100 - 5P = 20 + 3P$$

Solving for Equilibrium Price and Quantity

Solving for P^*:

$$100 - 5P = 20 + 3P$$

$$80 = 8P$$

$$P^* = 10$$
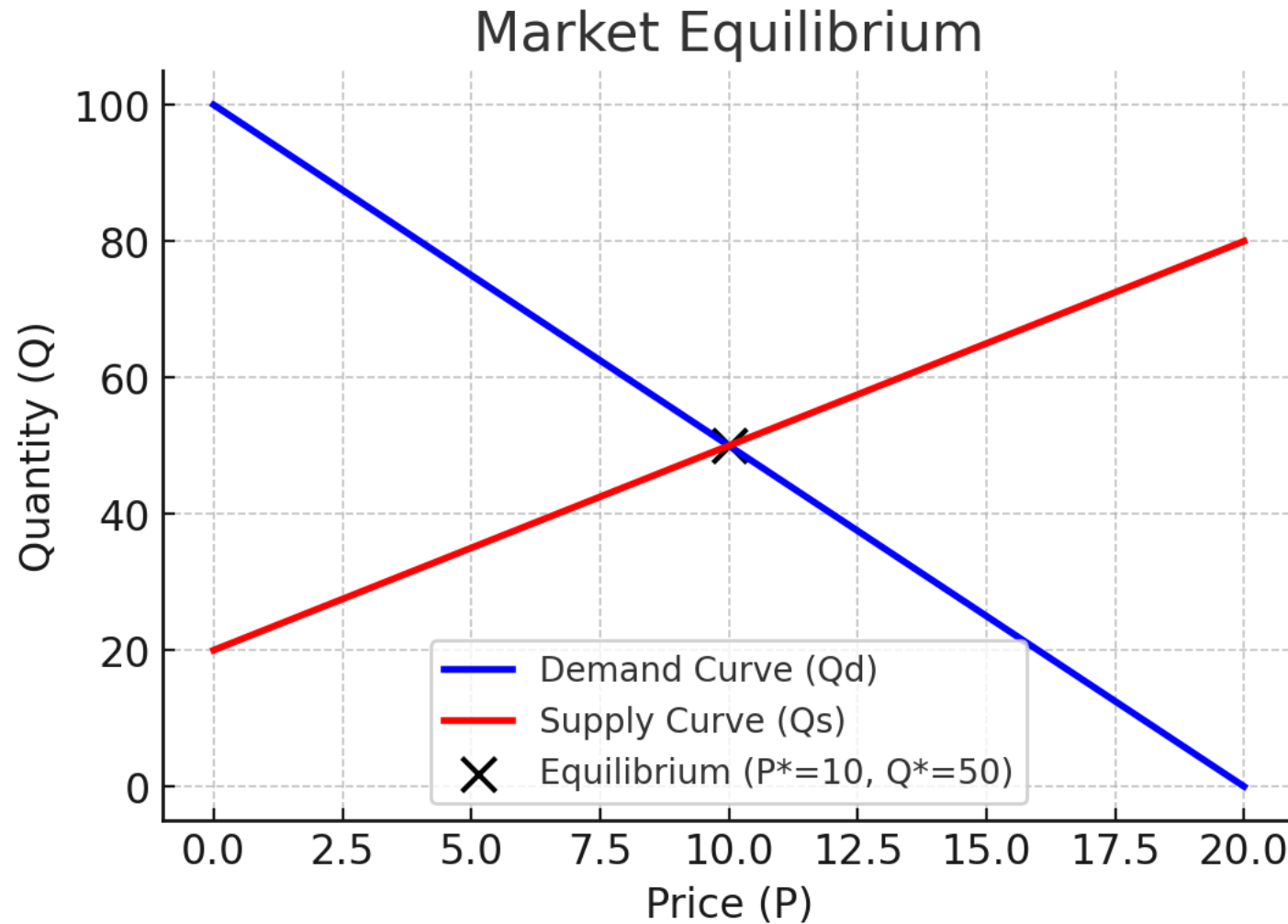
Substituting P^* into the demand equation:

$$Q^* = 100 - 5(10) = 50$$

Equilibrium Price: P^* = 10;
Equilibrium Quantity: Q^* = 50

Market Equilibrium Plot

Market Equilibrium

Your Turn: Practice Problem

Given the **demand and supply functions** for one good:

Demand Function:

$$Q_d = 50 - 2P$$

Supply Function:

$$Q_s = -10 + 3P$$

Interpret supply as zero until price is high enough that $Q_s \geq 0$.

1. **Find the equilibrium price** P^* and **quantity** Q^*.
2. **Plot the demand and supply functions.**
3. **Show the equilibrium point.**

Your Turn: Practice Problem (cont)

Given the **demand and supply functions** for one good:

Demand Function:

$$Q_d = 40 - 7P$$

Supply Function:

$$Q_s = -30 + 4P$$

1. Find the equilibrium price P^* and **quantity** Q^*.
2. Plot the demand and supply functions.
3. Show the equilibrium point.

Multi-Commodity Market Equilibrium

In **multi-commodity markets**, equilibrium is determined by **simultaneously solving multiple demand and supply equations**.

- **Each good has its own demand and supply function**.

- **Equilibrium occurs when** demand equals supply **for all goods**.

- **Requires solving a system of linear equations**.

Two-Commodity Market Equilibrium

Consider an economy with **two goods** where:

Demand Functions:

$$Q_{d1} = 50 - 2P_1 + P_2$$
$$Q_{d2} = 60 - P_1 - 3P_2$$

Supply Functions:

$$Q_{s1} = -10 + 3P_1$$
$$Q_{s2} = 5 + 2P_2$$

Find the equilibrium prices P_1^* and P_2^*.

Step 1: Rearrange into Standard Form

At **equilibrium**, demand = supply:

$$-5P_1 + P_2 = -60 \quad (1)$$
$$-P_1 - 5P_2 = -55 \quad (2)$$

Rearranged equations are now in standard form.

Step 2: Solve for P_2^*

Multiply **Equation (2)** by **5**:

$$-5P_1 + P_2 = -60$$
$$-5P_1 - 25P_2 = -275$$

Now subtract:

$$(-5P_1 - 25P_2) - (-5P_1 + P_2) = -275 + 60$$

$$-26P_2 = -215$$

$$P_2^* = \frac{-215}{-26} = 8.27$$

Equilibrium price for Good 2: $P_2^* = 8.27.$

Step 3: Solve for P_1^*

Substituting $P_2 = 8.27$ into **Equation (1)**:

$$-5P_1 + 8.27 = -60$$

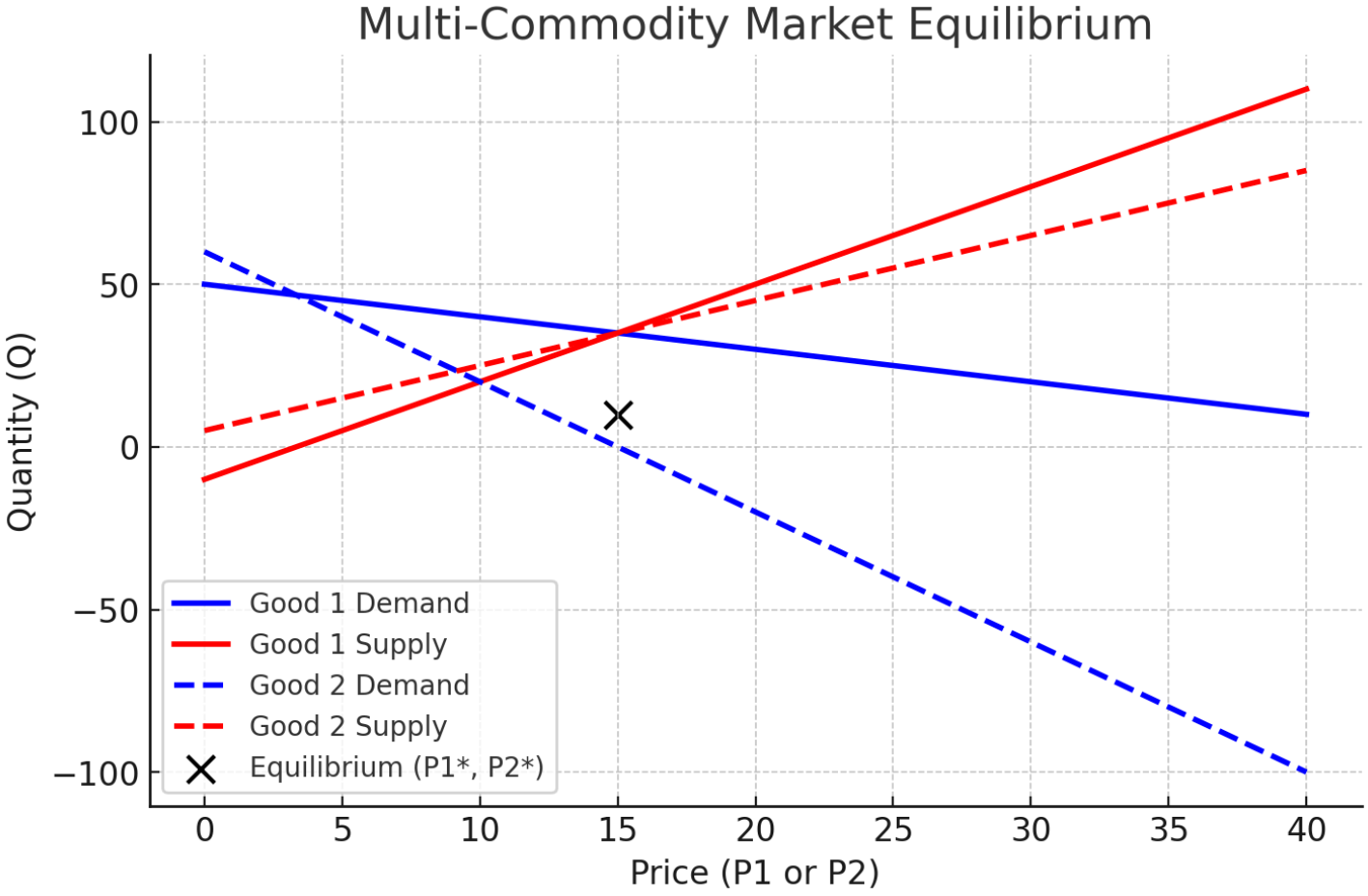$$-5P_1 = -60 - 8.27$$

$$P_1^* = \frac{-68.27}{-5} = 13.65$$

Equilibrium price for Good 1: $P_1^* = 13.65$.

Final Answer

$$ \boxed{ P_1^* = 13.65, \quad P_2^* = 8.27 } $$

These are the equilibrium prices where demand equals supply.

Graphical Solution

Multi-Commodity Market Equilibrium

Your Turn: Practice Problem

Solve for equilibrium prices P_1^* and P_2^* in the following two cases.

Example 1: Two-Commodity Market

Consider an economy with **two goods** where:

Demand Functions:

$$Q_{d1} = 80 - 4P_1 + 2P_2$$
$$Q_{d2} = 100 - 3P_1 - P_2$$

Supply Functions:

$$Q_{s1} = 20 + 5P_1$$
$$Q_{s2} = 10 + 4P_2$$

Find the equilibrium prices P_1^* and P_2^* by solving:

$$80 - 4P_1 + 2P_2 = 20 + 5P_1$$
$$100 - 3P_1 - P_2 = 10 + 4P_2$$

Example 2: Alternative Market System

A different economy has the following market equations:

Demand Functions:

$$Q_{d1} = 60 - 3P_1 + P_2$$
$$Q_{d2} = 90 - 2P_1 - 2P_2$$

Supply Functions:

$$Q_{s1} = -5 + 4P_1$$
$$Q_{s2} = 15 + 3P_2$$

Find the equilibrium prices P_1^* and P_2^* by solving:

$$60 - 3P_1 + P_2 = -5 + 4P_1$$
$$90 - 2P_1 - 2P_2 = 15 + 3P_2$$

Instructions

- **Rearrange equations** into standard form $Ax + By = C$.

- **Use elimination or substitution** to solve.

- **Interpret the results:** What do the equilibrium prices mean for the market?

2. Transposition of Formulae

What is Transposition of Formulae?

1. **Rearranging an equation to express one variable in terms of others.**

2. **Useful in economics, finance, and business analysis.**

3. **Allows solving for unknown variables in different contexts.**

Example:

$$A = B + C$$

To express C in terms of A and B:

$$C = A - B$$

Example 1: Solving for Price in Demand Function

Consider a **linear demand function**:

$$Q_d = a - bP$$

We want to solve for P:

$$P = \frac{a - Q_d}{b}$$

If $Q_d = 100 - 5P$, solve for P:

$$P = \frac{100 - Q_d}{5}$$

Now we can **calculate the price for any given quantity**.

Example 2: Solving for Time in Interest Formula

The compound interest formula is:

$$A = P(1+r)^t$$

Solve for t:

1. Divide both sides by P:

$$\frac{A}{P} = (1+r)^t$$

1. Take the natural logarithm:

$$\ln\left(\frac{A}{P}\right) = t\ln(1+r)$$

1. Solve for t:

$$t = \frac{\ln(A/P)}{\ln(1+r)}$$

3. National Income Determination

What is National Income?

- **Total value of goods and services produced** in a country over a period.

- **Measures economic activity** and **standard of living**.

- **Components of National Income:**

  - **Consumption** C: Spending by households.
  - **Investment** I: Spending by firms.
  - **Government Spending** G: Public expenditure.
  - **Net Exports** NX: Exports minus imports.

- **National Income Formula:**

$$Y = C + I + G + NX$$

The Consumption Function

The **linear consumption function:**

$$C = C_0 + cY$$

where:

- C = Total consumption
- C_0 = Autonomous consumption (spending even if income is zero)
- c = Marginal propensity to consume (MPC) (i.e., how much of each extra unit of income is consumed)
- Y = National income

Interpretation:

- If **income increases, consumption increases.**
- The slope c tells us **how much of each extra unit of income is consumed.**

Plotting the Consumption Function

Consumption Function
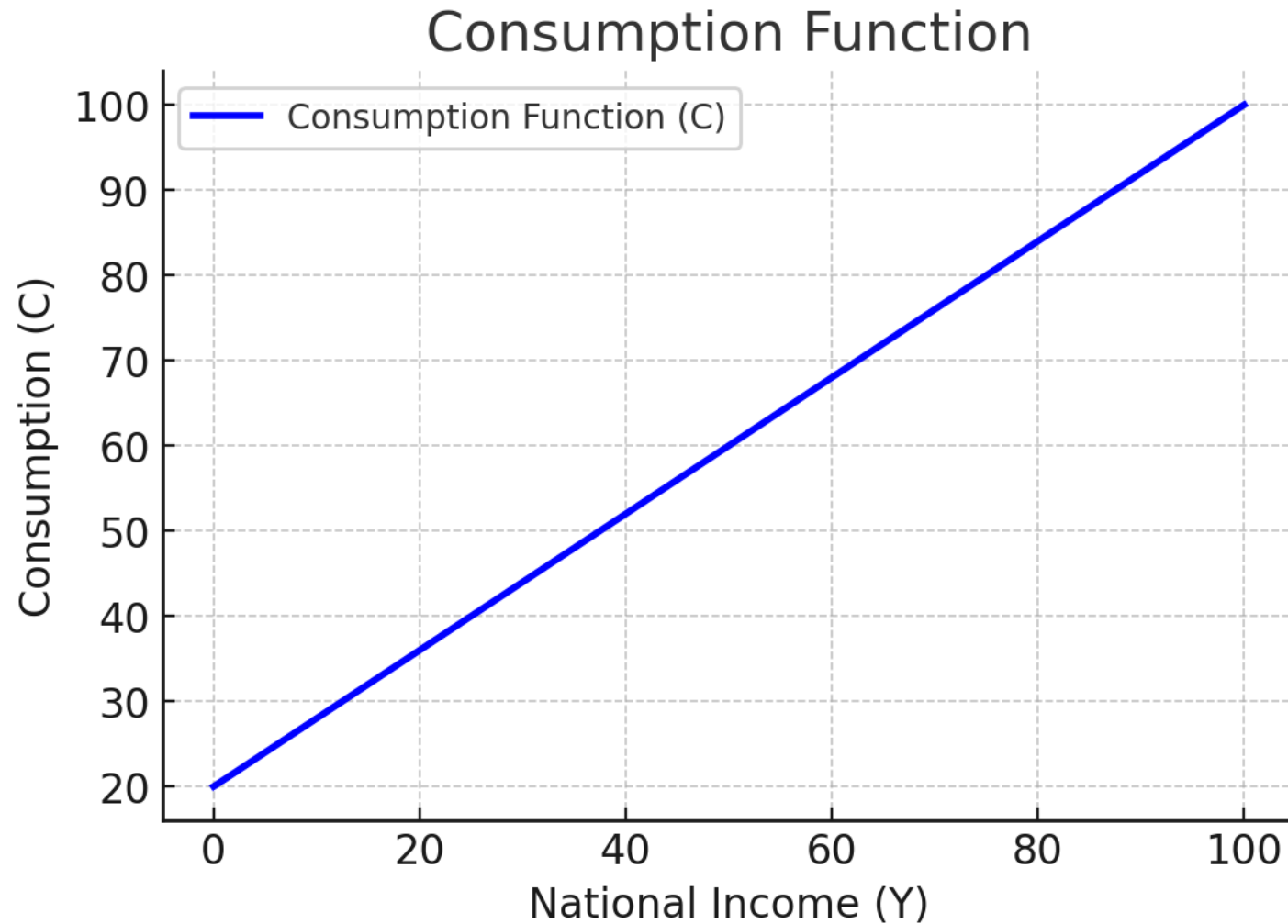
The Savings Function

In macroeconomics, **savings** is the portion of income that is **not spent on consumption.**

The **linear savings function** is:

$$S = S_0 + sY$$

where:

- S = **Total savings**
- S_0 = **Autonomous savings** (can be negative if dissaving occurs)
- s = **Marginal propensity to save (MPS)** (i.e., how much of each extra unit of income is saved)
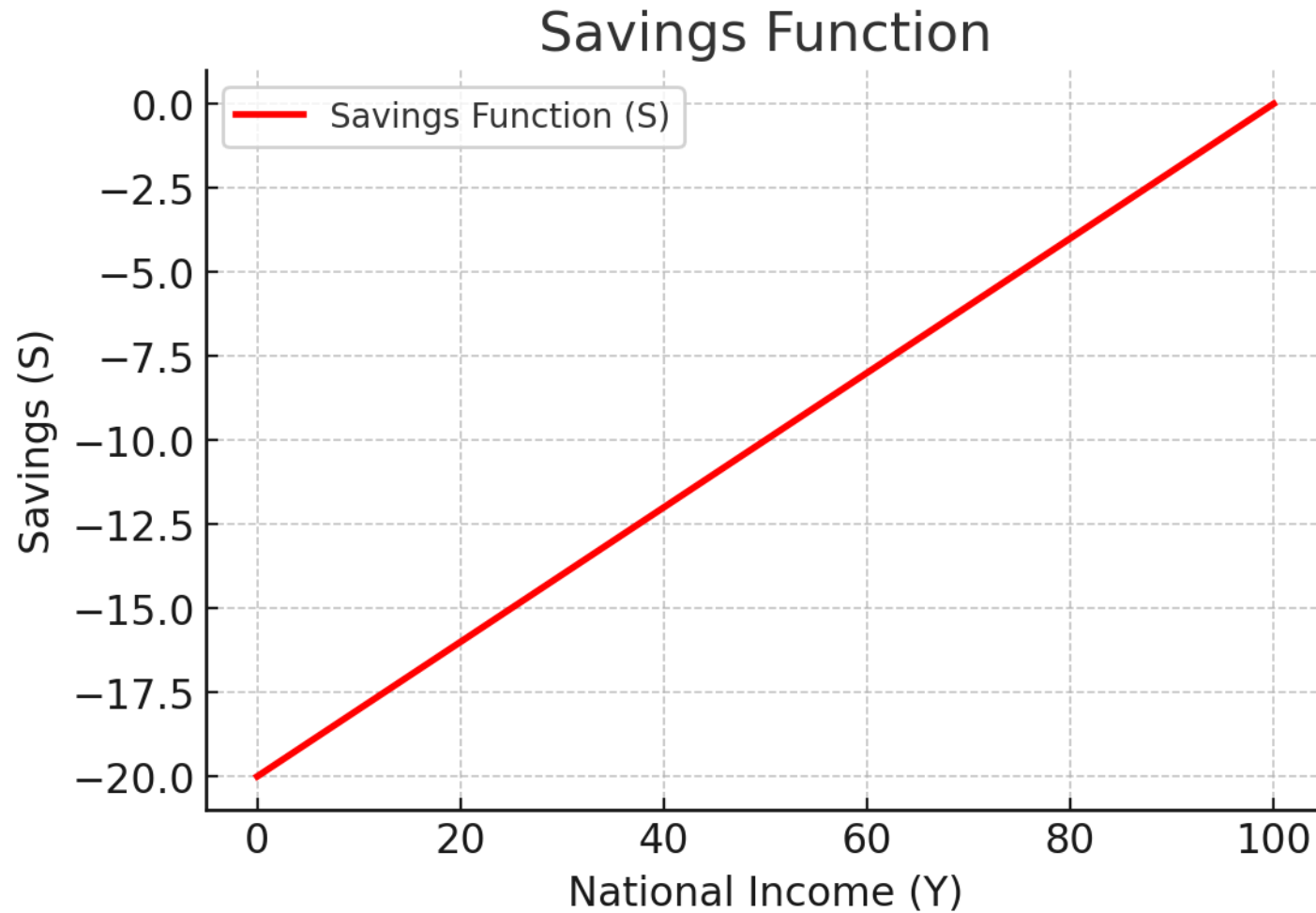- Y = **National income**

Since **income is either spent or saved**, we know:

$$MPC + MPS = 1$$

Interpretation:

- If **MPC = 0.8**, then **MPS = 0.2**.
- **Higher income leads to higher savings.**

Plotting the Savings Function

The Income-Expenditure Model

The **Income-Expenditure Model** determines **equilibrium national income**.

In a **simple Keynesian model**, total spending is:

$$Y = C + I + G + (X - M)$$

where:

- C = **Consumption**
- I = **Investment**
- G = **Government spending**
- $X - M$ = **Net exports** (ignored for a closed economy)

At **equilibrium**, income equals total spending:

$$Y = C_0 + cY + I + G$$

Solving for **equilibrium income**:

$$Y^* = \frac{C_0 + I + G}{1 - c}$$

Problem Statement

Given:

$$G = 40, \quad I = 55, \quad C = 0.8Y_d + 25, \quad T = 0.1Y + 10$$

Find the **equilibrium level of national income** Y^*.

Step 1: Define Disposable Income

Disposable income Y_d is the **income left after taxation**:

$$Y_d = Y - T$$

Using the tax function:

$$T = 0.1Y + 10$$

$$Y_d = Y - (0.1Y + 10) = 0.9Y - 10$$

Disposable income depends on total income Y.

Step 2: Write the Consumption Function

Consumption function is given by:

$$C = 0.8Y_d + 25$$

Substituting $Y_d = 0.9Y - 10$:

$$C = 0.8(0.9Y - 10) + 25$$

$$C = 0.72Y - 8 + 25$$

$$C = 0.72Y + 17$$

Consumption increases with national income Y.

Step 3: Write the Equilibrium Condition

In equilibrium:

$$Y = C + I + G$$

Substituting known values:

$$Y = (0.72Y + 17) + 55 + 40$$

$$Y = 0.72Y + 112$$

Now solve for Y.

Step 4: Solve for Equilibrium Income

Rearrange the equation:

$$Y - 0.72Y = 112$$

$$0.28Y = 112$$

$$Y^* = \frac{112}{0.28} = 400$$

Equilibrium national income is $Y^* = 400$.

Your Turn: Practice Problem

Given the following information:

$$G = 50, \quad I = 60, \quad C = 0.7Y_d + 30, \quad T = 0.15Y + 10$$

Find the **equilibrium level of national income** Y^*.

Summary

1. **Supply and Demand Analysis** helps us understand market equilibrium.

2. **Transposition of Formulae** allows us to solve for unknown variables in economic models.

3. **National Income Determination** helps us find equilibrium GDP in macroeconomic models.

Math is powerful—and fun!

Next Steps

1. Practice **problems from the textbook** (Jacques's 10ed, Sections 1.5-1.7).

2. Bring any questions to our **next class discussion!**

3. Work on your Home Assignment #1 (due next Friday, March 20, 13:30).

   - Chapter 1.1: Exercise 1.1, Problems 12, 14-16, 19 (p. 21-22)
   - Chapter 1.2: Exercise 1.2, Problems 9, 11, 14 (p. 39)
   - Chapter 1.3: Exercise 1.3, Problems 10 (p. 53-54)
   - Chapter 1.4: Exercise 1.4, Problem 4 (p. 65)
   - Chapter 1.5: Exercise 1.5, Problems 3, 5, 8 (p. 81)
   - Chapter 1.6: Exercise 1.6, Problem 5 (p. 92)
   - Chapter 1.7: Exercise 1.7, Problem 6 and 7 (p. 106)

Any QUESTIONS?

Thank you for your attention!

Next Class

- (Mar 17) Quadratic Functions (2.1), Revenue, Cost, and Profit (2.2)